Energy Flow in DIS at twist-4

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Progress report

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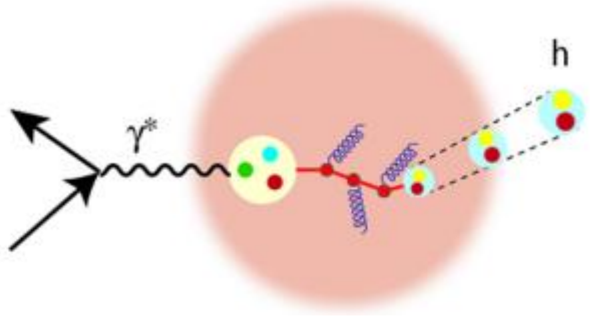
Introduction

- Multiple scatterings in deep inelastic scattering, hadron-nucleus, and heavy-ion collisions lead to important phenomena like **jet quenching** and **transverse momentum broadening** which serve as tools to study the properties of cold and hot nuclear matter.
- These effects have been observed at fixed-target experiments (DESY, Jefferson Lab, Fermilab) and collider experiments (RHIC, LHC), and will be a major focus at the future Electron-Ion Collider (EIC).



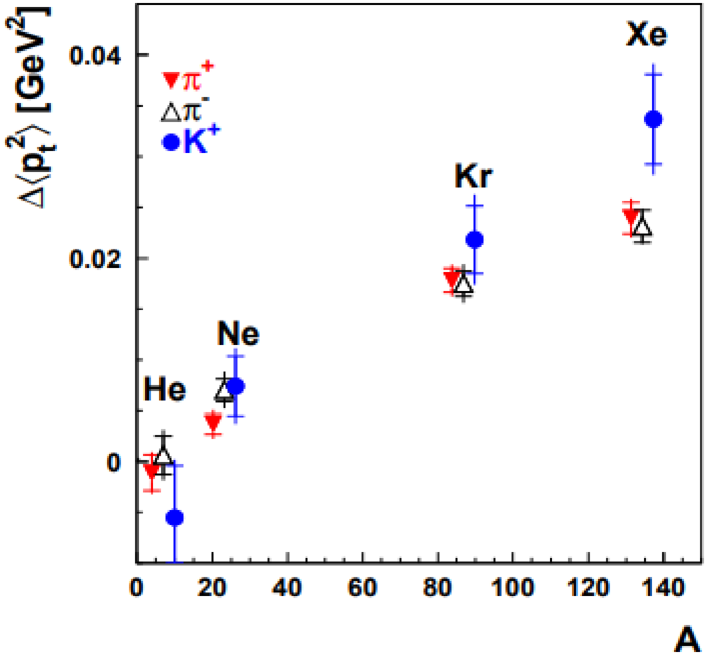
[1] Guiot, B.; Kopeliovich, B. (2020)

Higher Twist

- One of the approaches to study effects of multiple scatterings is based on a generalized high-twist factorization theorem.

- Most studies have focused on double parton scatterings and their effect on transverse momentum broadening, which leads to nuclear enhancement.

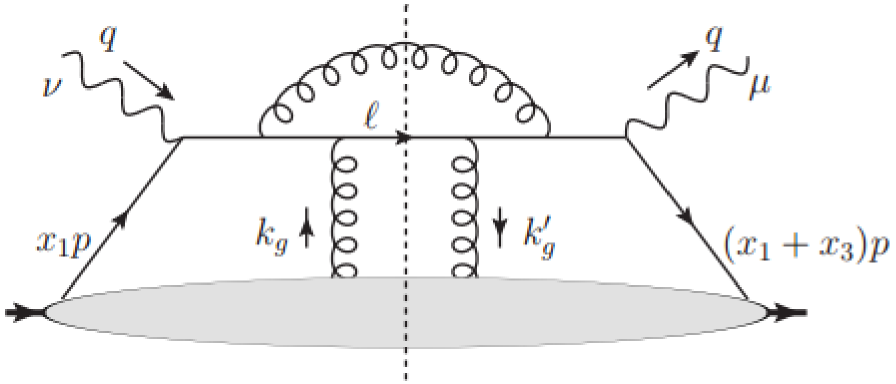
$$\Delta\langle p_t^2\rangle_A^h = \langle p_t^2\rangle_A^h - \langle p_t^2\rangle_D^h$$



[2] HERMES Collaboration, A. Airapetian et al., arXiv:0906.2478.

Transverse Momentum Broadening

- The transverse-momentum weighted NLO SIDIS cross section at twist-4 has been calculated by [Kang Z., Wang E., Wang X., Xing H. (2014)].
- It was shown that the cross-section factorizes.
- Soft divergences in real and virtual corrections cancel.
- Collinear divergences can be absorbed into the standard fragmentation function and/or the twist-4 parton correlation function of the nuclear state.

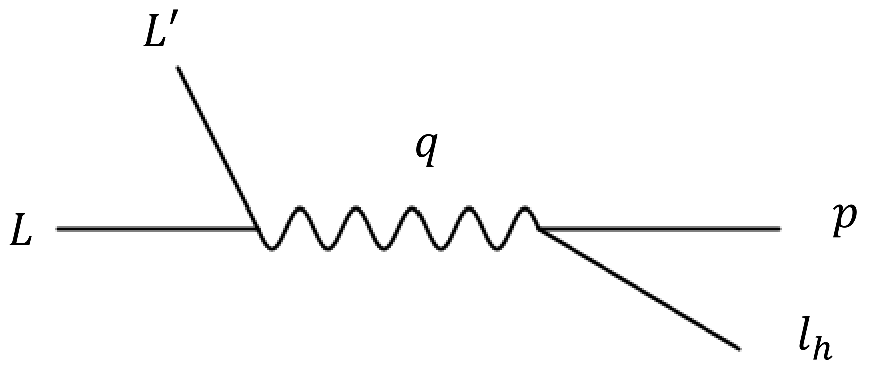


IR-safe quantity

- It is possible to consider quantities that measure the transverse spread of the QCD jets which do not depend on how partons fragment into hadrons. These quantities are infrared safe.
- Non-infrared-safe quantities need some information on what are the quark and gluon fragmentation functions, but no such information is needed for infrared-safe quantities.
- We want to use the angular spread of the energy flow as an illustration of an infrared-safe measure of transverse size.

Kinematics

- Consider a lepton l scattering off a large nucleus A.



$$l(L) + A(p) \rightarrow l(L') + h(l_h) + X$$

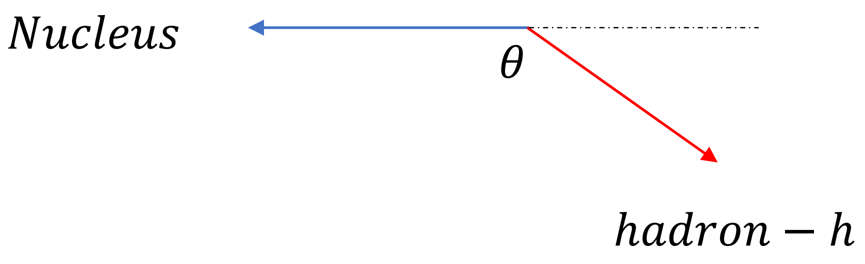
$$Q^2 = -q^2$$

- The usual SIDIS Lorentz-invariant variables are defined as follows:

$$x_B = \frac{Q^2}{2p.q} \qquad y = \frac{p.q}{p.L} \qquad z_h = \frac{p.l_h}{p.q}$$

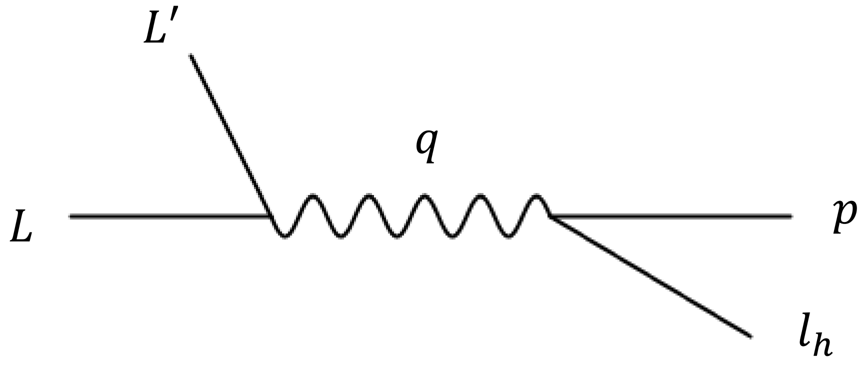
Energy Flow

- Energy Flow definition in SIDIS:



$$\frac{d\Sigma}{dx_B dQ^2 d\cos\theta} = \sum_h \int_0^1 dz_h\, z_h \left[\frac{d\sigma}{dx_B dQ^2 d\cos\theta\, dz_h}\right]$$

where θ is the angle between the hadron h and the nucleus.



Energy Flow

- Using the fact that the fragmentation functions obeys the momentum sum rule :

$$\sum_h \int_0^1 dz_h \, z_h \, D_{q/h}(z_h) = 1,$$

and one can show that $\frac{d\Sigma}{dx_B dQ^2 d\cos\theta}$ is free of the final-state divergence.

The width of Energy Flow

- The angular spread of a QCD jet can be described by

$$\langle \sin^2\theta \rangle = \frac{\int d cos\theta \, \sin^2\theta \, \frac{d\Sigma}{dx_B dQ^2 d cos\theta}}{\int d cos\theta \, \frac{d\Sigma}{dx_B dQ^2 d cos\theta}}$$

- Therefore, $\langle \sin^2\theta \rangle$, besides depending only on the parton structure functions, is normalized by the total cross section.
- Thus a calculation of $\langle \sin^2\theta \rangle$ will need less experimental input than what is needed for the average transverse momentum.

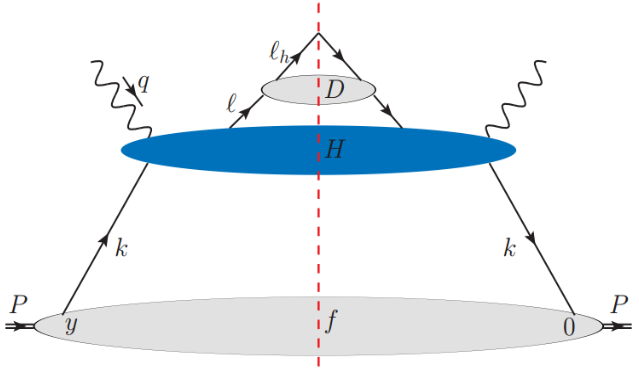
A measure of nuclear effects

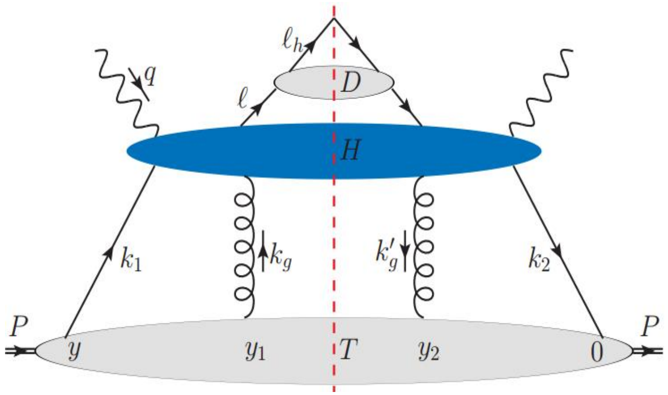
- We want to investigate the nuclear effects on the width of the energy flow using

$$\Delta\langle\sin^2\theta\rangle = \langle\sin^2\theta\rangle_{eA} - \langle\sin^2\theta\rangle_{\mathrm{ep}}$$

as function of θ at fixed x_B and Q^2.

$$d\sigma = d\sigma^{LT} + d\sigma^{T4} + \cdots$$





$$\boxed{\Delta\langle\sin^2\theta\rangle \approx \frac{\int d\cos\theta\,\sin^2\theta\,\frac{d\Sigma^{\mathrm{T4}}}{dx_B dQ^2 d\cos\theta}}{\int d\cos\theta\,\frac{d\Sigma}{dx_B dQ^2 d\cos\theta}}}$$

Another measure

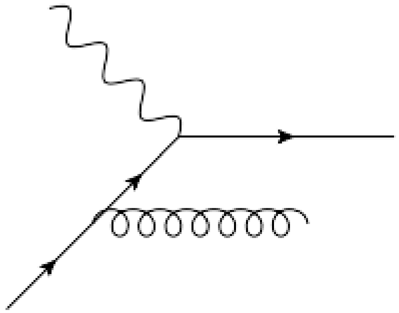
- One can also consider the ratio

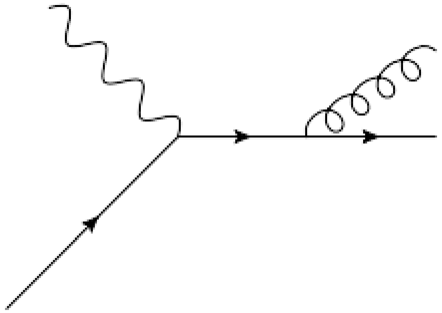
$$R_{eA/ep} = \langle \sin^2 \theta \rangle_{eA} / \langle \sin^2 \theta \rangle_{\mathrm{ep}}$$

to measure the in-medium modification to the width of the energy flow as function of θ at fixed x_B and Q^2.

How exactly does the final state divergence cancel?

- Example: NLO cross section at Leading Twist





- The Energy flow for $\gamma^* + q \ \rightarrow q + g$ should have two contributions:

$$\frac{d\Sigma}{dx_B dQ^2 dcos\theta} = \sum_h \int_0^1 dz_h\, z_h \left[\frac{d\sigma^{LT}}{dx_B dQ^2 dcos\theta dz_h}\right]_q + \sum_h \int_0^1 dz_h\, z_h \left[\frac{d\sigma^{LT}}{dx_B dQ^2 dcos\theta dz_h}\right]_g$$

NLO cross section at Leading Twist

- The cross section has both initial and final state IR logs that can be factorized.

- The initial and final state divergences can be combined with the bare PDF/FF to yield a $scale$-dependent PDF/FF.

- The weighting with z_h and the sum over all hadrons allows us to remove all reference to FF by using the momentum sum rule.

NLO at Twist-2

$$
\begin{aligned}
\frac{d\sigma}{d\mathcal{PS}} =&\sigma_0 \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} f_{q/A}(x,\mu_f^2) D_{h/q}(z,\mu_f^2)\delta(1-\hat{x})\delta(1-\hat{z}) \\
&+ \sigma_0 \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} f_{q/A}(x,\mu_f^2) D_{h/q}(z,\mu_f^2) \left\{ \ln\left(\frac{Q^2}{\mu_f^2}\right) \left[P_{qq}(\hat{x})\delta(1-\hat{z}) + P_{qq}(\hat{z})\delta(1-\hat{x})\right] + H_{T2-qq}^{NLO} \right\} \\
&+ \sigma_0 \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} f_{q/A}(x,\mu_f^2) D_{h/g}(z,\mu_f^2) \left[\ln\left(\frac{Q^2}{\mu_f^2}\right) P_{gq}(\hat{z})\delta(1-\hat{x}) + H_{T2-qg}^{NLO} \right] \\
&+ \sigma_0 \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} f_{g/A}(x,\mu_f^2) \left[D_{h/q}(z,\mu_f^2) + D_{h/\bar{q}}(z,\mu_f^2)\right] \left[\ln\left(\frac{Q^2}{\mu_f^2}\right) P_{qg}(\hat{x})\delta(1-\hat{z}) + H_{T2-gq}^{NLO} \right],
\end{aligned}
$$

- The IR-sensitive part has the form

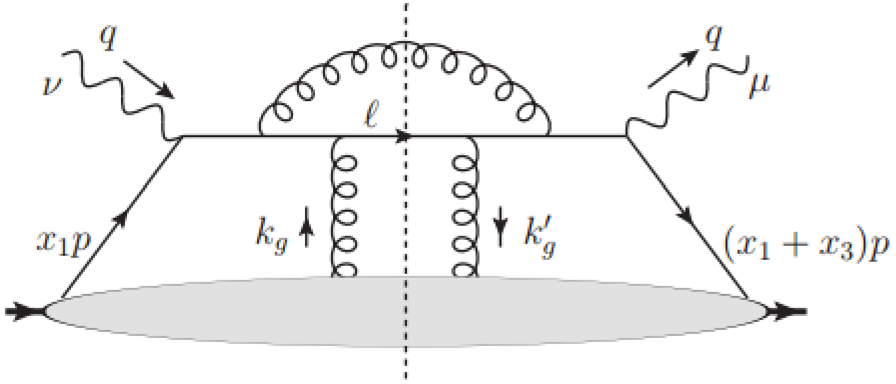
$$I_{IR} = -\frac{1}{\epsilon}\,\delta(1-\hat{x})\left[\int_0^1 d\hat{z}_q\;\hat{z}_q P_{qq}(\hat{z}_q)\;+\int_0^1 d\hat{z}_g\;\hat{z}_g P_{qg}(\hat{z}_g)\right]$$
$$= -\frac{1}{\epsilon}\,\delta(1-\hat{x})\left[\int_0^1 d\hat{z}_q\,\left(\hat{z}_q + 1 - \hat{z}_q\right)P_{qq}(\hat{z}_q)\,\right] = 0$$

$$\hat{z}_i = \frac{z_h}{z_i} \qquad P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right]$$

$$\hat{x} = \frac{x_B}{x} \qquad P_{qg}(x) = C_F\,\frac{1+(1-x)^2}{x}$$

NLO cross-section at twist-4

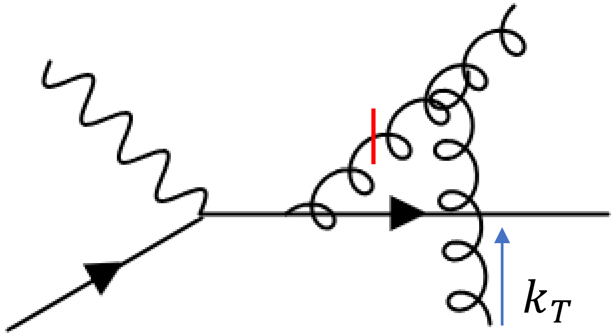
- Through explicit calculations of real and virtual corrections at twist-4, the transverse-momentum-weighted differential cross section due to double scattering is shown to factorize at NLO [Z.Kang, E. Wang, X. Wang, H. Xing arxiv:1409.1315].

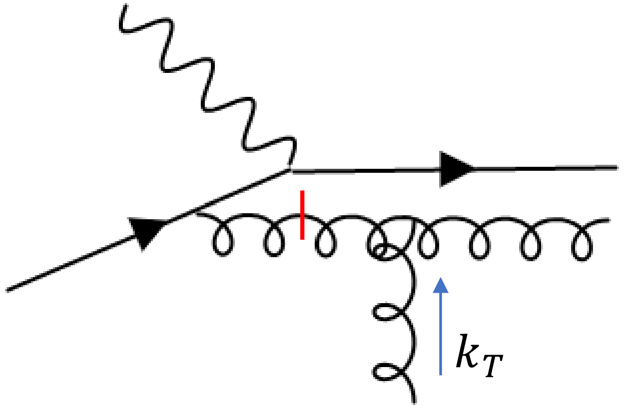


$$
\begin{aligned}
\frac{d\langle \ell_{hT}^2 \sigma^D \rangle}{d\mathcal{PS}} =& \sigma_h \sum_q e_q^2 \int_{x_B}^1 \frac{dx}{x} T_{qg}(x,0,0,\mu_f^2) \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z,\mu_f^2) \delta(1-\hat{x})\delta(1-\hat{z}) \\
&+ \sigma_h \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z,\mu_f^2) \int_{x_B}^1 \frac{dx}{x} \Big\{ \ln\left(\frac{Q^2}{\mu_f^2}\right) \Big[\delta(1-\hat{x}) P_{qq}(\hat{z}) T_{qg}(x,0,0,\mu_f^2) \\
&+ \delta(1-\hat{z}) \big(\mathcal{P}_{qg\to qg} \otimes T_{qg} + P_{qg}(\hat{x}) T_{gg}(x,0,0,\mu_f^2) \big) \Big] \\
&+ H_{qg}^{C-R} \otimes T_{qg} + H_{qg}^{C-V} \otimes T_{qg} - H_{qg}^A \otimes T_{qg}^A + H_{gg}^C \otimes T_{gg} \Big\},
\end{aligned}
$$

NLO at Twist-4: Gluon Fragmentation

- We need to calculate some additional diagrams that contribute to the gluon fragmentation:

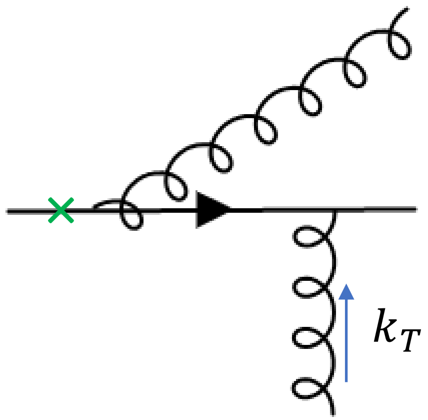


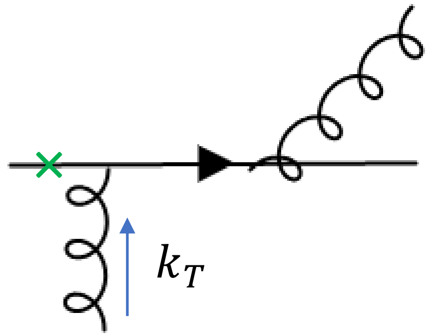


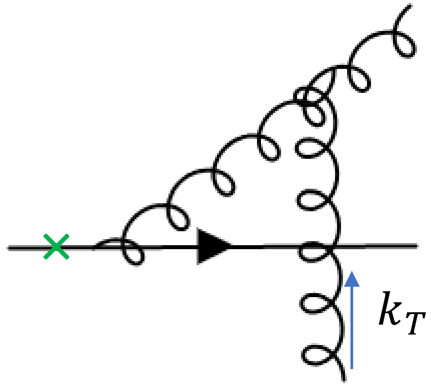
where the red bar represents where the soft pole arises. Soft here means the initial gluon momentum goes to 0 as k_T goes to 0.

Hard Scattering

- There are also other diagrams that are called hard scattering because the initial gluon momentum becomes finite as k_T goes to zero.



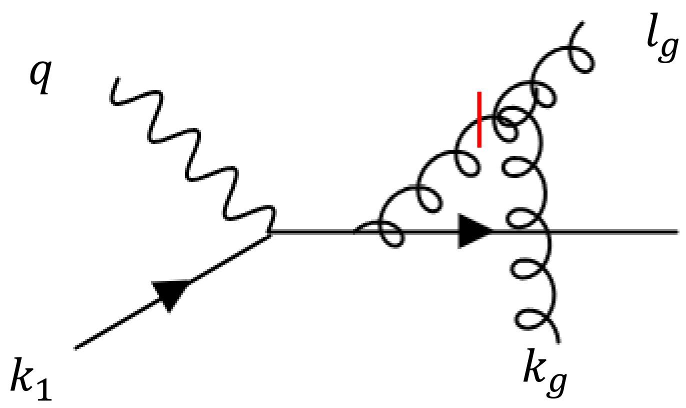




Poles

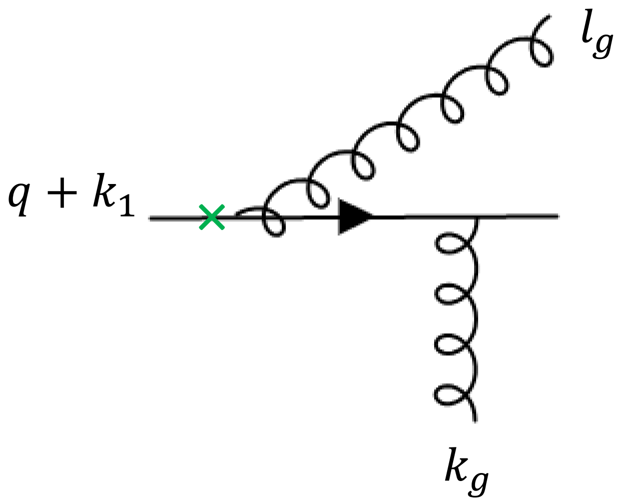
Soft

- $\dfrac{1}{\left(l_g - k_g\right)^2 + i\epsilon}$

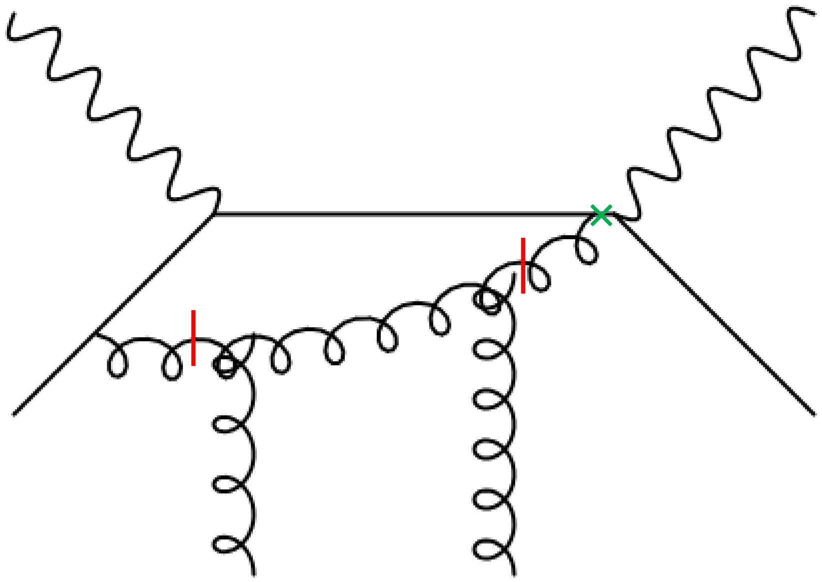


Hard

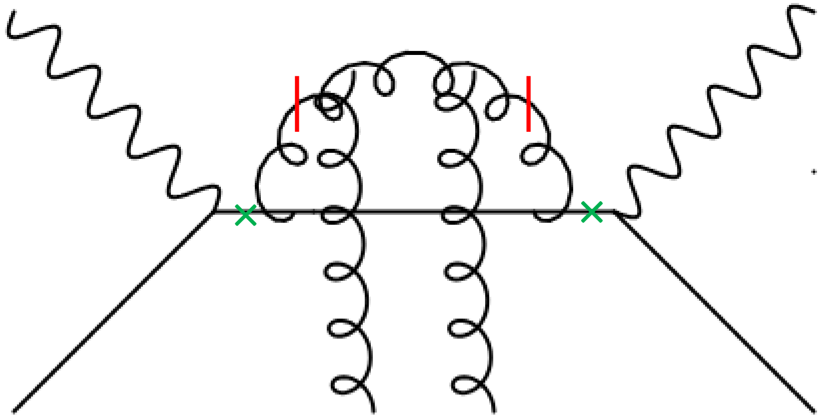
- $\dfrac{1}{(k_1 + q)^2 + i\epsilon}$



Diagrams:

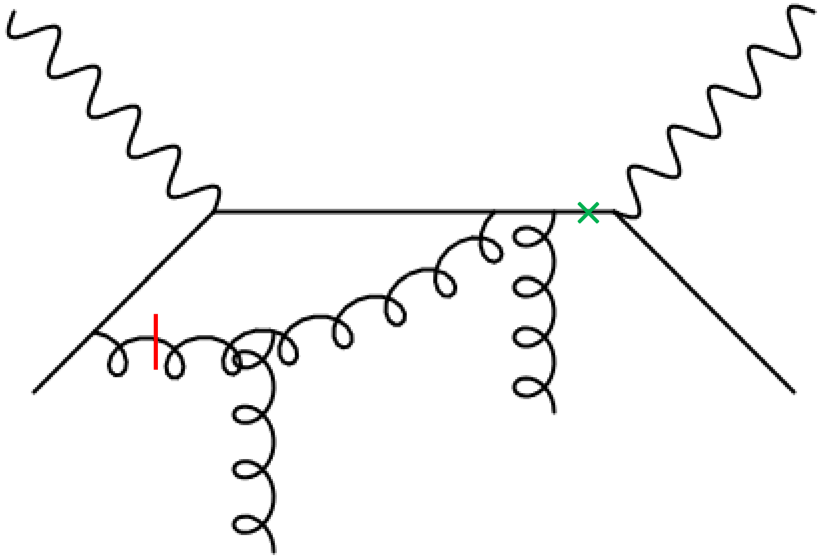


Soft-Soft & Soft-Hard

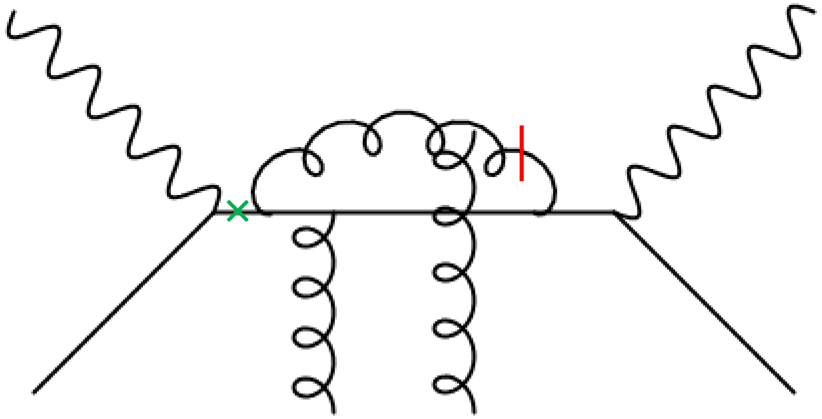


Soft-Soft & Soft-Hard & Hard-Soft & Hard-Hard

Diagrams:



Soft-Hard



Hard-Soft

Weighted cross section: Gluon Fragmentation

- We want to calculate

$$\Delta\langle\sin^2\theta\rangle_g = \int dcos\theta \sin^2\theta \left[\frac{d\Sigma^{\mathrm{T4}}}{dx_B dQ^2 dcos\theta}\right]_g$$

$$\frac{d\Sigma^{T4}}{dx_B dQ^2 d\cos\theta} = \sum_h \int_0^1 dz_h\, z_h \left[\frac{d\sigma^{T4}}{dx_B dQ^2 d\cos\theta\, dz_h}\right]_g$$

Current status

- We are now in the process of calculating these different diagrams: Hard-Hard, Soft-Soft, Hard-Soft and Soft-Hard.

- Then, one should extract the final-state divergence and add that to the final-stat divergent part from the quark contribution:

$$I_{IR} = -\frac{1}{\epsilon}\,\delta(1-\hat{x})\left[\int_0^1 d\hat{z}_q\;\hat{z}_q P_{qq}\left(\hat{z}_q\right)\right]$$

- This will verify the IR-safety of the width energy flow in SIDIS (NLO at twist-4).

Thanks