The application of machine learning in the holographic QCD

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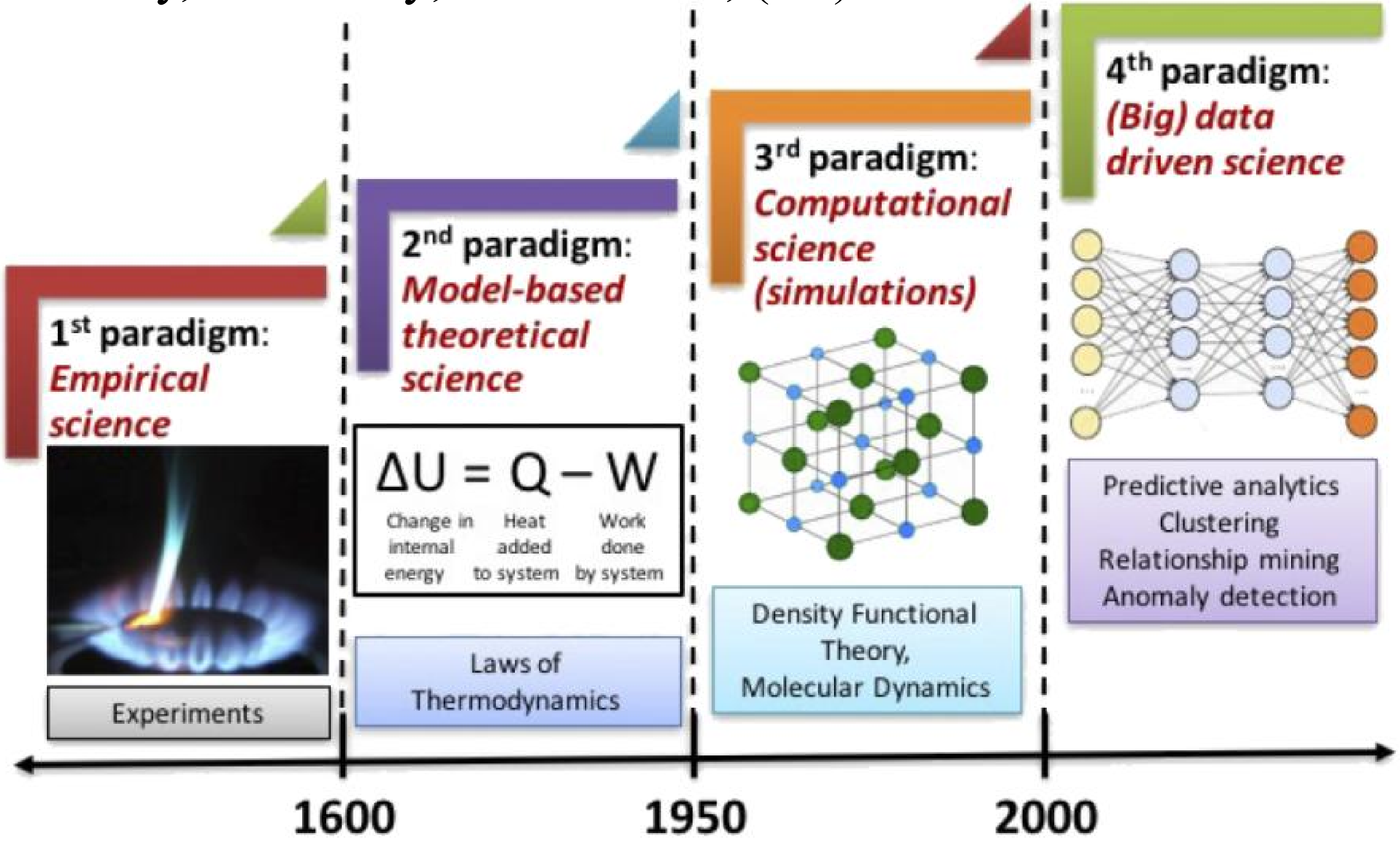
The Fourth Paradigm, Data-intensive Scientific Discovery

T. Hey, S. Tansley, and K. Tolle, (ed.)



The four paradigms of science: empirical, theoretical, computational, and data-driven.

Recent works about holographic QCD and machine learning

Koji Hashimoto,Sotaro Sugishita, Akinori Tanaka, Akio Tomiya, Deep learning and the AdS/CFT correspondence, Phys.Rev.D 98 (2018) 4, 046019

1、K. Li, **Y. Ling**, P. Liu and M. H. Wu, Phys. Rev. D 107 (2023) no.6, 066021.
2、K. Hashimoto, K. Ohashi and T. Sumimoto, PTEP 2023, no.3, 033B01 (2023).
3、Y. K. Yan, S. F. Wu, **X. H. Ge** and **Y. Tian**, Phys. Rev. D 102, no.10, 101902.
4、Byoungjoon Ahn, Hyun-Sik Jeong, **Keun-Young Kim**, Kwan Yun, arXiv: 2406.07395.
5、**Rong-Gen Cai**, **Song He**, **Li Li**, Hong-An Zeng, arXiv:2406.12772
……

Machine learning holographic black hole from lattice QCD equation of state
Xun Chen, Mei Huang, Phys.Rev.D 109 (2024) 5, L051902
ArXiv: 2401.06417

Flavor dependent Critical endpoint from holographic QCD through machine learning
Xun Chen, Mei Huang, JHEP (under review)
ArXiv: 2405.06179

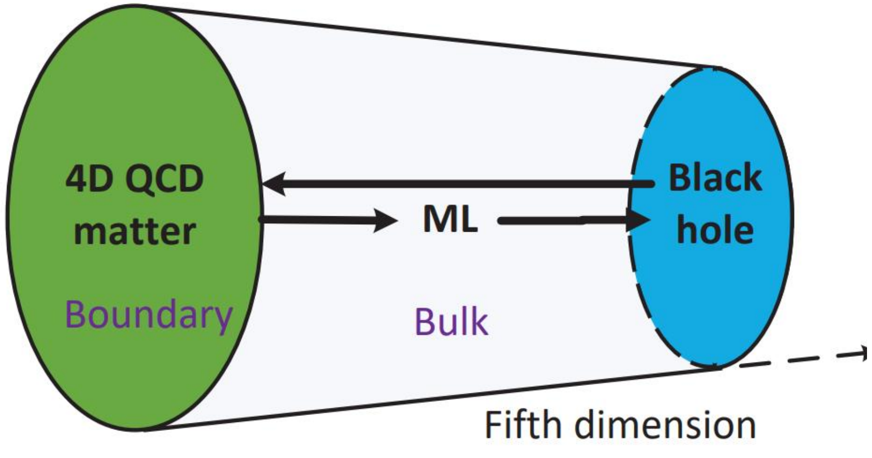


Figure 1. The sketch of holographic QCD and machine learning.

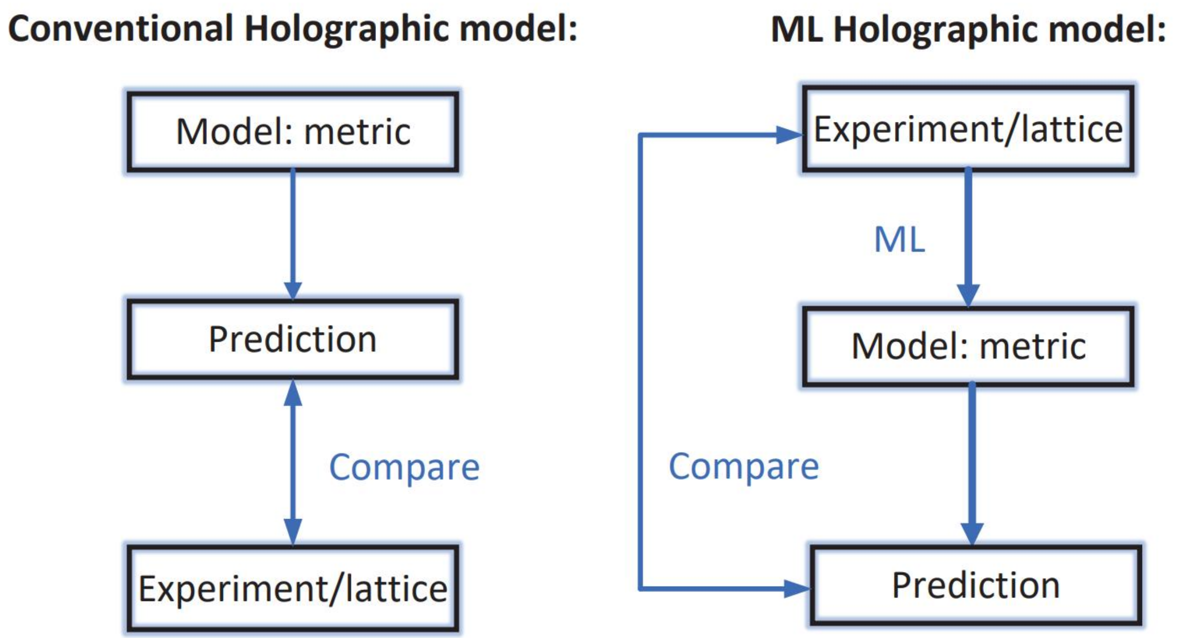
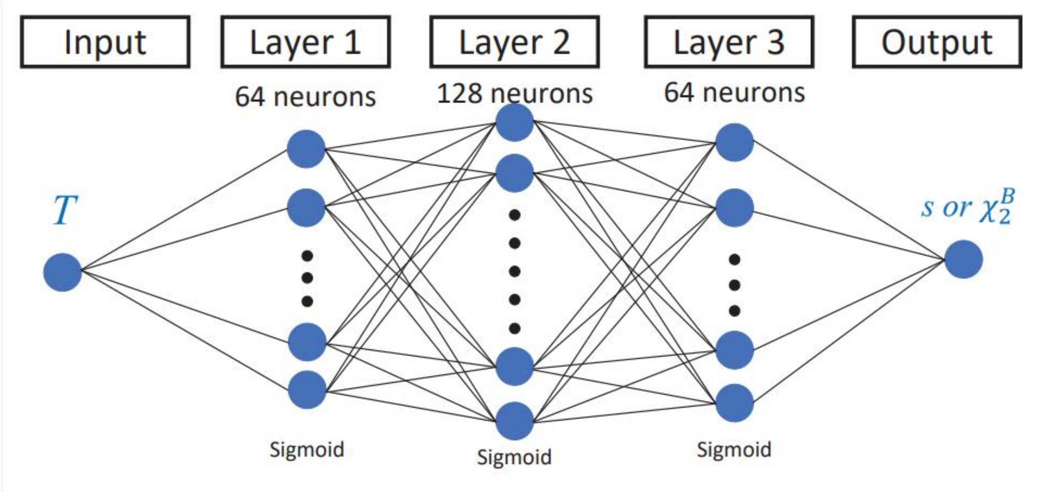


Figure 2. The difference between ML holographic model and the traditional holographic method.

Learning progress

First step：

2+1 flavor



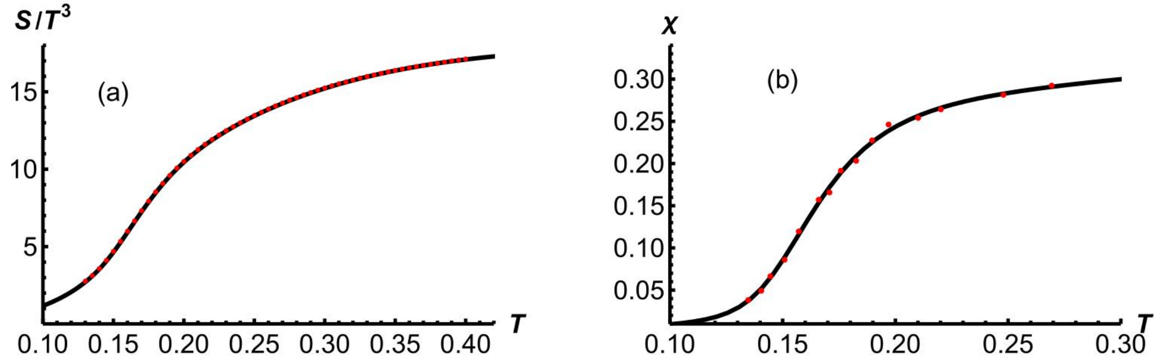


FIG. 1. (a) The entropy as a function of temperature. (b) The baryon susceptibility as a function of temperature. The dots are the results from the lattice and the black line is the prediction of the neural network. The unit of T is GeV.

We use ”TensorFlow” to build a neural network model for regression tasks.

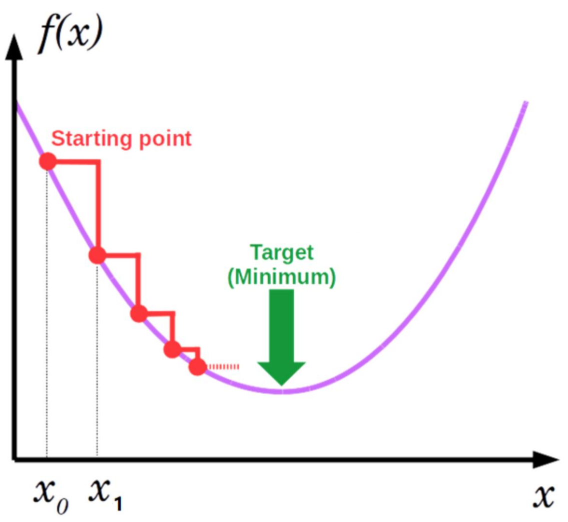
Activation function： Sigmoid

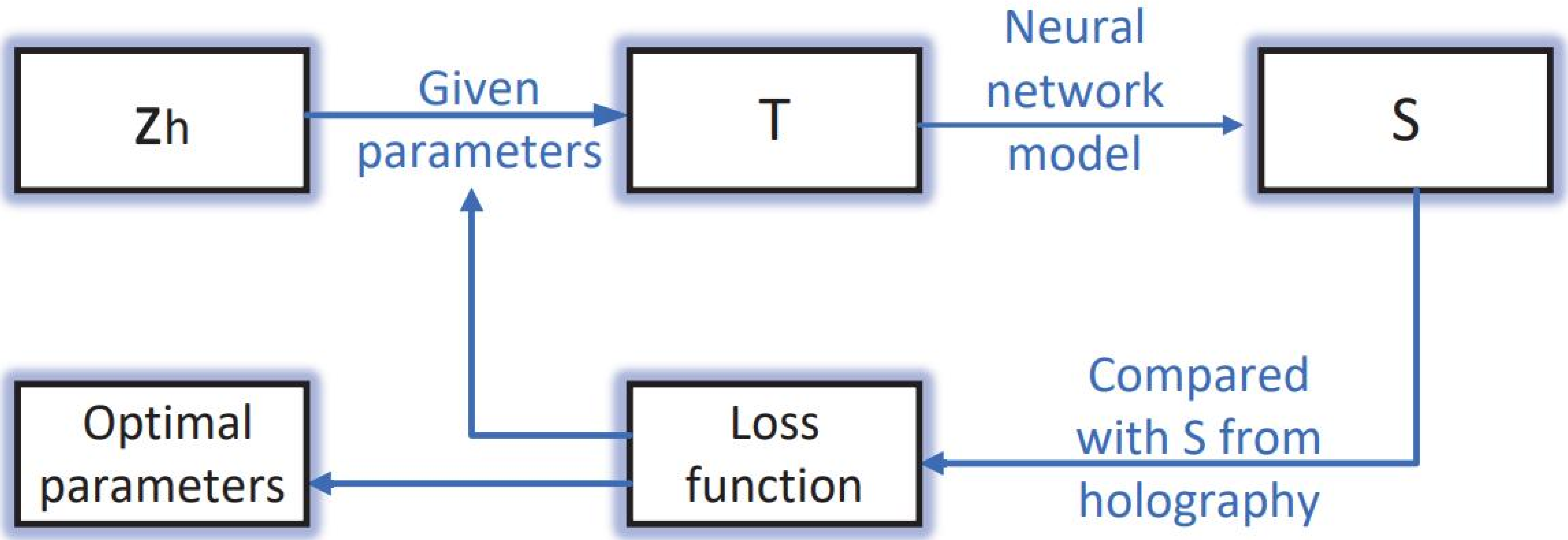
Optimizer: Adam

Learning progress

Second step：

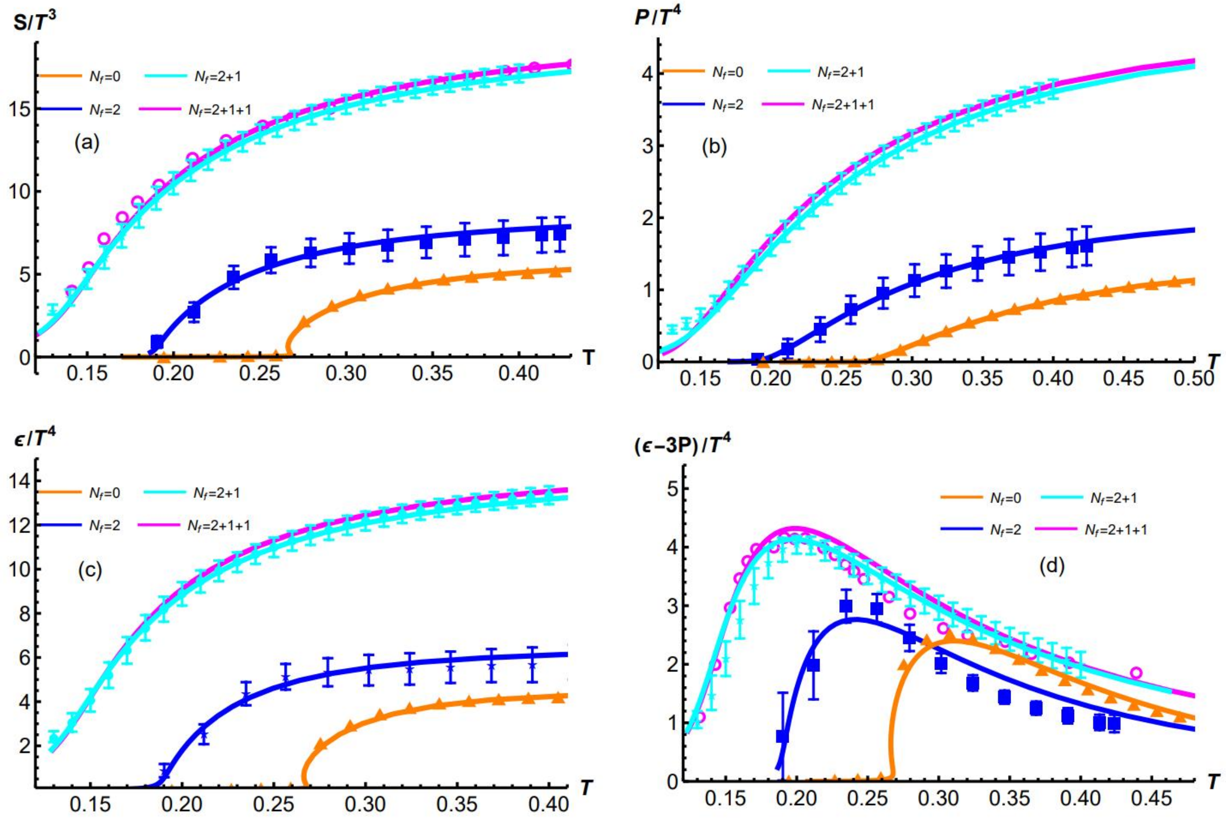
Gradient descent algorithm

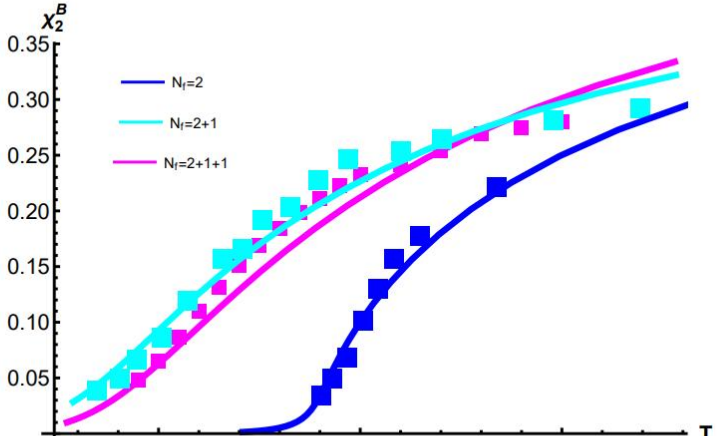


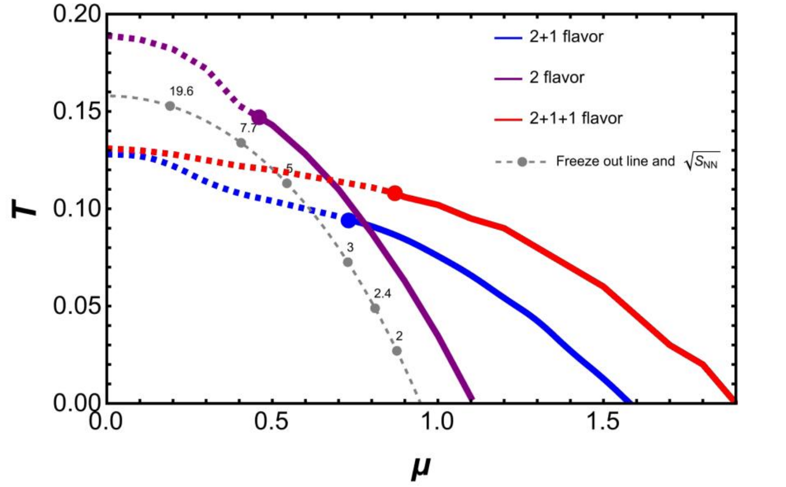


$$S_E = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{f(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right].$$

$$ds^2 = \frac{L^2 e^{2A(z)}}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + d\vec{x}^2\right] \qquad A(z) = d\ln\left(az^2+1\right) + d\ln\left(bz^4+1\right),\ f(z) = e^{cz^2 - A(z) + k}$$



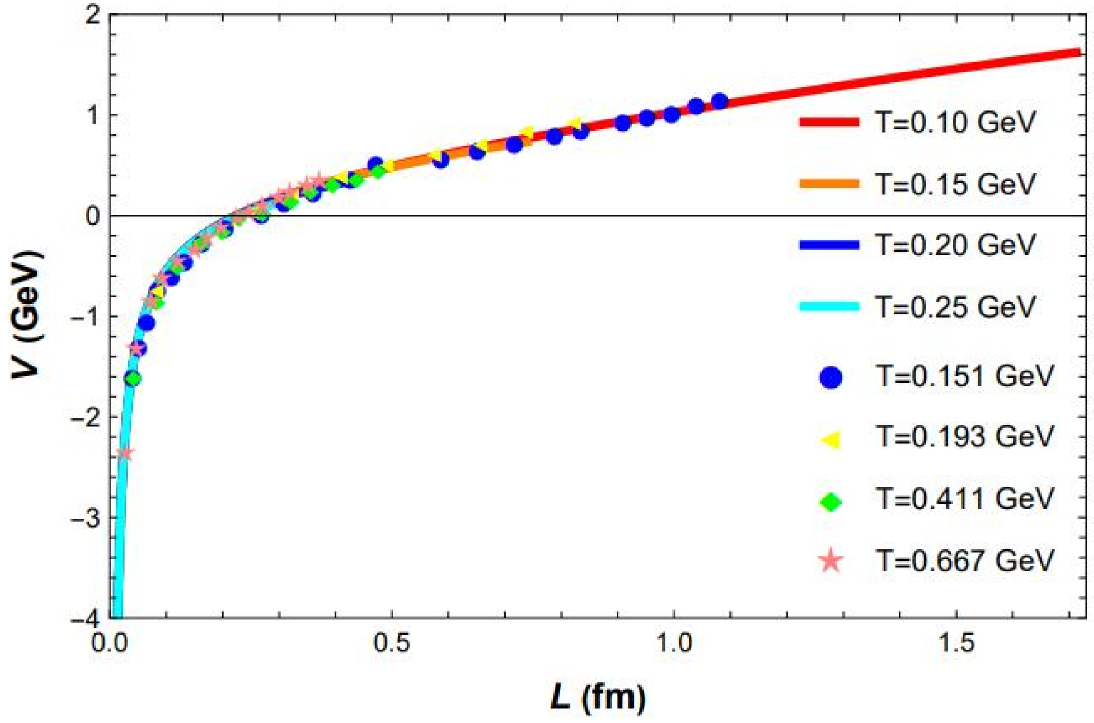


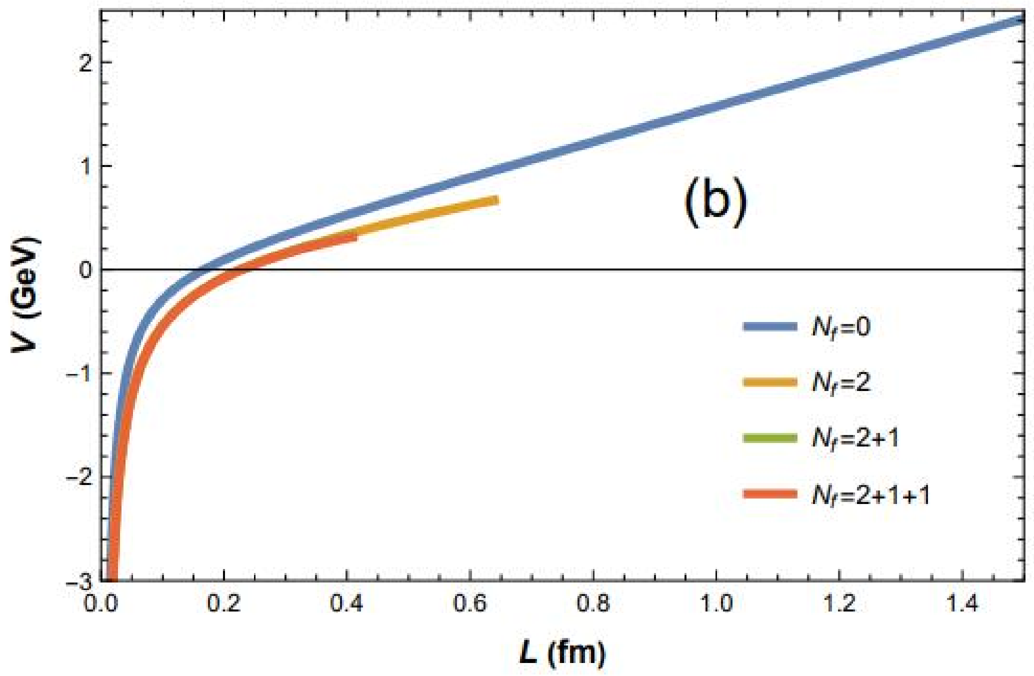


Potential energy of heavy quarkonium in flavor-dependent systems from a holographic model

Phys.Rev.D 110 (2024) 4, 046014 • e-Print: 2406.04650

Xi Guo, Xun Chen, Dong Xiang, Miguel Angel Martin Contreras, Xiao-Hua Li

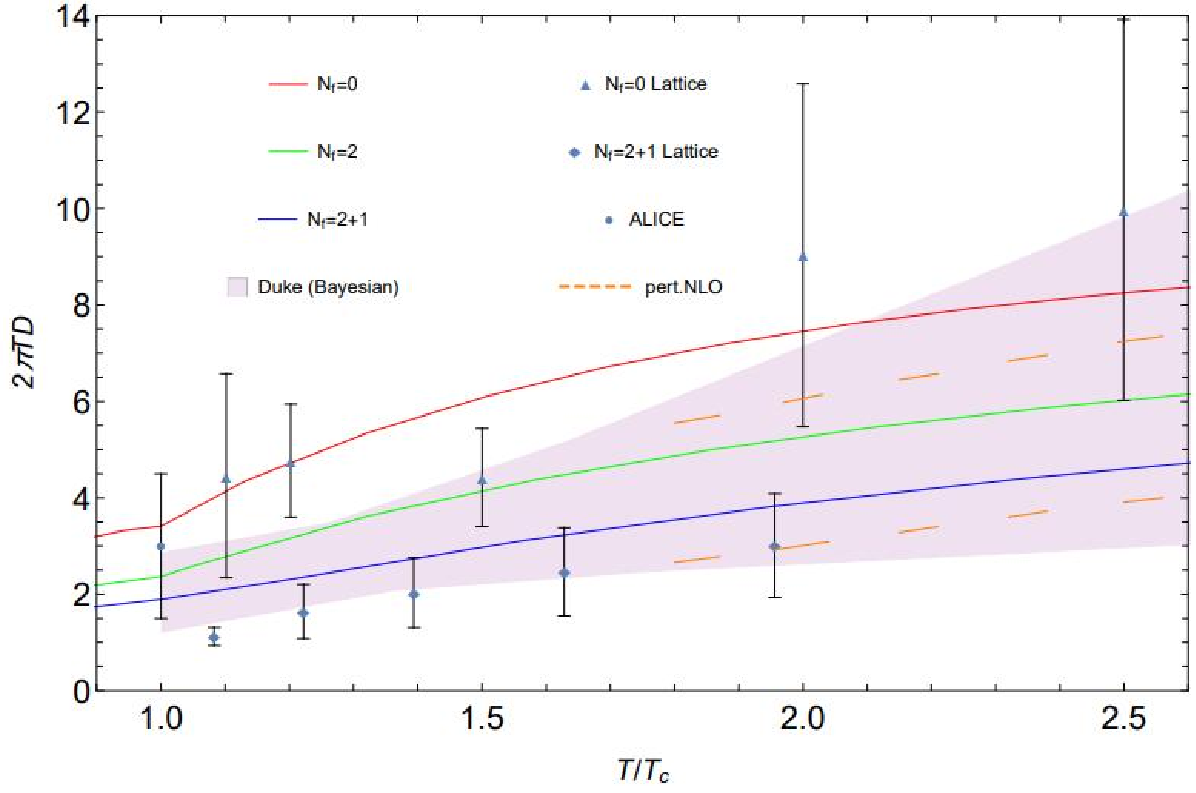


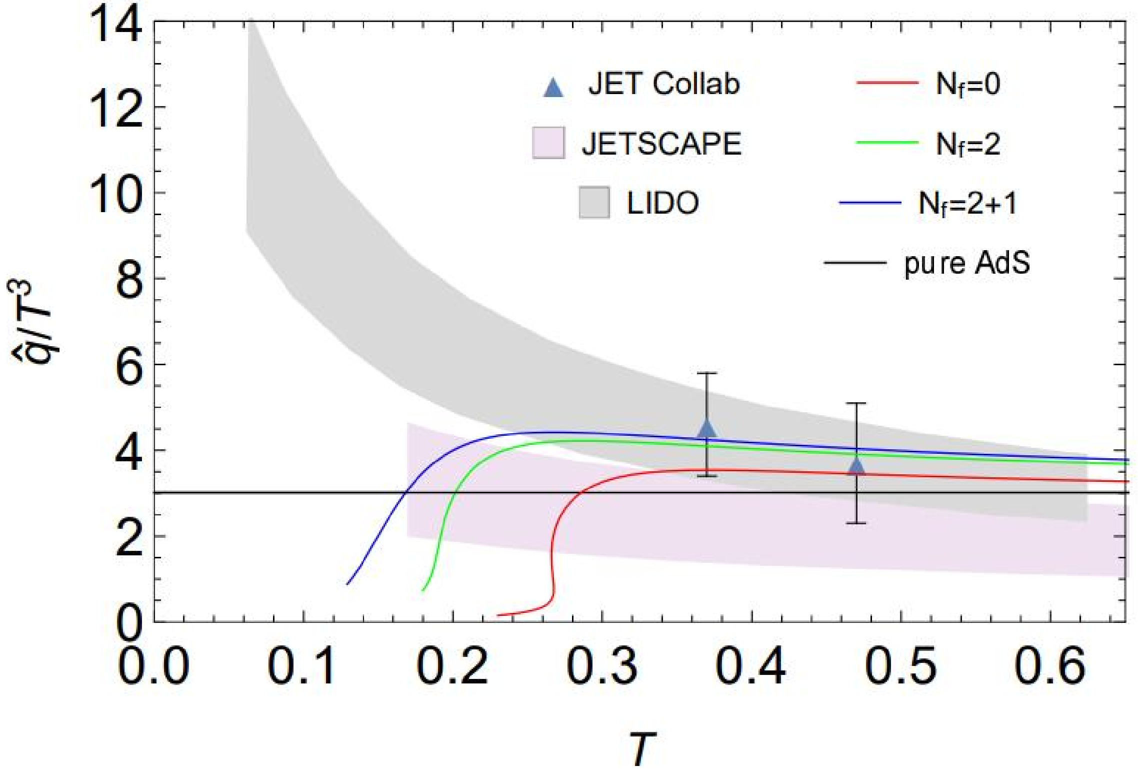


Exploring Transport Properties of Quark-Gluon Plasma with a Machine-Learning assisted Holographic Approach

e-Print: 2404.18217 (under review)

Bing Chen, Xun Chen, Xiaohua Li, Zhou-Run Zhu, Kai Zhou





Multi-Layer Perceptrons (MLP) and Kolmogorov-Arnold Networks (KAN)

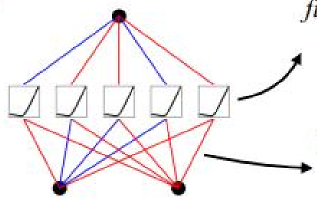
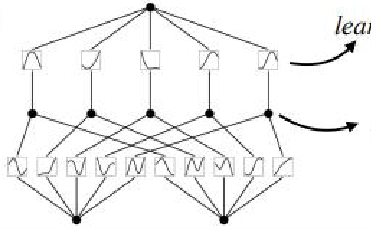
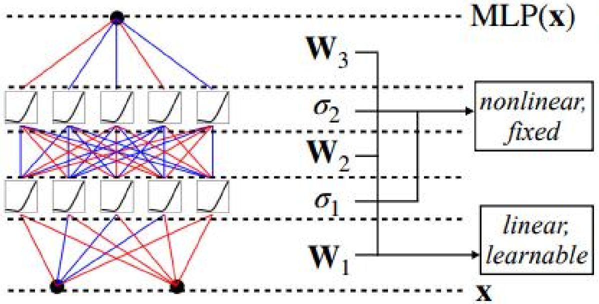
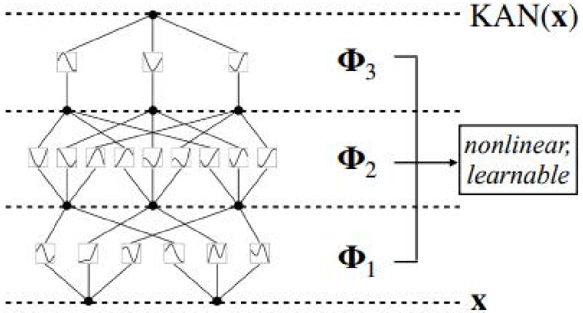
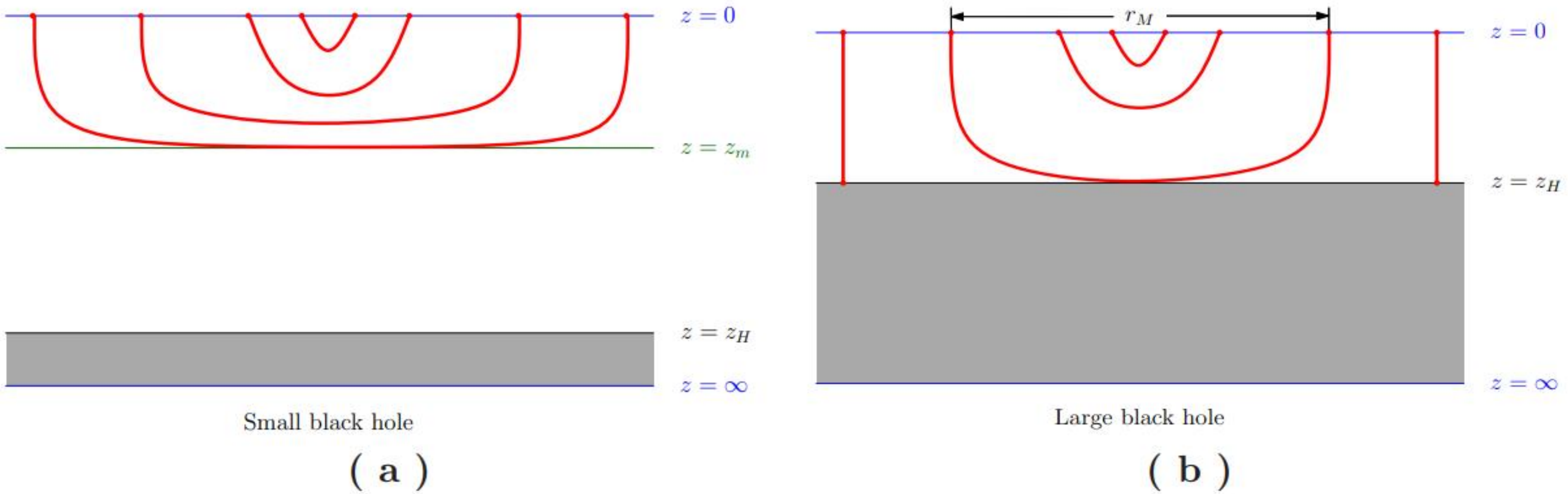
Model	Multi-Layer Perceptron (MLP)	Kolmogorov-Arnold Network (KAN)
Theorem	Universal Approximation Theorem	Kolmogorov-Arnold Representation Theorem
Formula (Shallow)	$f(\mathbf{x}) \approx \sum_{i=1}^{N(\epsilon)} a_i \sigma(\mathbf{w}_i \cdot \mathbf{x} + b_i)$	$f(\mathbf{x}) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right)$
Model (Shallow)	(a) 	(b) 
Formula (Deep)	$\mathrm{MLP}(\mathbf{x}) = (\mathbf{W}_3 \circ \sigma_2 \circ \mathbf{W}_2 \circ \sigma_1 \circ \mathbf{W}_1)(\mathbf{x})$	$\mathrm{KAN}(\mathbf{x}) = (\mathbf{\Phi}_3 \circ \mathbf{\Phi}_2 \circ \mathbf{\Phi}_1)(\mathbf{x})$
Model (Deep)	(c) 	(d) 

Figure 0.1: Multi-Layer Perceptrons (MLPs) vs. Kolmogorov-Arnold Networks (KANs)

Ziming Liu, et. al.
Arxiv: 2404.19756

Holographic heavy-quark potential



Andreev-Zakharov model
JHEP 04 (2007) 100

$$ds^2 = w(r)\frac{1}{r^2}\left[f(r)dt^2 + d\vec{x}^2 + f^{-1}(r)dr^2\right],$$

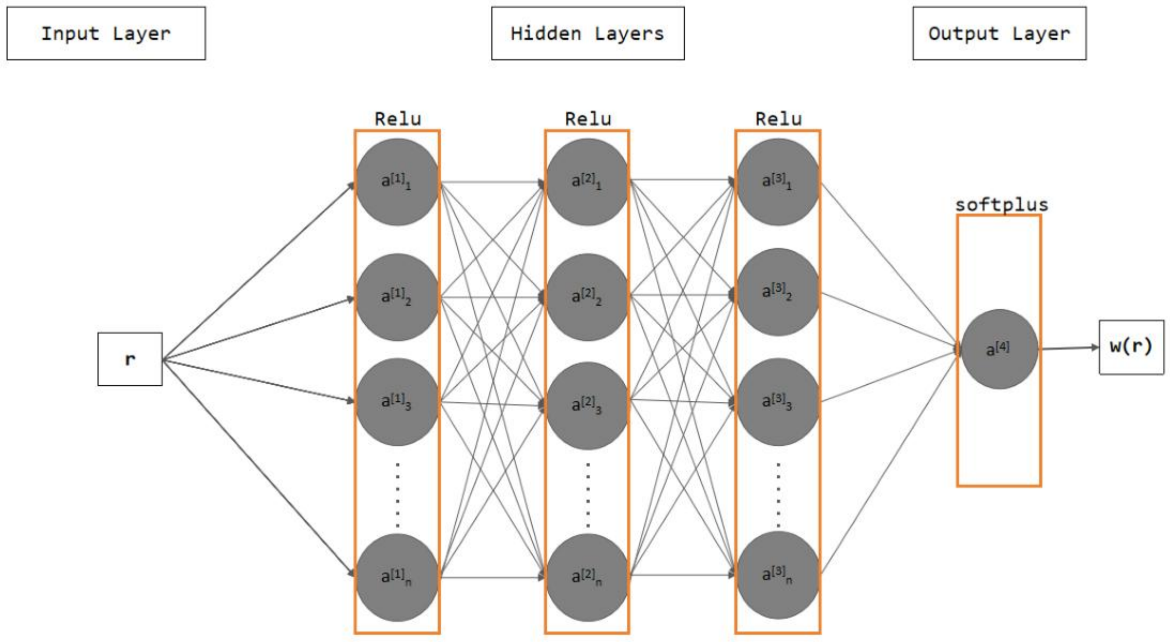
$$f(r) = 1 - \left(\frac{1}{r_h^4} + q^2 r_h^2\right) r^4 + q^2 r^6.$$

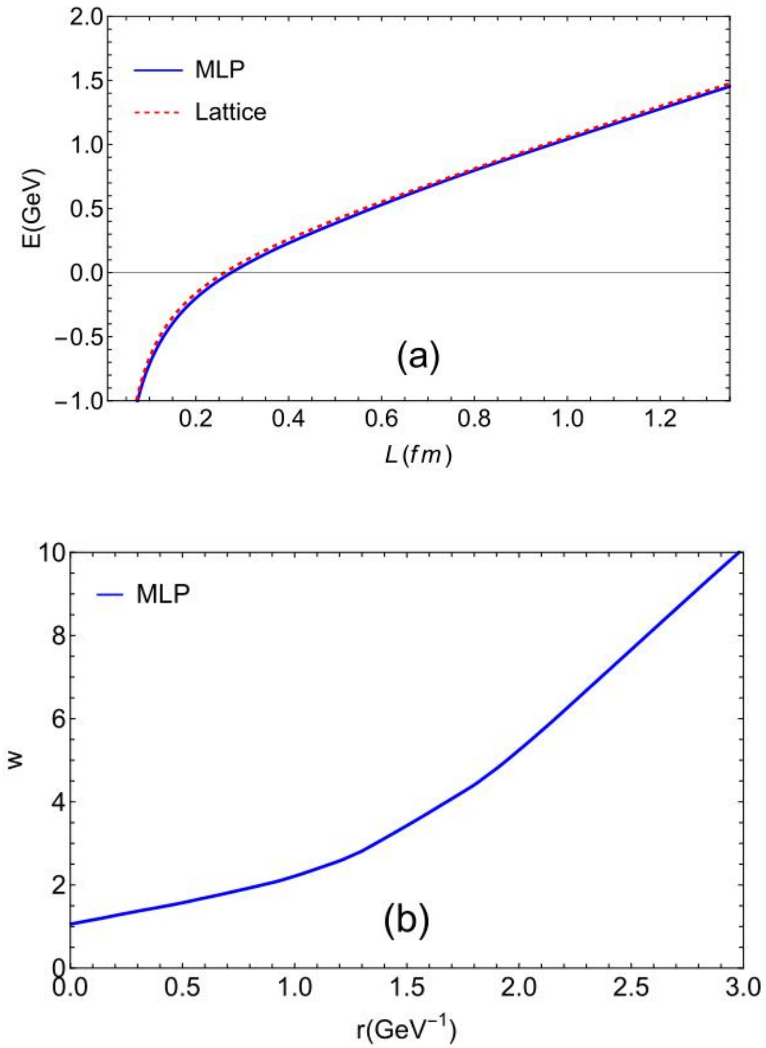
$$w(r) = 1.0e^{0.45r^2}$$

Neural Network Modeling of Heavy-Quark Potential from Holography

arXiv:2408.03784 (under review)

Ouyang Luo, **Xun Chen**, Fu-Peng Li, Xiao-hua Li, Kai Zhou





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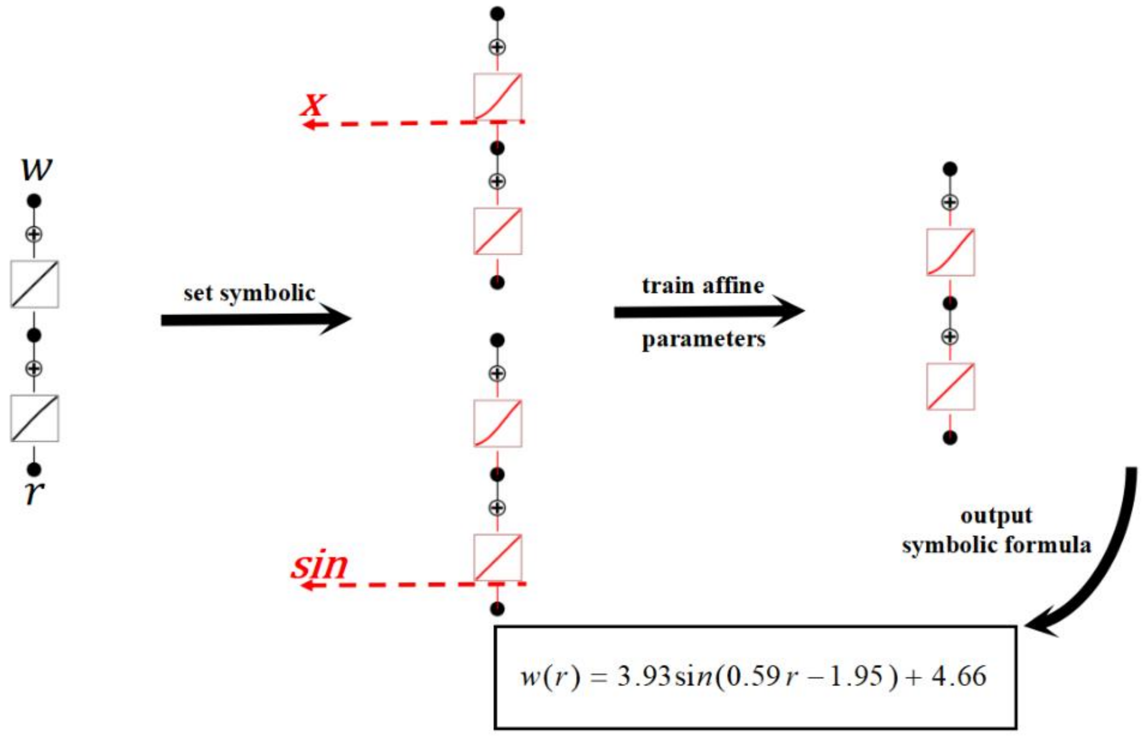
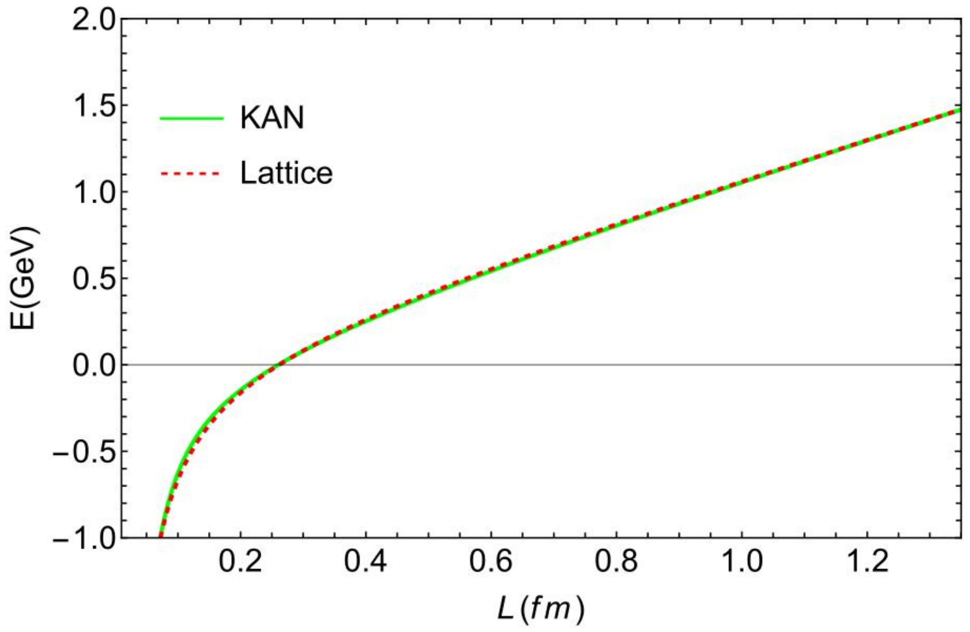
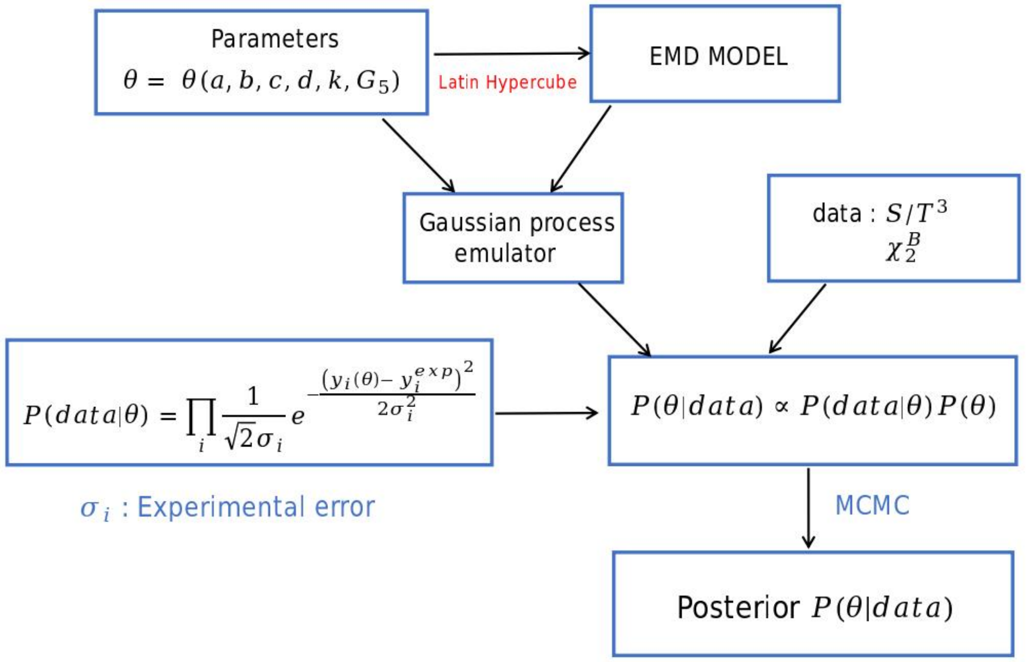




Bayesian inference of the critical endpoint for 2+1 flavor in the holographic model

In preparation,

Liqiang Zhu, **Xun Chen** , Kai Zhou, Hanzhong Zhan, and Mei Huang



$$S_E = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - \frac{f(\phi)}{4} F^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right].$$

$$A(z) = d\ln(az^2 + 1) + d\ln(bz^4 + 1),\ f(z) = e^{cz^2 - A(z) + k}$$

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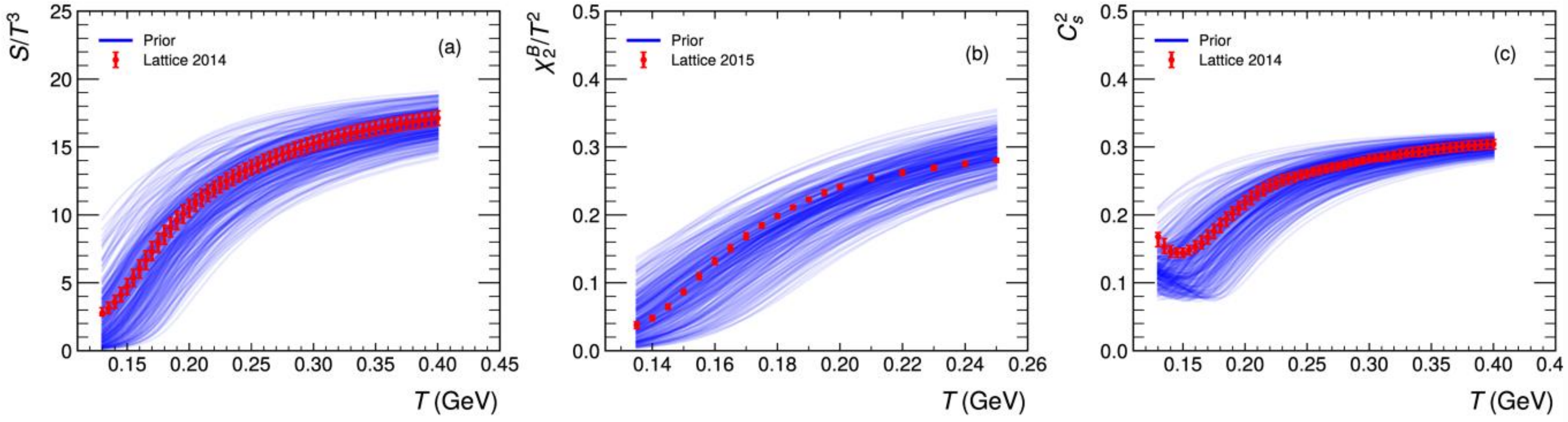
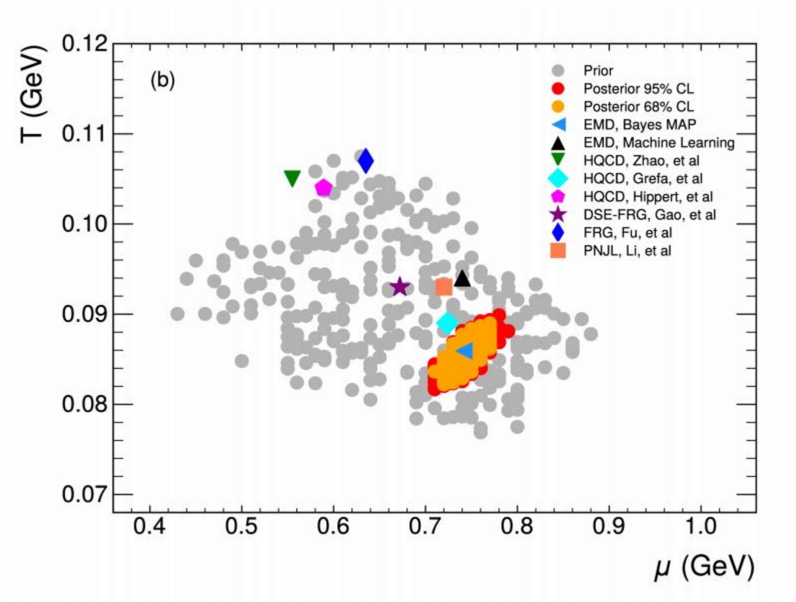




Holographic glueball spectrum and neural network

Dynamical holographic QCD model for glueball and light meson spectra

Danning Li, Mei Huang, JHEP 11 (2013) 088

Background
$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R_s + 4\partial_M\Phi\partial^M\Phi - V_G^s(\Phi)\right).$$

$$ds^2 = b_s^2(z)(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \quad b_s(z) \equiv e^{A_s(z)}.$$

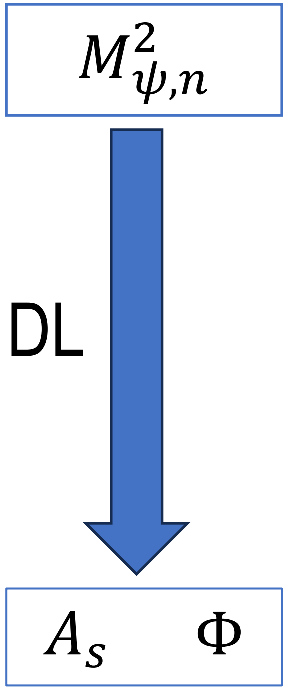
The scalar glueball $S_{\mathscr{G}} = \int d^5x\sqrt{g_s}\frac{1}{2}e^{-\Phi}\left[\partial_M\mathscr{G}\partial^M\mathscr{G} + M_{\mathscr{G},5}^2\mathscr{G}^2\right].$ EoM $-\mathscr{G}_n'' + V_{\mathscr{G}}\mathscr{G}_n = m_{\mathscr{G},n}^2\mathscr{G}_n,$

n(0^{++})	$\Phi = \mu_{G^2}^4 z^4$	
	$\mu_{G^2} = 650$	$\mu_{G^2} = 800$
0	1450	1784
1	3083	3795
2	4297	5289
3	5388	6632

n(0^{++})	$\Phi = \mu_G^2 z^2$		
	$\mu_G = 900$	$\mu_G = 1000$	$\mu_G = 1100$
0	1434	1593	1752
1	2356	2618	2880
2	2980	3311	3642
3	3489	3877	4264

Holographic glueball spectrum and neural network

Jia-Jie Jiang, **Xun Chen**, Mei Huang, in preparation



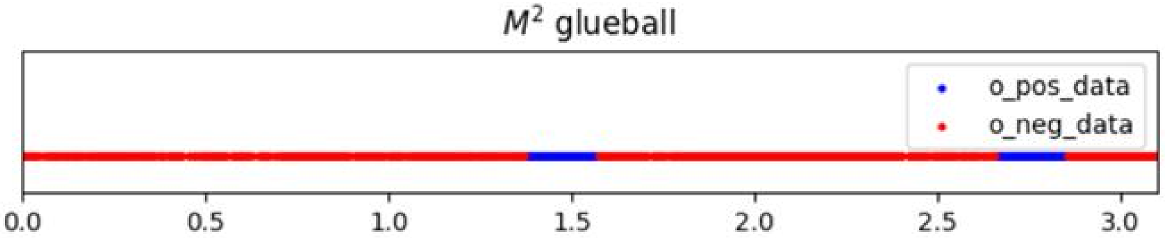
n(0^{++})	Lat1
	$N_c = 3$
1	1475(30)(65)
2	2755(70)(120)
3	3370(100)(150)
4	3990(210)(180)

Inverse problem

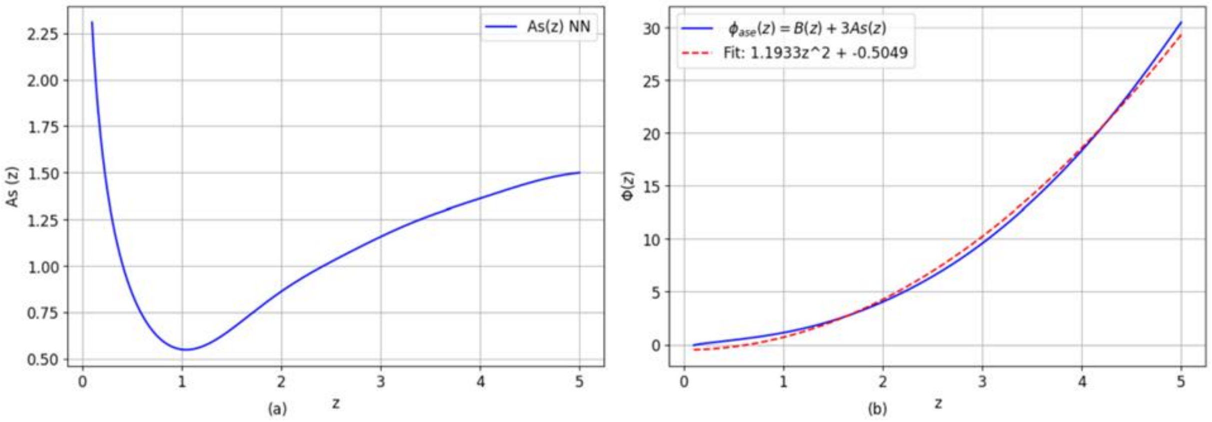
The Equation of motion for ψ has the form of

$$-e^{-(3As-\Phi)}\partial_z(e^{3As-\Phi}\partial_z\psi_n) = M^2_{\psi,n}\psi_n$$

$$-A_s''\text{-}\frac{4}{3}\Phi' A_s' + A_s'^2 + \frac{2}{3}\Phi'' = 0$$



n(0^{++})	Lat1	This Work	error
0	1.475	1.538	4.26%
1	2.755	2.710	1.62%
2	3.370	3.520	4.44%
3	3.990	4.219	5.75%



Summary

- By inputting the equation of state and baryon number susceptibility into the model, we construct the holographic model with machine learning. We calculated the results of heavy-quark potential and transport properties.

- Bayesian inference can also assist us in constructing the model by incorporating the error bars obtained from lattice data.

- Attempting to construct the holographic model using the MLP and KAN methods, based on the heavy-quark potential derived from lattice QCD.

- By inputting mass spectrum information, we aim to construct an effective holographic model.

Outlook

Can we desbribe all the sepctrums with the help of machine learning in the holographic framework.

Can we construct a holographic model with NN which can describe all the results (Include the sepctrums, EoS, Transport Properties......)?