Axions in a Cavity - A quantum picture

with Y. Gao, Z. Peng, Commun.Phys. 7 (2024) 277;

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CDM

- Accounts 23% of total energy density of universe while baryonic matter accounts 4%.
- Properties:

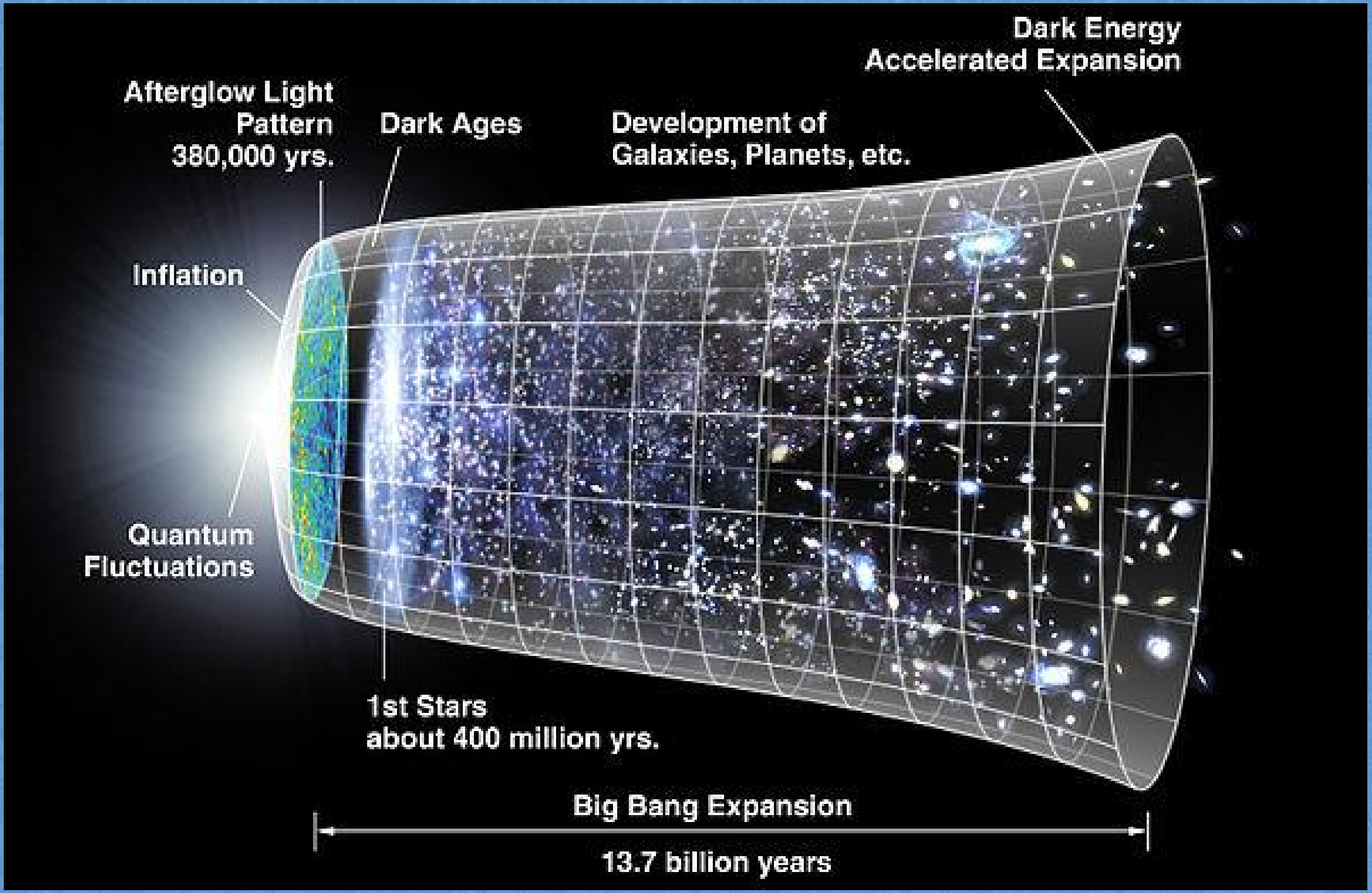
a. Pressureless

Primordial velocity is very small , at most $\sim 10^{-8}c$ today .

b.Collisionless

Cold dark matter is weakly interacting (so dark), except for gravity.

History of the Universe



picture by NASA

$$m_0 \approx 6 \times 10^{-5}\mathrm{eV}\left(\frac{10^{11}\mathrm{GeV}}{f_a}\right)$$

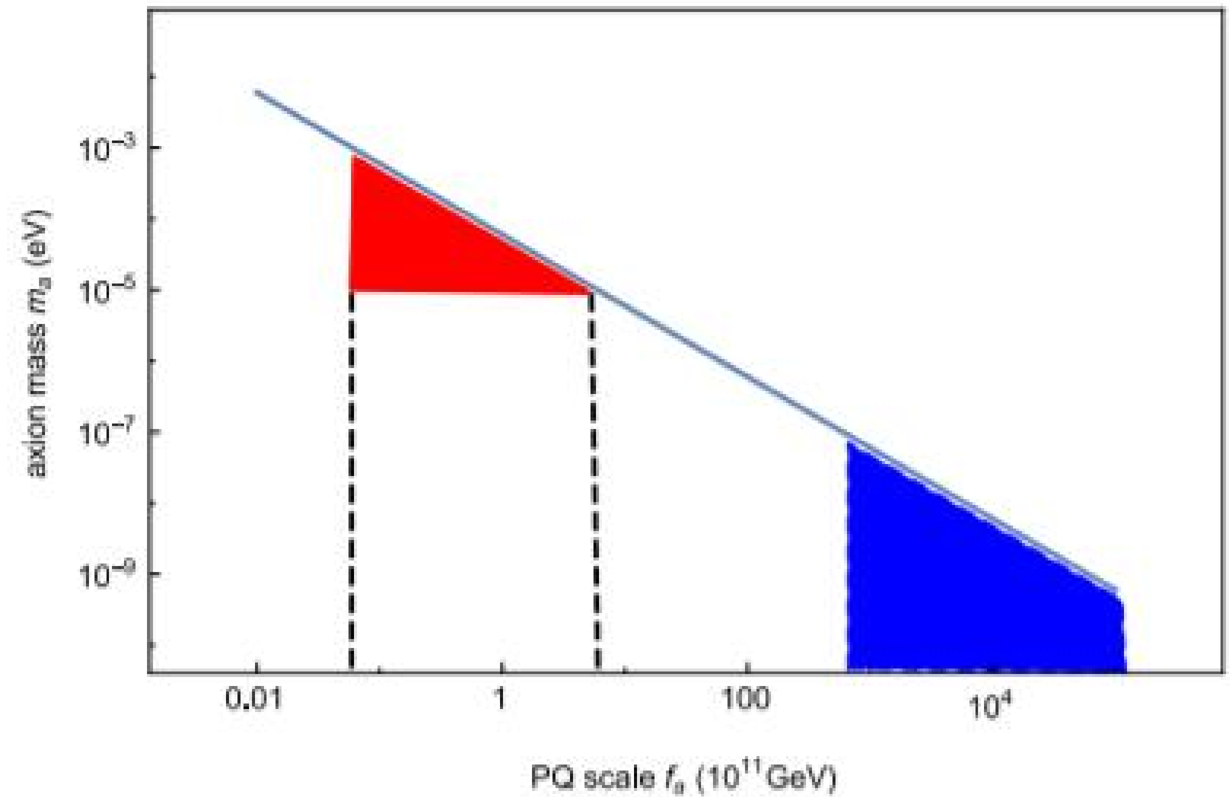


FIG. 1: The two possible windows of the dark matter axions. The upper-left one is often called the classical window and the lower-right one is the anthropic window assuming that $H_I < 10^{10}$GeV and the PQ symmetry was not restored after inflation.

Why resonant cavity?

If the resonance is tuned to the axion mass, the cavity acts like a forced oscillator and on resonance achieves a large axion-induced excitation.

The signal power:

$$\boldsymbol{\nabla} \times \partial_t \mathbf{B} - \partial_t^2 \mathbf{E} = -g_{a\gamma\gamma} B_0 \partial_t^2 a \, \hat{\mathbf{z}}$$

$$\Rightarrow -\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{E} - \partial_t^2 \mathbf{E} = -g_{a\gamma\gamma} B_0 \partial_t^2 a \, \hat{\mathbf{z}}$$

$$-\sum_m \left(\omega_m^2 + \partial_t^2\right) E_m(t) \, \mathbf{e}_m(\mathbf{x}) \cdot \mathbf{e}_n^*(\mathbf{x}) = -g_{a\gamma\gamma} B_0 \partial_t^2 a(t) \, \hat{\mathbf{z}} \cdot \mathbf{e}_n^*(\mathbf{x}).$$

1 question:

Does the axion-photon conversion rate increase for a single axion-photon transition?

2 question:

Does the axion-photon conversion rate increase for a non-coherent axion field?

a non-coherent wave function:

$$D(t) = \frac{\sqrt{\rho_{DM}}}{m} \sum_j \alpha_j \sqrt{f(v_j)\Delta v} \times \cos\left[m\left(1 + \frac{v_j^2}{2}\right) t + \phi_j \right]$$

The quantum properties of the Cavity

Modern cryogenic technology can sustain ~20mK or lower temperature

$$n(\omega_a, T) = \frac{1}{e^{\omega_a / k_B T} - 1}$$

The thermal photons have a very low occupation number n<<1 .

Thus it is useful to consider the quantum picture.

The Quantum picture of the Cavity (Feynman diagram method could not be used here)

The axion photon coupling is

$$\mathcal{L}_{a\gamma\gamma} = -g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

The Quantum picture of the Cavity

The interaction Hamiltonian is

$$
\begin{aligned}
H_I =& -\int d^3x \mathcal{L}_I \\
=& \chi\sqrt{\rho_{DM}} \cdot \sum_{jk} \alpha_j \sqrt{f(v_j)\Delta v} \cos\left(\omega_j t + \phi_j\right) \\
& \times \sqrt{\omega_k 2}\left(a_k e^{-i\omega_k t} U_k - c.c.\right) \ ,
\end{aligned}
$$

The Quantum picture of the Cavity

The electric field operator in the cavity can be expanded

$$\vec{E} = i \sum \sqrt{\frac{\omega_k}{2}} [a_k \vec{U}_k(\vec{r}) e^{-i\omega_k t} - a_k^\dagger \vec{U}_k^*(\vec{r}) e^{i\omega_k t}]$$

where $\vec{U}_k(\vec{r})$ is the cavity modes

The Quantum picture of the Cavity

The transition probability is

$$P \approx \left| \left\langle 1 \left| \int_0^t dt H_I \right| 0 \right\rangle \right|^2 = \frac{g_{a\gamma\gamma}^2 B_0^2 \rho_{DM}}{2m_a^2}$$
$$\times \left(\sum_{j,k} \sqrt{\omega_k} \alpha_j \sqrt{f(v_j)\Delta v} U_k^* \int_0^t dt \cos(\omega_j t + \phi_j)\, e^{i\omega_k t} \right)$$
$$\times \; c.c. \quad , \qquad (11)$$

The non-coherent terms canceled

$$\sum_{jj'kk'} \int_0^t dt \cos(\omega_j t + \phi_j) e^{i\omega_k t} \int_0^t dt \cos(\omega_{j'} t + \phi_{j'}) e^{-i\omega_{k'} t}$$

$$\approx \sum_{jj'kk'} \frac{1}{4} e^{-i(\phi_j - \phi_{j'})} \frac{[e^{i(\omega_k - \omega_j)t} - 1][e^{-i(\omega_{k'} - \omega_{j'})t} - 1]}{(\omega_k - \omega_j)(\omega_{k'} - \omega_{j'})}$$

$$\approx \sum_{j,k} \frac{\sin^2[(\omega_k - \omega_j)t/2]}{4[(\omega_k - \omega_j)/2]^2} \, .$$

The Quantum picture of the Cavity

The transition rate is

$$P = \frac{\pi g_{a\gamma\gamma}^2 B_0^2 \rho_{DM} t V}{4m_a^2} \sum_{jk} C_k \delta(\omega_k - \omega_j)\omega_k \alpha_j^2 f(v_j)\Delta v$$

$$\approx \frac{\pi g_{a\gamma\gamma}^2 B_0^2 \rho_{DM} t V \langle\alpha^2\rangle}{4m_a^2} \int dv f(v) \int d\omega_k C_k \frac{\omega_k}{d\omega_k} \delta(\omega_k - \omega_v)$$

$$\approx \frac{\pi g_{a\gamma\gamma}^2 B_0^2 t V}{2m_a^2} \rho_{DM} \int_0^{\Delta} dv_a f(v_a) C_{\omega_a} Q_c \ ,$$

The Quantum picture of the Cavity

The transition rate is enhanced by the cavity quality factor Q even for a single transition. Also, it is larger than the classical picture by a factor pi/2

Typical photon emitting rate of the cavity is order of 10Hz.

We have found that

1： The single ax-

ion to photon transition rate is enhanced by

confining the final photon states inside a cavity.

2：This en-

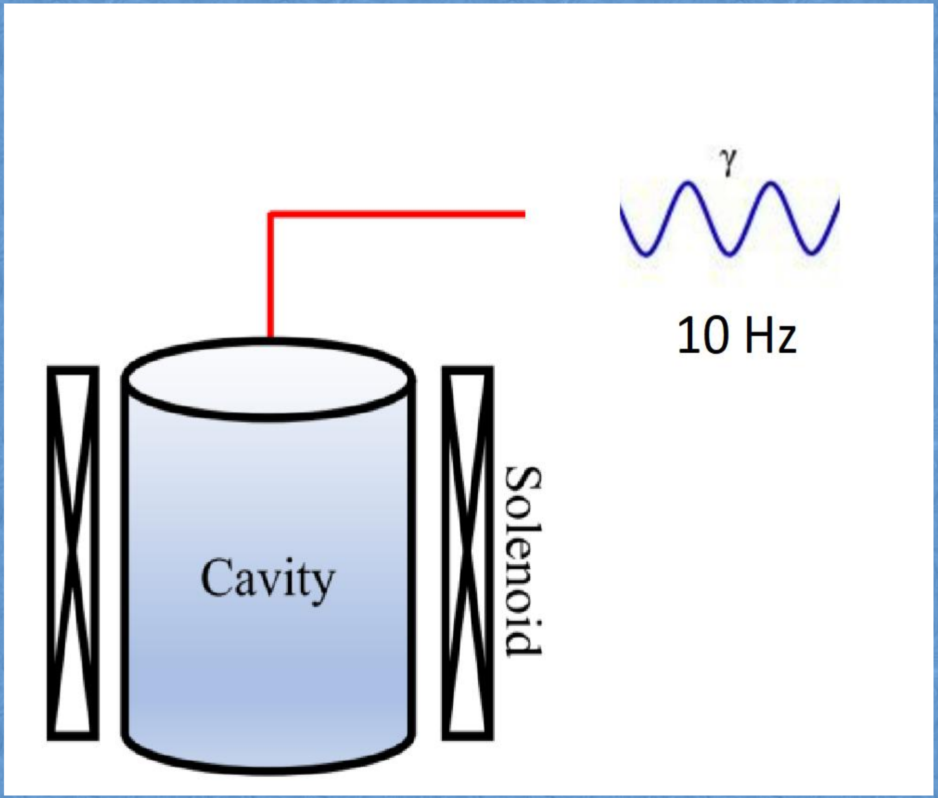
hancement remains even if the dark matter particles are

not in a coherent state during measurement。

The axion field is treated as a classical stochastic background, while the photon field is quantized. The key insight is that the transition rate enhancement arises from the confinement of the photon in the cavity (Purcell effect), which amplifies the density of photon states near resonance.

This enhancement is independent of dark matter phase coherence because the Hamiltonian sums of the interaction incoherently over all dark matter momentum modes

A cavity at quantum level can be regarded as a single photon emitter with a slow rate~ 10Hz.

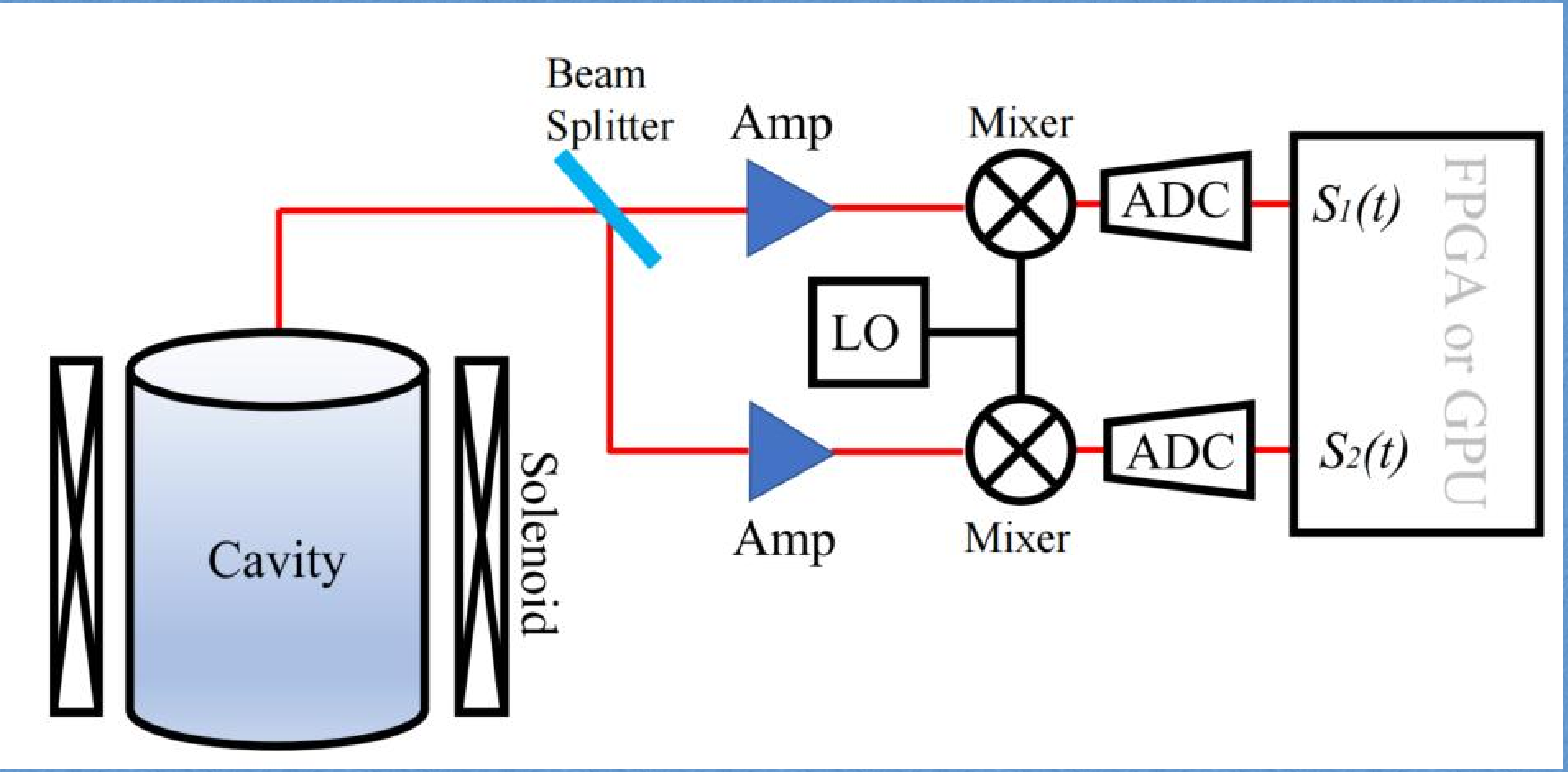


The bottleneck of the haloscope

The linear amplifier with a moderate bandwidth adds noise temperature order of T_eff=10K

$$\mathrm{SNR} = \frac{P_{sig}}{k_B T_{eff}} \sqrt{\frac{t}{b}}$$

The quantum interferometry



Simulated signal