中国科学院高能物理研究所
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Simulations of Fuzzy Dark Matter

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Outline

- **Theoretical background of Fuzzy Dark Matter (FDM)**
- **Self-consistent simulation of a FDM+stars system**
  - Initial condition
  - Evolution of the system
- **Simulation of tidal effect**
- **Summary**

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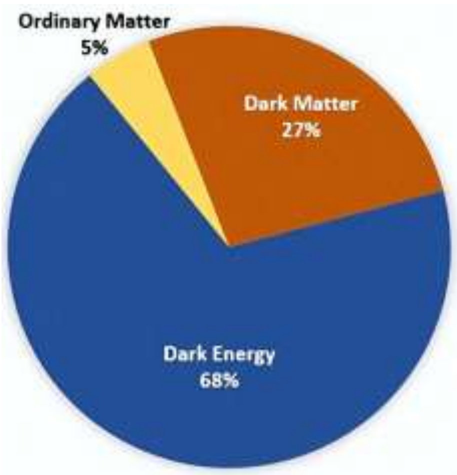
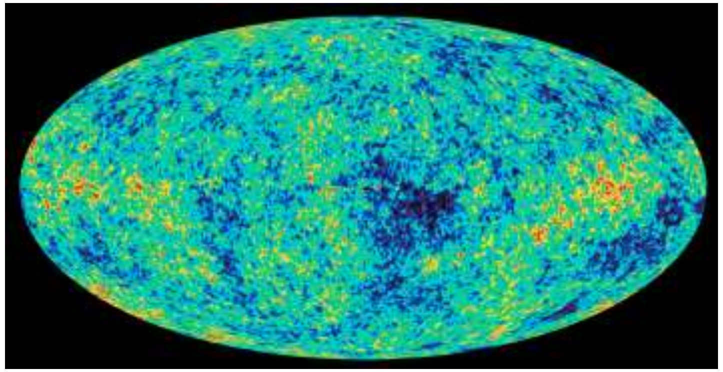
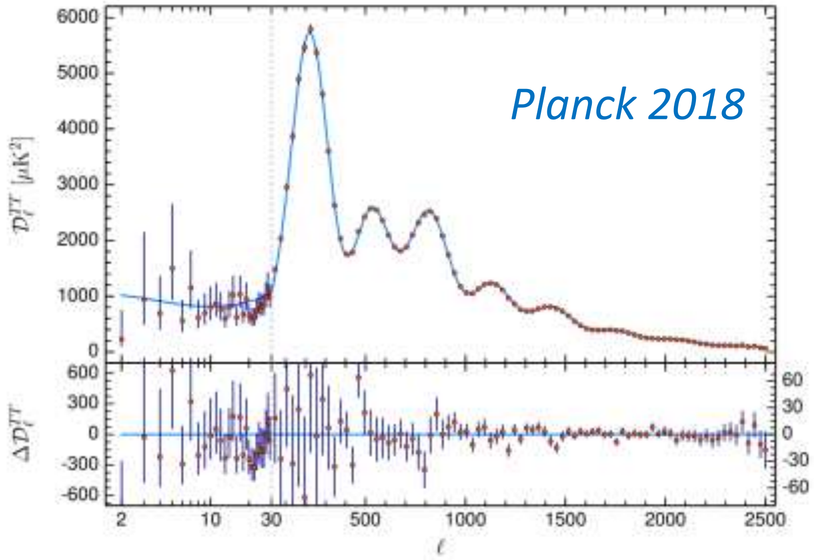
Theoretical background of Fuzzy Dark Matter (FDM)

ΛCDM model

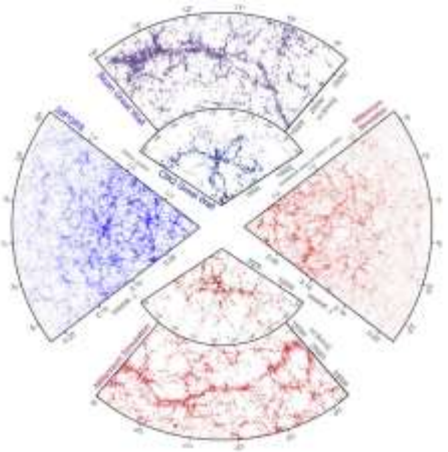


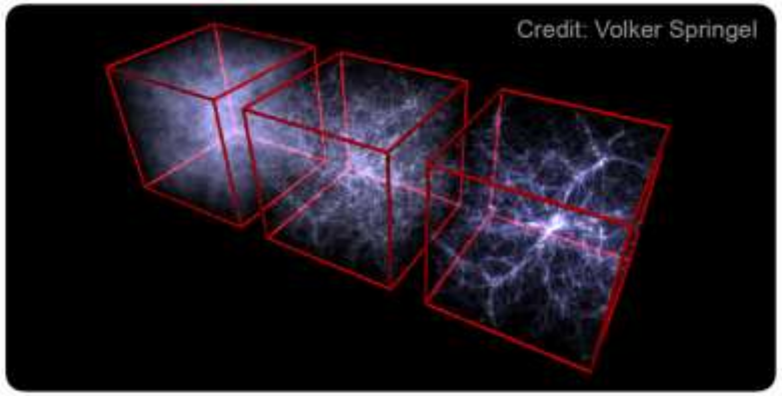
- Cosmological background radiation

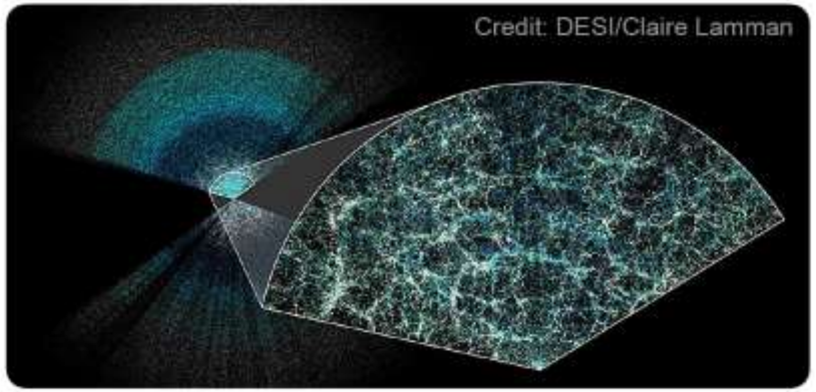




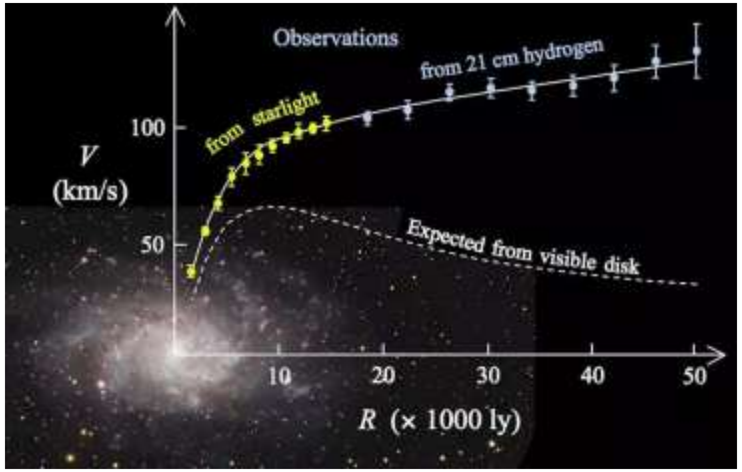
- Large-scale structure





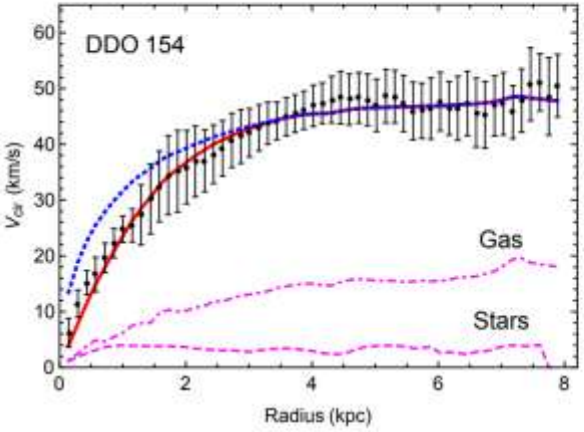


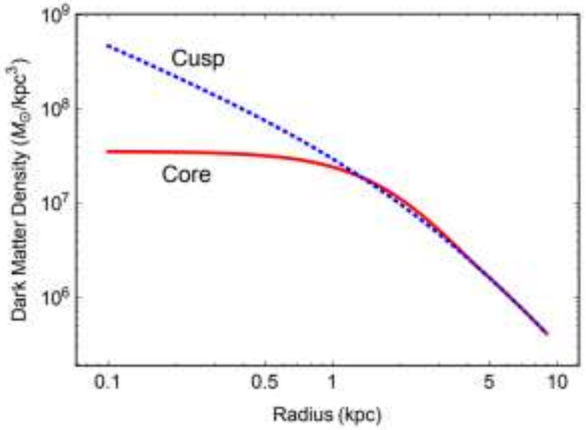
Theoretical background of Fuzzy Dark Matter (FDM)



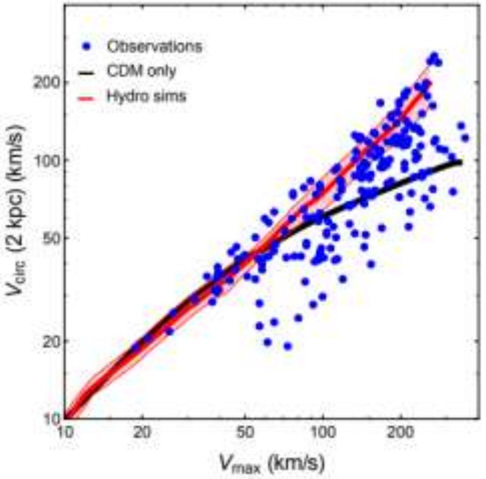
- Core-cusp problem

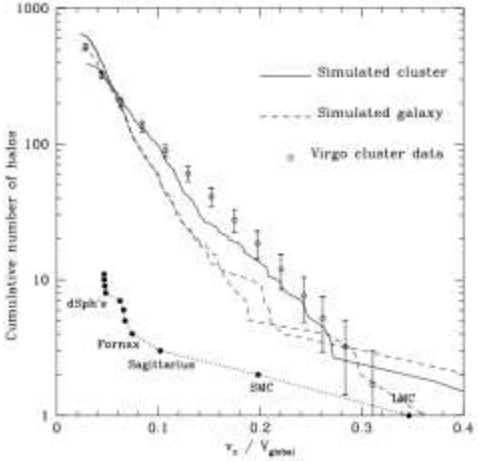
Tulin & Yu. Physics Reports 730 (2018)





- Diversity problem
- Missing satellites problem





- Too-big-to-fail problem, dark matter deficient galaxy ……

Theoretical background of Fuzzy Dark Matter (FDM)

- Fuzzy Dark Matter (FDM) *Hu, Rennan, and Gruzinov. PRL. 2000 Aug 7;85(6):1158.*
  - Equation of motion

Klein-Gordon equation

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi\right) - \sqrt{-g} m^2 \phi = 0$$

non-relativistic limit

$$\phi = \frac{1}{\sqrt{2m}} \left(\psi e^{-imt} + \psi^\star e^{imt}\right)$$

Schrödinger-Poisson (SP) equation

$$i\partial_t \psi = -\frac{\nabla^2}{2m}\psi + m\Phi\psi,$$

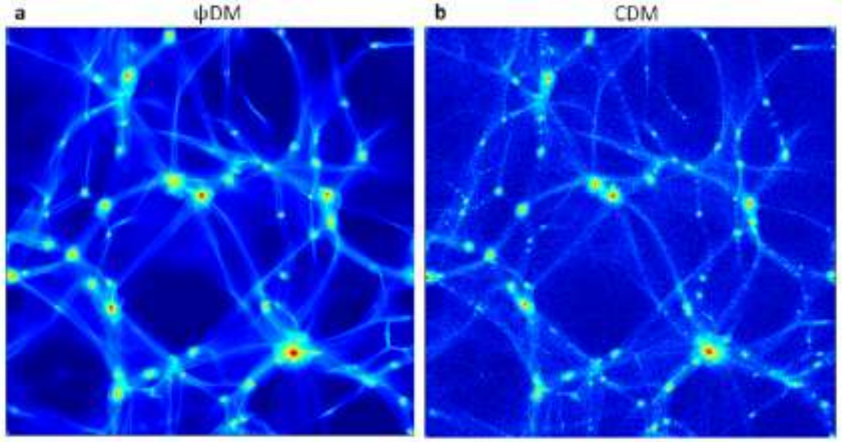
$$\nabla^2 \Phi = 4\pi G\rho, \quad \rho = m\,|\psi|^2 .$$

- de Broglie wavelength

$$\lambda_{\rm dB} \equiv \frac{2\pi}{mv} = 0.48\,{\rm kpc}\left(\frac{10^{-22}\ {\rm eV}}{m}\right)\left(\frac{250\,{\rm km\,s^{-1}}}{v}\right)$$

$$m \simeq 10^{-22} {\rm eV}$$

- Cosmological simulation

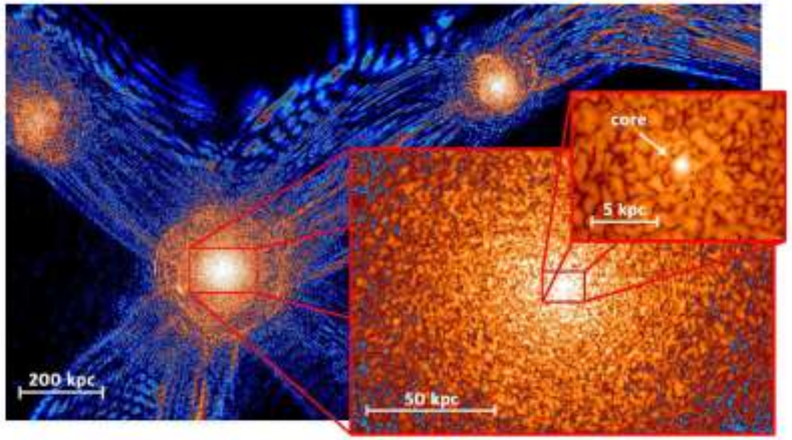


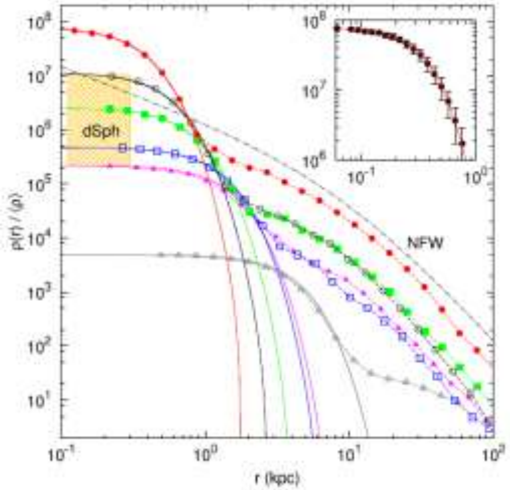
Schive et al. Nature Phys 10, 496–499 (2014).

Theoretical background of Fuzzy Dark Matter (FDM)

- FDM halo structure

Schive et al. Nature Phys 10, 496–499 (2014).



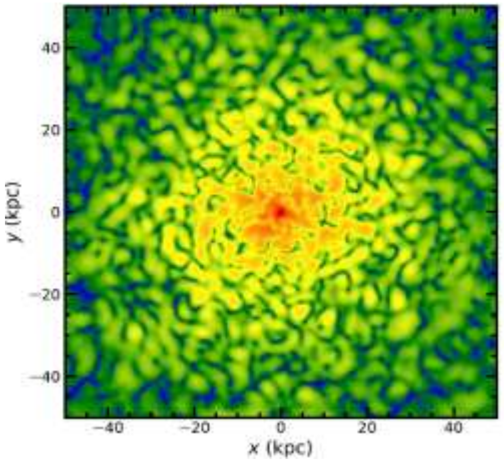


$$\rho_{\rm sol}(r) = \frac{\rho_c}{[1+0.091(r/r_c)^2]^8},$$

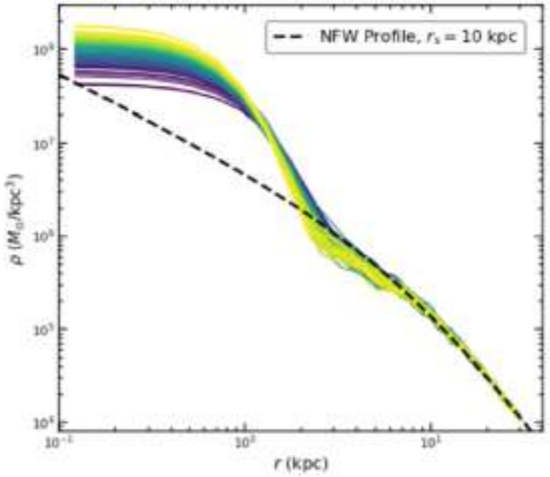
$$\rho_c = 1.95\times10^7 M_\odot {\rm kpc}^{-3}\left(\frac{m}{10^{-22}{\rm eV}}\right)^{-2}\left(\frac{r_c}{\rm kpc}\right)^{-4}$$

Core-cusp problem

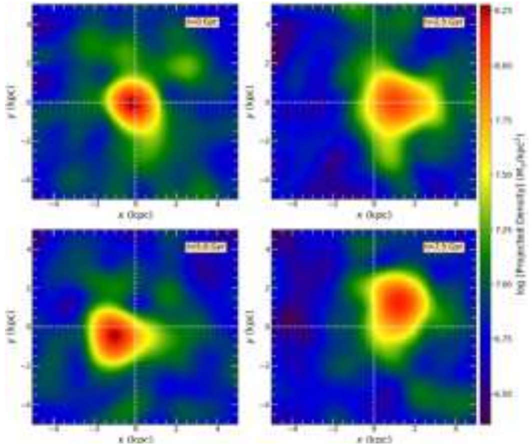
Interference



granules



soliton oscillation



soliton random walk

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Self-consistent simulation of a FDM+stars system

t $t+\Delta t$

virialised galaxy

FDM: $\psi(t, \boldsymbol{x})$ $\Downarrow$ $\Phi(t, \boldsymbol{x})$ $\Uparrow$ stars: $\{\boldsymbol{x}_i(t), \boldsymbol{v}_i(t)\}$ $i = 1, \ldots, N$

Schrödinger equation

Newton's second law

FDM: $\psi(t+\Delta t, \boldsymbol{x})$ $\Downarrow$ $\Phi(t+\Delta t, \boldsymbol{x})$ $\Uparrow$ stars: $\{\boldsymbol{x}_i(t+\Delta t), \boldsymbol{v}_i(t+\Delta t)\}$ $i = 1, \ldots, N$

Initial condition

- Construction of a FDM halo wave function

T.D. Yavetz, X Li, and L Hui, Phys.Rev.D 105 (2022) 2, 023512

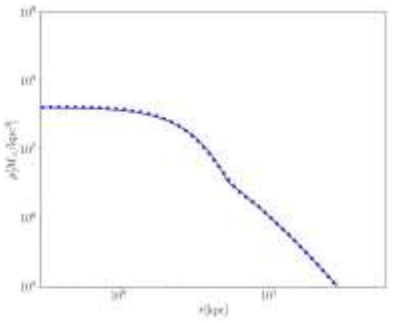
Soliton+NFW profile

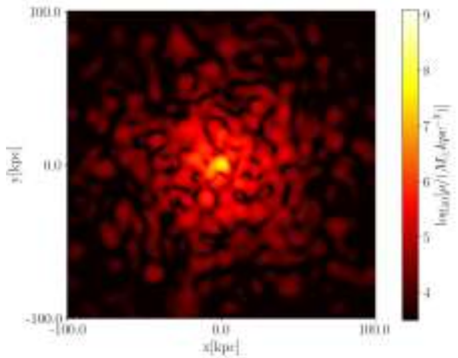
$$\rho_{\rm in}(r) = \begin{cases} \dfrac{\rho_c}{[1+0.091(r/r_c)^2]^8}, & r < kr_c \\ \dfrac{\rho_s}{(r/r_s)\,(1+r/r_s)^2}, & r \geq kr_c, \end{cases}$$

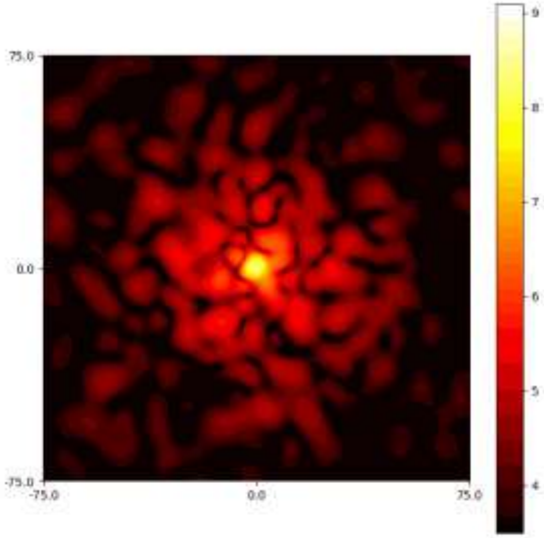
$$\psi(0,\mathbf{x}) = \sum_{nlm} |a_{nl}| e^{i\phi_{nlm}} \Psi_{nlm}(\mathbf{x}),$$

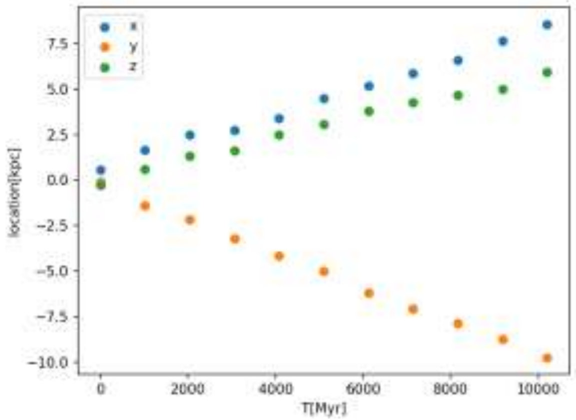
$$-\frac{\hbar^2}{2m_a}\nabla^2\Psi_{nlm}(\mathbf{x}) + m_a\Phi_{\rm in}(r)\Psi_{nlm}(\mathbf{x}) = E_{nl}\Psi_{nlm}(\mathbf{x}),$$

$$\rho_{\rm out}(r) = \frac{m_a}{4\pi}\sum_{nl}(2l+1)|a_{nl}|^2 R_{nl}^2(r).$$







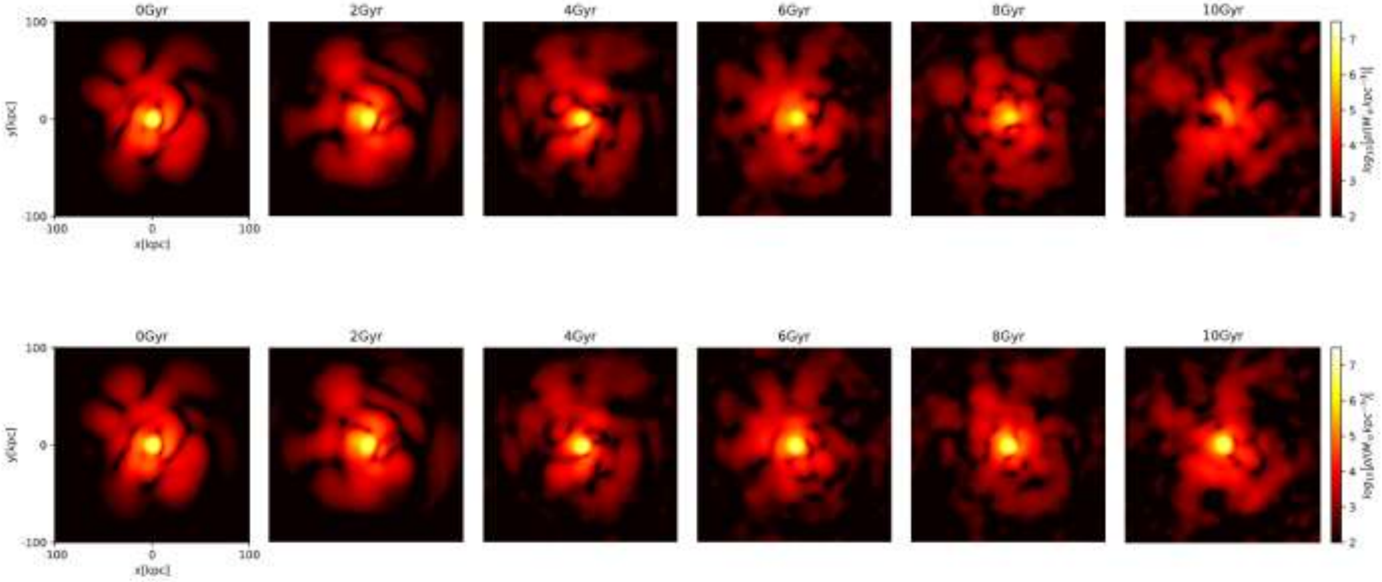


$$P_i = \frac{1}{2}\sum_{n_1,l_1;n_2,l_2} \frac{-\hbar}{\sqrt{(2l_1+1)(2l_2+1)}} |a_{n_1l_1}||a_{n_2l_2}| \times \left[\int_0^\infty (R_{n_1l_1}\partial_r R_{n_2l_2} - R_{n_2l_2}\partial_r R_{n_1l_1})\, r^2 dr\ \mathrm{I}_P + \int_0^\infty r R_{n_1l_1} R_{n_2l_2} dr\ \mathrm{II}_{P_i}\right]$$

$$\mathbf{v} = \frac{\mathbf{P}}{M_{\rm halo}}$$

$$\psi_A(t,\mathbf{x}) \to \widetilde{\psi}_A(t,\mathbf{x}) \equiv \psi_A(t,\mathbf{x}+\mathbf{v}t)e^{i(-m_a\mathbf{v}\cdot\mathbf{x}-\frac{1}{2}m_a\mathbf{v}^2t)/\hbar}$$

Initial condition



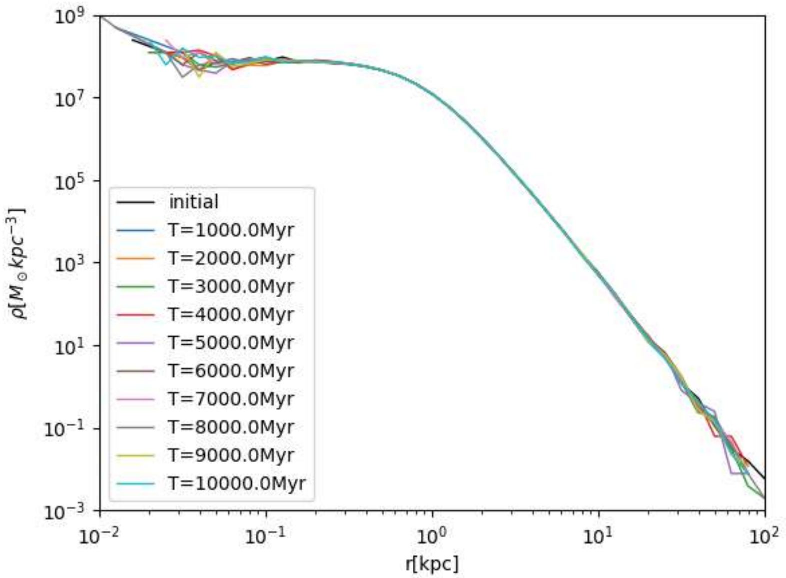
Y.M. Yang, X.J. Bi, and P.F. Yin, Phys.Rev.D 111 (2025) 6, 063013

- Stellar initial condition
  - Density profile

$$\rho_\star(r) = \frac{3M_\star}{4\pi a_i^3}\left(1 + \frac{r^2}{a_i^2}\right)^{-5/2}$$

  - Eddington formula

$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2}\frac{\mathrm{d}}{\mathrm{d}\mathcal{E}}\int_0^{\mathcal{E}}\frac{\mathrm{d}\Phi_0}{\sqrt{\mathcal{E}-\Phi_0}}\frac{\mathrm{d}\rho_\star}{\mathrm{d}\Phi_0},$$



Evolution of the system

- Dimensionless SP equation

$$i\dot{\psi}(t,\boldsymbol{x}) = -\frac{1}{2}\nabla^2\psi(t,\boldsymbol{x}) + \Phi(t,\boldsymbol{x})\psi(t,\boldsymbol{x})$$

$$\nabla^2\Phi(t,\boldsymbol{x}) = 4\pi\left(|\psi(t,\boldsymbol{x})|^2 - \langle|\psi(t,\boldsymbol{x})|^2\rangle\right)$$

- Newton's second law

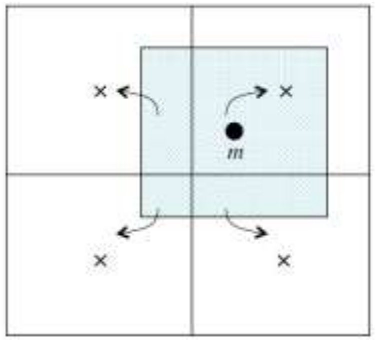
$$\frac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i$$

$$\frac{d\boldsymbol{v}_i}{dt} = \boldsymbol{a}_i = -\nabla\Phi(t,\boldsymbol{x}_i)$$

$i = 1, \dots, N$

Calculate ∇^2 in Fourier space!

$$\Phi_{\rm FDM}(t,\boldsymbol{x}) = \mathcal{F}^{-1}\left(-\frac{1}{k^2}\right)\mathcal{F}\,4\pi\,|\psi(t,\boldsymbol{x})|^2$$



Particle Mesh

t

$\psi(t,\boldsymbol{x})$

$\Phi_{\rm FDM}(t,\boldsymbol{x})$

$\Phi(t,\boldsymbol{x})$

$\Phi_{\rm stars}(t,\boldsymbol{x})$

$\{\boldsymbol{x}_i(t), \boldsymbol{v}_i(t)\}$
$i = 1, \dots, N$

$$\psi(t+\Delta t,\boldsymbol{x}) = \exp\left[-\frac{i\Delta t}{2}\Phi(t+\Delta t,\boldsymbol{x})\right]\mathcal{F}^{-1}\exp\left[-\frac{i\Delta t}{2}k^2\right]\mathcal{F}\exp\left[-\frac{i\Delta t}{2}\Phi(t,\boldsymbol{x})\right]\psi(t,\boldsymbol{x})$$

4th-order Runge-Kutta

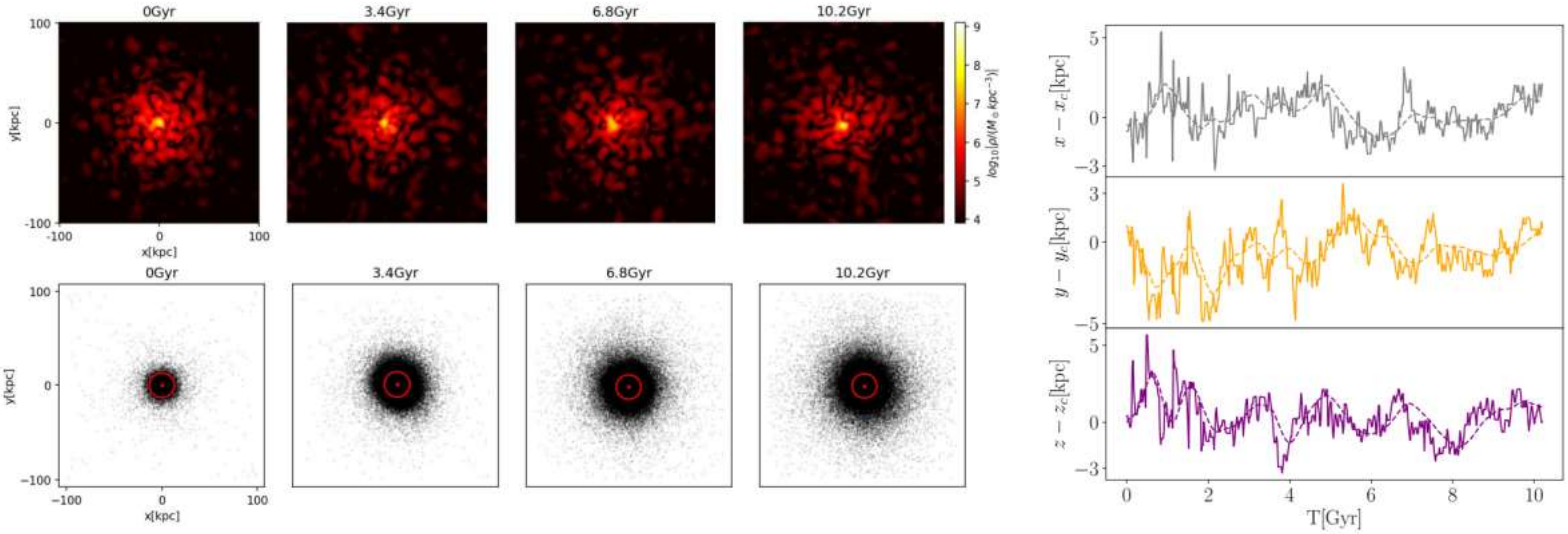
$t+\Delta t$

$\psi(t+\Delta t,\boldsymbol{x})$

$\Phi(t+\Delta t,\boldsymbol{x})$

$\{\boldsymbol{x}_i(t+\Delta t), \boldsymbol{v}_i(t+\Delta t)\}$
$i = 1, \dots, N$

Evolution of the system



Y.M. Yang, Z.C. Zhang, X.J. Bi, and P.F. Yin, 2025 ApJL 981 L26

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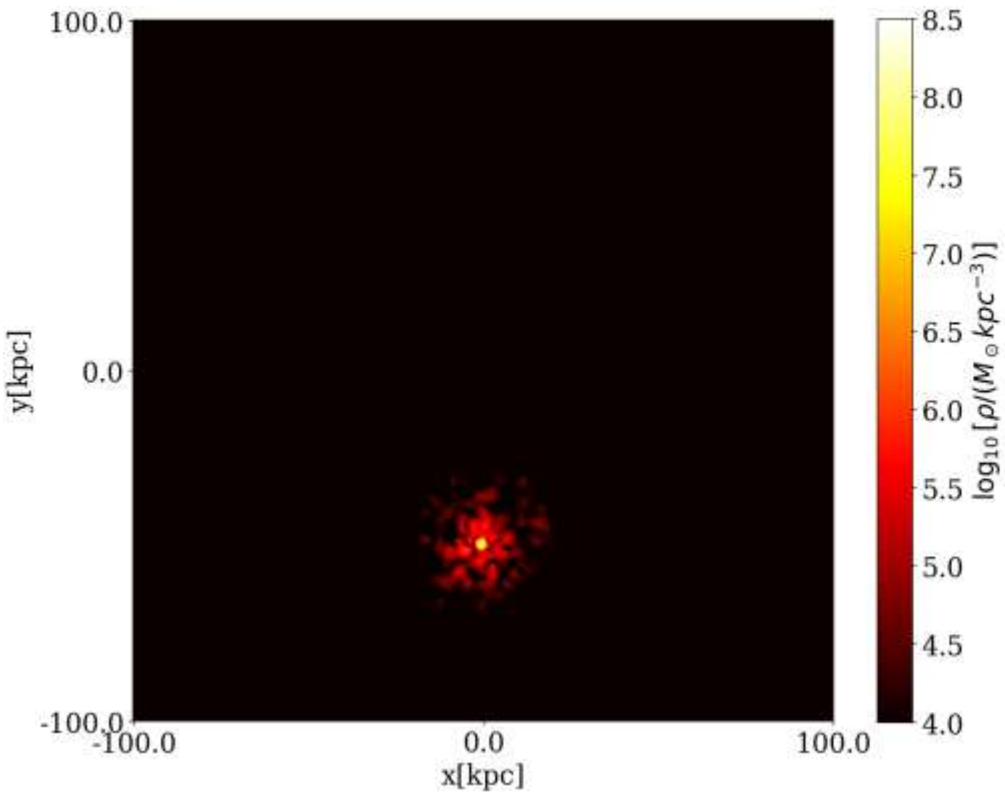
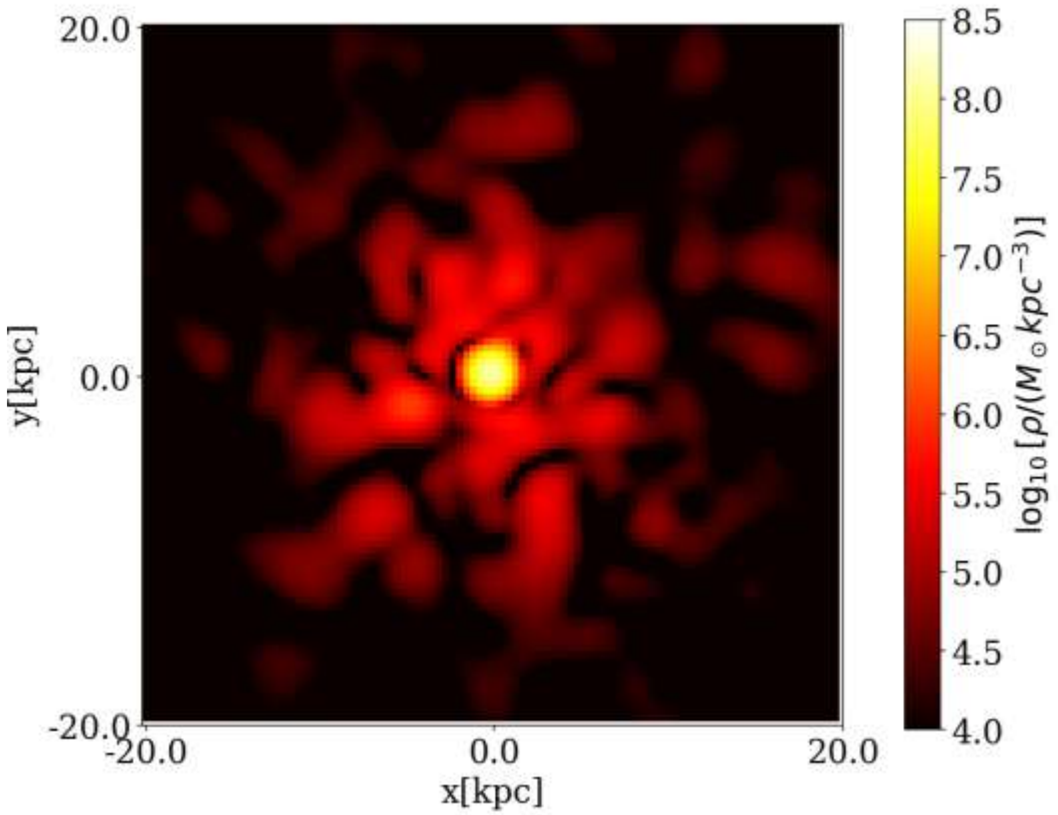
Simulation of tidal effect

- Host halo frame



- Subhalo frame



resol=512 ⟹ resol=100

Simulation of tidal effect

- Host halo frame

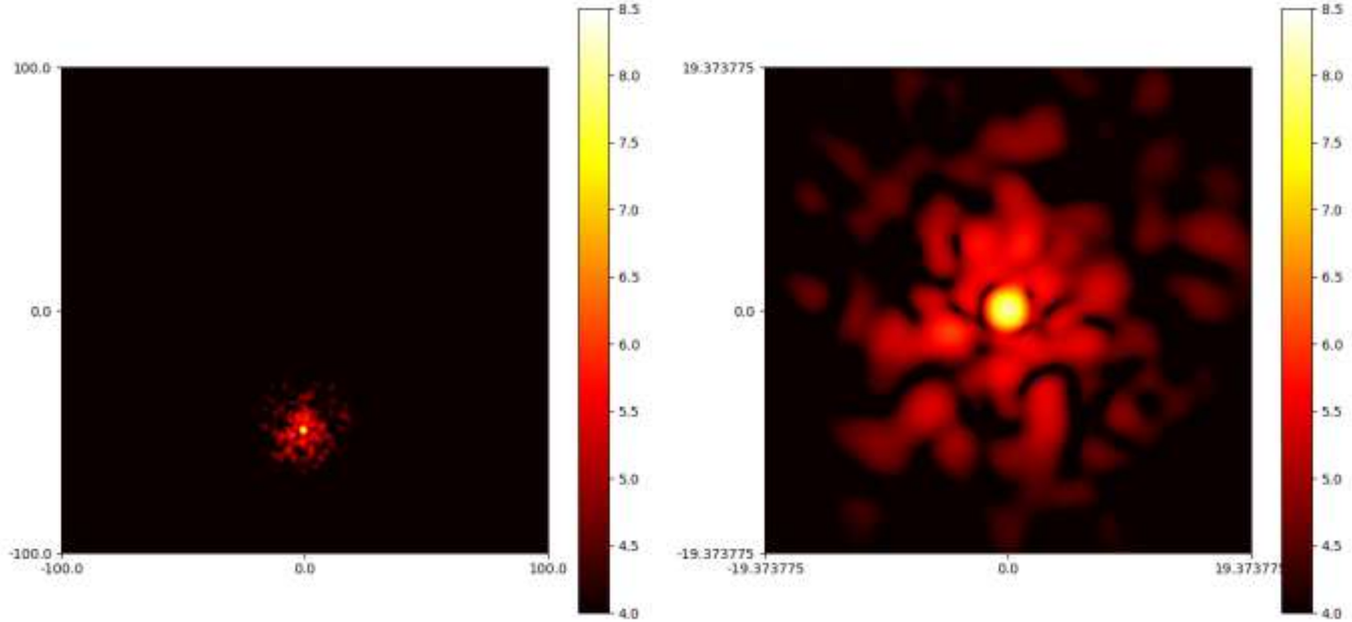
$\Phi(R)$: DM halo

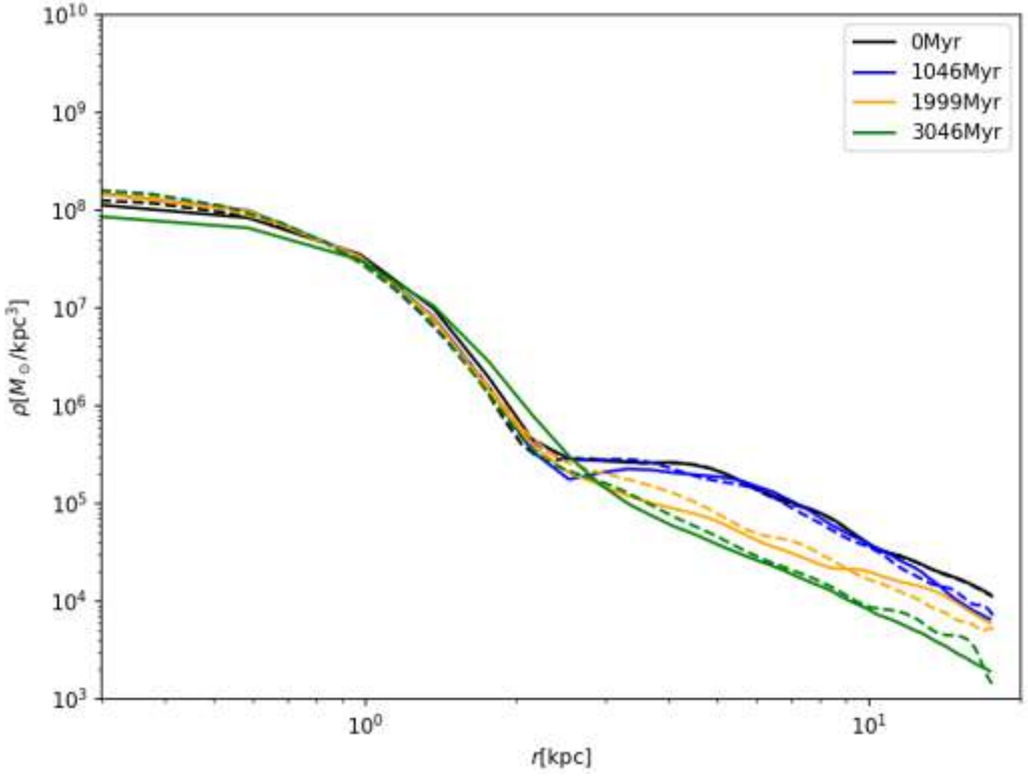
$\Phi(\rho, z)$: bulge, thick & thin stellar disks, HI & H_2 gas disks

- Subhalo frame

$$\Phi_{\rm tidal}(\boldsymbol{r}, \boldsymbol{R}) = \frac{1}{2}\left[\frac{\partial \Phi}{\partial R}\frac{r^2}{R} + \left(\frac{\partial^2 \Phi}{\partial R^2} - \frac{1}{R}\frac{\partial \Phi}{\partial R}\right)\frac{(\boldsymbol{r}\cdot\boldsymbol{R})^2}{R^2}\right]$$

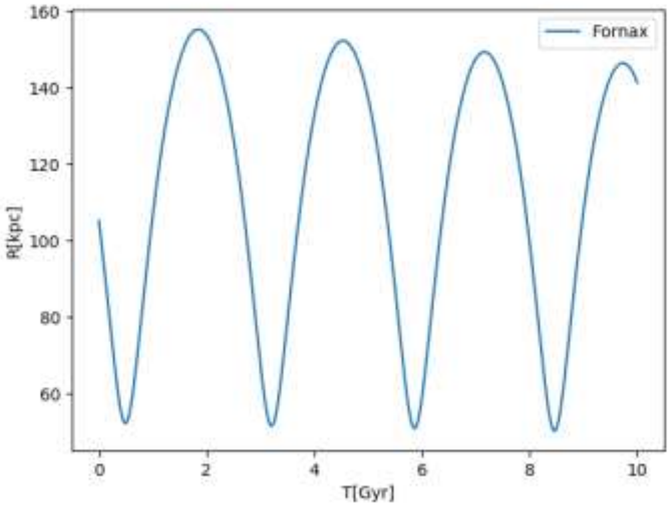
$$\Phi_{\rm tidal}(\boldsymbol{r}, \boldsymbol{R}) = \frac{1}{2}\left[\frac{\partial^2 \Phi}{\partial \rho^2}(\boldsymbol{r}\cdot\boldsymbol{e}_\rho)^2 + 2\frac{\partial^2 \Phi}{\partial \rho \partial z}(\boldsymbol{r}\cdot\boldsymbol{e}_\rho)(\boldsymbol{r}\cdot\boldsymbol{e}_z) + \frac{\partial^2 \Phi}{\partial z^2}(\boldsymbol{r}\cdot\boldsymbol{e}_z)^2 + \frac{\partial \Phi}{\rho \partial \rho}(\boldsymbol{r}\cdot\boldsymbol{e}_\theta)^2\right]$$



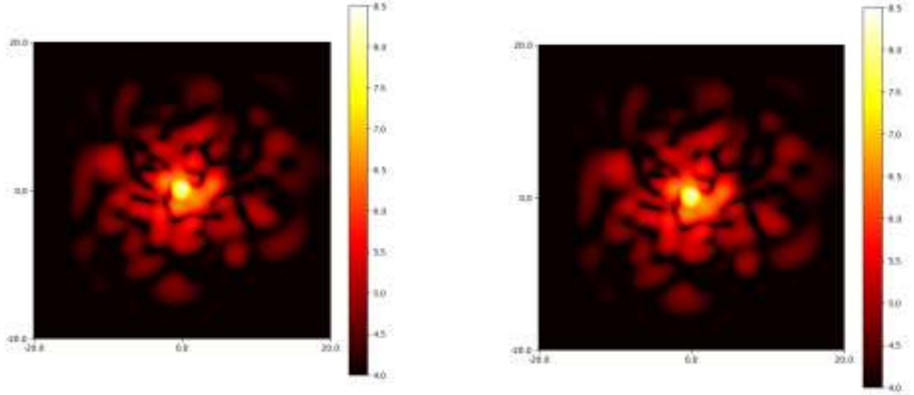


Simulation of tidal effect

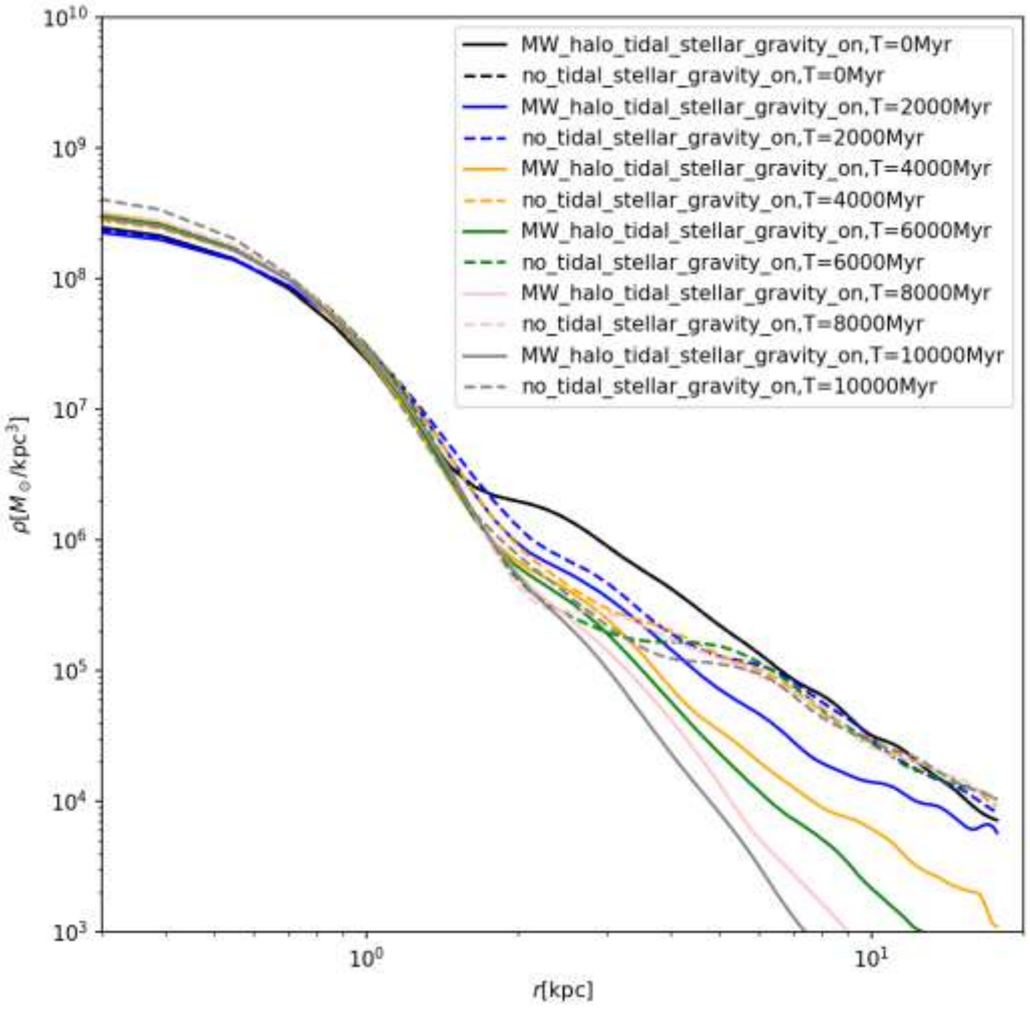
- Fornax in MW
  - Orbit



  - 3D FDM density field at z=0 plane



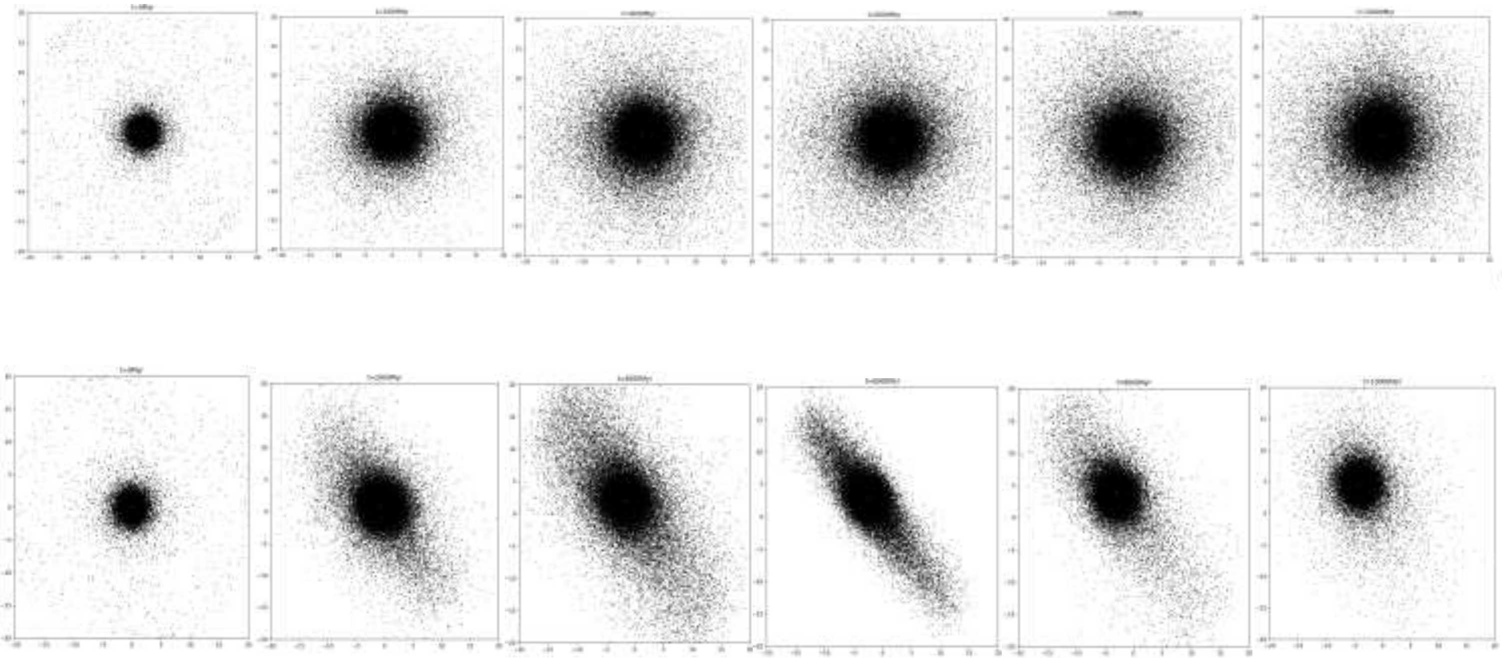
- Evolution of the FDM 3D density profile



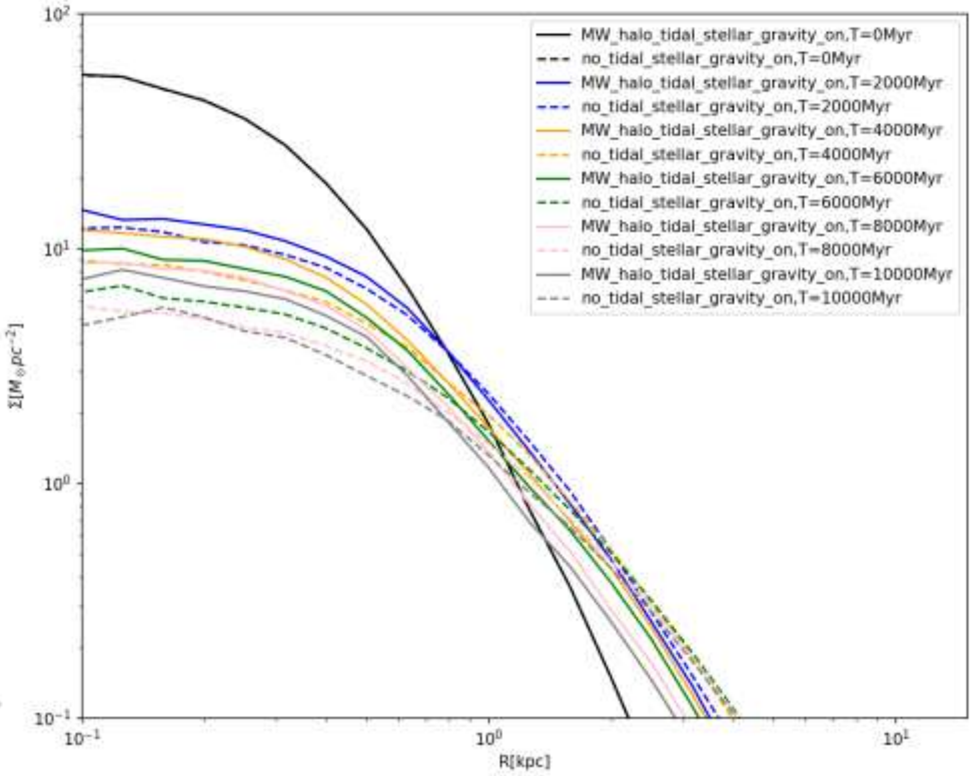
Simulation of tidal effect

➢Fornax in MW

➢ Stars projected location on the z=0 plane

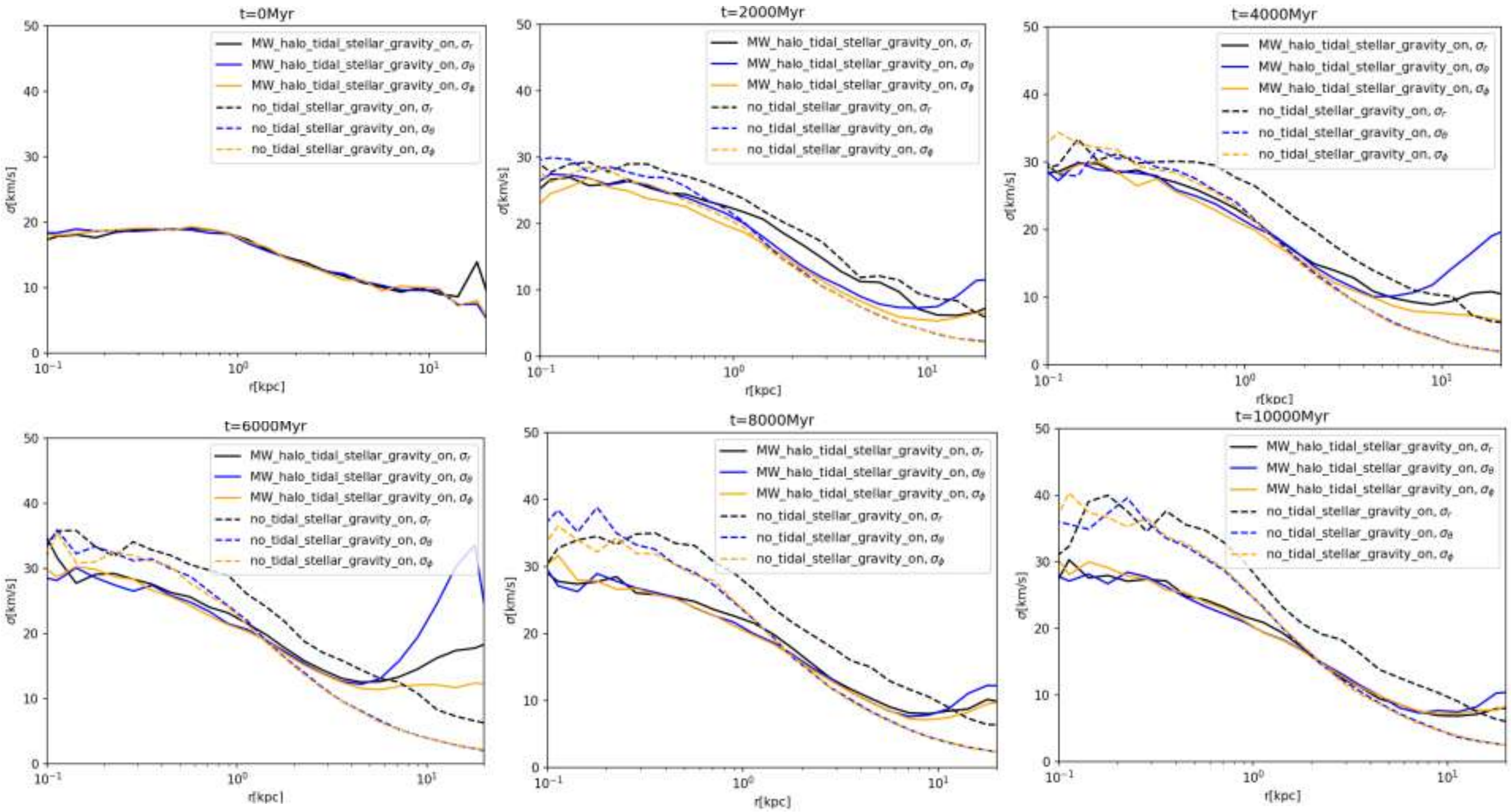


➢ Evolution of the 2D stellar density profile



Simulation of tidal effect

- Fornax in MW
  - Evolution of the stellar velocity dispersion



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Summary

- We effectively addressed the initial velocity issue in constructing the FDM halo wave function by applying a Galilean boost.
- We achieved a self-consistent simulation of a FDM+stars system by integrating wave function and particle simulations.
- Adopting the subhalo as a reference frame can significantly reduce the resolution requirements for tidal effect simulations.
- Under tidal forces, the inner soliton region of an FDM subhalo remains largely unaffected, while the outer NFW region is more susceptible to tidal stripping.
- Tidal effects can suppress the impact of dynamical heating.

Thank you!