# Gravitational Wave Birefringence in Fuzzy DM and Symmetron Cosmology


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# Content

- Introduction and Motivation
- Brief Review to Earlier work in CS Gravity
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- GW Birefringence in Symmetron Cosmology
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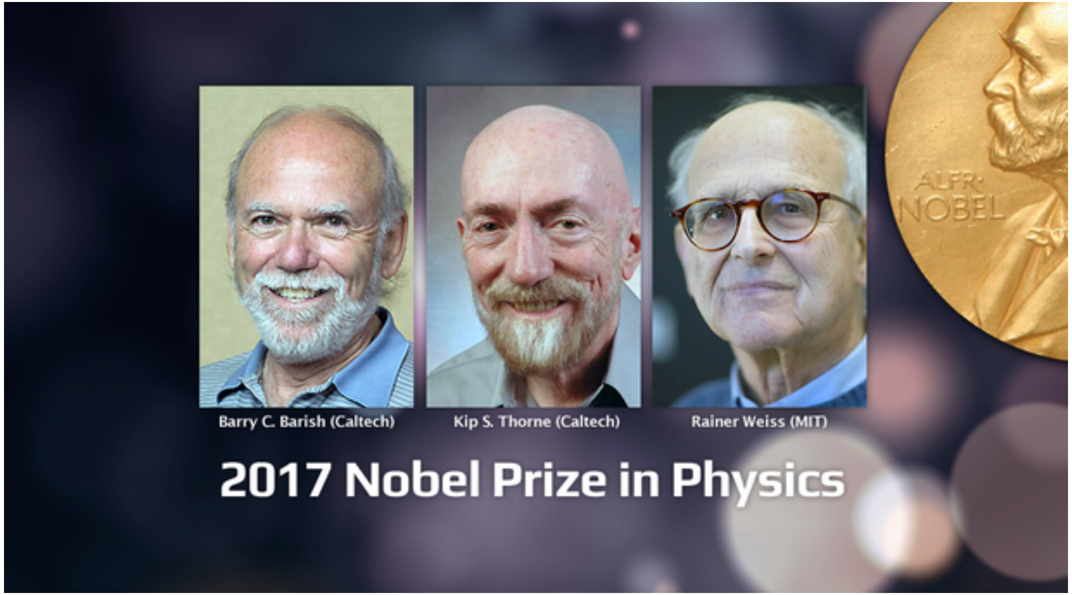
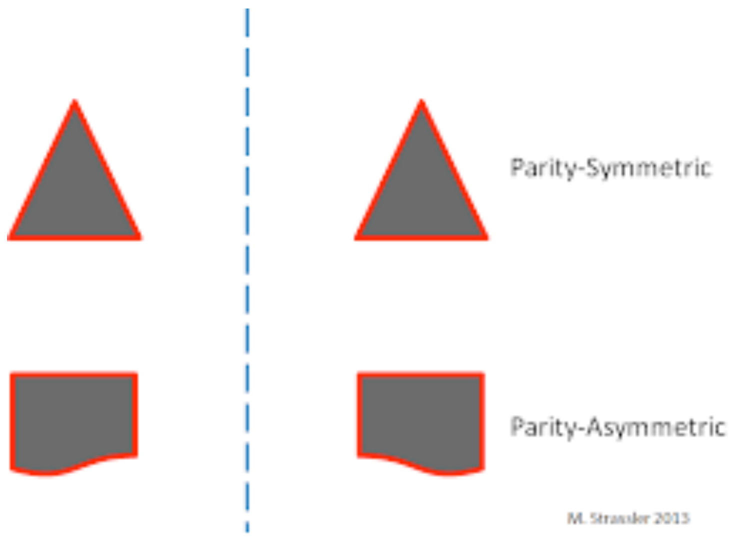
# Introduction

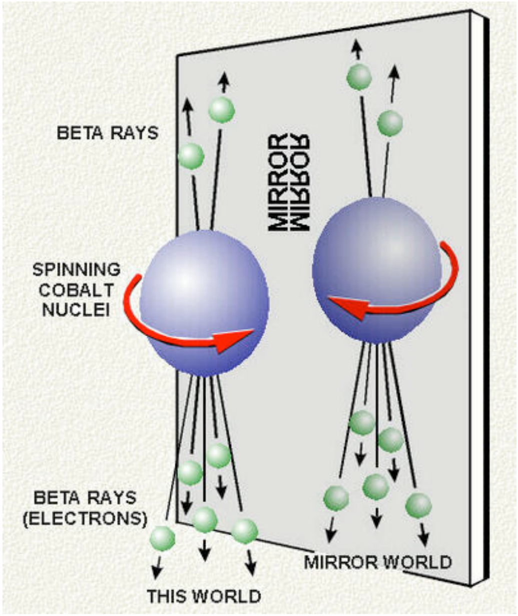
- Testing the nature of gravity is one of the key topics in the modern physics and astronomy.
- The direct detection of gravitational waves (GWs) by LIGO opened a new window to look into this important question.



# Motivation

- It is well-known that there are four fundamental forces in nature, such as electromagnetic, strong, weak and gravitational interactions;
- Also, parity conservation is violated by the weak interaction;



# Motivation

- **Question: Does gravity respect parity conservation?**

- In GR, parity reversal is a good symmetry;

- However, in many modified gravity theories, parity can be violated;

- GWs provide us a new tools to explore this fundamental symmetry in gravity.

**GW Birefringence**

# Benchmark Model: Chern- Simons Modified Gravity

- One typical parity-violating gravity is the Chern-Simons Gravity

Chern-Simons Coupling

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\kappa R + \boxed{\frac{\alpha}{4}\phi\, \tilde{R}^{\tau}{}_{\lambda\mu\nu} R^{\lambda}{}_{\tau}{}^{\mu\nu}}\right],$$

Pontryagin density

$$R\tilde{R} \equiv \frac{1}{2}\tilde{\epsilon}^{\mu\nu\rho\sigma} R^{\tau}{}_{\lambda\mu\nu} R^{\lambda}{}_{\tau\rho\sigma}$$

**Parity-odd**

where $\kappa \equiv (16\pi G)^{-1}$ with $G$ the Newton constant while $\alpha$ denotes the CS coupling with one length dimension.

- **ϕ** is an **axion field**, which can play a role of **dark matter** or **dark energy**;

- Due to the CS coupling, the axion background in the Universe behaves as the birefringence material for GW propagation.

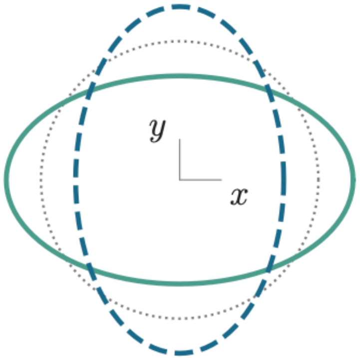
Jackiw & Pi (2003); Alexander & Yunes (2009)

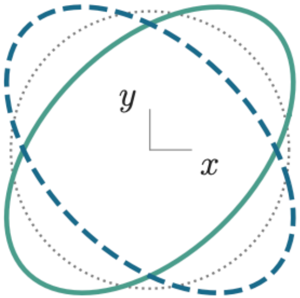
# Two GW Polarization Bases

- Linearly Polarized GWs: plus (+) and cross (×) polarizations

$$(h_{ij}) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

A GW moves along z-direction



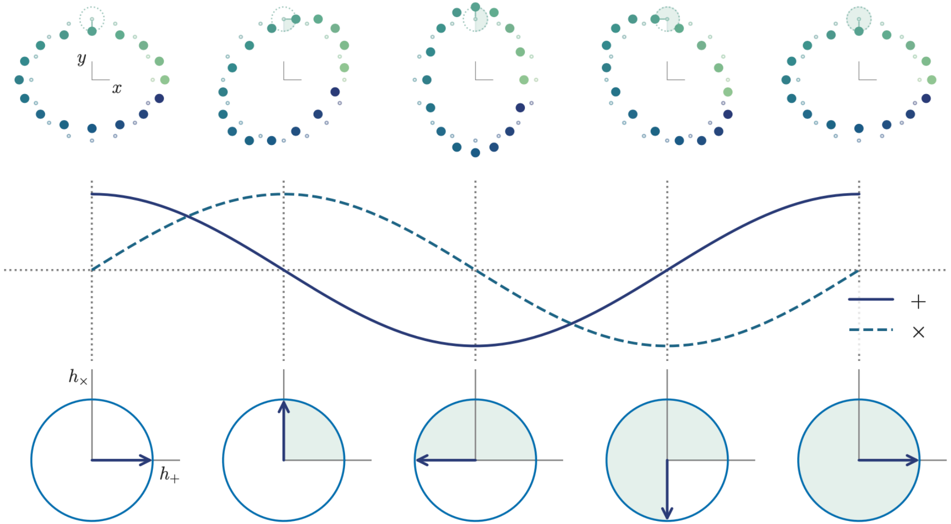



# Two GW Polarization Bases

➢ Circularly Polarized GWs: Left-handed (L) and Right-handed (R)

$$\tilde{h}_{R/L} = \frac{1}{\sqrt{2}}\left(\tilde{h}_+ \mp i\tilde{h}_\times\right),$$

$$h^R_{ij}(t; f_0) = \int_{-\infty}^{+\infty} \tilde{h}^R_{ij}(f)\, e^{-i2\pi f t}\, \mathrm{d}f$$
$$= \frac{1}{\sqrt{2}}\left(e^+_{ij}\cos\omega_0 t + e^\times_{ij}\sin\omega_0 t\right),$$




Right-handed

Credit: M. Isi(2022)

# GW Birefringence in CS Gravity: Earlier Studies

- CS gravity can generate GW birefringence in the axion background: the left- and right-handed circular polarizations propagate differently

- Background: FRW metric + spatially homogeneous scalar field

- Modified GW Equations of Motion

$$\boxed{ds^2 = a(\eta)^2[-d\eta^2 + (\delta_{ij} + h_{ij})]}$$

$$\Box h_{\mathrm{R,L}} = \boxed{-\frac{i\lambda_{\mathrm{R,L}}\alpha}{\kappa a^2}\left[-\frac{1}{a^2}(\phi'' - 2\mathcal{H}\phi')\partial_z h'_{\mathrm{R,L}} + \phi'\Box\partial_z h_{\mathrm{R,L}}\right]} \qquad \boxed{\lambda_{\mathrm{R,L}} = \pm 1}$$

Parity Violation

- Dispersion relation:

$$\boxed{\omega^2 = k^2 - i\omega[2\mathcal{H} + 4\lambda_{\mathrm{R,L}}\alpha\phi' k\mathcal{H}/(\kappa a^2)]\,,}$$

# GW Birefringence in CS Gravity: Earlier Studies

- Dispersion relation:

$$\boxed{\omega^2 = k^2 - i\omega[2\mathcal{H} + 4\lambda_{\mathrm{R,L}}\alpha\phi' k\mathcal{H}/(\kappa a^2)]\,,}$$

- Amplitude Birefringence:

$$\boxed{h^{\mathrm{obs}}_{R,L}(f) = h^{\mathrm{GR}}_{R,L}(f) \times \exp\left(\mp\kappa_A \times \frac{d_c}{\mathrm{Gpc}} \times \frac{f}{100\,\mathrm{Hz}}\right)}$$

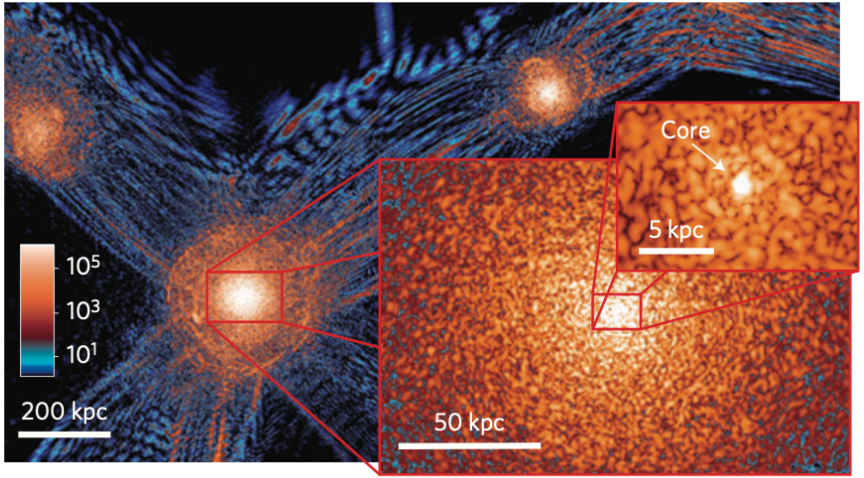
$$\boxed{\kappa_A \equiv 4\pi\alpha\dot{\phi}_0 H_0/\kappa\,,}$$

- We can constrain $\kappa_A$ by modifying the GW waveform template with this birefringence factor and comparing with the observed GW events.
  - GWTC-3 data: $\boxed{\kappa = -0.019^{+0.038}_{-0.029}}$ T.C.K.Ng, et al. (2023)
  - GW170817 (multi-messanger): $\boxed{\kappa_A = -0.12^{+0.60}_{-0.61}}$ M. Lagos, et al. (2024)

# GW Birefringence inside the Milky Way

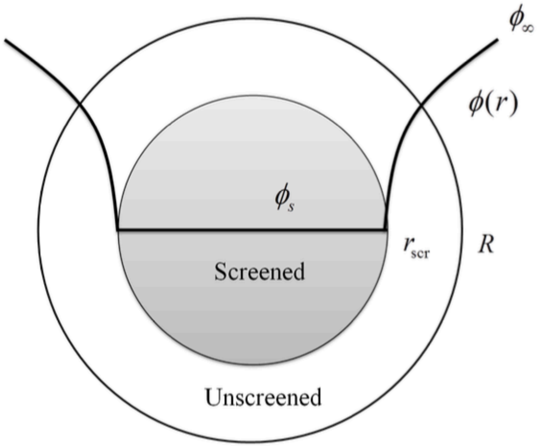
- In the real world, the ALP should have spatial dependence, no matter if it is **dark matter** or **dark energy** candidate.

**Fuzzy DM**




Solve small-scale structure problems

**Symmetron**




Screening to evade fifth force constraint

# Fuzzy Dark Matter: GW Birefringence inside the Milky Way

- GW birefringence in an FDM profile with spatial variations in CS gravity

$$\Box h_{\mathrm{R,L}} \mp i(\alpha/\kappa)\partial^{\alpha}\left[\partial_z\phi\partial_\alpha\partial_t - \partial_t\phi\partial_\alpha\partial_z\right] h_{\mathrm{R,L}} = 0\,.$$

- Since we are mostly interested in effects around the galactic scale, we can ignore the cosmological expansion, so that we take the flat metric.

- Further assume GW wavelength are much smaller than the variation of the FDM background profile, $\partial_{t,z}\phi \ll \omega, k.$ we can apply the **Eikonal Approximation** to perform our calculation.

# Eikonal Approximation

- GW waveform: $h_{\mathrm{R,L}} = h^0_{\mathrm{R,L}} e^{iS}$ , where the dominant evolution comes from the phase S, while $h^0_{\mathrm{R,L}}$ is slowly varying.

$$\omega = -\partial_t S\,, \quad k = \partial_z S$$

- Dispersion relations:

$$D^{\pm} = \underbrace{(\omega^2 - k^2)\left[1 \mp \frac{\alpha}{\kappa}(\omega\partial_z\phi + k\partial_t\phi)\right]}_{} \mp \underbrace{\frac{i\alpha}{\kappa}\left[(\omega^2 + k^2)\partial_t\partial_z\phi + \omega k(\partial_z^2\phi + \partial_t^2\phi)\right]}_{} = 0$$

$\boxed{\text{Real part}} \gg \boxed{\text{Imaginary part}}$ $\quad \boxed{\partial_{t,z}\phi \ll \omega, k.}$

- Real part: dispersion ➡ Velocity birefringence
- Imaginary part: dissipation ➡ Amplitude birefringence

# Eikonal Approximation: Dispersion

- Leading-order dispersion relation:

$$D^{\pm}=(\omega^2-k^2)\left[1\mp(\alpha/\kappa)(\omega\partial_z\phi+k\partial_t\phi)\right]=0\,.$$

- w=k is always the solution, which means that both polarizations of GWs move in the speed of light even there is an axion background.
- We can check this by examining the GW paths:

$$\frac{dx^i}{dt}=-\frac{\partial D^{\pm}/\partial k_i}{\partial D^{\pm}/\partial\omega}=\frac{2k\left[1\mp(\alpha/\kappa)(\omega\partial_z\phi+k\partial_t\phi)\right]\pm(\alpha/\kappa)(\omega^2-k^2)\partial_t\phi}{2\omega\left[1\mp(\alpha/\kappa)(\omega\partial_z\phi+k\partial_t\phi)\right]\mp(\alpha/\kappa)(\omega^2-k^2)\partial_z\phi}\delta^i_z\approx\delta^i_z\,,$$

$$\frac{dk_i}{dt}=\frac{\partial D^{\pm}/\partial x^i}{\partial D^{\pm}/\partial\omega}=\frac{\mp(\alpha/\kappa)(\omega^2-k^2)(\omega\partial_z\partial_i\phi+k\partial_i\partial_t\phi)}{2\omega\left[1\mp(\alpha/\kappa)(\omega\partial_z\phi+k\partial_t\phi)\right]\mp(\alpha/\kappa)(\omega^2-k^2)\partial_z\phi}=0\,,$$

$$\frac{d\omega}{dt}=-\frac{\partial D^{\pm}/\partial t}{\partial D^{\pm}/\partial\omega}=\frac{\pm(\alpha/\kappa)(\omega^2-k^2)(\omega\partial_z\partial_t\phi+k\partial_t^2\phi)}{2\omega\left[1\mp(\alpha/\kappa)(\omega\partial_z\phi+k\partial_t\phi)\right]\mp(\alpha/\kappa)(\omega^2-k^2)\partial_z\phi}=0\,,$$

The frequency and wavenumber of GW keep the same, and GWs still travel along a straight line for both polarizations.

# Eikonal Approximation: Dissipation

- Dissipation effects:

$$D^{\pm} = (\omega^2 - k^2)\left[1 \mp \frac{\alpha}{\kappa}(\omega\partial_z\phi + k\partial_t\phi)\right] \mp \frac{i\alpha}{\kappa}\left[(\omega^2 + k^2)\partial_t\partial_z\phi + \omega k(\partial_z^2\phi + \partial_t^2\phi)\right] = 0$$

- Here w and k can take complex values

$$\begin{aligned}
\frac{dx^i}{dt} &= -\frac{\partial D^{\pm}/\partial k_i}{\partial D^{\pm}/\partial\omega} = \frac{k\delta_z^i\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \pm i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (\omega/k)(\partial_z^2\phi + \partial_t^2\phi)]\right\}}{\omega\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \mp i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (k/\omega)(\partial_z^2\phi + \partial_t^2\phi)]\right\}} \\
&\approx \left\{1 \pm i(\alpha/\kappa)[2\partial_t\partial_z\phi + (\partial_t^2\phi + \partial_z^2\phi)]\right\}\delta_z^i\,, \\
\frac{dk_i}{dt} &= \frac{\partial D^{\pm}/\partial x^i}{\partial D^{\pm}/\partial\omega} = \frac{\mp i(\alpha/\kappa)\left[(\omega^2 + k^2)\partial_t\partial_z\partial_i\phi + \omega k\partial_i(\partial_z\phi + \partial_t^2\phi)\right]}{2\omega\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \mp i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (k/\omega)(\partial_z^2\phi + \partial_t^2\phi)]\right\}} \\
&\approx \mp i(\alpha\omega/2\kappa)\left[2\partial_t\partial_z\partial_i\phi + \partial_t^2\partial_i\phi + \partial_z^2\partial_i\phi\right]\,, \\
\frac{d\omega}{dt} &= -\frac{\partial D^{\pm}/\partial t}{\partial D^{\pm}/\partial\omega} = \frac{\pm i(\alpha/\kappa)\left[(\omega^2 + k^2)\partial_t^2\partial_z\phi + \omega k(\partial_t^3\phi + \partial_t\partial_z^2\phi)\right]}{2\omega\left\{1 \mp (\alpha/\kappa)(\omega\partial_z\phi + k\partial_t\phi) \mp i(\alpha/2\kappa)[2\partial_t\partial_z\phi + (k/\omega)(\partial_z^2\phi + \partial_t^2\phi)]\right\}} \\
&\approx \pm i(\alpha\omega/2\kappa)\left[2\partial_t^2\partial_z\phi + \partial_t\partial_z^2\phi + \partial_t^3\phi\right]\,,
\end{aligned} \tag{36}$$

# Eikonal Approximation: Dissipation

- Define
$$\dot{\phi} \equiv \frac{d\phi}{dt} = \partial_t \phi + \partial_z \phi \frac{dz}{dt} \approx \partial_t \phi + \partial_z \phi \,,$$

- Integration of above equations gives:
$$\boxed{\begin{aligned} \Delta k_i &= \mp i \alpha \omega \partial_i \dot{\phi}/(2\kappa) \,, \\ \Delta \omega &= \pm i \alpha \omega \partial_t \dot{\phi}/(2\kappa) \,. \end{aligned}}$$

- The phase of the GW varies:
$$\Delta S = -\int_{t_e}^{t_o} dt \Delta\omega + \int_{\mathbf{x}_e}^{\mathbf{x}_o} dx^i \Delta k_i = \mp i \alpha \omega (\dot{\phi}_o - \dot{\phi}_e)/(2\kappa) \,,$$

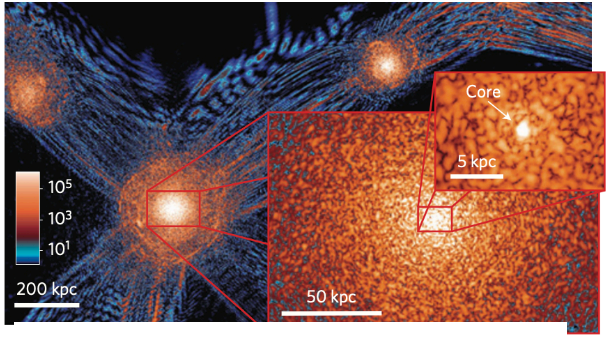
- GW amplitude birefringence:
$$\boxed{h_{\mathrm{R,L}} = h^0_{\mathrm{R,L}} \exp(i\Delta S) = h^0_{\mathrm{R,L}} \exp\left( \pm \alpha \omega (\dot{\phi}_o - \dot{\phi}_e)/(2\kappa) \right) \,,}$$

# Fuzzy Dark Matter: GW Birefringence inside the Milky Way

- Locally, the fuzzy DM profile is given by

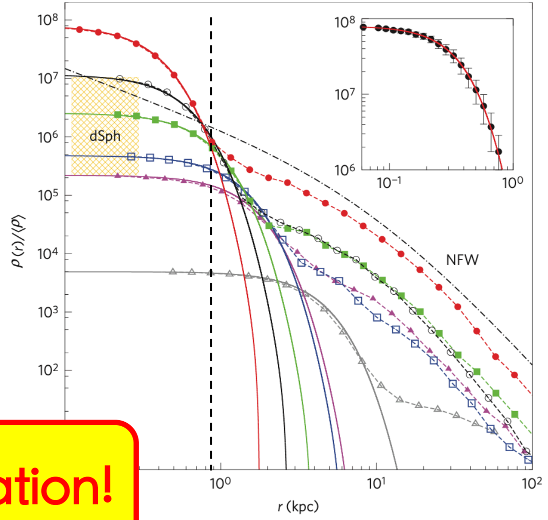
$$\phi(t,\mathbf{x}) = \frac{\sqrt{2\rho_{\mathrm{NFW}}}}{m_\phi}\cos\left(m_\phi t + \alpha(\mathbf{x})\right)\,.$$

A. Khmelnitsky & V. Rubakov (2014)




- Due to $\partial_t\boldsymbol{\phi} \gg \partial_z\boldsymbol{\phi}$ for $m_{\boldsymbol{\phi}}$ ~$10^{-22}$ eV

$$\dot{\phi}_o \approx \partial_t\phi = \sqrt{2\rho_{\mathrm{NFW}}}\cos\left(m_\phi t + \alpha(\mathbf{x})\right)$$




- The magnitude birefringence can be given by

$$\boxed{h^{\mathrm{obs}}_{R,L}(f) = h^{\mathrm{GR}}_{R,L}(f)\times\exp\left(\pm\kappa'_A\times\frac{f}{100\ \mathrm{Hz}}\right)}$$

$$\boxed{\kappa'_A \equiv \pi(\alpha/\kappa)\sqrt{2\rho_\odot}\,\sin(m_\phi t+\alpha_0)\,.}$$

**Time Modulation!**

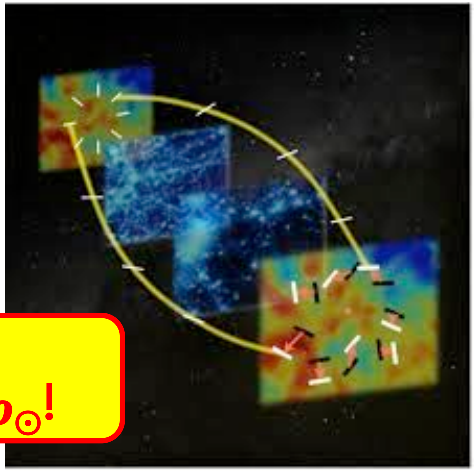
Z.-X. Xiong & **DH**: 2406.13394

# Fuzzy Dark Matter: Extra-Galactic GW Birefringence

- FDM Field in Cosmology: $$\boxed{\phi(t) = \phi_0 \left(1/a\right)^{3/2} \cos(m_\phi t + \alpha_c)\,,}$$ $$\boxed{\phi_0 = \sqrt{2\rho_{\rm DM}}/m_\phi}$$
- The field amplitude and phase should change at different spatial points, with the typical scale as the inverse of de Broglie wavelength $m_\phi v$. For $v \ll 1$, such variations can be ignored compared with $m_\phi$.
- GW propagation: $$\Box h_{\rm R,L} = \pm \frac{i\alpha}{\kappa a^2}\left[\frac{1}{a^2}(\phi'' - 2\mathcal{H}\phi')\partial_z h'_{\rm R,L} - \phi'\Box\partial_z h_{\rm R,L}\right]\,,$$
- GW birefringence:

$$\boxed{h^{\rm ex}_{\rm R,L}(f) = h^0_{\rm R,L}(f)\exp\left(\pm\frac{\kappa^{\rm ex}_A}{1\ {\rm Gpc}} \times \frac{f}{100\ {\rm Hz}}\right)\,,}$$

$$\boxed{\kappa^{\rm ex}_A \equiv \alpha\pi\sqrt{2\rho_{\rm DM}}\sin(m_\phi t + \alpha_c)/\kappa\,.}$$



Ignored for $\rho_{\rm DM} \ll \rho_\odot$!

Z.-X. Xiong & **DH**: 2406.13394

# Introduction to Symmetron

- **Symmetron** can be a **dark energy** candidate, which can avoid the strong fifth force constraints in the solar system by screening the light scalar field.

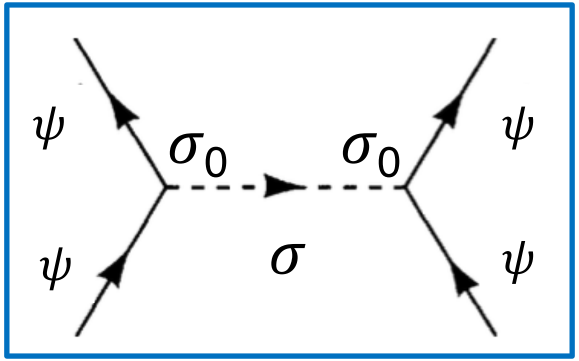
- Symmetron Scalar $\sigma$ + $Z_2$ Symmetry: $\sigma \to -\sigma$

$$S_0 = \int d^4x\sqrt{-g}\left[\kappa R - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma)\right] + \int d^4x\sqrt{-\tilde{g}}\mathcal{L}_{\rm m}(\psi, \tilde{g}_{\mu\nu})$$

- To guarantee the Weak Equivalence Principle, the matter fields $\psi$ should couple universally to the Jordan-frame metric $\tilde{g}_{\mu\nu} \equiv A^2(\sigma)g_{\mu\nu}$

- Due to the $Z_2$ symmetry, the coupling function $A(\sigma)$ is

$$\boxed{A(\sigma) = 1 + \frac{\sigma^2}{2M^2}\,,}$$



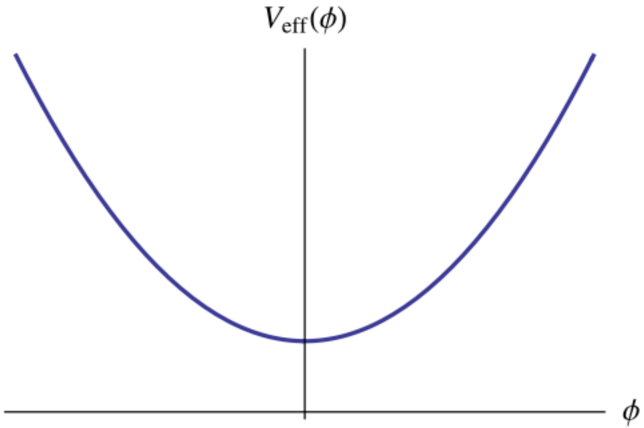

➡ The symmetron-matter coupling $\propto$ VEV of $\sigma$

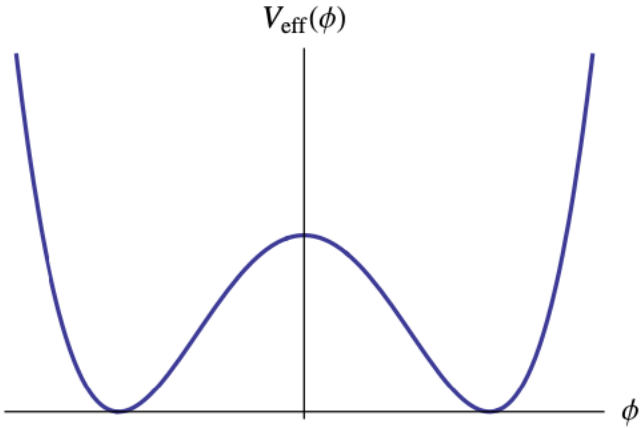
K.Hinterbichler & J. Khoury, et al. (2010,2011)

# Introduction to Symmetron

- Symmetron Effective Potential $V_{\text{eff}}(\sigma) = V(\sigma) + \sum_i A^{1-3w_i}(\sigma)\rho_i$

$$V(\sigma) = -\frac{1}{2}\mu^2\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{\mu^4}{4\lambda} = \frac{\lambda}{4}\left(\sigma^2 - \frac{\mu^2}{\lambda}\right)^2$$




High density → $\sigma_0=0$ → suppressed scalar force

Low density → $\sigma_0 \neq 0$ → scalar force is effective

K. Hinterbichler & J. Khoury, et al. (2010,2011)

# Symmetron: GW Birefringence inside the Milky Way

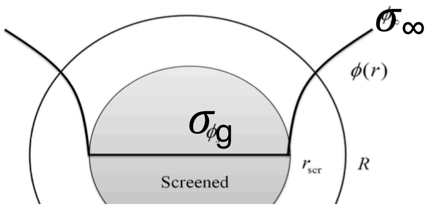
$Z_2$ Symmetry → $$S_{\rm CS} = \int dx^4 \, \sqrt{-g} (\alpha/4) \sigma^2 R^{\tau}{}_{\lambda\mu\nu} \widetilde{R}^{\lambda}{}_{\tau}{}^{\mu\nu}$$

- Different from original CS gravity, this new term explicitly breaks parity
- Directly apply the results in the conventional CS gravity by $\phi \rightarrow \sigma^2$.
- Amplitude Birefringence by eikonal approximation

$$h_{\rm R,L}(f) = h^{\rm GR}_{\rm R,L}(f) \exp\left( i \int_{\mathbf{x}_{\rm out}}^{\mathbf{x}_{\rm in}} \Delta k_i dx^i \right) = h^{\rm GR}_{\rm R,L}(f) \exp\left( \lambda_{\rm R,L} \kappa'_A \frac{f}{100{\rm Hz}} \right)$$

where $$\kappa'_A \equiv (2\pi\alpha/\kappa)(\sigma_{\rm in}\partial_z\sigma_{\rm in} - \sigma_{\rm out}\partial_z\sigma_{\rm out})$$

Screening → Suppress GWB!

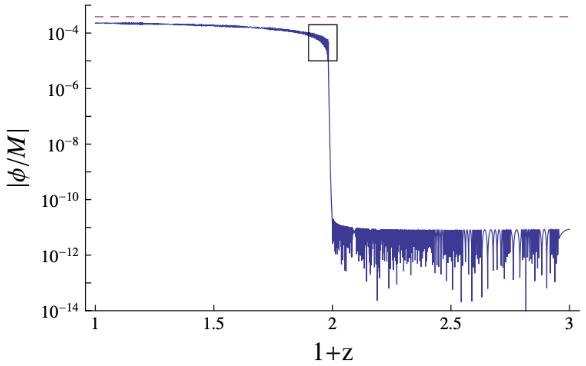



$$\partial_z \sigma_{in} \approx \partial_z \sigma_{out} \approx 0$$

# Symmetron: Extra-Galactic GW Birefringence

- Cosmological Symmetron Profile: adiabatic solution to its effective potential

$$V_{\text{eff}} = V(\sigma) + A(\sigma)\rho_m + A^4(\sigma)\rho_{\text{de}}$$
$$= V(\sigma) + \rho_c\left[A(\sigma)\Omega_m a^{-3} + A^4(\sigma)(1-\Omega_m)\right]$$




$$\boxed{\sigma^2 = \frac{1}{\lambda}\left[\mu^2 - \frac{\rho_c(\Omega_m a^{-3} + 4\Omega_\Lambda)}{M^2}\right]}$$

$$\Box h_{\rm R,L} = -\frac{i\lambda_{\rm R,L}\alpha}{\kappa a^2}\left[-\frac{1}{a^2}\left[(\sigma^2)'' - 2\mathcal{H}(\sigma^2)'\right]\partial_z h'_{\rm R,L} + (\sigma^2)'\Box\partial_z h_{\rm R,L}\right] \quad \boxed{\kappa_A \equiv \frac{72\pi\alpha\Omega_m H_0^4}{\lambda M^2}}$$

- Amplitude Birefringence: $h_{\rm R,L}(f) = h^{\rm GR}_{\rm R,L}e^{i\Delta S_c} \approx h^{\rm GR}_{\rm R,L}\exp\left(-\lambda_{\rm R,L}\kappa_A\frac{f}{100\ \text{Hz}}\frac{d_c}{1\ \text{Gpc}}\right)$
  - GWTC-3 data: T.C.K.Ng, et al. (2023)

$$\kappa = -0.019^{+0.038}_{-0.029} \quad \Rightarrow \quad \alpha(M/M_{\rm Pl})^4 \lesssim 8\times 10^{-22}\ \text{Gpc}^2$$

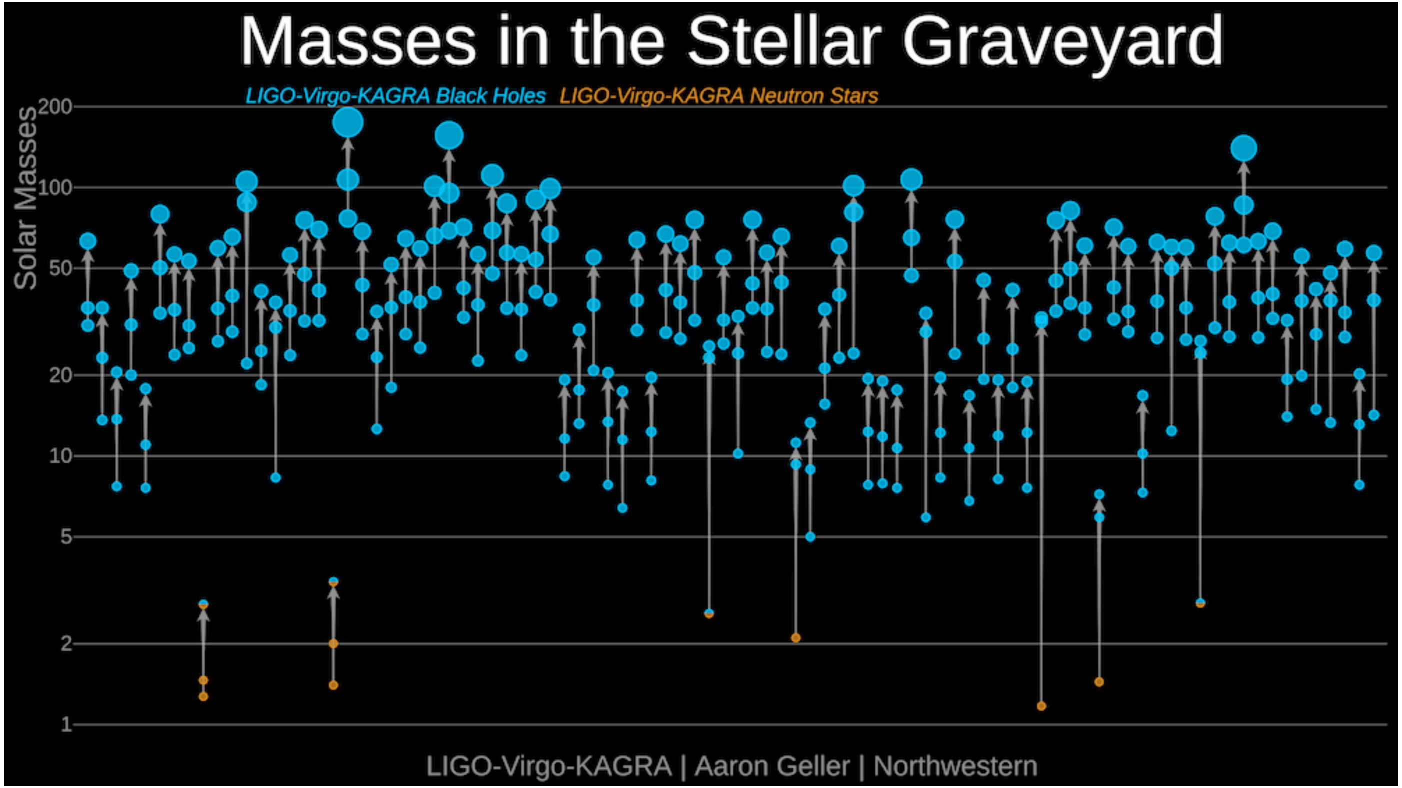
# Conclusions

- Observation of **GW birefringence** is a remarkable way to test parity violation in gravity;
- We have studied the GW birefringence over a nontrivial spatial distribution of the fuzzy DM and symmetron in the Milky Way ;
- It is found that both GW circular polarizations moves with the speed of light in both models, while their relative amplitudes would be changed, generating the amplitude  birefringence!
- For the fuzzy DM, its galactic distribution produces the dominant effect, which shows a remarkable time modulation.
- In the symmetron model,  we introduce a new $Z_2$-symmetric CS-like interaction, which generates the GWB. It is interesting to note that the galactic contribution is suppressed due to its screening mechanism.

THANK YOU !

# Introduction

- Up to O3 run, the LVK Collaboration observed 90 GW events.



# Directional Dependence of GW Birefringence in the MW

- Subdominant Spatial Dependence: $\partial_z\boldsymbol{\phi} \ll \partial_t\boldsymbol{\phi}$ for $m_{\boldsymbol{\phi}}$ ~$10^{-22}$ eV

$$\boxed{\kappa'_A = \pi(\alpha/\kappa)\partial_r\phi(R_\odot)\cos\langle\mathbf{k},\mathbf{r}\rangle\,.}$$

where $\langle\mathbf{k},\mathbf{r}\rangle$ represents the angle between the incident GW and the radial direction of the Solar system in the Milky Way. The spatial variation of the Solar system r is given by

$$\partial_r\phi(R_\odot) = -\frac{1}{m_\phi R_\odot}\sqrt{\frac{\rho_\odot}{2}}\left(\frac{1+3R_\odot/r_s}{1+R_\odot/r_s}\right)\cos(m_\phi t+\alpha_0)\,, \tag{57}$$

- For $m_{\boldsymbol{\phi}}$ ~$10^{-22}$ eV & $R_\odot$ = 8 kpc

$$\frac{\partial_z\phi}{\partial_t\phi} \sim \frac{1}{m_\phi R_\odot} \sim \mathcal{O}(10^{-5})\,.$$