Examining the Timing Signal and Its Synergy with Polarization Signals of Ultralight Axion-Like Dark Matter

2025 Axion Dark Matter: Theory and Phenomenology, Qingdao

Ximeng Li (李西蒙), IHEP

Based on Ximeng Li, Yong-Hao Liu, et.al, arXiv 2506.abcde

Outline

1. Motivation: polarization and timing signal
2. Correlation of the signal
3. Data analysis framework
4. Summary

Motivation

Pulsar timing signal

Wave-like DM → oscillation energy momentum tensor

$$a(\mathbf{x},t) = A(\mathbf{x})\cos(m_a t + \alpha(\mathbf{x}))$$

$$\Longrightarrow T_{\mu\nu} = \partial_\mu a \partial_\nu a - \frac{1}{2} g_{\mu\nu}((\partial a)^2 - m_a^2 a^2)$$

Oscillation redshift

$$\partial_t(\Delta t) \equiv \frac{\nu(t) - \nu_0}{\nu_0} \approx \Psi(\mathbf{x}_e, t_e) - \Psi(\mathbf{x}_p, t_p)$$

Timing signal: $\Delta t \propto P_{DM} \propto a^2$

Pulsar timing signal from ultralight scalar dark matter

Andrei Khmelnitsky[a] and Valery Rubakov[b,c]

Pulsar polarization signal

CP-odd wave-like ALDM → non-trivial dispersion relation

$$\mathcal{L} \subset \frac{1}{2} a g_{a\gamma\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\Longrightarrow \omega_\pm \simeq k \pm g_{a\gamma\gamma}\left(\dot{a} + \nabla a \cdot \frac{\mathbf{k}}{k}\right)$$

Oscillation polarization

$$\Delta \mathrm{PA} = g_{a\gamma\gamma}\left[a(\mathbf{x}_p, t_p) - a(\mathbf{x}_e, t_e)\right]$$

Polarization angle signal: $\Delta \mathrm{PA} \propto a$

PHYSICAL REVIEW D VOLUME 41, NUMBER 4 15 FEBRUARY 1990

Limits on a Lorentz- and parity-violating modification of electrodynamics

Sean M. Carroll and George B. Field
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138

Roman Jackiw*
Department of Physics, Columbia University, New York, New York 10027
(Received 5 September 1989)

CLT: The axion field a:
Gaussian distribution with zero mean

Random phase:
uniform distribution in $[0, 2\pi)$

$$a(\mathbf{x}, t) \approx \frac{\sqrt{\rho(\mathbf{x})}}{m_a} \sum_{\mathbf{v}\in\Omega} (\Delta v)^{3/2} \alpha_{\mathbf{v}} \sqrt{f_{\mathbf{x}}(\mathbf{v})} \cos[\omega t - \mathbf{k}\cdot\mathbf{x} + \phi_{\mathbf{v}}]$$

Rayleigh
distribution: $\alpha = 2$

Standard halo model (SHM)

$$f(\mathbf{v}) = \frac{1}{\pi^{3/2} v_0^3} \exp\left[-\frac{(\mathbf{v}+\mathbf{v}_\odot)^2}{v_0^2}\right]$$

Timing signal: $\Delta t \propto P_{\mathrm{DM}} \propto a^2$

↓

Non-Gaussian distribution

Polarization angle signal: $\Delta \mathrm{PA} \propto a$

↓

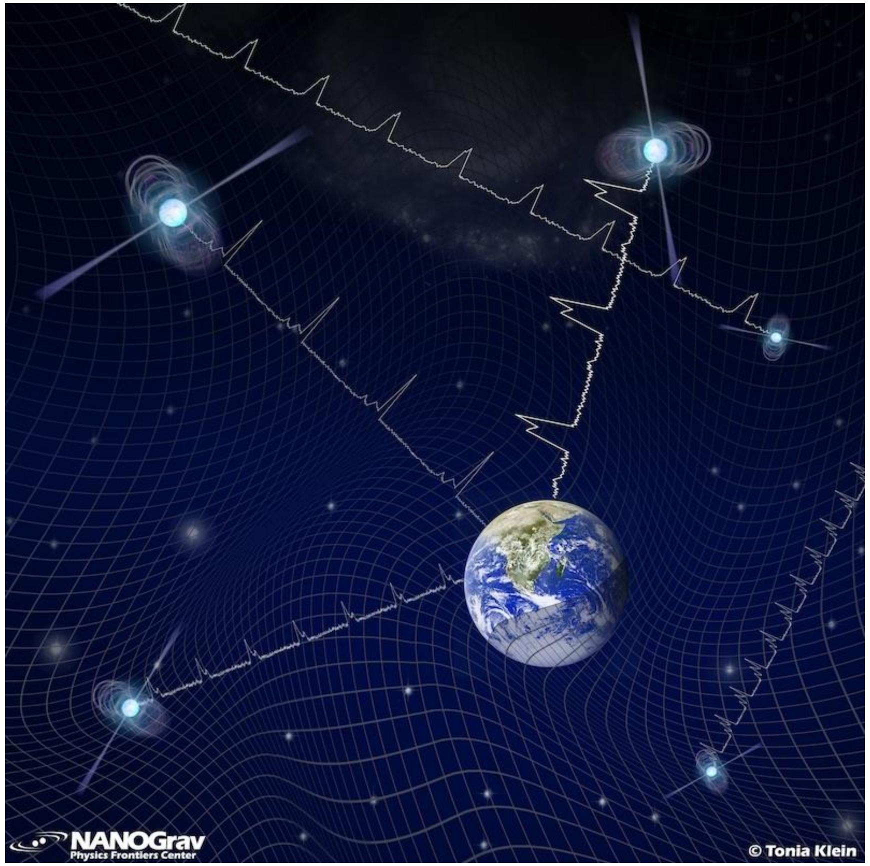
Gaussian distribution

LCTP-17-08, MIT-CTP 4964

Revealing the Dark Matter Halo with Axion Direct Detection

Joshua W. Foster,[1] Nicholas L. Rodd,[2] and Benjamin R. Safdi[1]
[1] *Leinweber Center for Theoretical Physics, Department of Physics, University of Michigan, Ann Arbor, MI 48109*
[2] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139*

Questions



1. The influence of the non-Gaussianity for data analysis?

2. How to describe the $\Delta t - \Delta \mathrm{PA}$ cross-correlation?

Correlation of the signal

I. Polarization signal

$$\mathbf{\Delta PA}^a = (\Delta\mathrm{PA}^a_{1,1}, \ldots, \Delta\mathrm{PA}^a_{1,N_1}, \ldots, \Delta\mathrm{PA}^a_{p,n}, \ldots, \Delta\mathrm{PA}^a_{\mathcal{N},1}, \ldots, \Delta\mathrm{PA}^a_{\mathcal{N},N_\mathcal{N}})^T$$

Auxiliary Gaussian parameter X, one signal with zero mean

$$\Delta\mathrm{PA}_{p,n} \approx g_{a\gamma\gamma}\Big[a(\mathbf{x}_{\mathrm{psr}}, t_{\mathrm{psr}}) - a(\mathbf{x}_{\mathrm{e}}, t_{\mathrm{e}})\Big]_{p,n} \equiv -\frac{g_{a\gamma\gamma}}{m_a}\left\{\sqrt{\rho_{\mathrm{psr}}}X^{\mathrm{psr}}_{p,n} - \sqrt{\rho_{\mathrm{e}}}X^{\mathrm{e}}_{p,n}\right\}$$

$$X^{(i)}_{p,n} \equiv \sum_{\mathbf{v}\in\Omega}(\Delta v)^{3/2}\alpha_{\mathbf{v}}\sqrt{f(\mathbf{v})}\cos\left[m_a\left(t^{(i)} - \mathbf{v}\cdot\mathbf{x}^{(i)}\right) + \phi_{\mathbf{v}}\right]$$

Encode space-time information in Gaussian parameter

The covariance matrix

$$\boldsymbol{C}^{(a)}_{\mathrm{PA}} = \langle \mathbf{\Delta PA}^a(\mathbf{\Delta PA}^a)^T\rangle = \frac{g^2_{a\gamma\gamma}}{m_a^2}\sum_{i,j}(-1)^{i+j}\sqrt{\rho(\mathbf{x}^{(i)}_p)\rho(\mathbf{x}^{(j)}_q)}\,\boldsymbol{C}^{(ij)}_X$$

$$(C^{(ij)}_{\mathbf{X}})_{pn,qm} \equiv \langle X^{(i)}_{p,n}X^{(j)}_{q,m}\rangle = e^{-\frac{1}{4}(y^{ij}_{pq})^2}\cos\left[m_a(t^{(i)}_{p,n} - t^{(j)}_{q,m}) + m_a\mathbf{v}_\odot\cdot\mathbf{x}^{(ij)}_{pq}\right]$$

$$y_{ij} \equiv |\mathbf{x}_i - \mathbf{x}_j|\,/l_c$$

Spatial correlation between two pulsars

Wave DM: separation $\ll$ de Broglie wavelength ($l_c \equiv (m_a v_0)^{-1}$)

II. Timing signal

$$\mathbf{\Delta t}^a = (\Delta t^a_{1,1}, ..., \Delta t^a_{1,N_1}, ..., \Delta t^a_{p,n}, ..., ..., \Delta t^a_{\mathcal{N},1}, ..., \Delta t^a_{\mathcal{N},N_\mathcal{N}})^T$$

Quadratic form of Gaussian distribution with zero mean

$$\Delta t \approx -\frac{\pi G}{2m_a^2}\left[\dot{a}(\mathbf{x}_p, t-L_p)a(\mathbf{x}_p, t-L_p) - \dot{a}(\mathbf{x}_e, t)a(\mathbf{x}_e, t)\right]$$

$$\Delta t_{p,n} \equiv \frac{\pi G}{2m_a^3}\left[\rho_\mathrm{e} X^\mathrm{e}_{p,n} Y^\mathrm{e}_{p,n} - \rho_\mathrm{psr} X^\mathrm{psr}_{p,n} Y^\mathrm{psr}_{p,n}\right] \qquad Y^{(i)}_{p,n} = \cdots \cos \to \sin$$

$$\boldsymbol{C}_t^{(a)} = \langle \mathbf{\Delta t}^a (\mathbf{\Delta t}^a)^T \rangle = \frac{\pi^2 G^2}{4m_a^6} \sum_{i,j} (-1)^{i+j} \rho(\mathbf{x}_p^{(i)}) \rho(\mathbf{x}_q^{(j)}) \left[\boldsymbol{C}_X^{(ij)} \odot \boldsymbol{C}_X^{(ij)} - \boldsymbol{C}_{XY}^{(ij)} \odot \boldsymbol{C}_{XY}^{(ij)}\right]$$

Cross-correlation between two pulsars

Anti-symmetry

$$\left\langle \Delta t^a_{p,n} \Delta t^a_{q,m} \right\rangle = \frac{\pi^2 G^2}{4m_a^6} \sum_{i,j} (-1)^{i+j} \rho(\mathbf{x}_p^{(i)}) \rho(\mathbf{x}_q^{(j)}) e^{-\frac{1}{2}\left(y^{ij}_{pq}\right)^2} \cos\left[2m_a\left(t^{(i)}_{p,n} - t^{(j)}_{q,m}\right) + 2m_a \mathbf{v}_\odot \cdot \mathbf{x}^{(ij)}_{pq}\right]$$

A comparison between ALDM PTA and SGWB

	PTA	SGWB
Spectrum	Monotonic, $2\pi f \approx 2m_a$	Power law
Coherent length	Non-relativistic, $\sim\mathrm{kpc}$	Relativistic, $\sim\mathrm{pc}$
Signal strength	Pulsar term, enhanced with large ρ_{psr}	Only earth term
Space correlation	Depend on pulsar positions	Depend on pulsar angular separations (H-D curve)

III. Δt − $\Delta \mathrm{PA}$ correlation

No two-point correlation

Three Gaussian variables

$$\langle \Delta t^a_{p,n} \Delta \mathrm{PA}^a_{q,m} \rangle = -\frac{\pi G g_{a\gamma\gamma}}{4 m_a^4} \sum_{i,j} (-1)^{i+j} \rho(\mathbf{x}_p^{(i)}) \sqrt{\rho(\mathbf{x}_q^{(j)})} \, \langle \underline{X^{(i)}_{p,n} Y^{(i)}_{p,n} X^{(j)}_{q,m}} \rangle = 0$$

LO: three-point correlation

$$\langle \Delta \mathrm{PA}^a_{p,n} \Delta \mathrm{PA}^a_{q,m} \Delta t^a_{r,l} \rangle = \frac{\pi G g^2_{a\gamma\gamma}}{2 m_a^5} \sum_{i,j,k} (-1)^{i+j+k} \sqrt{\rho(\mathbf{x}_p^{(i)}) \rho(\mathbf{x}_q^{(j)})} \rho(\mathbf{x}_r^{(k)}) \left\langle \underline{X^{(i)}_{p,n} X^{(j)}_{q,m} X^{(k)}_{r,l} Y^{(k)}_{r,l}} \right\rangle$$

$$= -\frac{\pi G g^2_{a\gamma\gamma}}{2 m_a^5} \sum_{i,j,k} (-1)^{i+j+k} \sqrt{\rho(\mathbf{x}_p^{(i)}) \rho(\mathbf{x}_q^{(j)})} \rho(\mathbf{x}_r^{(k)}) \boxed{e^{-\frac{1}{4}\left(y^{ik}_{pr}\right)^2} e^{-\frac{1}{4}\left(y^{jk}_{qr}\right)^2}}$$

$$\times \sin\left[m_a (t^{(i)}_{p,n} + t^{(j)}_{q,m} - 2 t^{(k)}_{r,l}) + m_a \mathbf{v}_\odot \cdot (\mathbf{x}^{(ik)}_{pr} + \mathbf{x}^{(jk)}_{qr}) \right].$$

The product of two exponential factors

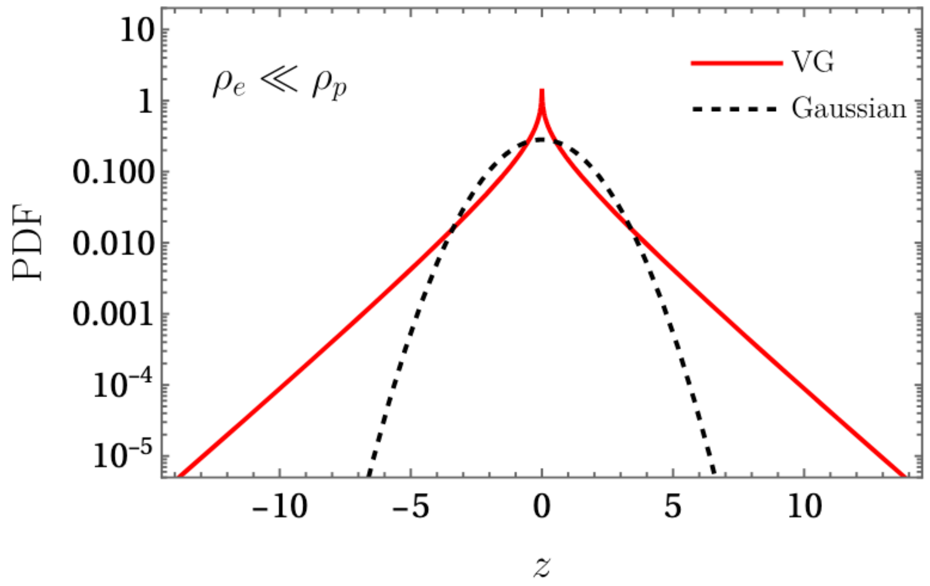
Correlation between PA and timing!

I. Non-Gaussian timing signals: quadratic forms

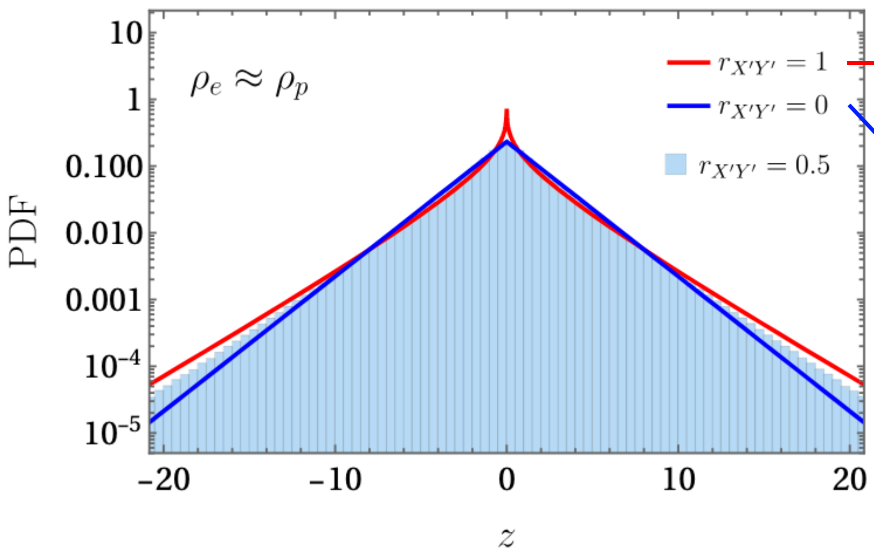
$$\Delta t_{p,n} \equiv \frac{\pi G}{2m_a^3}\left[\rho_e X^e_{p,n} Y^e_{p,n} - \rho_{psr} X^{psr}_{p,n} Y^{psr}_{p,n}\right]$$

(1) $\rho_{psr} \gg \rho_e$ limit $\quad \Delta t_{p,n} \equiv c_1 XY$

(2) $\rho_{psr} \approx \rho_e$ limit $\quad \Delta t_{p,n} \equiv c_2(XY - UV)$



$$f_{\Delta t}(z) = \frac{1}{\pi\sigma} K_0\left(\frac{|z|}{\sigma}\right), \ \sigma^2 = c_1$$



$$f_{\Delta t}(z) = \frac{1}{\pi\sigma} K_0\left(\frac{|z|}{\sigma}\right)$$

$$f_{\Delta t}(z) = \frac{1}{2b} e^{-\frac{|z|}{b}}$$

$$\mathrm{CF}_{\Delta t}(\omega) = \frac{\pi^2}{L^2} \frac{1}{\sqrt{(\omega^2 + K_1{}^2)(\omega^2 + K_2{}^2)}}$$

Obtain PDF by Fourier transform

The non-Gaussianity as small correction

Expand case (1) PDF with Gauss-Hermite series

$$f_X(x) = \left\{ \sum_{n=0}^{\infty} \frac{\alpha_n}{2^n n! \sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{4\sigma^2}\right) H_n\left(\frac{x}{\sqrt{2}\sigma}\right) \right\}^2$$

Non-negative truncated series

Hermite polynomials: orthogonal and complete basis

Gaussian function

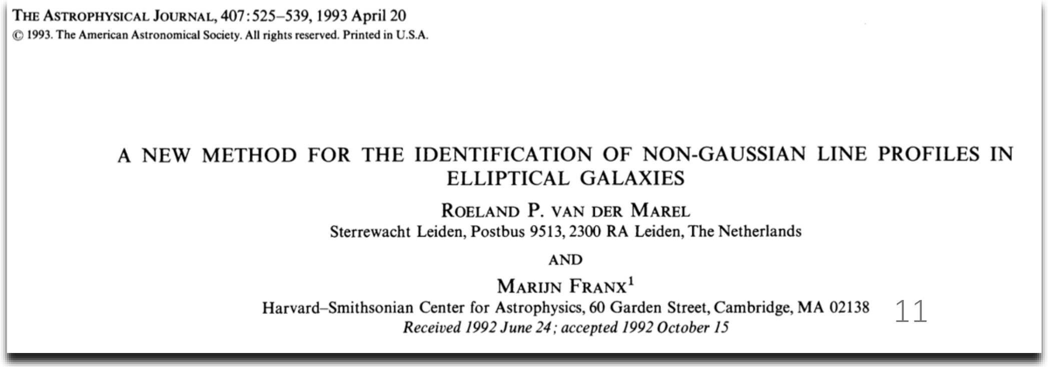
$$f_{\Delta t}(z) = \frac{1}{\pi} K_0\left(|z|\right)$$

$$\Longrightarrow \alpha_0 \approx 0.96,\ \alpha_2 \approx -0.059,\ \alpha_4 \approx 0.19,\ \alpha_6 \approx -0.053,\ \cdots$$

LO: Gaussian

Smaller non-Gaussianities, converge at large n

Process the timing signal under Gaussian limit?


THE ASTROPHYSICAL JOURNAL, 407:525–539, 1993 April 20
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A NEW METHOD FOR THE IDENTIFICATION OF NON-GAUSSIAN LINE PROFILES IN ELLIPTICAL GALAXIES

ROELAND P. VAN DER MAREL
Sterrewacht Leiden, Postbus 9513, 2300 RA Leiden, The Netherlands

AND

MARIJN FRANX[1]
Harvard–Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138
Received 1992 June 24; accepted 1992 October 15

II. PTA analysis under Gaussian limit

Difficulties

1. Popular methods (e.g. Copula) dependent on models
2. Numerical methods (e.g. MCMC): cost too much

First approach: $\Delta t \sim$ Gaussian distribution

Deterministic noise

$$\mathbf{\Delta t}^{\text{obs}} = \boxed{\mathbf{\Delta t}^{w} + \mathbf{\Delta t}^{r}} + \boxed{\mathbf{\Delta t}^{\text{det}}} + \boxed{\mathbf{\Delta t}^{a}}$$

Assume: Gaussian noise

ALDM signal

$$f_{\mathbf{\Delta t}^a}(\boldsymbol{z}) = \frac{1}{\sqrt{\det\left(2\pi \boldsymbol{C}_t^{(a)}\right)}} \exp\left[-\frac{1}{2}\boldsymbol{z}^T (\boldsymbol{C}_t^{(a)})^{-1}\boldsymbol{z}\right]$$

Likelihood

$$\mathcal{L}_m = \frac{1}{\sqrt{\det\left(2\pi \boldsymbol{C}_t^{(n)}\right)}} \int \exp\left[-\frac{1}{2}(\mathbf{\Delta t}_o - \boldsymbol{z})^T (\boldsymbol{C}_t^{(n)})^{-1}(\mathbf{\Delta t}_o - \boldsymbol{z})\right] f_{\mathbf{\Delta t}^a}(\boldsymbol{z}) d\boldsymbol{z}$$

Gaussian approximation

$$\Longrightarrow \mathcal{L}_m^{(g)} = \frac{1}{\sqrt{\det(2\pi \boldsymbol{C}_t)}} \exp\left[-\frac{1}{2}\mathbf{\Delta t}_o^T \boldsymbol{C}_t^{-1}\mathbf{\Delta t}_o\right]$$

Another approach: realistic distribution, but small-signal limit

$$\mathbf{\Delta t}^{\text{obs}} = \mathbf{\Delta t}^{w} + \mathbf{\Delta t}^{r} + \mathbf{\Delta t}^{\text{det}} + \mathbf{\Delta t}^{a}$$

Marginalized the Gaussian likelihood for $\rho^e \ll \rho^p$

$$\mathcal{L}_m = \frac{1}{\sqrt{\det(2\pi \boldsymbol{C}_t^{(n)})}} \int \exp\left[-\frac{1}{2}(\mathbf{\Delta t}_o - c_1 \boldsymbol{D}_x \boldsymbol{y})^\top (\boldsymbol{C}_t^{(n)})^{-1} (\mathbf{\Delta t}_o - c_1 \boldsymbol{D}_x \boldsymbol{y})\right] \exp\left[-\frac{1}{2}\boldsymbol{x}^\top \boldsymbol{S}^{-1}\boldsymbol{x}\right]$$
$$\times \exp\left[-\frac{1}{2}\boldsymbol{y}^\top \boldsymbol{S}^{-1}\boldsymbol{y}\right] \exp\left[\frac{1}{2}(-\boldsymbol{y}^\top \boldsymbol{S}^{-1}\boldsymbol{C}_{XY}\boldsymbol{C}_X^{-1}\boldsymbol{x} + \boldsymbol{x}^\top \boldsymbol{C}_X^{-1}\boldsymbol{C}_{XY}\boldsymbol{S}^{-1}\boldsymbol{y})\right] d\boldsymbol{x} d\boldsymbol{y}$$

$$\Delta t^a = c_1 XY$$

By taking small-signal limit, the Gaussian likelihood

$$\mathcal{L}_m^{(g)} \propto \exp\left[-\frac{1}{2}\mathbf{\Delta t}_o^T (\boldsymbol{C}_t^{(n)})^{-1}\mathbf{\Delta t}_o\right]\left[1 + \frac{1}{2}c_1^2 \mathbf{\Delta t}_o^T (\boldsymbol{C}_t^{(n)})^{-1}\boldsymbol{C}_t^{(a)}(\boldsymbol{C}_t^{(n)})^{-1}\mathbf{\Delta t}_o + \mathcal{O}(c_1^3)\right]$$

LO: Gaussian

NLO: 2-point correlation: $\langle \Delta t \Delta t \rangle$

The same LO!

The noise matrix

$$\boldsymbol{C}_t^{-1} = (\boldsymbol{C}_t^w + \boldsymbol{C}_t^r + \boldsymbol{C}_t^a)^{-1}$$

- ALDM timing signals $\quad (\boldsymbol{C}_t^a)_{p,n;q,m} \equiv \langle \Delta t^a_{p,n} \Delta t^a_{q,m} \rangle = (\boldsymbol{F}^a_{pn})^T \boldsymbol{\Phi}^a_{pq} \boldsymbol{F}^a_{qm}$ 2D-DFT
- Correlated (between pulsars) red noise $\quad (\boldsymbol{C}_t^r)_{p,n;q,m} = (\boldsymbol{F}^r_{pn})^T \boldsymbol{\Phi}^r_{pq} \boldsymbol{F}^r_{qm}$
- Uncorrelated white noise $\quad (\boldsymbol{C}_t^w)_{p,n;q,m} \propto \delta_{pq}\delta_{mn}$

In practice, combine ALDM signal and red noise

$$\boldsymbol{\Phi} = \mathrm{diag}\{\boldsymbol{\Phi}^a, \boldsymbol{\Phi}^r\},$$

$$\boldsymbol{F} = \begin{pmatrix} \boldsymbol{F}^a \\ \boldsymbol{F}^r \end{pmatrix} = \begin{pmatrix} \boldsymbol{F}^a \\ \mathrm{diag}\{\boldsymbol{F}^r_1, \boldsymbol{F}^r_2, \ldots, \boldsymbol{F}^r_{\mathcal{N}}\} \end{pmatrix}$$

$$\Longrightarrow \quad \boldsymbol{C}_t^r + \boldsymbol{C}_t^a = \boldsymbol{F}^T \boldsymbol{\Phi} \boldsymbol{F}$$

$$\boldsymbol{C}_t^{-1} \approx (\boldsymbol{C}_t^w)^{-1} - (\boldsymbol{C}_t^w)^{-1} \boldsymbol{F}^T \left(\boldsymbol{\Phi}^{-1} + \boldsymbol{F} (\boldsymbol{C}_t^w)^{-1} \boldsymbol{F}^T\right)^{-1} \boldsymbol{F} (\boldsymbol{C}_t^w)^{-1}$$

II. PTA-PPA analysis

Core concept: PTA and PPA are independent.

Approach I: Gaussian likelihood

$$\mathcal{L} = \mathcal{L}_t \mathcal{L}_{\rm PA} \propto \exp\left[-\frac{1}{2}(\mathbf{\Delta t}_o - \mathbf{\Delta t}^a)^T (\boldsymbol{C}_t^{(n)})^{-1} (\mathbf{\Delta t}_o - \mathbf{\Delta t}^a)\right] \times \exp\left[-\frac{1}{2}(\mathbf{\Delta PA}_o - \mathbf{\Delta PA}^a)^T (\boldsymbol{C}_{\rm PA}^{(n)})^{-1} (\mathbf{\Delta PA}_o - \mathbf{\Delta PA}^a)\right]$$

Approach II: integrate out the auxiliary Gaussian parameters

$$\mathcal{L}_m \propto \int d\boldsymbol{x}\, d\boldsymbol{y} \exp\left[-\frac{1}{2}(\mathbf{\Delta t}_o - c_1 \boldsymbol{D}_x \boldsymbol{y})^T (\boldsymbol{C}_t^{(n)})^{-1} (\mathbf{\Delta t}_o - c_1 \boldsymbol{D}_x \boldsymbol{y})\right] \times \exp\left[-\frac{1}{2}(\mathbf{\Delta PA}_o - c_2 \boldsymbol{x})^T (\boldsymbol{C}_{\rm PA}^{(n)})^{-1} (\mathbf{\Delta PA}_o - c_2 \boldsymbol{x})\right] \times \exp\left[-\frac{1}{2}\left(\boldsymbol{x}^T \boldsymbol{S}^{-1} \boldsymbol{x} + \boldsymbol{y}^T \boldsymbol{S}^{-1} \boldsymbol{y} + \boldsymbol{y}^T \boldsymbol{S}^{-1} \boldsymbol{C}_{XY} \boldsymbol{C}_X^{-1} \boldsymbol{x} - \boldsymbol{x}^T \boldsymbol{C}_X^{-1} \boldsymbol{C}_{XY} \boldsymbol{S}^{-1} \boldsymbol{y}\right)\right]$$

With

$$\mathcal{L}_m \propto \exp\left[-\frac{1}{2}\left(\mathbf{\Delta t}_o^T(\boldsymbol{C}_t^{(n)})^{-1}\mathbf{\Delta t}_o + \mathbf{\Delta PA}_o^T(\boldsymbol{C}_{\mathrm{PA}}^{(n)})^{-1}\mathbf{\Delta PA}_o\right)\right]$$

Gaussian likelihood

$$\times \exp\left[\frac{1}{2}c_2^2\mathbf{\Delta PA}_o^T(\boldsymbol{C}_{\mathrm{PA}}^{(n)})^{-1}\boldsymbol{C}_X(\boldsymbol{C}_{\mathrm{PA}}^{(n)})^{-1}\mathbf{\Delta PA}_o\right]$$

$$\times \exp\left[\frac{1}{2}c_1c_2^2\mathbf{\Delta PA}_o^T(\boldsymbol{C}_{\mathrm{PA}}^{(n)})^{-1}(\boldsymbol{C}_X\boldsymbol{D}_t\boldsymbol{C}_{XY} - \boldsymbol{C}_{XY}\boldsymbol{D}_t\boldsymbol{C}_X)(\boldsymbol{C}_{\mathrm{PA}}^{(n)})^{-1}\mathbf{\Delta PA}_o\right]$$

Two-point function $\langle \mathbf{\Delta PA}_n^a \mathbf{\Delta PA}_m^a \rangle$

Three-point function $\langle \mathbf{\Delta PA}_n^a \mathbf{\Delta PA}_m^a \mathbf{\Delta t}_l^a \rangle$

Summary

- ALDM-induced polarization and timing signals display distinct spatial and temporal dependencies

- Polarization: multivariate Gaussian distribution; first PPA limits on ALDM established with the PPTA collaboration

- Timing: non-Gaussian behavior due to quadratic dependence on Gaussian variables; however, a Gaussian approximation remains a good starting point for PTA data analysis (analysis ongoing with the PPTA collaboration)

- Polarization-timing: 1^{st} non-trivial correlation is the three-point functions $\langle \Delta\mathrm{PA}\, \Delta\mathrm{PA}\, \Delta t \rangle$, which determines the LO correction to the marginalized likelihood in combined PTA-PPA analysis

Thanks for your attention