2025.5 I Qingdao I Axion Dark Matter: Theory and Phenomenology

Oscillations of the black hole **photon ring** as a probe of **ultralight dilaton** fields

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Based on arXiv: 2503.07947

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CONTENTS

Supermassive black hole image as a probe of ultralight particles

- Superradiance mechanism

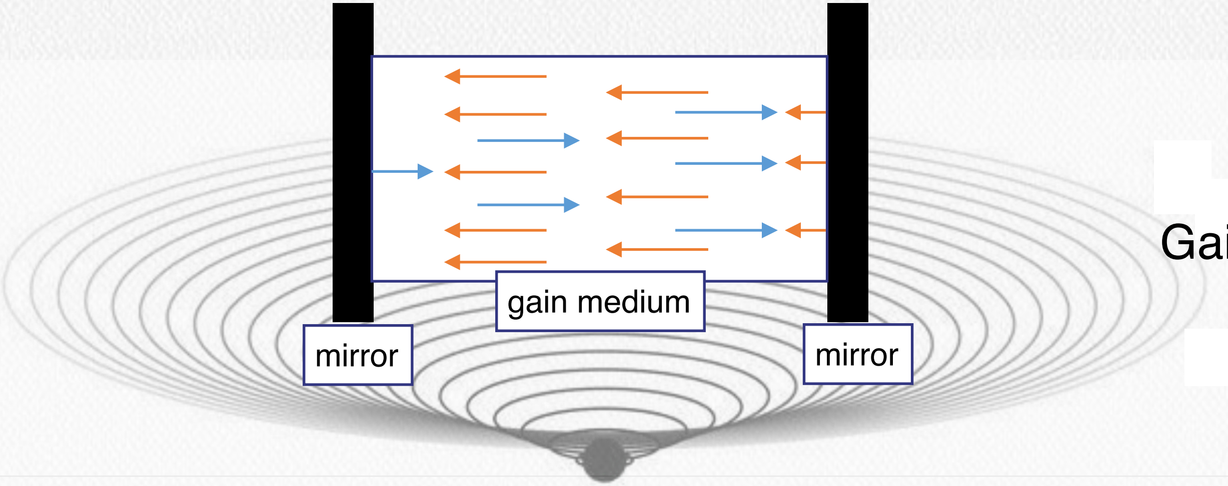
$$\omega^r_{nlm} = \mu \left(1 - \frac{\alpha^2}{2n^2} + \mathcal{O}\left(\alpha^4\right) \right)$$

$$\left(\nabla^\mu \nabla_\mu - \mu^2\right)\phi = 0 \;\Rightarrow\; \phi(t, r, \theta, \varphi) = e^{-i\omega t + im\varphi} R_{nlm}(r) S_{lm}(\theta) \;\Rightarrow\; \omega_{nlm} = \omega^r_{nlm} + i\omega^i_{nlm}$$

$$\Rightarrow\; \omega^i_{nlm} \propto \alpha^{4l+5}\left(m\frac{a}{2r_+} - \omega^r_{nlm}\right)(1 + \mathcal{O}(\alpha)) \;\Rightarrow\; if\ \frac{a}{2r_+} > \frac{\omega^r_{nlm}}{m} \;\Rightarrow\; \text{Exponential growth of } |\phi|$$

$$r_+ \equiv r_g + r_g\sqrt{1 - a^2}$$

Similar to **laser** generation and amplification:



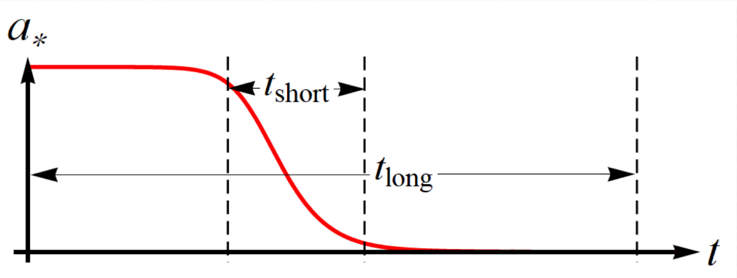
Mirror	Gravitational potential barrier
Gain medium	Black hole horizon
ϕ	Laser

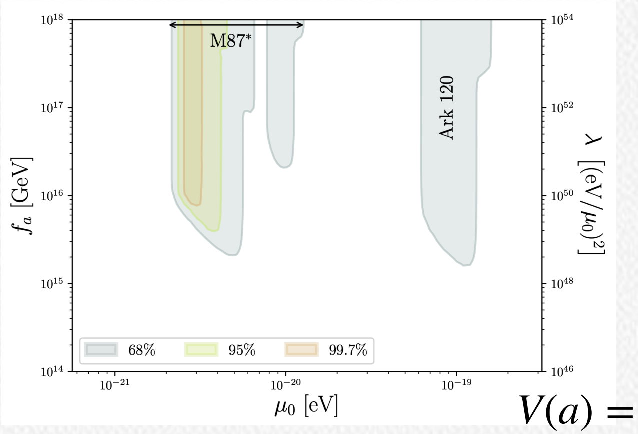
Y. B. Zel'Dovich, "Generation of Waves by a Rotating Body", Soviet Journal of Experimental and Theoretical Physics Letters 14, 180 (1971)
W. H. Press and S. A. Teukolsky, "Floating Orbits, Superradiant Scattering and the Black-hole Bomb", Nature 238, 211 (1972)

Supermassive black hole image as a probe of ultralight particles

- Spin evolution

BH spin energy $\longrightarrow$ boson cloud





SMBH mass: $10^6 M_\odot$ to $10^{10} M_\odot$

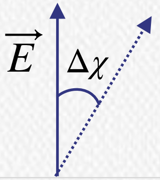
Boson mass: 10^{-20}eV to 10^{-16}eV

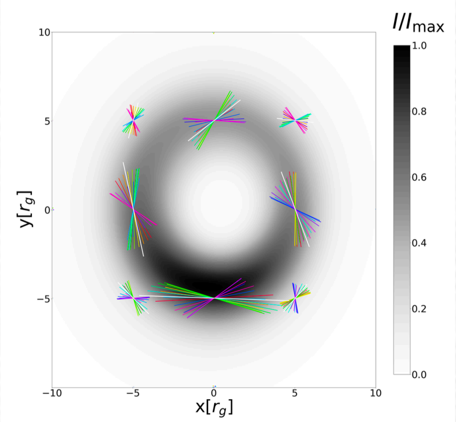
$$V(a) = \mu^2 f_a^2 \left(1 - \cos\left[a/f_a\right]\right)$$

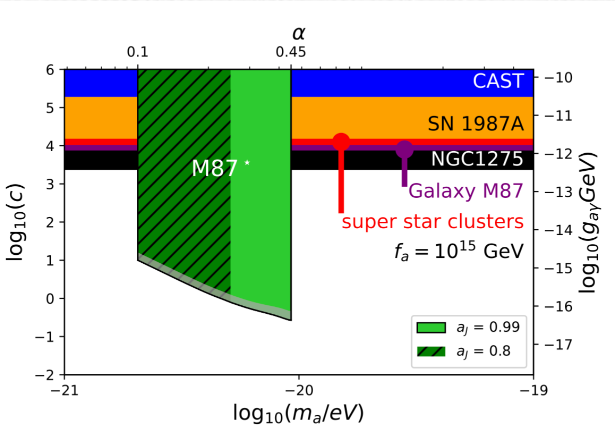
- Radiative transfer

$$\mathscr{L}_{int} = g_{a\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}/2$$

$$\Delta\chi = g_{a\gamma}\left[a\left(t_{obs}, \mathbf{x}_{obs}\right) - a\left(t_{emit}, \mathbf{x}_{emit}\right)\right]$$







$$c \equiv 2\pi g_{a\gamma} f_a$$

Rittick Roy et al., "Superradiance evolution of black hole shadows revisited", Phys.Rev.D 105 8, 083002, (2022)
M. J. Stott, "Ultralight Bosonic Field Mass Bounds from Astrophysical Black Hole Spin", arXiv: 2009.07206
Yifan Chen et al., "Stringent axion constraints with Event Horizon Telescope polarimetric measurements of M87", Nature Astron. 6 5, 592-598, (2022)

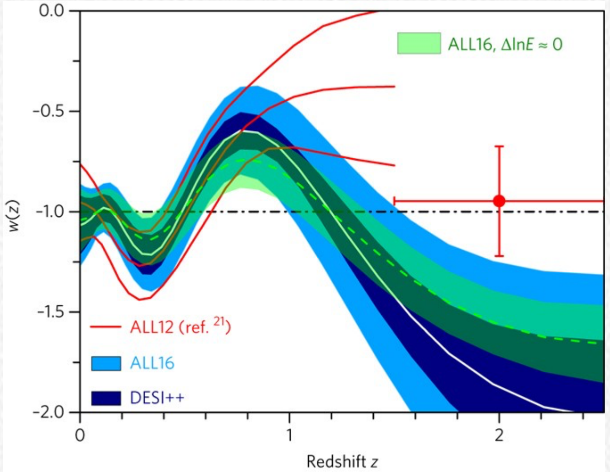
Introduction to dilaton

$$\mathscr{L} = \frac{1}{2\kappa^2}\sqrt{-g}\left[f(\phi)R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{4}e^{-2a\phi}F_{\mu\nu}F^{\mu\nu}\right]$$

- Scalar partner to the graviton in string theories

- Dynamical dark energy & dark matter & inflation

Evolution of the gravitational constant $G(t)$ or the vacuum energy Λ

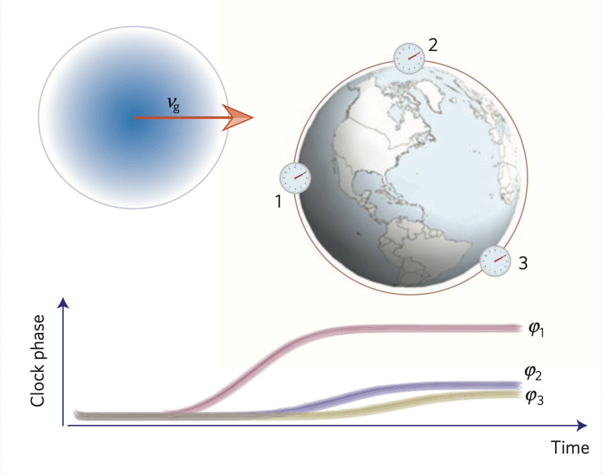


$$w = \frac{p}{\rho}$$

- Cosmological evolution and oscillating variations of fundamental constants

$$-\mathscr{L}_{\rm int} = \phi^2\left(\frac{m_e\bar{\Psi}_e\Psi_e}{\Lambda_e^2} + \frac{m_p\bar{\Psi}_p\Psi_p}{\Lambda_p^2} - \frac{1}{4\Lambda_\gamma^2}F_{\mu\nu}^2 + \cdots\right)$$

$$\Rightarrow \quad m_{e,p}^{\rm eff} = m_{e,p}\left(1 + \frac{\phi^2}{\Lambda_{e,p}^2}\right) \qquad \alpha^{\rm eff} = \frac{\alpha}{1 - \phi^2/\Lambda_\gamma^2}$$



Gongbo Zhao et al., “Dynamical dark energy in light of the latest observations”, Nature Astronomy, 1, 627-632 (2017)
A. Derevianko and M. Pospelov, “Hunting for topological dark matter with atomic clocks”, Nature Physics 10, 933–936 (2014)

Geometric approximation

- In flat spacetime

$$\mathscr{L} = -\frac{1}{4} f(\phi) F_{\mu\nu} F^{\mu\nu} - e A_\mu J^\mu \qquad f(\phi) = 1 - g_{\phi\gamma}\phi \qquad e: \ electron$$

J^μ: plasma effects $\quad \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0 \quad \vec{J} = i\omega_p^2/\omega \vec{E} \quad \rho = e n_e$

$$\omega_p^2 = n_e e^2/m_e$$

Plasma frequency
=Effective mass of photons

$$\Rightarrow \nabla^2 \vec{E} - \frac{\partial^2 \vec{E}}{\partial t^2} = g_{\phi\gamma}^2 (1 - g_{\phi\gamma}\phi)^{-2} \left(\frac{\partial \phi}{\partial t} \nabla\phi \times \vec{B} - \left(\frac{\partial \phi}{\partial t}\right)^2 \vec{E} + \nabla\phi \cdot (\nabla\phi \cdot \vec{E}) \right)$$
$$+ g_{\phi\gamma}(1 - g_{\phi\gamma}\phi)^{-1} \left(\nabla \frac{\partial \phi}{\partial t} \times \vec{B} - \frac{\partial^2 \phi}{\partial t^2} \vec{E} + \vec{E} \times (\nabla \times \nabla\phi) + (\vec{E} \cdot \nabla)\nabla\phi \right)$$
$$+ (1 - g_{\phi\gamma}\phi)^{-1} \left(\frac{\partial \vec{J}}{\partial t} + \nabla\rho \right) + g_{\phi\gamma}(1 - g_{\phi\gamma}\phi)^{-2} \left(\frac{\partial \phi}{\partial t} \vec{J} + \rho \nabla\phi \right)$$
$$+ g_{\phi\gamma}(1 - g_{\phi\gamma}\phi)^{-1} \left((\nabla\phi \cdot \nabla)\vec{E} - \frac{\partial \phi}{\partial t} \frac{\partial \vec{E}}{\partial t} \right).$$

Protons mass $m_p \gg m_e$, making their collective dynamics negligible.

Geometric approximation

- Assumption:

$$\vec{E} = \vec{E}_0 e^{iS}, \vec{B} = \vec{B}_0 e^{iS} \qquad (\omega, \vec{k}) = (\partial_t S, \nabla S)$$

Only consider $\lambda \ll l$? l**: the characteristic variation scale of ϕ, n_e and $g_{\mu\nu}$**

Photon wavelength

A. Non-relativistic dilaton field ϕ $\qquad \mu^2 = \omega^2 + k_\phi^2$

$$k_\phi \sim |\nabla \phi|/\phi \ll 1/r_g \qquad r_g = GM$$

$$\Rightarrow \lambda \ll k_\phi^{-1}, r_g$$

B. Plasma

$$n_e(r) \propto \left(r/r_g\right)^h \qquad k_e \sim \left|\nabla n_e\right|/n_e \sim 1/r_e \quad \Rightarrow \quad \lambda \ll k_e^{-1}$$

J. Frank, A. R. King, and D. Raine, "Accretion power in astrophysics", Cambridge university press, 2002

Geometric approximation

$$next\ order :\sim \mathrm{i}E_i k^{\mu}\partial_{\mu}\phi$$

$$\nabla^2\vec{E}-\frac{\partial^2\vec{E}}{\partial t^2}=g_{\phi\gamma}^2(1-g_{\phi\gamma}\phi)^{-2}\left(\frac{\partial\phi}{\partial t}\nabla\phi\times\vec{B}-\left(\frac{\partial\phi}{\partial t}\right)^2\vec{E}+\nabla\phi\cdot(\nabla\phi\cdot\vec{E})\right)$$
$$+g_{\phi\gamma}(1-g_{\phi\gamma}\phi)^{-1}\left(\nabla\frac{\partial\phi}{\partial t}\times\vec{B}-\frac{\partial^2\phi}{\partial t^2}\vec{E}+\vec{E}\times(\nabla\times\nabla\phi)+(\vec{E}\cdot\nabla)\nabla\phi\right)$$
$$+(1-g_{\phi\gamma}\phi)^{-1}\left(\frac{\partial\vec{J}}{\partial t}+\nabla\rho\right)+g_{\phi\gamma}(1-g_{\phi\gamma}\phi)^{-2}\left(\frac{\partial\phi}{\partial t}\vec{J}+\rho\nabla\phi\right)$$
$$+g_{\phi\gamma}(1-g_{\phi\gamma}\phi)^{-1}\left((\nabla\phi\cdot\nabla)\vec{E}-\frac{\partial\phi}{\partial t}\frac{\partial\vec{E}}{\partial t}\right).$$

$$\Rightarrow\quad -E_j k_\mu k^\mu=\omega_p^2E_j(1-g_{\phi\gamma}\phi)^{-1}$$

$$\Downarrow$$

$$\mathscr{H}=\vec{k}^2-\omega^2+\omega_p^2(1-g_{\phi\gamma}\phi)^{-1}$$

Modify the effective mass of photons

- Including $g_{\mu\nu}$

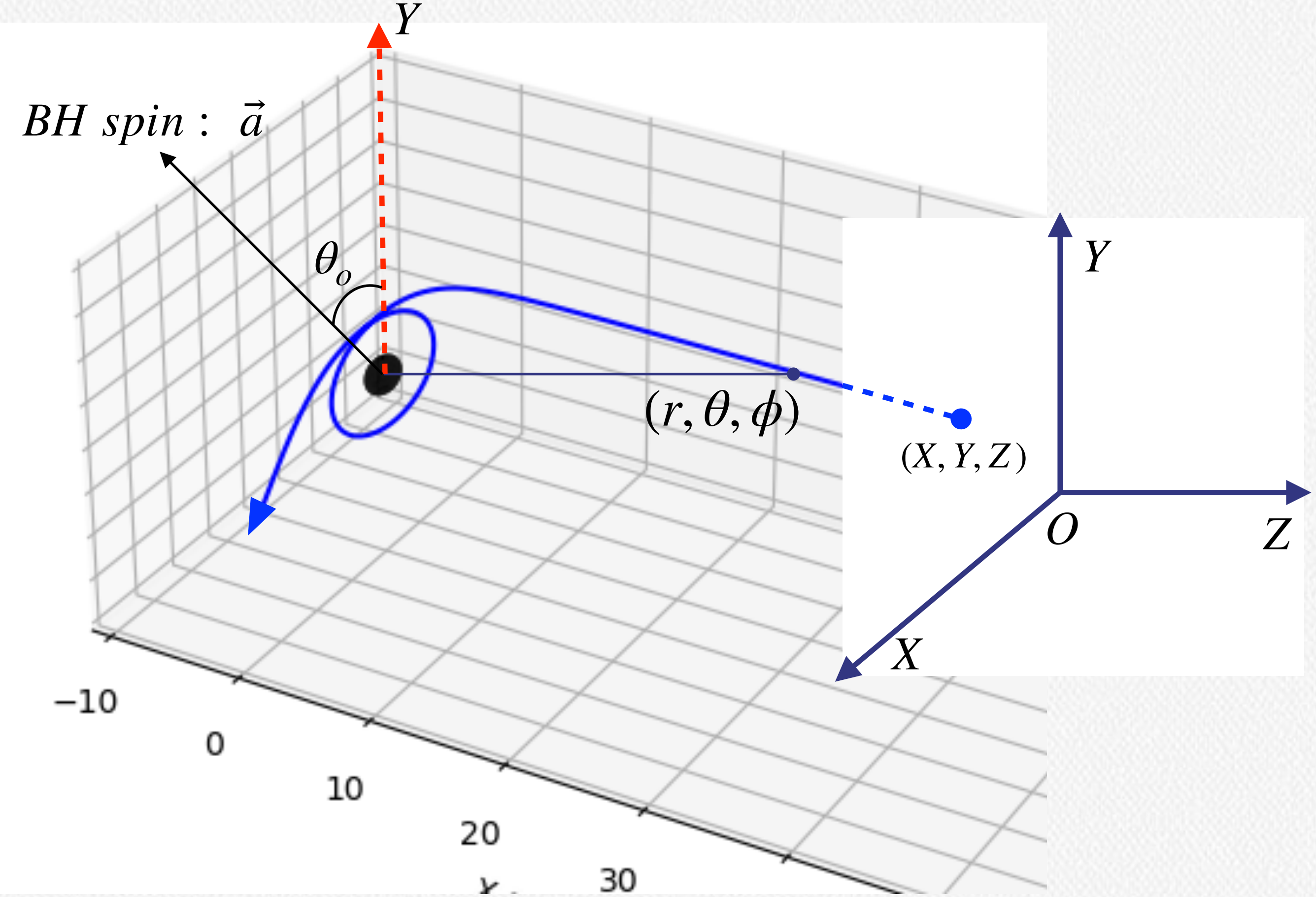
$$\mathscr{L}=-\frac{1}{4}\sqrt{-g}f(\phi)F_{\mu\nu}F^{\mu\nu}$$

$$\Rightarrow\quad \mathscr{H}=\frac{1}{2}\left(g^{\mu\nu}k_\mu k_\nu+\omega_p^2(1-g_{\phi\gamma}\phi)^{-1}\right)$$

$$\frac{dx^\mu}{d\lambda}=\frac{\partial\mathscr{H}}{\partial k_\mu},\quad \frac{dk_\mu}{d\lambda}=-\frac{\partial\mathscr{H}}{\partial x^\mu}\quad\Rightarrow\quad \frac{d^2x^\sigma}{d\lambda^2}+\Gamma^\sigma_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}+\frac{1}{2}g^{\rho\sigma}\frac{\partial}{\partial x^\rho}\left(\omega_p^2(1-g_{\phi\gamma}\phi)^{-1}\right)=0$$

Ray tracing

For a given point in the image space: (X, Y, Z)



$$x = r_o \sin\theta_o + Z \sin\theta_o - Y \cos\theta_o,$$
$$y = X,$$
$$z = r_o \cos\theta_o + Z \cos\theta_o + Y \sin\theta_o,$$

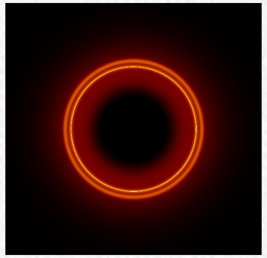
$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \arccos\frac{z}{r}$$
$$\phi = \arctan\frac{y}{x}$$

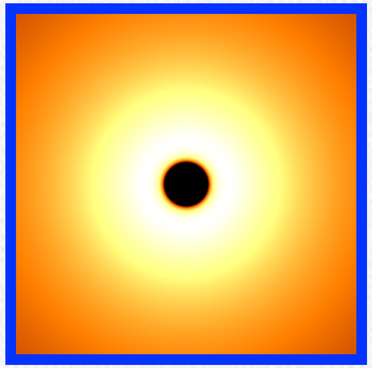
(r, θ, ϕ) BL coordinate of black hole

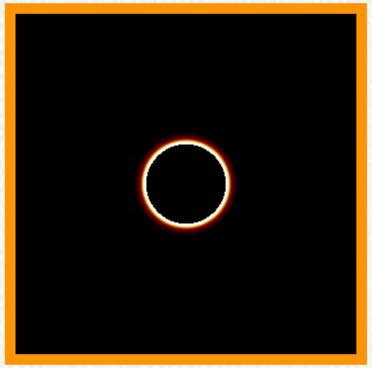
$$\dot{r} = \dot{Z}\cos\theta\cos\theta_o - p_0 \sin\theta\sin\theta_o\cos\phi$$
$$\dot{\theta} = \frac{\dot{Z}}{r}\left[\sin\theta_o\cos\theta\cos\phi - \sin\theta\cos\theta_o\right]$$
$$\dot{\phi} = -\frac{\dot{Z}}{r\sin\theta}\sin\theta_o\sin\phi$$
$$\dot{t} = \beta + \sqrt{\beta^2 + \gamma}, \quad \beta \equiv -\frac{g_{ti}\dot{x}^i}{g_{tt}}, \gamma \equiv -\frac{g_{ij}\dot{x}^i\dot{x}^j}{g_{tt}}$$

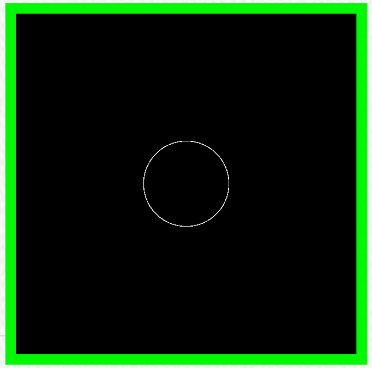
Ray tracing

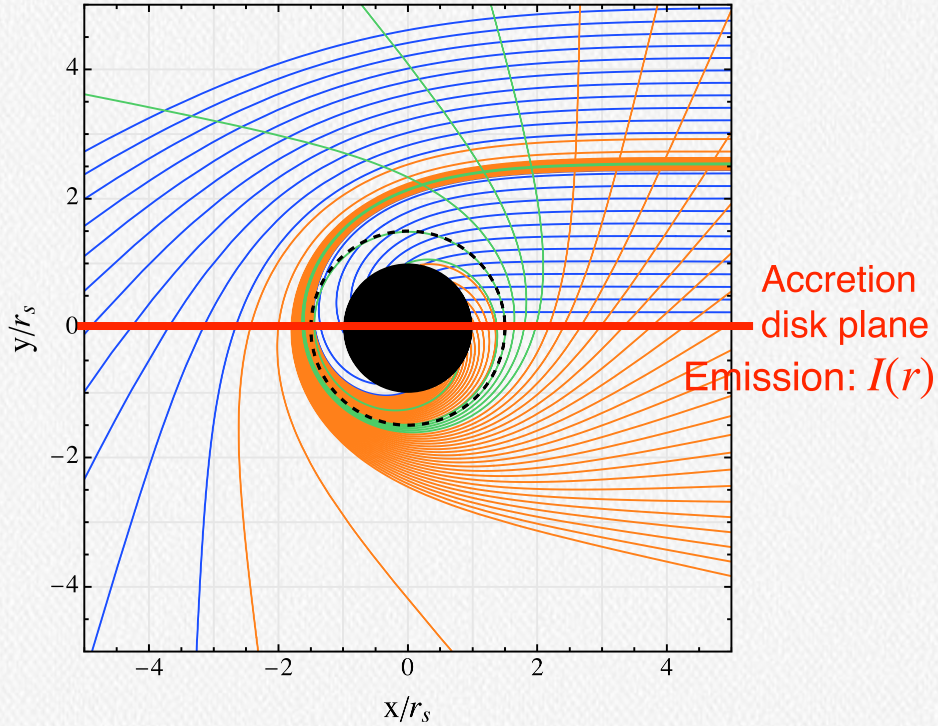
- Formation of photon ring: $$I^{\mathrm{obs}}(X,Y) = \sum_m g(r)^4 I(r)\Big|_{r=r_m(X,Y)}$$

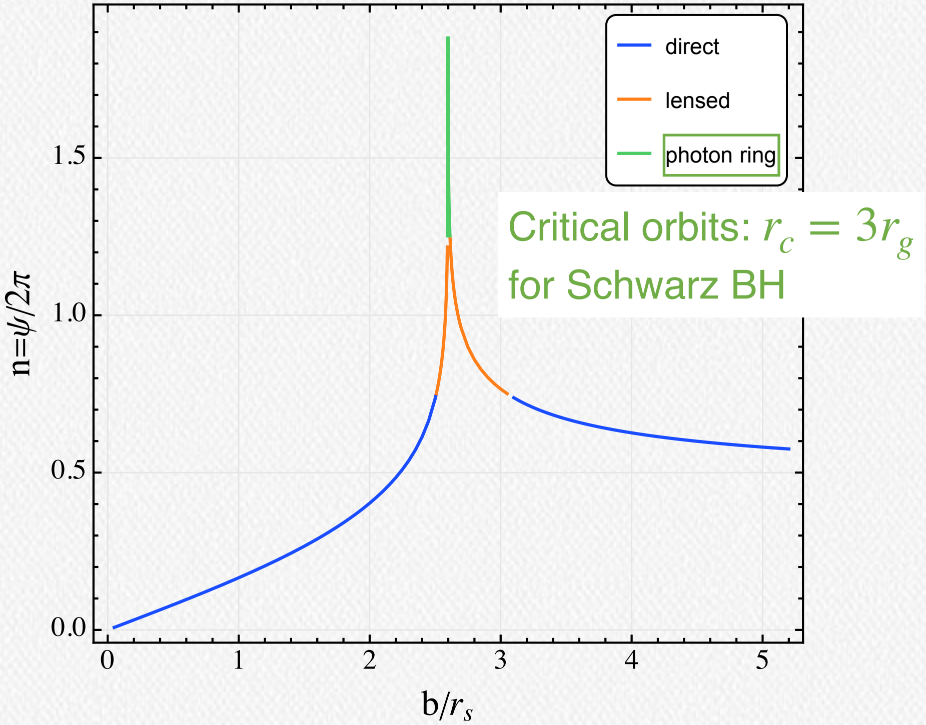












S. E. Gralla, D. E. Holz, and R. M. Wald, “Black hole shadows, photon rings, and lensing rings”, Phys. Rev. D 100, 024018 (2019)

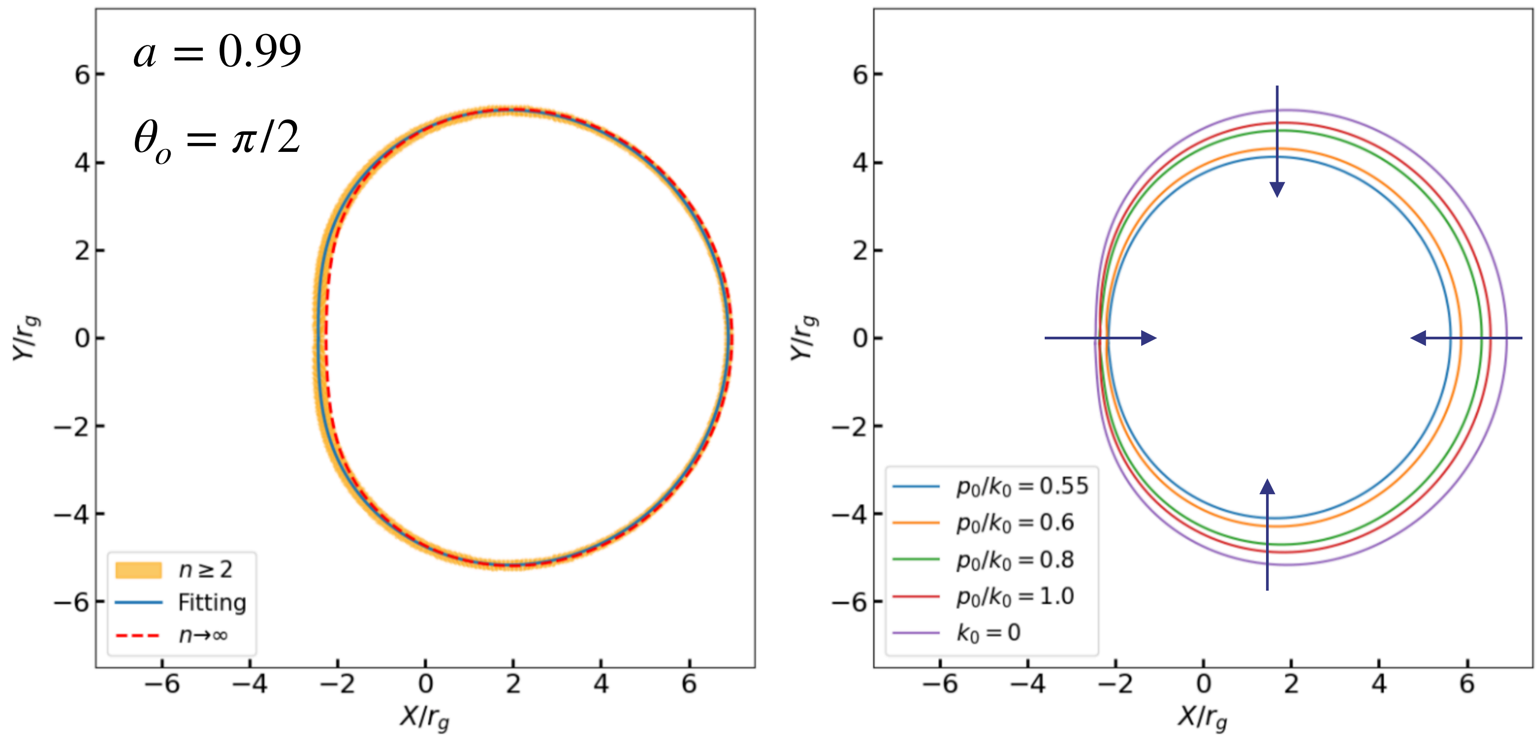
Ray tracing

- Plasma effects:

$$\omega_p^2 = \frac{k_0^2}{(r/r_g)^h}$$

$h = 1.5$ for RIAF model

p_0: photon frequency

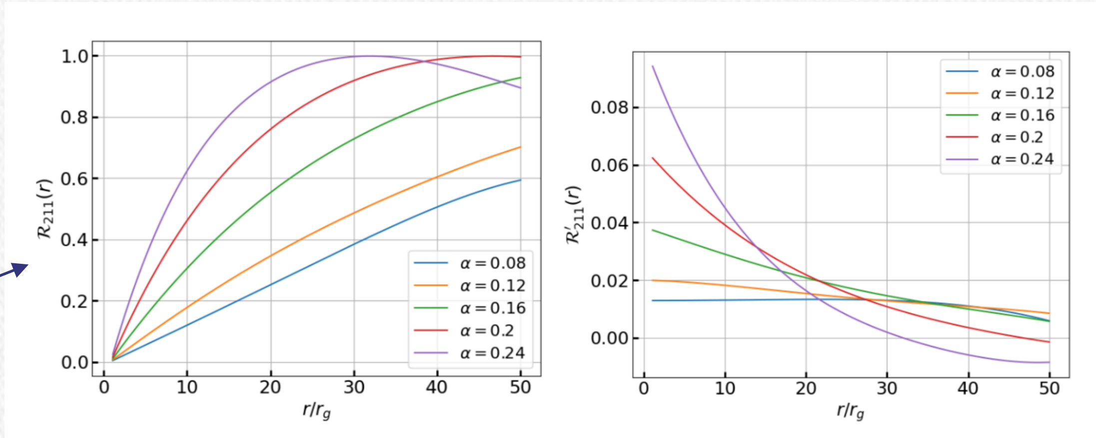


Plasma effects cause the photon ring to shrink.

Dilaton Effects

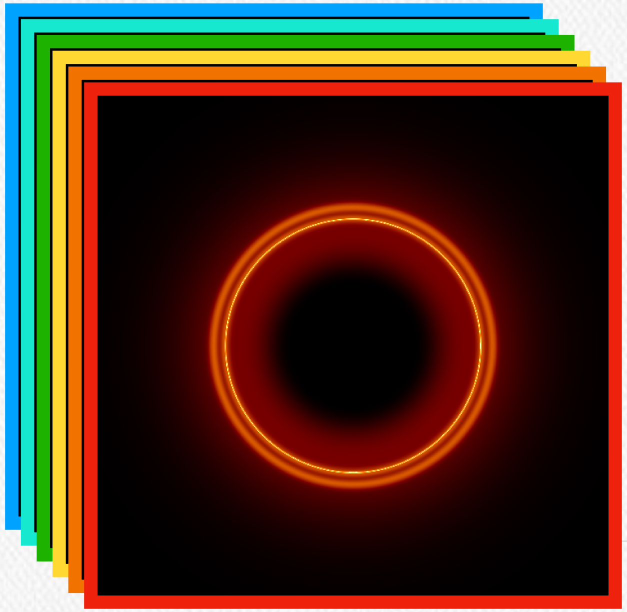
$$\phi(t,\mathbf{r}) = \phi_{\max} e^{-i\omega t + im\varphi}\mathscr{R}_{211}(r)\sin(\theta)$$

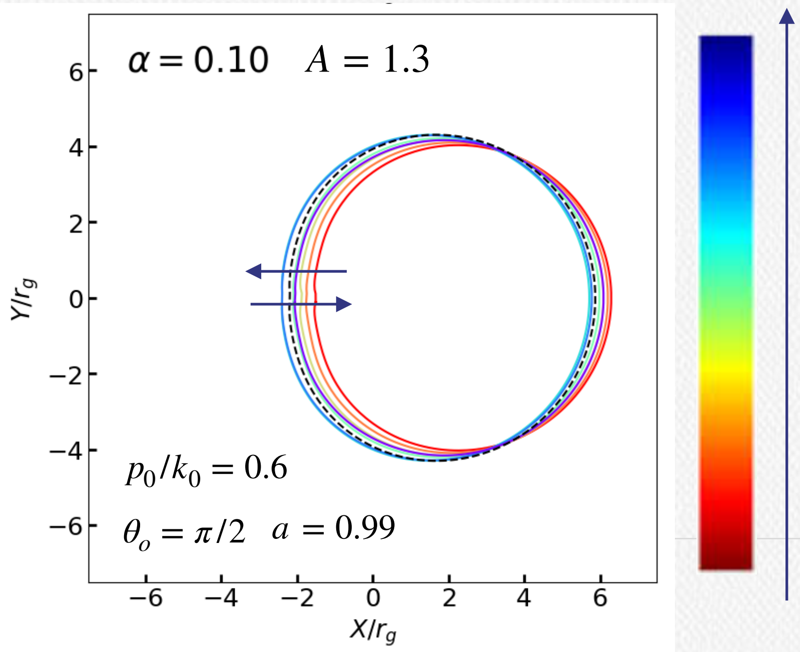
$$\mathscr{R}_{211}(r) \equiv R_{211}(r)/R_{211}\left(r_{\max}\right)$$



Parameters: $A \equiv g_{\phi\gamma}\phi_{\max}$

$$\alpha = \mu r_g$$



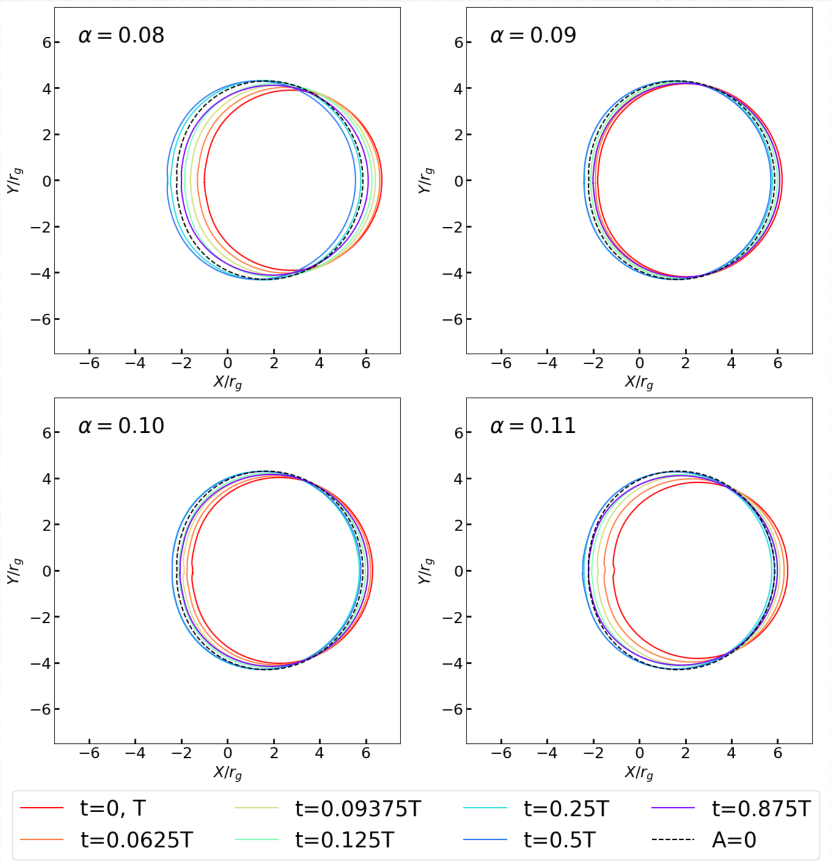


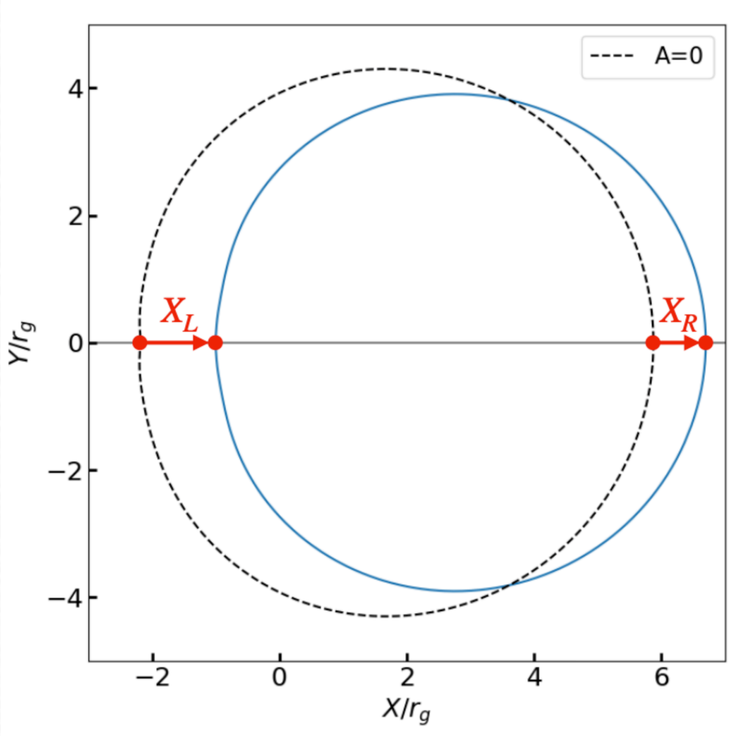
The photon ring oscillates with a period $T/r_g = 2\pi/\alpha$

For $M87^\star (6.5\times 10^9 M_\odot)$ and $\alpha = 0.1$, $T = 23\ days$

Dilaton Effects

- Influence of dilaton mass





$\delta_\phi \equiv X_R - X_L$ Overall size changing

$\epsilon_\phi \equiv X_R + X_L$ Transverse shift

Dilaton Effects

Photon velocity modification caused by dilaton:

$$\Delta v \sim \int_L \underline{D(t(\lambda))} d\lambda$$

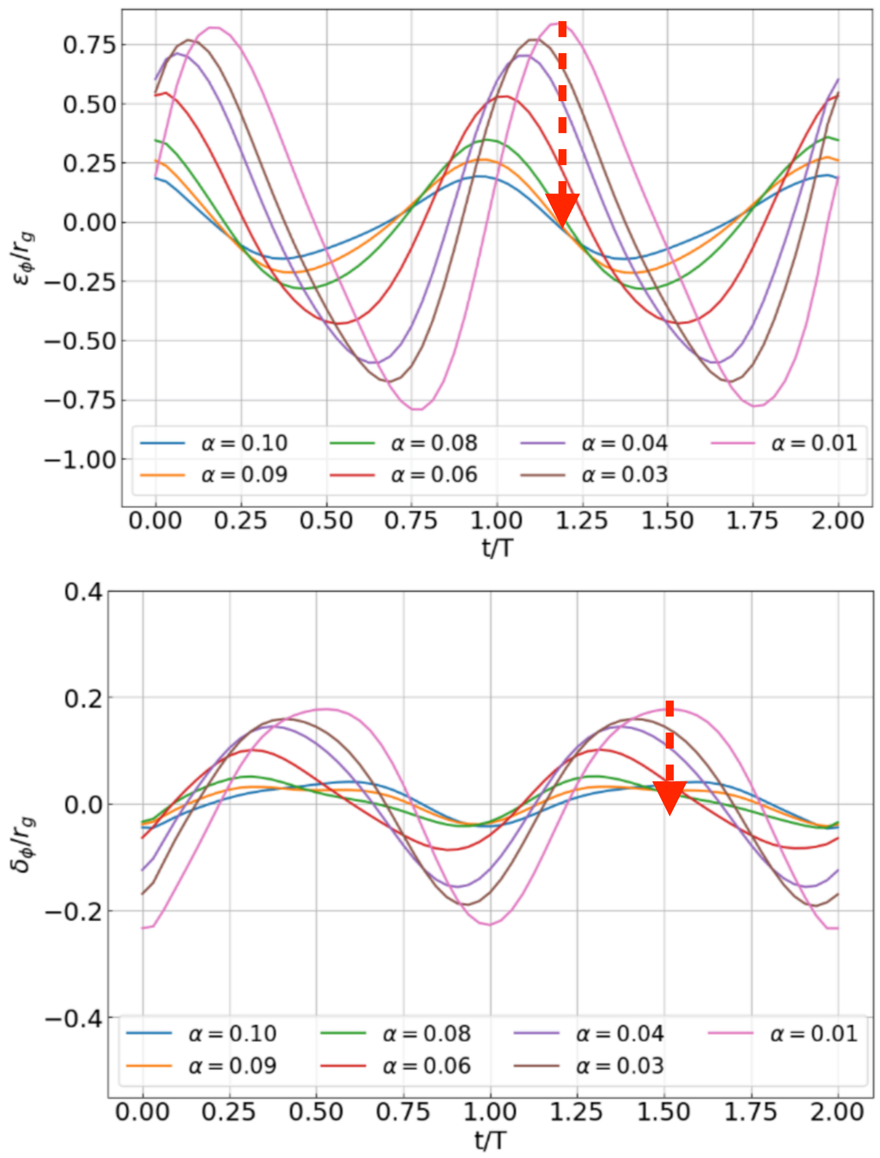
$\propto \cos(\mu t)$ dilaton oscillation

L: photon path affected by the dilaton field

$L \sim O(10) r_g$

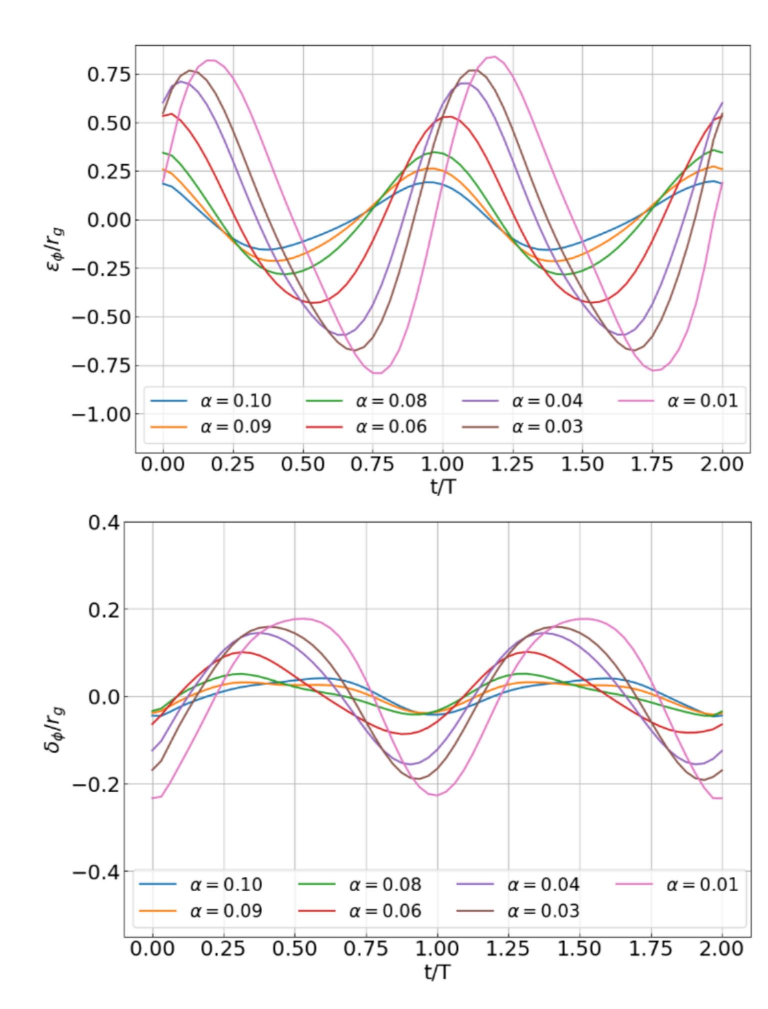
If $T \ll L$, Δv will be averaged out.

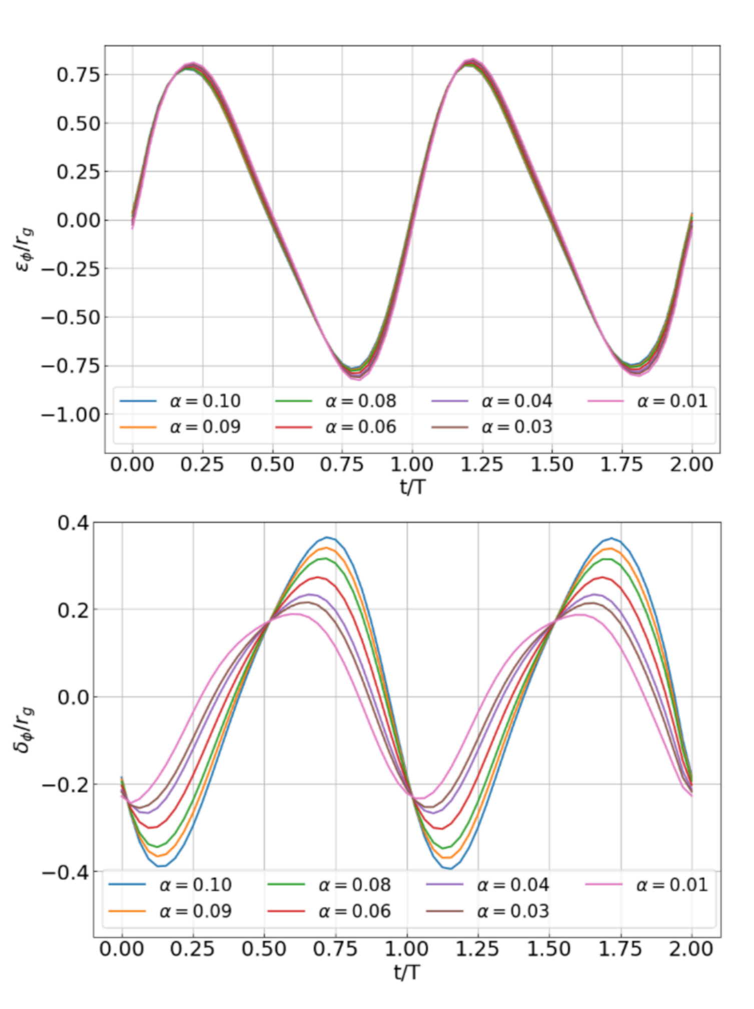
$fix\ R(r)$



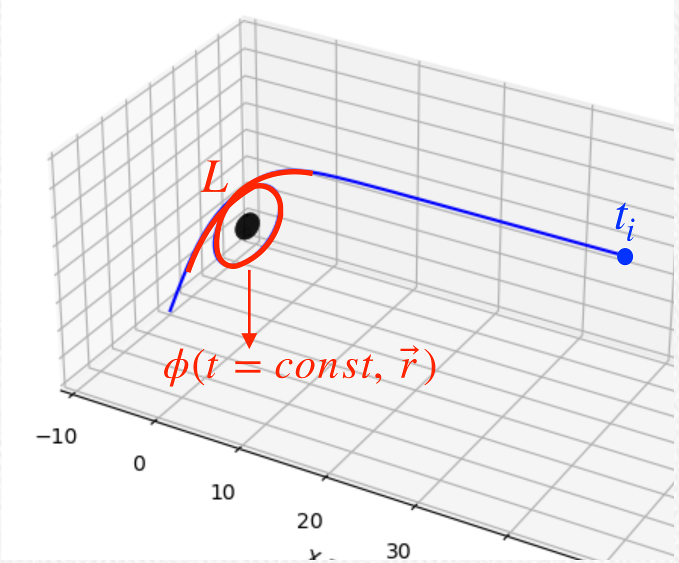
As α (or μ, dilaton Mass) increases, the amplitude of the photon ring decreases.

Dilaton Effects





Quasi-static limit:
$\alpha \to 0 \ (T \gg L)$



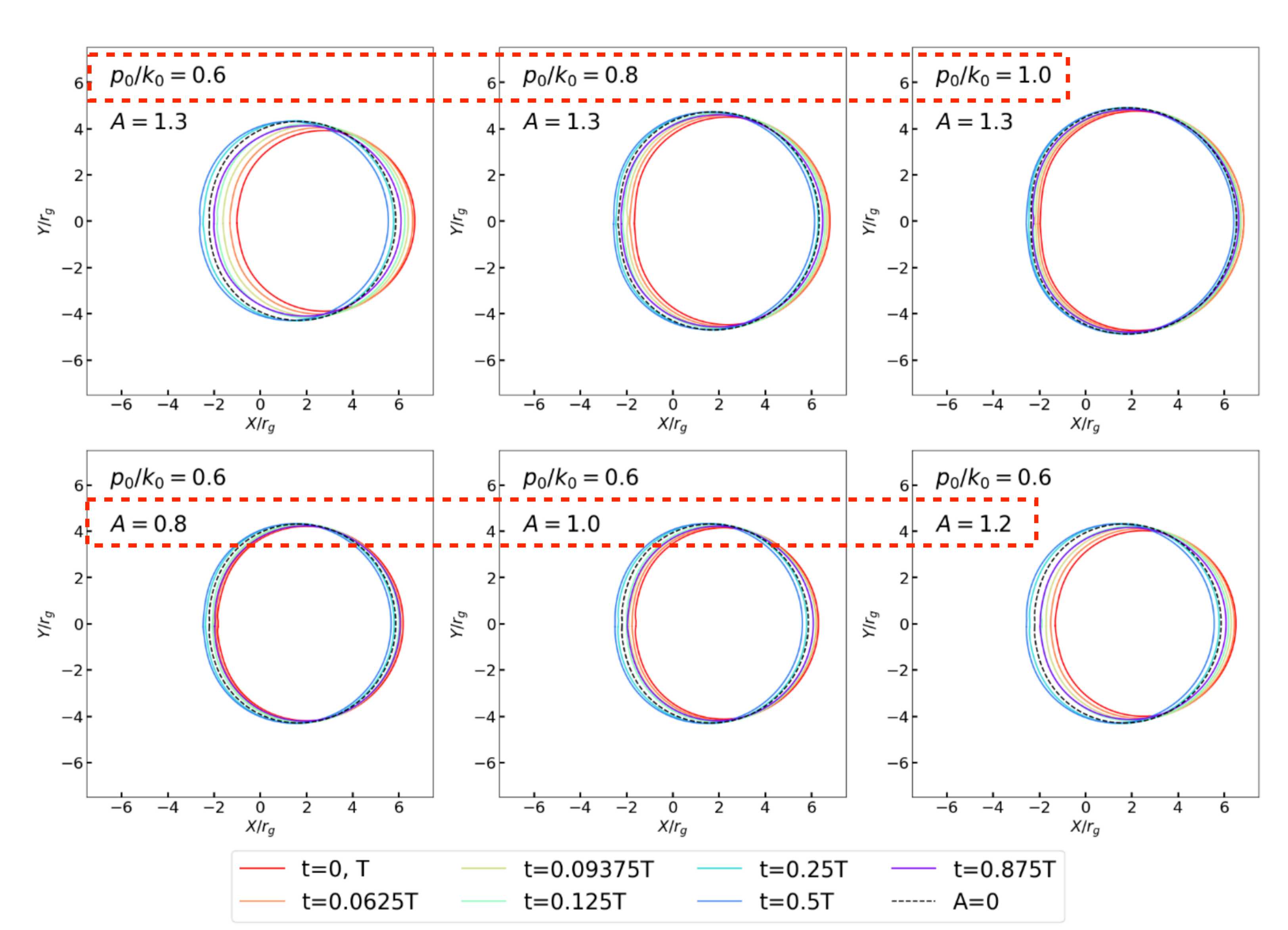
Dilaton Effects

- Influence of A and p_0/k_0

$$\omega_p^2 = \frac{k_0^2}{(r/r_g)^h}$$

p_0: photon frequency

$$A \equiv g_{\phi\gamma}\phi_{\max}$$



Detectability

- By applying the **quasi-static limit**, an upper bound on the dilaton effects can be obtained.

$$\frac{d}{r_g} = 3\sqrt{3}\left(1 - \delta_p - \delta_\phi\right) \qquad \delta_\phi \equiv 3^{-h-1}\frac{k_0^2}{\omega_0^2}g_{\phi\gamma}\bar{\phi}$$

- For spherically symmetric accretion of a perfect fluid without pressure：

$$k_0^2 = \frac{e^2 L}{\sqrt{2}\eta m_e m_p r_g^2}$$

$$\omega_p^2 = \frac{k_0^2}{(r/r_g)^h}$$

$n_e \sim 10^7 cm^{-3}$ for $M87^\star$ at $3r_g$

$$\omega_p^2 = n_e e^2/m_e$$

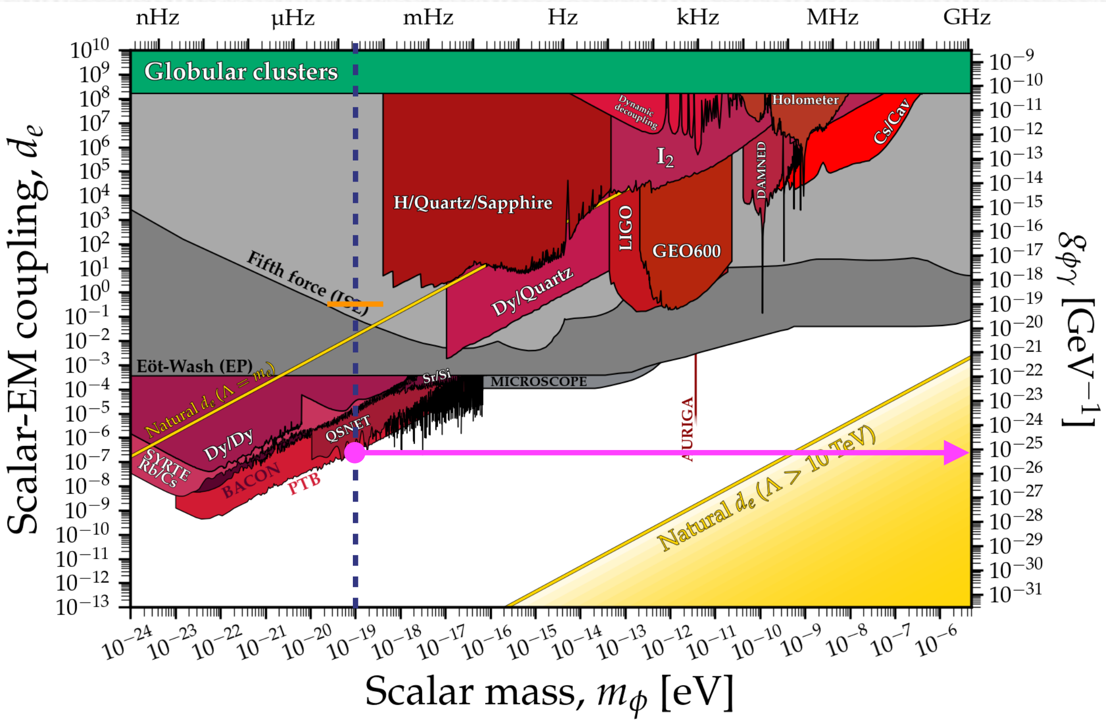
- The energy density of ϕ：

$$M_{cloud} = 10\,\%\,M_{SMBH}$$

$\rho(r)$ is confined within a sphere with radius $\sim \lambda_c$.

$$\bar{\rho} \sim \frac{M_{cloud}}{(4\pi/3)\lambda_c^3} \longrightarrow \bar{\phi} \sim 10^{25}\mathrm{eV} \quad for\ \alpha \sim 0.01$$

Detectability



Cite from https://github.com/cajohare/AxionLimits

$$g_{\phi\gamma}\bar{\phi} = \mathcal{O}(1)\left(\frac{\delta_\phi}{0.01}\right)\left(\frac{\eta}{10^{-4}}\right)\left(\frac{\lambda_0}{1\mathrm{cm}}\right)^{-2}\left(\frac{L}{10^6 L_\odot}\right)^{-1}\left(\frac{M}{4\times 10^6 M_\odot}\right)^{2}$$

For $r_o = 8.3kpc$, angular resolution $\delta\beta = 10^{-3}\mu as$

Currently: $\delta\beta = \lambda/D \approx 20\mu$as

$D = 1.3\times 10^4 km,\ \lambda = 1.3mm$

BACON Collaboration, "Frequency ratio measurements at 18-digit accuracy using an optical clock network", Nature 591, 564-569 (2021)
M. Filzinger et al., "Improved Limits on the Coupling of Ultralight Bosonic Dark Matter to Photons from Optical Atomic Clock Comparisons", PRL 130, 253001 (2023)

Summary

- A superradiant dilaton cloud can induce periodic variations in the black hole photon ring, with a period $T = 2\pi/\mu$.
- An increase in $\alpha = \mu r_g$ leads to a reduction in the oscillation amplitude of the photon ring. (Wash-out effects)
- In the case of $SgrA^\star$, achieving an angular resolution better than 10^{-3}µas is necessary to surpass the constraints set by ground-based atomic clock experiments.