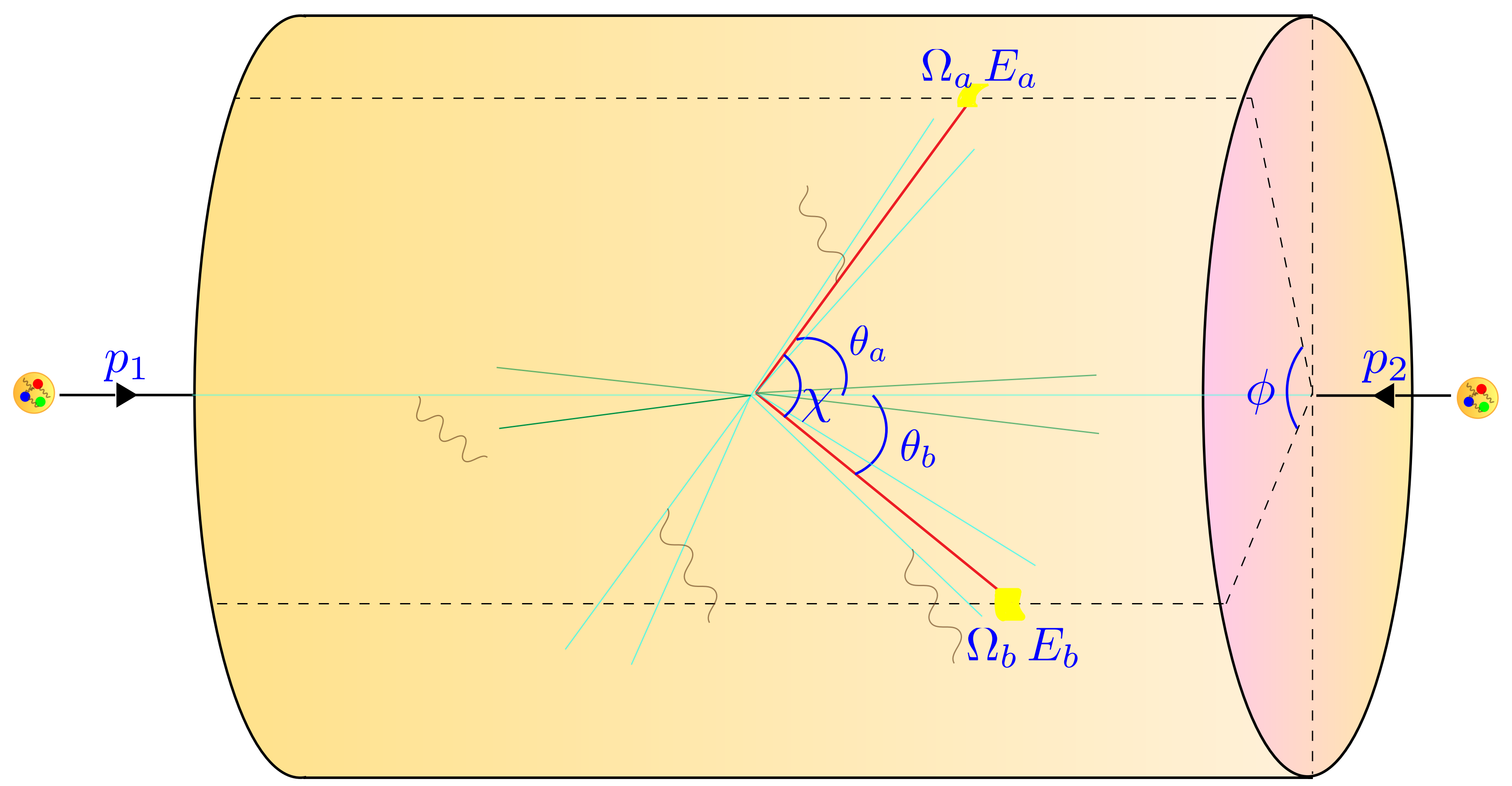
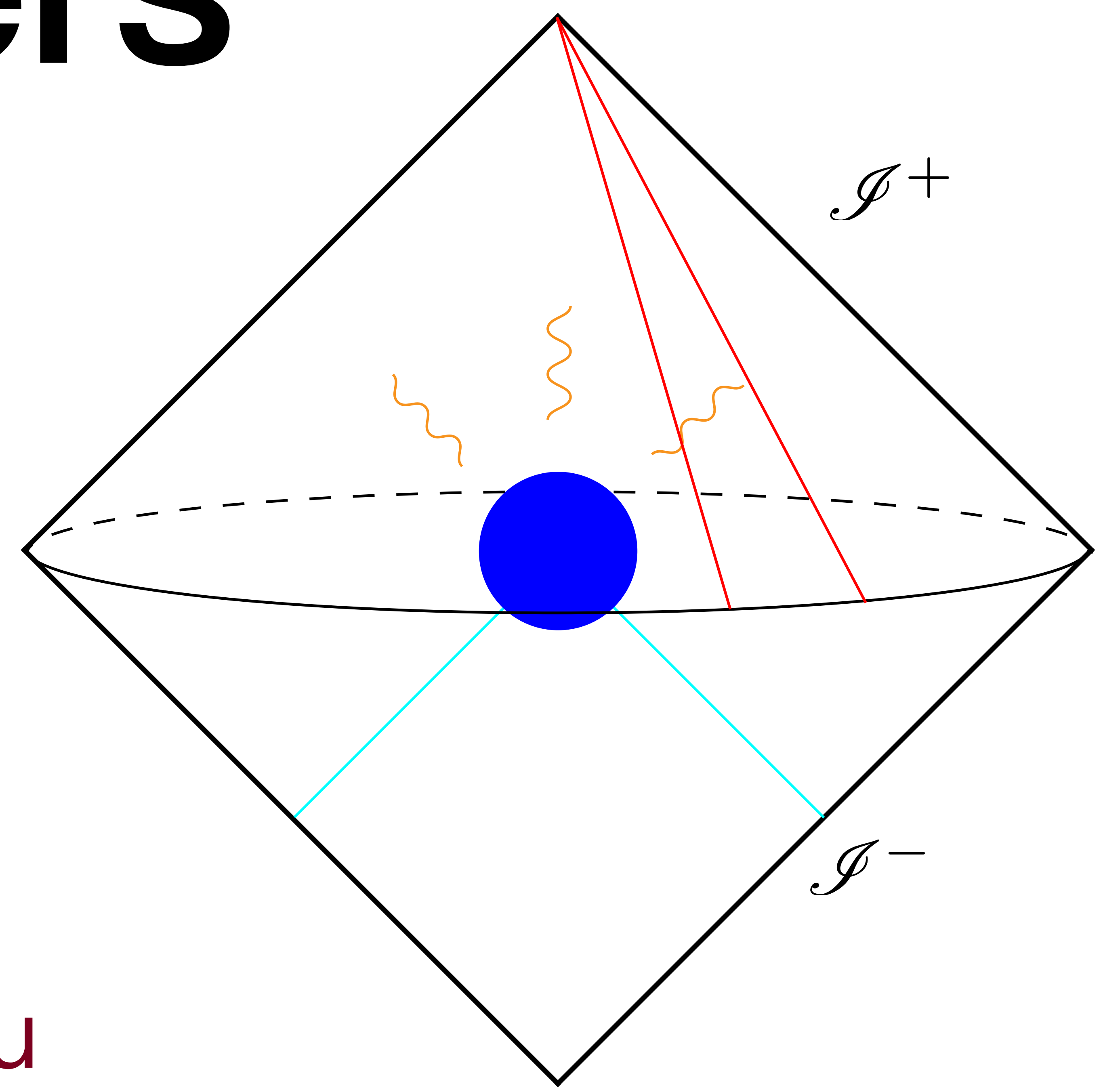
EEC in pure gluon scattering at Hadron Colliders



Hongyi Ruan
Peking University



Based on:
work with Hao Chen and Hua Xing Zhu
ongoing project with Hua Xing Zhu

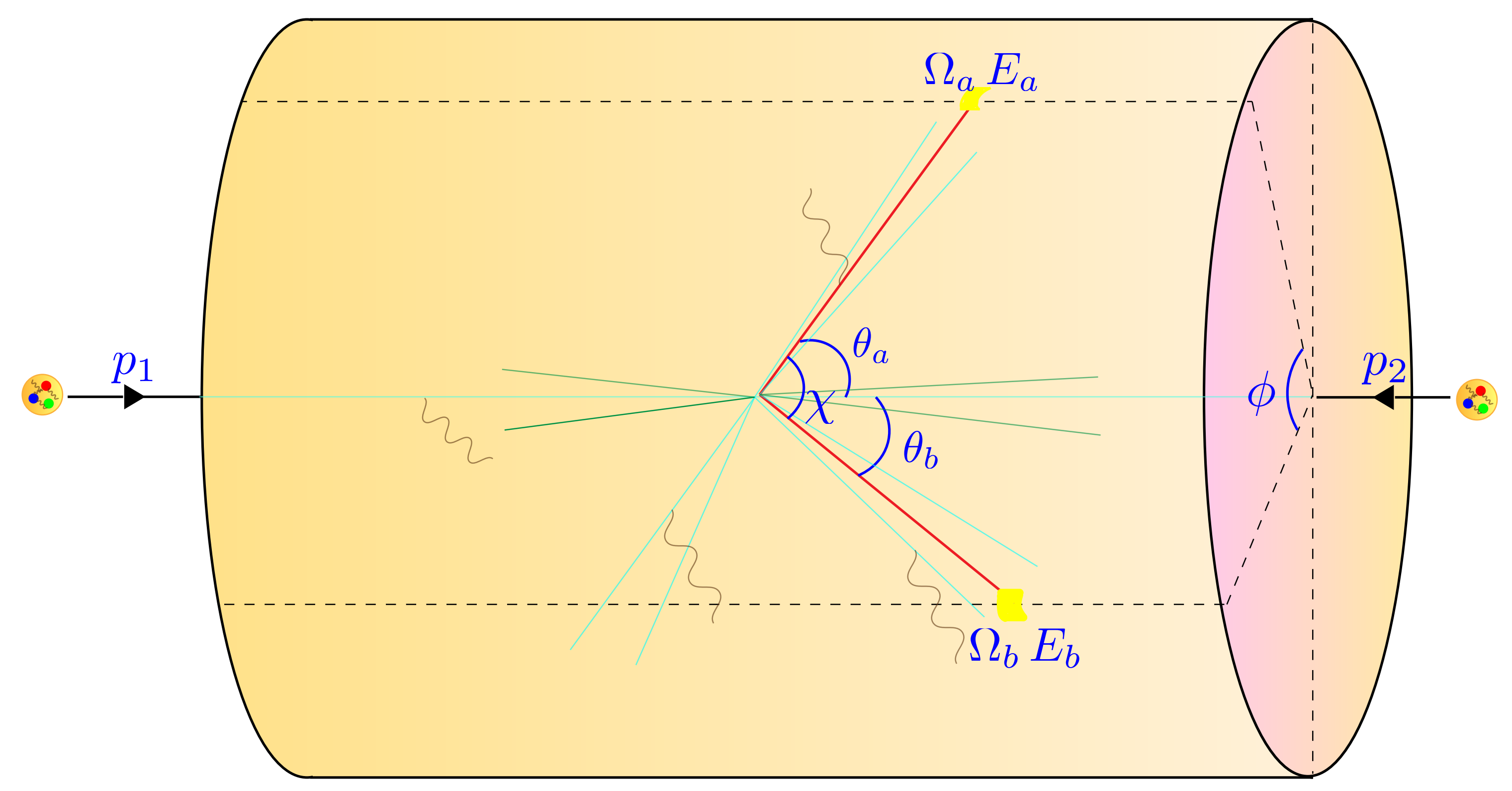
New Opportunities in Particle and Nuclear Physics with Energy Correlators

May 14, 2025

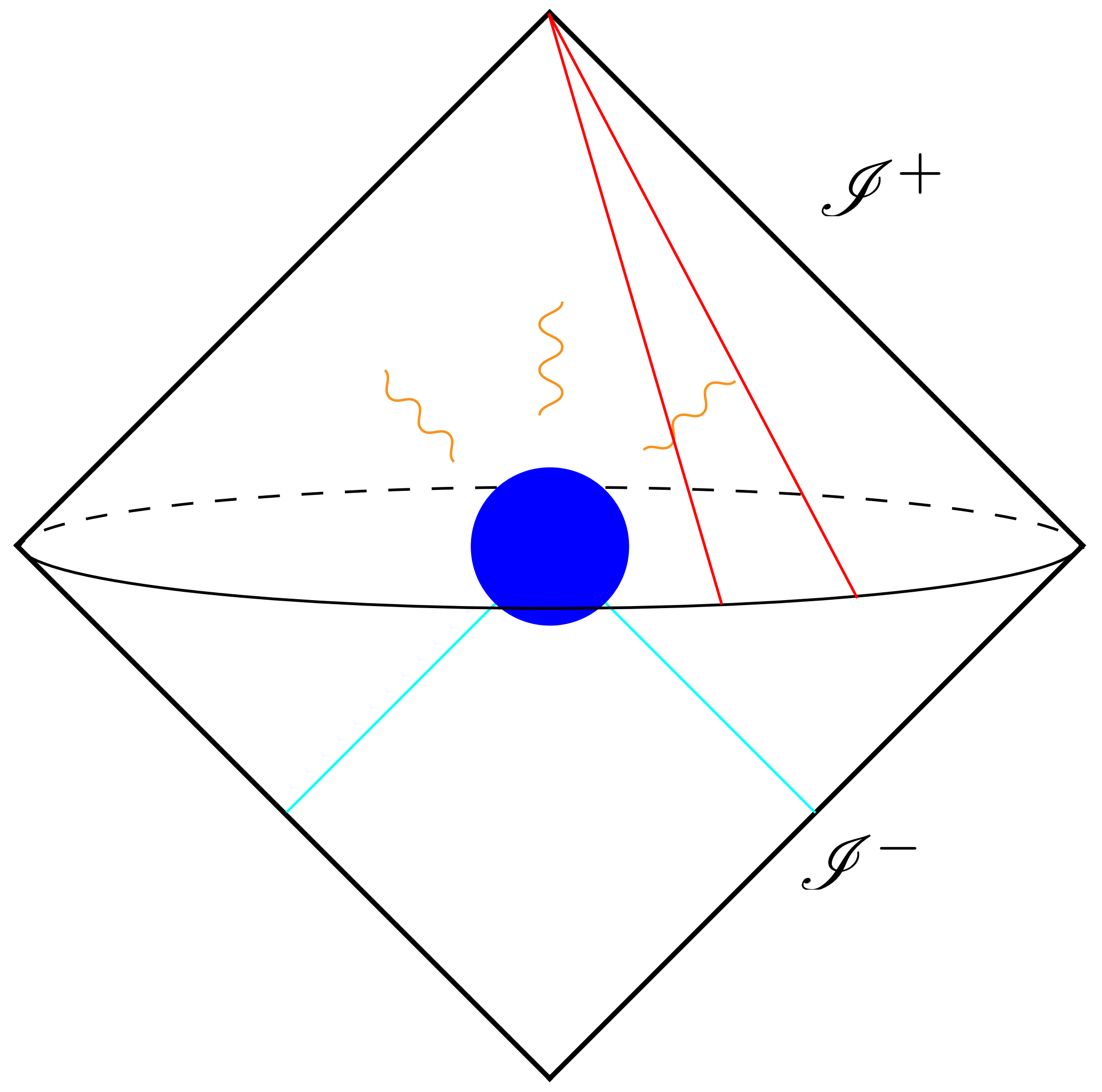
Outline

EEC at hadron colliders

- Collider Physics Aspects
  - Motivation and Definition
  - Results and Singular Approximations
  - Evolution Equation in the Regge Limit



- Formal Theory Aspects
  - Celestial Blocks for Hadron Colliders
  - Analyticity in Spin



Motivation

increased information via symmetry breaking

- $\mathcal{N}=4$ SYM： "Hydrogen Atom"

  EEC: NLO [Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2014]

  NNLO [Henn, Sokatchev, Yan, Zhiboedov, 2019]

  E3C: LO, Collinear [Chen, Luo, Moult, Yang, Zhang, Zhu, 2019]

  LO [Yan, Zhang, 2022]

  E4C: LO, Collinear [Chicherin, Moult, Sokatchev, Yan, Yunyue, 2024]

- QCD in electron–positron colliders: "Metal Atom"

  EEC: LO [Basham, Brown, Ellis, Love, 1978]

  NLO [Dixon, Luo, Shtabovenko, Yang, Zhu, 2018]

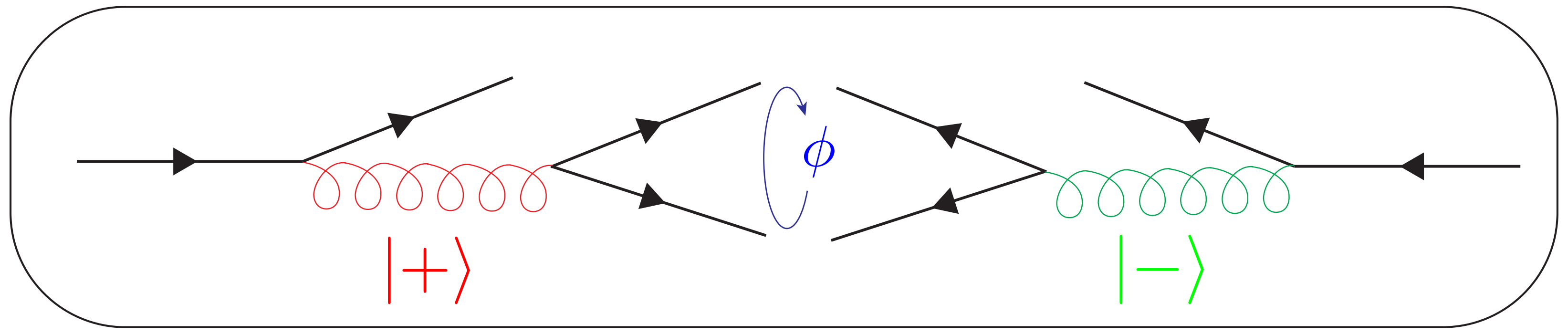
  E3C: LO, Collinear [Chen, Luo, Moult, Yang, Zhang, Zhu, 2019]

- QCD in hadron colliders: "Zeeman effect"

  TEEC: Back–to–Back [Gao, Li, Moult, Zhu, 2019; 2023]

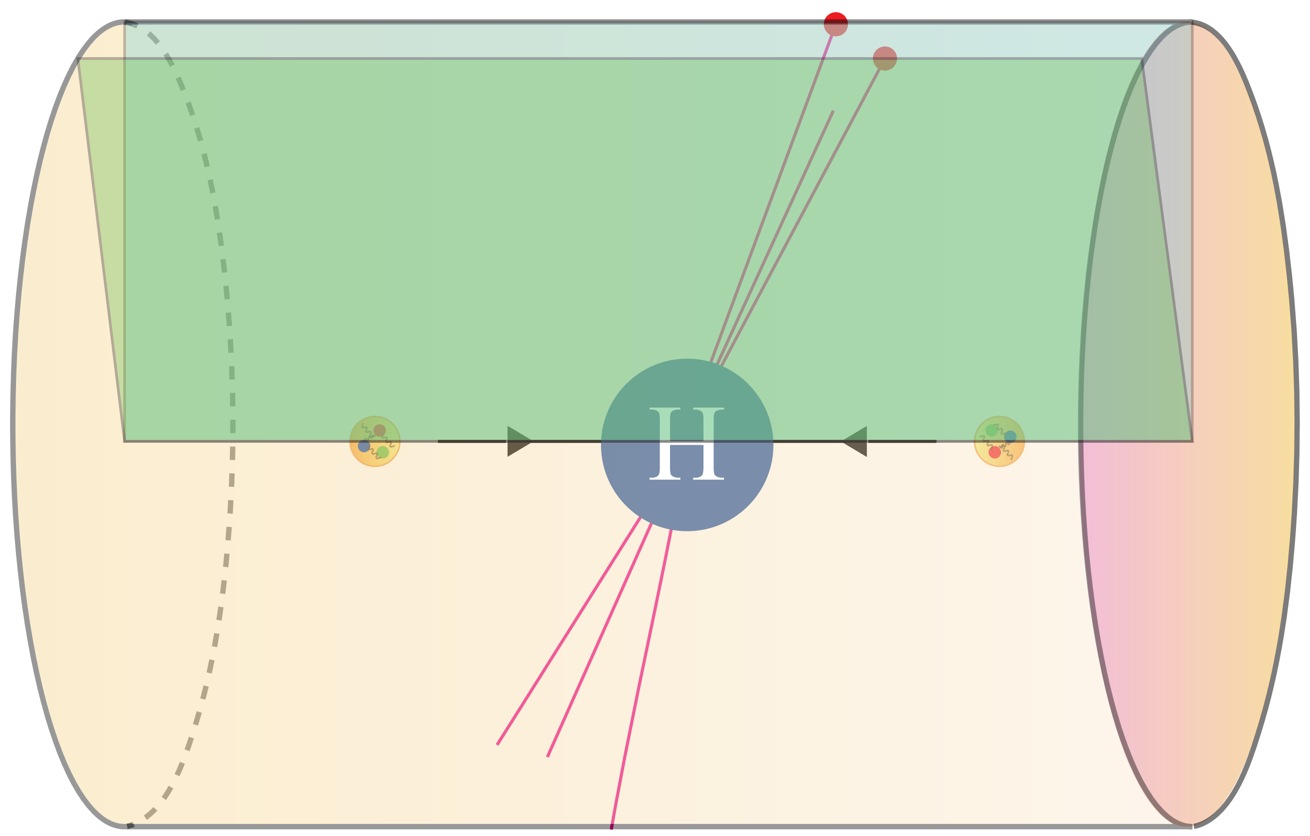
  FEEC: LO, Full angle [Chen, Ruan, Zhu, forthcoming]

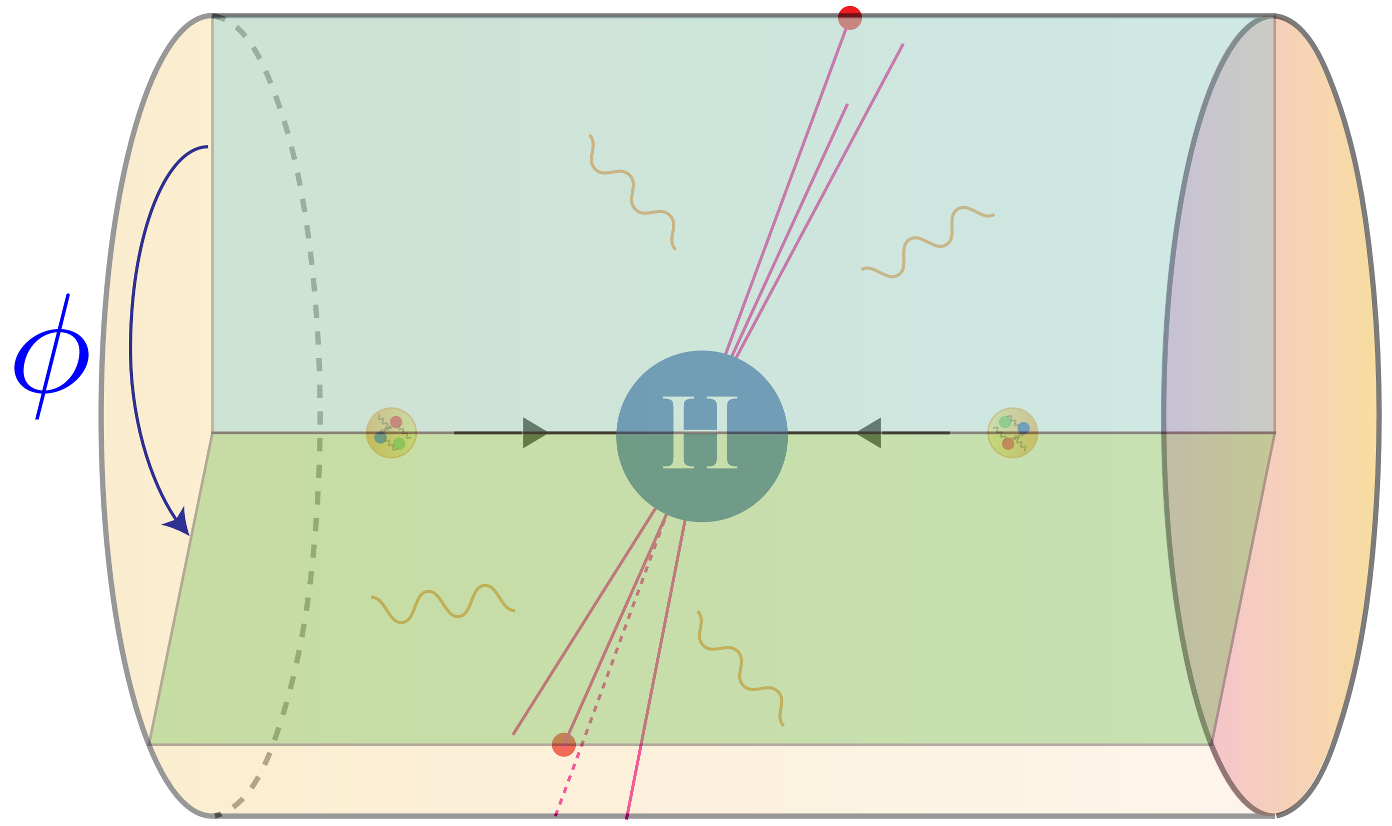
Interference Effect



An interference effect arises between intermediate gluons in the squeezed limit, a phenomenon observed in QCD but absent in $\mathcal{N}=4$ SYM.

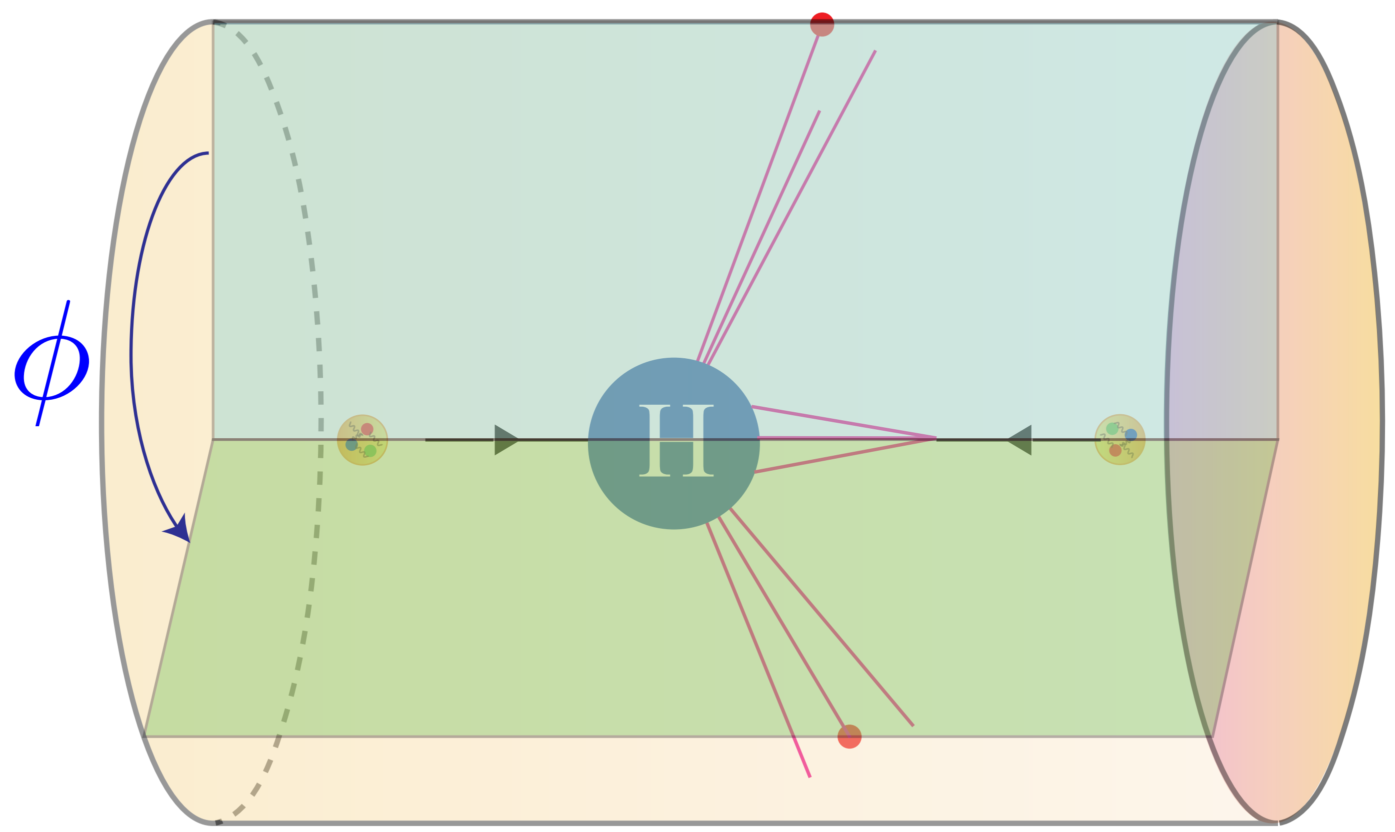
What "Splits Out" ?

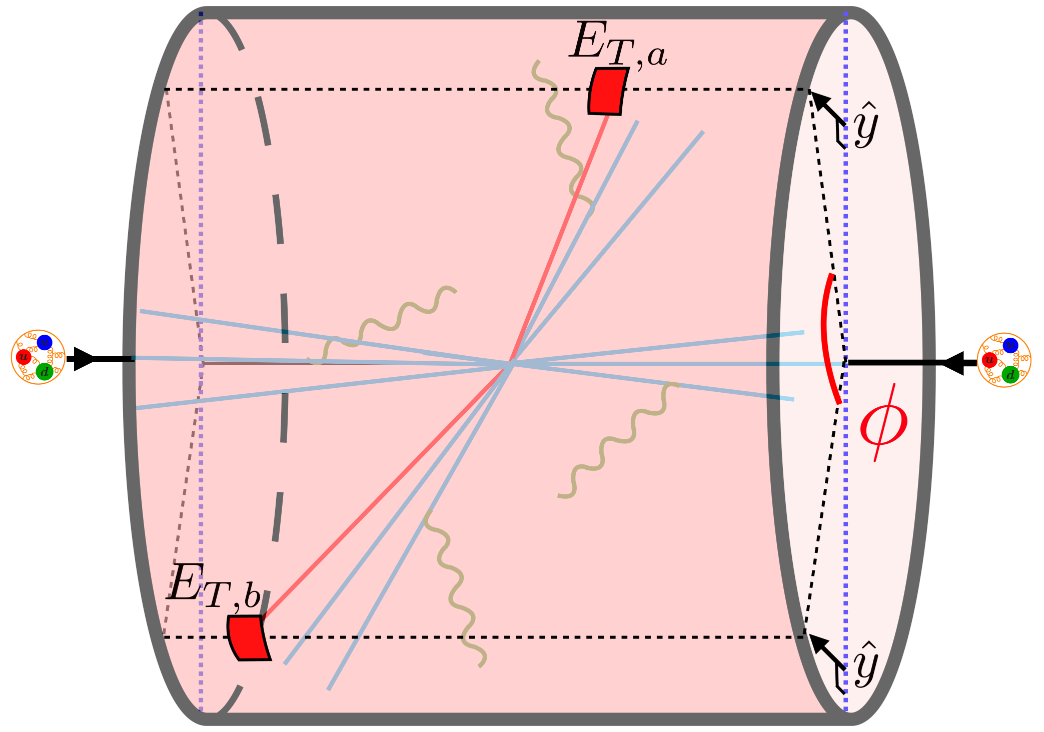




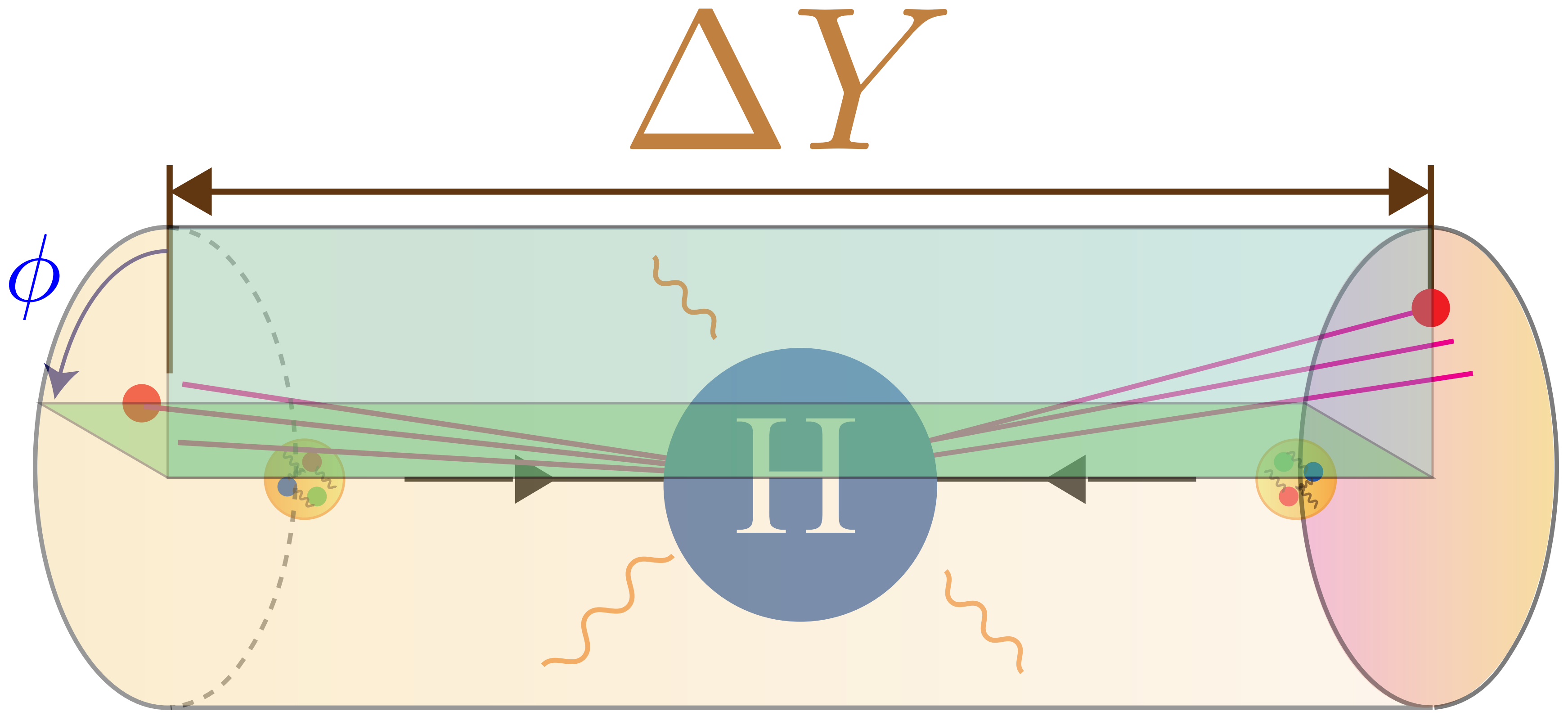
Collinear limit and back–to–back limit, well understood in the EEC of e^+e^- colliders

opposite coplanar limit, well understood as the back–to–back limit in the TEEC of pp colliders





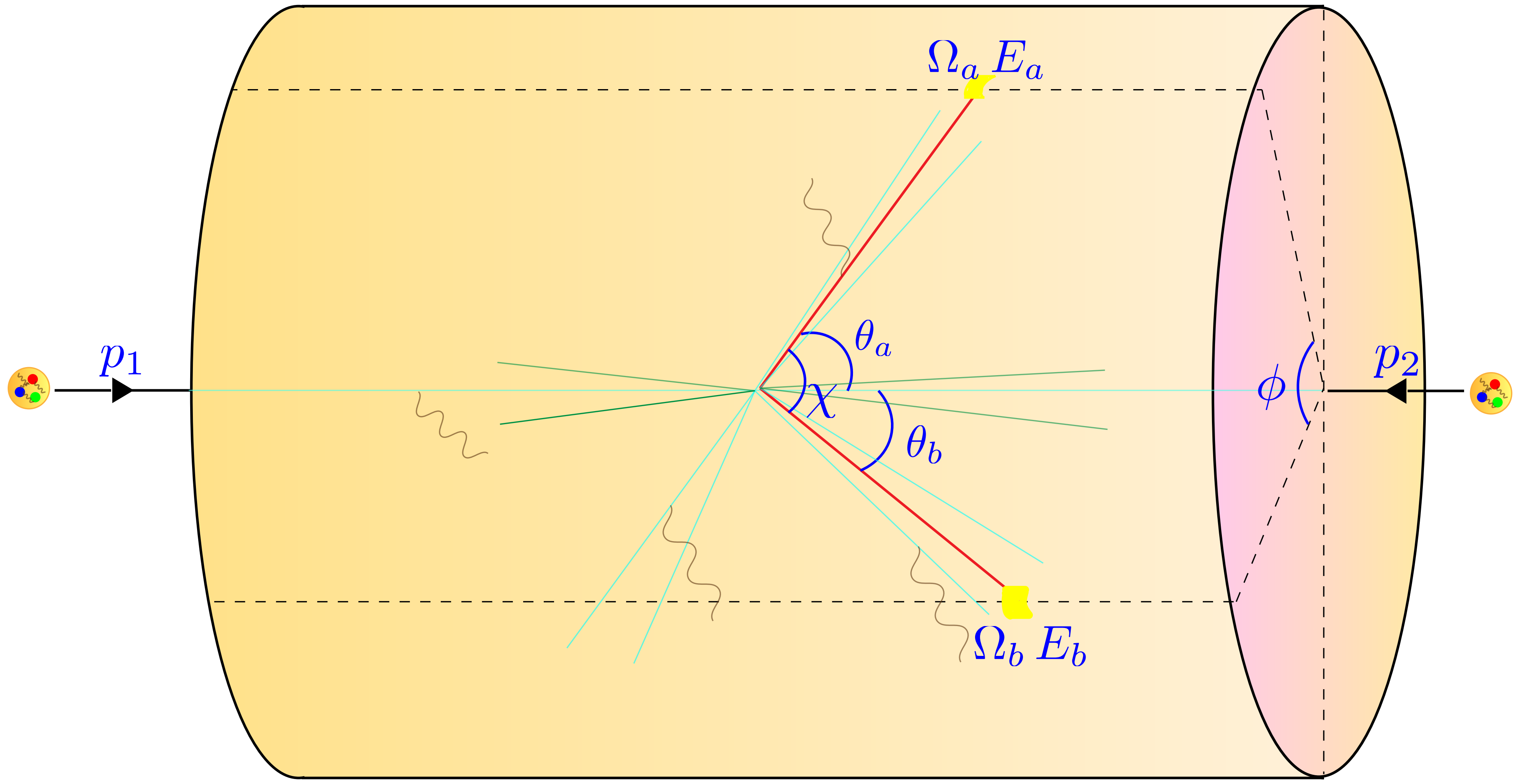
art from [Gao, Li, Moult, Zhu, 2023]



The Regge limit, characterized by a large rapidity difference (ΔY) between the two detectors, remains unexplored in the context of energy correlators.

EEC at Hadron Colliders

in pure gluon scattering



EEC at Hadron Colliders is defined as:

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \sum_{i,j} \int d\sigma_{pp\to i+j+X}\, E_i E_j\, \delta^{(2)}(\Omega_a - \Omega_{p_i})\delta^{(2)}(\Omega_b - \Omega_{p_j})$$

with dependence on three angles: θ_a, θ_b, χ.

The collision axis breaks spherical symmetry down to axial symmetry, rendering the final result considerably more complex, yet providing additional insights.

Using operator language, EEC can be expressed as a matrix element, which schematically factorizes as:

Parton Matrix Element

For now, just consider the pure gluon scattering.

$$\langle P_1 P_2 | \mathcal{E}(n_a)\mathcal{E}(n_b) | P_1 P_2 \rangle = \sum_{\alpha,\beta} \int_0^1 dx_1 dx_2\, x_1 x_2 f_\alpha(x_1) f_\beta(x_2) \langle \alpha(p_1)\beta(p_2) | \mathcal{E}(n_a)\mathcal{E}(n_b) | \alpha(p_1)\beta(p_2) \rangle \Big|_{p_1 = x_1 P_1,\, p_2 = x_2 P_2}$$

$$\begin{cases} w_H \equiv \dfrac{P_1 \cdot n_a}{P_2 \cdot n_a} = e^{-2Y_{a|H}} \\ w_P \equiv \dfrac{p_1 \cdot n_a}{p_2 \cdot n_a} = e^{-2Y_{a|P}} \end{cases} \Longrightarrow \frac{w_P}{w_H} = \frac{x_1}{x_2}$$

The detector rapidities $Y_{a|P}$ and $Y_{a|H}$ are related by a boost. EEC in the hadron COM frame can be reconstructed from the parton matrix element computed in the parton frame.

EEC Results

for pure gluon scattering

LO EEC for gluon scattering:

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \frac{Q^2}{16384\pi^5}\int_0^1 dx\,\frac{(1-x)^2x^2}{(1-x\zeta)^3}\overline{\sum_{\text{h,c}}\left|\mathcal{A}_5^{\text{full,tree}}\right|^2}$$

x: Energy fraction $\qquad \zeta = \dfrac{1-\cos\chi}{2}$

The averaged squared amplitude:

$$\overline{\sum_{\text{h,c}}\left|\mathcal{A}_5^{\text{full,tree}}\right|^2} = \frac{27g^6}{16}\frac{\sum_{1\le i<j\le 5} s_{ij}^4}{\prod_{1\le i<j\le 5} s_{ij}}\sum_{\sigma\in S_4}\left(s_{1\sigma(2)}s_{\sigma(2)\sigma(3)}s_{\sigma(3)\sigma(4)}s_{\sigma(4)\sigma(5)}s_{\sigma(5)1}\right)$$

← Compact and symmetric in amplitude level

The final result spans over pages, so we only present its main structure:

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \frac{9g^6}{(8\pi)^5(c_{\Delta Y}-c_\phi)^5(c_{\Delta Y}+c_Y)(c_Y+c_\phi)(c_{\Delta Y}c_\phi+c_{\Delta Y}c_Y-c_Yc_\phi-1)}$$
$$\times\Bigg[C_0 + C_1\ln\left(\frac{c_Y+c_\phi}{c_{\Delta Y}+c_Y}\right) + C_2\ln\left(\frac{(c_{\Delta Y}+s_{\Delta Y})(c_Y+s_Y)+1}{c_{\Delta Y}+c_Y+s_{\Delta Y}+s_Y}\right)s_{\Delta Y}s_Y$$
$$+C_3\,\phi\csc(\phi) + C_4\,\Delta Y s_{\Delta Y} + C_5\arccos\left(\frac{c_Yc_\phi+1}{c_Y+c_\phi}\right)\csc(\phi)s_Y\Bigg]$$

To compare with experimental data, we use:

- rapidity sum: Y
- rapidity difference: ΔY
- azimuthal angle separation: ϕ

Shorthand notation:

$c_Y = \cosh Y,\quad c_{\Delta Y} = \cosh\Delta Y,\quad c_\phi = \cos\phi,$ etc.

Redundancy of EEC

in the rational term

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = \left[A_0 \ln(1-\zeta) + A_1 \ln\left(\frac{1-y_b}{1-y_a}\right) + A_2 \arccos\left(\frac{2-y_a-y_b-2\zeta}{2\sqrt{(1-y_a)(1-y_b)(1-\zeta)}}\right) + A_3\right]$$
$$\times \frac{9g^6}{8\left(8\pi(1-y_a^2)(1-y_b^2)\zeta\right)^5(1-\zeta)\left((y_a-y_b)^2+4\zeta y_a y_b\right)\left((y_a-y_b)^2-4\zeta(1-y_a y_b-\zeta)\right)}$$
$$+(y_a \leftrightarrow y_b) + (y_a \to -y_a, y_b \to -y_b) + (y_a \to -y_b, y_b \to -y_a).$$

absorbed into $\tilde{A}_3$ for simplicity

Shorthand notation:

$$y_a = \cos\theta_a,\ y_b = \cos\theta_b,\ \zeta = \frac{1-\cos\chi}{2}.$$

Ansatz: $\tilde{A}_3 \sim \sum_{n_1,n_2,n_3=0}^{\infty} c_{n_1,n_2,n_3}\, \zeta^{n_1} y_a^{n_2} y_b^{n_3}$

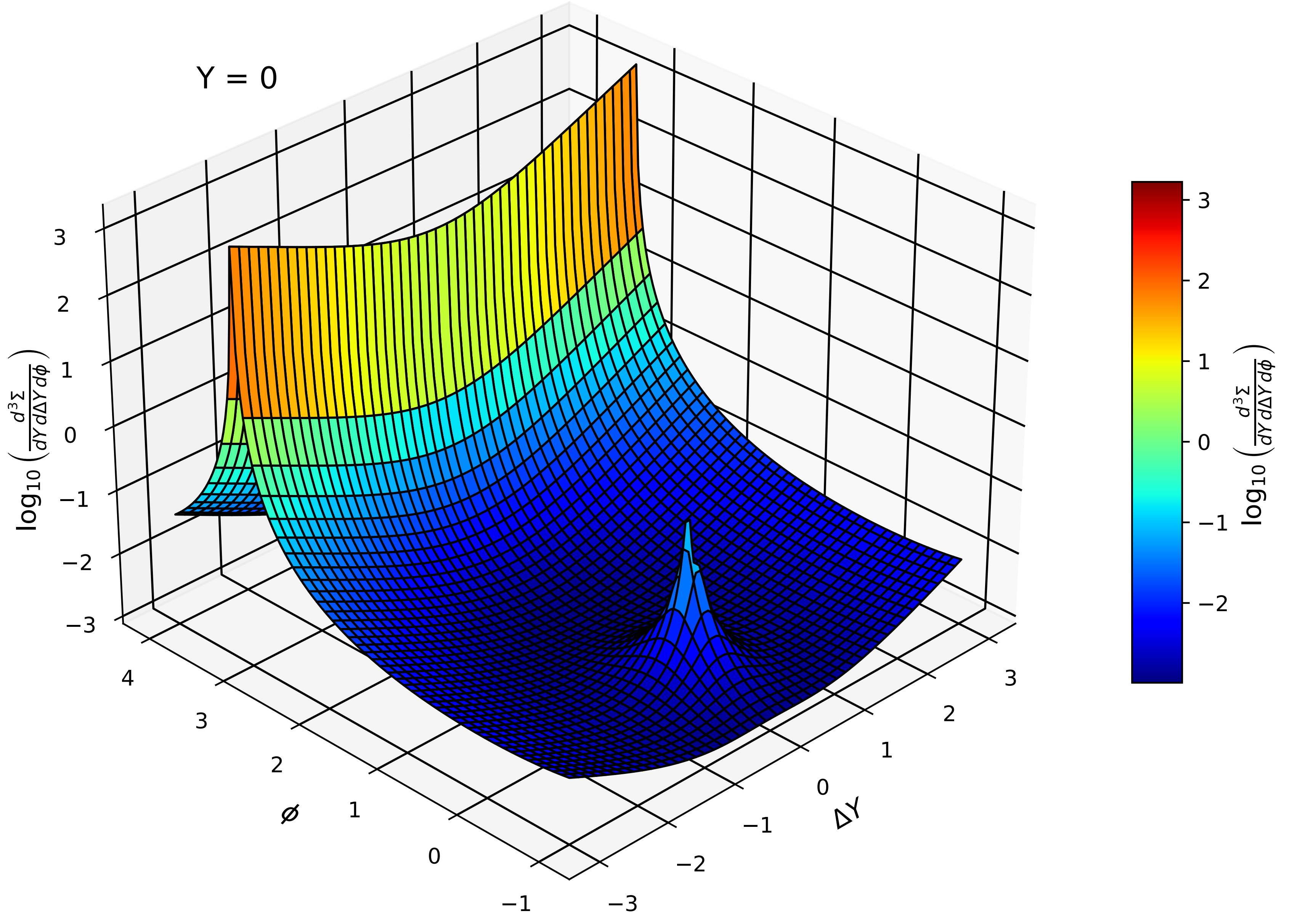
- Constraints on n_i (210 terms):
  - $\zeta \to \infty$, EEC ~ $\frac{1}{\zeta}$: $n_1 \le 4$.
  - $y_a \to \infty$, EEC ~ y_a^3: $n_2 \le 11$.
  - $y_b \to \infty$, EEC ~ y_b^3: $n_3 \le 11$.
  - symmetric under $y_a \leftrightarrow y_b$: $n_2 \leftrightarrow n_3$
  - symmetric under $y_a \to -y_a, y_b \to -y_b$: EEC vanishes when $n_2 + n_3$ is odd.

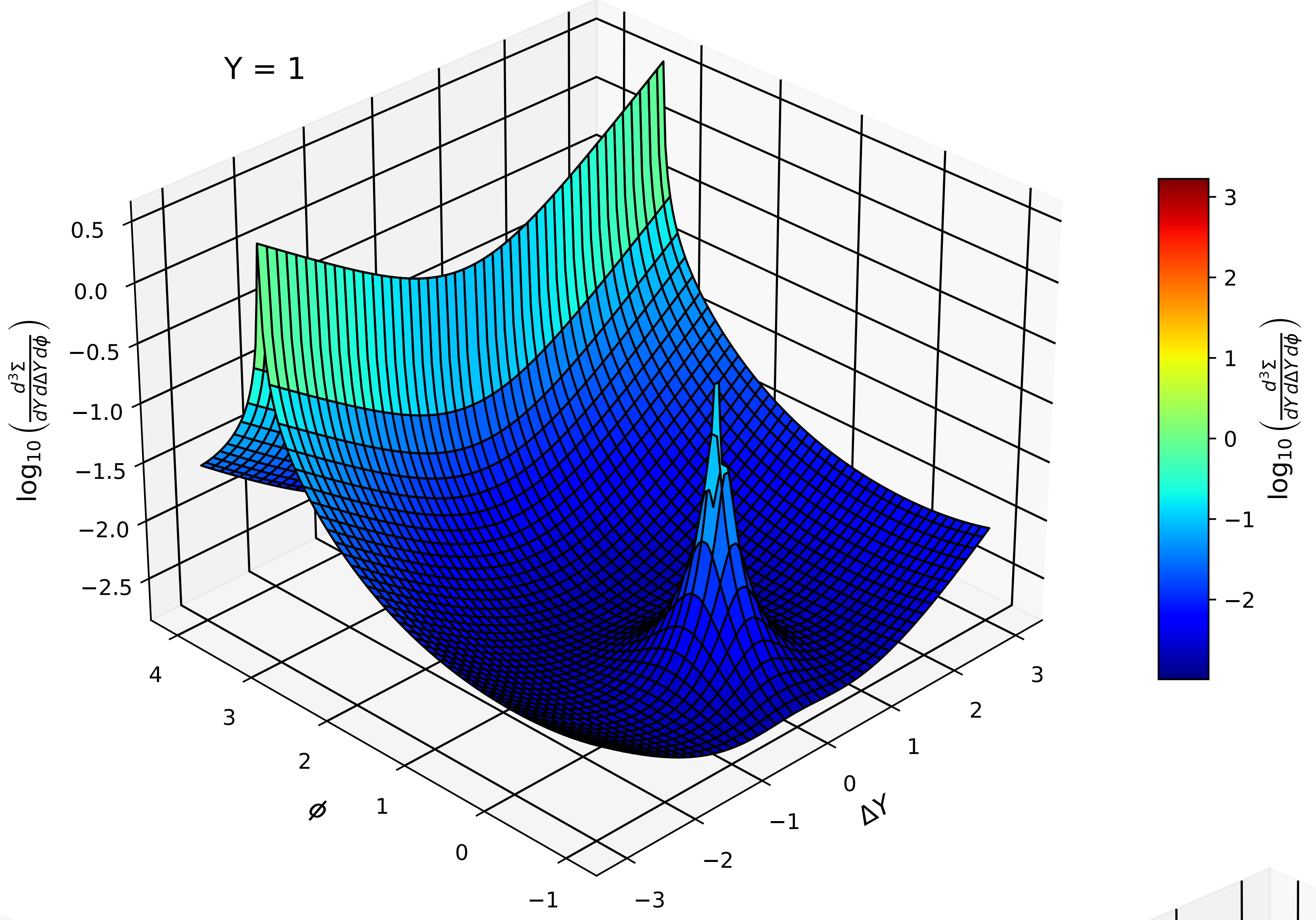
- Constraints on coefficients:
  - $\zeta \to 0$, EEC ~ $\frac{1}{\zeta}$: c_{n_1,n_2,n_3} fixed for $n_1 \le 2$. 84 terms left.
  - $y_a \to \pm 1, y_b \to \pm(1-2\zeta)$, EEC ~ $\frac{1}{1 \mp y_a}$. Same for $a \leftrightarrow b$.
  - Residues in $\zeta \to 0, 1;\ y_{a,b} \to \pm 1;\ y_a \to \pm 1, y_b \to \mp 1, z \to 1$.

  14 terms left.

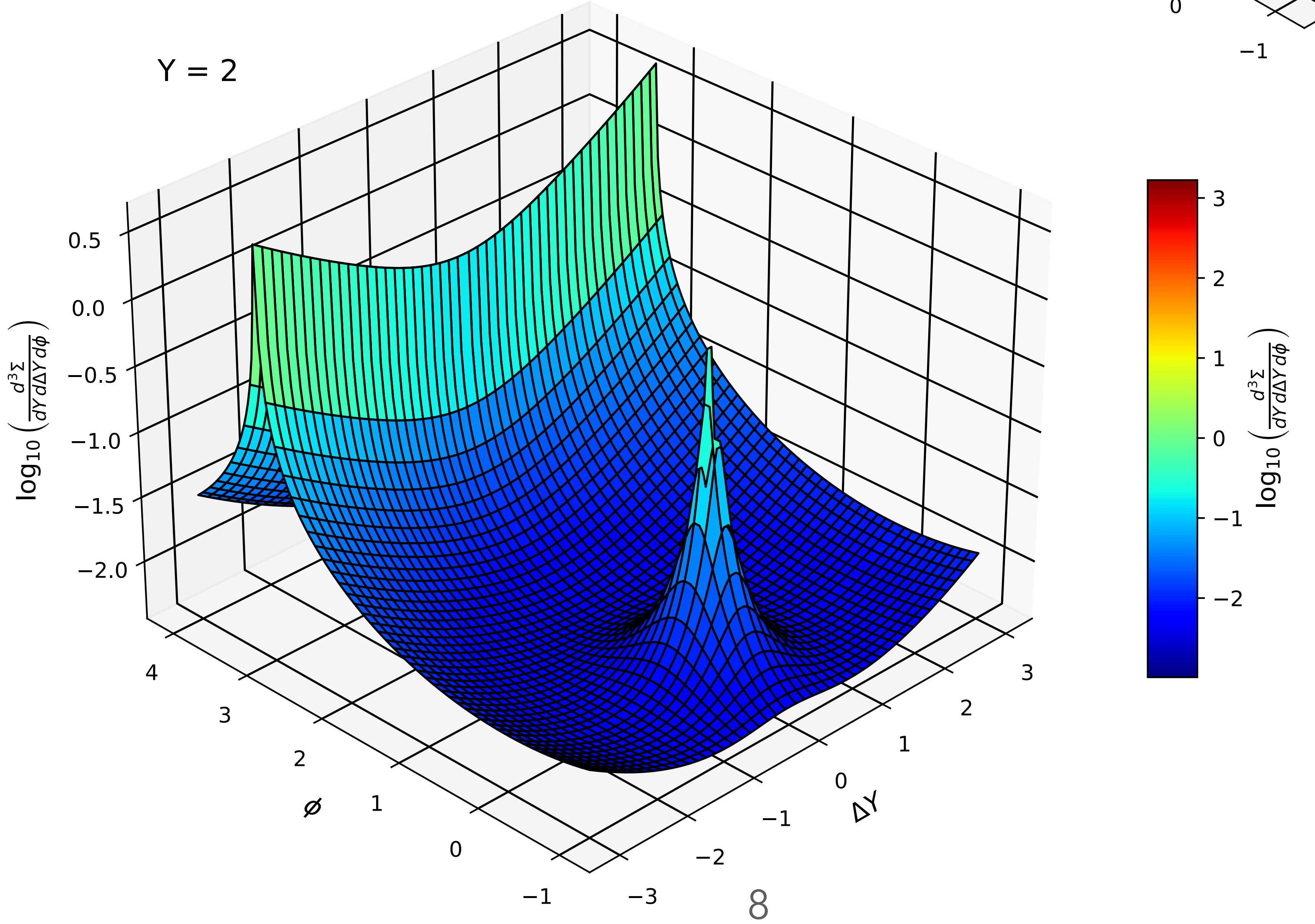
The EEC Landscape

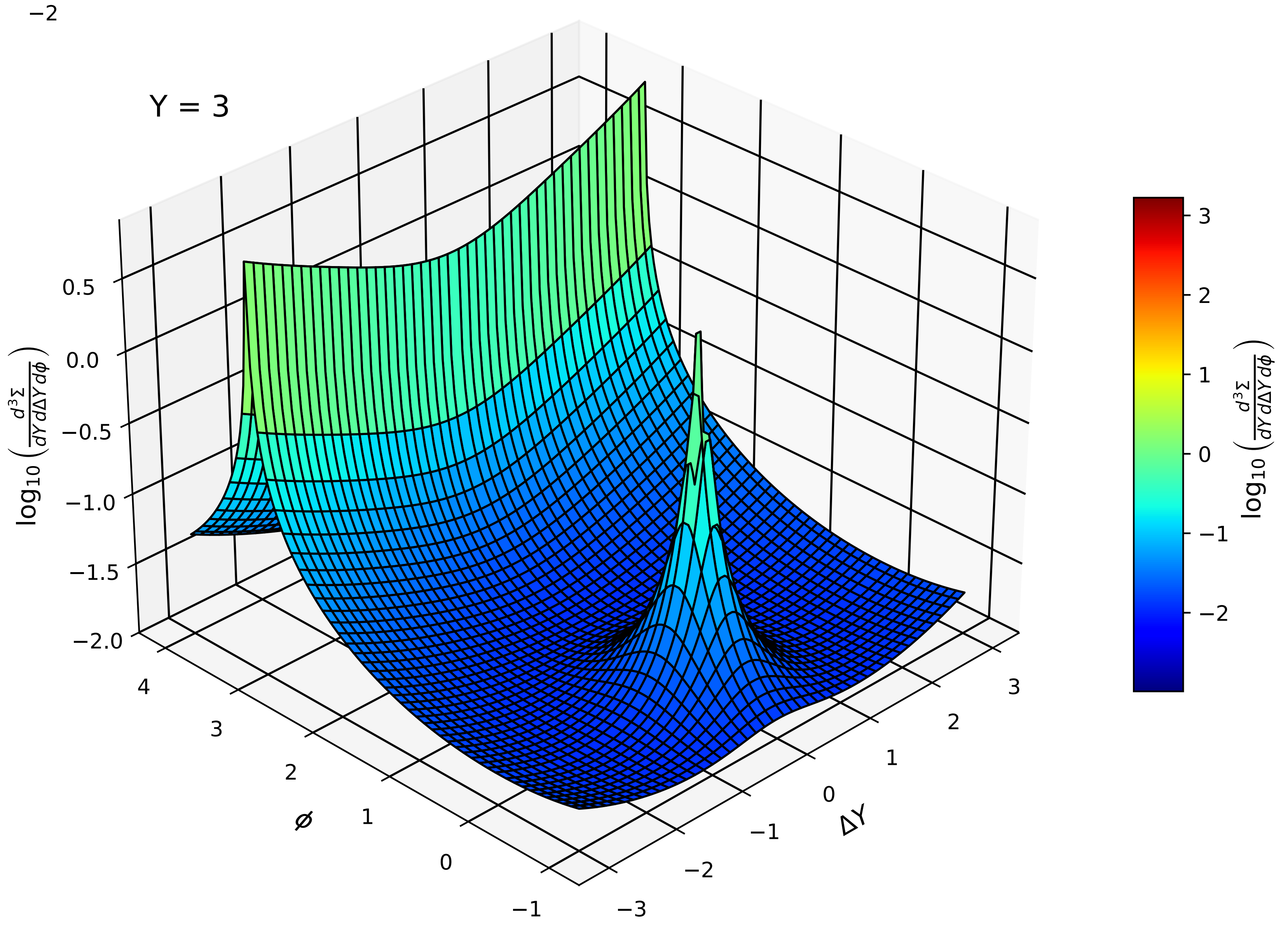
at parton level





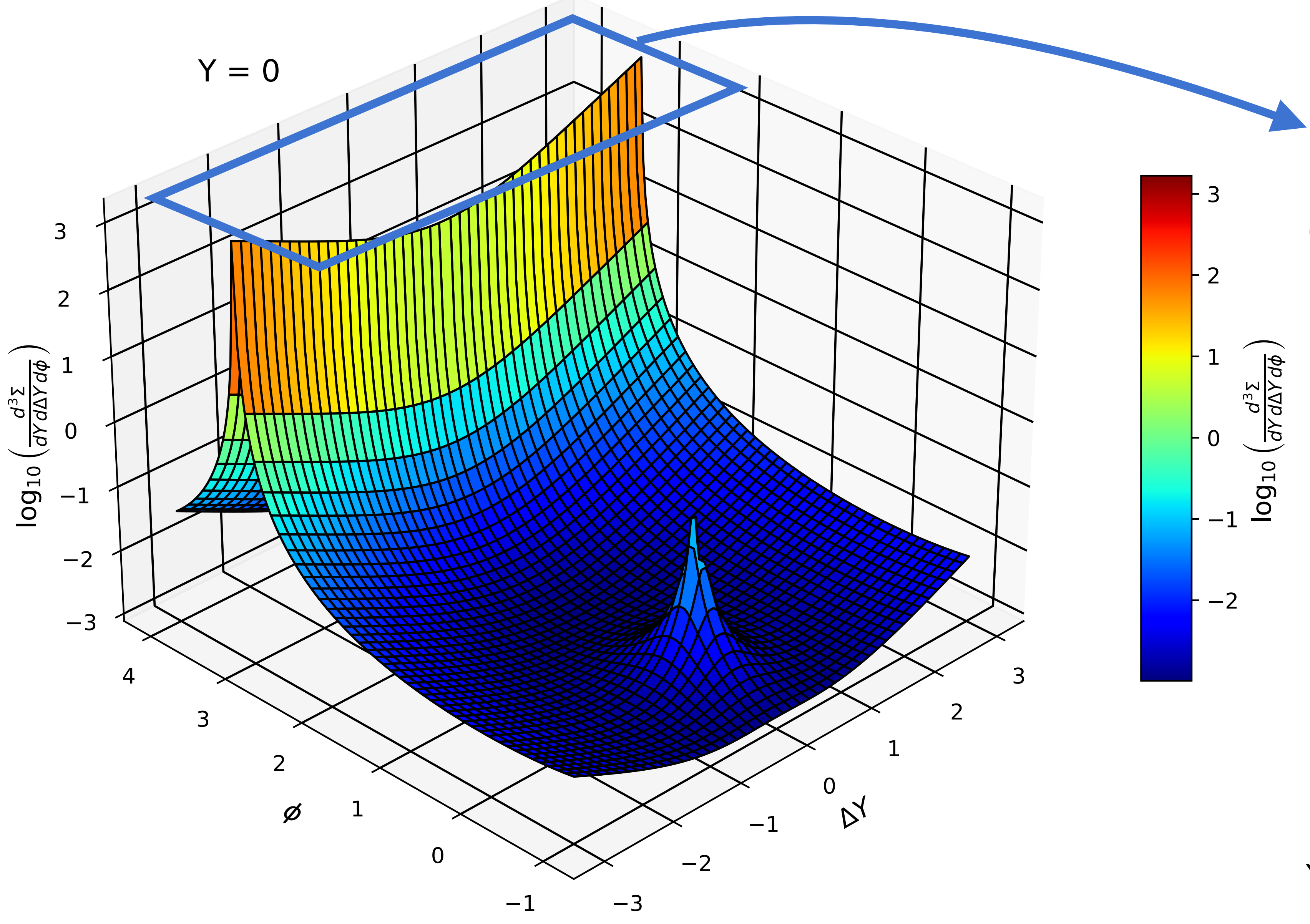
$$\frac{d^3\Sigma}{dY\,d\Delta Y\,d\phi} = \pi(1-y_a^2)(1-y_b^2)\frac{d^2\Sigma}{d\Omega_a\,d\Omega_b}$$
$$= \frac{16\pi e^{2(Y+\Delta Y)}}{(e^Y+e^{\Delta Y})^2(1+e^{Y+\Delta Y})^2}\frac{d^2\Sigma}{d\Omega_a\,d\Omega_b}$$

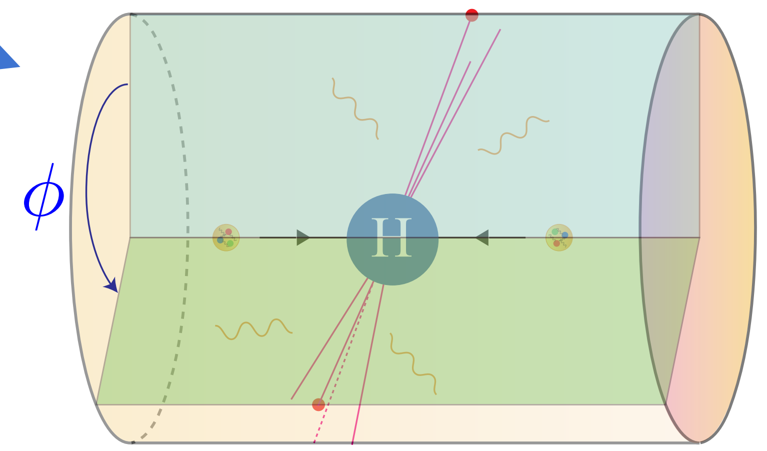


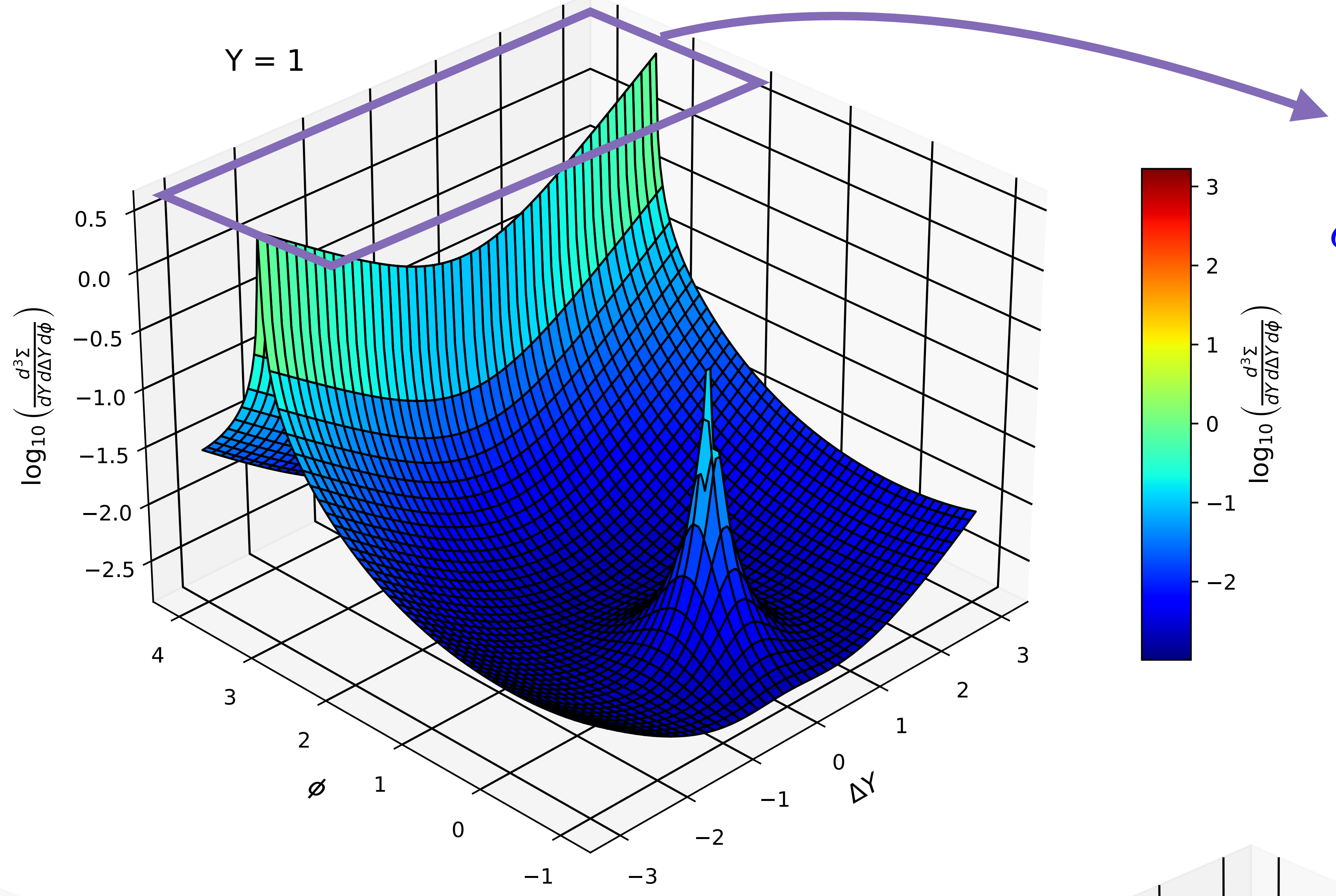


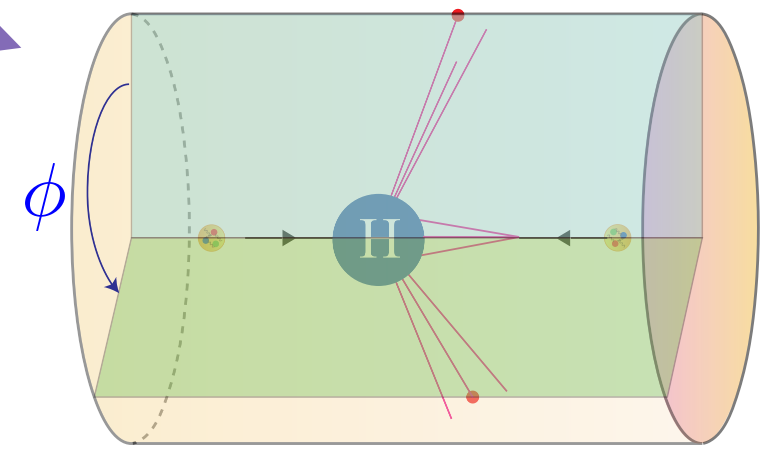
The EEC Landscape

at parton level



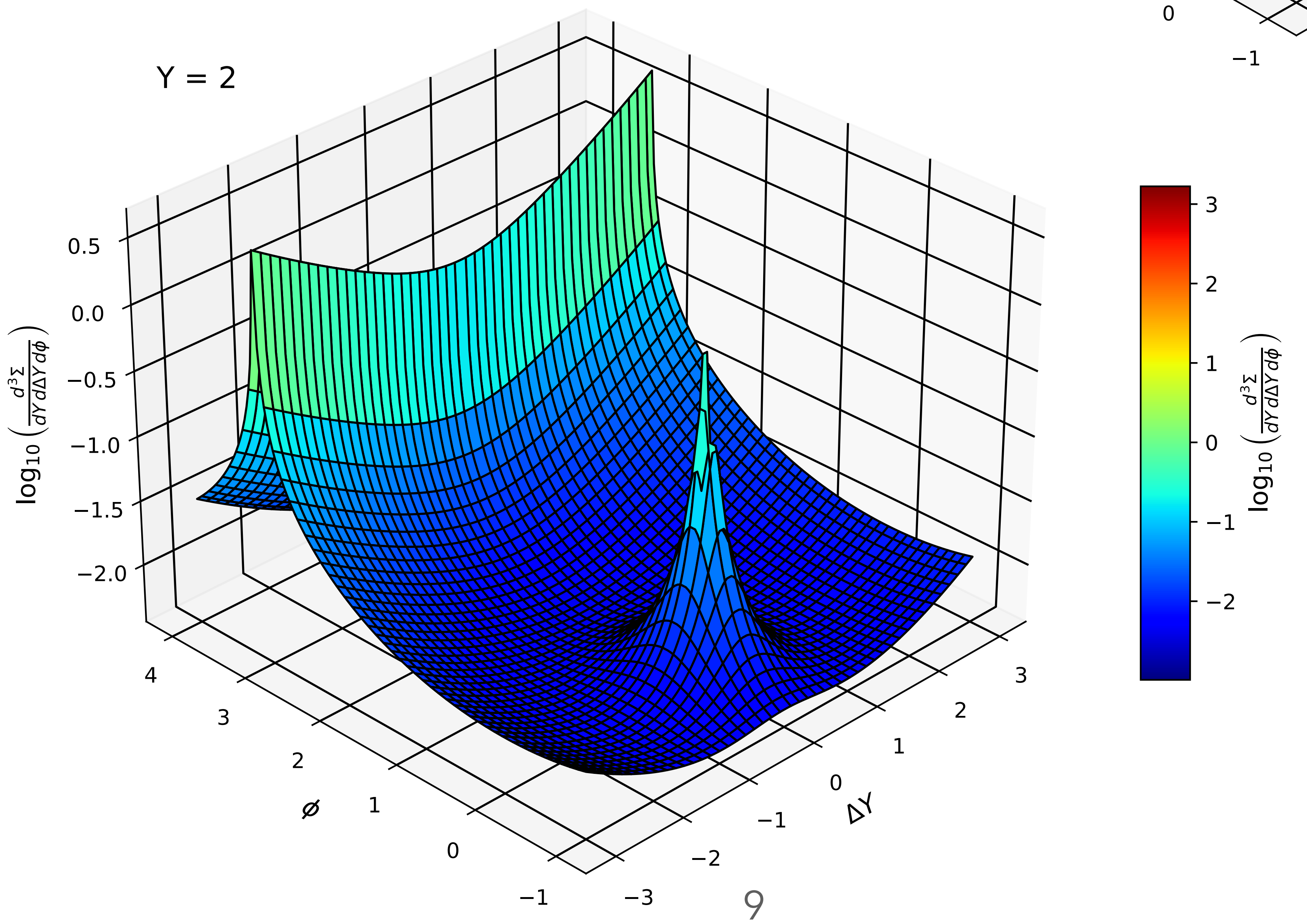


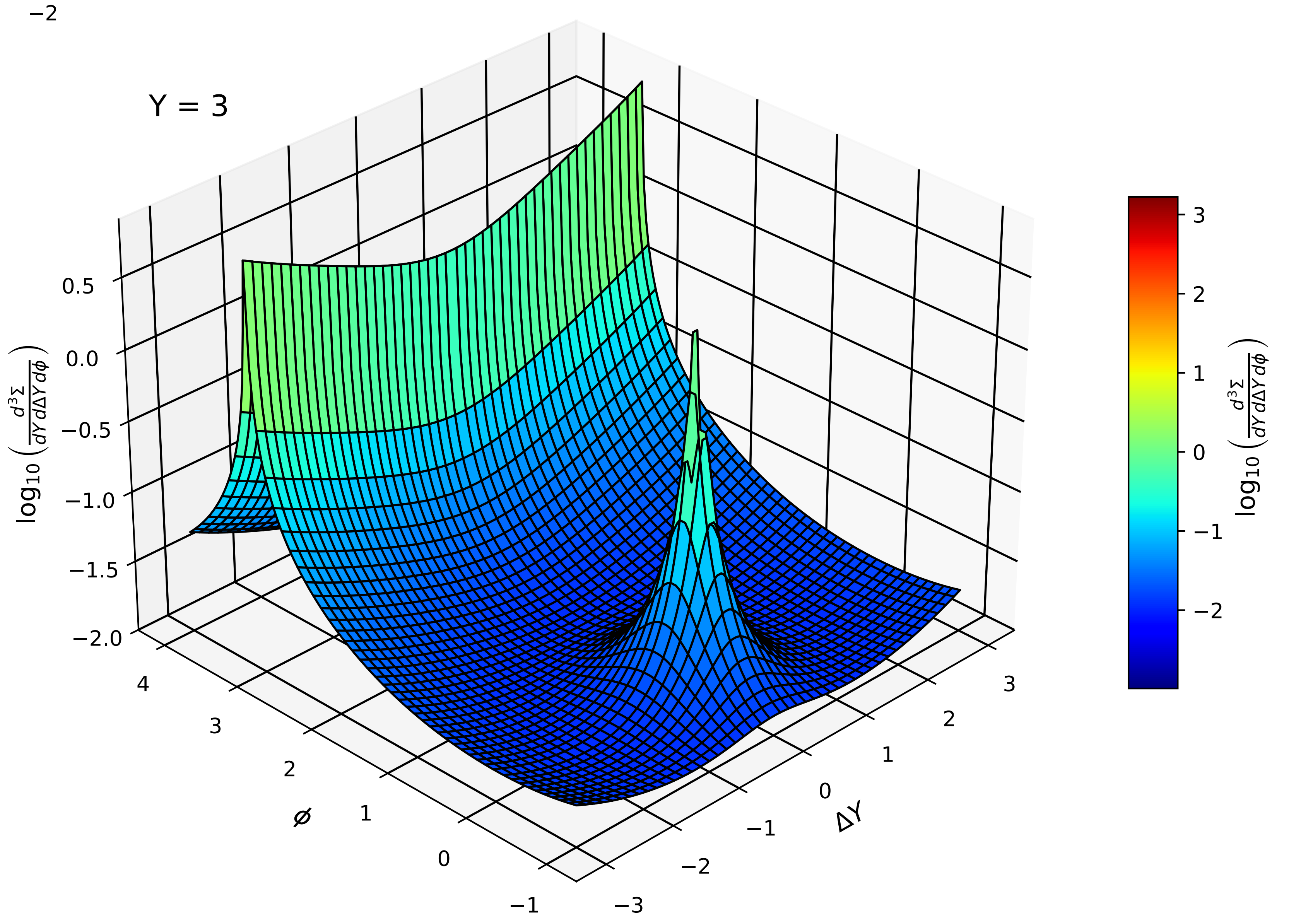




$$\frac{d^3\Sigma}{dY\,d\Delta Y\,d\phi} = \pi(1-y_a^2)(1-y_b^2)\frac{d^2\Sigma}{d\Omega_a\,d\Omega_b}$$

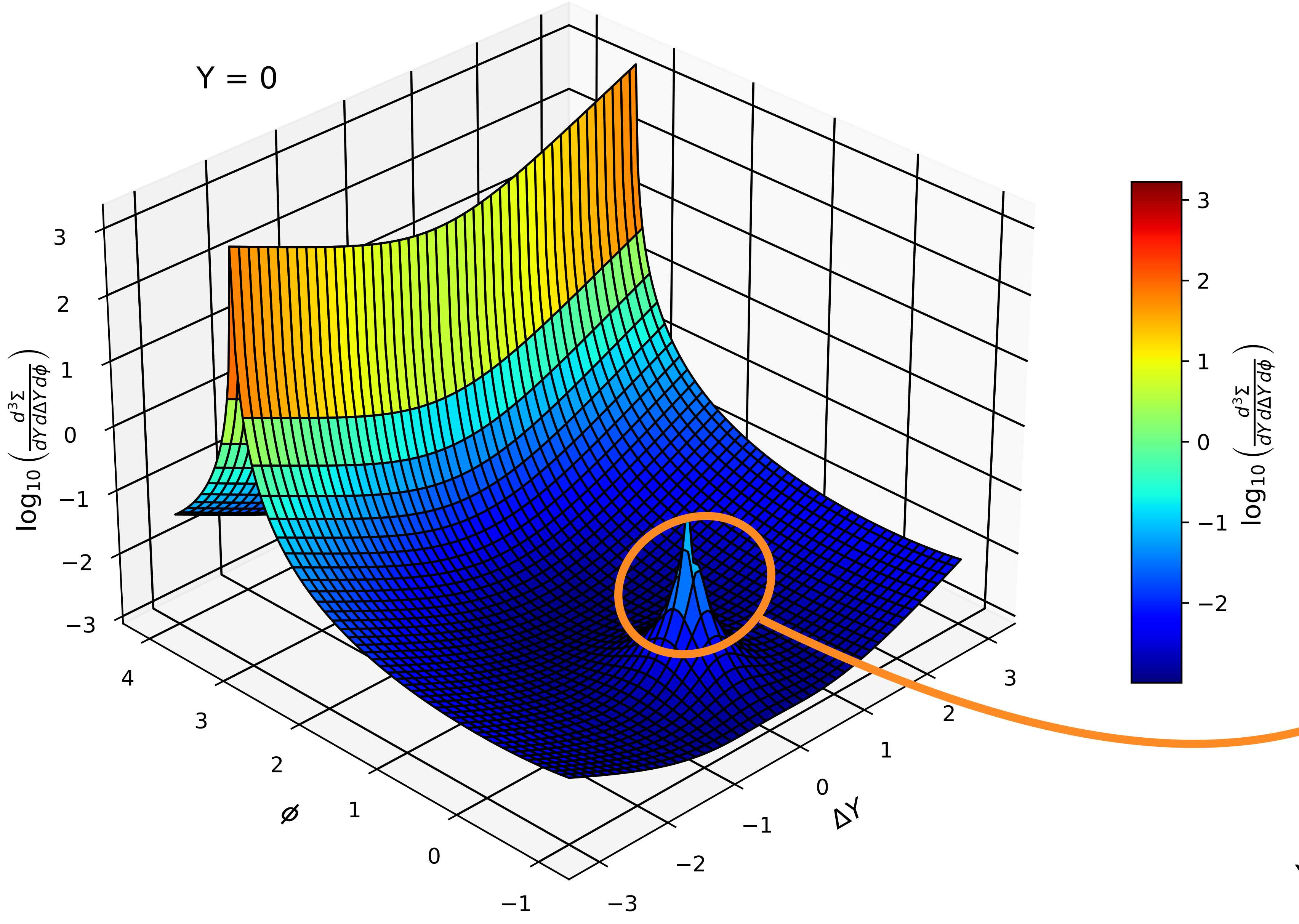
$$= \frac{16\pi e^{2(Y+\Delta Y)}}{(e^Y+e^{\Delta Y})^2(1+e^{Y+\Delta Y})^2}\frac{d^2\Sigma}{d\Omega_a\,d\Omega_b}$$

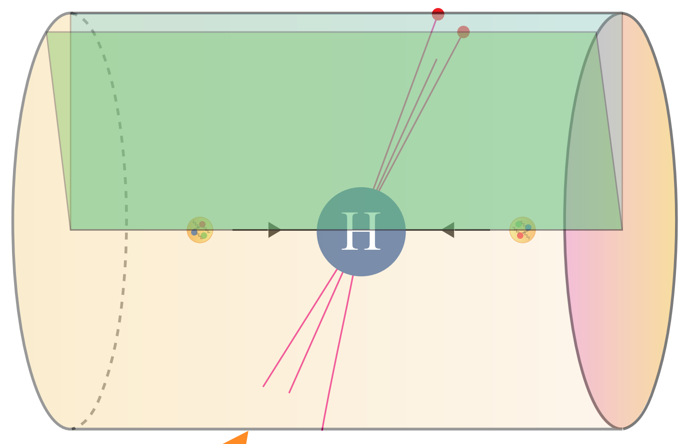


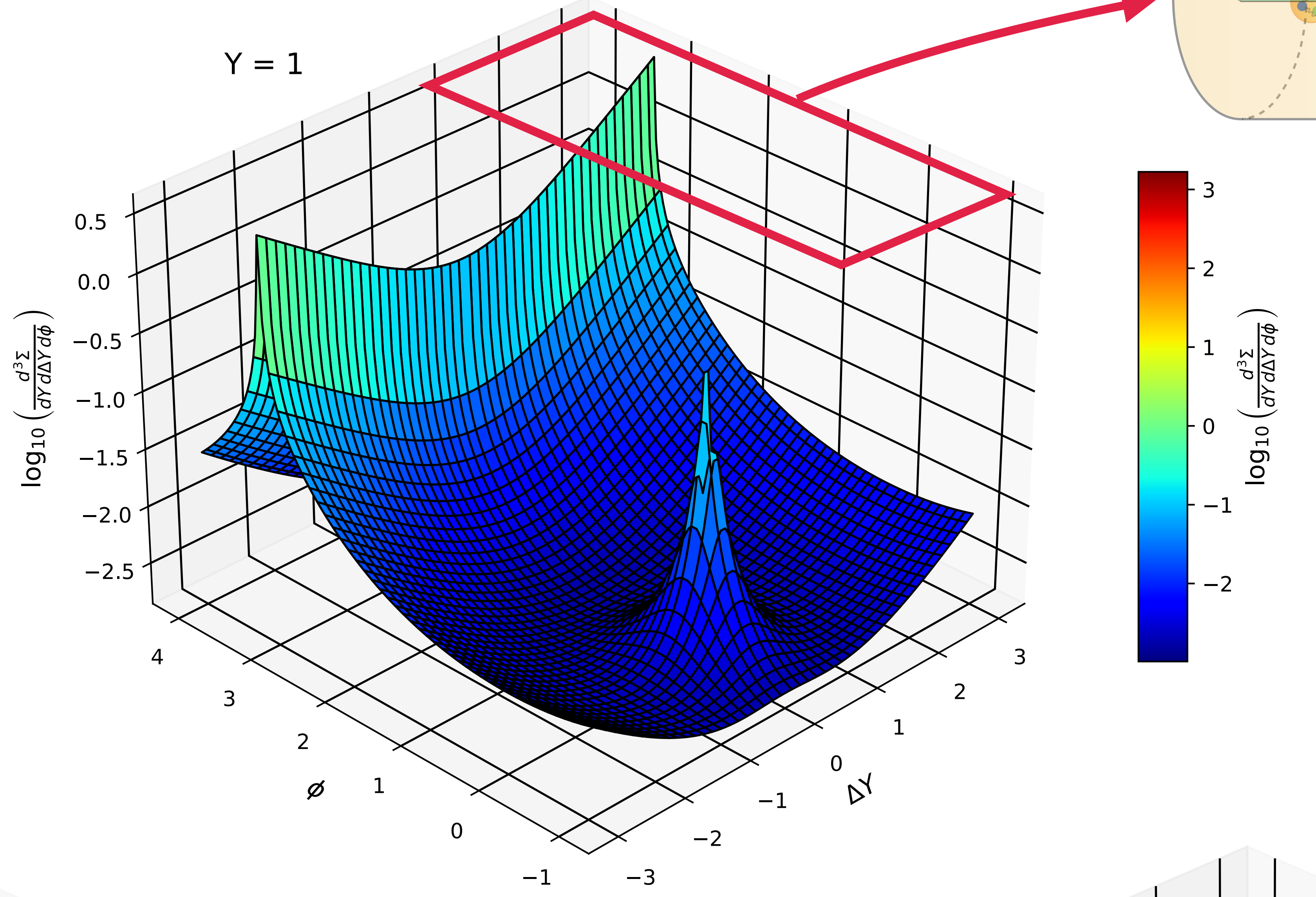


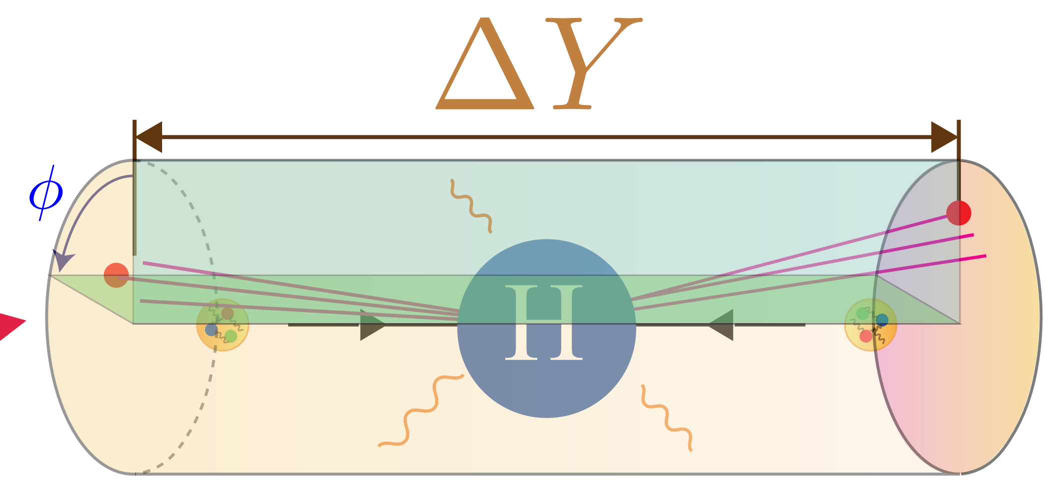
The EEC Landscape

at parton level



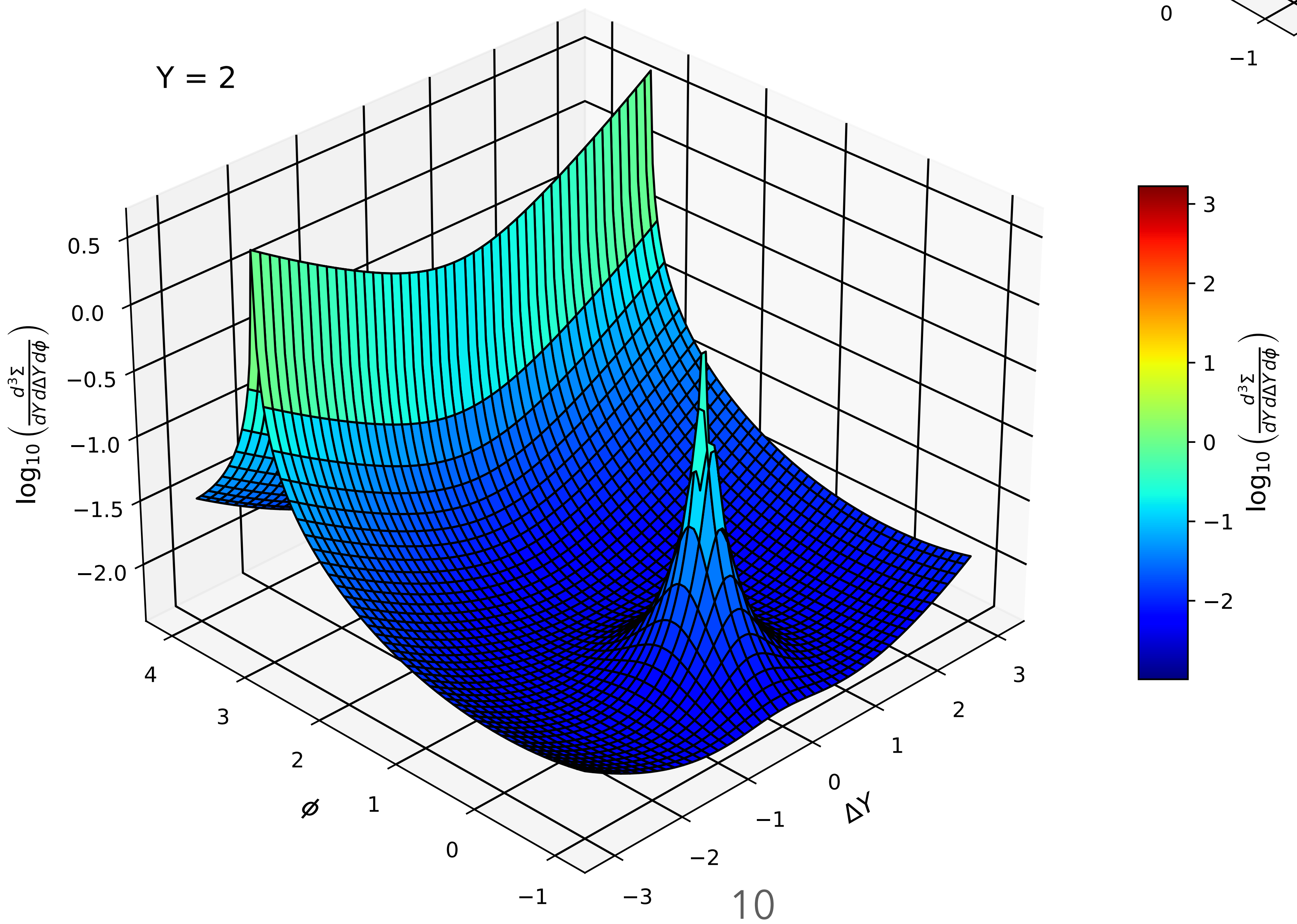


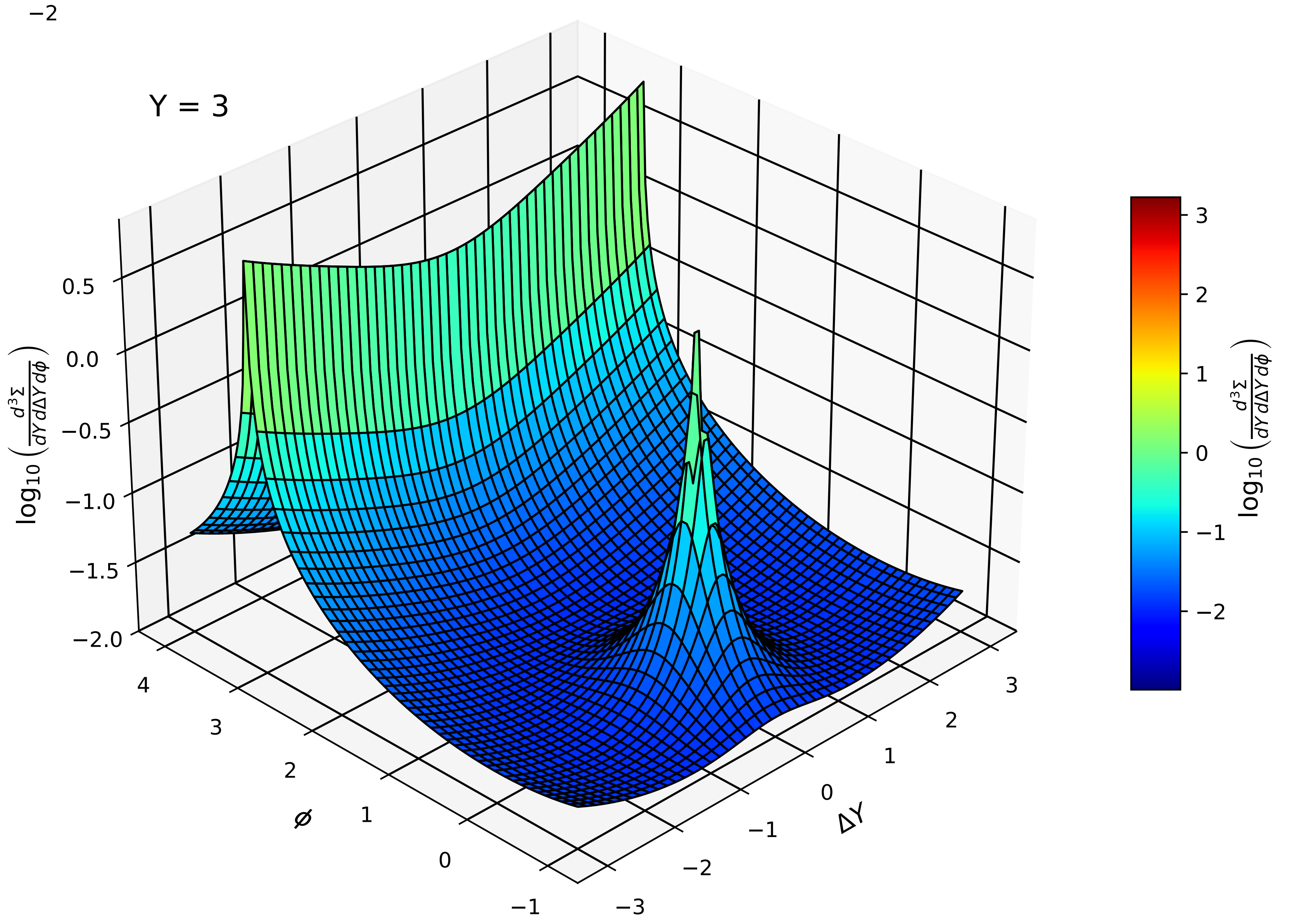




$$\frac{d^3\Sigma}{dY\,d\Delta Y\,d\phi} = \pi(1-y_a^2)(1-y_b^2)\frac{d^2\Sigma}{d\Omega_a\,d\Omega_b}$$

$$= \frac{16\pi e^{2(Y+\Delta Y)}}{(e^Y+e^{\Delta Y})^2(1+e^{Y+\Delta Y})^2}\frac{d^2\Sigma}{d\Omega_a\,d\Omega_b}$$

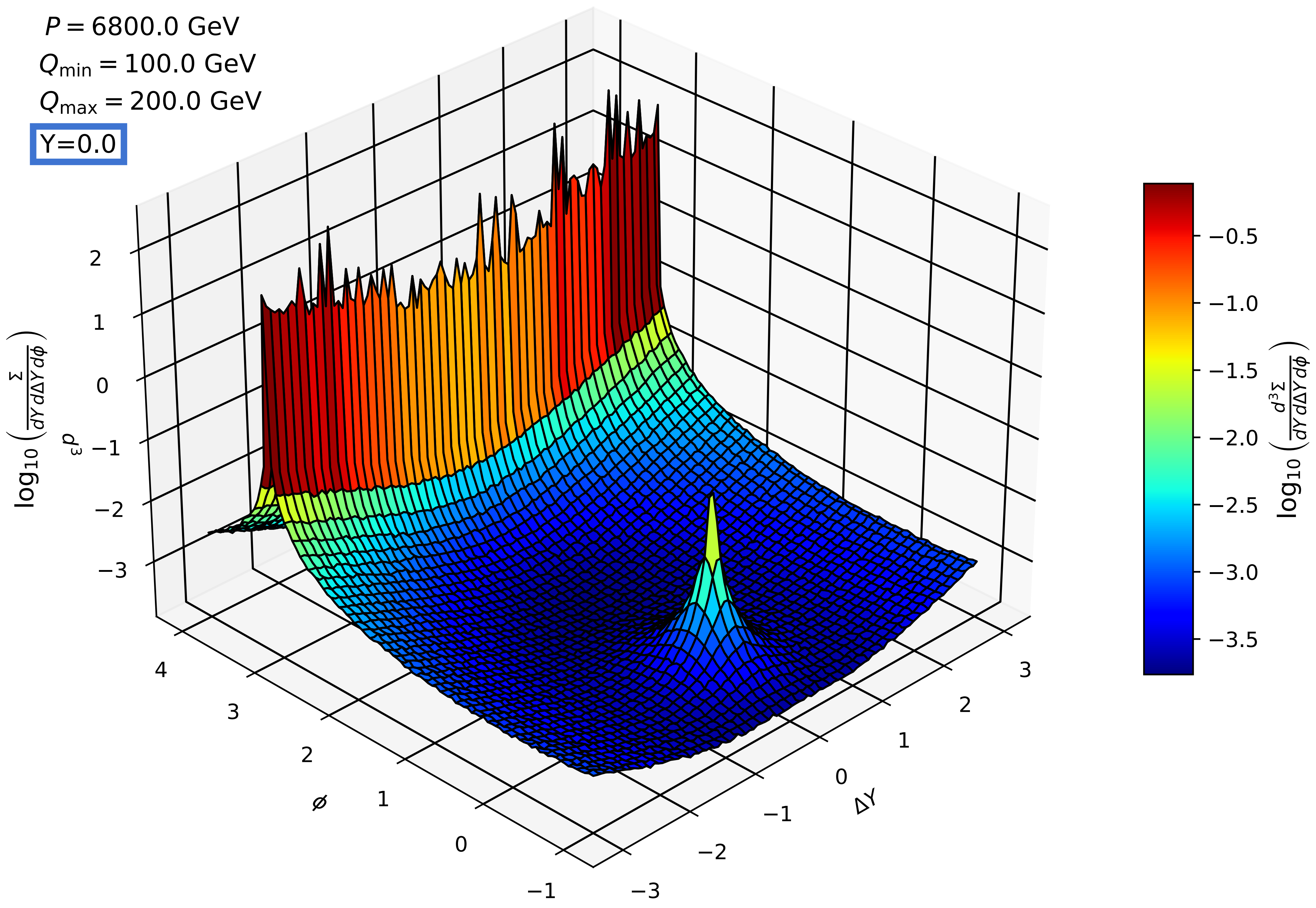


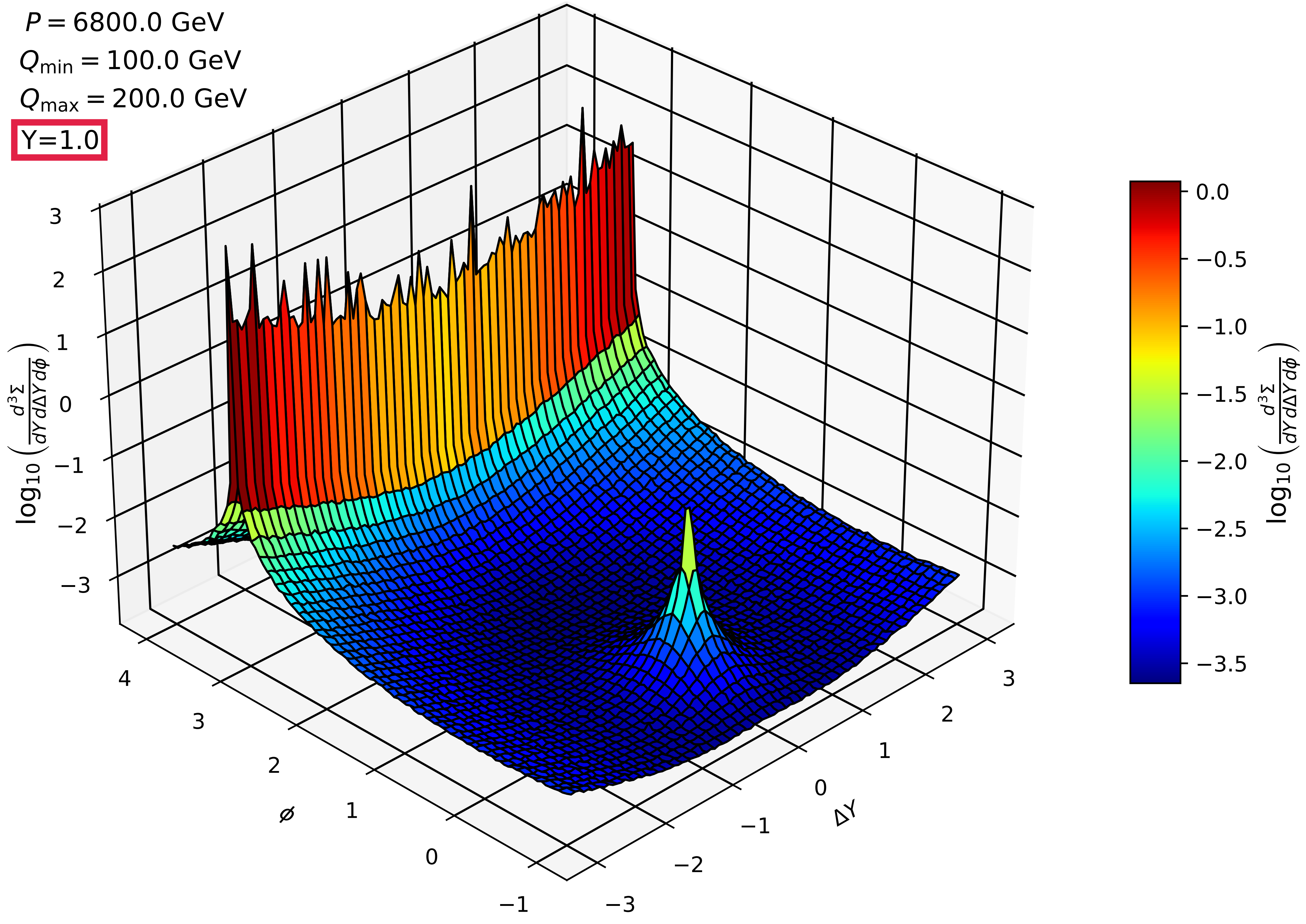


The EEC Landscape

in dijet production

Contribution from pure gluon scattering: $$\int_0^1 dx_1 dx_2\, x_1 x_2 f_g(x_1) f_g(x_2) \langle g(p_1)g(p_2)|\mathcal{E}(n_a)\mathcal{E}(n_b)|g(p_1)g(p_2)\rangle \Big|_{\text{constrains}}$$

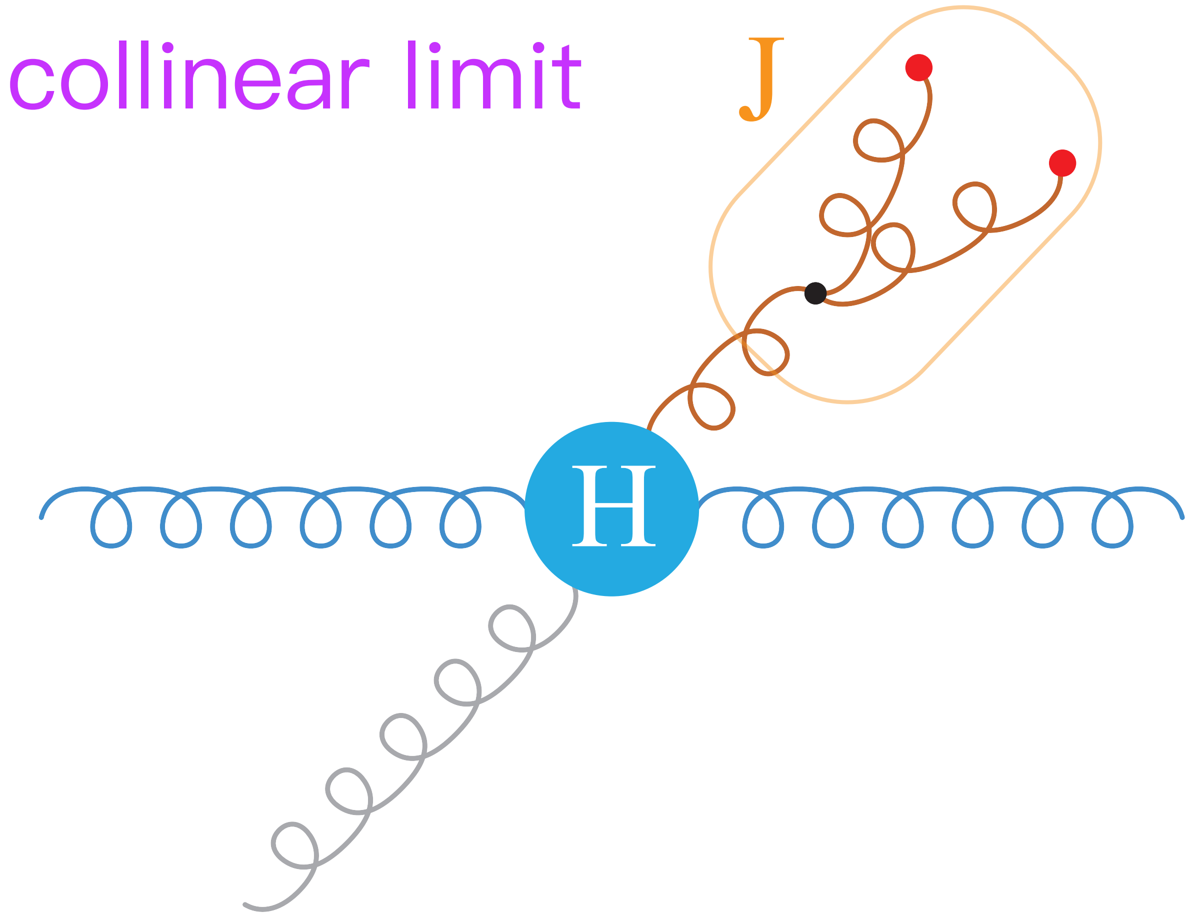




Limiting Behavior of EEC

Singular Approximations in Various Limits

collinear limit

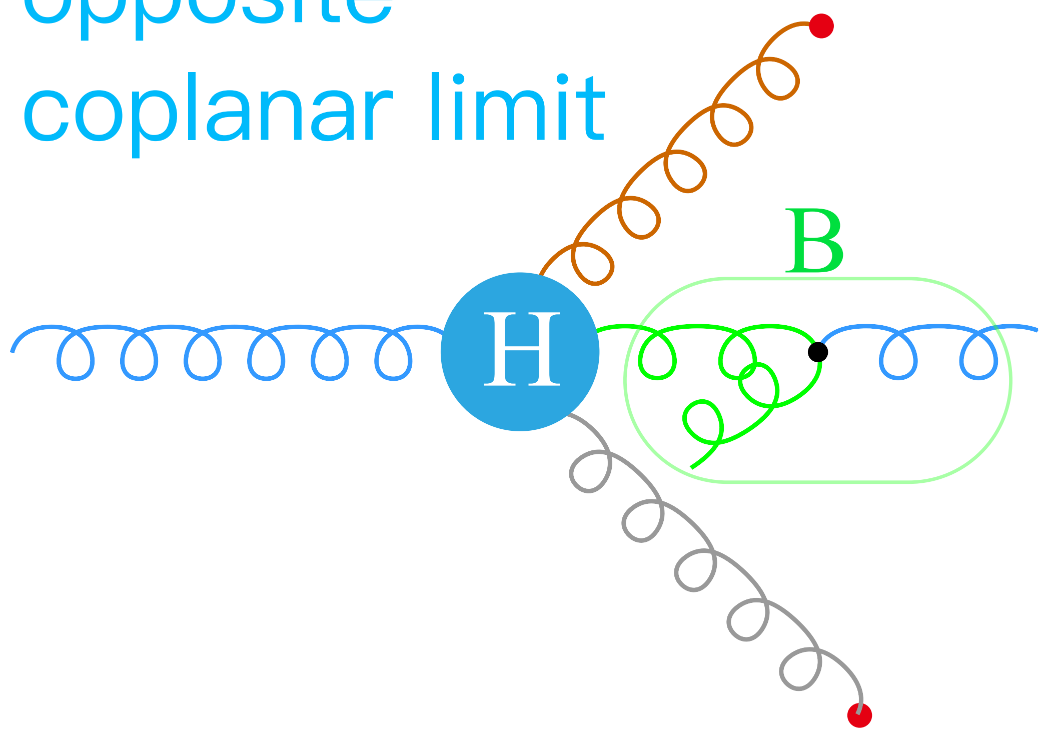


$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} \xrightarrow{\zeta\to 0} \frac{Q^2}{16384\pi^5} \overline{\sum_{\text{h,c}} \left|\mathcal{A}_4^{\text{full,tree}}\right|^2} J_{gg}$$

$$J_{gg} = \int_0^1 dx \frac{2g^2(1-x)x}{Q^2\zeta} P_{g\leftarrow g}(x)$$

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} \xrightarrow{\zeta\to 0} \frac{189 g^6 \left(e^Y+e^{-Y}+1\right)^3}{81920\pi^5 \left(\phi^2+\Delta Y^2\right)}$$

opposite coplanar limit

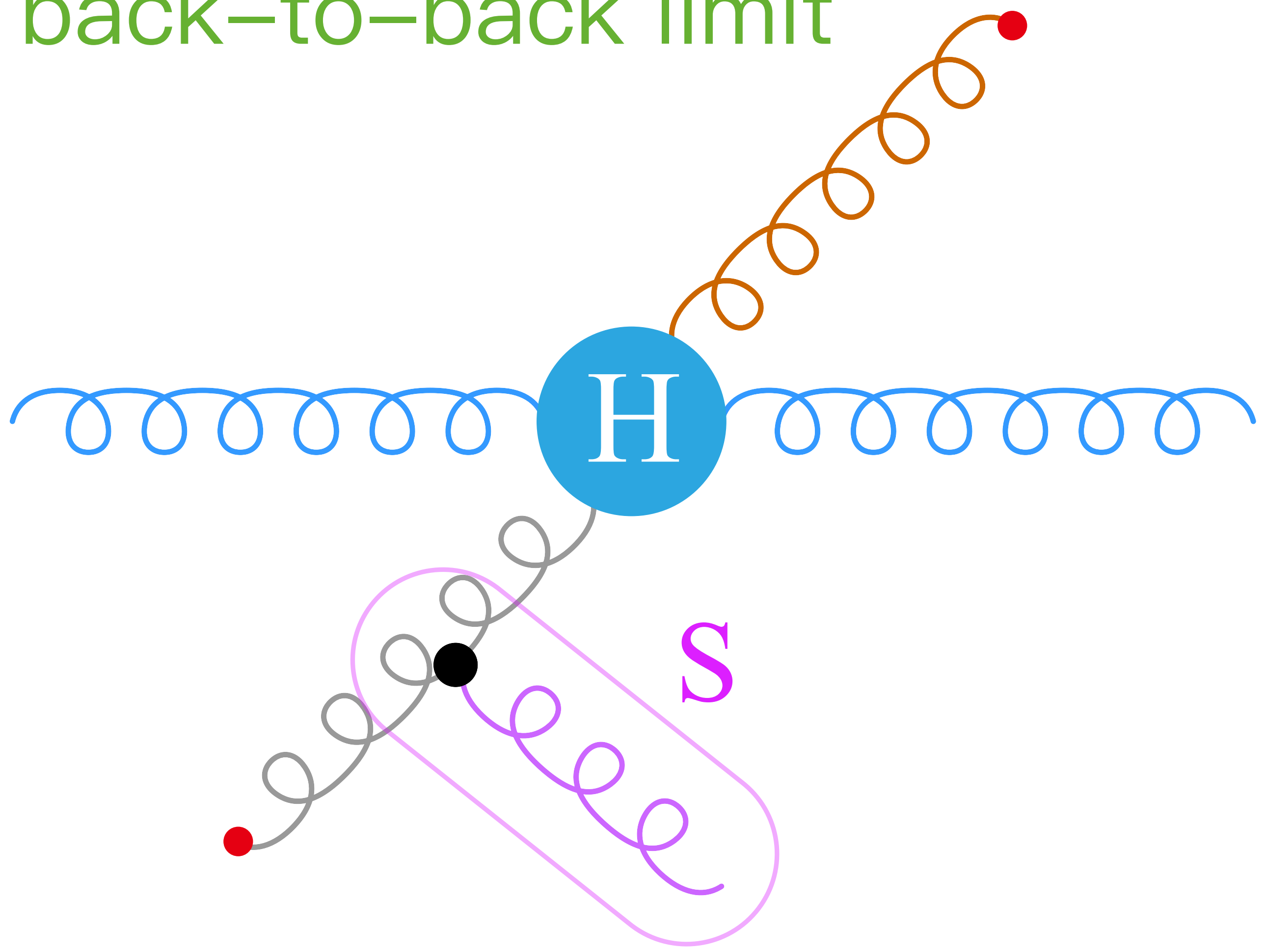


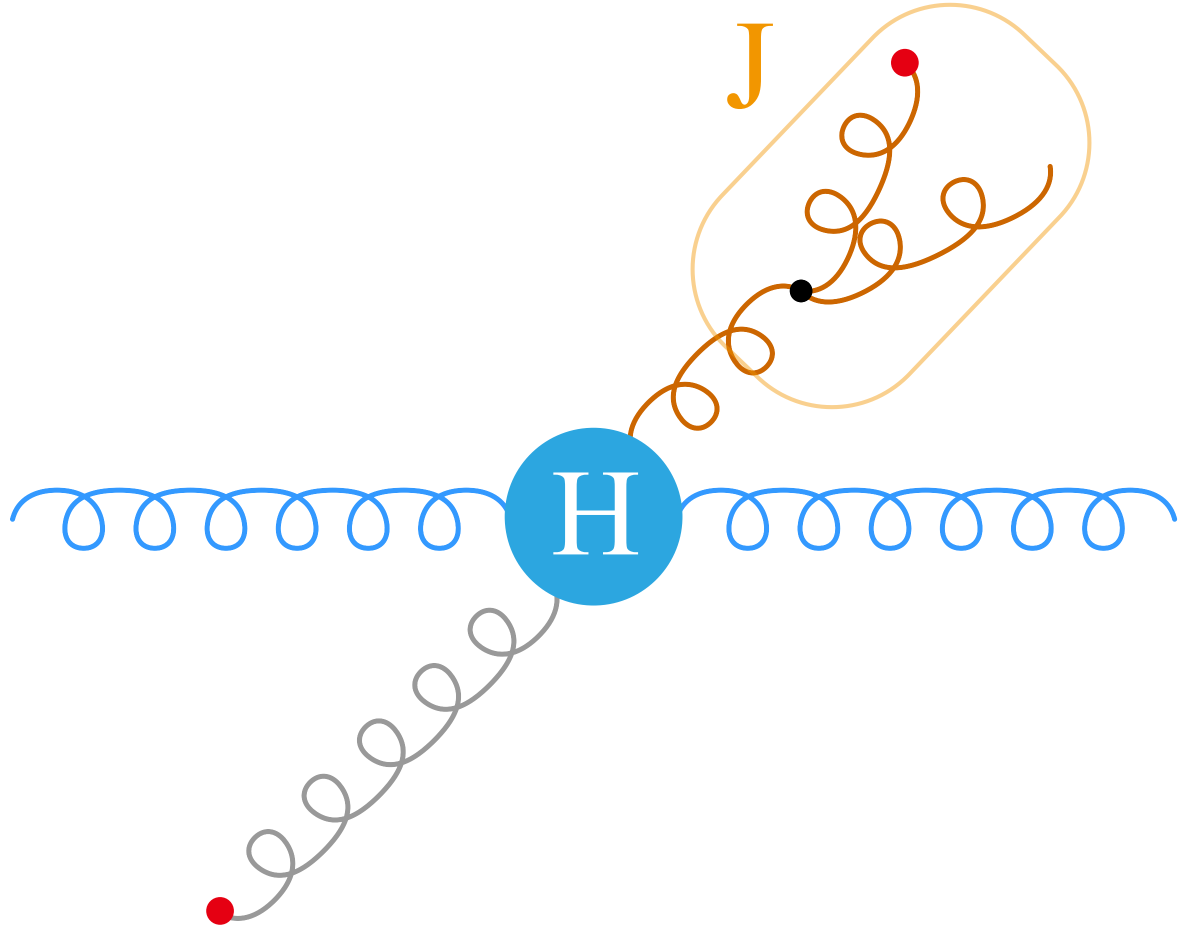
$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} \xrightarrow{\phi\to \pi} \frac{Q^2}{16384\pi^5} \overline{\sum_{\text{h,c}} \left|\mathcal{A}_4^{\text{full,tree}}\right|^2} B_{gg}$$

$$B_{gg} = \int_0^1 dx \frac{(1-x)x}{(1-x\zeta)^3} \frac{2g^2 p_\perp^2}{p_c^4} P_{g\leftarrow g}(x)$$

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} \xrightarrow{\phi\to \pi} \frac{(e^{\Delta Y}+e^Y)^3 (e^{Y+\Delta Y}+1)^3}{(e^{\Delta Y}+1)^6 (e^Y-1)\, e^{4Y}} \times \frac{27\, g^6 (e^{\Delta Y}+e^{-\Delta Y}+1)^3 (e^Y+e^{-Y}-1)^2}{8192\, \pi^4 (\pi-\phi)}$$

back–to–back limit





$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} \xrightarrow{\delta r\to 0} \frac{Q^2}{16384\pi^5} \left(\overline{\sum_{\text{h,c}} \left|\mathcal{A}_4^{\text{full, tree}}\right|^2} \mathcal{C}_{gg} + \overline{\sum \mathcal{H}_{4g} \mathcal{S}_{gg}} \right)$$

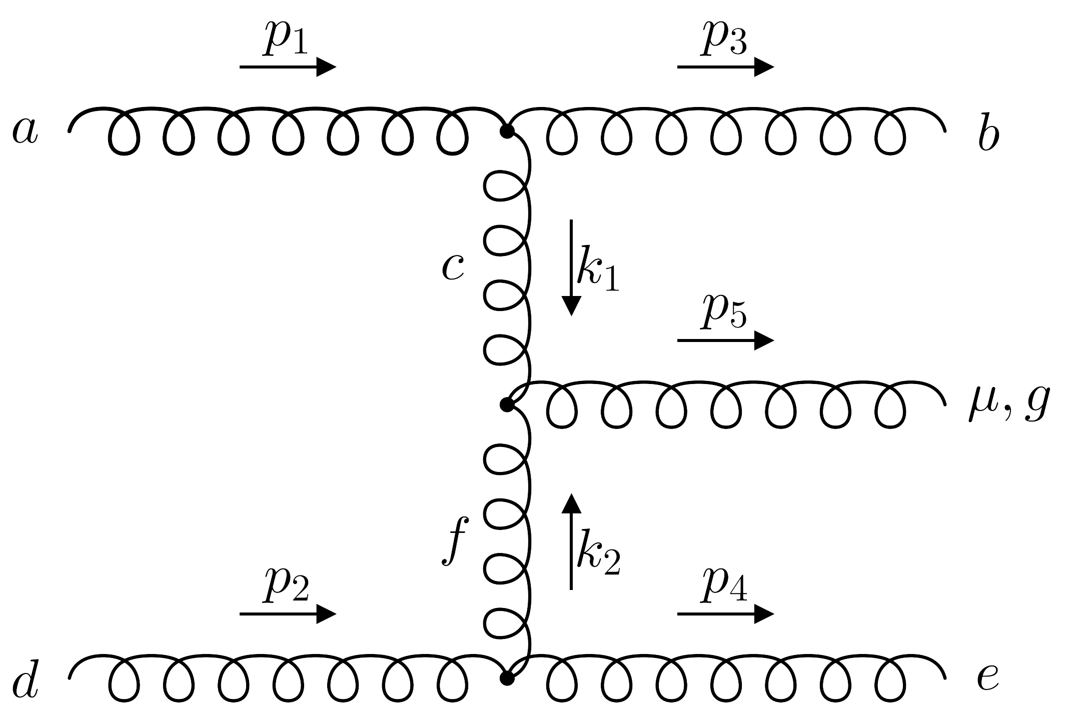
LP LL

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} \xrightarrow{\delta r\to 0} -\frac{9g^6 \left(e^{2\Delta Y}+e^{\Delta Y}+1\right)^3 \left(12 \ln \delta r - 6 \ln\left(e^{\Delta Y}+e^{-\Delta Y}+2\right) - 12\varphi \tan\varphi + 11\right)}{16384\pi^5 e^{3\Delta Y} \delta r^2} + \frac{27 g^6 \left(e^{2\Delta Y}+e^{\Delta Y}+1\right)^2 \left(2\left(e^{2\Delta Y}+1\right)\varphi \sin 2\varphi + \left(e^{2\Delta Y}-1\right)\Delta Y \cos 2\varphi\right)}{16384\pi^5 e^{2\Delta Y} \delta r^2 \left(e^{2\Delta Y} - 2e^{\Delta Y}\cos 2\varphi + 1\right)}$$

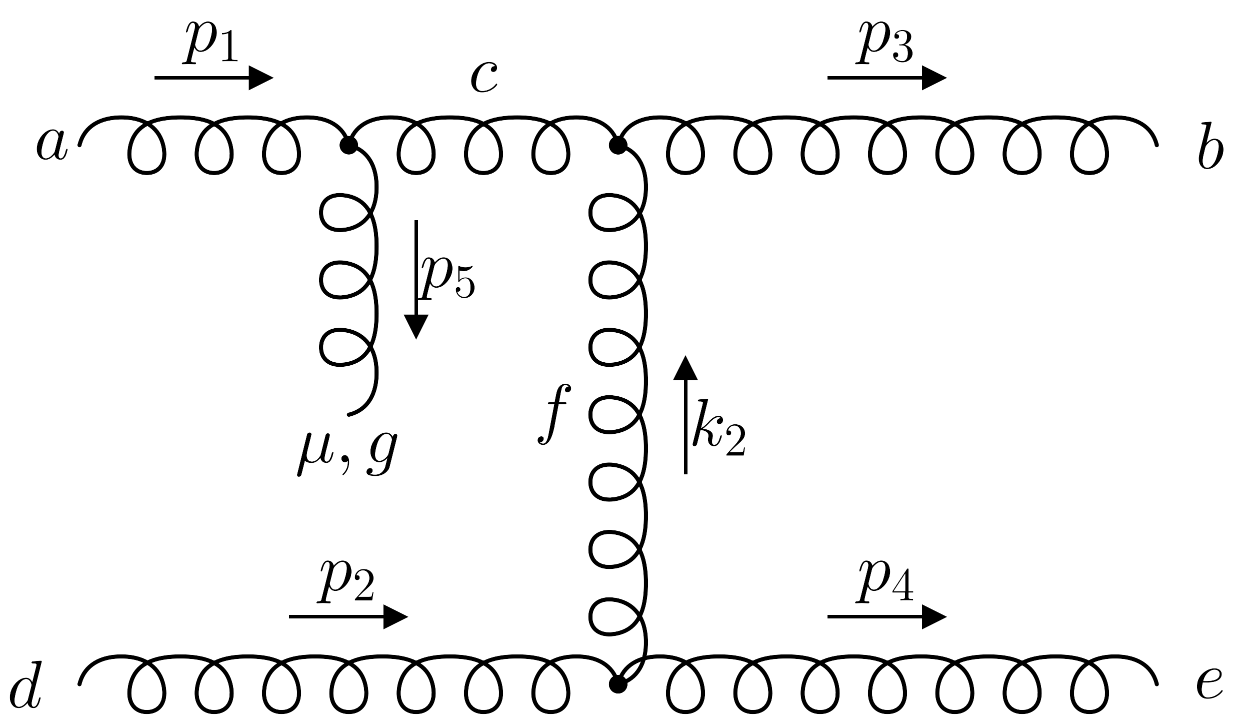
$$\delta r = \sqrt{(\pi-\phi)^2 + (e^Y-1)^2}, \quad \varphi = \arctan\left(\frac{e^Y-1}{\pi-\phi}\right)$$

EEC in the Regge Limit

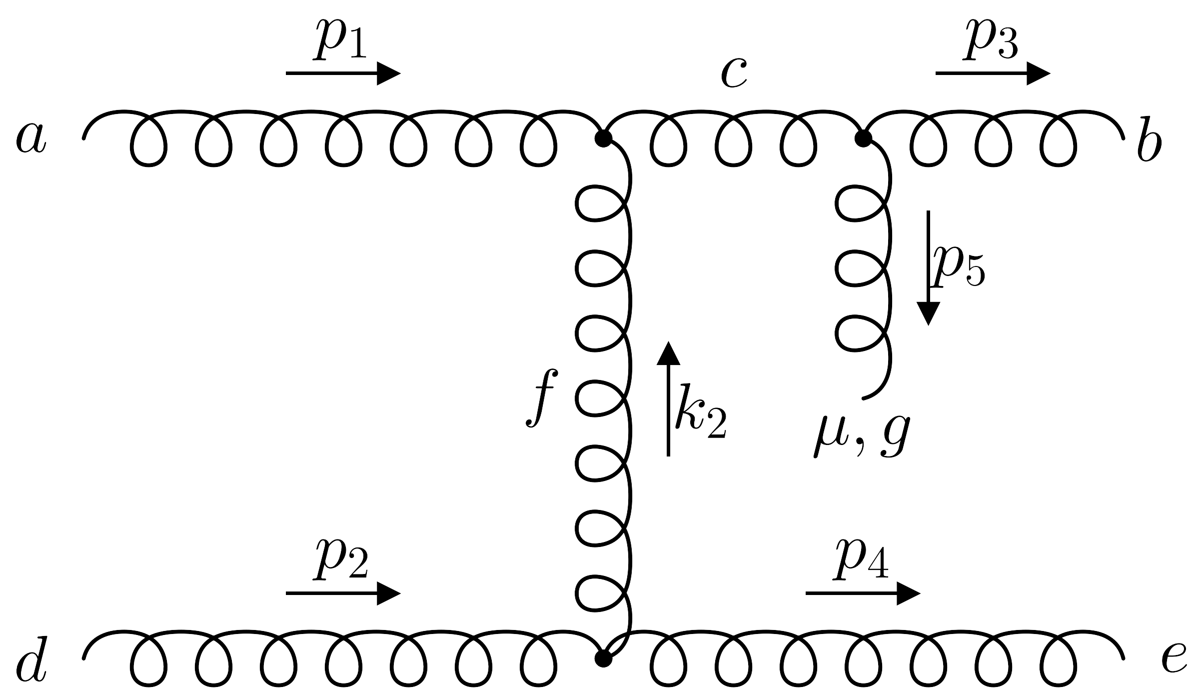
Leading Power Leading Log Approximation



a



b



c

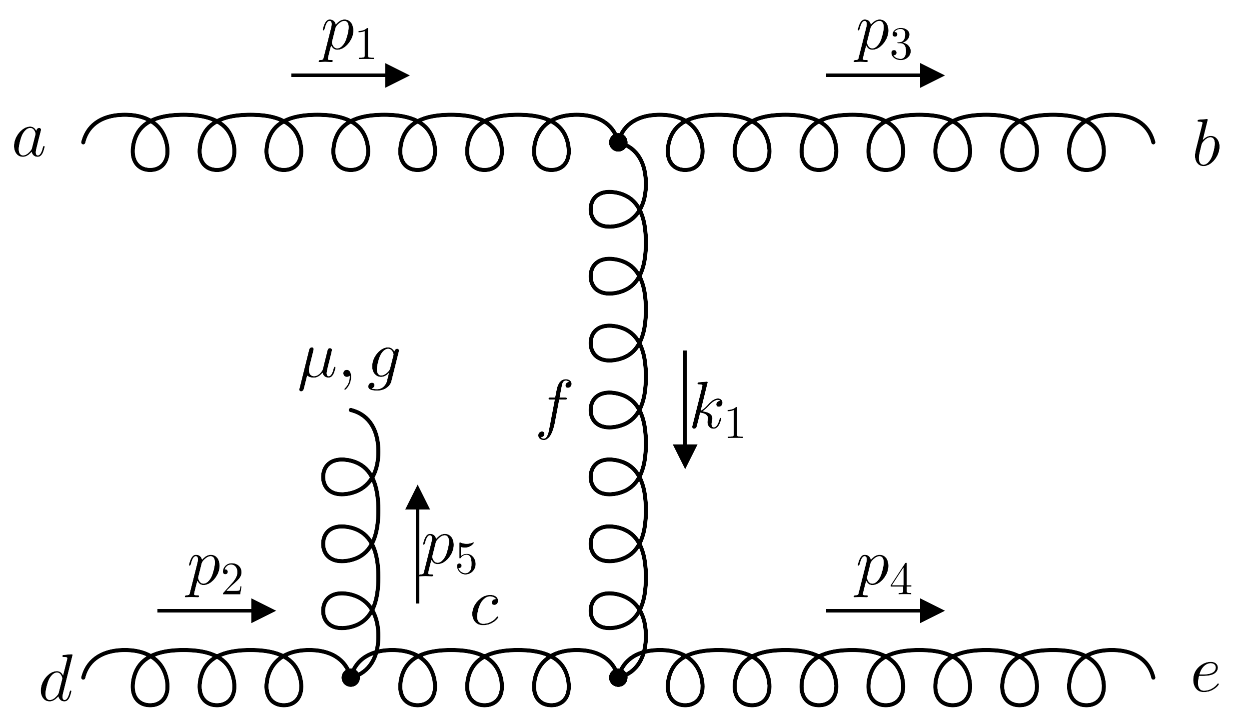
$$iM^\mu_{b,c} = -\frac{4g^3}{\rho_2\lambda_2 s} p_1^\mu \eta^{h_2h_4} \eta^{h_1h_c}\eta^{h_ch_3} f_{abc}f_{def}f_{cfg}$$

$$k_i^\mu = \rho_i p_1^\mu + \lambda_i p_2^\mu + k_{i\perp}^\mu$$

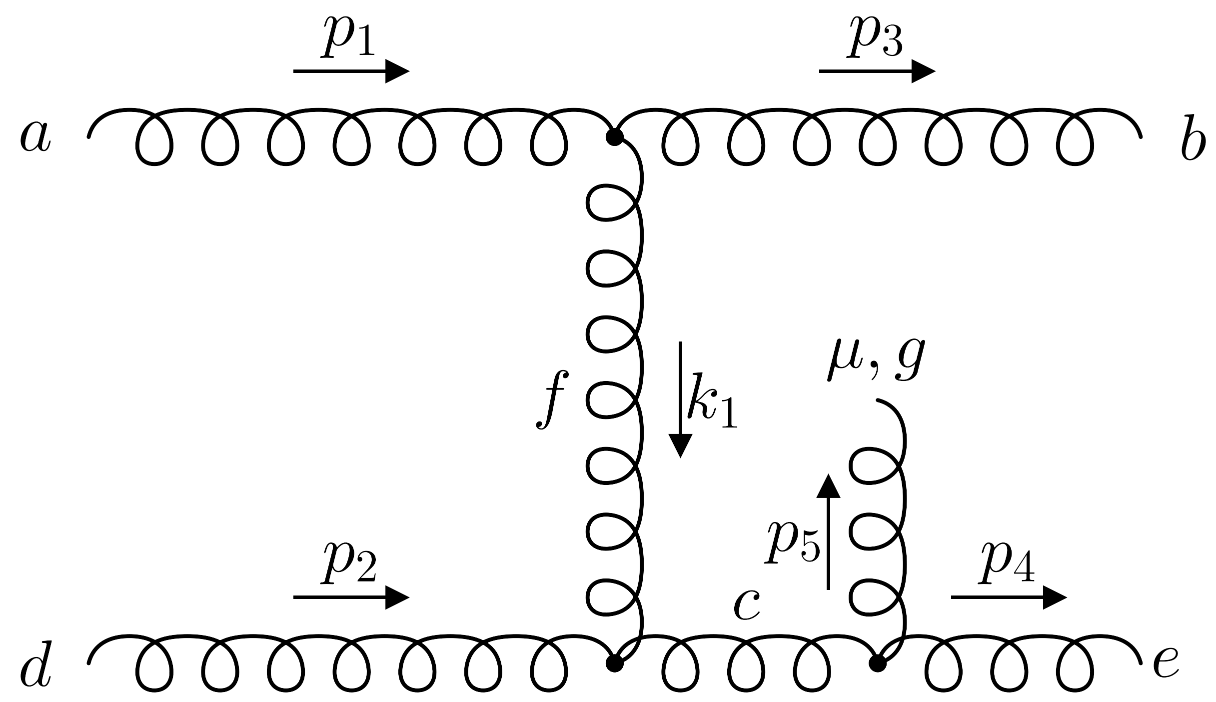
$$1 \gg |\rho_1| \gg |\rho_2|$$

$$1 \gg |\lambda_2| \gg |\lambda_1|$$

$$iM_a^\mu = \frac{2g^3}{\lambda_1\rho_2 s}\eta^{h_1h_3}\eta^{h_2h_4} f_{abc}f_{def}f_{cfg} \left(\rho_1 p_1^\mu - \lambda_2 p_2^\mu - k_{1\perp}^\mu + k_{2\perp}^\mu\right)$$



d



e

$$iM^\mu_{d,e} = \frac{4g^3}{\rho_1\lambda_1 s} p_2^\mu \eta^{h_1h_3} \eta^{h_2h_c}\eta^{h_ch_4} f_{abc}f_{def}f_{cfg}$$

$$iM^\mu = \frac{2g^3}{s} f_{abc}\, f_{def}\, f_{cfg}\, \eta^{h_1h_3}\, \eta^{h_2h_4}\, \Gamma^\mu(k_1, k_2)$$

Lipatov's effective vertex

$$\longrightarrow \frac{1}{\lambda_1\rho_2}\left(\left(\rho_1 + \frac{2\lambda_1}{\lambda_2}\right)p_1^\mu - \left(\lambda_2 + \frac{2\rho_2}{\rho_1}\right)p_2^\mu - k_{1\perp}^\mu + k_{2\perp}^\mu\right)$$

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} \xrightarrow{\Delta_\eta\to\infty} \frac{27 g^6 \eta\, \Delta_\eta^3 \ln \Delta_\eta}{8192\pi^5\left(1 + 2\eta\cos\phi + \eta^2\right)}$$

$$\eta = e^Y, \qquad \Delta_\eta = e^{\Delta Y}$$

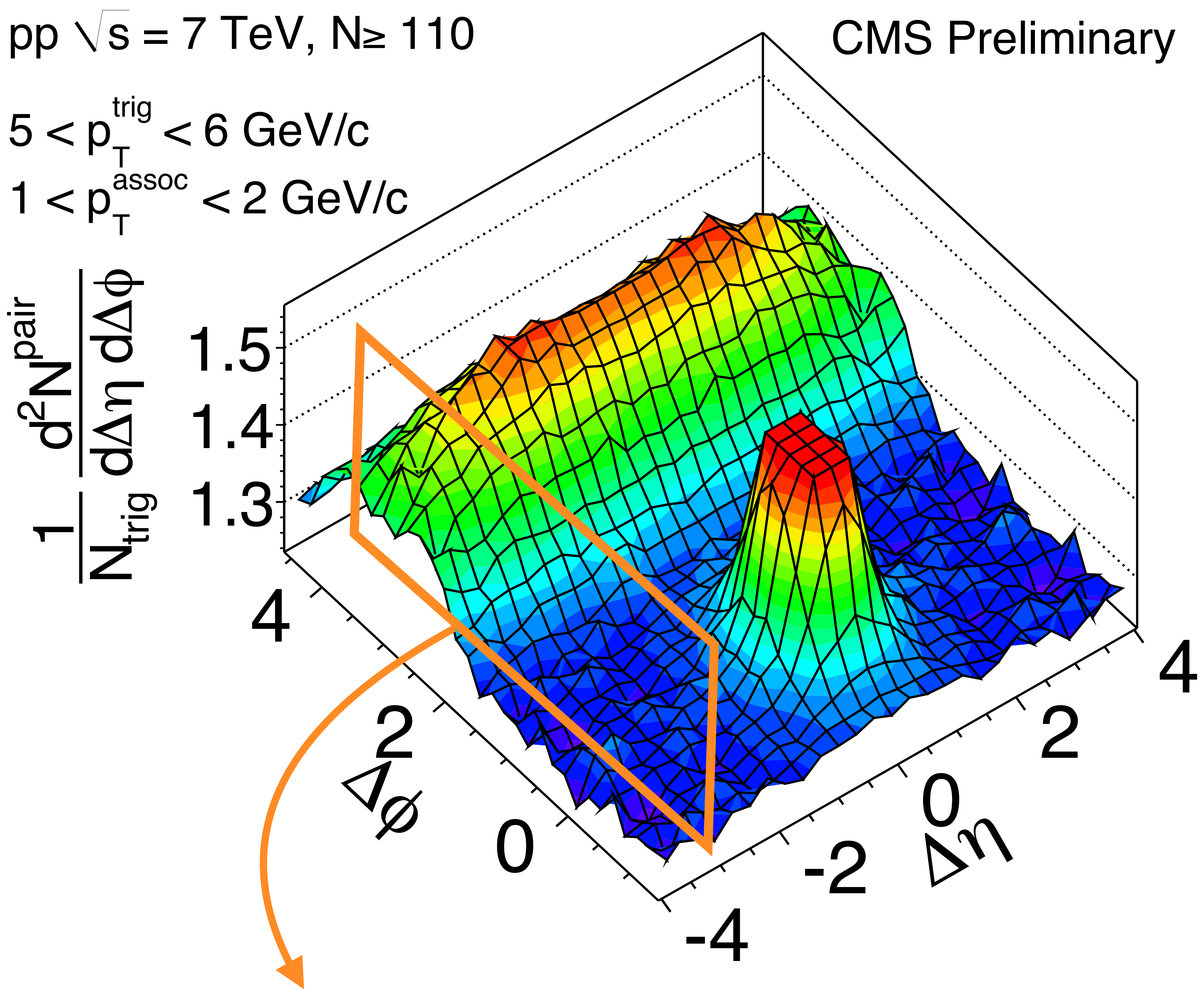
EEC in the Regge Limit:

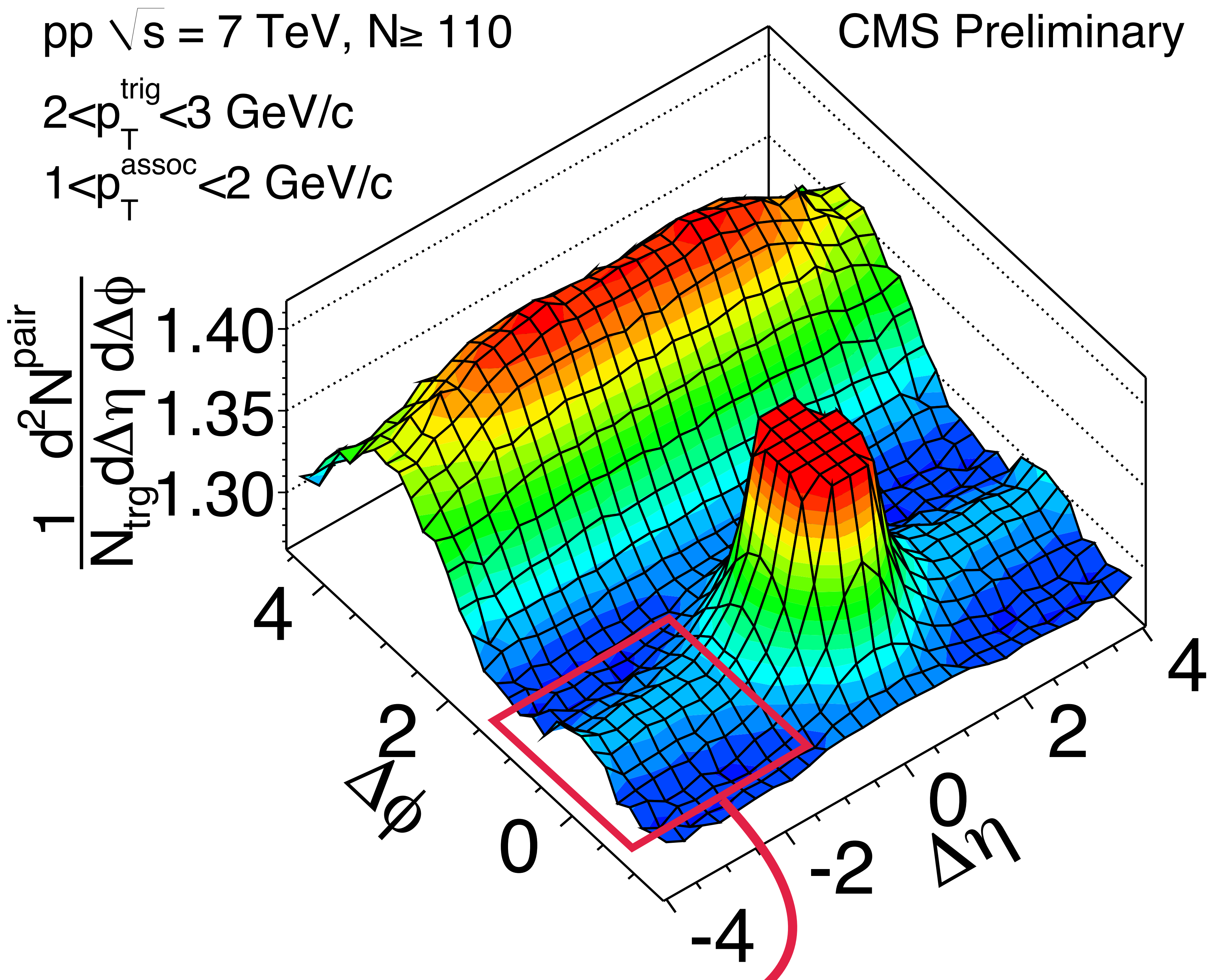
Toward LL Resummation

A Mysterious Ridge Effect

in proton–proton collision

[CMS, 2011]



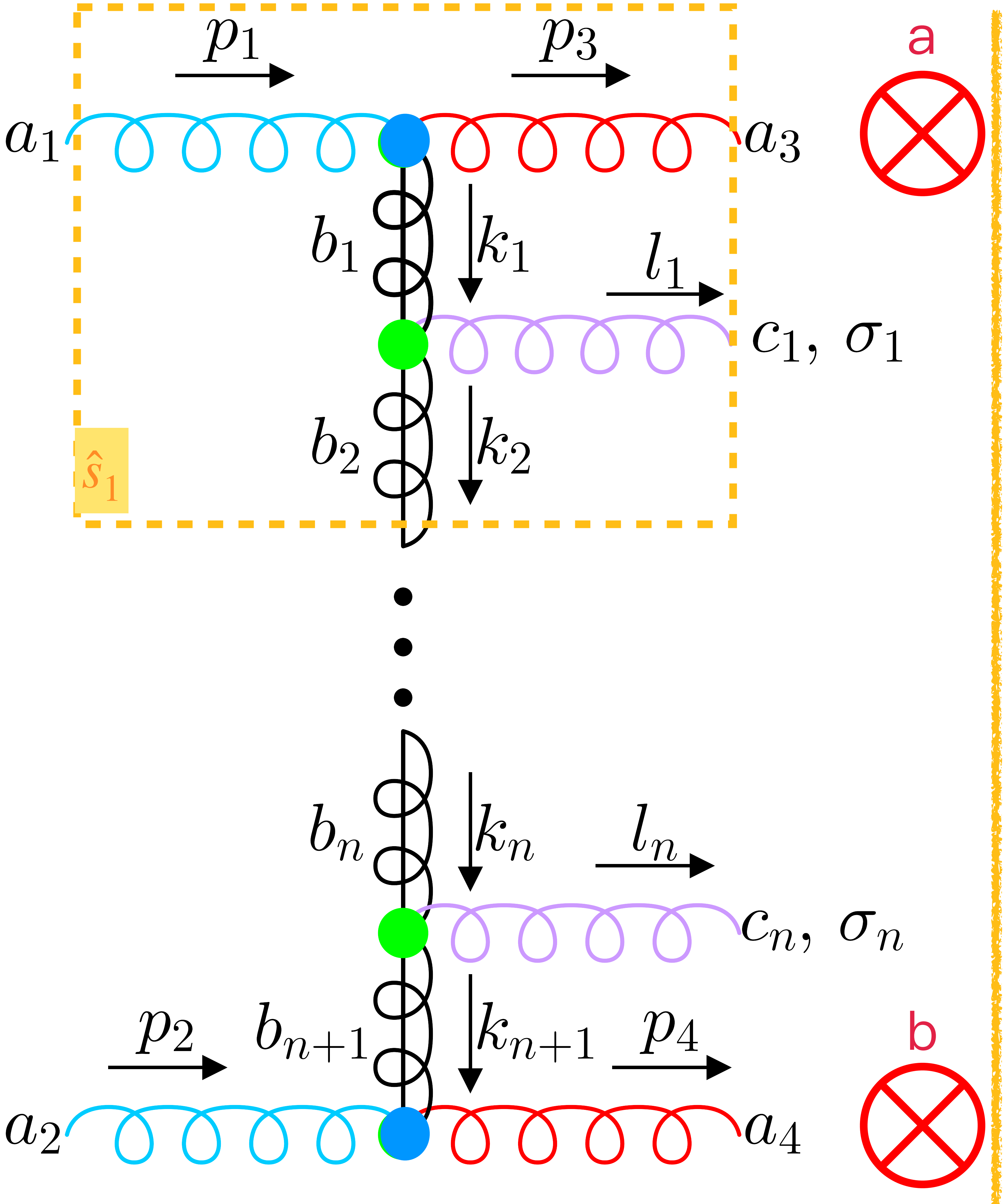


Corresponding to the Regge limit in the context of EEC

Long–range enhancement on the near side

Amplitude in Multi Regge Limit

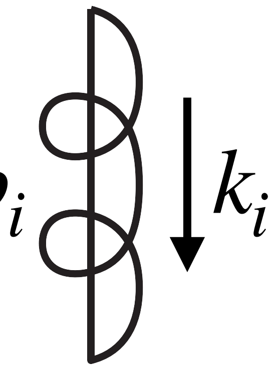
Leading Logarithm Approximation



- $k_i^\mu = \rho_i p_1^\mu + \lambda_i p_2^\mu + k_{i\perp}^\mu, \quad \rho_0 \equiv 1 \gg \rho_1 \gg \cdots \gg \rho_{n+1}, \quad |\lambda_{n+2}| \equiv 1 \gg |\lambda_{n+1}| \gg \cdots \gg |\lambda_1|$
- On shell condition: $\left(\vec{k}_{i\perp} - \vec{k}_{i+1\perp}\right)^2 \approx -\rho_i \lambda_{i+1} s, \quad \vec{k}_{1\perp}^2 \approx -\lambda_1 s, \quad \vec{k}_{n+1\perp}^2 \approx \rho_{n+1} s$
- $\dfrac{1-y_3}{1+y_3} = \dfrac{-\lambda_1}{1-\rho_1} \approx -\lambda_1, \quad \dfrac{1+y_4}{1-y_4} = \dfrac{\rho_{n+1}}{1+\lambda_{n+1}} \approx \rho_{n+1}$ ⇒ k_1, k_{n+1} are fixed by detector!

- ● : eikonal vertex (polarization dotted in) $2gf_{a_1a_3b_1}\eta^{h_1h_3}p_1^\mu,\ 2gf_{a_2a_4b_{n+1}}\eta^{h_2h_4}p_2^\mu$
- ● : Lipatov's effective vertex

$$\Gamma_i^\mu(k_i, k_{i+1}) = \frac{1}{\lambda_i \rho_{i+1}}\left(\left(\rho_i + \frac{2\lambda_i}{\lambda_{i+1}}\right)p_1^\mu - \left(\lambda_{i+1} + \frac{2\rho_{i+1}}{\rho_i}\right)p_2^\mu - k_{i\perp}^\mu + k_{i+1\perp}^\mu\right)$$

- b_i  k_i : reggeized gluon $\tilde{D}_{\mu\nu}(\hat{s}_i, k_i^2) = \dfrac{ig_{\mu\nu}}{\vec{k}_{i\perp}^2}\left(\dfrac{\hat{s}_i}{\vec{k}_{i\perp}^2}\right)^{\epsilon_G(k_i^2)} = \dfrac{ig_{\mu\nu}}{\vec{k}_{i\perp}^2}\left(\dfrac{\rho_{i-1}}{\rho_i}\right)^{\epsilon_G(k_i^2)}$

$$\epsilon_G(k_i^2) \equiv -\frac{Ng^2}{16\pi^3}\int d^2\vec{k}\frac{\vec{k}_i^2}{\left(\vec{k}_i - \vec{k}\right)^2\vec{k}^2} = -\frac{Ng^2}{16\pi^3}\int d^2\vec{k}\frac{2k_i^2}{\left(\vec{k}_i - \vec{k}\right)^2\left(k^2 + \left(\vec{k}_i - \vec{k}\right)^2\right)}$$

$$\overline{\sum |M_{2\to 2+n}|^2} = \frac{Ng^2(N^2-1)s}{64}\left(\frac{-4Ng^2}{s}\right)^{n+1}\prod_{i=1}^{n+1}\frac{1}{\rho_i\lambda_i}\left(\frac{\rho_{i-1}}{\rho_i}\right)^{\epsilon_G(k_i^2)}$$

EEC in Multi Regge Limit

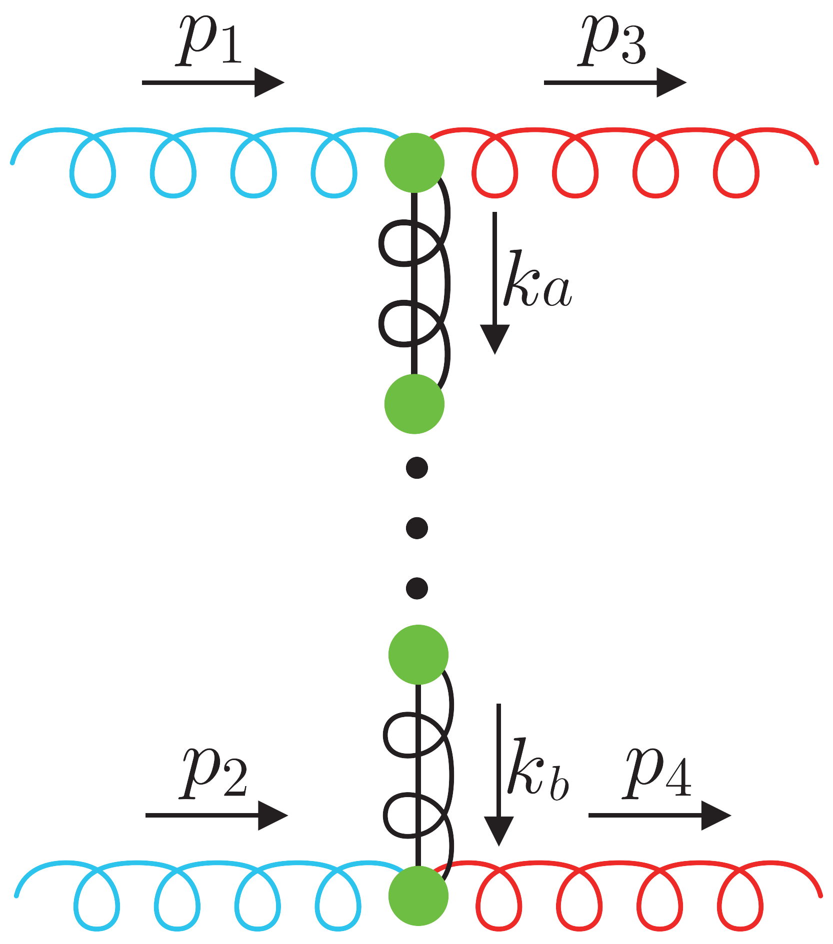
Leading Logarithm Approximation

- Phase Space Integral:

$$\int d\Pi_{n+2}\delta^{(2)}(\Omega_a-\Omega_3)\delta^{(2)}(\Omega_b-\Omega_4) \approx \frac{s}{8(4\pi)^2}\prod_{i=2}^{n+1}\int\frac{d^2\vec{k}_{i\perp}}{2(2\pi)^3}\prod_{i=1}^{n}\int_{\rho_{i+1}}^{1}\frac{d\rho_i}{\rho_i}\int_0^1 d\rho_{n+1}\,\delta\left(\rho_{n+1}-\frac{1+y_4}{1-y_4}\right)\delta^{(2)}(\vec{k}_b-\vec{k}_{n+1})$$

$s/\vec{k}_b^2$

nested integral



- $2 \to 2+n$ EEC:

$$\frac{d^2\Sigma_n}{d\Omega_a d\Omega_b} = \frac{1}{2s}\int d\Pi_{n+2}\overline{\sum |M_{2\to 2+n}|^2} p_3^0 p_4^0 \delta^{(2)}(\Omega_a-\Omega_3)\delta^{(2)}(\Omega_b-\Omega_4)$$

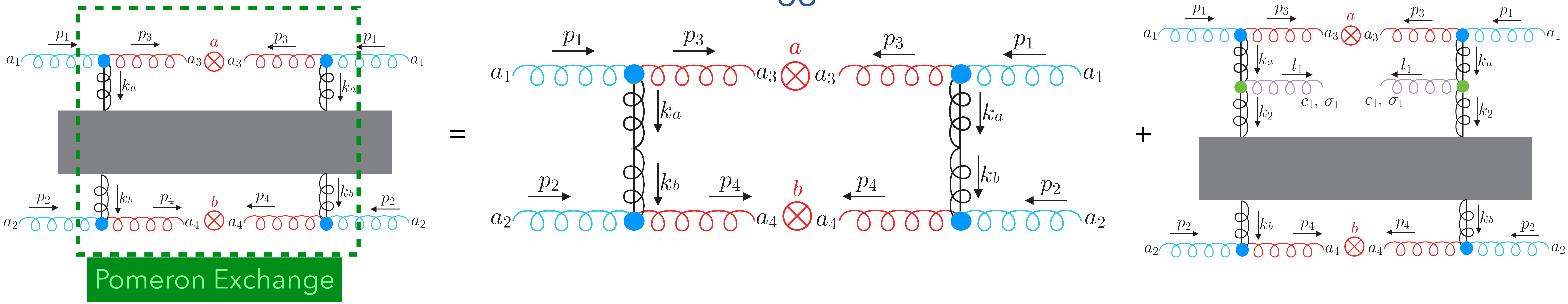
$$\equiv \tilde{f}_n(\vec{k}_a,\vec{k}_b,s) = f_n\left(\eta=\frac{k_b}{k_a}, \Delta_\eta=\frac{s}{k_a k_b}, \phi=\phi_a-\phi_b-\pi\right)$$

- EEC in Mellin Space:

$$F_n(\vec{k}_a,\vec{k}_b,\omega) = \int_1^\infty d\left(\frac{s}{k_b^2}\right)\left(\frac{s}{k_b^2}\right)^{-\omega-1}\frac{\tilde{f}_n(\vec{k}_a,\vec{k}_b,s)}{s^3} \quad \longleftrightarrow \quad \frac{\tilde{f}(\vec{k}_a,\vec{k}_b,s)}{s^3} = \frac{1}{2\pi i}\int_C d\omega\left(\frac{s}{k_b^2}\right)^{\omega} F(\vec{k}_a,\vec{k}_b,\omega)$$

Evolution Equation for EEC

in the Regge limit



$$\omega F(\vec{k}_a,\vec{k}_b,\omega) = \frac{Ng^2(N^2-1)\left(4Ng^2\right)}{(256\pi)^2 k_a^2 k_b^2}\delta^{(2)}(\vec{k}_a-\vec{k}_b) + 4Ng^2\int \frac{d^2\vec{k}_2}{2(2\pi)^3}\frac{1}{(\vec{k}_2-\vec{k}_a)^2}\left[\frac{k_2^2}{k_a^2}F(\vec{k}_2,\vec{k}_b,\omega) - \frac{k_a^2}{k_2^2+(\vec{k}_2-\vec{k}_a)^2}F(\vec{k}_a,\vec{k}_b,\omega)\right]$$

Vanish as $\vec{k}_2 \to \vec{k}_a$, manifestly convergent

Redefine: $G(\vec{k}_a,\vec{k}_b,\omega) \equiv \frac{(256\pi)^2 k_a^2 k_b^2}{4N^2(N^2-1)g^4}F(\vec{k}_a,\vec{k}_b,\omega)$

$$\omega G(\vec{k}_a,\vec{k}_b,\omega) = \delta^{(2)}(\vec{k}_a-\vec{k}_b) + \frac{Ng^2}{4\pi^3}\int \frac{d^2\vec{k}_2}{(\vec{k}_a-\vec{k}_2)^2}\left[G(\vec{k}_2,\vec{k}_b,\omega) - \frac{k_a^2}{k_2^2+(\vec{k}_2-\vec{k}_a)^2}G(\vec{k}_a,\vec{k}_b,\omega)\right]$$

LL BFKL equation with zero momentum transfer

NNLO LL EEC in the Regge limit

iterative solution to the BFKL equation

$$G(\omega, \vec{k}_a, \vec{k}_b) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} d\nu \left(\frac{k_a^2}{k_b^2}\right)^{i\nu} \frac{e^{in(\phi_a - \phi_b)}}{2\pi^2 k_a k_b} \frac{1}{\omega - \bar{\alpha}_s \chi_n(\nu)}$$

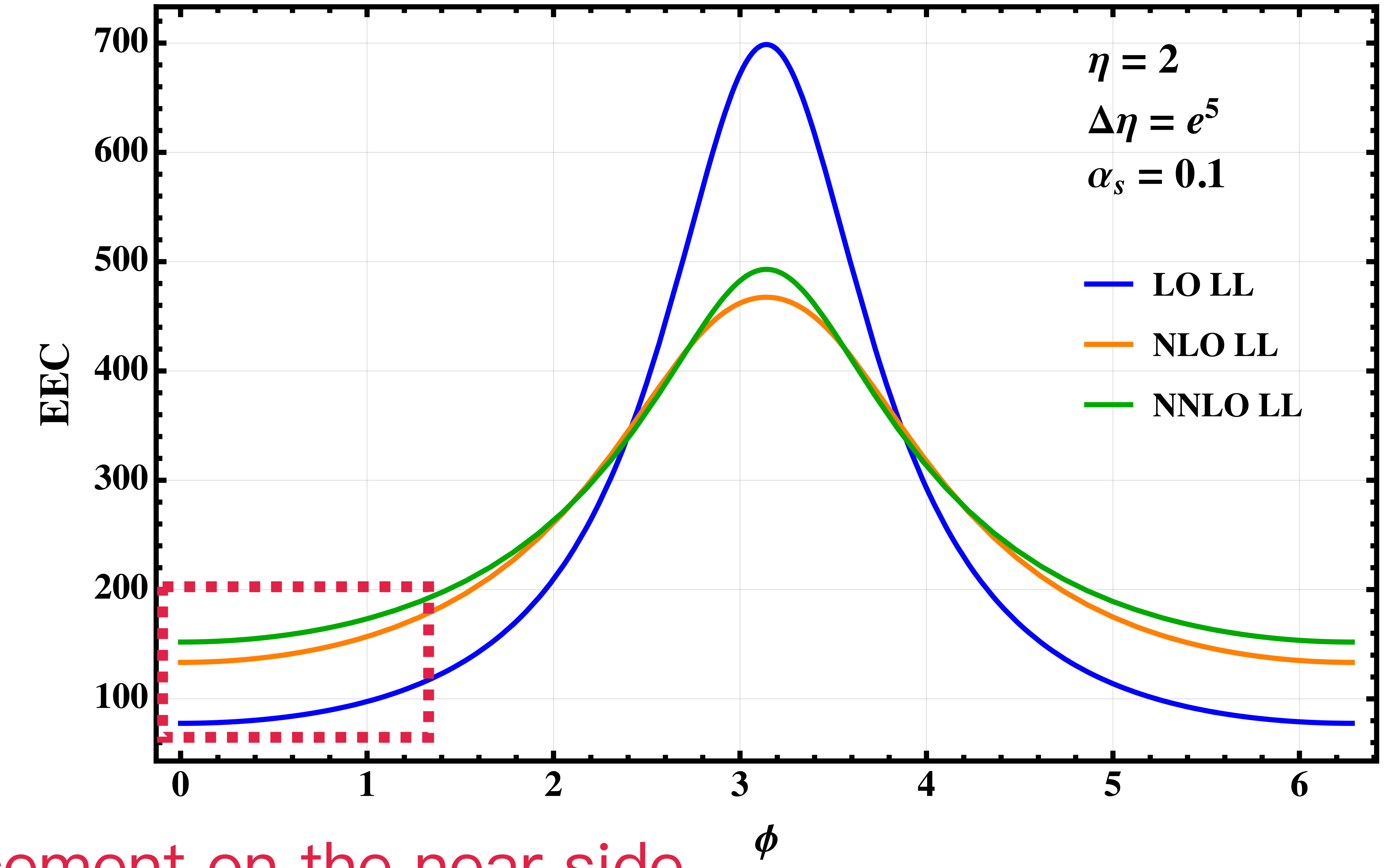
$$\bar{\alpha}_s = \frac{Ng^2}{4\pi^2}, \quad \chi_n(\nu) = 2\left(-\gamma_E - \mathscr{Re}[\psi((n+1)/2 + i\nu)]\right)$$

decrease with increasing n

To see the ϕ dependence, we use iterative solution up to N3LO and find that:

$$f(\eta, \Delta\eta, \phi) = \frac{27 g^6 \Delta\eta^3 \ln \Delta\eta}{8192 \pi^5 \tau}$$
$$\times \left(1 + \frac{3g^2 \ln \Delta\eta}{4\pi^2} \ln \tau + \frac{1}{2}\left(\frac{3g^2 \ln \Delta\eta}{4\pi^2}\right)^2 \left(\ln^2 \tau - \frac{1}{3} \ln^2 \eta\right)\right)$$

with $\tau \equiv \dfrac{1 + 2\eta\cos(\phi) + \eta^2}{\eta}$.



enhancement on the near side

Celestial Block

For Hadron Colliders

Conformal and Lorentz Symmetry

From 4D Minkowski to Celestial Sphere

Lorentz

$$L_{\mu\nu} = M_{\mu\nu}$$

$$L_{-1,0} = D$$

$$L_{0,\mu} = \frac{1}{2}(P_\mu + K_\mu)$$

$$L_{-1,\mu} = \frac{1}{2}(P_\mu - K_\mu)$$

Conformal

$$M^{03} = -\sin\theta\,\partial_\theta,\ \cdots$$

$$M^{12} = \partial_\phi,\ \cdots$$

Celestial Blocks for Hadron Colliders

solving the Casimir equation

Embedding Formalism

Lorentz Symmetry in 4d Minkowski Space ⟷ Conformal Symmetry on 2d Celestial Sphere

EEC at hadron colliders has the general form:

$$\frac{\langle P_1 P_2 | \mathcal{E}(n_a)\mathcal{E}(n_b) | P_1 P_2 \rangle}{\langle P_1 P_2 | P_1 P_2 \rangle} = \frac{P_1 \cdot P_2}{(n_a \cdot n_b)^3} F(u, v, w)$$

(Projective null vector)

$$n^2 = 0, \quad n \sim \lambda n$$

$$u = \frac{(n_a \cdot n_b)(n_1 \cdot n_2)}{(n_1 \cdot n_a)(n_2 \cdot n_b)}, \quad v = \frac{(n_2 \cdot n_a)(n_1 \cdot n_b)}{(n_1 \cdot n_a)(n_2 \cdot n_b)}, \quad w = \frac{P_1 \cdot n_a}{P_2 \cdot n_a}$$

Breaking the boost symmetry along the beam direction

Casimir Equation:

$$\left[(1-z)z^2\partial_z^2 + (w\partial_w - 1)z^2\partial_z + (z \leftrightarrow \bar{z})\right] F_{h,\bar{h}}(z,\bar{z},w) = \left(h(h-1) + \bar{h}(\bar{h}-1)\right) F_{h,\bar{h}}(z,\bar{z},w)$$

Solution

$$F(z,\bar{z},w) = \sum_{\delta,j} \int \frac{d\gamma}{2\pi i} c_{\delta,j,\gamma}\, w^\gamma G^{(\gamma)}_{\delta,j}(z,\bar{z})$$

2d conformal block:

$$\left[z^h\, {}_2F_1(h, h-\gamma; 2h; z)\right]\left[\bar{z}^{\bar{h}}\, {}_2F_1(\bar{h}, \bar{h}-\gamma; 2\bar{h}; \bar{z})\right] + (z \leftrightarrow \bar{z})$$

Celestial Block Decomposition

Applied to the Result of the EEC

$$\frac{d^2\Sigma}{d\Omega_a d\Omega_b} = -\frac{27g^6}{2(8\pi)^5(n_a\cdot n_b)^3}\Bigg[\Bigg[\int\frac{d\gamma}{2\pi i}\,\pi\csc(\pi\gamma)\frac{14\gamma(\gamma^4+55\gamma^2+304)}{75}w_p^\gamma G_{4,0}^{(\gamma)}(z,\bar z) - \frac{112}{5}G_{4,0}^{(0)}(z,\bar z)$$

twist–2, transverse spin–0

$$+\int\frac{d\gamma}{2\pi i}\,\pi\csc(\pi\gamma)\frac{\gamma(122\gamma^6+8897\gamma^4+157493\gamma^2+612168)}{11025}w_p^\gamma G_{6,0}^{(\gamma)}(z,\bar z) - \frac{664}{35}G_{6,0}^{(0)}(z,\bar z)$$

twist–4, transverse spin–0

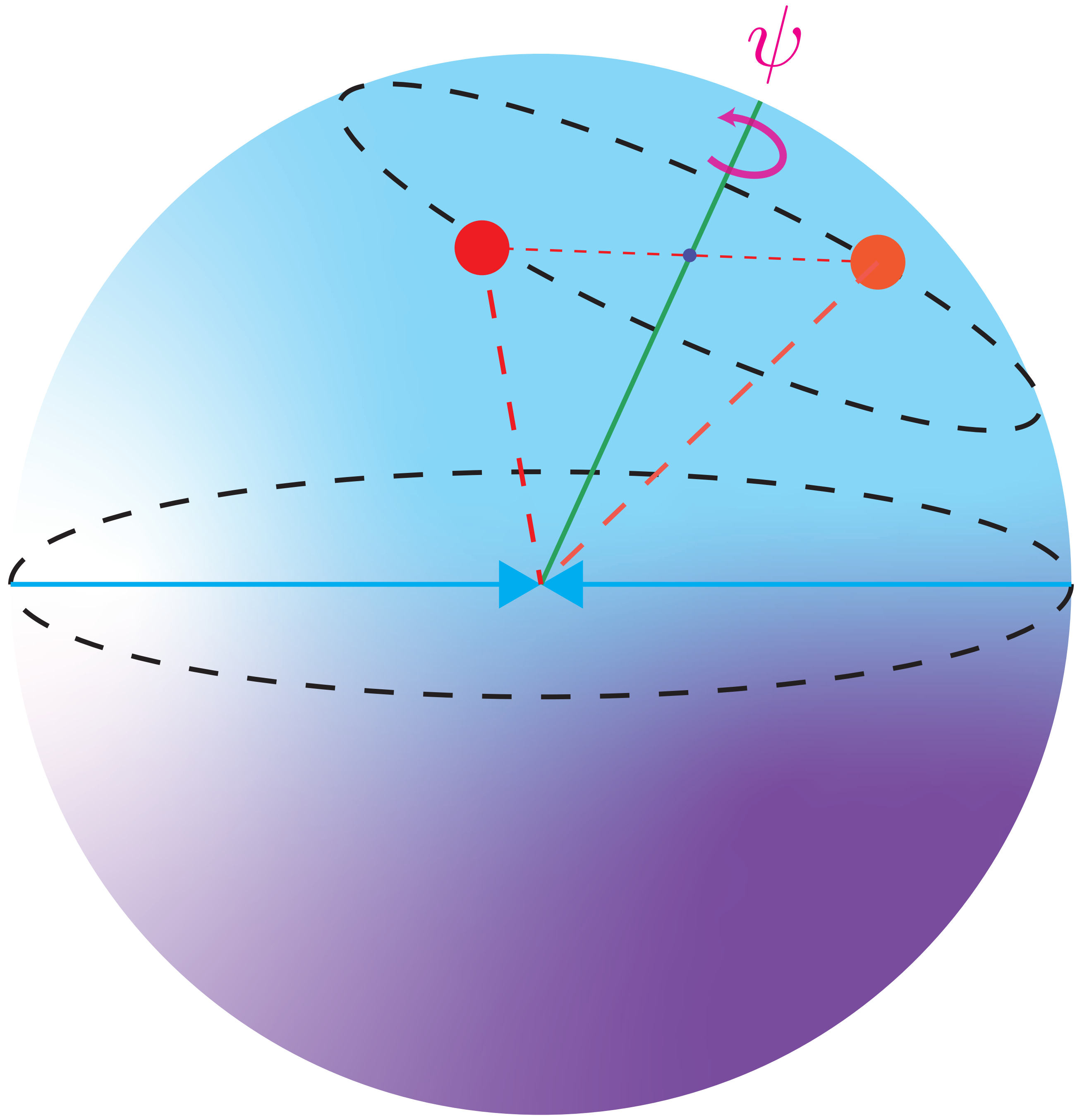
$$+\int\frac{d\gamma}{2\pi i}\,\pi\csc(\pi\gamma)\frac{\gamma(641\gamma^6+50456\gamma^4+875819\gamma^2+2994204)}{110250}w_p^\gamma G_{6,2}^{(\gamma)}(z,\bar z) - \frac{1544}{175}G_{6,2}^{(0)}(z,\bar z)$$

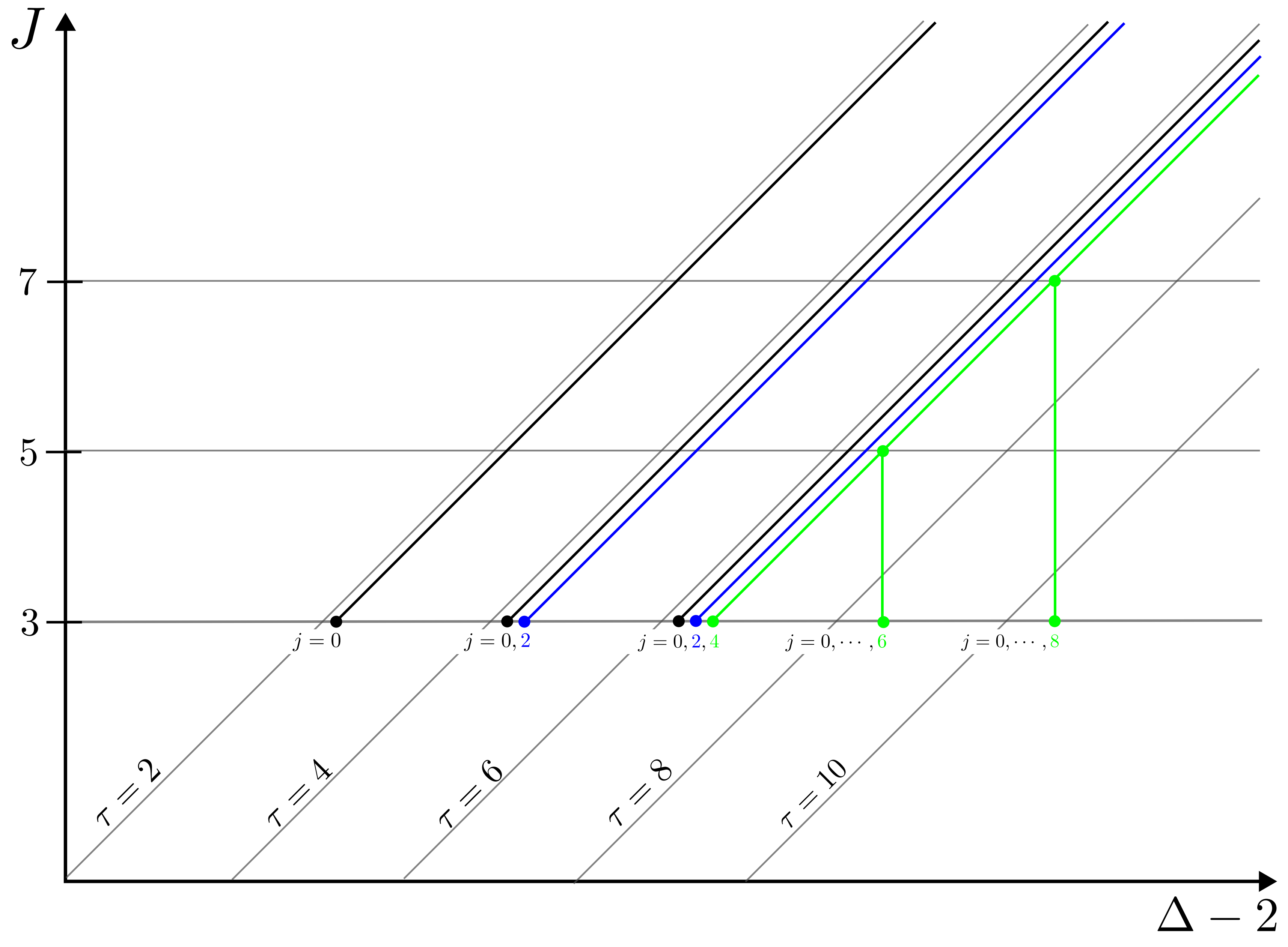
twist–4, transverse spin–2

$$+\cdots\Bigg]$$

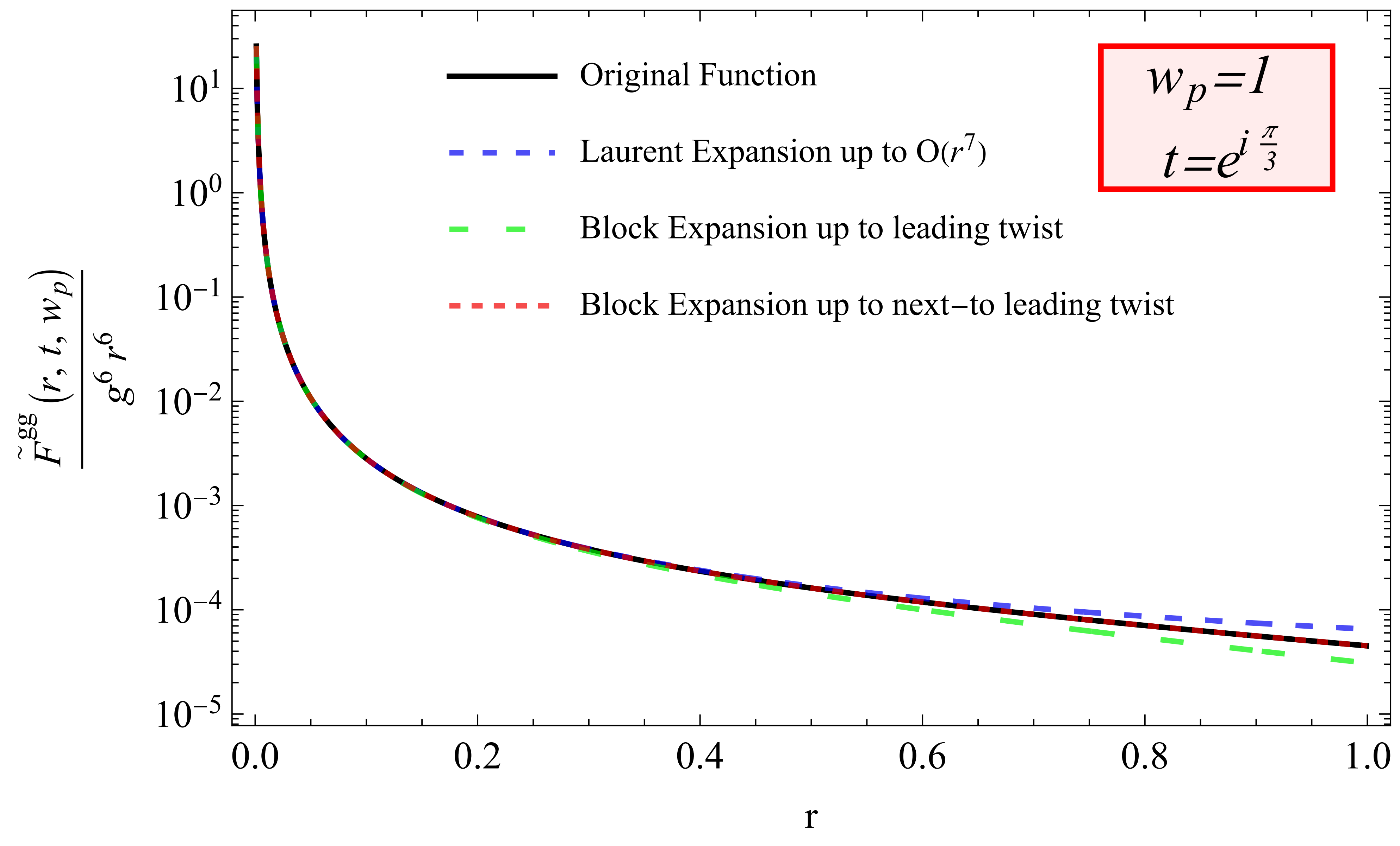
higher twist

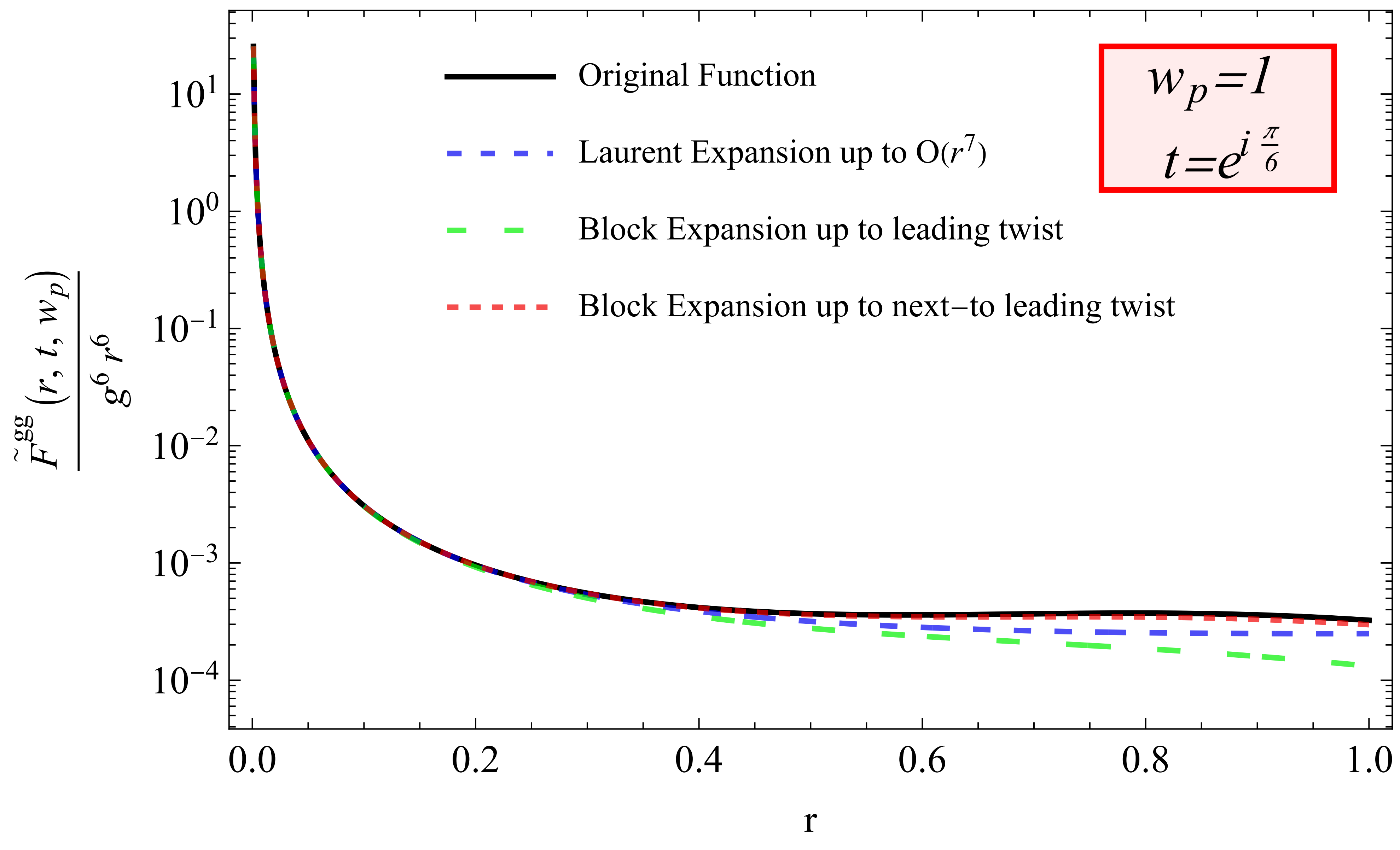
Light–ray OPE:

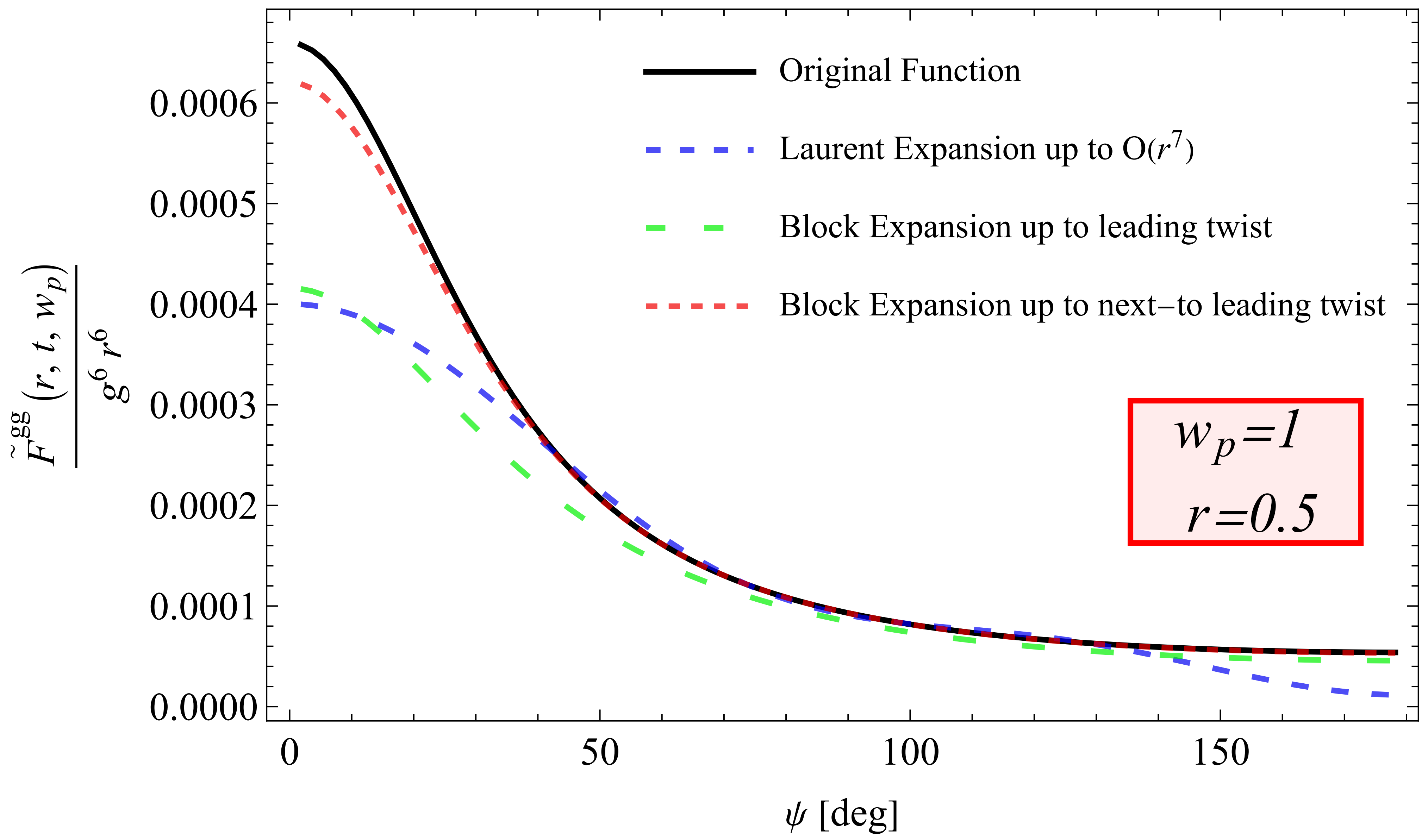


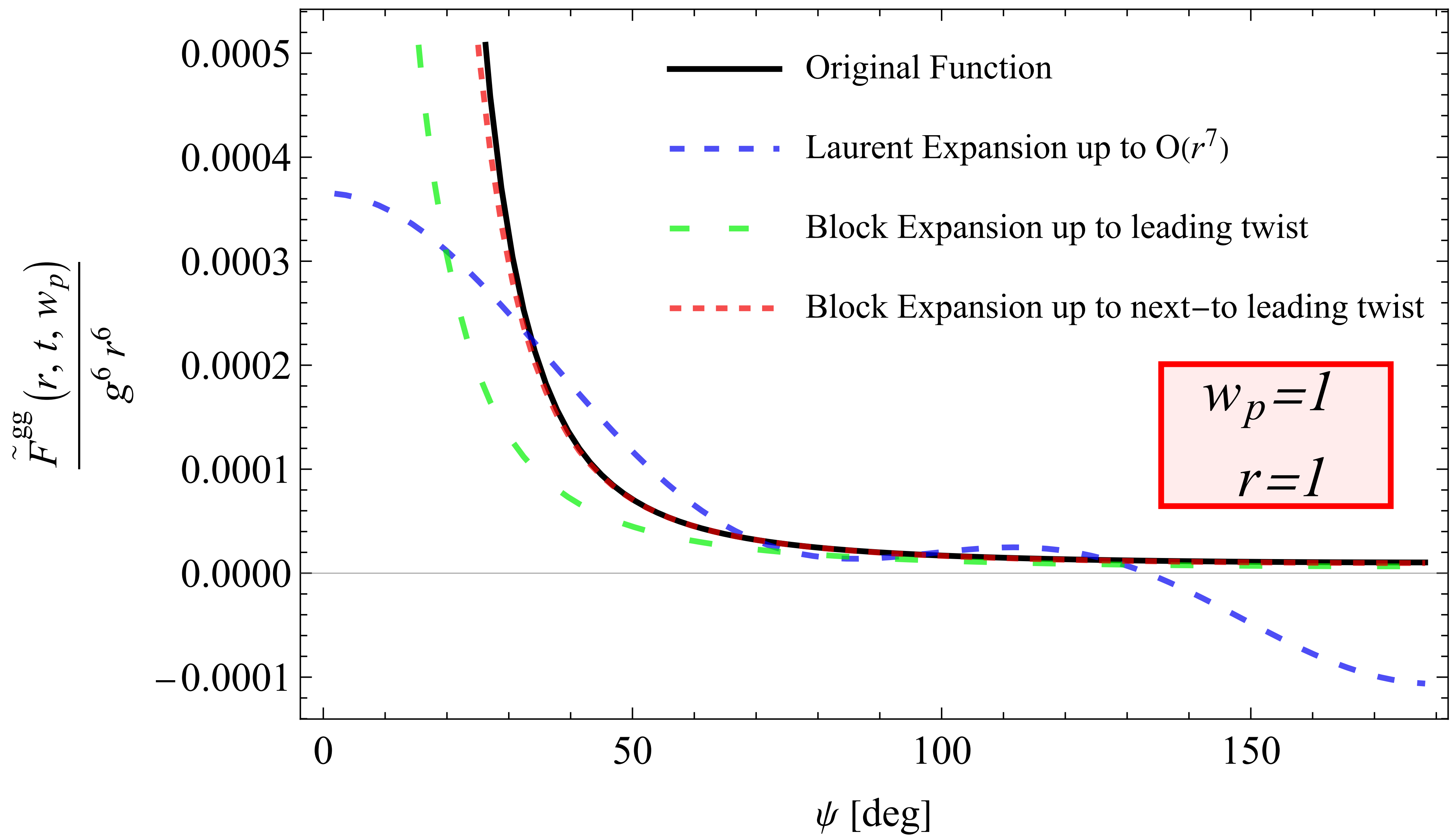


Block Expansion vs Series Expansion



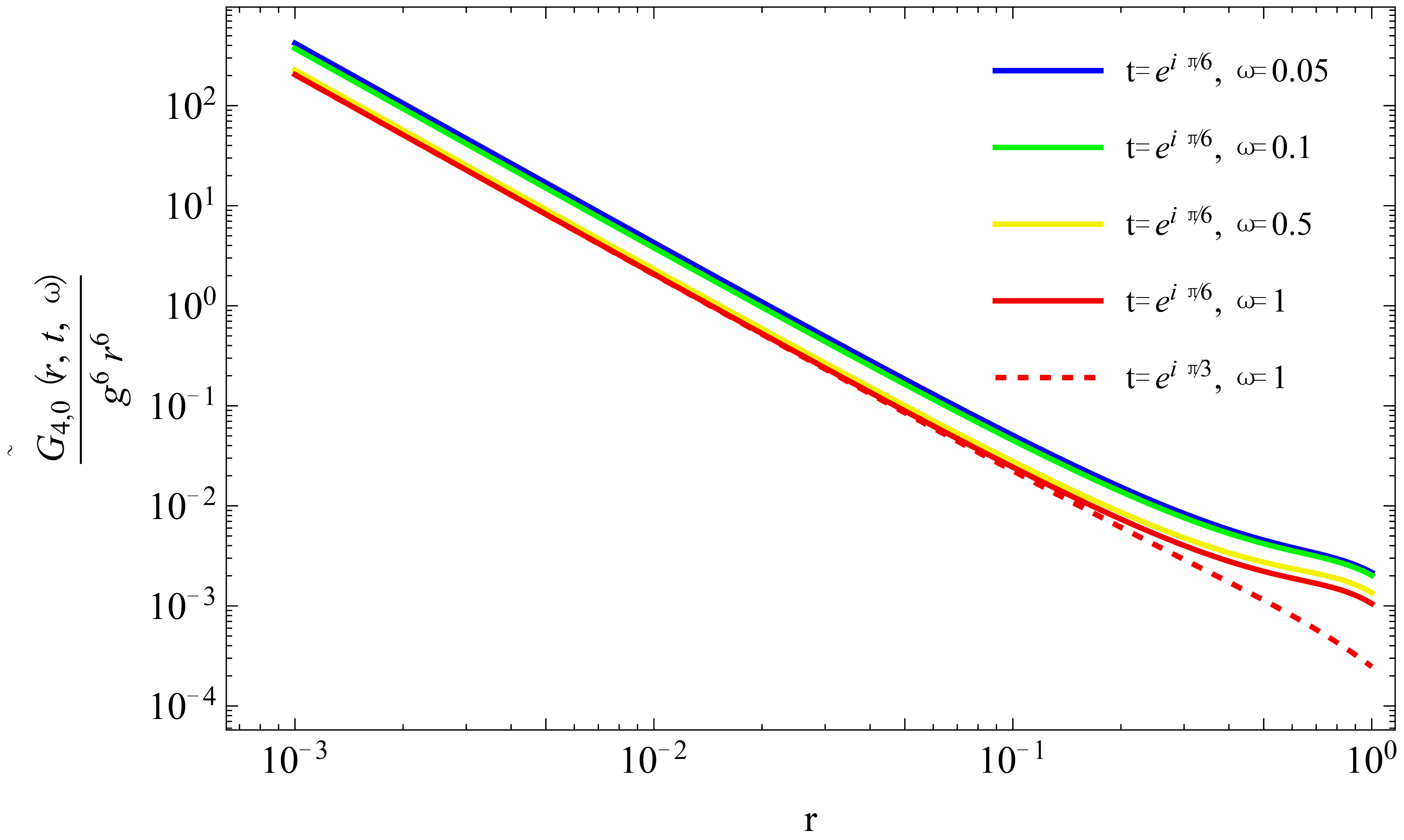


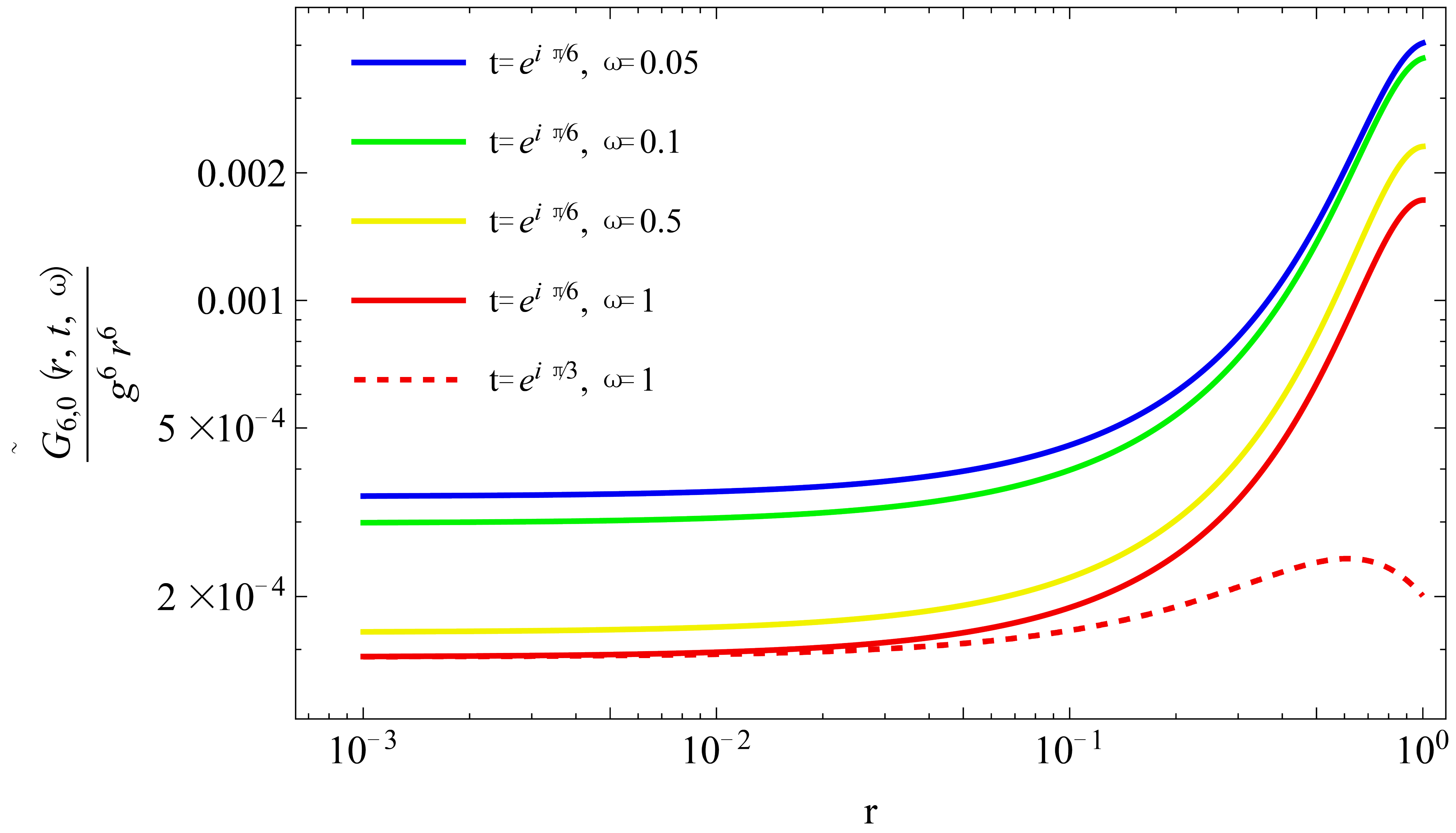


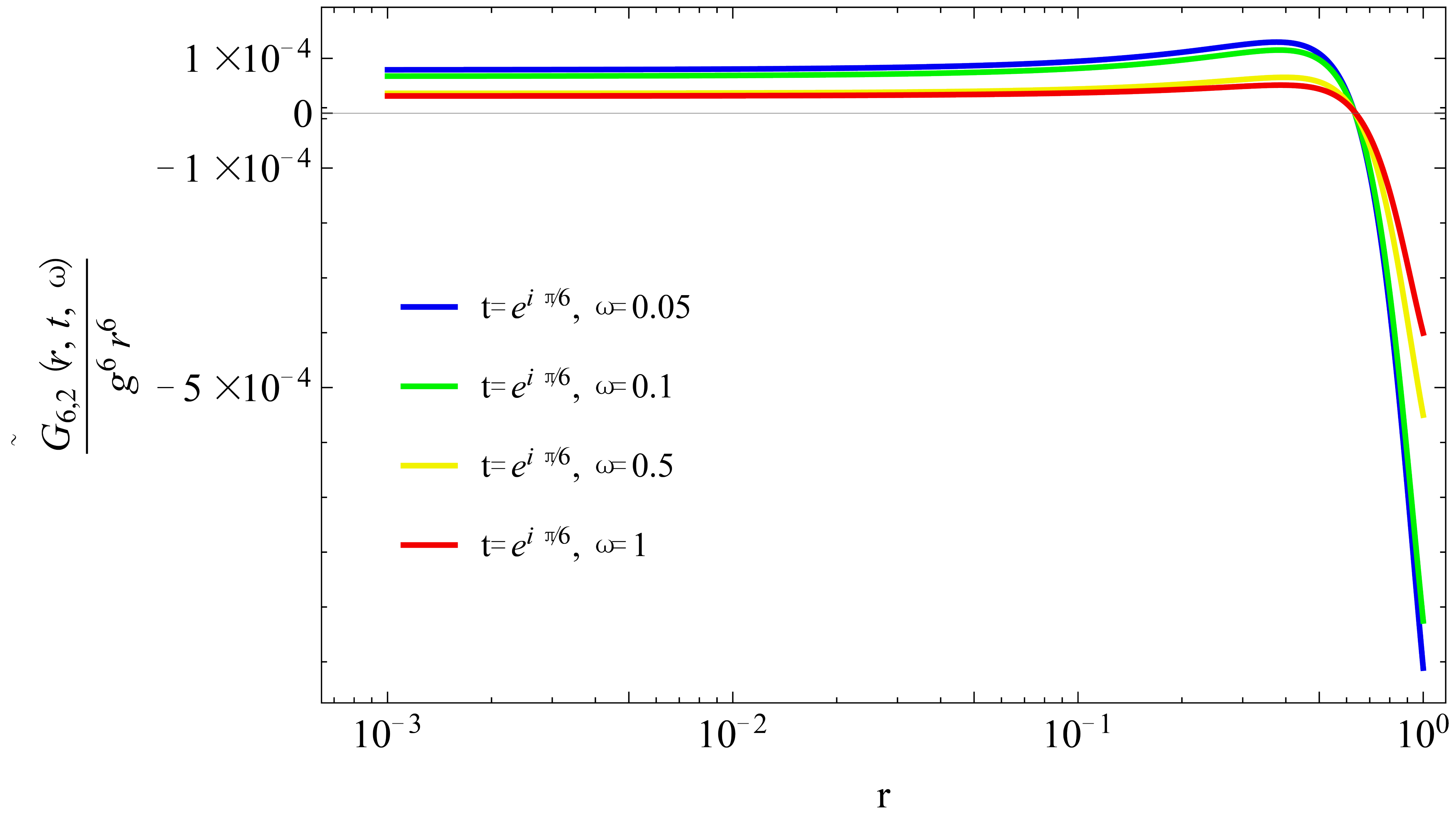


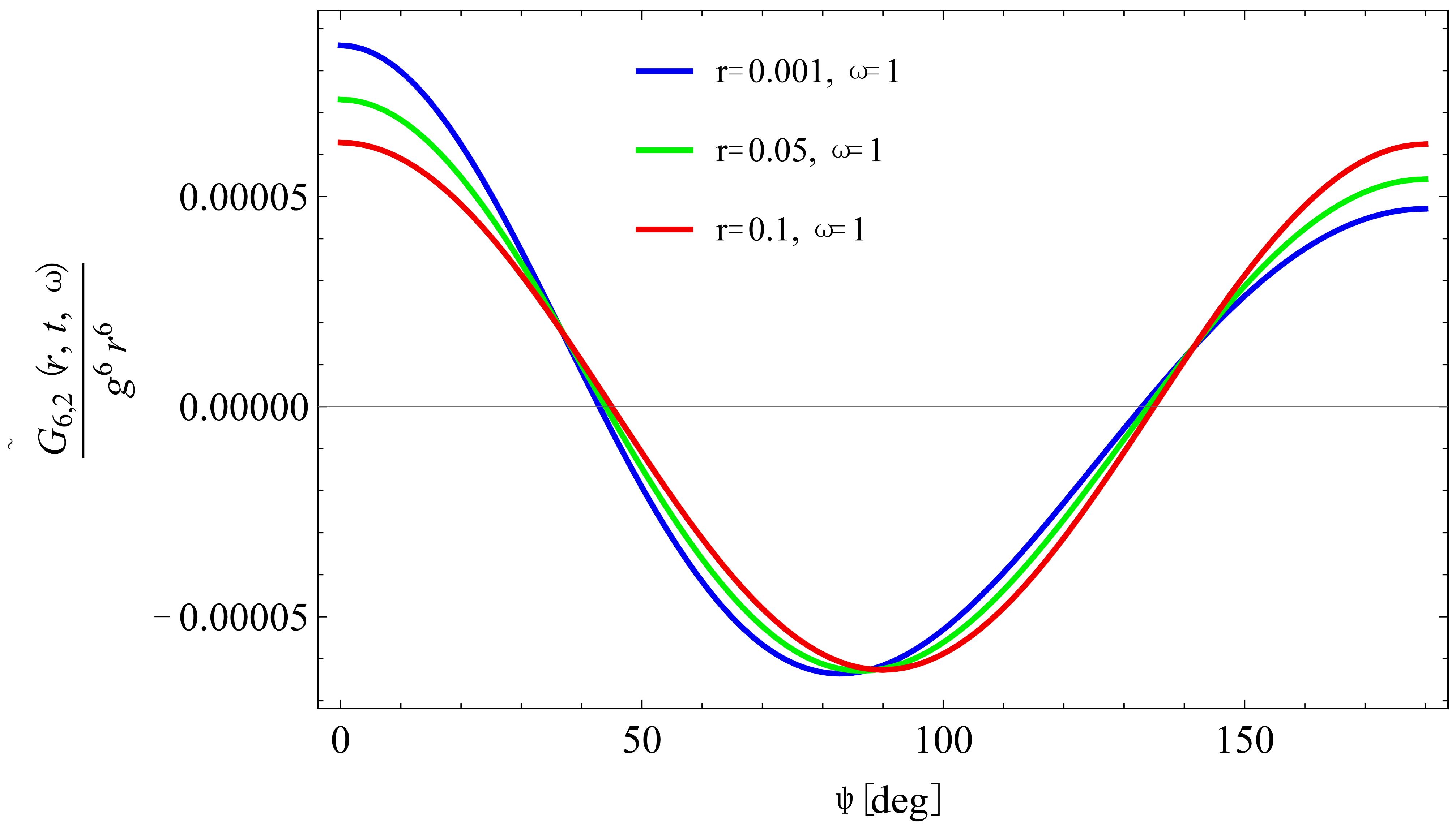
Celestial Block Dependence

A Window to Twist-4 Operators









Analyticity in Spin

collinear and transverse

Analyticity in Collinear Spin

even– and odd–signature

For 2–to–2 scattering: $\mathscr{A}(s,t) = \frac{1}{2i}\oint_C dJ \frac{2J+1}{\sin \pi J} \sum_{\eta=\pm 1} \frac{\eta + e^{-i\pi J}}{2} a^{(\eta)}(J,t) P(J, 1+2s/t)$

partial wave decomposition at amplitude level

Recall the celestial block expansion: $F(z,\bar{z},w) = \sum_{\delta,j} \int \frac{d\gamma}{2\pi i} c_{\delta,j,\gamma}\, w^{\gamma}\, G^{(\gamma)}_{\delta,j}(z,\bar{z})$

partial wave decomposition at cross–section level

γ: collinear spin difference between interfering partial waves.

Applying collinear boost $\Lambda_Y = e^{-iY\hat{z}\cdot\vec{\mathbf{K}}}$:

$$\langle P_1P_2|\Lambda_Y^{-1}|\Psi^{(i_1)}_{J_1}\rangle \ldots \langle \Psi^{(i_2)}_{J_2}|\Lambda_Y|P_1P_2\rangle = e^{-(J_1-J_2)Y}\langle P_1P_2|\Psi^{(i_1)}_{J_1}\rangle \ldots \langle \Psi^{(i_2)}_{J_2}|P_1P_2\rangle$$

collinear spin

$F(z,\bar{z},w) \rightarrow F(z,\bar{z},e^{-2Y}w)$ ➡ $\gamma = \frac{J_1 - J_2}{2}$

$$\tilde{c}^{gg}_{\delta,j,\gamma} = \frac{\pi\gamma}{\sin(\pi\gamma)} P(\gamma) - \# \cdot i\pi\delta(\gamma)$$

integer pole in γ

⬇

no interference between odd and even collinear spins!

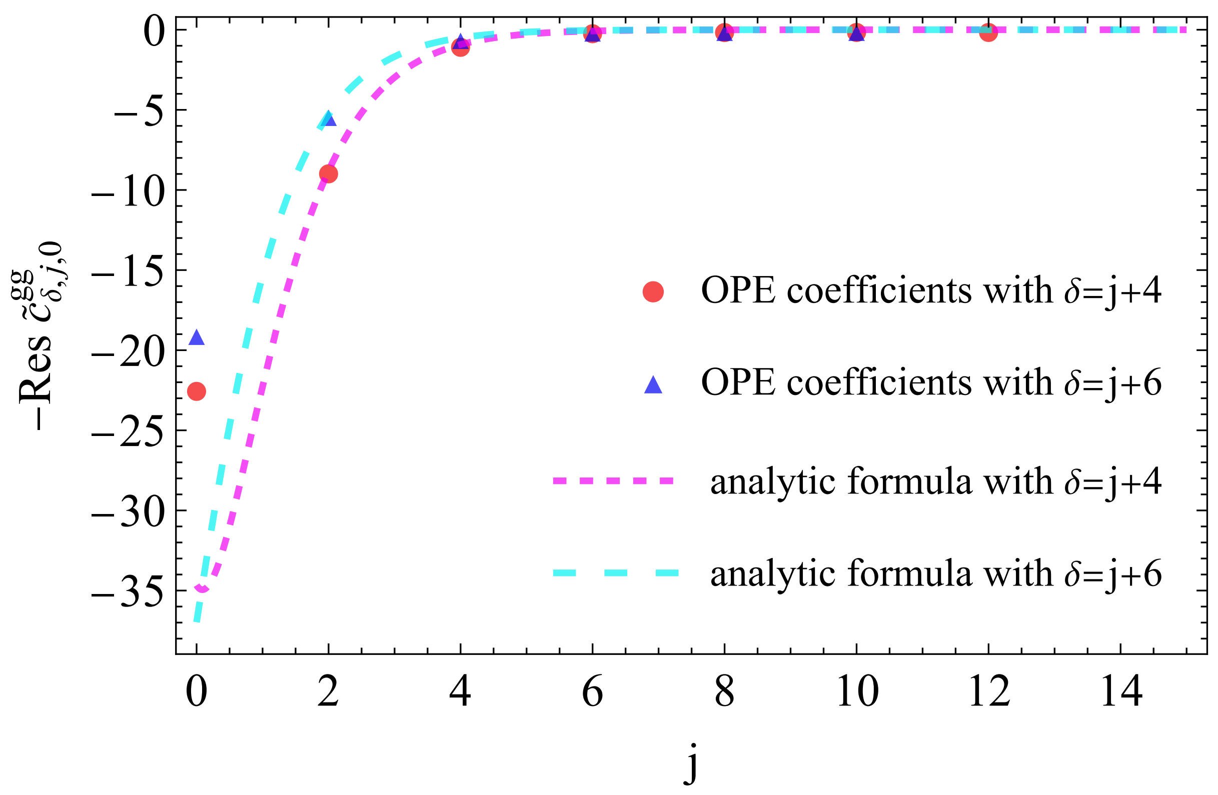
Analyticity in Transverse Spin

Lorentzian inversion formula

[Caron–Huot, 2018]

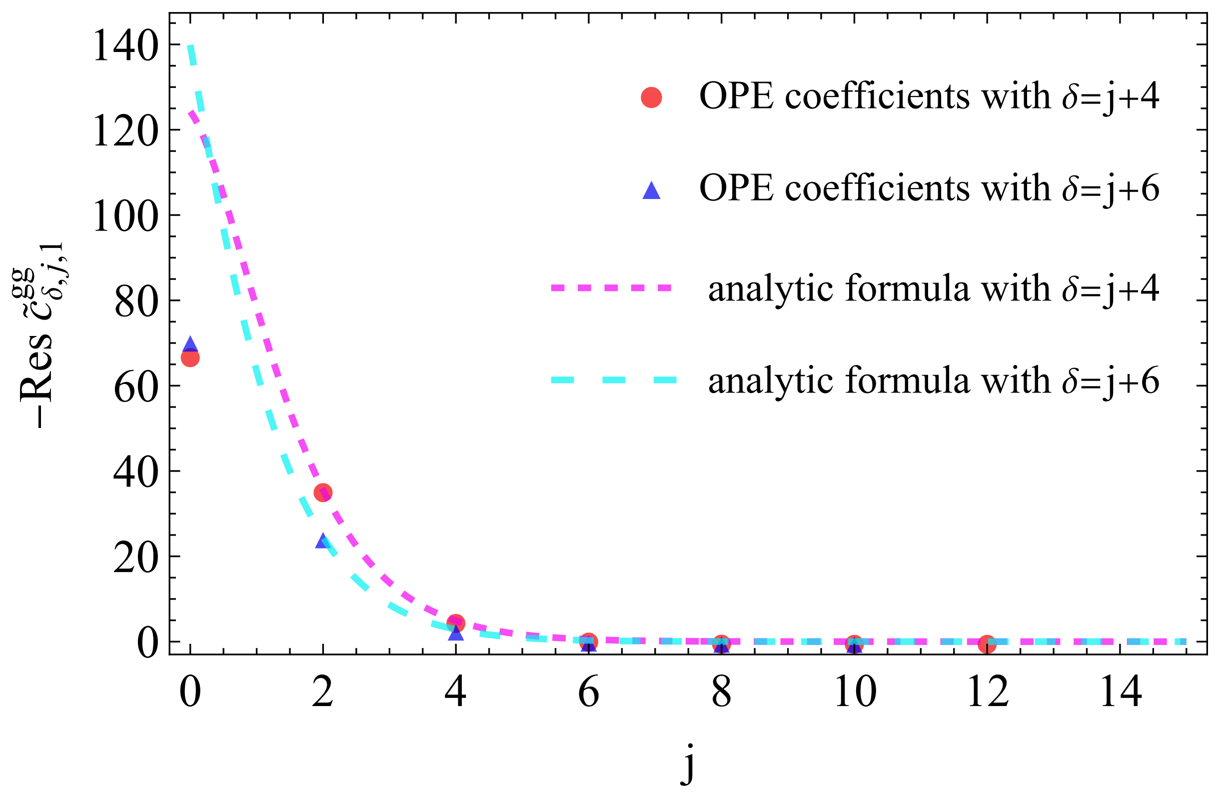
$$F(z,\bar{z},w) = \sum_{m=0}^{\infty} w^m \sum_{\delta,j} \left[-\underset{\gamma=m}{\mathrm{Res}}\, c_{\delta,j,\gamma} \right] G^{(m)}_{\delta,j}(z,\bar{z})$$

→ Analytic function of transverse spin j



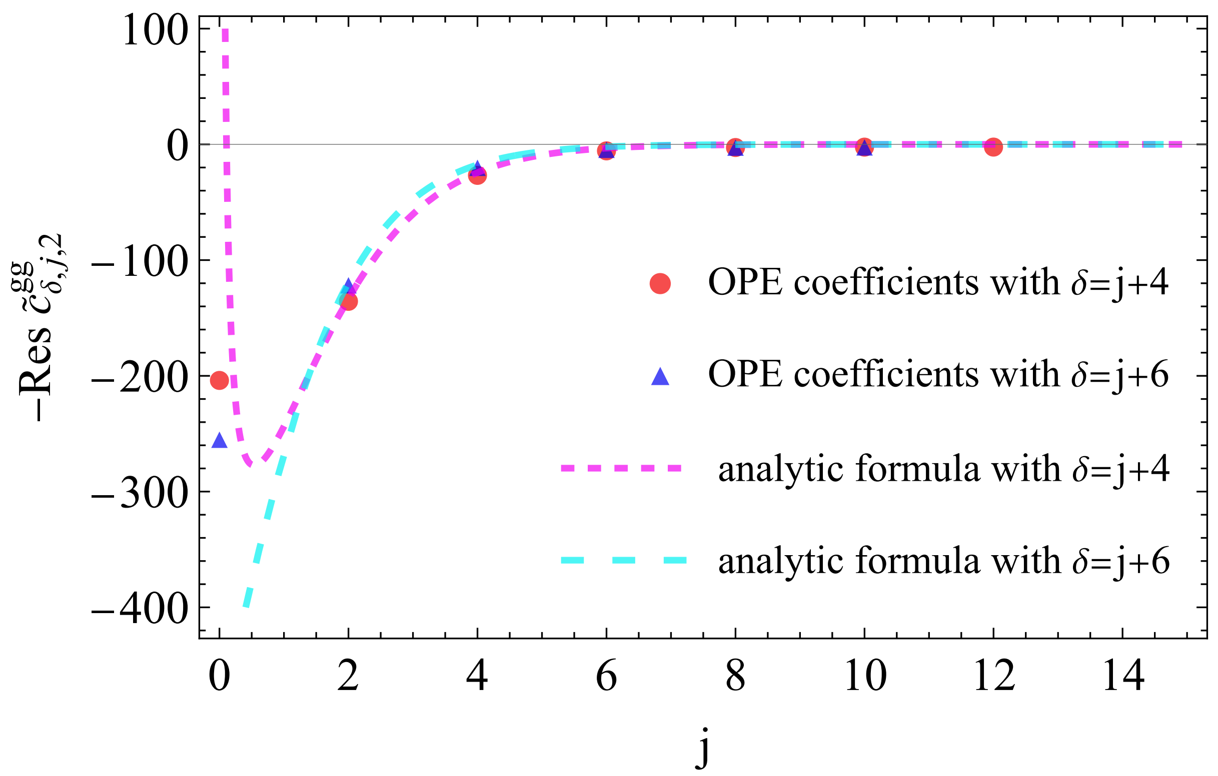
$$-\underset{\gamma=0}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+4,j,\gamma}\Big|_{j\geq 4} = -\frac{16}{3}\frac{\Gamma(j+2)^2}{\Gamma(2j+3)}(24H_{j+1}-11)$$

$$-\underset{\gamma=0}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+6,j,\gamma}\Big|_{j\geq 4} = -\frac{128}{15}\frac{\Gamma(j+3)^2}{\Gamma(2j+5)}(30H_{j+2}-19)$$



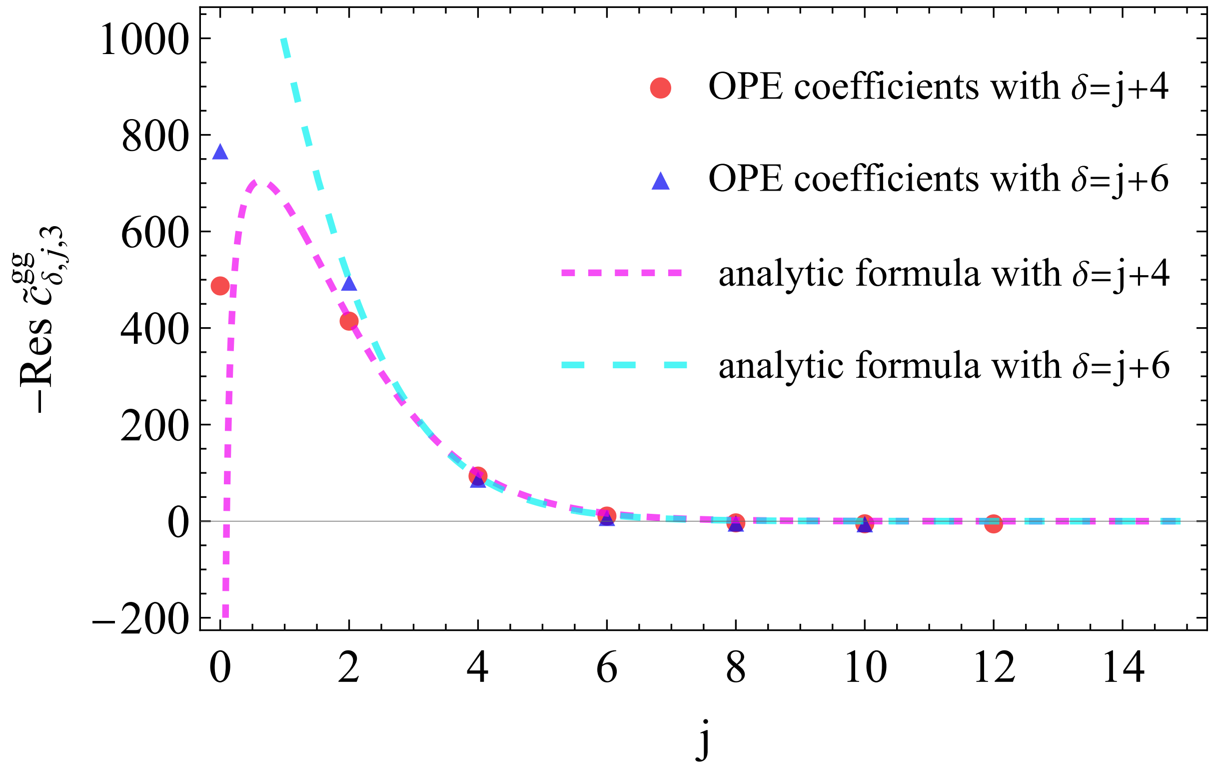
$$-\underset{\gamma=1}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+4,j,\gamma}\Big|_{j\geq 4} = \frac{8}{15}\frac{\Gamma(j+2)\Gamma(j+3)}{\Gamma(2j+3)}\left(120H_{j+1}+113\right)$$

$$-\underset{\gamma=1}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+6,j,\gamma}\Big|_{j\geq 4} = \frac{4}{5}\frac{\Gamma(j+3)\Gamma(j+4)}{\Gamma(2j+5)}\left(120H_{j+2}-\frac{80}{j+2}+\frac{80}{j+3}+183\right)$$



$$-\underset{\gamma=2}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+4,j,\gamma}\Big|_{j\geq 4} = \frac{\Gamma(j+2)\Gamma(j+4)}{\Gamma(2j+3)}\left(-32H_{j+1}+\frac{48}{j(j+3)}-\frac{80}{(j+1)(j+2)}-\frac{1088}{15}\right)$$

$$-\underset{\gamma=2}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+6,j,\gamma}\Big|_{j\geq 4} = \frac{\Gamma(j+3)\Gamma(j+5)}{\Gamma(2j+5)}\left(-32H_{j+2}+\frac{96}{(j+1)(j+4)}-\frac{64}{(j+2)(j+3)}-\frac{22412}{105}\right)$$



$$-\underset{\gamma=3}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+4,j,\gamma}\Big|_{j\geq 4} = \frac{\Gamma(j+2)\Gamma(j+5)}{\Gamma(2j+3)}\left(\frac{32}{3}H_{j+1}-\frac{24}{j(j+3)}+\frac{40}{(j+1)(j+2)}+\frac{15566}{315}\right)$$

$$-\underset{\gamma=3}{\mathrm{Res}}\, \tilde{c}^{gg}_{\delta=j+6,j,\gamma}\Big|_{j\geq 4} = \frac{\Gamma(j+3)\Gamma(j+6)}{\Gamma(2j+5)}\left(\frac{16}{3}H_{j+2}-\frac{28}{(j+1)(j+4)}+\frac{4}{(j+2)(j+3)}+\frac{7069}{45}\right)$$

Summary and Outlook

- For hadron colliders, we investigate the EEC with **full angular dependence**, achieving:
  - The first Analytic LO Result in pure gluon scattering.
  - Singular Approximations in several limits.
  - Evolution equation in the Regge Limit.
- From a formal theoretical perspective:
  - Celestial Block for hadron colliders.
  - Analyticity in both collinear and transverse spin.
- Potential directions for further research:
  - EEC in Other Scattering Channels.
  - LL Resummation in the Regge limit.
  - Operator Language of the pomeron exchange.


Thanks

Table of Labels

Operator	Dimension Δ	Spin J	Twist τ	Celestial Dimension δ (Collinear Spin)	Celestial Spin (Transverse Spin) j	Celestial Twist
$T^{\mu\nu}$	4	2	2			
$\mathcal{E}$	1	3	2	3	0	3
$\mathcal{O}_g^{[J],ij}$	J+2	J	2	(J+1) Labeled by J	(0/2)	
$\mathbb{O}_g^{[J],ij}(\vec{n})$	J–1	J+1	2	J+1 Labeled By J	0/2	J+1/J–1