The Energy-Energy Correlator on Tracks: Collinear Limit

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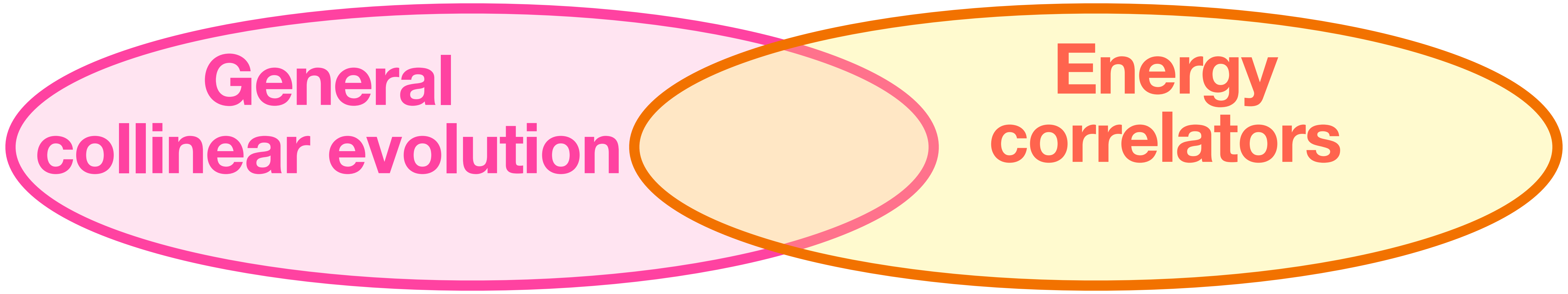
Based on [2108.01674], [2201.05166], [2210.10061], [2210.10058],
and the ongoing project with Max Jaarsma, Ian Moult, Wouter Waalewijn, Hua Xing Zhu

Outline



General motivation

- **What** does the general collinear evolution mean?
- **Why and where** should we use the general collinear evolution?
- **Why** do we like energy correlators?

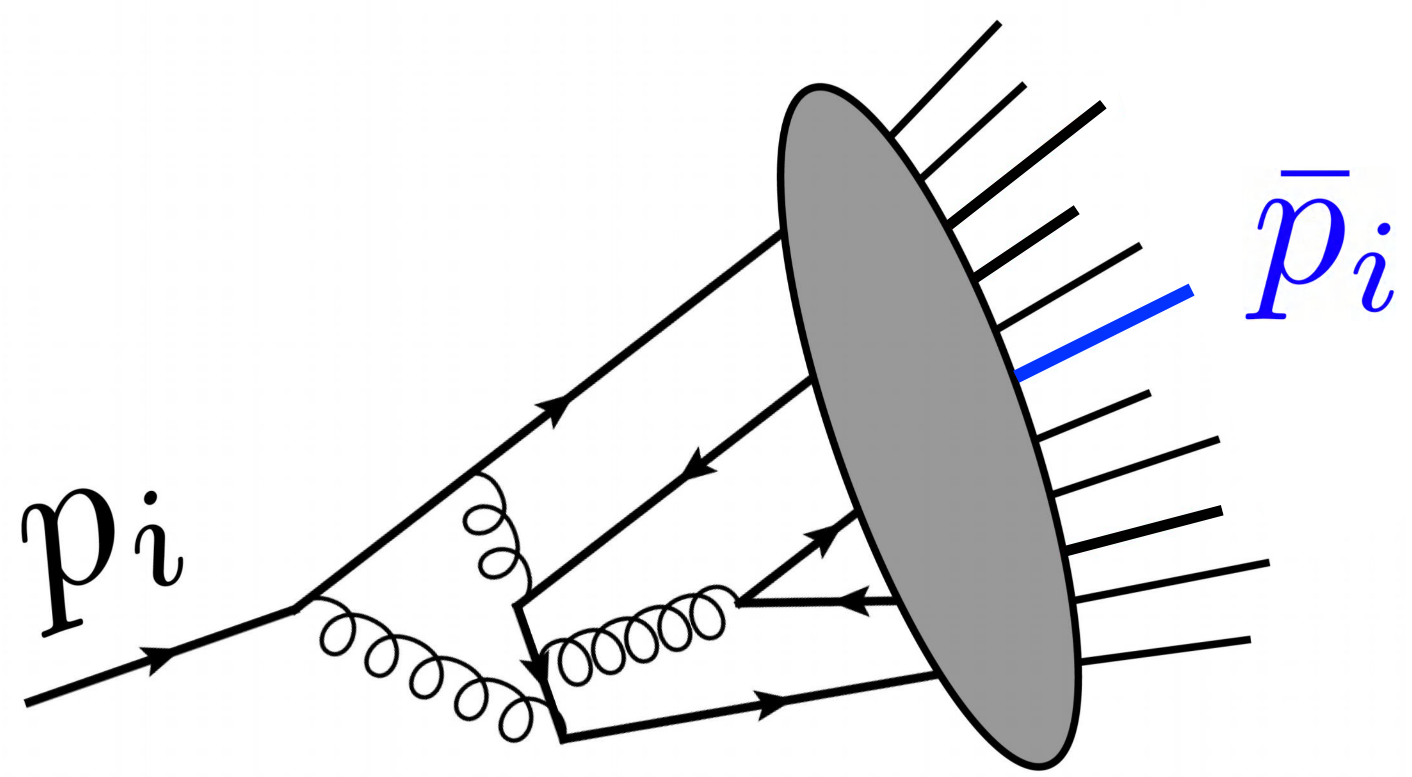
Scheme in detail

- **How** do we apply it?
  - Track function formalism.
  - Application: track EEC.

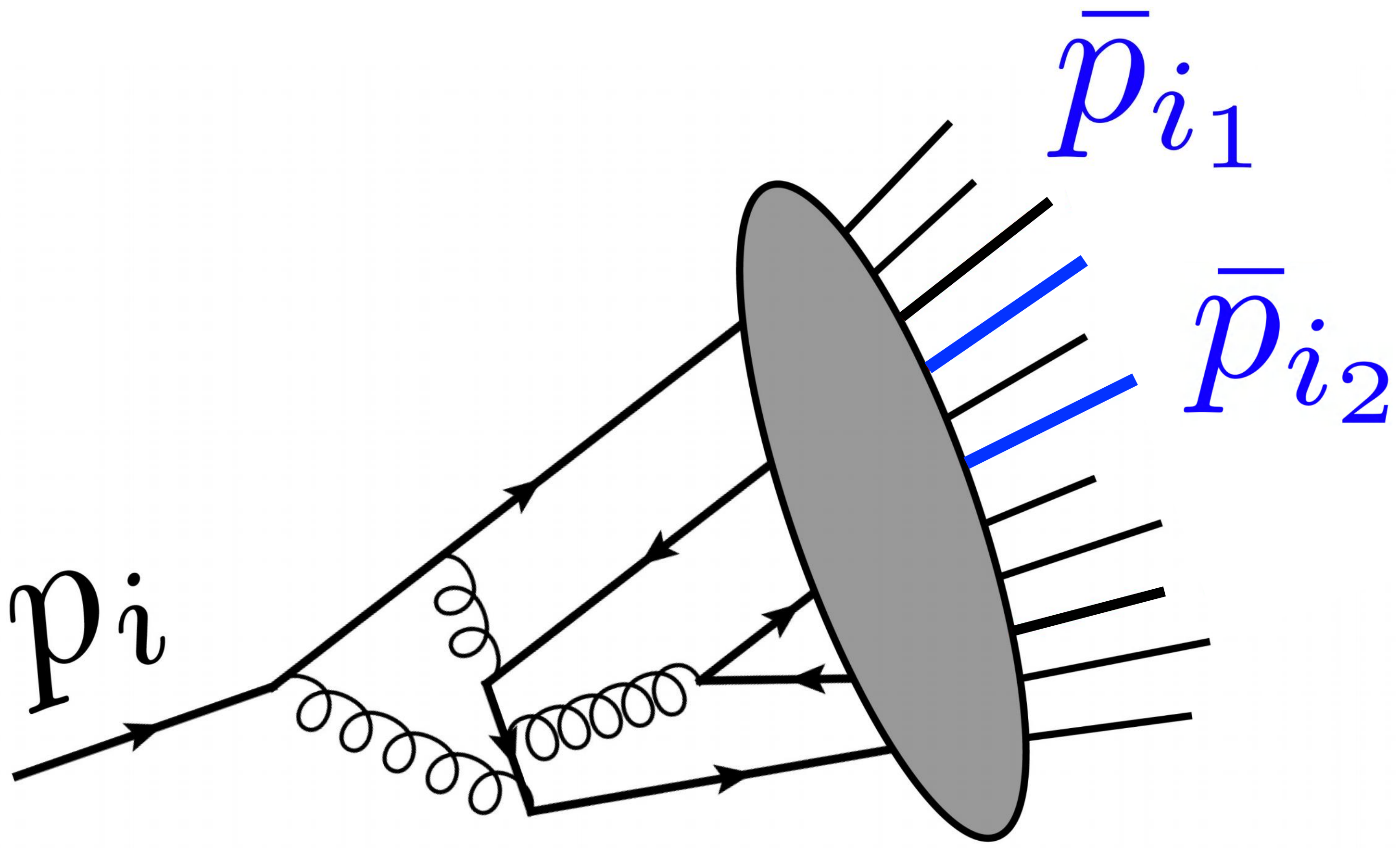
Collinear evolution

Collinear factorization theorem: [Collins, Soper, Sterman `84, 85, 88]

Separating the perturbatively calculable part of a process from universal non-perturbative objects describing the hadronization process.



single-hadron
fragmentation function (FF)



di-hadron FF

DGLAP evolution

non-linear

Collinear evolution

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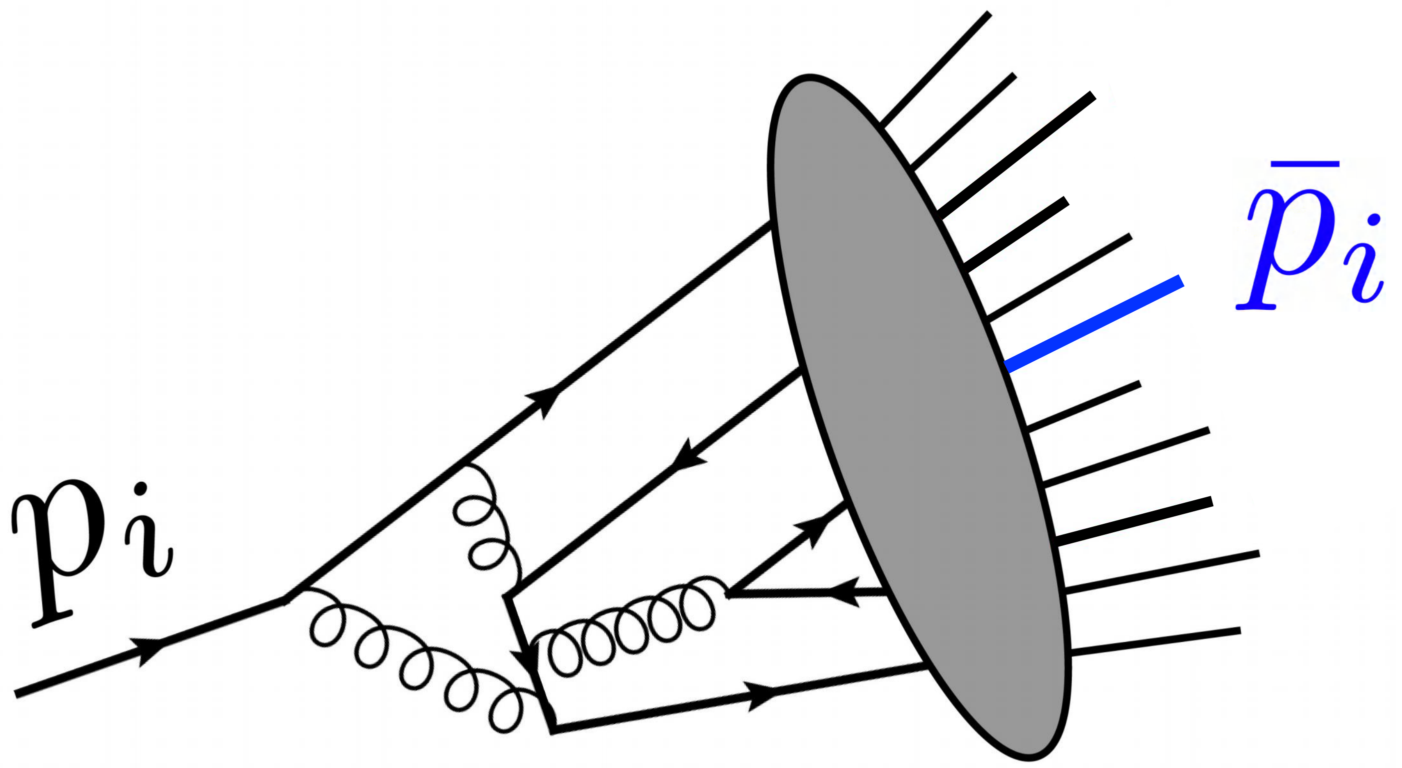

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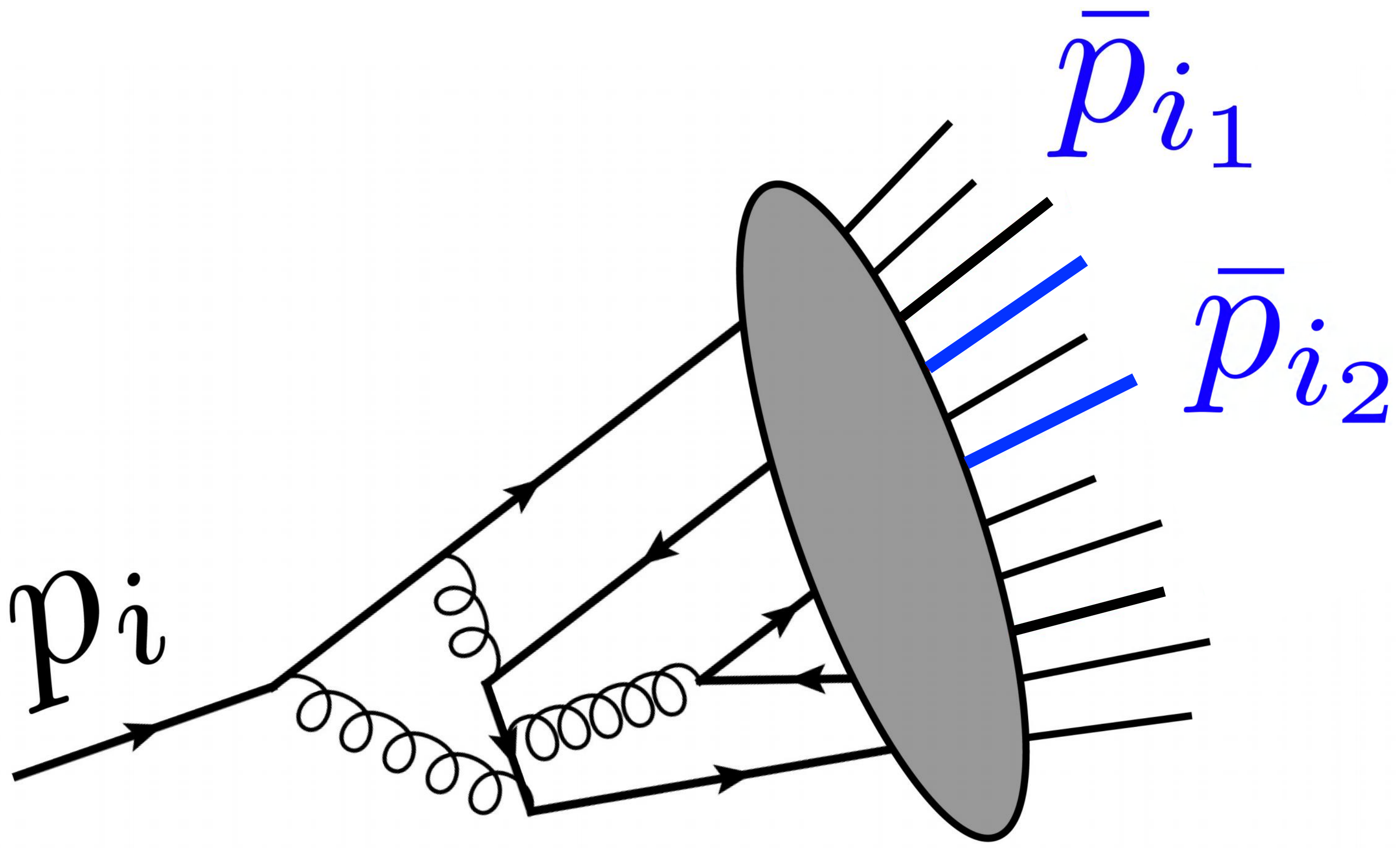

di-hadron FF

an arbitrary number; the total kinematics

?

DGLAP evolution

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Collinear evolution

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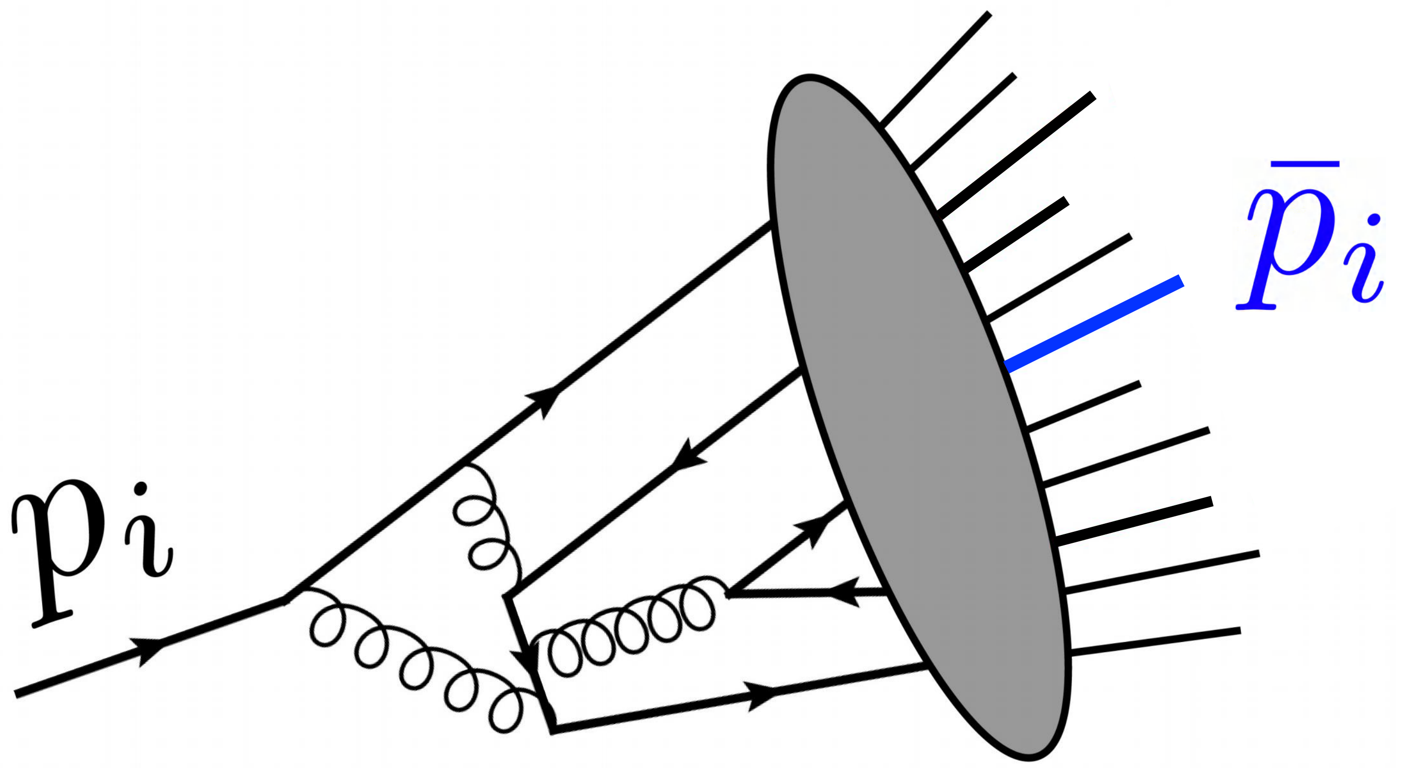

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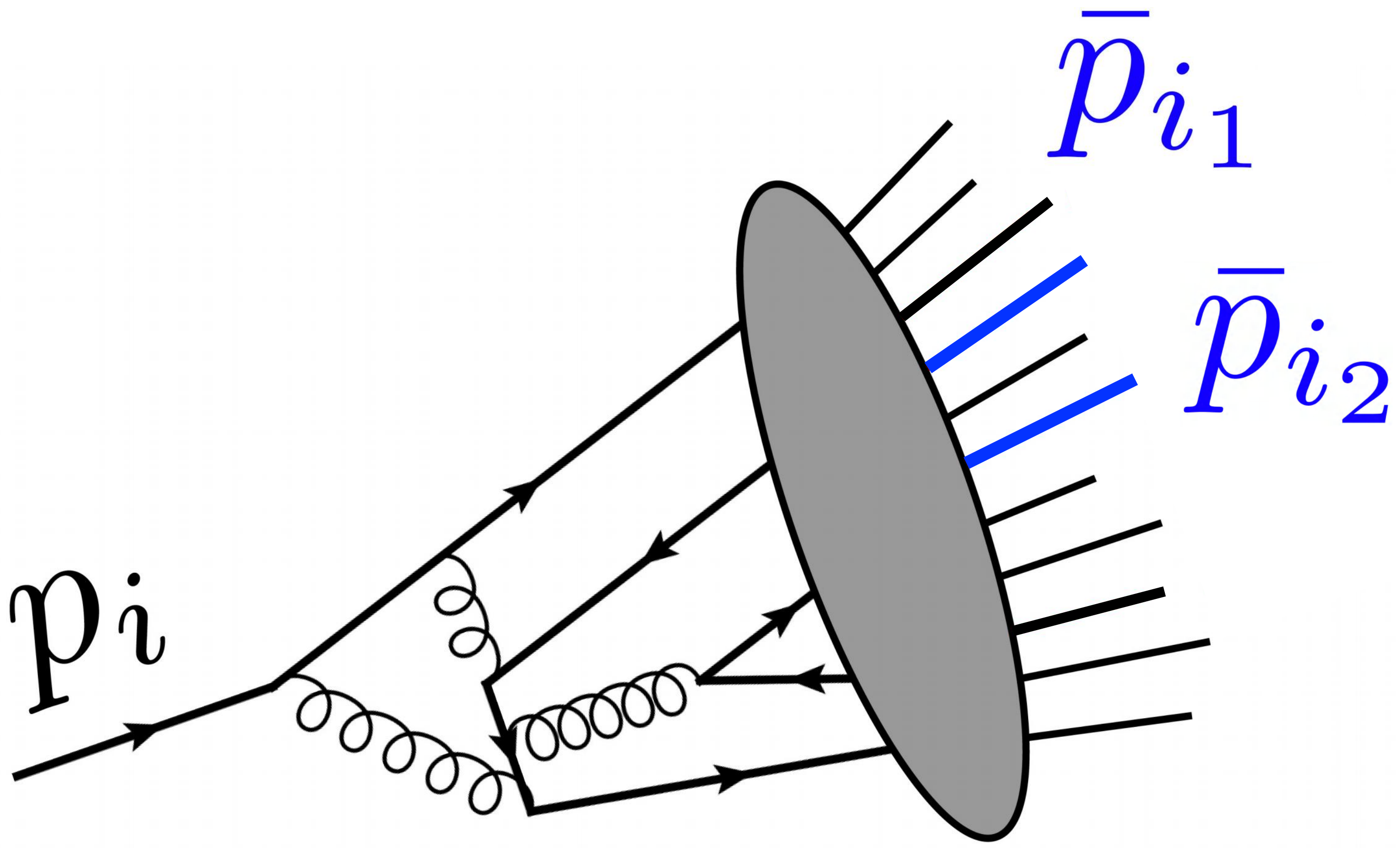

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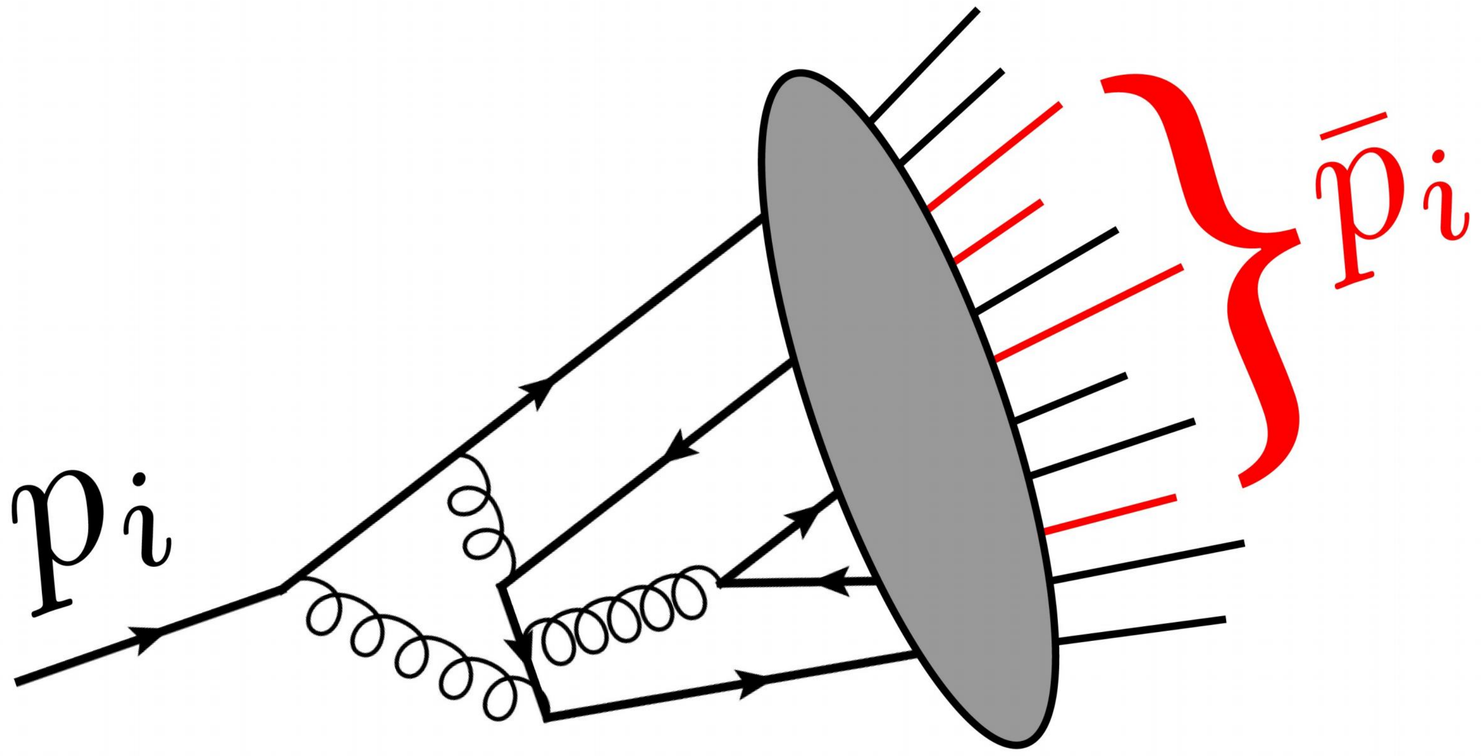

track function (generalized FF)

DGLAP evolution — non-linear → **general collinear evolution**

Correlations of energy fluxes

- Many observables fundamentally characterize **energy flow**, eg.:

[Chen, Moult, Zhang, Zhu `20; Larkoski, Moult, Nachman `17]

- *k*-point energy correlators:

[Basham, Brown, Ellis, Love `78-79; Sveshnikov, Tkachov `95; Belitsky, Korchemsky, Sterman `01]

$$\sigma_{\omega_k} = \frac{1}{Q^k}\int \mathrm{d}^4x\, e^{iQ\cdot x}\langle 0|\mathcal{O}(x)\mathcal{E}(\vec{n}_1)\cdots\mathcal{E}(\vec{n}_k)\mathcal{O}^\dagger(0)|0\rangle$$

$$\frac{1}{Q}\mathcal{E}(\vec{n})|X\rangle = \sum_i \frac{E_i}{Q}\delta^{(2)}(\Omega_{\vec{p}_i} - \Omega_{\vec{n}})|X\rangle$$

- 2-point: EEC

$$\frac{1}{Q^2}\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|X\rangle \sim \sum_{i_1,i_2}\frac{E_{i_1}}{Q}\frac{E_{i_2}}{Q}\delta(\theta_{12} - \theta_{i_1 i_2})|X\rangle$$

- k-point $k(k-1)/2$ angular vars

- Angularity-type:

[Farhi `77; Rakow, Webber `81; Ellis, Webber `86; Berger, Kucs, Sterman `03; Almeida, Lee, Perez, Sterman, Sung, Virzi `09; Ellis, Vermilion, Walsh, Hornig, Lee `10; …]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_k} = \int \mathrm{d}^4x\, e^{iQ\cdot x}\langle 0|\mathcal{O}(x)\delta(\tau_k - \hat{\tau}_k)\,\mathcal{O}^\dagger(0)|0\rangle$$

$$\hat{\tau}_k|X\rangle \sim \frac{1}{Q}\sum_i E_i\,\theta_i^k|X\rangle$$

$$\delta(\tau_k - \hat{\tau}_k) = \sum_{n=0}^{\infty}\frac{(\hat{\tau}_k)^n}{n!}\delta^{(n)}(\tau_k)$$

$$\int (\tau_k)^n \frac{\mathrm{d}\sigma}{\mathrm{d}\tau_k} = \int \mathrm{d}^4x\, e^{iQ\cdot x}\langle 0|\mathcal{O}(x)(\hat{\tau}_k)^n\mathcal{O}^\dagger(0)|0\rangle$$

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- Any observable based on final-state kinematics involves a finite or infinite number of energy correlators. → **a basis for IRC-safe observables**

$\langle\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\cdots\mathcal{E}(\vec{n}_k)\rangle$

[See also *energy flow polynomials*: Komiske, Metodiev, Thaler `17]

Why go beyond the DGLAP paradigm?

◉ Extending observables

- From fully inclusive energy flow **to** a (proper) subset (R) of final-state hadrons.

$$\mathcal{E}(\vec{n}) \downarrow \mathcal{E}_R(\vec{n})$$

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◉ Factorization

- Exclusive energy flow correlations are not IRC safe. [The KLN theorem]
- Regularize collinear divergences.

↓ absorbed (factorized)

Non-perturbative functions with RG evolution ← **Essential for prediction!**

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◉ Multi-particle correlations → general collinear evolution

- k-point correlators ($k \geq 2$) encode multi-particle correlations.
- Require non-perturbative objects with multi-particle correlations, necessitating **collinear evolution beyond DGLAP**. ← **All embedded in track function RGE.**

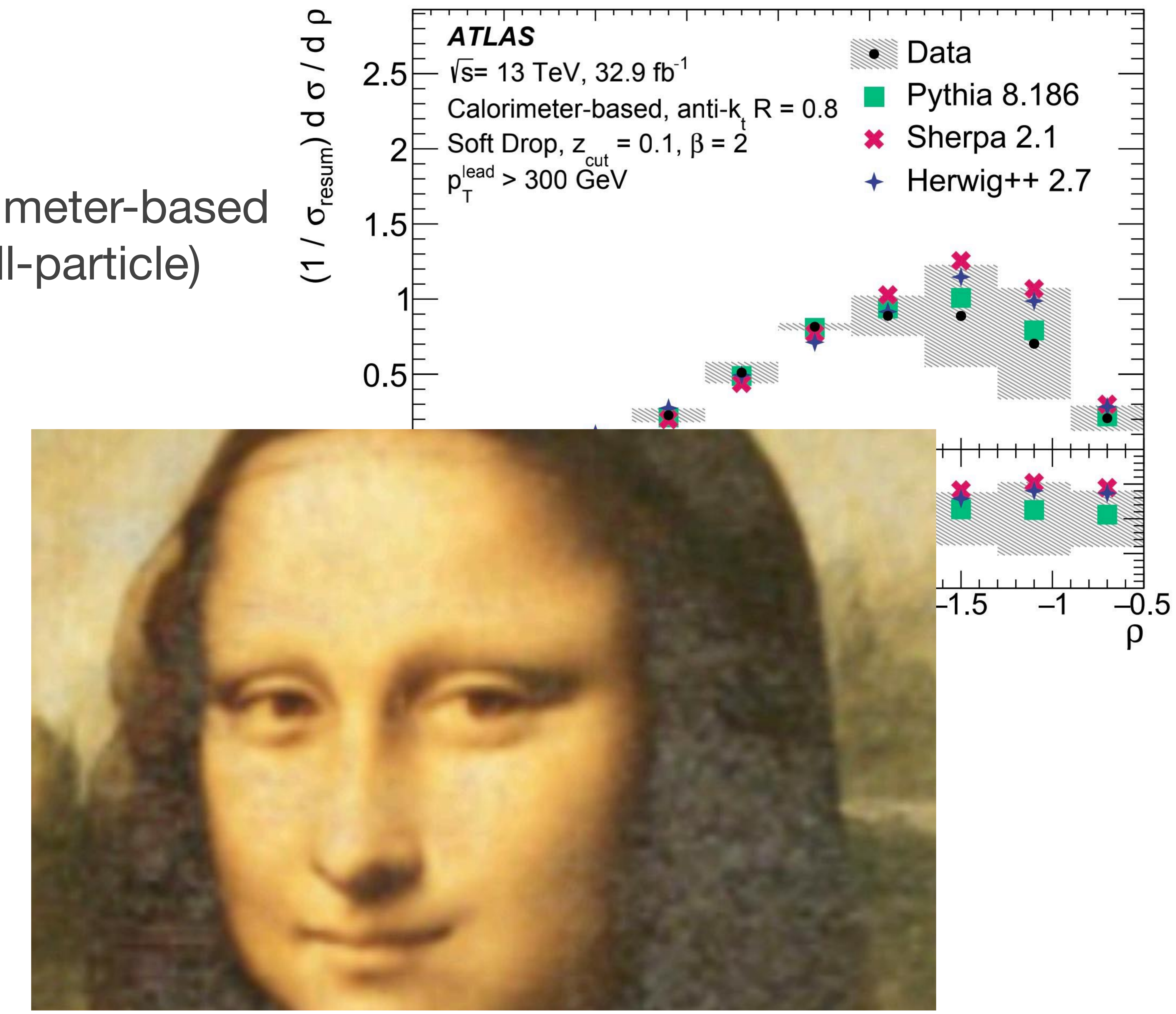
Ex: track-based observables

- Tracking system: superior angular resolution

  & pileup mitigation.

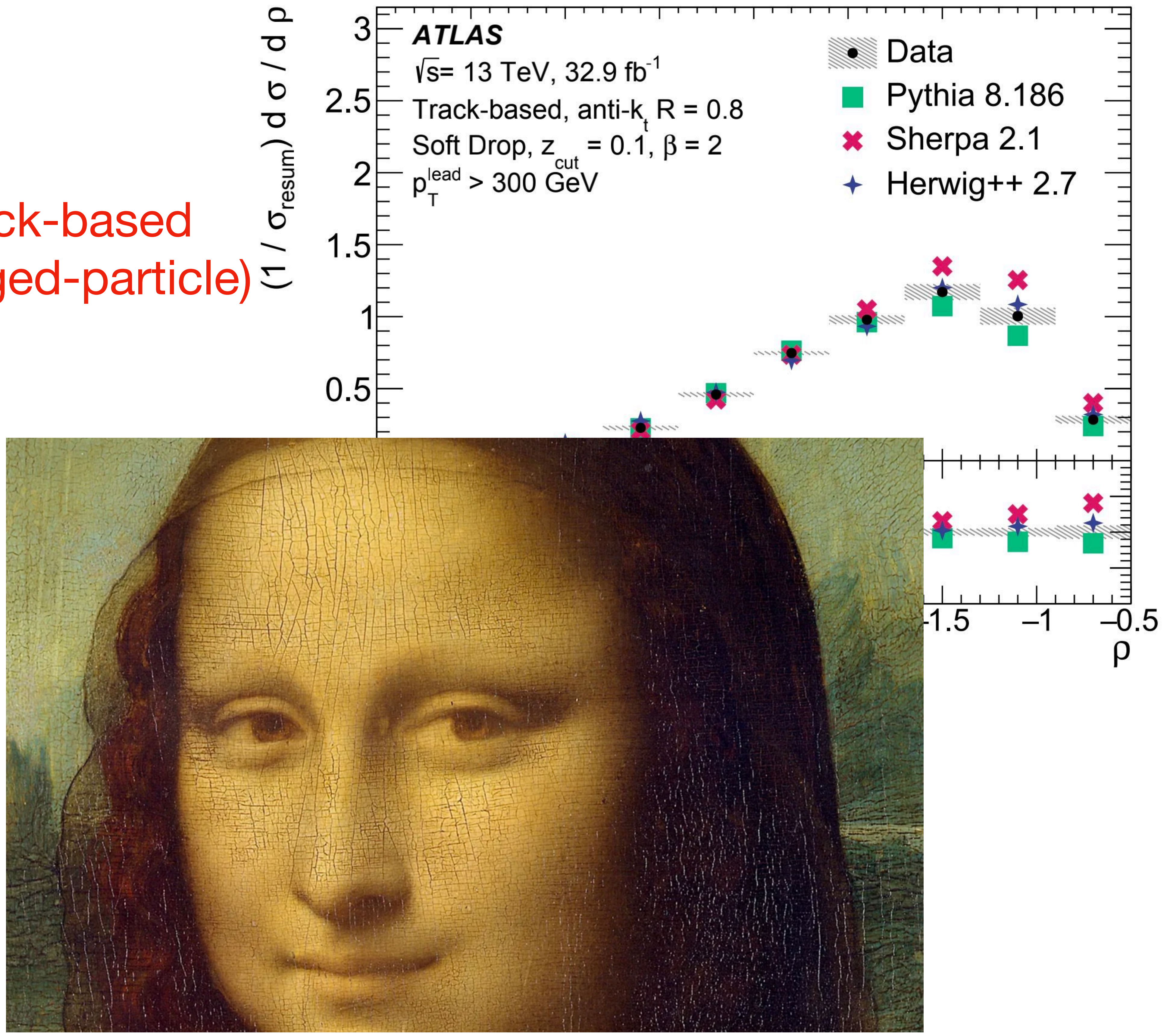
- Experimentally cleaner to measure.

calorimeter-based
(all-particle)



[ATLAS Collaboration `19]

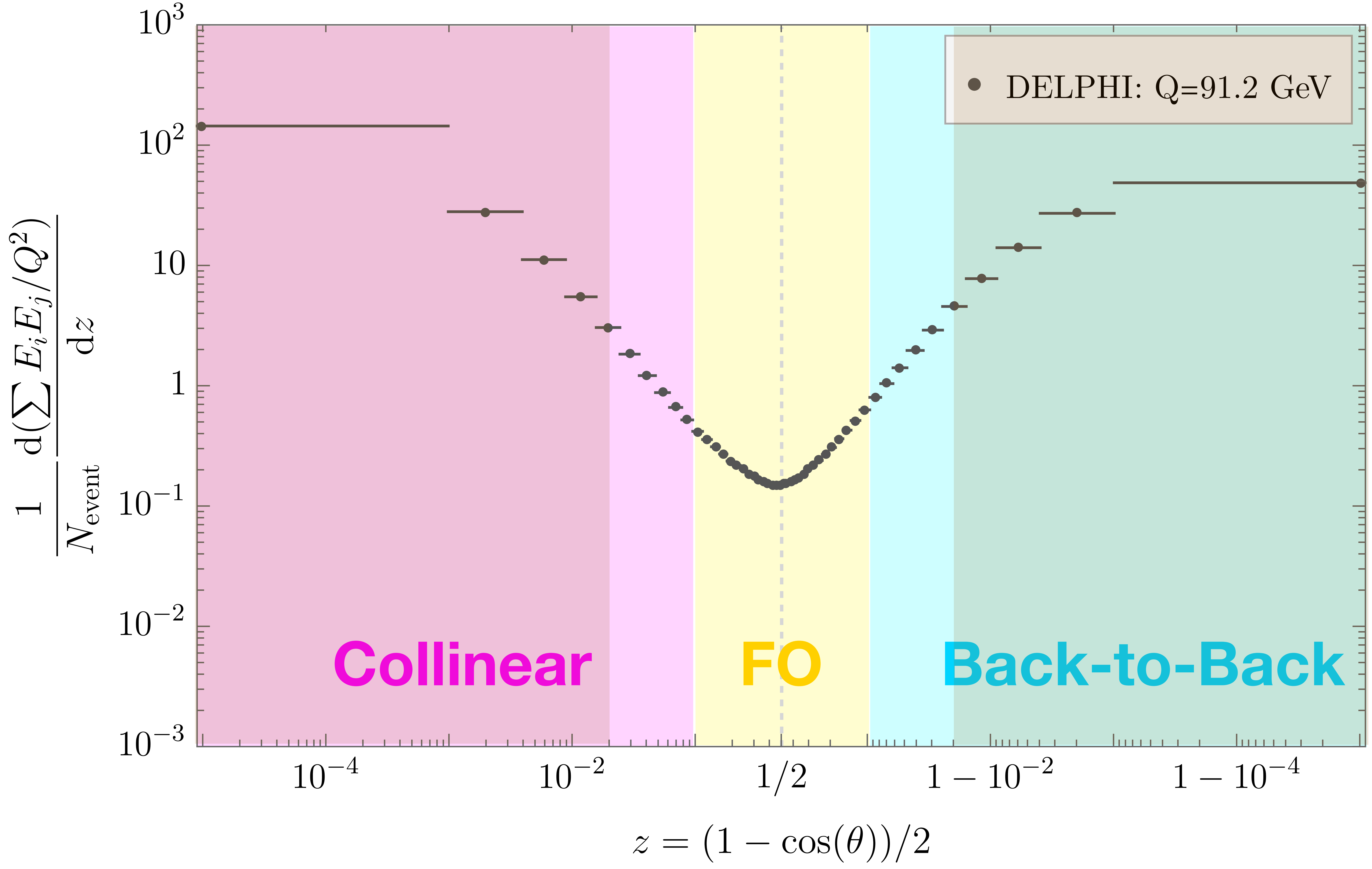
track-based
(charged-particle)

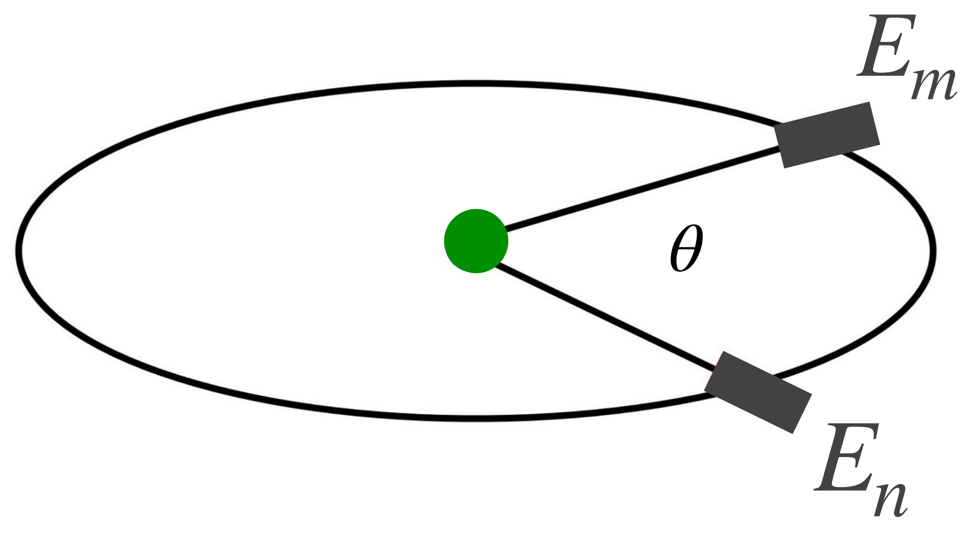


[Illustration idea inspired by Jingjing Pan’s talk at BOOST2024 :)]

Ex: track-based observables

Two-point energy correlator (EEC): all-hadron



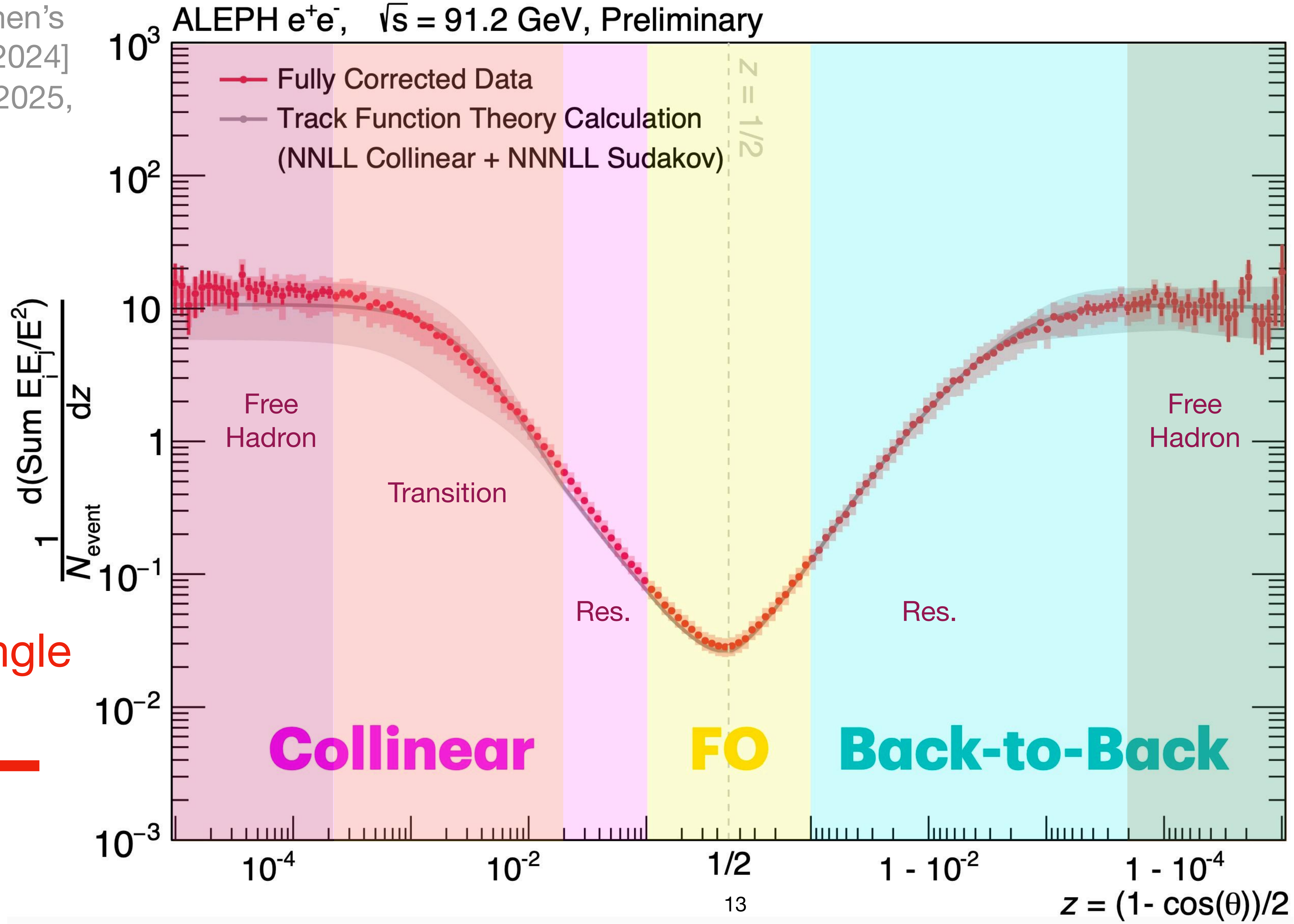


Ex: track-based observables

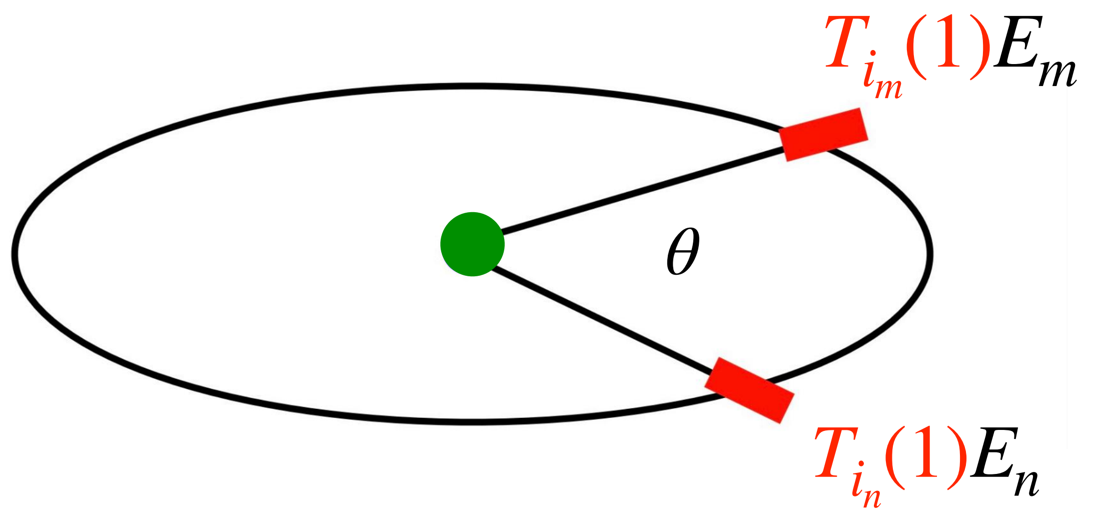
Two-point energy correlator (EEC): track

Use track functions $T(x)$ to calculate!

[Yu-Chen (Janice) Chen's talk at Hard Probes 2024]
[Max's talk at SCET 2025, and his talk today!]



[Theory inputs: Jaarsma, **Li**, Moult, Waalewijn, Zhu `25] [Coming soon]



Resolve small-angle information

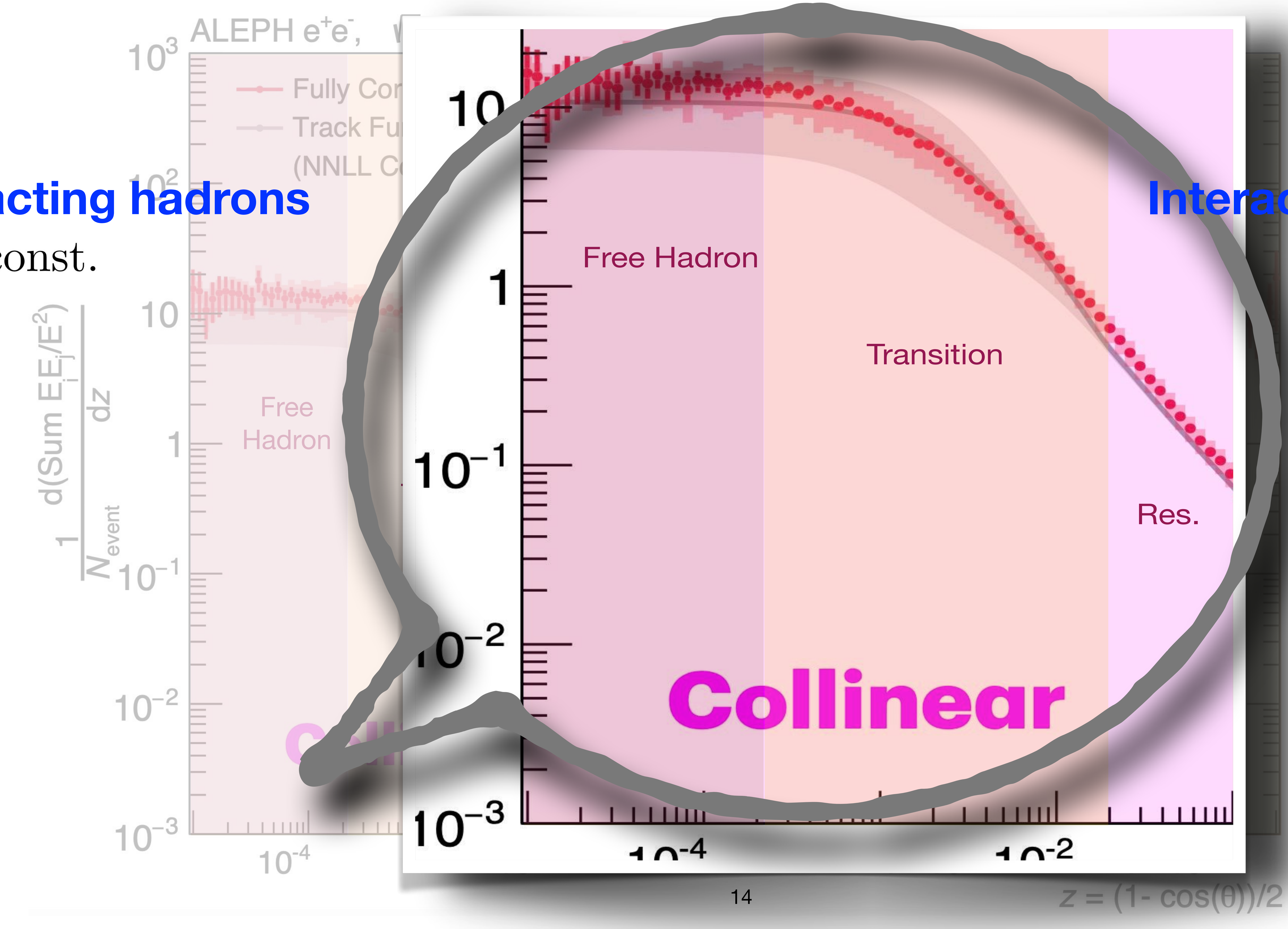
Small q_T region

Zooming in

◉ "Imaging" of confinement transition

Non-interacting hadrons

$\mathrm{EEC}(z) = \mathrm{const.}$



Interacting quarks & gluons

The OPE limit:

$\mathrm{EEC}(z) \sim 1/z^{1-\gamma(3)}$

[Dixon, Moult, Zhu `19;
Chen, Moult, Zhang, Zhu `20;
Lee, Mecaj, Moult `22;
Chen `23]

Zooming in

Probe rich physics of QCD through its dependence on z.

◉ "Imaging" of confinement transition

Non-interacting hadrons

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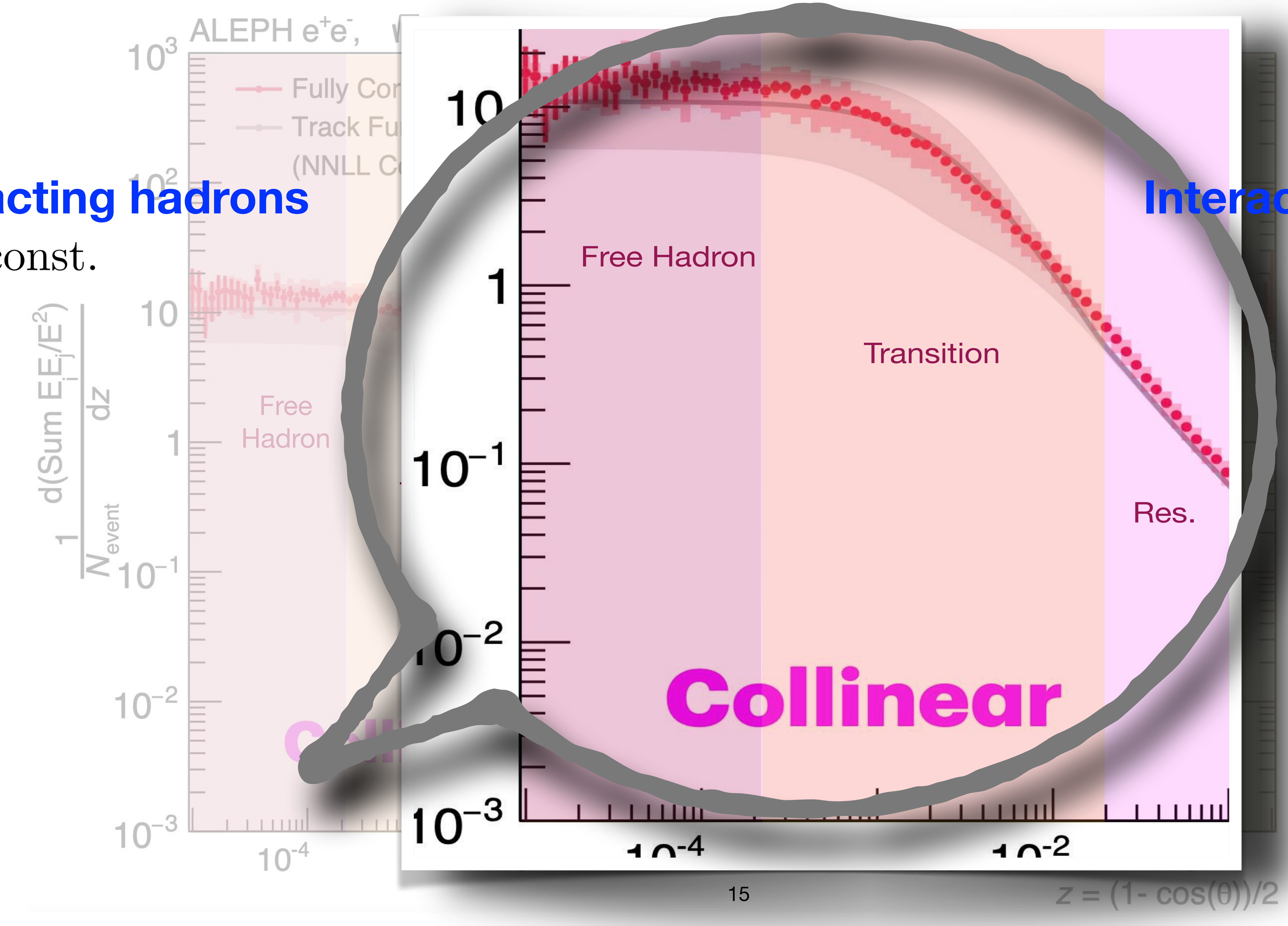

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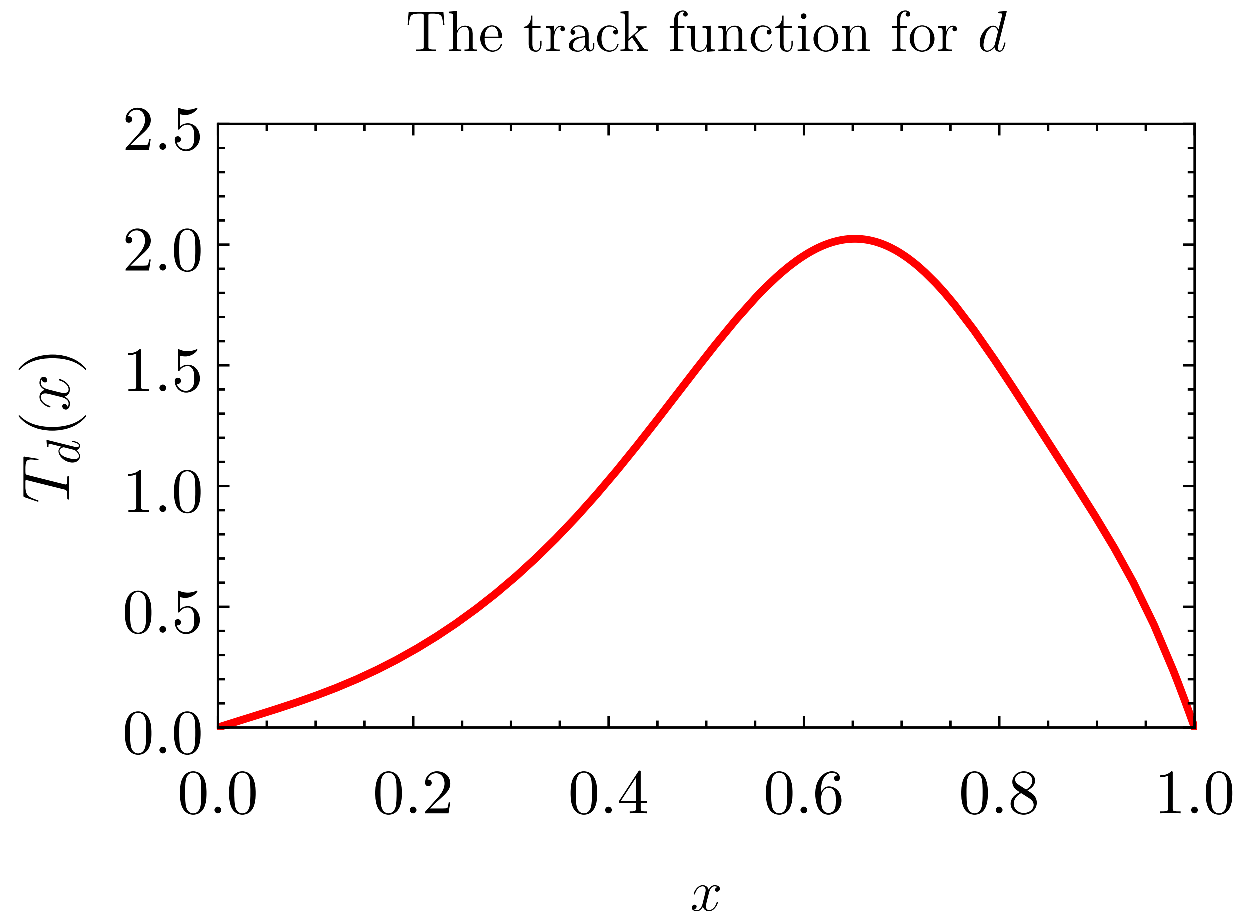
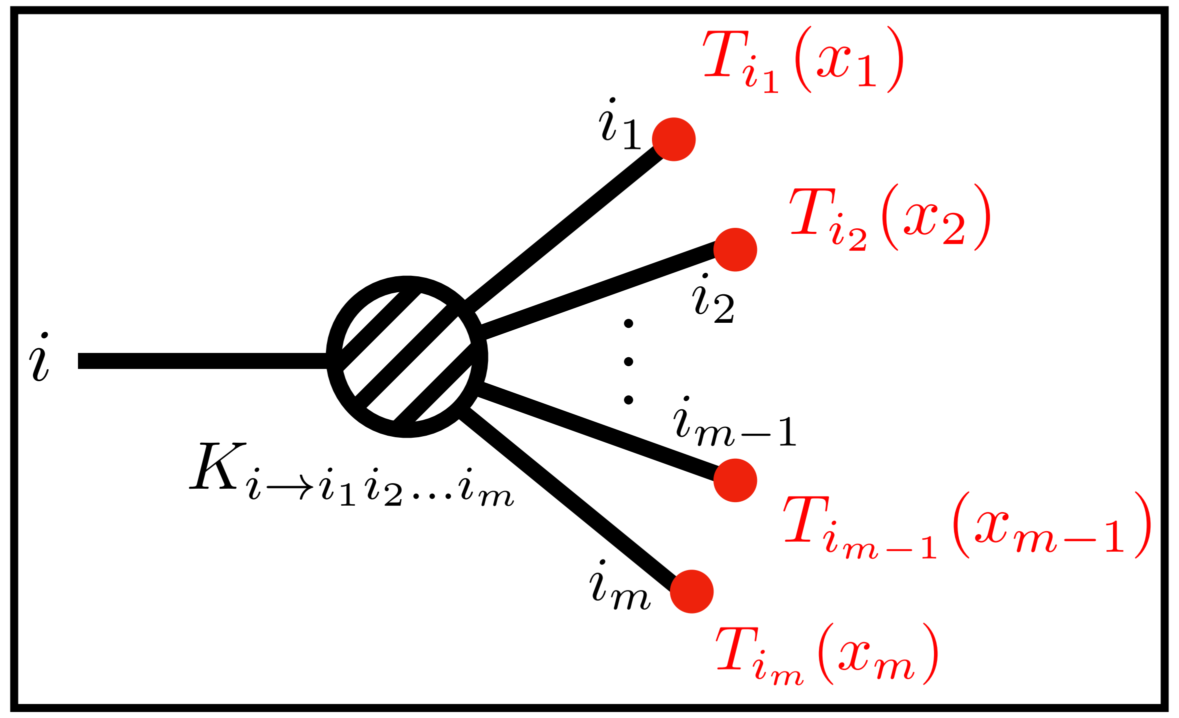
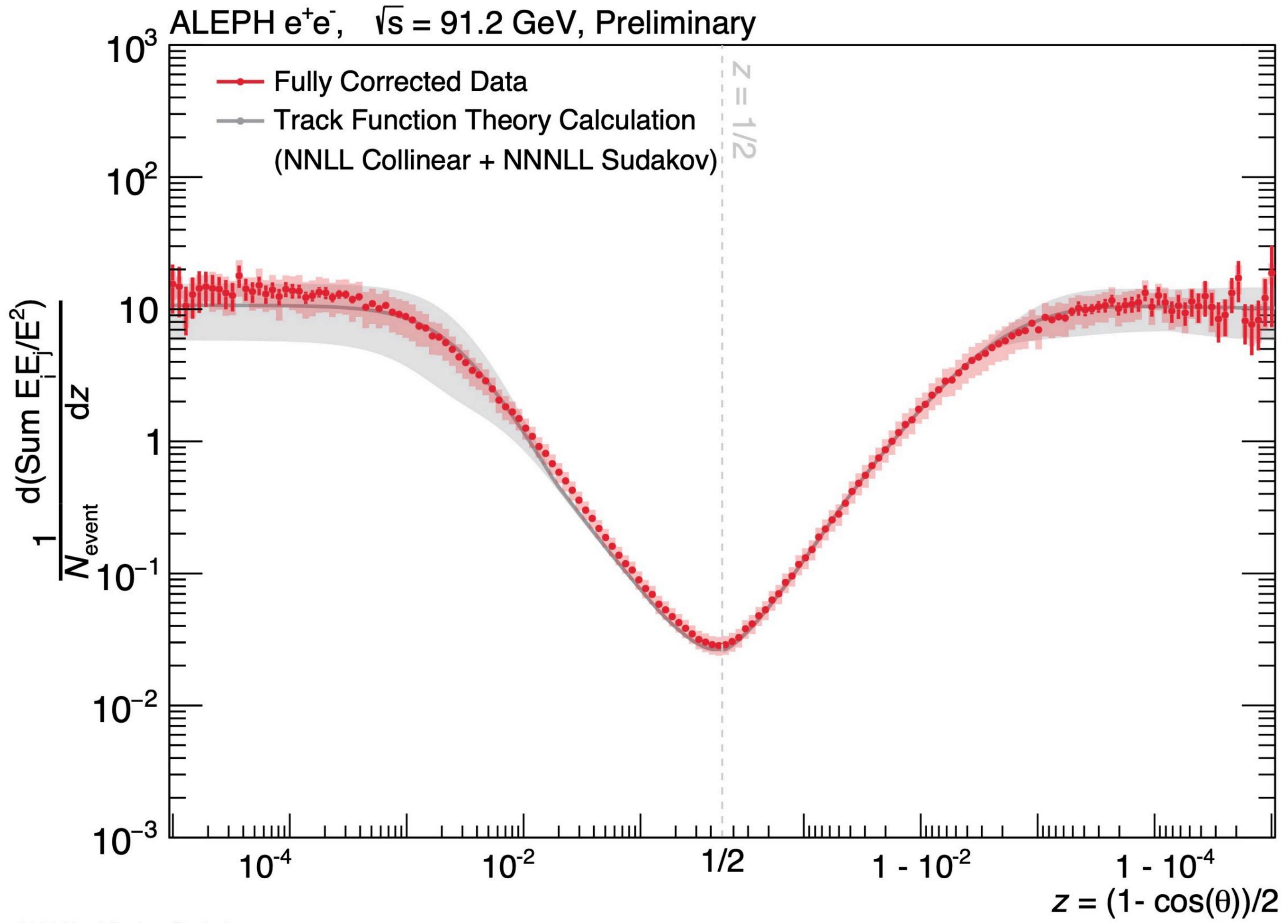
How does it work?

- Track function formalism

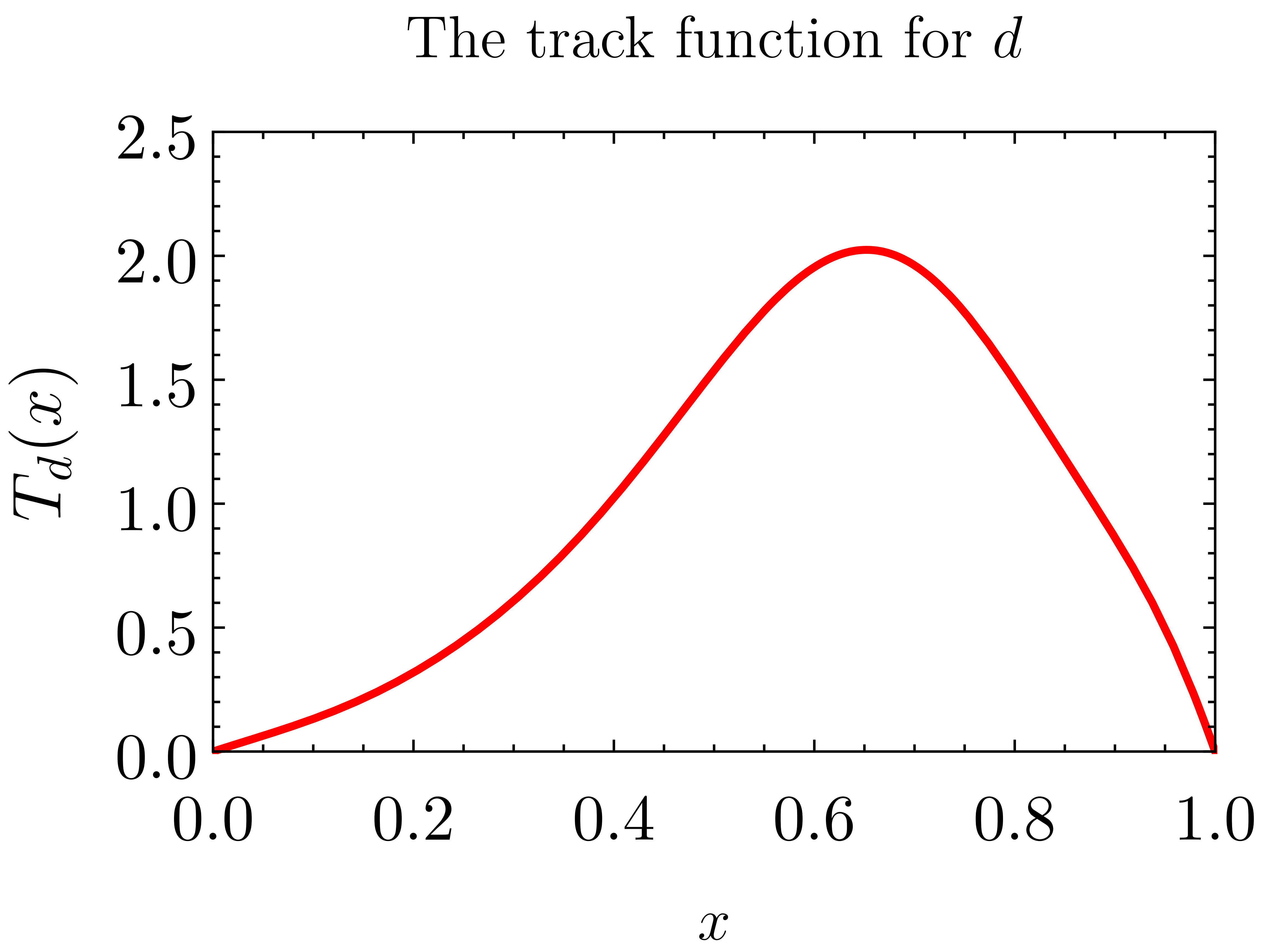




- Full-range energy-energy correlation in e^+e^-



Track function formalism

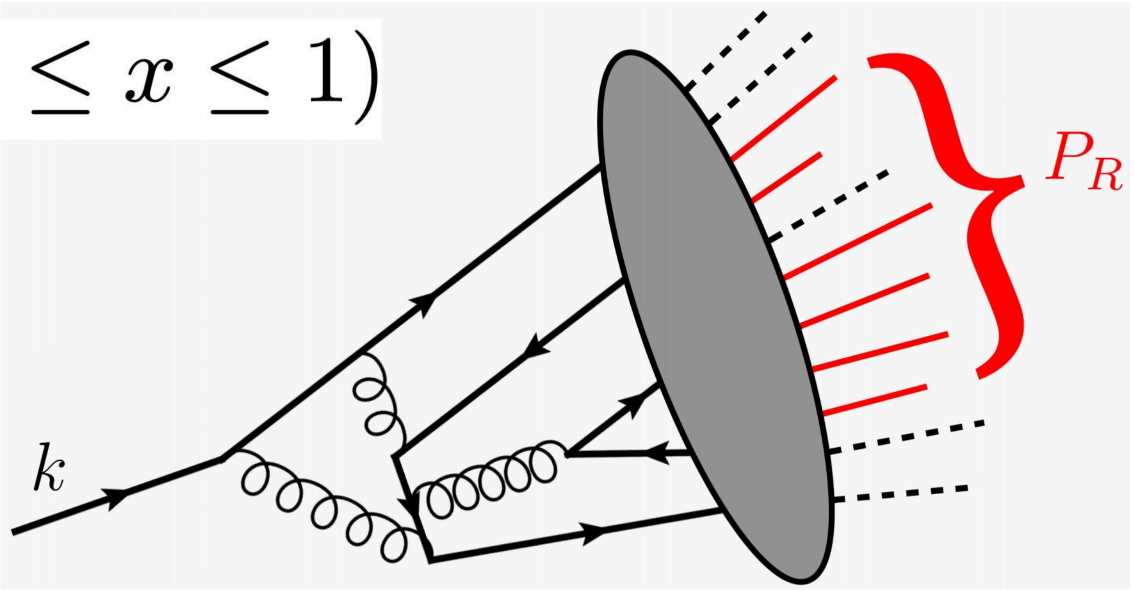


Track functions $T_i(x,\mu)$

[Chang, Procura, Thaler, Waalewijn `13]
[Chen, Jaarsma, **Li**, Moult, S. van Velzen, Waalewijn, Zhu `21-23]

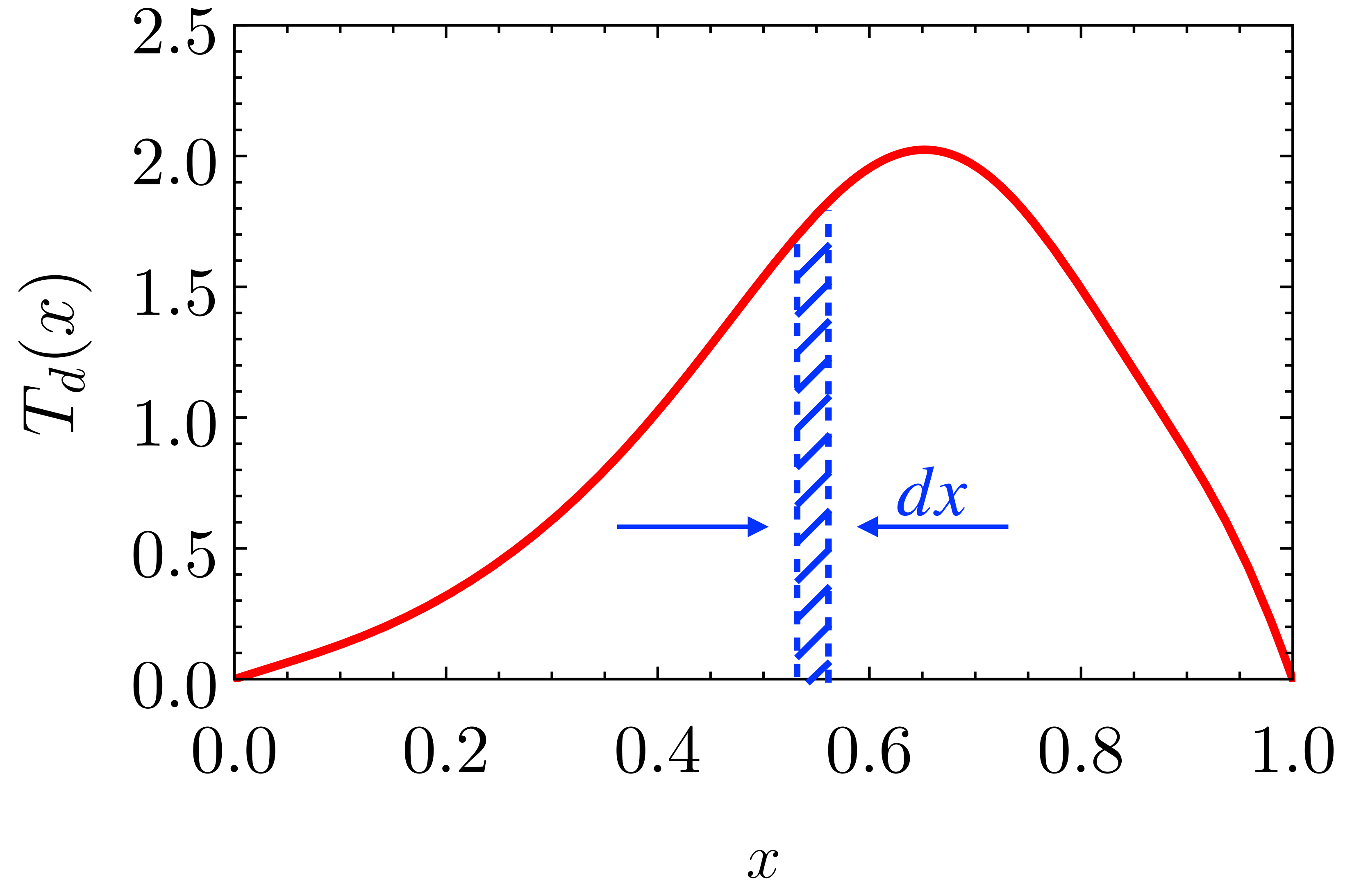
- Describes the total momentum fraction x of a restricted subset R of final-state hadrons in a jet initiated by a hard parton i.

$$P_R^\mu = x k^\mu + \mathcal{O}(\Lambda_{\rm QCD}) \quad (0 \le x \le 1)$$



$$T_q(x) = \int \mathrm{d}y^+ \mathrm{d}^2 y_\perp e^{ik^- y^+/2} \frac{1}{2N_c} \sum_{R,\bar{R}} \delta\left(x - \frac{P_R^-}{k^-}\right) \mathrm{tr}\left[\frac{\gamma^-}{2} \langle 0|\psi(y^+,0,y_\perp)|R\bar{R}\rangle\langle R\bar{R}|\bar{\psi}(0)|0\rangle\right]$$

- E.g., R = charged particles (tracks).
- Nonperturbative.
- Perturbatively calculable scale (μ) dependence.
- Sum rule: $\int_0^1 \mathrm{d}x\, T_i(x,\mu) = 1$



Incorporating tracks

- For a δ-function type observable τ measured on partons:

$$\frac{d\sigma}{d\tau} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta\left[\tau - \hat{\tau}(\{p_i^\mu\})\right]$$

tracks

full functional form of *T*

$$\frac{d\sigma}{d\bar{\tau}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \prod_{i=1}^{N} dx_i T_i(x_i) \times \delta\left[\bar{\tau} - \hat{\tau}(\{x_i p_i^\mu\})\right]$$

- For a k-point energy correlator :

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\cdots\mathcal{E}(\vec{n}_k)\rangle$$

tracks

$$\langle \mathcal{E}_R(\vec{n}_1)\mathcal{E}_R(\vec{n}_2)\cdots\mathcal{E}_R(\vec{n}_k)\rangle$$

- Related to partonic-level correlation functions by moments of *T*:

$$\langle \mathcal{E}_R(\vec{n}_1)\mathcal{E}_R(\vec{n}_2)\cdots\mathcal{E}_R(\vec{n}_k)\rangle = \sum_{i_1,i_2,\cdots,i_k} T_{i_1}(1)\cdots T_{i_k}(1)\langle \mathcal{E}_{i_1}(\vec{n}_1)\mathcal{E}_{i_2}(\vec{n}_2)\cdots\mathcal{E}_{i_k}(\vec{n}_k)\rangle + [\text{contact terms}]$$

dependent on higher moments of T

k-th moment:

$$T_i(k) \equiv \int_0^1 dx\, x^k\, T_i(x)$$

Incorporating tracks

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Simpler!

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$$+[\text{contact terms}]$$

dependent on higher moments of T

k-th moment:

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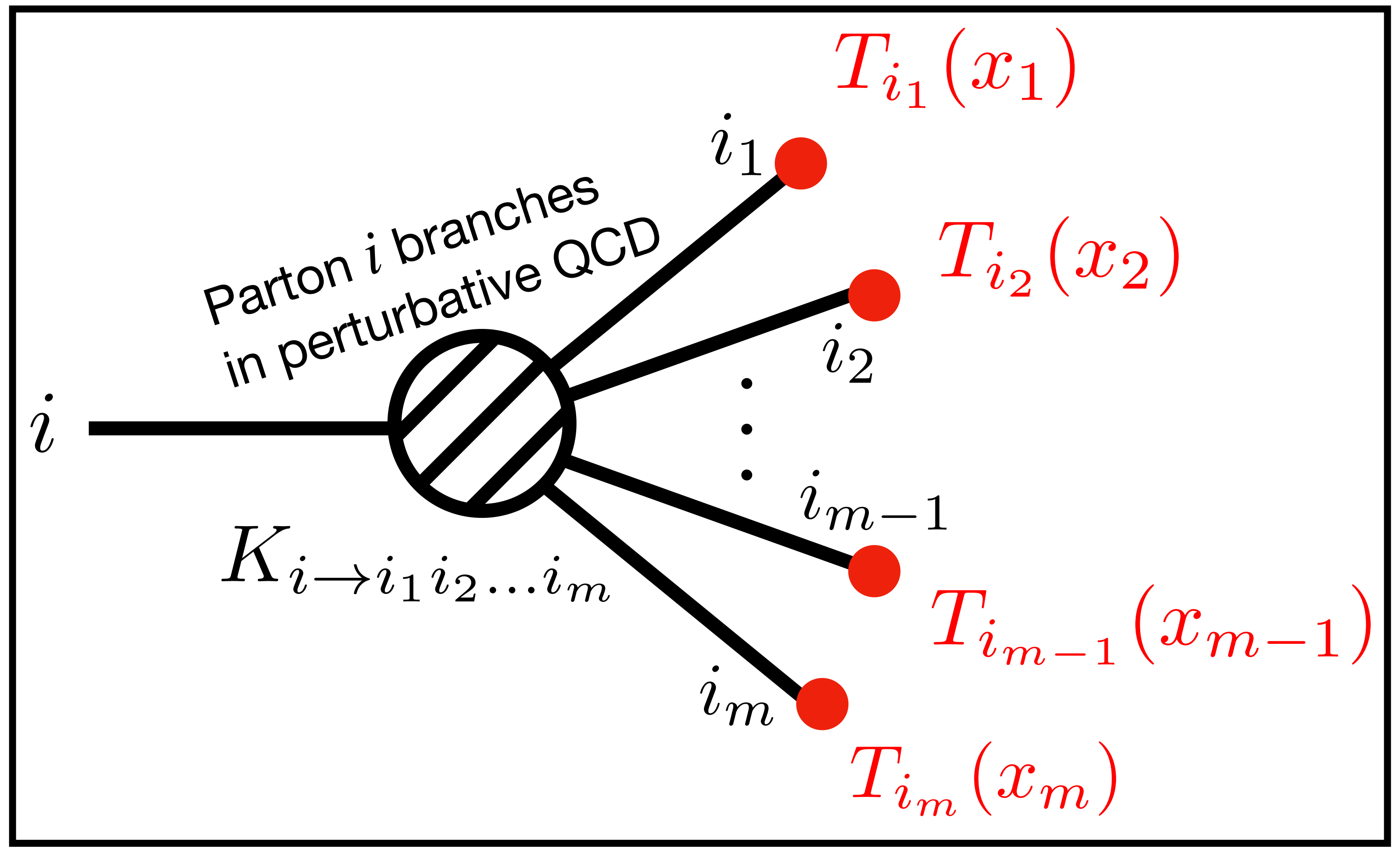
Track function evolution

Universal

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x) = \sum_M \sum_{\{i_f\}} \left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}z_m\right]\delta\left(1-\sum_{m=1}^{M} z_m\right) \boxed{K_{i\to\{i_f\}}(\{z_f\})}$$
$$\times \left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}x_m\, T_{i_m}(x_m)\right]\delta\left(x-\sum_{m=1}^{M} z_m x_m\right)$$

$(i, i_f = g, u, \bar{u}, d, \cdots)$



- **Nonlinear**, involving contributions from all branches of splittings.
- **In moment space,**

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\mathbf{T}_n(\mu) = \boxed{\widehat{R}_n}\mathbf{T}_n(\mu) \qquad \boxed{\widehat{R}_n} = \sum_{L=1}^{\infty} a_s^L \widehat{R}_n^{(L)}$$

$$\mathbf{T}_n = \{T_i(n), \cdots, T_{i_1}(k)T_{i_2}(n-k), \cdots, T_{i_1}(1)\cdots T_{i_n}(1)\}^{\mathrm{t}}$$

— Combinations of moments with a total weight *n*

- For single-hadron FFs: Only one branch observed → **Linearity**

✓ NLO evolution fully derived. [Chen, Jaarsma, **Li**, Moult, Waalewijn, Zhu `22]

Shift symmetry

- Energy conservation implies the evolution is shift-symmetric: $x \to x+a$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x+a) = \sum_M \sum_{\{i_f\}} \left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}z_m\right] \delta\left(1-\sum_{m=1}^{M} z_m\right) K_{i\to\{i_f\}}(\{z_f\}) \left[\prod_{m=1}^{M}\int_0^1 \mathrm{d}x_m\, T_{i_m}(x_m+a)\right] \delta\left(x-\sum_{m=1}^{M} z_m x_m\right)$$

- This invariance constrains the form of the evolution of track function moments:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\Delta = \left[-\gamma_{qq}(2) - \gamma_{gg}(2)\right]\Delta\ ,$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}\begin{bmatrix}\sigma_g(2)\\ \sigma_q(2)\end{bmatrix} = \begin{bmatrix}-\gamma_{gg}(3) & -\gamma_{qg}(3)\\ -\gamma_{gq}(3) & -\gamma_{qq}(3)\end{bmatrix}\begin{bmatrix}\sigma_g(2)\\ \sigma_q(2)\end{bmatrix} + \begin{bmatrix}\gamma^g_{\Delta^2}\\ \gamma^q_{\Delta^2}\end{bmatrix}\Delta^2$$

shift-invariant objects:

$$\Delta := T_q(1) - T_g(1)$$

$$\sigma_i(2) := T_i(2) - T_i(1)^2$$

- In charged-hadron case, the small Δ implies that the $\sigma(2)$ evolution is dominated by the DGLAP kernels.

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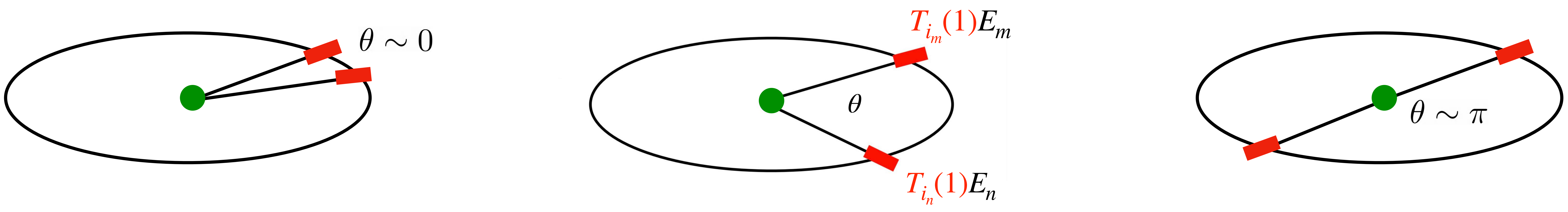
Extend the NLO second-moment evolution to NNLO using three-loop DGLAP kernels.

🚀🚀🚀

Track EEC at NNLL

Full-range track EEC



collinear

fixed-order

back-to-back

A sketch

- NNLL resummation
  [3-loop DGLAP: Moch, Vogt `07; Chen, Yang, Zhu, Zhu `20]
- 2-loop analyt.
- 3-loop
  [CoLoRFulNNLO]
- N4LL Sudakov resummation
  [Ingredients: …; Billis, Michel, Tackmann `24]
- NP Collins-Soper kernel piece from lattice and data
  [Moos, Scimemi, Vladimirov, Zurita `23]
  [Avkhadiev, Shanahan, Wagman, Zhao `24]
- NP effects in TMDs
  […; Li, Makris, Vitev `21]
- NP power correction: Ω_1
  […; Lee, Pathak, Schindler, Stewart, Sun `24]
- Col plateau ~ b2b plateau

Collinear **FO** **Back-to-Back**

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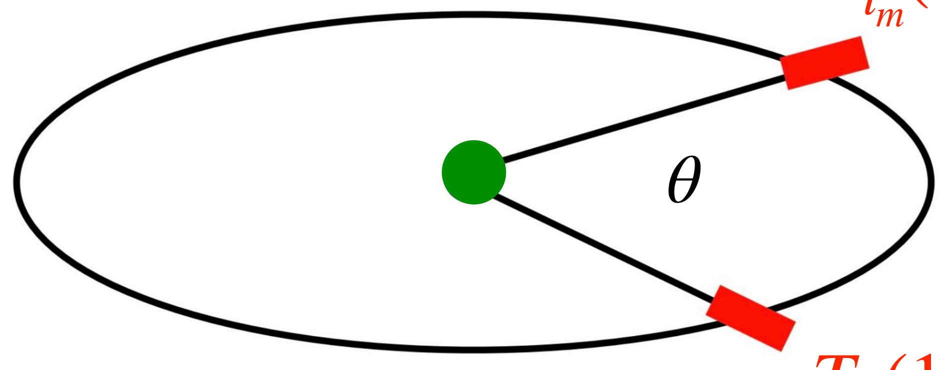
Collinear **FO** **Back-to-Back**

See Max's talk this afternoon!

Track EEC

Definition

◉ **Partonic EEC:** $$\frac{d\sigma}{dz} = \frac{1}{\sigma_0}\sum_{m,n}\int d\sigma \frac{E_m E_n}{Q^2}\delta\left(z - \frac{1-\cos\theta_{mn}}{2}\right)$$



◉ **Track EEC:**

$$\text{Track EEC}(z) = \frac{1}{\sigma_0}\left[\sum_{m\neq n} T_{i_m}^{(0)}(1)T_{i_n}^{(0)}(1)\int d\sigma \frac{E_m E_n}{Q^2}\delta\left(z - \frac{1-\cos\theta_{mn}}{2}\right) + \sum_m T_{i_m}^{(0)}(2)\int d\sigma \frac{E_m^2}{Q^2}\delta(z)\right]$$

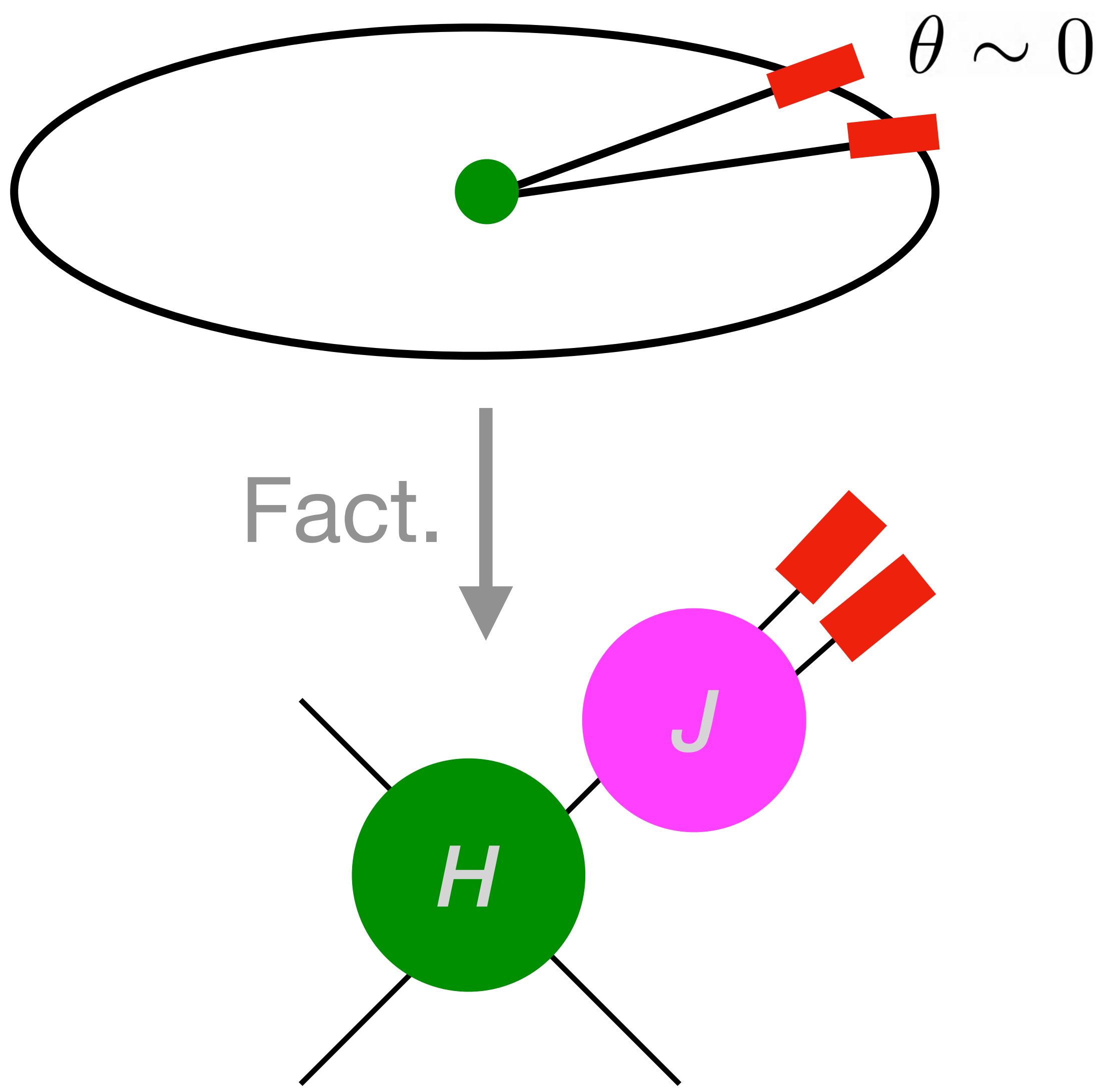
◉ **Fixed-order calculation:**

- Renormalization of the coupling constant and track function moments.

$$\mathbf{T}_n^{\text{bare}} = \mathbf{T}_n(\mu) + a_s \frac{\widehat{R}_n^{(1)}}{\epsilon}\mathbf{T}_n(\mu) + \frac{1}{2}a_s^2\left(\frac{\widehat{R}_n^{(2)}}{\epsilon} + \frac{\widehat{R}_n^{(1)}\widehat{R}_n^{(1)} - \beta_0\widehat{R}_n^{(1)}}{\epsilon^2}\right)\mathbf{T}_n(\mu) + \mathcal{O}(a_s^3)$$

Collinear resummation

- **Collinear limit:** $\theta \to 0, z \to 0.$
- **Physical scales involved:** $Q, \sqrt{z}Q, \Lambda_{\text{QCD}}$.
- **Hard-collinear factorization:** $Q \gg \sqrt{z}Q \gg \Lambda_{\text{QCD}}$,



$$\text{EEC}^{\text{fact.}}_{z\to 0}(z) = \frac{\mathrm{d}}{\mathrm{d}z}\int_0^1 \mathrm{d}x\, x^2 \vec{J}\left(\ln\frac{zx^2Q^2}{\mu^2}, \mu\right)\cdot \vec{H}\left(x, \ln\frac{Q^2}{\mu^2}, \mu\right)$$

- $\mu_H \sim Q$, $\mu_J \sim \sqrt{z}Q$.

Large logs $\ln(z)$

- Single logs:

$$a_s^L\left(\underbrace{C_{L,L}\left[\frac{\ln^{L-1} z}{z}\right]_+}_{\text{LL}} + \underbrace{C_{L,L-1}\left[\frac{\ln^{L-2} z}{z}\right]_+}_{\text{NLL}} + \underbrace{C_{L,L-2}\left[\frac{\ln^{L-3} z}{z}\right]_+}_{\text{NNLL}} + \cdots\right)$$

- Soft insensitivity: natural jet substructure observable.

Also true for higher-point correlators.

Collinear resummation

◉ **Techniques:**

- Evolve the jet function from $\mu_J \sim \sqrt{z}Q$ to $\mu_H \sim Q$.

- The jet function at N^{l}LL accuracy:

$$J_i^{\mathrm{N}^l\mathrm{LL}}\left(a_s(\mu), \mathbf{T}_2(\mu), \ln\frac{\mu_J^2}{\mu^2}\right) = \sum_{L=0}^{\infty} a_s^L(\mu)\left[\sum_{m=L-l}^{L} \mathbf{j}_{i,m}^{[2],(L)} \cdot \mathbf{T}_2\ \ln^m\left(\frac{\mu_J^2}{\mu^2}\right)\right]$$

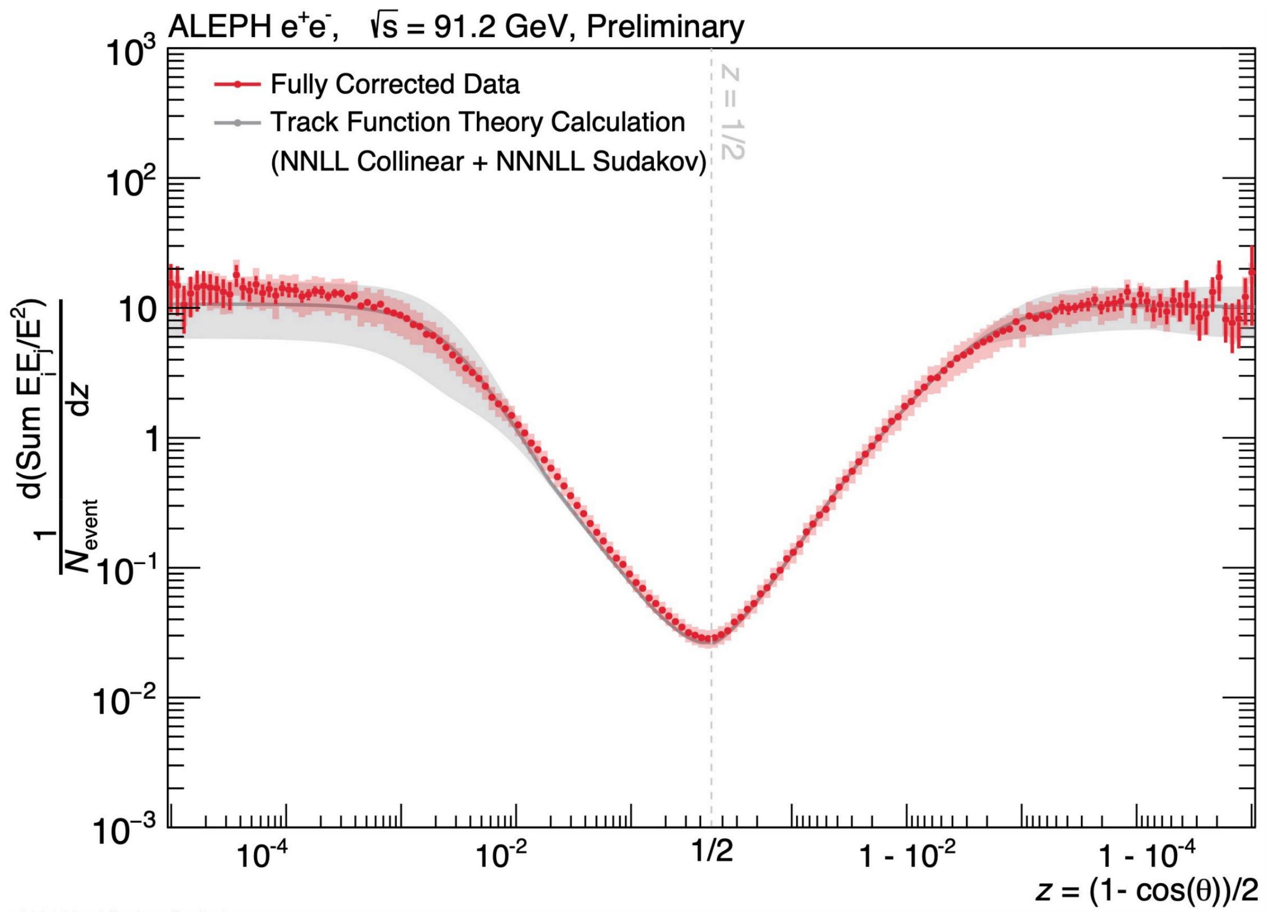
- Iteratively solve the RG equation for the perturbative coefficients.

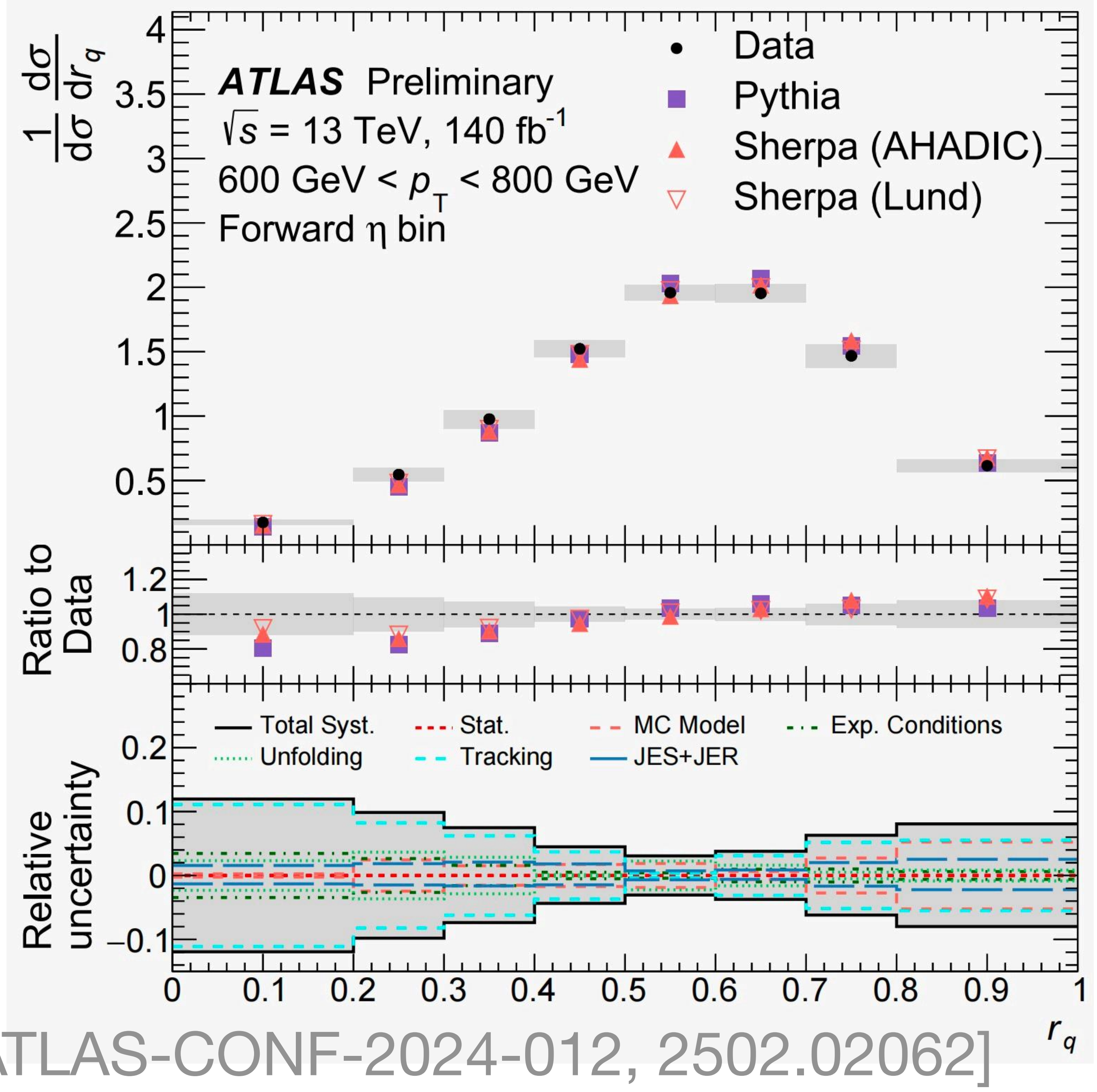
- A truncated solution up to order-a_s^{25}.

Summary

- The general collinear evolution with track functions.

- Application to full-range track EEC.





[ATLAS-CONF-2024-012, 2502.02062]

Measurement of track functions starts!

- Use track energy correlators to probe into both perturbative and nonperturbative QCD.

Outlook

- Precision phenomenology with all particles ⇨ $\langle \mathcal{E}_R(\vec{n}_1)\mathcal{E}_R(\vec{n}_2)\cdots\mathcal{E}_R(\vec{n}_k)\rangle$

  ⇨ $\langle \mathcal{E}_{R_1}(n_1)\,\mathcal{E}_{R_2}(n_2)\cdots\mathcal{E}_{R_k}(n_k)\rangle$ [Lee, Moult `23]

  Including extra quantum info.

- Data and pheno for multi-hadron fragmentation functions.

  [Recent: di-hadron: Cocuzza, Metz, Pitonyak, et al `23-24]

- A benchmark for triple collinear evolution in parton showers.

  [Mrinal Dasgupta's talk at Fragmentation 2025, Zurich]

- Including quark mass effect.

- Non-perturbative power corrections.

Outlook

◉ More formal aspects:

[Hao Chen’s talk: Chang, Chen, Kravchuk, Simmons-Duffin, Zhu: coming soon]

- Moments of (single-hadron) FFs are related to light-ray operators. Their analytic continuation gives DGLAP/BFKL mixing trajectory.

- DGLAP evolution: RG of twist-2 light-ray operators ⇨

  Connections between the non-linear kernels and higher-twist operators.

Thanks!