Differentiating Energy-Energy Correlators with Charged Particle Multiplicities within a Jet

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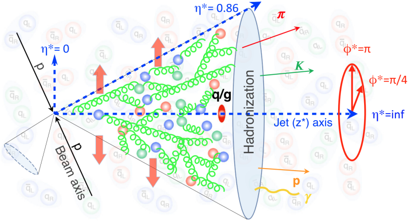
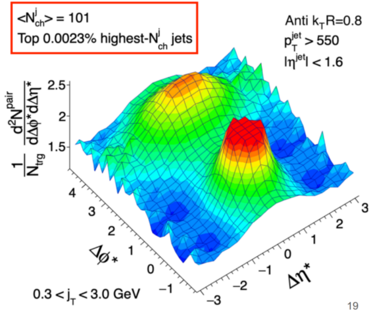
Introduction





- Jet with extreme multiplicities have been measured and display non-trivial particle correlations. A new system to study in the ‘many’ limit and attracts theoretical interest.
- However, PYTHIA8 (MPI + colour reconnection) underestimates N_{ch} distribution in the interested region by order of magnitude. Also, it is hard to trigger on jets with high multiplicity with event generator.
- Develop theoretical tools to study jet substructure evolution with multiplicity, focusing on EEC in this study.

CMS, *Phys. Rev. Lett.* 133, 142301

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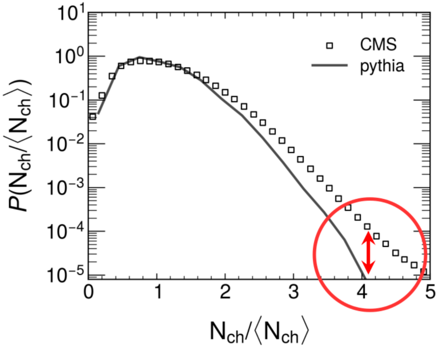

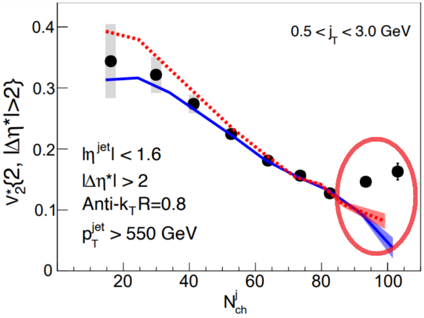

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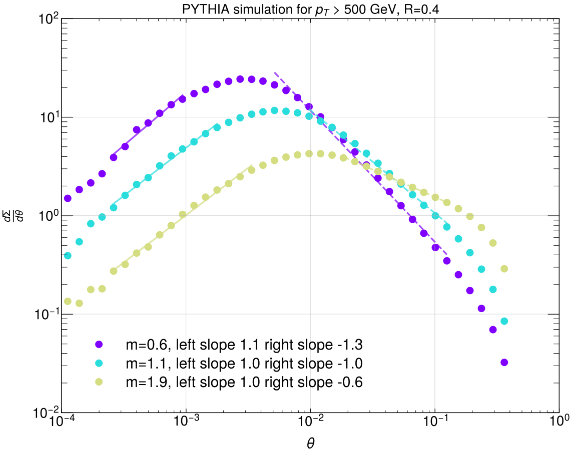
Why Energy-Energy Correlators?

- EEC is typically defined for a jet as the measurement of energy flow into two different directions separated by an angle θ.

$$\frac{\mathrm{d}\Sigma(\theta)}{\mathrm{d}^2\vec{\theta}} = \frac{\mathrm{d}\sigma_{\text{jet}}}{\mathrm{d}PS_{\text{jet}}} \sum_{i,j} \frac{\mathrm{d}N}{\mathrm{d}PS_i \mathrm{d}PS_j} \frac{E_i E_j}{E_{\text{jet}}^2} \delta^{(2)}(\vec{\theta} - \vec{\theta}_{i,j}) \mathrm{d}PS_i \mathrm{d}PS_j$$

- It is an infrared and collinear safe observable.
- Probes both perturbative and non-perturbative QCD dynamics at the scale θE.
- We define the **multiplicity class** as: $m = \frac{N_{\text{ch}}}{\langle N_{\text{ch}} \rangle}$, where N_{ch} is the number of charged particles inside the jet.
- Our goal is to study how EEC behaves with multiplicity class.

EEC from PYTHIA Simulation in pp Collision at 13 TeV



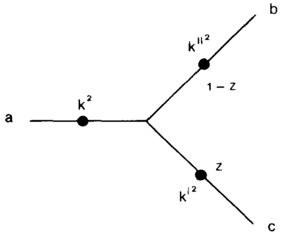
The simulation shows that the EEC (or in general, jet substructure) is very sensitive to multiplicity. So, can we study the substructure EEC by dividing the events according to multiplicity classes?

Multiplicity Generating Function (Partition Function)

- We define the multiplicity generating function as

$$Z(s) = \sum_{n=0}^{\infty} e^{-ns} P(n), \quad P(n) = \int_{s_0 - i\pi}^{s_0 + i\pi} \frac{\mathrm{d}s}{2\pi i} Z(s) e^{ns}$$

- With the help of the generating function, the convolution of the distributions becomes a product.



$$P_a(n) = \sum_{m=0}^{n} P_b(m) P_c(n-m) \Longleftrightarrow Z_a(s) = Z_b(s) Z_c(s)$$

Jet Multiplicity Evolution Equation

Take the pure gluon system as an example:

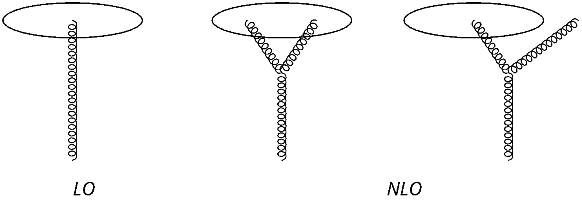
- Let $M_i(s, z, \omega_J, R)$ represents a jet with energy ω_J and radius R, carrying a fraction z of the large light-cone momentum component of the parton i that initiates the jet, with final charged multiplicity as N.
- For LO, a single parton carries all its energy to form a jet, which then fragments into N charged particles:

$$M_g^{(0)}(s, z, \omega_J, R) = \delta(1-z) Z_g^{(0)}(s, \omega_J, R)$$

- For NLO, parton undergoes a splitting

$$M_{g\to gg}^{(1)} \propto \delta(1-z)\frac{\alpha_s}{\pi}\int \mathrm{d}x \frac{1}{2} P_{gg}(x,\epsilon) Z_g^{(0)}(s, x\omega_J) Z_g^{(0)}(s,(1-x)\omega_J) \int \frac{\mathrm{d}q_\perp}{q_\perp^{1+2\epsilon}} \Theta_{\text{alg}}^{<R}$$

$$M_{g\to g(g)}^{(1)} \propto \frac{\alpha_s}{\pi} P_{gg}(z,\epsilon) Z_g^{(0)}(s, z\omega_J) \int \frac{\mathrm{d}q_\perp}{q_\perp^{1+2\epsilon}} \Theta_{\text{alg}}^{>R}$$



Jet Multiplicity Evolution Equation

We can give the following factorization as in the pure gluon system:

$$M_{g\to g} = \left[J^{(0)}_{g\to g} + J^{(1)}_{g\to gg} + J^{(1)}_{g\to g(g)}\right] \times \left[Z^{(0)}_g + Z^{(1)}_g\right]$$

The final evolution equation

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2}\begin{pmatrix} M_q(s,z,\omega_J,\mu) \\ M_g(s,z,\omega_J,\mu)\end{pmatrix} = \frac{\alpha_s(\mu)}{2\pi}\begin{pmatrix} P_{qq}(z) & P_{gq}(z) \\ 2n_f P_{qg}(z) & P_{gg}(z)\end{pmatrix} \otimes \begin{pmatrix} M_q(s,z,\omega_J,\mu) \\ M_g(s,z,\omega_J,\mu)\end{pmatrix}$$

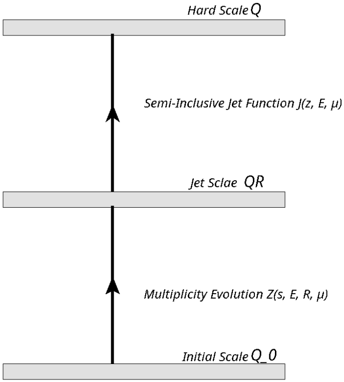
because multiplicity is sensitive to soft radiation, use improved evolution equation with angular ordering:

$$\frac{\partial Z_g}{\partial \ln\zeta} = \frac{1}{2\pi}\int \mathrm{d}x\,\alpha_s(k_\perp^2)\left\{\frac{1}{2}P_{gg}(x)Z_g\cdot Z_g + n_f P_{qg}(x)Z_q\cdot Z_q\right\}\Theta\left(k_\perp^2 - Q_0^2\right)$$

$$\frac{\partial Z_q}{\partial \ln\zeta} = \frac{1}{2\pi}\int \mathrm{d}x\,\alpha_s(k_\perp^2)P_{qq}(x)Z_q\cdot Z_g\Theta\left(k_\perp^2 - Q_0^2\right)$$

where $\zeta = 1 - \cos\theta$.

A. Bassetto, M. Ciafaloni, G. Marchesini, *Physics Reports*, Volume 100, Issue 4, 1983



Non-Perturbative Modeling

- A parton with virtuality Q_0 and energy ω:
  - In its rest frame ($\omega = Q_0$), it fragments isotropically into hadrons with $E \sim \Lambda_{\text{QCD}}$.
  - When boosted ($\omega \gg Q_0$), hadrons are squeezed to a cone of size $\theta_0 \sim Q_0/\omega$.
- Ansatz for average charged hadron count in a cone:

$$\langle n\rangle(\omega, Q_0, \theta_0) = \frac{n_0(Q_0)}{1 + cQ_0^2/(\omega^2\theta_0^2)}$$

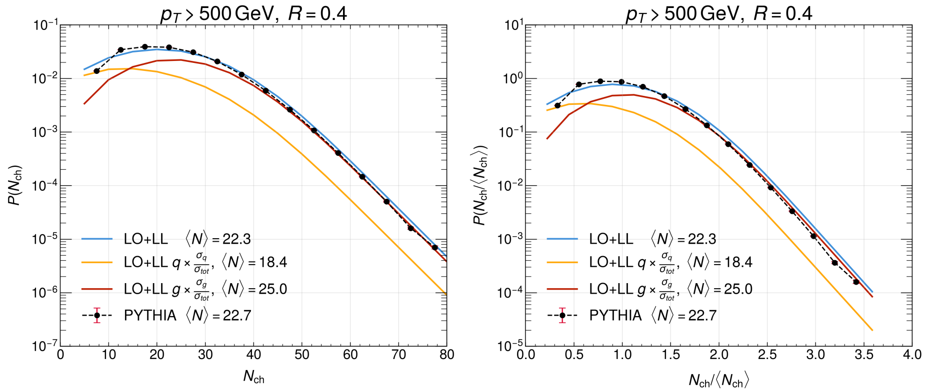
- Full distribution modeled as binomial:

$$P(n) = \binom{N_{\max}}{n} p^n (1-p)^{N_{\max}-n}, \quad p = \frac{\langle n\rangle}{N_{\max}}$$

- Laplace transform:

$$Z(s) = (pe^{-s} + 1 - p)^{N_{\max}}$$

Multiplicity Distribution in pp Collision at 13 TeV



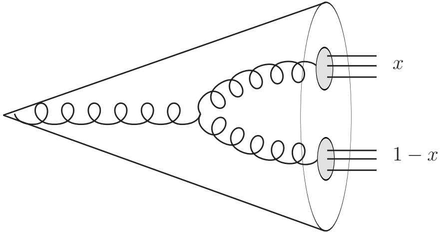
- The LO+LL result agrees with PYTHIA simulations.
- The high-multiplicity tail mainly comes from gluon jets, due to their larger color charge, higher splitting rate, and larger jet cross section at this region.

EEC Evolution Equation

Take exclusive process as example:

- For NLO, the parton splits into two partons, each carrying energy fraction x and $1-x$, respectively:

$$\frac{\mathrm{d}\Sigma^{(1)}_{g\to gg}}{\mathrm{d}\vec{\theta}} \propto \delta(1-z)\frac{g_s^2}{2\pi}\int \mathrm{d}x \int \frac{\mathrm{d}^{2-2\epsilon}k_\perp}{(2\pi)^{2-2\epsilon}k_\perp^2}$$
$$\left\{\left[x^2+(1-x)^2\right]\frac{1}{2}P_{gg}(x,\epsilon)Z_g^{(0)}\cdot Z_g^{(0)}\delta^{(2)}(\vec{\theta})\right.$$
$$\left.+\left[x(1-x)\right]\frac{1}{2}P_{gg}(x,\epsilon)Z_g^{(0)}\cdot Z_g^{(0)}\delta^{(2-2\epsilon)}\left(\vec{\theta}-\frac{\vec{k_\perp}}{x(1-x)\omega_J}\right)\right\}\Theta_{\mathrm{alg}}^{<R}$$



EEC Evolution Equation

Evolution equation from EEC scale θQ to jet scale RQ:

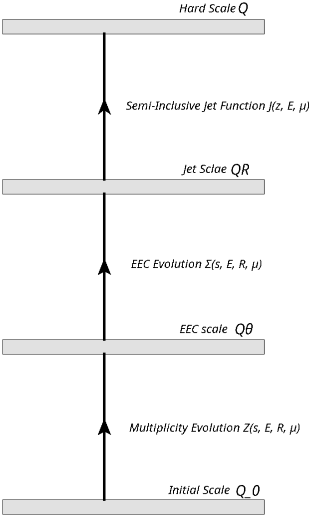
$$\frac{\partial \Sigma_g}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int \mathrm{d}x \left[x^2 + (1-x)^2\right] \left\{ \frac{1}{2} P_{gg}(x) Z_g \cdot Z_g + n_f P_{qg}(x) Z_q \cdot Z_q \right\}$$

$$\frac{\partial \Sigma_q}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int \mathrm{d}x \left[x^2 + (1-x)^2\right] P_{qq}(x) Z_q \cdot Z_g$$

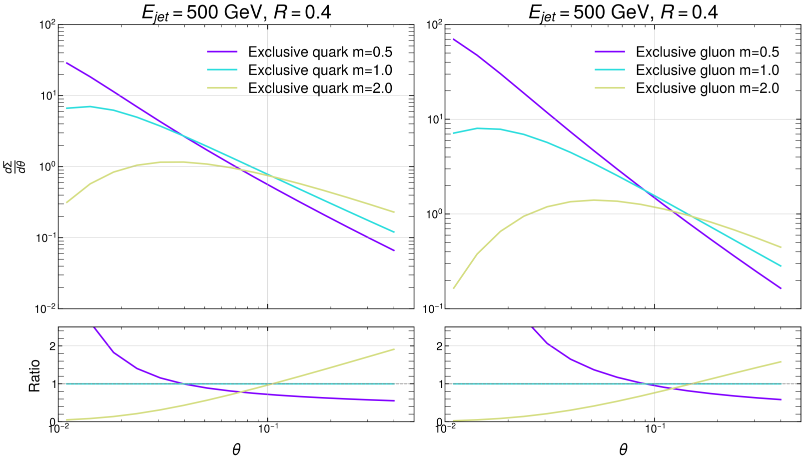
Initial condition of EEC at θQ:

$$\Sigma_g(\theta) = \frac{\alpha_s}{\pi} \frac{1}{\theta} \int \mathrm{d}x [x(1-x)] \left\{ \frac{1}{2} P_{gg}(x) Z_g \cdot Z_g + n_f P_{qg}(x) Z_q \cdot Z_q \right\}$$

$$\Sigma_q(\theta) = \frac{\alpha_s}{\pi} \frac{1}{\theta} \int \mathrm{d}x [x(1-x)] P_{qq}(x) Z_q \cdot Z_g$$

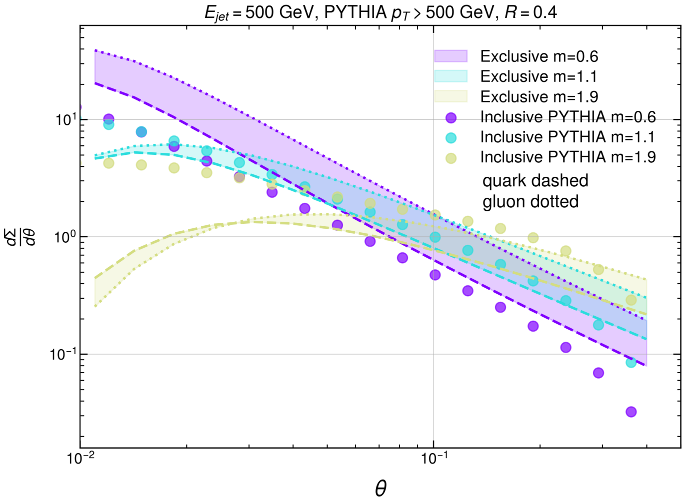


Exclusive Jet EEC



- The EEC slope increases at perturbative region as jet multiplicity rises.
- For $m = 0.5/m = 1$, we observe a decrease at large angles.
- For $m = 2/m = 1$, we observe an increase at large angles.

Exclusive Jet EEC



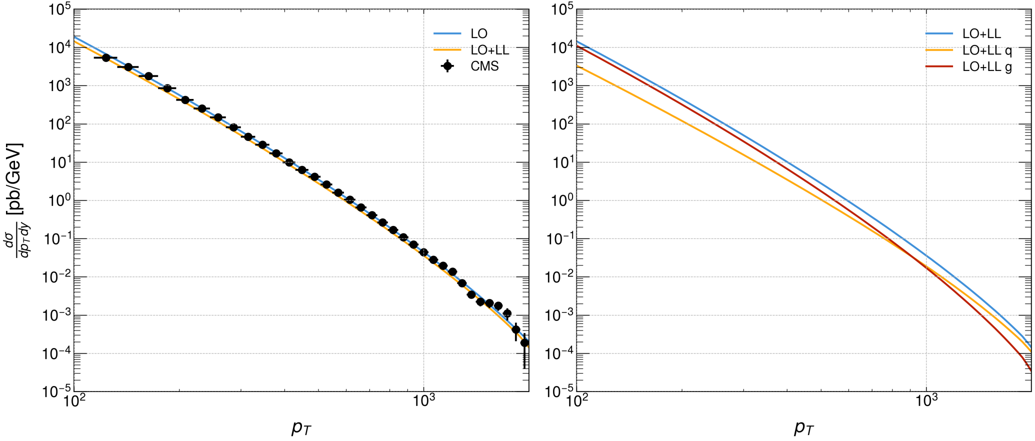
- Our exclusive results reasonably capture the slope-changing behavior when compared to the inclusive PYTHIA simulation.

Conclusion

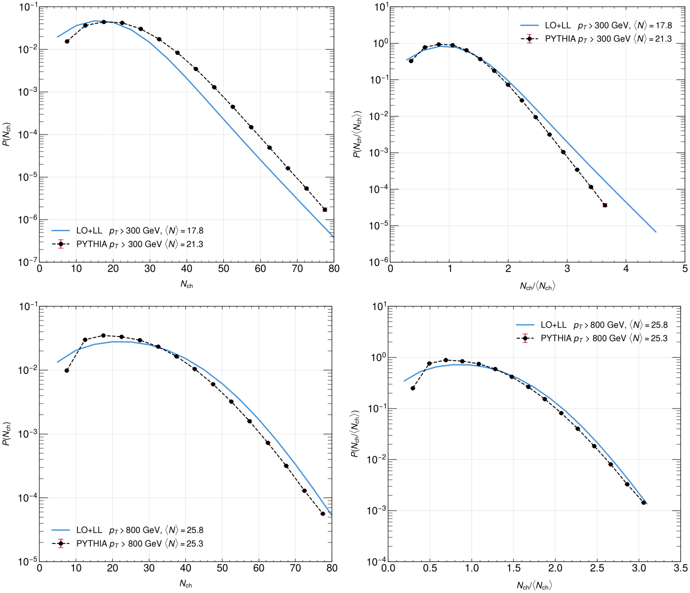
- Multiplicity selection provides a powerful tool to distinguish between quark and gluon jets, separate jets with many perturbative splittings from those with only a few, and can test non-perturbative hadronization mechanism. This gives us a flexible way to study and control jet substructure in detail.
- Remarkably, this method supports the study of jet substructure under extreme particle number fluctuations within just 10 seconds (with GPU-acceleration). In the future, it offers a hopeful path to investigate whether final-state interactions can modify the substructure in high-multiplicity jets.
- **Outlook**: In order to analyse the multiplicity distribution and energy-energy correlations (EEC) after medium modification, we can further incorporate the medium's partition function, giving a combined framework $Z = Z_{\text{jet}} \otimes Z_{\text{med}}$. This opens the way for applying first-principles theory directly to the study of multiplicity-selected jets in a medium.

Backup

Jet Cross Section in pp Collision at 13 TeV



Multiplicity Distribution



EEC at Higher Multiplicity Class