“引力原子”中由视界反射引起的矢量云“超辐射增长反常”

Superradiant Growth Anomaly in Vectorial Bosonic Condensates Around Kerr Black Holes With Near-Horizon Reflections

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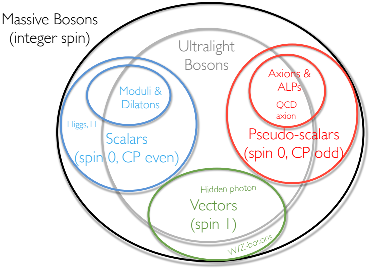
  - The superradiance modifier and the anomalous growth
  - Anomaly features and GW waveform deformation
- Outlook and Conclusion

Ultralight bosons as dark matter candidates

- Axions
  - Proposed to solve the Strong-CP problem
  - Assumed to be ultralight
- Axion-like Particles (ALPs)
  - With integer spins
  - Wave-like behavior when close to black holes

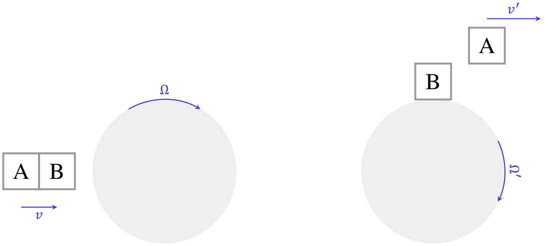
A Classification of Bosonic Dark Matters



Taken from "Axion Dark Matter: What is it and Why Now?" arXiv: 2105.01406.

Detection in Laboratory: extremely challenging
Black hole superradiance: indirect evidence.

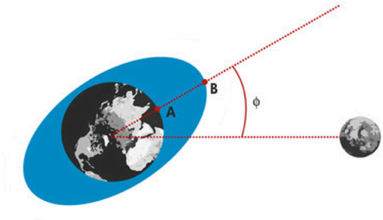
Rotational superradiance — a classical analog



- A **rotating** and **absorbing** central object.
- A and B are weakly connected
- B is sticked to the surface, A and B are disconnected
- A gets energy and flies away faster

Superradiance and dissipation

Tidal drag in Earth-Moon system



Key ingredients: Friction between the Earth and the tidal bulge —**Dissipation.**

BH superradiance — Penrose Process for a particle

$E = -p^\mu t_\mu$,
t_μ — killing vector of Kerr spacetime, spacelike in ergosphere
E — can be positive or negative

$E = E_1 + E_2$, with $E_2 < 0$ and $E_1 > E$, energy extracted from rotating BH.
Dissipation: negative energy states in ergosphere.

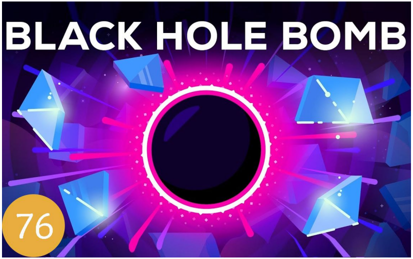
Superradiance as a multiple process

Different channels

- Fission process — forbidden
- With a reflective boundary at Infinity:
  - enclosed in a mirror, ideal, toy model
  - AdS boundary, theoretical applications
  - barrier from particle mass, most astrophysically realistic!

Bouncing Penrose process — Black hole bomb

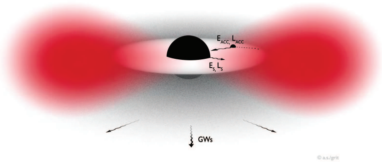
Massless Photons: amplitude amplified after Penrose process, happens only once.



If we put a mirror surrounding the BH, bouncing the photons back, they get re-amplified, so on and so forth, energy inside the mirror gets higher and higher, and finally boom!

BH superradiance for ultralight bosons — Gravitational Atom

The particle mass as the barrier/mirror to drive the bosons back.



The mass coupling constant is defined as

$$\alpha = \frac{GM\mu}{\hbar c} = \frac{r_g}{\lambda_c} = \frac{\lambda_c}{r_B} \simeq 0.02 \left(\frac{M}{3M_\odot}\right)\left(\frac{\mu}{10^{-12}\text{eV}}\right)$$

- λ_c — Compton wavelength of bosons
- r_B — Bohr radius for the cloud, the larger, the smaller α is
- $r_g \equiv GM$ — Gravitational radius

BH superradiance for ultralight bosons — Properties

Superradiance is a thermodynamic process, it obeys the BH first and second law.

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega_{\mathrm{H}}\delta J$$

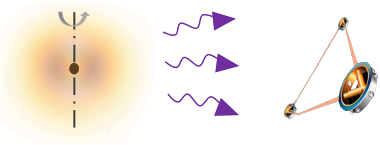
$$\delta J/\delta M = m/\omega \quad \Longrightarrow \quad \delta A = \frac{8\pi}{\kappa}\frac{(\omega - m\Omega_{\mathrm{H}})}{\omega}\delta M,$$

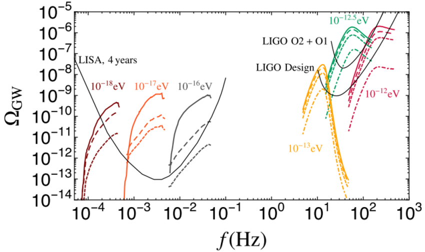
$$\delta A > 0 \;\&\; \delta M < 0 \quad \Longrightarrow \quad \omega < m\Omega_{\mathrm{H}},$$

GW signals from the gravitational atom

- Particle occupation number growth: $\frac{\mathrm{d}N}{\mathrm{d}t} = \Gamma N$
- Hydrogenic energy levels: $\omega \approx \mu \left(1 - \frac{\alpha^2}{2\bar{n}^2}\right)$
- $\omega_{\mathrm{t}} = \omega_{\mathrm{e}} - \omega_{\mathrm{g}} \simeq \frac{1}{2}\mu\alpha^2 \left(\frac{1}{\bar{n}_{\mathrm{e}}^2} - \frac{1}{\bar{n}_{\mathrm{g}}^2}\right)$
- $\omega_{\mathrm{a}} = 2\omega_{\mathrm{a}} \simeq 2\mu \left(1 - \frac{\alpha^2}{2\bar{n}_{\mathrm{a}}^2}\right)$

- Near-monochromatic GW

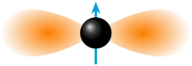




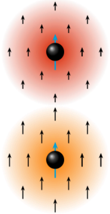
$$f_{\mathrm{GW}} \sim \omega_R/\pi \sim 5\,\mathrm{kHz}\left(\frac{\mu\hbar}{10^{-11}\mathrm{eV}}\right)$$

[Brito et al., 2017]

BH angular momentum, double channels to transfer:
orbital angular momentum, intrinsic spin.



	l=0	l=1	l=2
Scalar: $\vert nlm\rangle_{(j=l)}$		$\vert 211\rangle$	$\vert 322\rangle$
Vector: $\vert nljm\rangle$	$\vert 1011\rangle$	$\vert 2122\rangle$	$\vert 3233\rangle$
Tensor: $\vert nljm\rangle$	$\vert 1022\rangle$	$\vert 2133\rangle$	$\vert 3244\rangle$



Bosonic fields with spin: field configuration closer to BH, more sensitive to horizon geometry.

$$\Gamma_{nljm} \propto (m\Omega_{\rm H} - \omega)\ \alpha^{2(l+j)+5}, \qquad j \in (|l-s|, l+s)$$

smaller $l+j$, larger rate $\Longrightarrow$ **vector** dominant mode most significant! **(NOT tensor)**

Superradiant “instability”

The stability of electron distribution:
  regular boundary condition at the proton,
$\omega = E$, only a real part.

The instability of bosonic cloud distribution:
  purely ingoing boundary condition at the horizon
$\omega = E + i\Gamma$, imaginary part appears.

What about a B.C. with a reflection at the horizon?
We will investigate $\Gamma(\mathcal{R})$ in the vector case.

Proca equation separation in Kerr spacetime

A Kerr BH with mass M and angular momentum $J \equiv Ma$,

$$\begin{aligned} \mathrm{d}s^2 = &\left(1 - \frac{2Mr}{\Sigma}\right)\mathrm{d}t^2 + \frac{4aMr}{\Sigma}\sin^2\theta \mathrm{d}t\,\mathrm{d}\varphi - \frac{\Sigma}{\Delta}\mathrm{d}r^2 \\ &- \Sigma \mathrm{d}\theta^2 - \left[\left(r^2 + a^2\right)\sin^2\theta + 2\frac{Mr}{\Sigma}a^2\sin^4\theta\right]\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

with

$$\begin{aligned} \Delta &\equiv r^2 - 2Mr + a^2, \\ \Sigma &\equiv r^2 + a^2\cos^2\theta. \end{aligned} \tag{2}$$

A bosonic vector field obeying Proca EoM,

$$\nabla_\mu F^{\mu\nu} + \mu^2 A^\nu = 0, \tag{3}$$

The separable ansatz:

$$A^\mu = B^{\mu\nu}\nabla_\nu Z,\;\; Z(t, \mathbf{r}) = \mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}m\varphi}R(r)S(\theta). \tag{4}$$

Matched asymptotics in three pieces

Angular function $S(\theta)$ known, radial function $R(r)$ a bit complicated...

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\left(\frac{1}{z}+\frac{1}{z+1}\right)\frac{\mathrm{d}}{\mathrm{d}z}-\frac{j\,(j+1)}{z(z+1)}+\frac{P_+^2}{z^2}+\frac{P_+^2}{(z+1)^2}-\frac{2P_+^2}{z}+\frac{2P_+^2}{z+1}\right]R_0^{\rm near}=0,$$

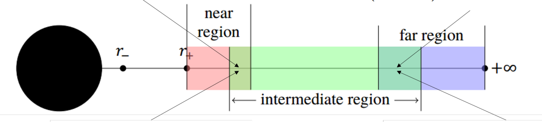
$$\left[\frac{\mathrm{d}^2}{\mathrm{d}y^2}+\left(\frac{2}{y}-\frac{2y}{\lambda_0^2+y^2}\right)\frac{\mathrm{d}}{\mathrm{d}y}-\frac{\lambda_0\,(\lambda_0-1)}{y^2}-\frac{2\lambda_0}{\lambda_0^2+y^2}\right]R_0^{\rm int}=0,$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\frac{2}{x}\frac{\mathrm{d}}{\mathrm{d}x}+\frac{\nu_0}{x}-\frac{\ell\,(\ell+1)}{x^2}-\frac{1}{4}\right]R_0^{\rm far}=0.$$

Due to existence of extra poles, radial functions must be solved in three pieces, and coefficients must be matched in two overlap regions...

$$R_0^{\rm far}(r)\sim C_0^{\rm far}\left[\left(2\sqrt{\mu^2-\omega^2}r\right)^{\ell+1}\frac{(-1)^{1+\ell-\nu_0}(\ell+\nu_0)!}{2(1+2\ell)!}+\ldots+\left(2\sqrt{\mu^2-\omega^2}r\right)^{-\ell}(-1)^{\ell-\nu_0}\delta\nu_0(2\ell)!(-1-\ell+\nu_0)!+\ldots\right]$$

$$R_0^{\rm near}(z)\sim A_0^{\rm near}\left[-(z+1)\right]^{j}+B_0^{\rm near}\left[-(z+1)\right]^{-1-j}$$



$$R_0^{\rm int}(y)\sim C_0^{\rm int}y^{-\lambda_0}+D_0^{\rm int}y^{-1+\lambda_0}\lambda_0^2\,(2\lambda_0+1)$$

$$R_0^{\rm int}(r)\sim C_0^{\rm int}\,(r-r_+)^{\ell+1}+D_0^{\rm int}\,(r-r_+)^{-\ell}\,(-3-2\ell)$$

Perform two matchings and determine the matching coefficients

$$A_0^{\rm near}=\frac{(-1)^{\nu_0}(\ell+\nu_0)!}{2(2\ell+1)!}\left[2\sqrt{\mu^2-\omega^2}\,(r_+-r_-)\right]^{\ell+1}$$

$$B_0^{\rm near}=(-1)^{\nu_0}\delta\nu_0\frac{(2\ell+1)\,(\ell+1)^2\,(2\ell)!(-1-\ell+\nu_0)!}{(2\ell+3)}\left[2\sqrt{\mu^2-\omega^2}\,(r_+-r_-)\right]^{-\ell}$$

Boundary condition with a reflection

And we have to piece the solution to a boundary condition...

$$\lim_{r_* \to -\infty} R(r_*) \sim \left[\mathrm{e}^{-\mathrm{i}(\omega - m\Omega_H) r_*} + \mathcal{R} \mathrm{e}^{\mathrm{i}(\omega - m\Omega_H) r_*} \right],$$

with $\mathcal{R}$ a phenomenological parameter denoting reflection.
Energy level shifts due to reflection:

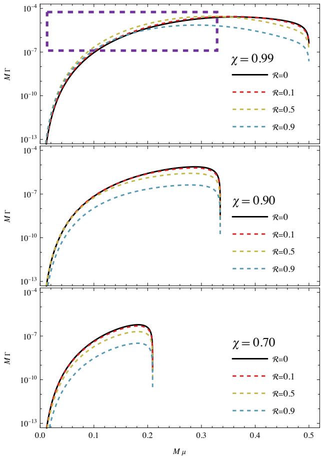
$$\delta\nu = \delta\bar{\nu} \frac{1 - \mathcal{R} \exp\left(2\mathrm{i} \sum_{q=1}^{j} \phi_q\right)}{1 + \mathcal{R} \exp\left(2\mathrm{i} \sum_{q=1}^{j} \phi_q\right)}, \text{ with } \phi_q \equiv \arctan \frac{2P_+}{q},$$

with $\delta\bar{\nu}$ the energy level without reflection,

$$\delta\bar{\nu} = \mathrm{i} P_+ \mu^{2j-2l} (2k)^{2l+1} (r_+ - r_-)^{2j+1} \frac{(l+\bar{n})!}{(\bar{n}-l-1)!} \times \left[\frac{l!}{(l+j)!(l+j+1)!} \right]^2 \times \prod_{\mathrm{i}=1}^{j} \left(\mathrm{i}^2 + 4P_+^2 \right), \; j = l \pm 1.$$

Anomalous boost to superradiance rate

By $\omega = E + i\Gamma = \mu\sqrt{1 - \frac{\alpha^2}{\nu^2}}$, and $\nu = \bar{n} + \delta\nu$, we can have $\Gamma(\mathcal{R})$.

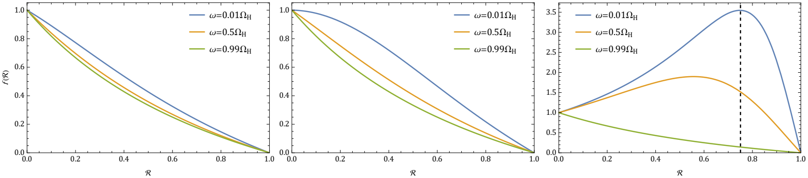


- Consider fastest-growing mode $\{n, l, j, m\} = \{0, 0, 1, 1\}$
- Intuition: reflection near horizon $\rightarrow$ less "absorbing" $\rightarrow$ lower superrdiance rate
- Result: $\Gamma_{njlm} = f(\mathcal{R})\bar{\Gamma}_{njlm}$, the "modifier" $f(\mathcal{R})$ can be greater than 1.

The reflection modifier

We extract the superradiant modifier as

$$f(\mathcal{R}) \equiv \Gamma_{njlm}/\bar{\Gamma}_{njlm} = \frac{1-\mathcal{R}^2}{\mathcal{R}^2+1+2\mathcal{R}\cos\left(2\phi_1\right)},$$



BH spin $\chi = 0.5$ (left), $\chi = 1/\sqrt{2}$ (middle), $\chi = 0.99$ (right).

Three descriptions

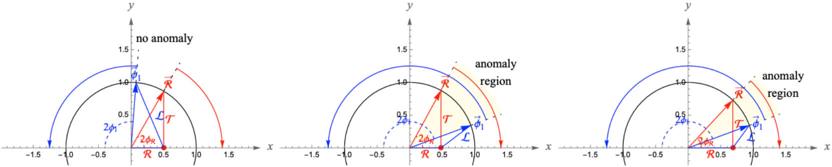
Requiring $f(\mathcal{R}) > 1$ would result in —

Mathematical:

$$2P_+ = \underbrace{\frac{\chi}{\sqrt{1-\chi^2}}\left(1-\frac{\omega}{\Omega_{\rm H}}\right) > \sqrt{\frac{1+\mathcal{R}}{1-\mathcal{R}}}} \geqslant 1,$$

Geometrical:

$$\phi_{\mathcal{R}} + \phi_1 > \frac{\pi}{2}, \qquad \text{with} \qquad \begin{cases} 2P_+ = \tan(\phi_1) \\ \mathcal{R} = \cos(2\phi_{\mathcal{R}}) \end{cases}$$



Three descriptions

Physical:

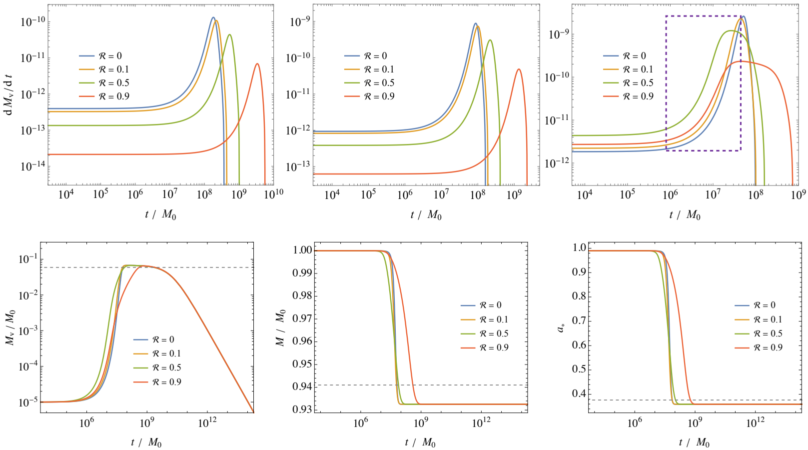
$$2P_+ = \frac{m\Omega - \omega}{\kappa_+} = -\frac{\omega}{8\pi}\frac{\delta A}{\delta M} = \frac{1}{2\pi k_{\mathrm{B}}}\frac{\delta S_{\mathrm{BH}}}{\delta N} = -\frac{1}{2\pi k_{\mathrm{B}}}\frac{\delta S_{\mathrm{c}}}{\delta N} = \frac{1}{2\pi k_{\mathrm{B}}}\frac{\delta I}{\delta N}$$

$$\Longrightarrow \frac{\delta I}{\delta N} > 2\pi k_{\mathrm{B}} \cot \phi_{\mathcal{R}}.$$

Information per particle carries exceeds a certain value determined by the reflection parameter.

May be a “rotation-relativistic” effect of an extremal BH.

Superradiant evolution and anomaly behavior



(a) (b) (c)

Condensate speed and mass/angular momentum evolution at different BH spin.

"The Tortoise and the Hare" and anomaly features

Generally, the reflection will lead to an even slower tortoise (superradiance rate).

a_{*0}	$\mathcal{R}$	Anomaly features		
		$\left(\Delta \frac{\mathrm{d}M_\mathrm{v}}{\mathrm{d}t}\right)_{\max}$	$\left(\Delta M_\mathrm{v}\right)_{\max}/M_0$	τ_adv/M_0
0.99	0	-	-	-
	0.1	3.23×10^{-10}	1.36×10^{-2}	7.70×10^{7}
	0.5	6.16×10^{-10}	2.16×10^{-2}	5.66×10^{7}
	0.9	4.06×10^{-11}	9.36×10^{-4}	3.27×10^{7}

$$t_{(\Delta \dot{M}_v)_{\max}} < t_{(\Delta M_\mathrm{v})_{\max}} < t_\mathrm{adv}$$

"The Tortoise and the Hare"

Image generated by AI

What we've found is a temporally running faster "Hare", though who eventually reaches almost the same destination (final state).

Imprints in GW waveform deformation

This will effectively influence the GW waveform!

$$h \propto \sqrt{\dot{E}_{\mathrm{GW}}} \propto M_{\mathrm{v}}$$

$$r_h(\mathcal{R}, t) = \frac{h_{\mathrm{v}}(\mathcal{R}, t)}{h_{\mathrm{v}}(\mathcal{R}=0, t)} = \frac{M_{\mathrm{v}}(\mathcal{R}, t)}{M_{\mathrm{v}}(\mathcal{R}=0, t)}$$

$$\begin{aligned}
t_{(\Delta \dot{M}_v)\mathrm{max}} &\rightarrow \; [f(\mathcal{R}, t)]_{\mathrm{max}}, \; \dot{r}_h(\mathcal{R}, t) > 0, \quad \text{anomalous growth rate reaches its maximum.} \\
t_{(\Delta M_v)\mathrm{max}} &\rightarrow \; f(\mathcal{R}, t) = 1, \; [r_h(\mathcal{R}, t)]_{\mathrm{max}}, \quad \text{reflective GW strain is most distinguishable.} \\
t_{\mathrm{adv}} &\rightarrow \; f(\mathcal{R}, t) < 1, \; r_h(\mathcal{R}, t) = 1, \quad \text{GW strains return the same, anomalous signal ends.}
\end{aligned}$$

When $\chi = 0.99$ and $\omega/\Omega_{\mathrm{H}} = 0.01$, the reflection-induced GW strain magnification is about $r_h(\mathcal{R}, t) \approx f(\mathcal{R} \simeq 0.75) \simeq 3.5$ times.

Anomaly magnification in vector case

This “anomaly”, already exists in scalar case, is largely magnified in vector case due to a faster growing rate in dominant mode 1s.

For $M_0 = 100 M_\odot$ and $\mathcal{R} = 0.5$,

$\left(\varDelta \frac{\mathrm{d}M_{\mathrm{V}}}{\mathrm{d}t}\right)_{\max} = 2.49 \times 10^{26}\ \mathrm{kg} \cdot \mathrm{s}^{-1}$ (100 times of Earth mass),

$(\varDelta M_{\mathrm{v}})_{\max} = 4.30 \times 10^{30}\ \mathrm{kg}\ (2.16 M_\odot)$,

$\tau_{\mathrm{adv}} = 2.79 \times 10^{4}\ \mathrm{s}$ (7.75 hours).

$$\frac{h_{\mathrm{v}}(\mathcal{R})}{h_{\mathrm{s/t}}(\mathcal{R})} \approx \frac{\Gamma_{\mathrm{v}}}{\Gamma_{\mathrm{s/t}}} \approx \alpha^{-2} \simeq 2500 \left(\frac{3M_\odot}{M}\right)^2 \left(\frac{10^{-12}\mathrm{eV}}{\mu}\right)^2$$

Future Outlook

- Physical origin still to be explored:
  A possible postulation:
  the existence of reflection term plays two roles, one of which is to occupy the negative energy orbits which leads to attenuating the instability rate, the other is to release negative energy states by modify the BH area thus effectively enlarging the ergosphere volume.

- Potential Applications:
  In joint with the Kuchiev spectrum, the medium mass selection can be done:

$$\mathcal{R}(\omega) = \exp(4\pi P_+).$$

Conclusion

- Vector Superradiance with Reflective Near-horizon B.C.
  - Low BH spin: superradiance rate decays with reflection parameter.
  - High BH spin: in earlier stage, reflection temporally enhances superradiance. (anomalous situation)
  - Physical origin still to be explored.
- Black Hole-Condensate System Evolution
  - Although anomaly exists, almost the same final states are reached.
  - Three anomaly features for the "tortoise and hare" evolution, imprints on GW waveform deformation.
  - In joint with specific spectrum, medium mass selection can be done.

Thank you for listening!

Comments are welcome!