

### 轴子暗物质: 理论与观测 青岛/2025.5.11

# **Primordial** gravitational atoms 康召毛

Based on work *JCAP* 11 (2024) 039, ZK, Tianjun Li & Weitao Ye

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# Part I I (primordial) Black hole: A super star from (cosmology) gravity

### Why interest in PBH if you are a particle worker?





### Primordial fluctuations as seeds of PBH

\* perturbation on scale k with density contrast  $\delta_k \equiv (\rho - \bar{\rho})/\bar{\rho}$ 

1. Before entering horizon, the physical scale of perturbation continues to expand according to a(t)/k

2. Entering horizon as  $\frac{a(t_H(k))}{k} = \frac{1}{H(t_H(k))}$ , with density contrast  $\delta_H$  following  $P(\delta_H) = \frac{1}{\sqrt{2\pi}\sigma_H} \exp(-\delta_H^2/2\sigma_H^2)$ 



3. Perturbation gradually decouples from the Hubble flow due to gravity, making  $\delta \propto \delta_H t^{2/3}$  until  $t_d$  when  $\delta \sim 1$ and perturbation begins to collapse

4. PBH mass, the Hubble mass at horizon crossing:  $M_{\rm BH} = \frac{M_{\rm PL}^2}{2H(t_H(k))} \propto t_H(k)$ 



### PBH formation in the dust-like era (for spin)

No pressure to against gravity, collapse to PBH is enhanced

 $\approx \beta(k)$ : probability of perturbation k collapse to a PBH

1. Overestimates the probability in the spherical limit  $\beta_0 \sim (\sigma_H/\delta_c) \exp(-\delta_c^2/2\sigma_H^2)$ 

2. Nonspherical effects lead to  $\beta_{ani} \approx 0.056 \sigma_H^5$  for  $\sigma_H \ll 1$ 

by Jeans criterion 
$$\delta_c \simeq \left(\frac{3(1+w)}{5+3w}\right) \sin^2 \left[\frac{\pi\sqrt{w}}{1+3w}\right]$$
 for  $p = w\rho$ , so the dust limit  $w \to 0$  gives  $\delta_c \to 0$ , i.e., slightly overdense would collapse to BH

Zel'dovich approximation (1970) describes the nonlinear evolution of density perturbation + hoop conjecture for formation of BH horizon (Thorne, 1972)

#### <u>Harada etc.,PRD, 2016</u>

3. inhomogeneous effects yields a further suppression by  $\beta_{inhom} \sim 3.7 \sigma_H^{3/2}$ 

#### PBH formation in the dust-like era (for spin) PBH born in the MD era tends to develop maximal $\tilde{a} \equiv J/M_{BH} \rightarrow 1!$ T. Harada, etc., PRD 2017

PBH born in the RD era usually has  $\tilde{a} \ll 1$ , but Hawking radiation may lead to sizable  $\tilde{a}$ 

 accretion of nonrotating matter may fast reduce ã
Eloy de Jong, etc, 2023

So, a short MD era⇒ monochromatic spectrum

It picks the perturbation scale that collapse at the end of MD, with  $\sigma_{end} \simeq 0.08 f_{PBH}^{4/29} \left(\frac{M_{\rm BH}}{M}\right)^{2/29}$ 





# I PBH meets a light bosonic field

### Penrose process (1969)

#### Extracting rotating kinematic from rotating Kerr BH



\* negative energy orbital within the ergosphere allows an incoming particle with energy E to escape carrying larger energy:  $E_+ = E - E_- > E$ 

Both BH's angular & mass are reduced by eating the negative energy particle

### More realistic: Zeldovich superradiance (1971)

Extracting rotating energy from rotating BH via wave



If the surface rotates fast enough, the frequency of the incoming wave will change from positive to negative, obtaining energy from the rotating surface.

The underlying principle is the rotational Doppler effect

 superradiant instability and black bomb Press and Teukolsky, 1972
quasi-normal bound states given a light massive bosonic wave
Th Damour, etc., 1976

test using the sound wave (2013) . A test using EM wave? 10<sup>9</sup>rotation/*s*!

Reviewed by R. Brito and P. Pani

#### BH-boson gravity system ~ nucleus-electron EM system



Black denotes the Kerr BH with gravitational/horizon scale  $r_{BH} = \frac{M_{BH}}{M_{PL}^2}$ 

1. A strong gravitational field around BH may bound a light particle

2. According to QM, fermions are limited by the Pauli exclusion principle and do not form macroscopic clouds, while bosons can form BEC and accumulate in clouds

3. Bohr radius of the BH-boson system should exceed BH horizon:  $r_B = \frac{\lambda_S}{\alpha} \sim \frac{r_{BH}}{\alpha^2} \gg r_{BH} \Rightarrow \alpha < 1 \Leftrightarrow \mu_S \ll \alpha \frac{M_{PL}^2}{M_{BH}}$  Gravitational fine structure constant  $\alpha = \frac{\mu_S M_{BH}}{M_{PL}^2} \sim \frac{r_{BH}}{\lambda_S}$ 

gravitational potential of BH is Coulomb-like at large distances, thus Hydrogen-like

#### quantify the boson cloud (a generalized BEC)

1. KG equation in the Kerr spacetime:  $(\nabla^2 - \mu_S^2)\Phi = 0$ 

2. Solution:  $\Phi_{\omega lm} = e^{-i\omega t} e^{-im\phi} S_{lm}(\theta) R_{\omega lm}(r) + c.c.$  with  $\omega$  complex

3. quasi-bound states  $|nlm\rangle$  at energy level  $\omega_{R,nlm} = \mu_S(1 - \alpha^2/2n^2)$  with  $n = 0, 1, 2 \dots, l = 0, 1, \dots n - 1$ , with  $m = -l, -l + 1, \dots l$  the boson angular momentum along the BH spin

4.  $\omega_{I,nlm} = \tilde{r}_+ C_{nl} G_{lm} (m \Omega_H - \omega_R) a^{4l+5} > 0$  with  $\Omega_H = \frac{\tilde{a}}{2r_{BH}\tilde{r}_+}$  the BH horizon angular velocity and  $\tilde{r}_+ = 1 + \sqrt{1 - \tilde{a}^2}$  indicates a cloud with growing energy  $\frac{dM_C}{dt} = 2\omega_{I,nlm} M_C$ 

1. superradiant condition:  $\tilde{a} > \frac{4\alpha/m}{4\alpha^2/m^2+1} \&\& \alpha < m/2$  2. The fastest-growing mode is |n = l + 1 = 2, 1 = 1, m = 1

3. Cloud exponential growth time scale  $1/\omega_{I,211} \sim \alpha^{-9} r_{BH}$ 

4. Cloud saturates with  $M_C^{\max} \sim \alpha M_{BH} \tilde{\alpha}$ 

#### Cloud decay via GW emission

1. condensation oscillates as  $\Phi_{\omega lm} \propto \sin \omega_R t$  with angular frequency  $\omega_R \simeq \mu_S$ , thus, emitting GW at frequency  $\mu_S$ 

2. Cloud energy  $M_C$  decays as  $\frac{dM_C}{dt} = -\frac{M_{PL}^2}{M_{BH}^2} \frac{d\tilde{E}}{dt} M_C^2$  with  $\frac{d\tilde{E}}{dt}(\alpha)$ , with solution  $M_C(t) \approx M_C^{max}/(1 + (t - t_0)/\tau_{GW})$ 

 $\frac{dM_C}{dt}$  is also the power of radiated GWs

3. characteristic time for the consumption of the scalar cloud through GW emission is estimated as

 $\tau_{\rm GW} \sim 6 \times 10^{-12} \text{ years} \frac{M_{\rm BH}}{M_{\rm orbit}} \alpha^{-15} \tilde{a}^{-1}.$ 

 $\frac{d\tilde{E}}{dt}$  can be determined numerically, or analytically for small  $\alpha$ 

Huang Yang and Fa Peng Huang. PRD, 2023

Which is much longer than the cloud growing time scale  $\tau_{grow}$ 

$$\tau_{\rm grow} \sim 7 \times 10^{-12} \text{years} \frac{M_{\rm BH}}{M_{\rm solar}} \alpha^{-9} \tilde{a}^{-1},$$

## Part III

Till now, almost all studies are interested in GA with an astrophysical BH core, since PBH was believed to have a small dimensionless spin

SGWB from primordial gravitational atoms (PGA)

#### SGWB from the PGA

1. Initial GW produced at z with uniform frequency  $\frac{\mu_S}{\pi}$ , redshifted to  $f = \frac{\mu_S/\pi}{1+z}$ 

 $\Omega_{\rm GW}(f) = \theta \left(\frac{1}{2} - \alpha\right) \frac{M_{\rm PL}^2}{M_{\rm PU}^3} \frac{\Omega_{\rm DM} f_{\rm PBH}}{|\dot{z}|} \frac{d\tilde{E}}{dt} \int_{4\alpha/(4\alpha^2+1)}^1 P(\tilde{a}) d\tilde{a} \left(M_C^2[t(f)]\right),$ 

 $\frac{dt}{1+\underline{z}} = -df/f\dot{z}$ 

2. isotropically and homogeneously distributed monochromatic PBHs

Random PBH spin direction and GW phase, the GWs emitted by clouds overlap to form an isotropic and homogeneous SGWB

 $P(\tilde{a})$  the PBH spin distribution function, taking two examples

BEC cloud  $M_c^2$  as a giant GW laser, radiating coherent GWs

#### SGWB from the PGA: spectrum shape analysis



1. Shape of GW produced in the RD  $\Omega_{GW}(f) \propto \frac{M_C^2}{|\dot{z}|} \propto \frac{f^3}{(1 + \frac{\tilde{\tau}}{\tau_{GW}} \frac{f^2}{\mu^2})^2}.$ 2.  $f_{peak} \sim \mu_S \sqrt{3\tau_{GW}/\tilde{\tau}}$  with  $\tilde{\tau} \sim 10^{12}$  year Quite different between PGA-0 and -1

#### 3. Rising & falling power

$$_{\rm W} \propto \begin{cases} f^3 \\ f_{\rm peak}^4 f^{-1} \end{cases}$$

$$\begin{split} t_0 < t(f) \lesssim 3\tau_{\rm GW} \\ 3\tau_{\rm GW} \lesssim t(f) < t_{\rm eq}. \end{split}$$

#### SGWB from the PGA: current constraints and prospects



**SGWB from the PGA-0**: new constraints on PBH

> The first taking spin distribution from short MD scenario; while for others we take several fixed spin for demonstration

For  $M_{BH} \sim 1 - 100 M_{sun}$ , which subject to existing relatively weak constraints, the SGWB signal is already able to yield stronger constraints provided the scalar clouds with  $\mu_{s} \sim 10^{-11} - 10^{-13} \text{eV}$ 



 $\tilde{a} = 1$ 

-2

-2

-1

0

-1

 $\tilde{a} = 0.2$ 

0

2

2

\* SGWB from the PGA: constraints & prospects in the  $\mu_S - M_{BH}$  plane, a narrow stripe

> Green: the current bound; yellow: the future prospect. One needs a  $\tilde{a} \gtrsim 0.1$

For the PGA-1, the situation can be significantly improved



Again, spin?? To be continued...

**Untion!!** 

### Conclusion & outlook

Both PBH and light bosonic fields may appear in the early universe, and we proposed the scenario of primordial gravitational atom

How to analytically calculate the PBH spin produced in the MD era including accretion effect ???

Other scenarios of PBH? More solid estimate the SGWB with a general setup, and many other works can be explored