Small Radius Inclusive Jet Production through NNLO+NNLL QCD

Based on work by

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New Opportunities in Particle and Nuclear Physics
with Energy Correlators

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UNIVERSITY OF CAMBRIDGE

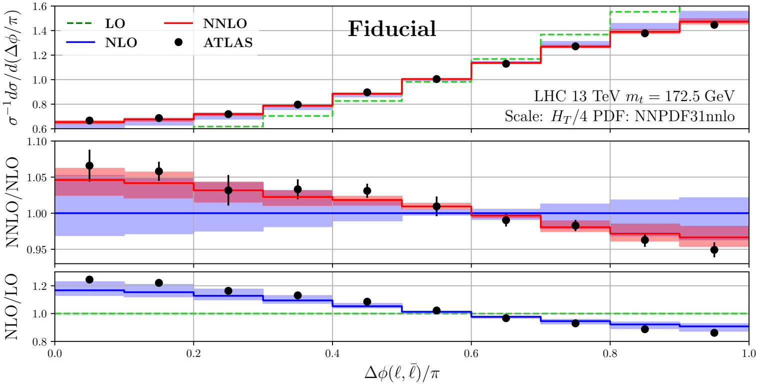
Introduction to the software

NNLO computations with STRIPPER

- STRIPPER framework: Monte Carlo code for the numerical computation of NNLO cross sections
- Designed to handle the real phase space of any NNLO cross section
- Fully general: only process-specific part: two-loop amplitudes
- Underlying technology: sector-improved residue subtraction scheme
  Czakon (2010, 2011); Czakon, Heymes (2014); Czakon, van Hameren, Mitov, Poncelet (2019)
- Completely takes care of all soft and/or collinear divergences
- Fully differential: can study any (IRC-safe) parton-level observable

Examples of results

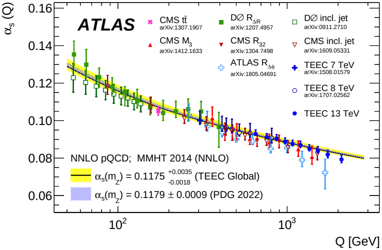
Angular correlations between leptons in $p\,p \to t\,\bar{t} + X \to \ell\bar{\ell} + Y$
Cuts on (b-)jets: non-trivial result!

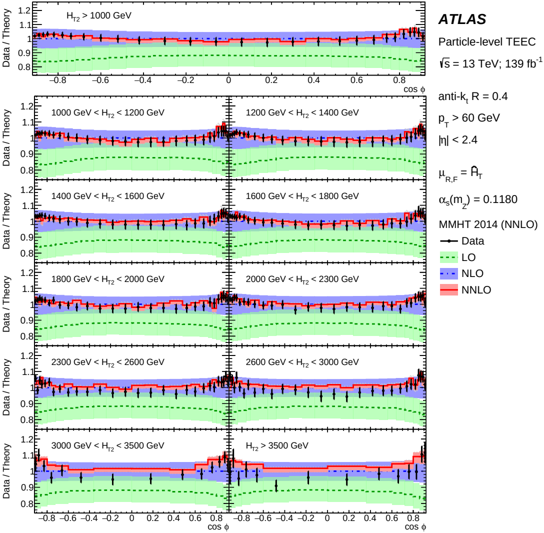


Behring, Czakon, Mitov, Papanastasiou, Poncelet (2019)

Examples of results

$$\frac{1}{\sigma}\frac{\mathrm{d}\Sigma}{\mathrm{d}\cos\phi} \equiv \frac{1}{\sigma}\sum_{ij}\int \frac{\mathrm{d}\sigma}{\mathrm{d}x_{\mathrm{T}i}\mathrm{d}x_{\mathrm{T}j}\mathrm{d}\cos\phi} x_{\mathrm{T}i}x_{\mathrm{T}j}\mathrm{d}x_{\mathrm{T}i}\mathrm{d}x_{\mathrm{T}j} = \frac{1}{N}\sum_{A=1}^{N}\sum_{ij}\frac{E^A_{\mathrm{T}i}E^A_{\mathrm{T}j}}{\left(\sum_k E^A_{\mathrm{T}k}\right)^2}\delta(\cos\phi - \cos\varphi_{ij})$$





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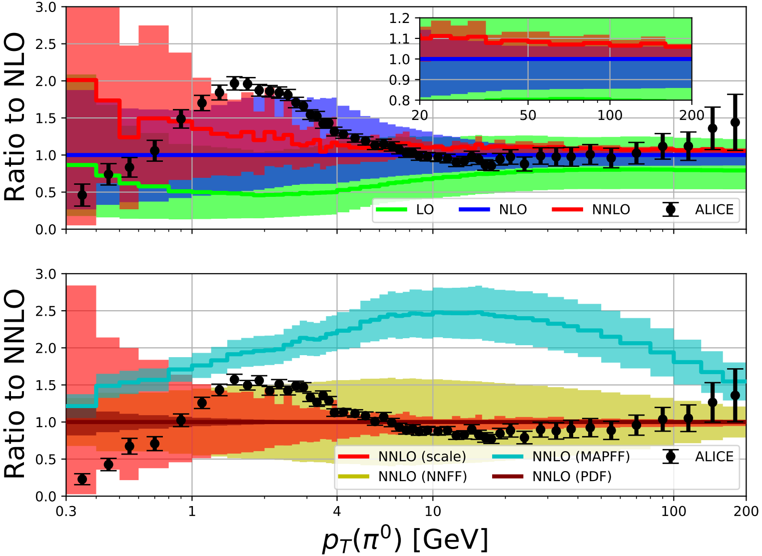
STRIPPER and fragmentation

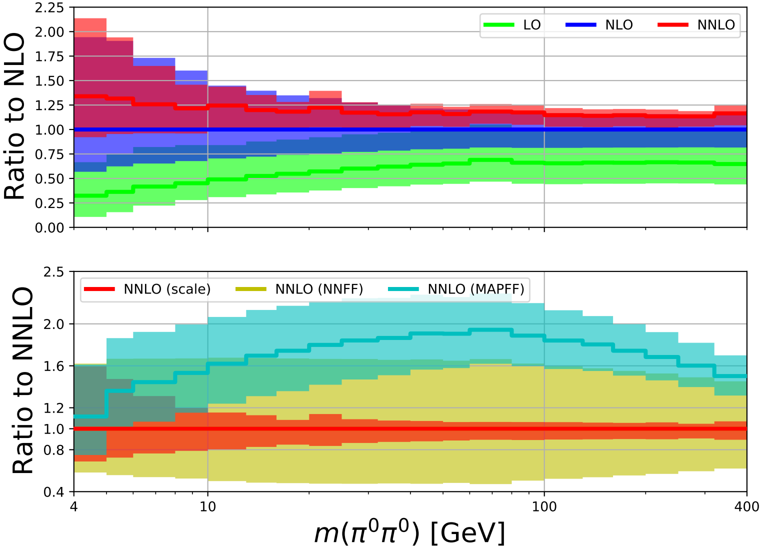
- Original STRIPPER implementation: parton-level final states only
- Extended to support fragmentation a few years ago

  Czakon, TG, Mitov, Poncelet (2021)
- As for the base code: fully general implementation
- I.e.: any process with any number of identified hadrons supported!

Example: π^0 p_T spectrum at 8 TeV LHC (ALICE)



Example: $\pi^0\pi^0$ invariant mass spectrum at 13 TeV LHC



Small radius jets at the LHC

Goal and background

- High-precision predictions for small-radius jet production
- NLO+NLL threshold and $\ln R$ resummation already available

  NLL threshold: Kidonakis, Oderda, Sterman (1998); Kidonakis, Owens (2000)
  NLL $\ln R$: Dasgupta, Dreyer, Salam, Soyez (2014,2016); Kang, Ringer, Vitev (2016); Dai, Kim, Leibovich (2016)
  NLL threshold+$\ln R$: Liu, Moch, Ringer (2017, 2018); Moch, Eren, Lipka, Liu, Ringer (2018)
- Fixed-order NNLO inclusive jet and dijet also available

  Currie, Glover, Pires (2016); Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires (2017, 2018); Czakon, van Hameren, Mitov, Poncelet (2019); Chen, Gehrmann, Glover, Huss, Mo (2022)
- Now: resum $\ln^m R$ to NNLL and match to NNLO
- Factorisation very similar to fragmentation: Lee, Moult, Zhang (2024)

$$\frac{d\sigma_{\text{jet}}}{dp_T}(p_T, R) = \sum_i \int_0^1 \frac{dz}{z} \frac{d\sigma_i}{dp_T}(p_T/z, \mu_J) J_i \left(z, \ln \frac{p_T R}{z\,\mu_J}\right) + \mathcal{O}(R^2 \ln^m R)$$

- J_i is the 'FF' for producing a jet with radius R from parton i
- $\Rightarrow$ Can repurpose fragmentation implementation in STRIPPER!

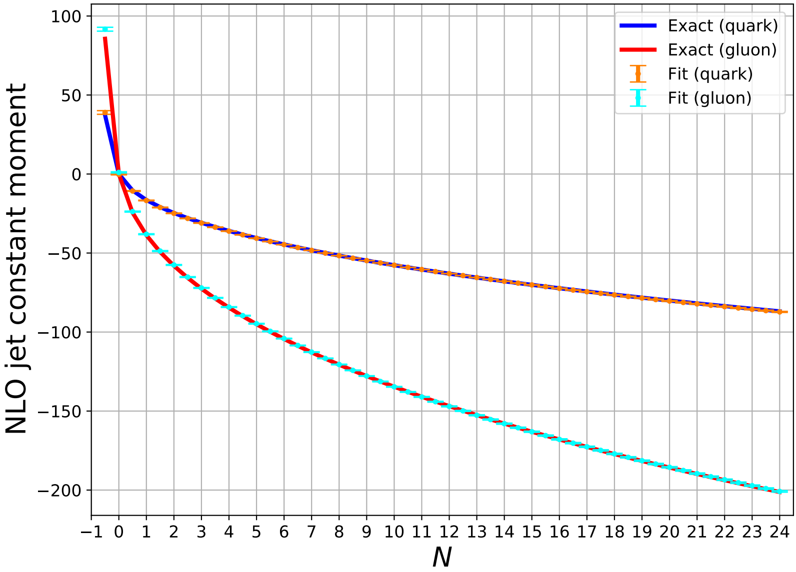
Approach

- DGLAP evolution performed by truncating at high order
- Converges well and gives precise control over included terms
- Matching trivial: $\sigma = (\text{exact NNLO}) + (\text{LP beyond NNLO})$
- Requires convolutions with many different distributions
- In practice: α_s^5 for LL and NLL and α_s^4 for NNLL terms
- $\Rightarrow$ Need convolutions with $\left(\frac{\ln^5(1-x)}{1-x}\right)_+$
  $\Rightarrow$ Need very robust and stable code
- STRIPPER generalised to support arbitrary distributions

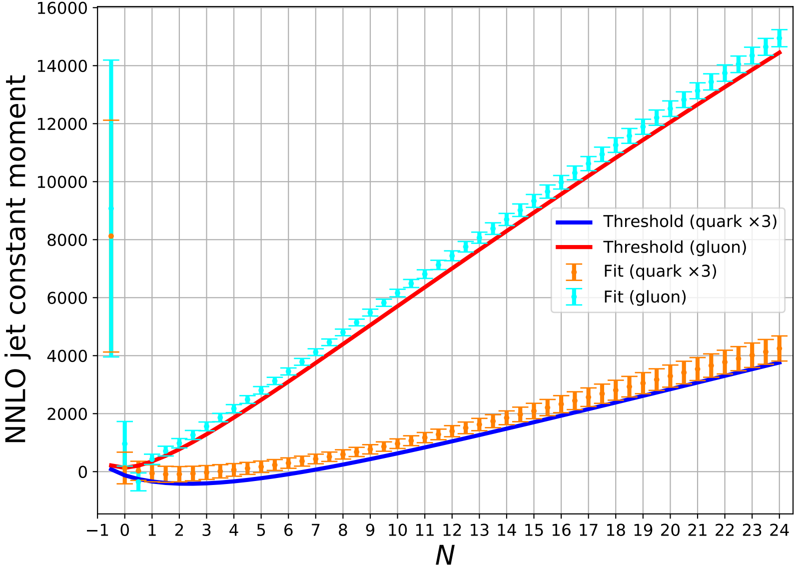
NNLO jet constant

- NNLL part of NNLO jet functions, i.e. $\mathcal{O}(\ln^0 R)$ not known
- But: can compute exact, fixed-order NNLO cross section
- Extract unknown terms by comparing exact and factorised result!
- In practice: cross section moments double-differential in y and $\hat{H}_T$
- Also split up cross section according to initial-state partons
- Allows to disentangle quark and gluon-initiated jet very well
- Computed at $R = 0.1$, power corrections found to be negligible
- Obtained the first 50 half-integer moments of both J_q and J_g

Cross-check: NLO jet constant



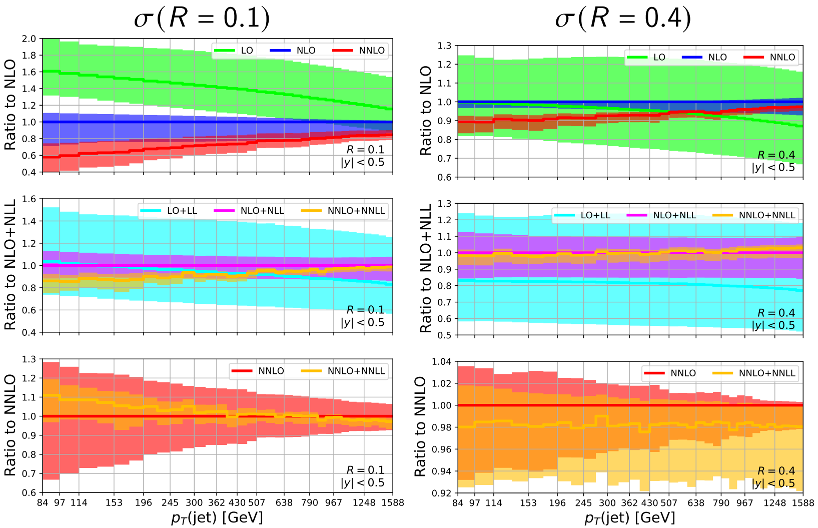
Result: NNLO jet constant



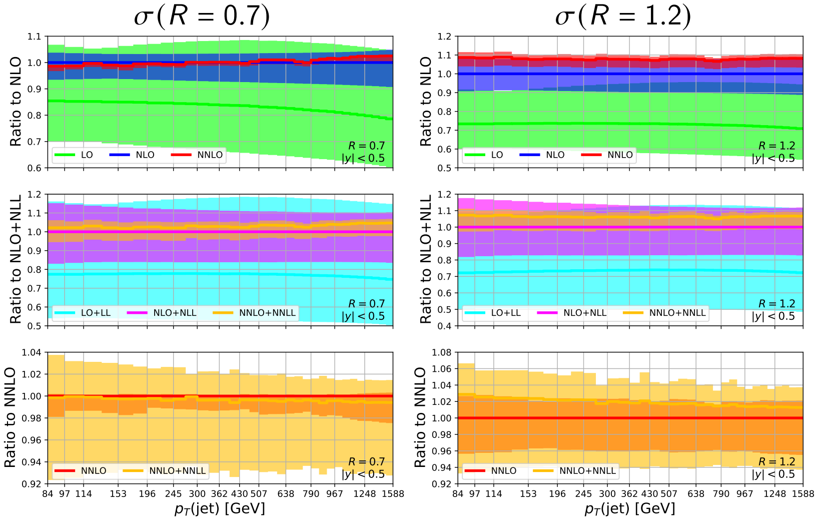
The measurement

- Ultimately want to compare to data
- arXiv:2005.05159: '3D' measurement of inclusive jets by CMS
- Double-differential in p_T and y for $R = 0.1, 0.2, ..., 1.2$
- Absolute spectra not provided; only ratio's w.r.t. $R = 0.4$
- Will use same binning and cuts to facilitate comparison

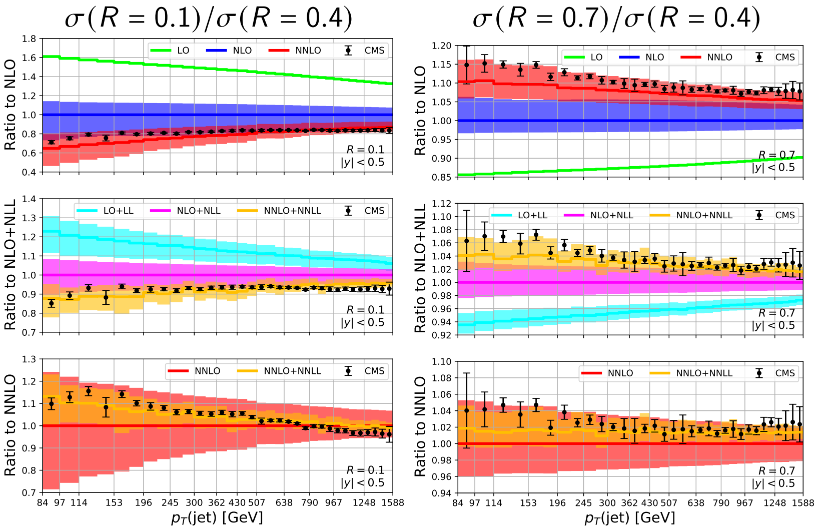
Results: cross sections at 13 TeV LHC



Results: cross sections at 13 TeV LHC



Results: cross section ratios at 13 TeV LHC



Conclusion & outlook

- Highlighted the STRIPPER framework
- Can compute any fully differential IRC-safe NNLO cross section
- Can now convolve with any (1D) distribution - useful for many things!
- Here: first NNLO+NNLL calculation of small radius jets at the LHC
- Reduced or more reliable uncertainties w.r.t. FO NNLO
- Better agreement with data w.r.t. both FO NNLO and NLO+NLL
- Important part of energy correlator calculations

Conclusion & outlook

Many directions to explore:

- For e.g.: N-point energy correlators in the collinear limit:

$$\Sigma^{[N]}\left(R_0, R_L, \ln\frac{p_T^2}{\mu^2}\right) = \int_0^1 dx\, x^N \vec{J}^{[N]}\left(\ln\frac{R_L^2 x^2 p_T^2}{\mu^2}\right)\cdot\vec{H}\left(R_0, x, \ln\frac{p_T^2}{\mu^2}\right)$$

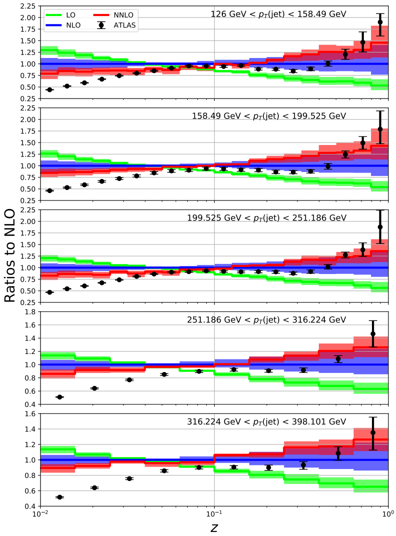
- Can achieve NNLO+NNLL using STRIPPER
- Straightforward application of existing implementation - stay tuned!
- Can also convolve with two or more functions (e.g. small radius dijet)
- Generalisable in many directions: track functions, di-hadron FFs, fragmenting jet functions, ...

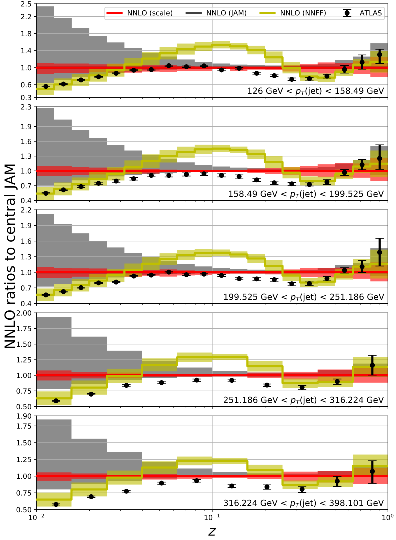
We would love to hear any thoughts and ideas you may have!

Backup

Example: charged hadrons at 5.02 TeV LHC (ATLAS)

Jets with $R = 0.4$ anti-k_T, $z = p_T(h)\cos(\Delta R)/p_T(\text{jet})$





Not quite fragmentation

- Kinematics-dependent cutoff scale $R\, p_T$ in the jet functions
- $J_i\left(z, \ln \frac{p_T(\text{jet}) R}{z\, \mu_J}\right) = J_i\left(z, \ln \frac{p_T(\text{parton}) R}{\mu_J}\right)$
- Choice: fix p_T(parton) or p_T(jet)?
- Leads to difference in convolution structure / DGLAP evolution

  van Beekveld, Dasgupta, El-Menoufi, Helliwell, Karlberg, Monni (2024); van Beekveld, Dasgupta, El-Menoufi, Helliwell, Monni, Salam (2024); Lee, Moult, Zhang (2024)
- I.e. when fixing p_T(parton):

$$\frac{d\vec{J}\left(z, \ln \frac{p_T(\text{parton}) R}{\mu_J}, \mu\right)}{d \ln \mu^2} = \int_z^1 \frac{dy}{y} \vec{J}\left(\frac{z}{y}, \ln \frac{y^2 p_T(\text{parton}) R}{\mu_J}, \mu\right) \cdot \widehat{P}_T(y)$$