# Exploring the Saturation effect via the heavy quark/meson pair photo-production


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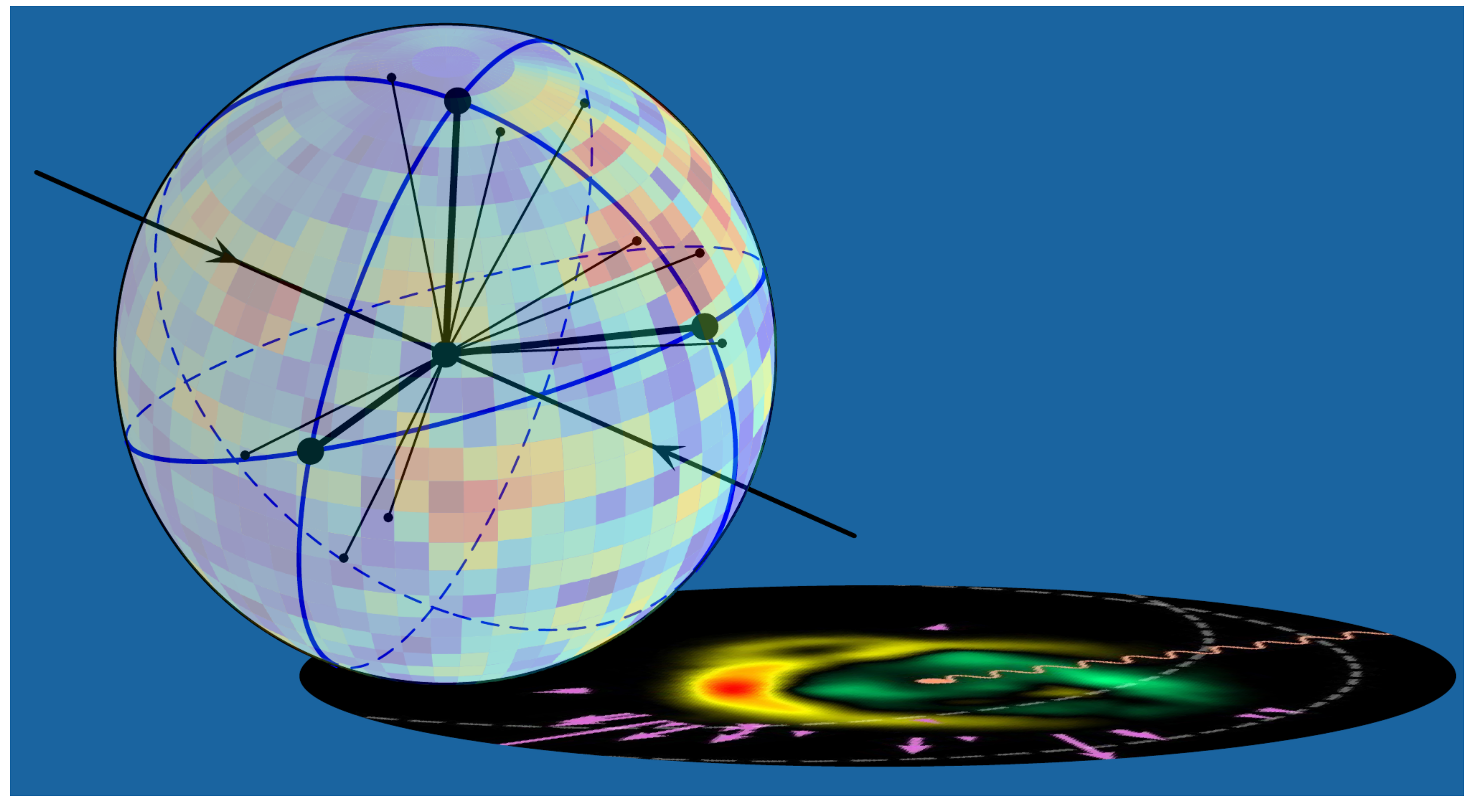


- Cyrille Marquet, YS, Cheng Zhang, in preparation

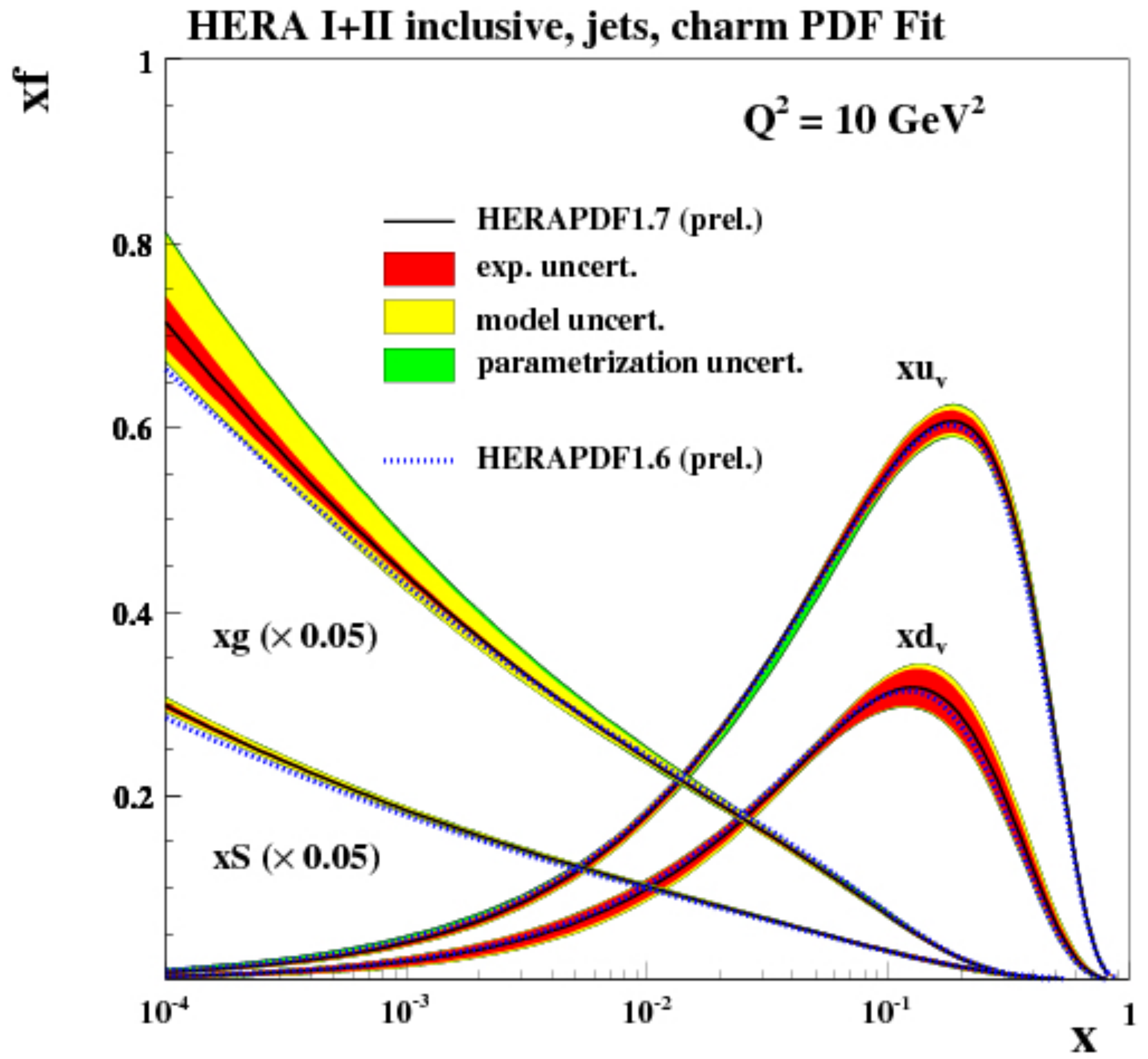
New Opportunities in Particle and Nuclear Physics with Energy Correlators

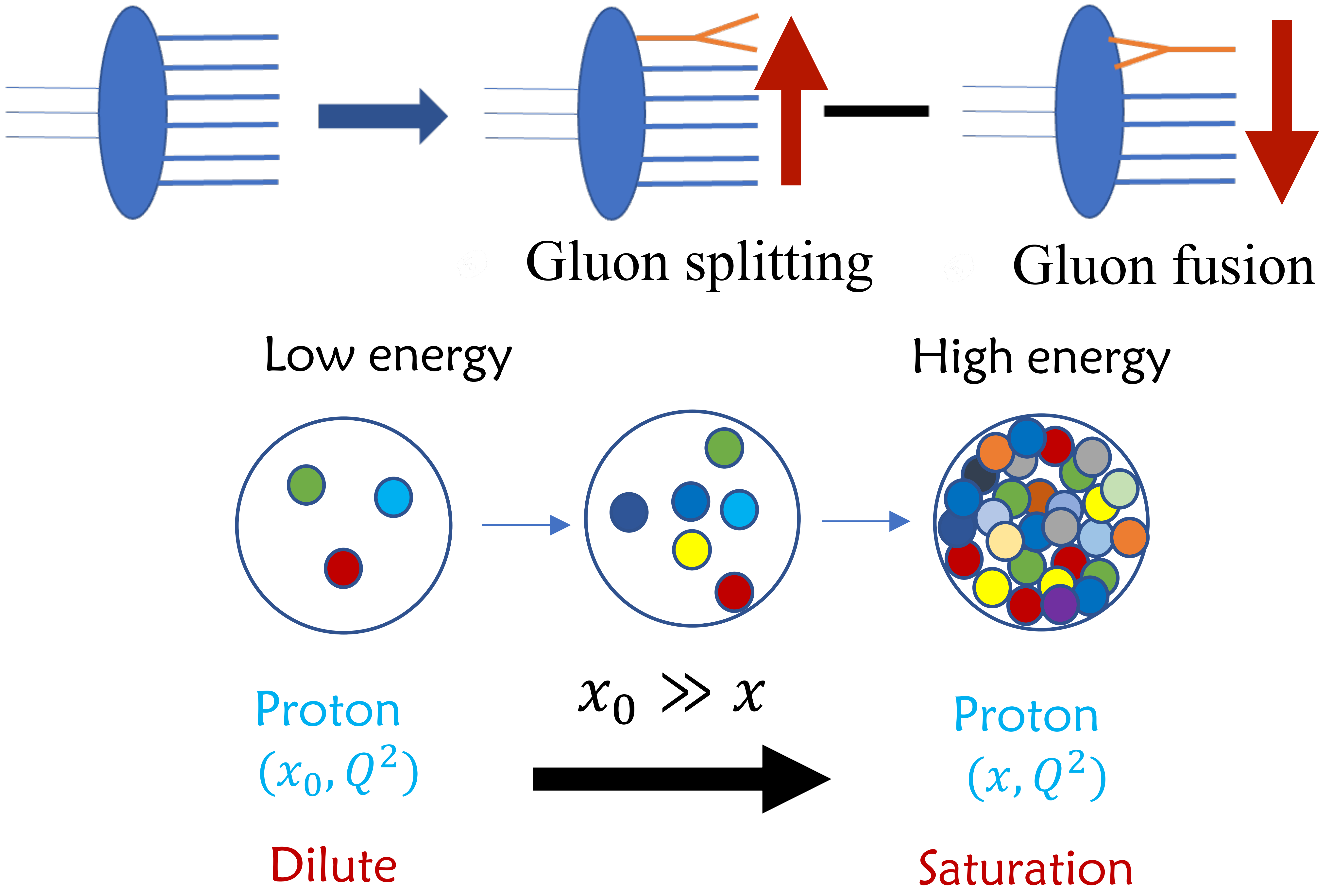
June 6, 2025

# Outline

1) Introduction
2) Heavy quark/meson pair photo-production in small-x framework
3) Azimuthal Asymmetry
4) Transverse energy-energy correlator

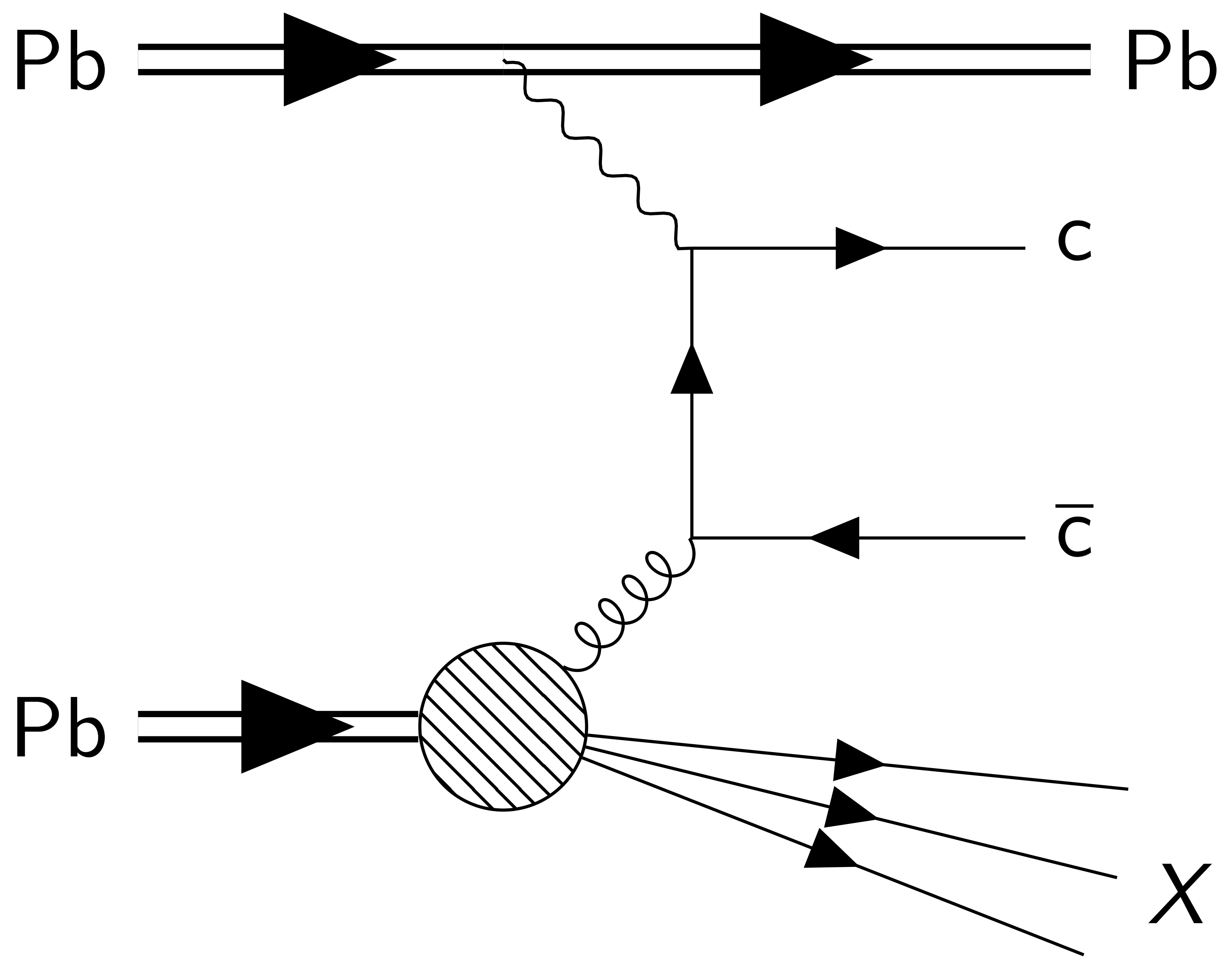
# Gluon Saturation in the small-x



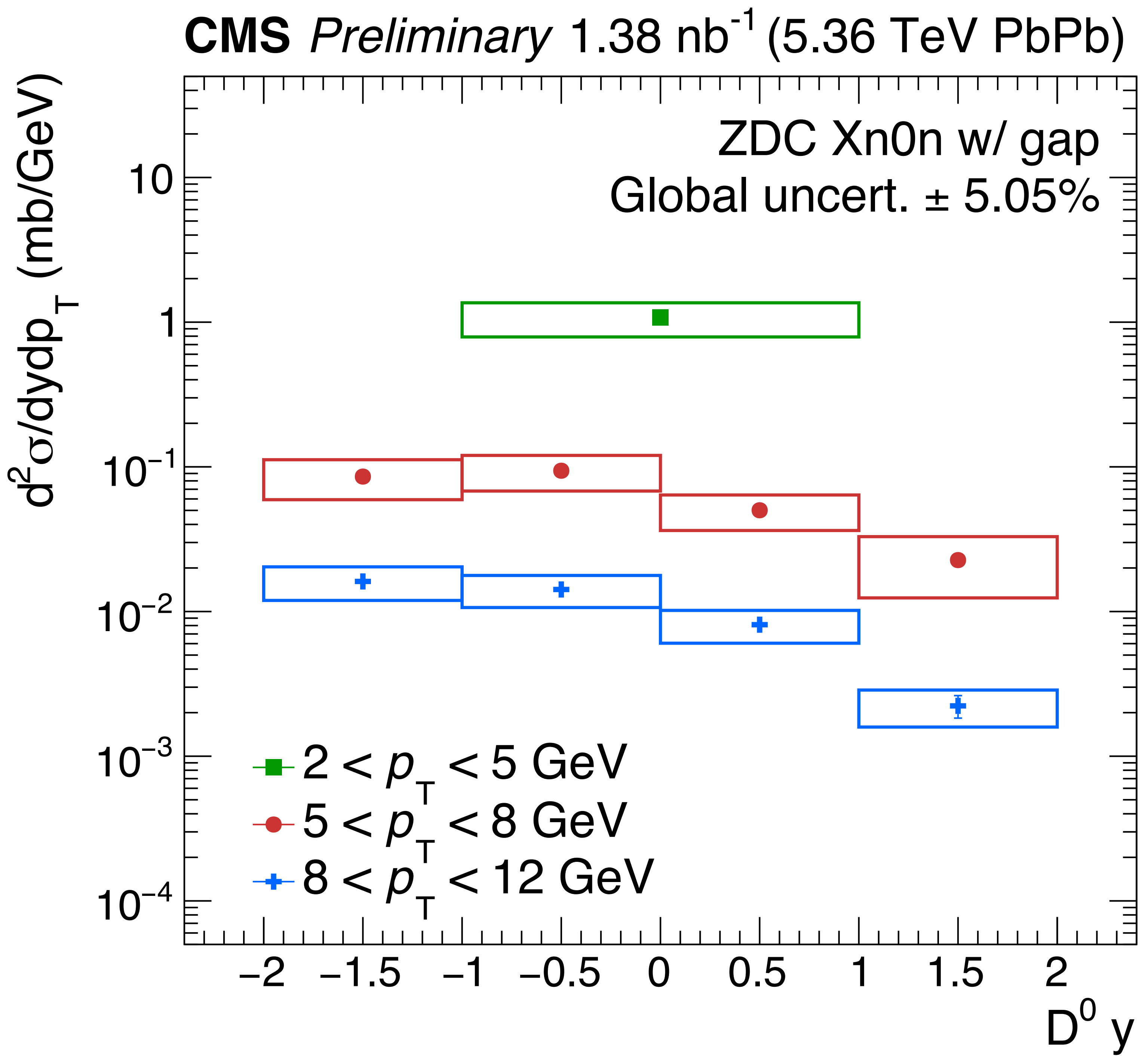




- dense gluon field —gluon saturation— Color Glass Condensate (CGC)
- multiple gluon scattering + small-x non-linear evolution (BK/JIMWLK)

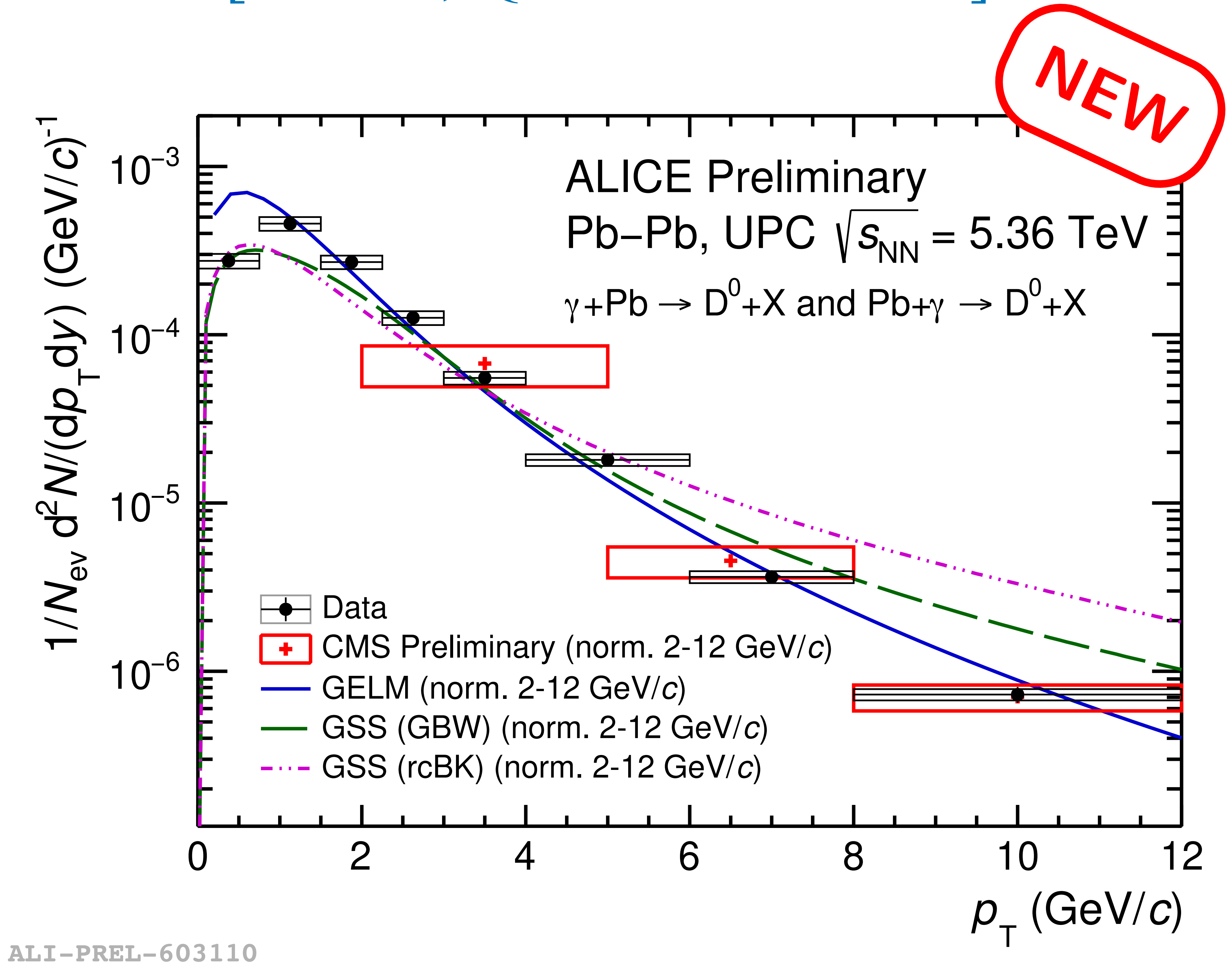
# The heavy quark pair in UPC




[CMS, Quark matter 2025]



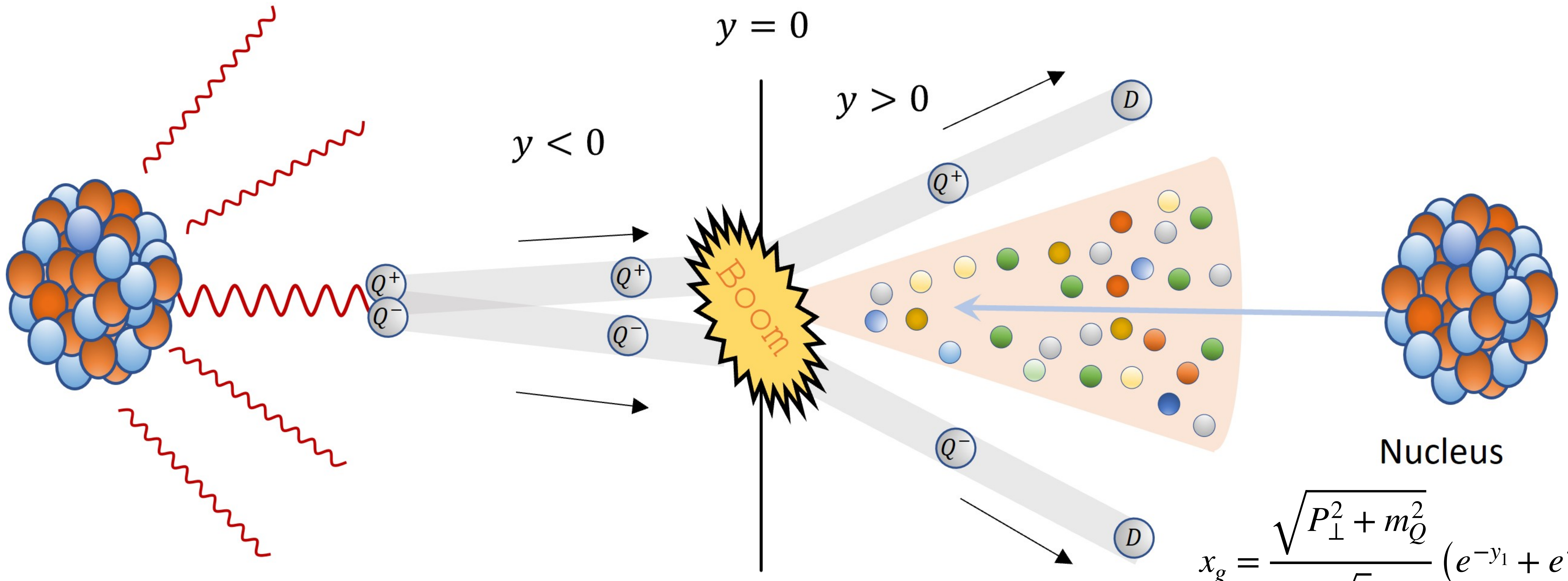

[ALICE, Quark matter 2025]




- CMS will measure heavy quark di-jets/hadrons with higher-statistics Run 3 data.
- High-precision experimental data will be available in the near future !!!

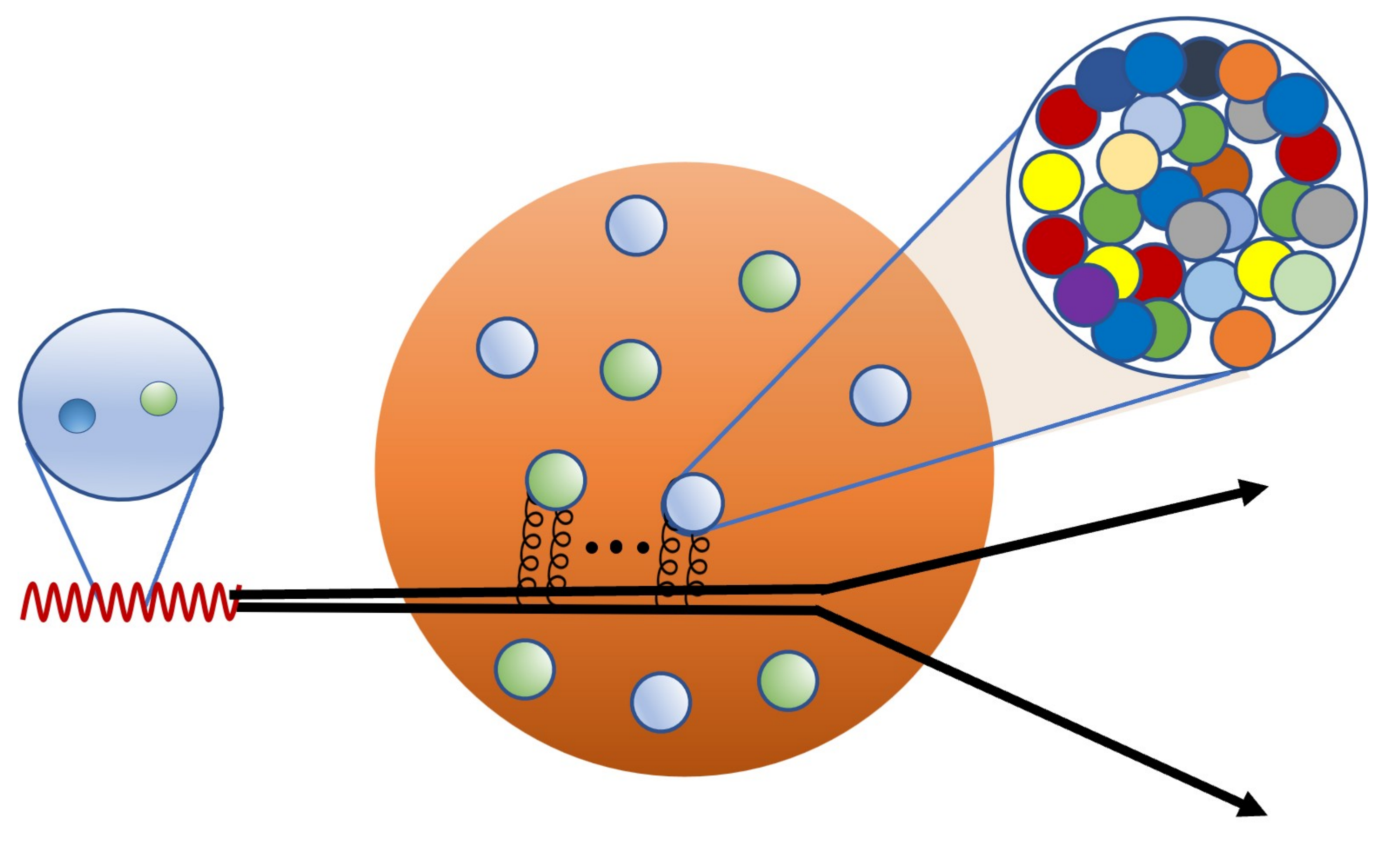
2) The heavy quark/meson pair photo-production in small-x framework

# The heavy quark pair in UPC




$$x_g = \frac{\sqrt{P_\perp^2 + m_Q^2}}{\sqrt{s}} \left(e^{-y_1} + e^{-y_2}\right) \ll 1$$

ultra-dense gluonic matter



nucleus

quasi-real photon    heavy quark pair    small-*x* gluon! (Dense)

- Studying this process can help us to understand the gluon saturation effect.

# Factorization of heavy quark pair photo-production

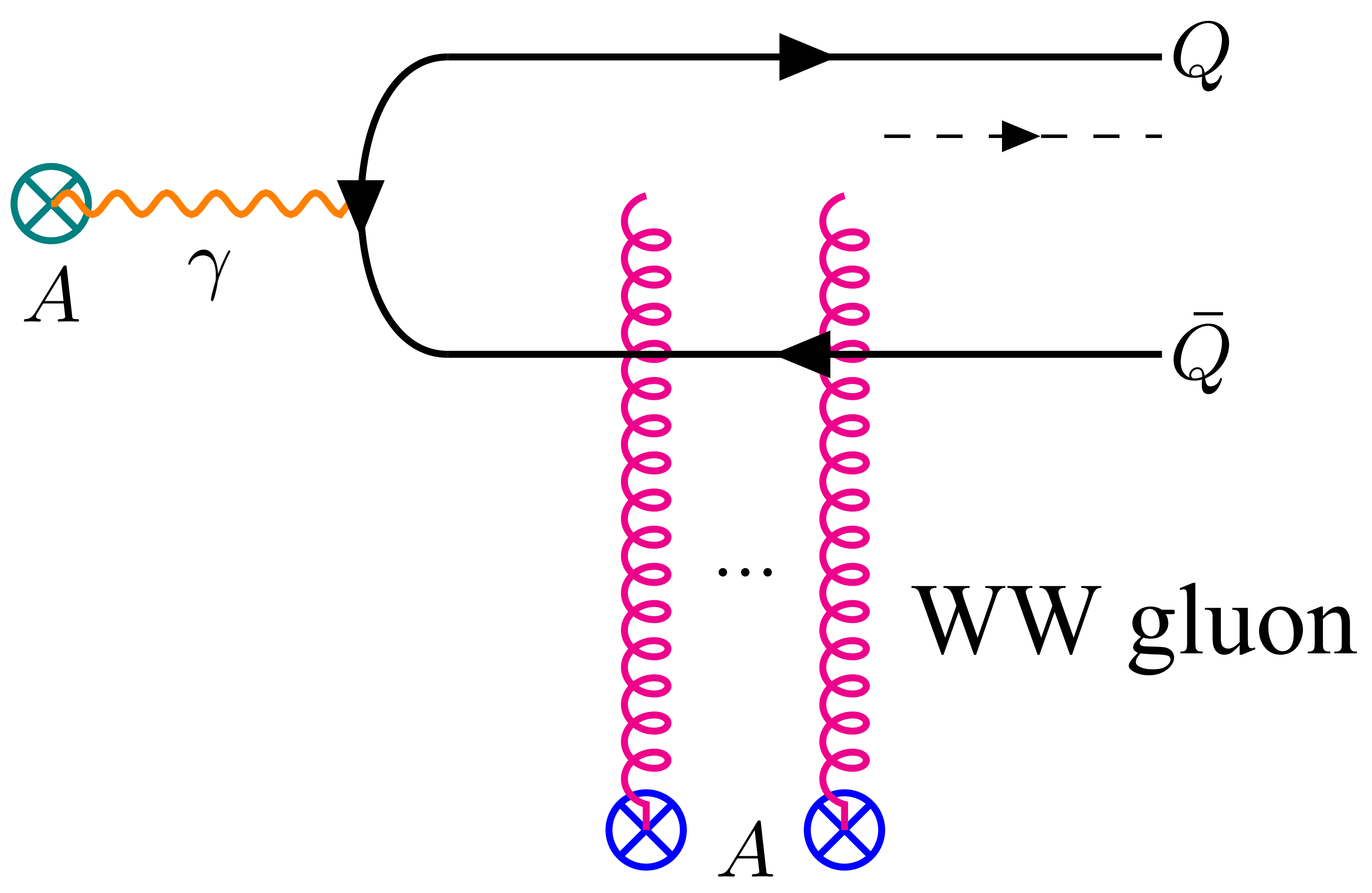
In the back-to-back limit ($\boldsymbol{q} \ll \boldsymbol{P}$), it can be factorized as TMD hard factor ⊗ WW gluon dis

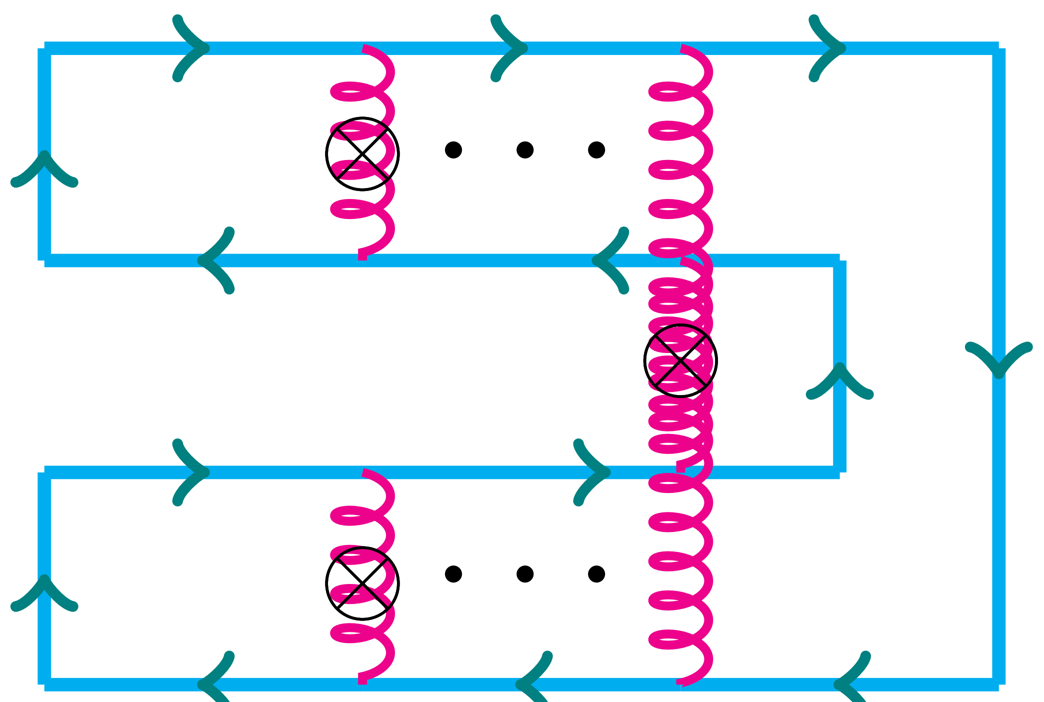
$$\frac{d\sigma_{\rm LO}^{\gamma A \to Q\bar{Q}X}}{dy_1 dy_2 d^2\boldsymbol{P} d^2\boldsymbol{q}} = x_\gamma f(x_\gamma) \boxed{O_{\rm TMD}(\boldsymbol{P})} \left[ \boxed{F_{gg}^{(3)}(x_g, \boldsymbol{q})} + f_1 \cos(2\phi_{Pq}) \boxed{H_{gg}^{(3)}(x_g, \boldsymbol{q})} \right]$$

TMD Hard factor

un-polarized WW gluon

linearly-polarized WW gluon





Quadrupole (WW gluon)

Momentum imbalance $\boldsymbol{q} = \boldsymbol{k}_1 + \boldsymbol{k}_2$

Relative momentum $\boldsymbol{P} = (1-z)\boldsymbol{k}_1 - z\boldsymbol{k}_2$

longitudinal momentum fraction

$$z = \frac{k_1^+}{p_\gamma^+}$$

- Golden channel to study Weizsacker-Williams (WW) gluon dis.
- WW gluon dis encodes dense gluon info.

[Dominguez, Marquet, Xiao, Yuan, Phys.Rev.D 83 (2011) 105005; Metz, Zhou, Phys.Rev.D 84 (2011) 051503 ; Dominguez, Qiu, Xiao, Yuan, Phys.Rev.D 85 (2012) 045003]

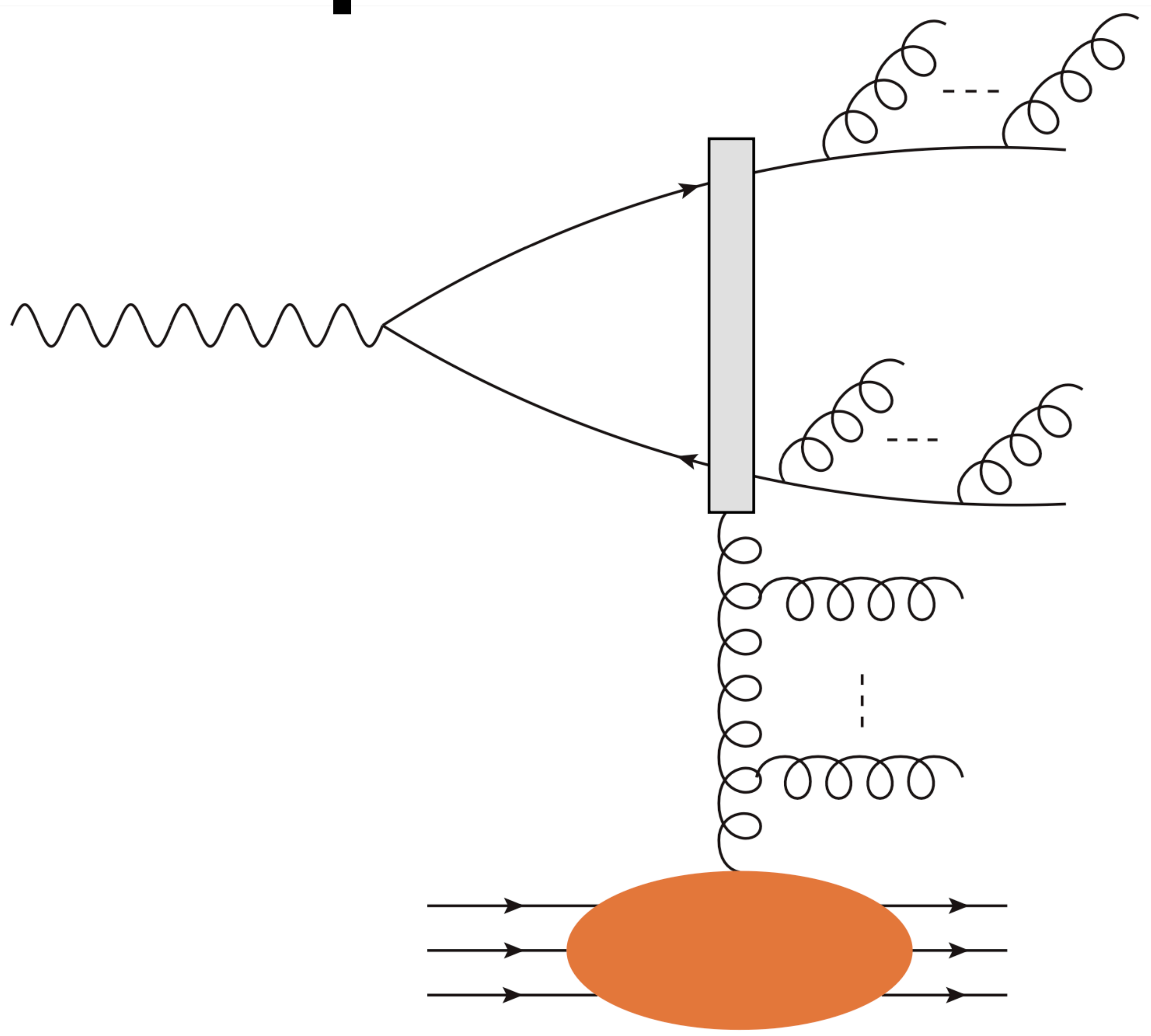
# Soft gluon radiations and qt broadening

LO

azimuthal asymmetry

$$\frac{d\sigma^{LO}}{d^2\boldsymbol{q}} \propto F_{gg}^{(3)}(\boldsymbol{q}) + f_1 \boxed{\cos(2\phi_{Pq})} H_{gg}^{(3)}(\boldsymbol{q})$$

($\boldsymbol{q}$ ~ Saturation scale $Q_s$ )



Final radiations from the quark pair

Initial radiation from the gluon

LO+ Soft-gluon radiations

$$\frac{d\sigma}{d^2\boldsymbol{q}} \propto \int \frac{d\sigma_{\mathrm{LO}}}{d^2\boldsymbol{q}'} \otimes \mathcal{S}_r(\boldsymbol{k}_g)\delta^{(2)}\left(\boldsymbol{q} - \boldsymbol{k}_g - \boldsymbol{q}'\right)$$

Soft-gluon radiations

Saturation effect

Soft gluon radiations

$$\mathcal{S}_r(\boldsymbol{k}_g) \propto \frac{\bar{\alpha}_s}{\boldsymbol{k}_g^2} \ln \frac{M_{in}^2}{\boldsymbol{k}_g^2} + \ldots \longrightarrow \sum_{n=0}^{\infty} \frac{1}{n!}(\alpha_s L)^n = e^{\alpha_s L}$$

[Muller, Xiao, Yuan, PRD88, 114010 (2013); Stasto, Wei, Xiao, Feng Yuan, Phys.Lett.B 784 (2018) 301-306; Hatta, Xiao, Yuan, Zhou, PRD104, 054037 (2021); Shao, YS, Zhang, Zhou, Zhou, JHEP 07 (2024) 189]

# small-x and the Sudakov resummation

- The resummation improved cross-section

[Marquet, YS, Zhang, in preparation]

Soft gluon radiations also generate azimuthal asymmetry!!!

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{P} d^2\boldsymbol{q}} = x_\gamma f(x_\gamma) \int \frac{d^2\boldsymbol{b} d^2\boldsymbol{k}}{(2\pi)^2} e^{i\boldsymbol{q}\cdot\boldsymbol{b} - i\boldsymbol{q}'\cdot\boldsymbol{b}} e^{-\mathrm{Sud}(\boldsymbol{b}, M_{\mathrm{in}})} \left[1 - c_2 \frac{2C_f \alpha_s(\mu_b)}{\pi} \cos(2\phi_{bP})\right]$$

$$\times \left[O_{\mathrm{TMD}}(\boldsymbol{P}) F_{gg}^{(3)}(x_g, \boldsymbol{q}') + \cos(2\phi_{Pq'}) \Omega_{\mathrm{TMD}}(\boldsymbol{P}) H_{gg}^{(3)}(x_g, \boldsymbol{q}')\right]$$

Linearly-polarized gluon generate azimuthal asymmetry.

- the Sudakov factor

$$\mathrm{Sud}(\boldsymbol{b}, M_{\mathrm{in}}) = \mathrm{Sud}_i(\boldsymbol{b}, M_{\mathrm{in}}) + 2\mathrm{Sud}_f(\boldsymbol{b}, M_{\mathrm{in}}) + \mathrm{Sud}_{\mathrm{NP}}^i(\boldsymbol{b}, M_{\mathrm{in}})$$

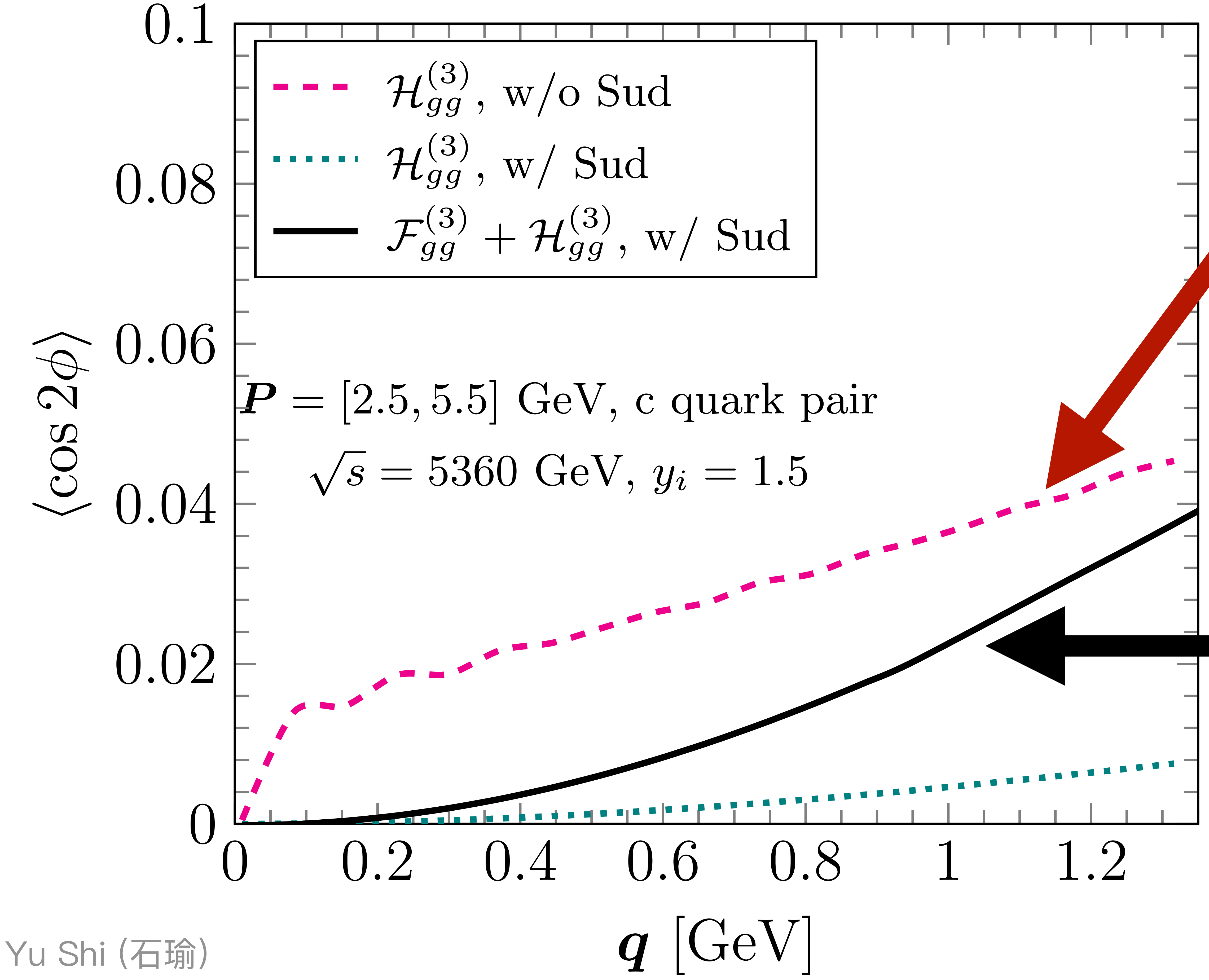
Initial gluon radiation

Final heavy quarks radiation

Non-perturbative effects

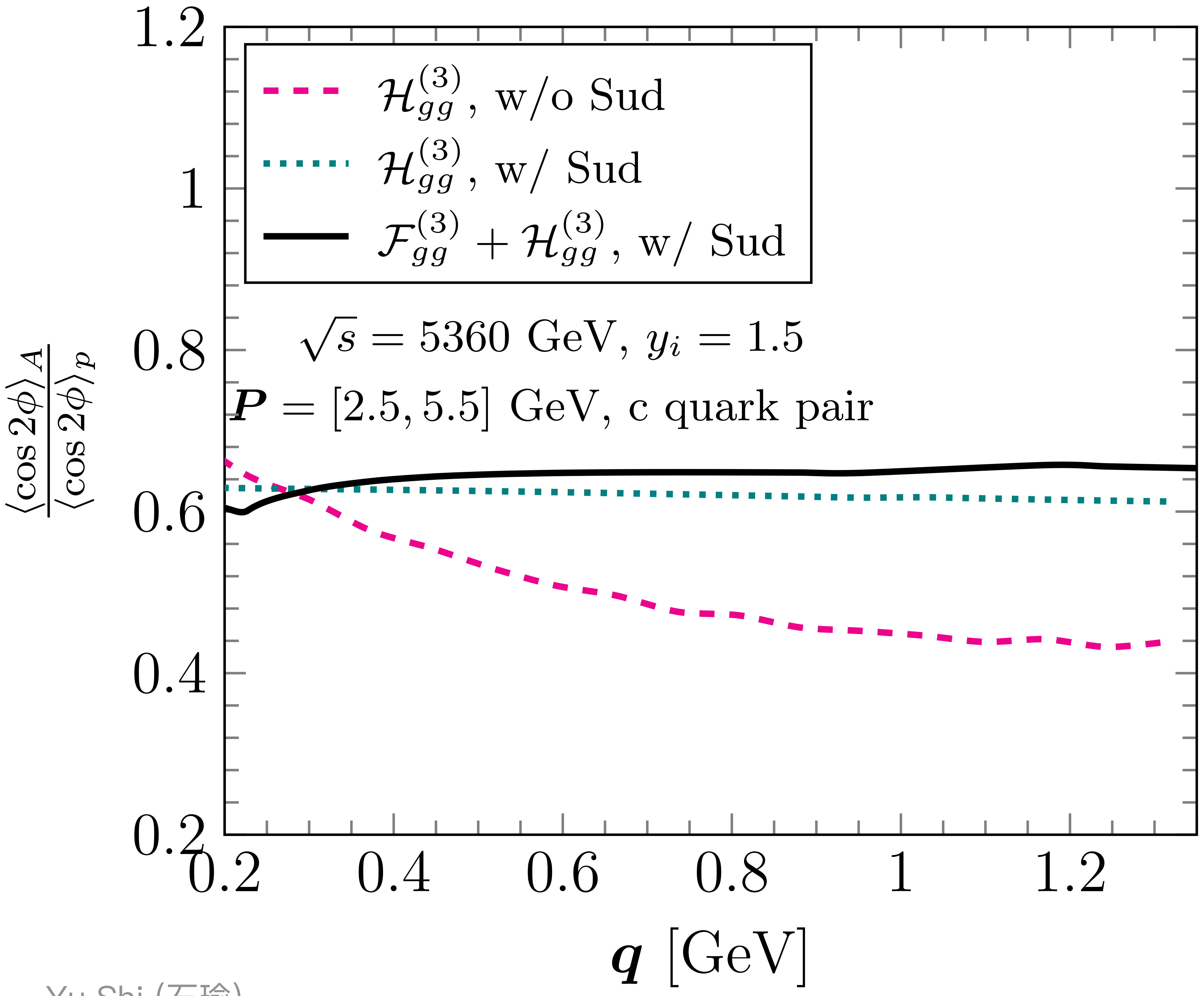
# Preliminary results for heavy quark pair photo-production

$$\langle \cos 2\phi \rangle = \frac{\int d\mathscr{P}\,.\,\mathscr{S}\,.\,\cos(2\phi)\frac{d\sigma}{d\mathscr{P}\,.\,\mathscr{S}\,.}}{\int d\mathscr{P}\,.\,\mathscr{S}\,.\,\frac{d\sigma}{d\mathscr{P}\,.\,\mathscr{S}\,.}}$$




- without Sudakov effect, only linearly polarised WW gluon $\mathcal{H}_{gg}^{(3)}$ generates $\langle \cos 2\phi \rangle$.
- With Sudakov effect, the linearly polarised WW has been suppressed, and the soft-gluon radiation dominates.

# Preliminary results for heavy quark pair photo-production



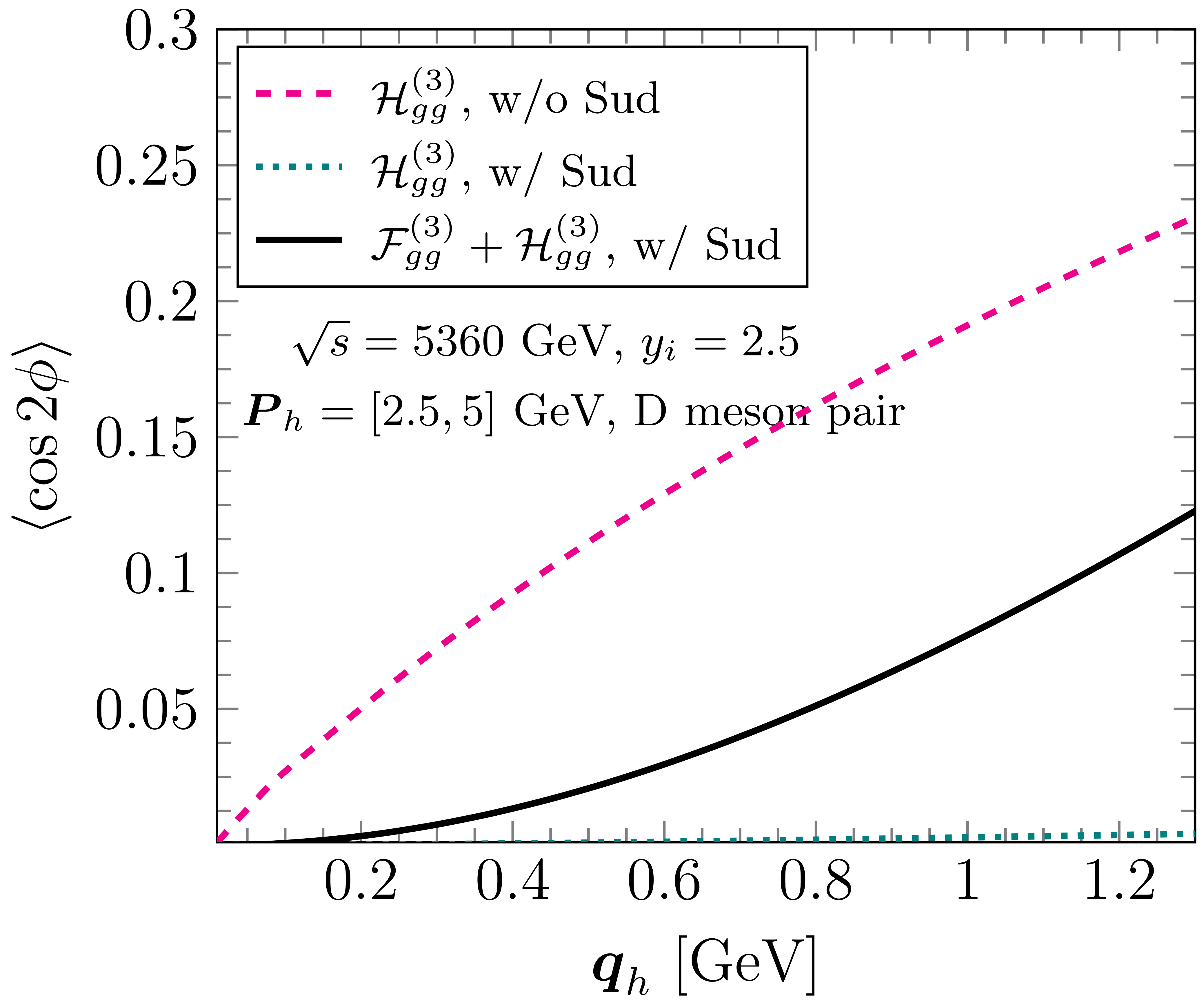

$$\frac{\langle\cos 2\phi\rangle_{\gamma A}}{\langle\cos 2\phi\rangle_{\gamma p}} = 1 \rightarrow$$ No saturation effect

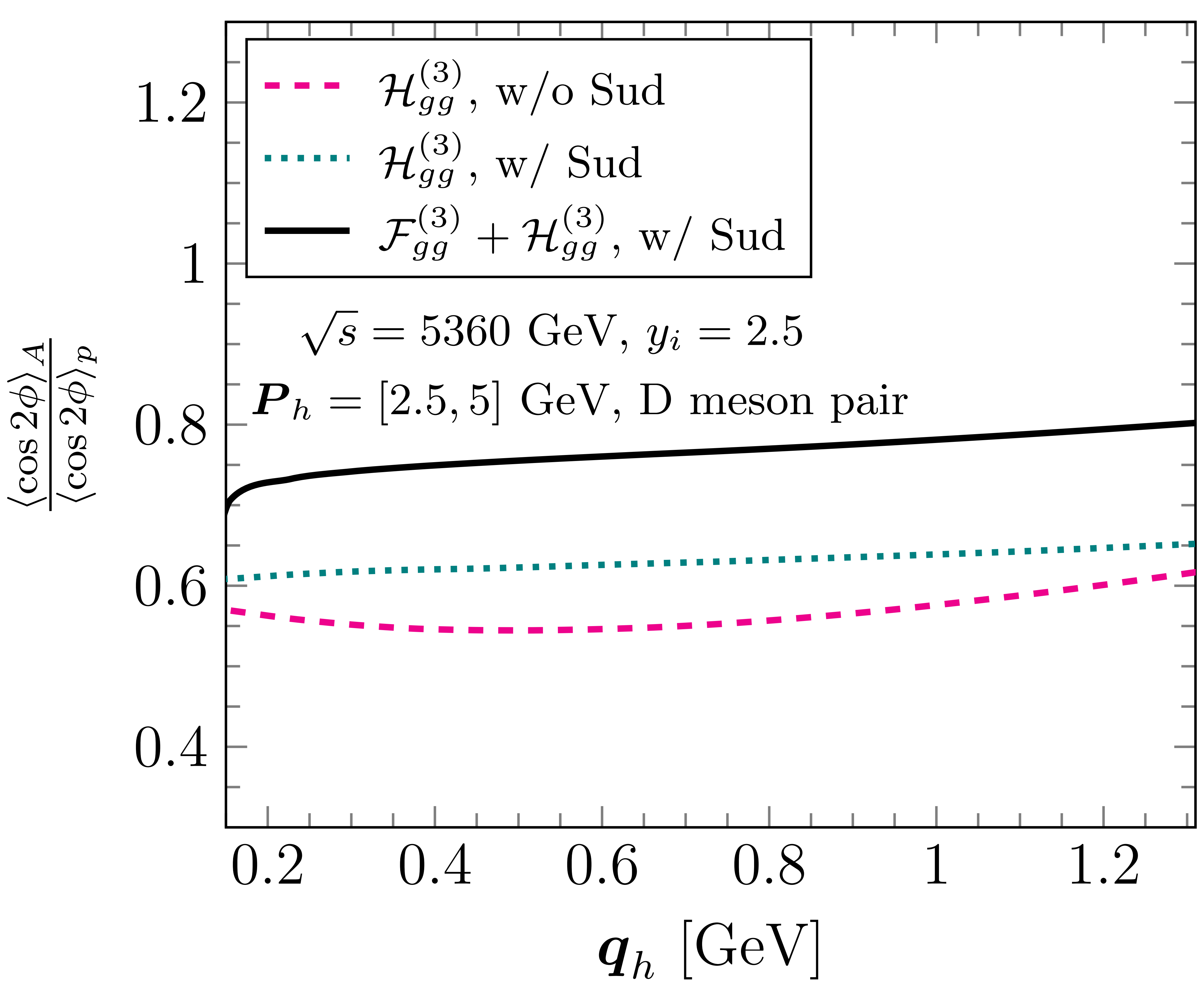
$$\frac{\langle\cos 2\phi\rangle_{\gamma A}}{\langle\cos 2\phi\rangle_{\gamma p}} < 1 \rightarrow$$ saturation effect emerges

strong saturation effect!!!

# Preliminary results for open heavy flavour pair photo-production

$$\frac{d\sigma^{\gamma A\to D\bar{D}X}}{dy_1 dy_2 d^2\boldsymbol{p}_1 d^2\boldsymbol{p}_2} = \int D_{D/c}(z_D,\mu)\otimes D_{\bar{D}/\bar{c}}(z_{\bar{D}},\mu)\otimes \frac{d\sigma^{\gamma A\to Q\bar{Q}X}}{dy_1 dy_2 d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2}$$




- The $\cos(2\phi)$ ratio may serve as a good process to detect the saturation effect.

2) The transverse energy-energy correlator in heavy quark pair photo-production

# The TEEC in heavy quark pair photo-production

[ Ali, Pietarinen, Stirling, Phys.Lett.B 141 (1984) 447-454; Gao, Li, Moult, Zhu, 2019, 2023]

- The Transverse Energy-Energy Correlator (TEEC)

$$\frac{d\Sigma^{\gamma A}}{d\tau} = \boxed{\sum_{h_1,h2}} \int dy_1 dy_2 d^2\boldsymbol{p}_1 d^2\boldsymbol{p}_2 \frac{d\sigma^{\gamma A \to hhX}}{dy_1 dy_2 d^2\boldsymbol{p}_1 d^2\boldsymbol{p}_2} \boxed{\frac{E_{h_1 T} E_{h_2 T}}{\left(\sum_{h_1'/T} E_{h_1'/\bar{c}T}\right)\left(\sum_{h_2'/T} E_{h_2'/\bar{c}T}\right)}} \delta(\tau - \frac{1+\cos\phi_{ab}}{2})$$

**Sum over all hadrons**

**Transverse Energy weighted**

Heavy quark in collinear limit

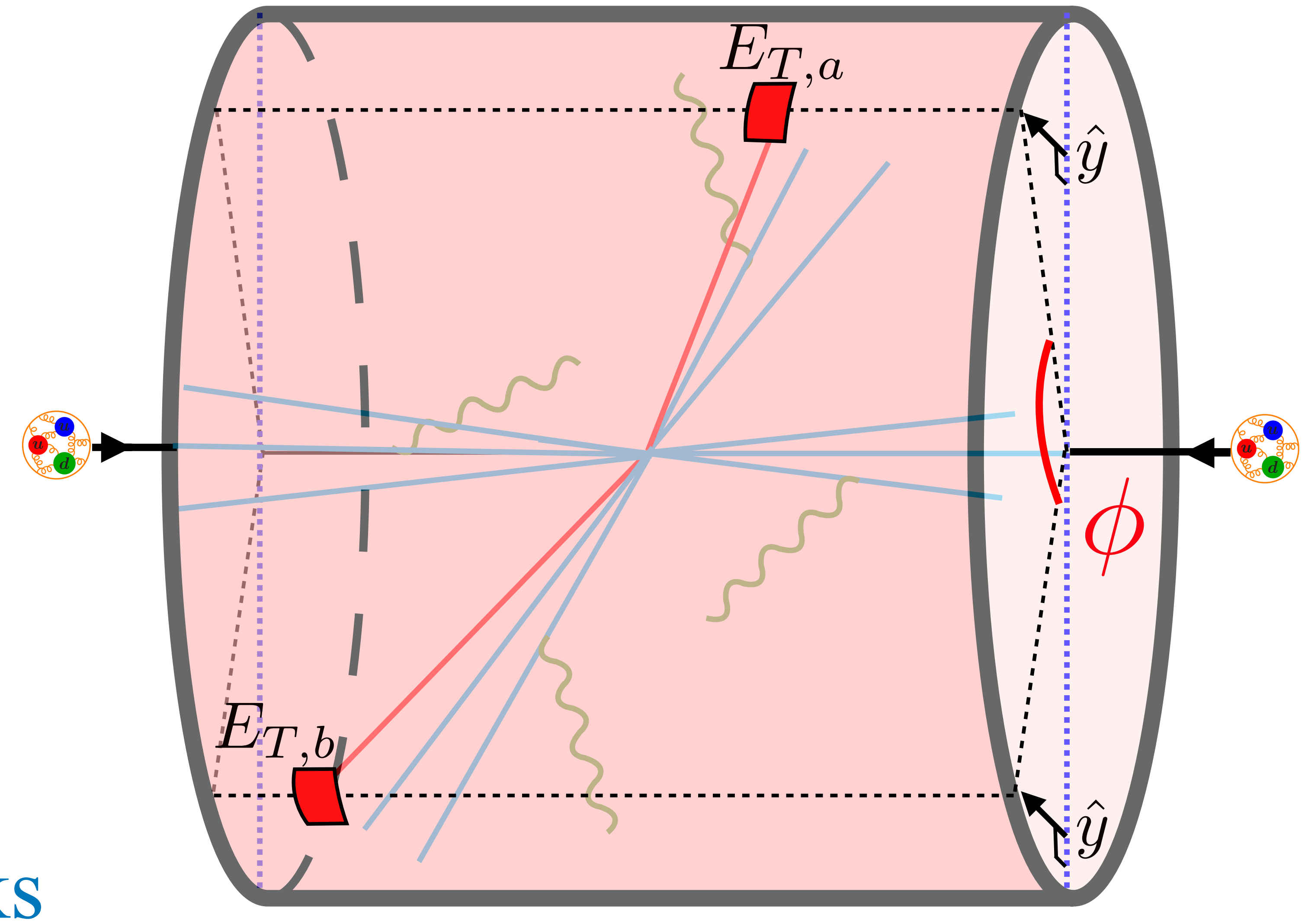
**Azimuthal angle correlations**

[ Andres, Dominguez, Holguin, Marquet, Moult, PRD, 2024; Xing, Cao, Qin, Wang, PRL, 25]

- Collinear limit $\tau \to 1$: Probe jet substructure.
- Back-to-back limit $\tau \to 0$: Probe TMD physics.
- Recent developments (EEC/TEEC in TMD/Small-x)

[Liu, Liu, Pan, Yuan, Zhu, PRL, 2023; Liu, Zhu, 24; Liu, Vogelsang, Yuan, Zhu, PRL, 25; Kang, Lee, Shao, Zhao, JHEP, 24; Kang, Penttala, Zhao, Zhou, PRD, 2024; Devereaux, Fan, Ke, Lee, Moult, 23; Chen, Ma, Tong, JHEP, 2024; Mantysaari, Tawabutr, Tong, 25; Bhattacharya, Kang, Padilla, Penttala, 2025; Kang, Kao, Li, Penttala, 25;]

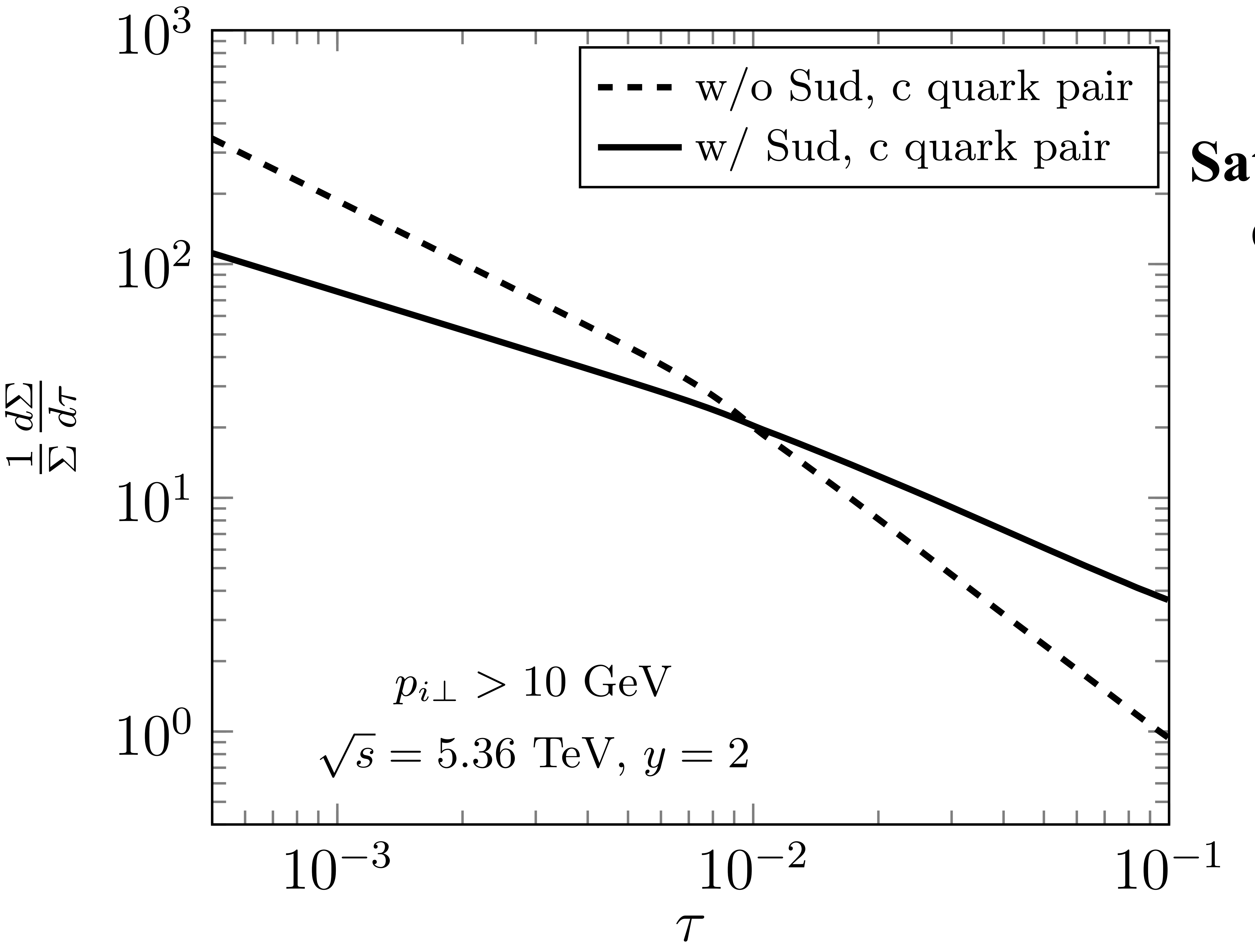
And many other works

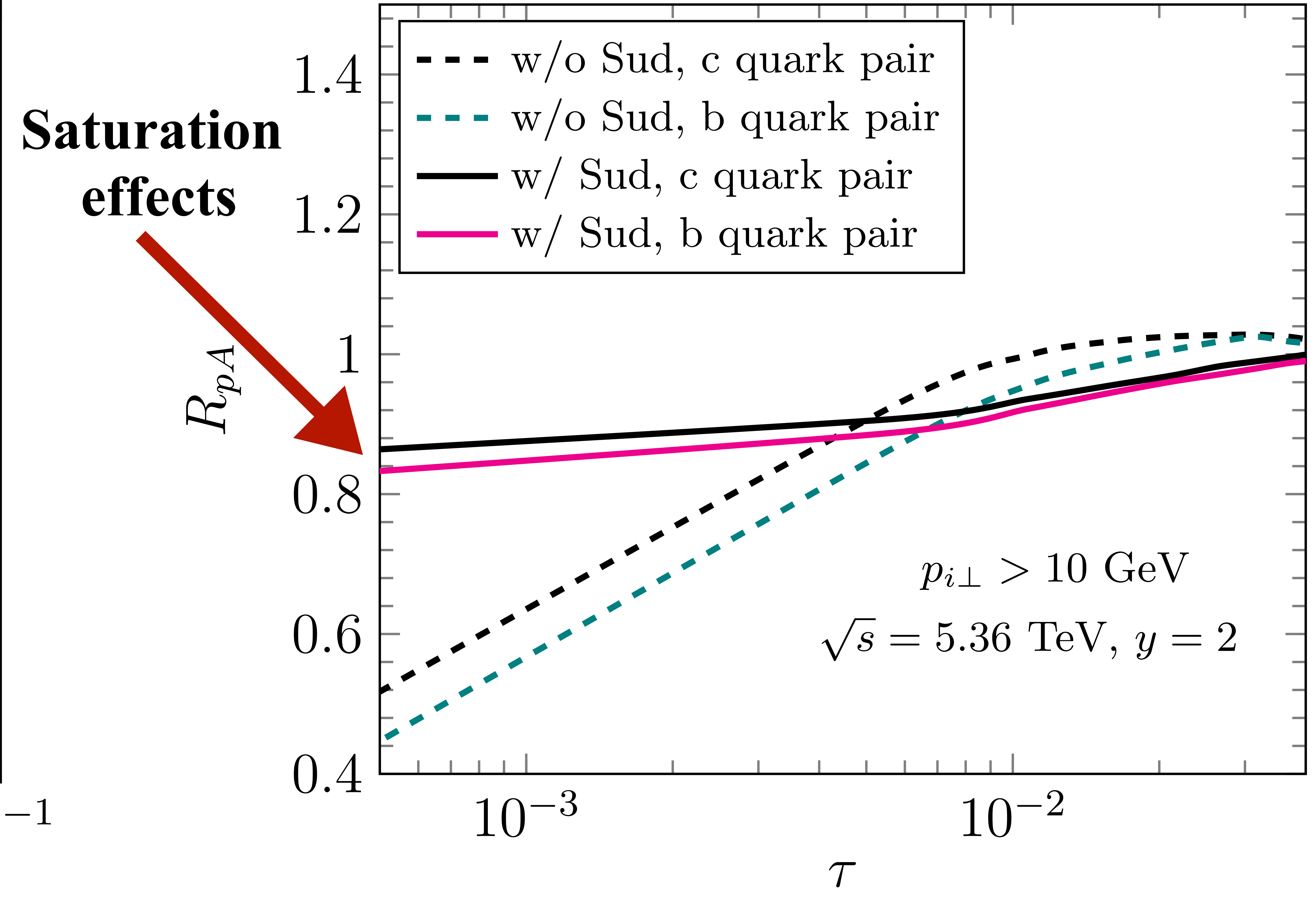


# Preliminary results for transverse energy-energy correlator

- In the back-to-back region

Nuclear modification factor $R_{pA} = \frac{1}{A}\frac{\sigma_{\gamma A}}{\sigma_{\gamma p}}$



# Summary

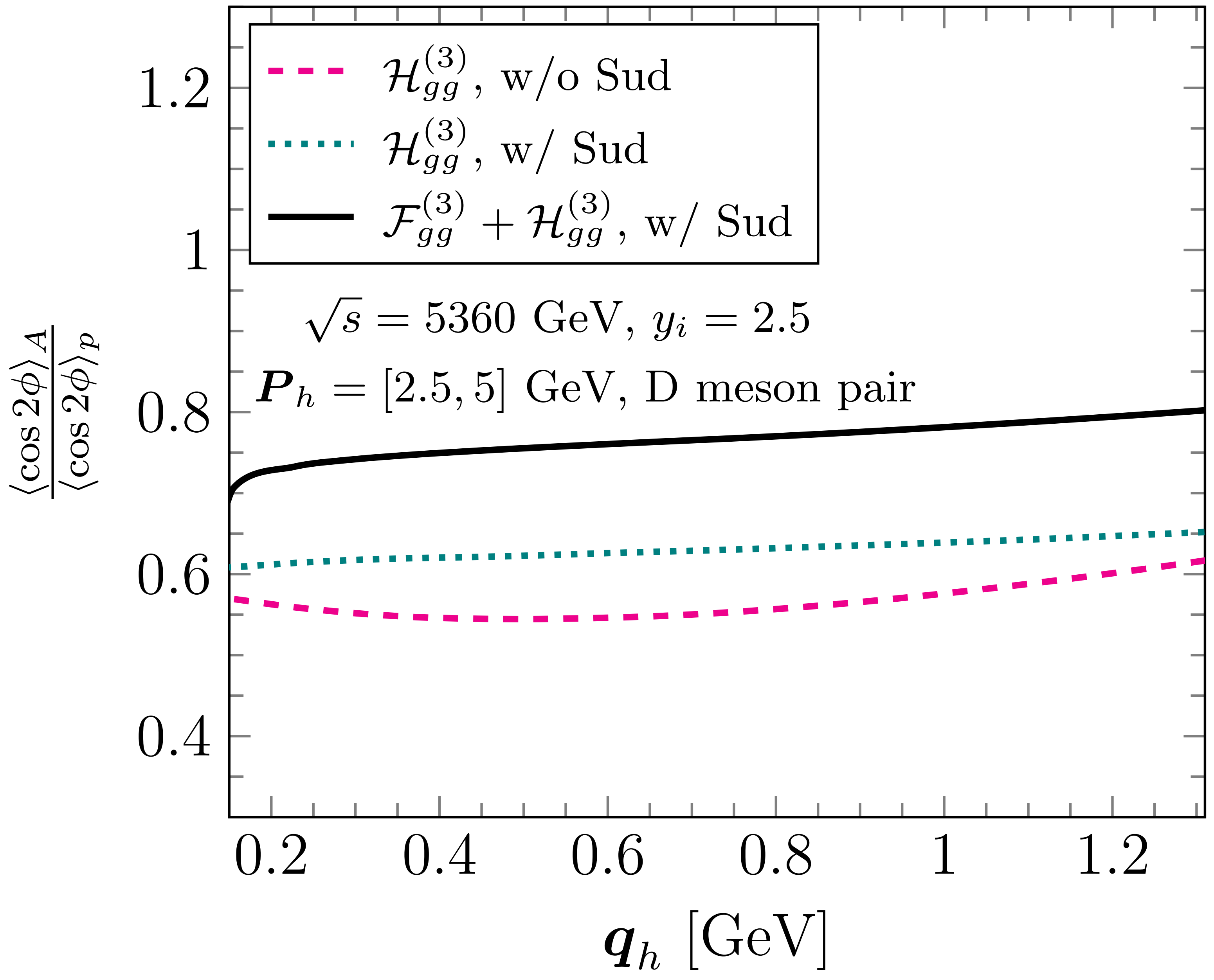
- We investigate the impact of the multi-soft-gluon radiation on heavy quark/meson pair photo-production (small x + Sudakov).
- Two observables: azimuthal Asymmetry and TEEC.
- Linearly-polarized WW gluon has been suppressed by the Sudakov effect.

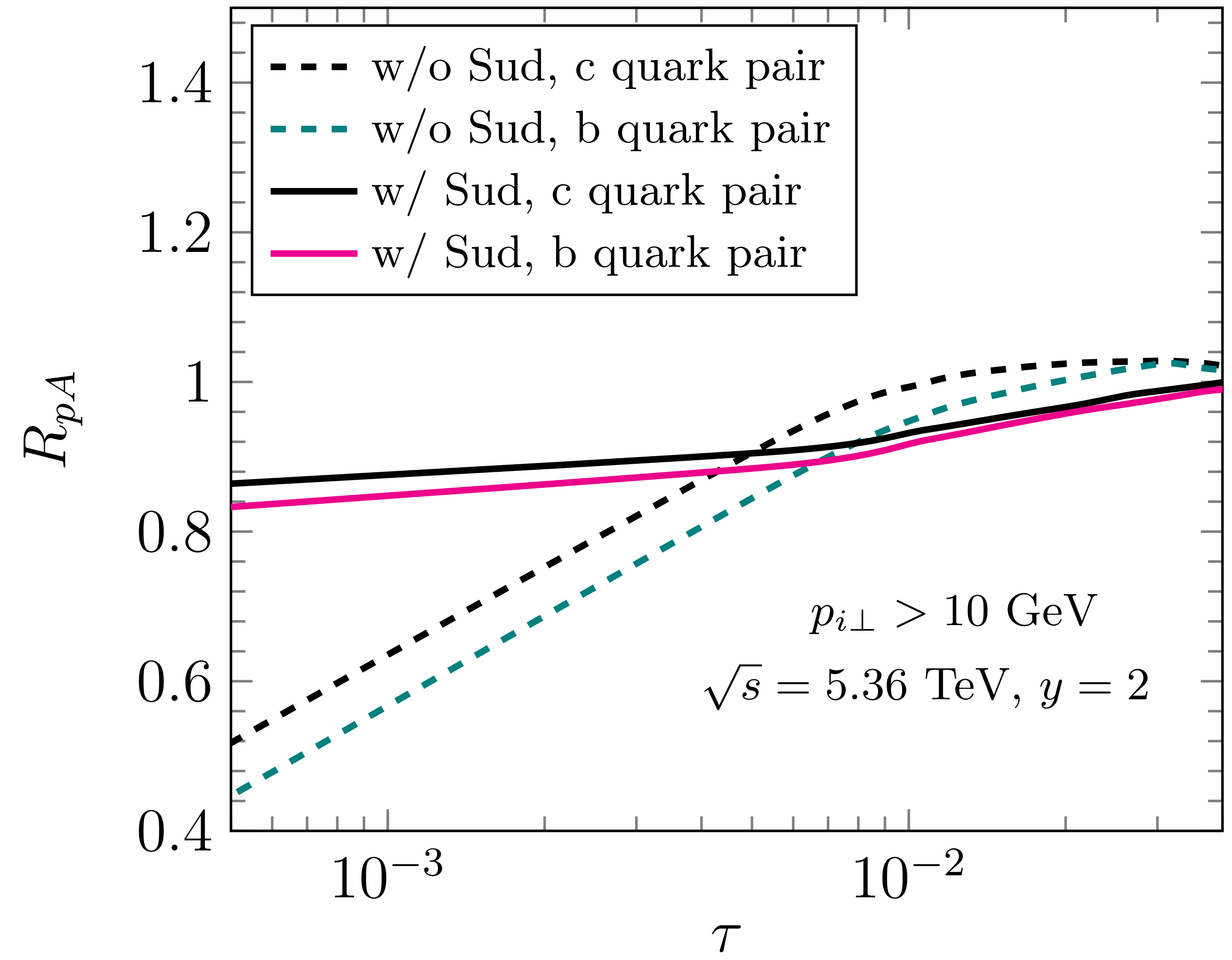
  Question: how to detect the linearly-polarized WW gluon?

- Sensitivity of the $\dfrac{\langle \cos 2\phi \rangle_A}{\langle \cos 2\phi \rangle_p}$ ratio to saturation effect.

- For the TEEC, the suppression of 15 – 20% in $R_{pA}$.

- Great potential in searching for compelling evidences of gluon saturation in the near future.



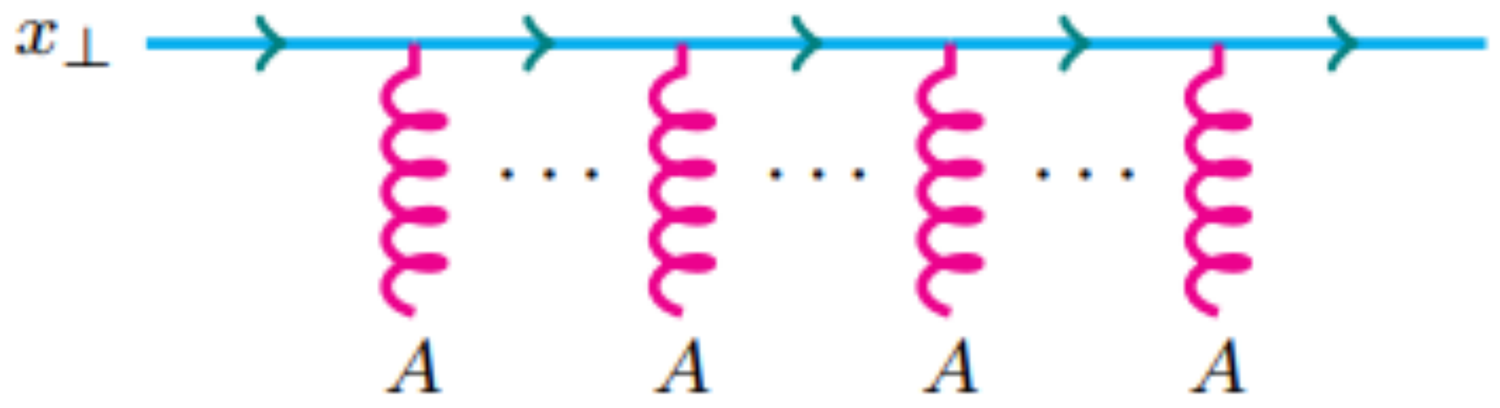



# backup

# Wilson lines in CGC

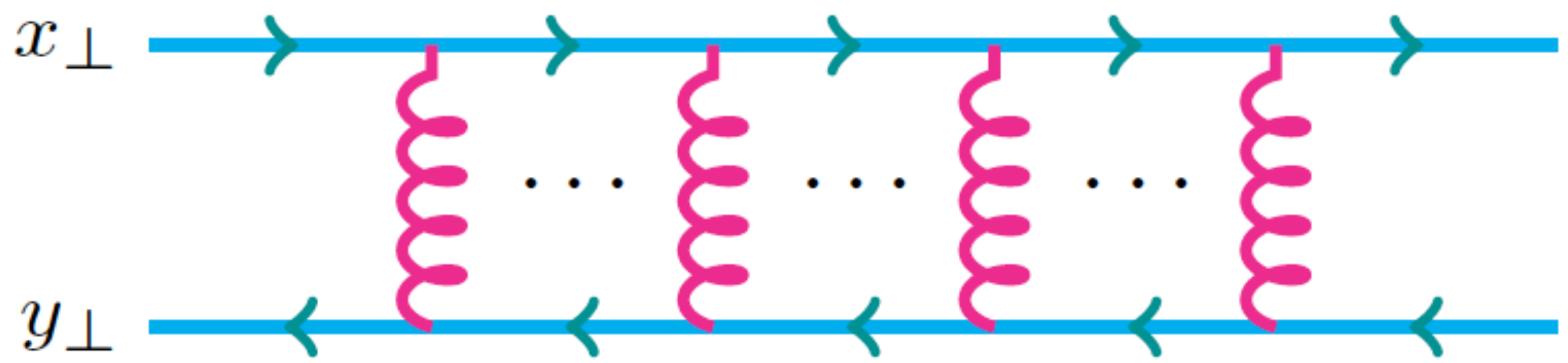
- Consider the multiple scattering between a fast quark and target background gluon fields.

$$U(x_\perp) = \mathscr{P} \exp\left(-ig\int dz^+ A^-(x_\perp, z^+)\right)$$




- The Wilson loop (color singlet dipole)

$$\frac{1}{N_c}\langle \mathrm{Tr}\left[U_x U_y^\dagger\right]\rangle$$

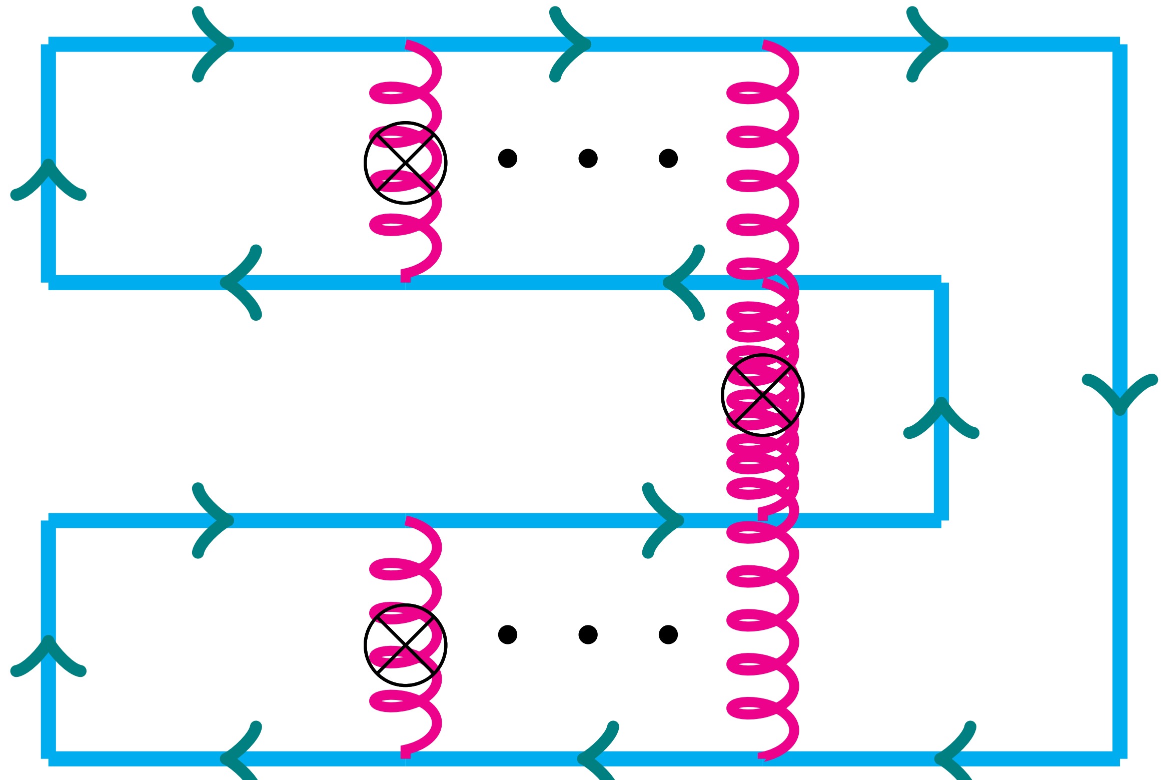



- Weizsacker-Williams (WW) gluon → Quadrupole

$$xG_{WW}(x_g, \boldsymbol{q}) = \frac{4}{g_s^2}\int \frac{d^2\boldsymbol{x}d^2\boldsymbol{y}}{(2\pi)^3} e^{-\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})}\langle \mathrm{Tr}\left[(\partial_i U_x)U_y^\dagger(\partial_i U_y)U_x^\dagger\right]\rangle$$

$$= \mathscr{F}_{gg}^{(3)}(x_g, \boldsymbol{q}) + \left(\frac{2\boldsymbol{q}_i\boldsymbol{q}_j}{\boldsymbol{q}^2} - \delta_{ij}\right)\mathscr{H}_{gg}^{(3)}(x_g, \boldsymbol{q})$$

un-polarized linearly-polarized



Quadrupole (WW gluon)

# rcBK equation and power-law fit

The impact parameter independent rcBK equation likes

$$\frac{\mathrm{d}N(\eta, r_\perp)}{\mathrm{d}\eta} = \int \mathrm{d}^2 r_{1\perp} \mathcal{K}_{\mathrm{BK}}(r_\perp, r_{1\perp}, r_{2\perp})[N(\eta, r_{1\perp}) + N(\eta, r_{2\perp}) - N(\eta, r_\perp) - N(\eta, r_{1\perp})N(\eta, r_{2\perp})]$$

We use the modified MV model as the initial condition [Albacete, Armesto, Milhano, Quiroga-Arias and Salgado, 11]

$$N(\eta = 0, r_\perp) = 1 - \exp\left[-\frac{1}{4}(r_\perp^2 Q_{s0}^2)^\gamma \ln(e + \frac{1}{|r_\perp|\Lambda_{\mathrm{BK}}})\right]$$

Taking Fourier transforms to $k_\perp$ space

$$F(\eta, k_\perp) = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} \left(1 - N(\eta, r_\perp)\right)$$

At large $k_\perp$, the power-law behaviour emerge

$$F(\eta, k_\perp) \sim \frac{C(\eta, \beta)}{k_\perp^{2\beta}}$$