Resolving Jets with energy correlators

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Nikhef

New Opportunities in Particle and Nuclear Physics with Energy Correlators

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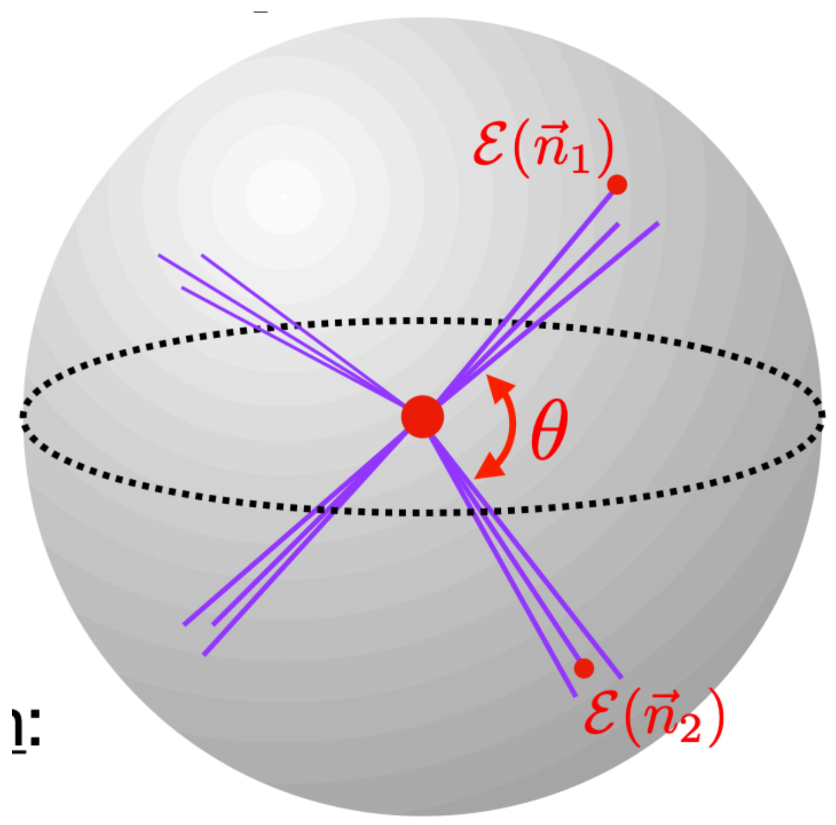
In collaboration with: Wouter Waalewijn, Jesse Thaler, Hao Chen, Samuel Alipour-fard and Isabelle Pels

Outline

- Introduction to Energy correlators
- Projected correlators and their analytic continuation
- A new parameterization
- Extension beyond projected correlators
- Summary & Outlook



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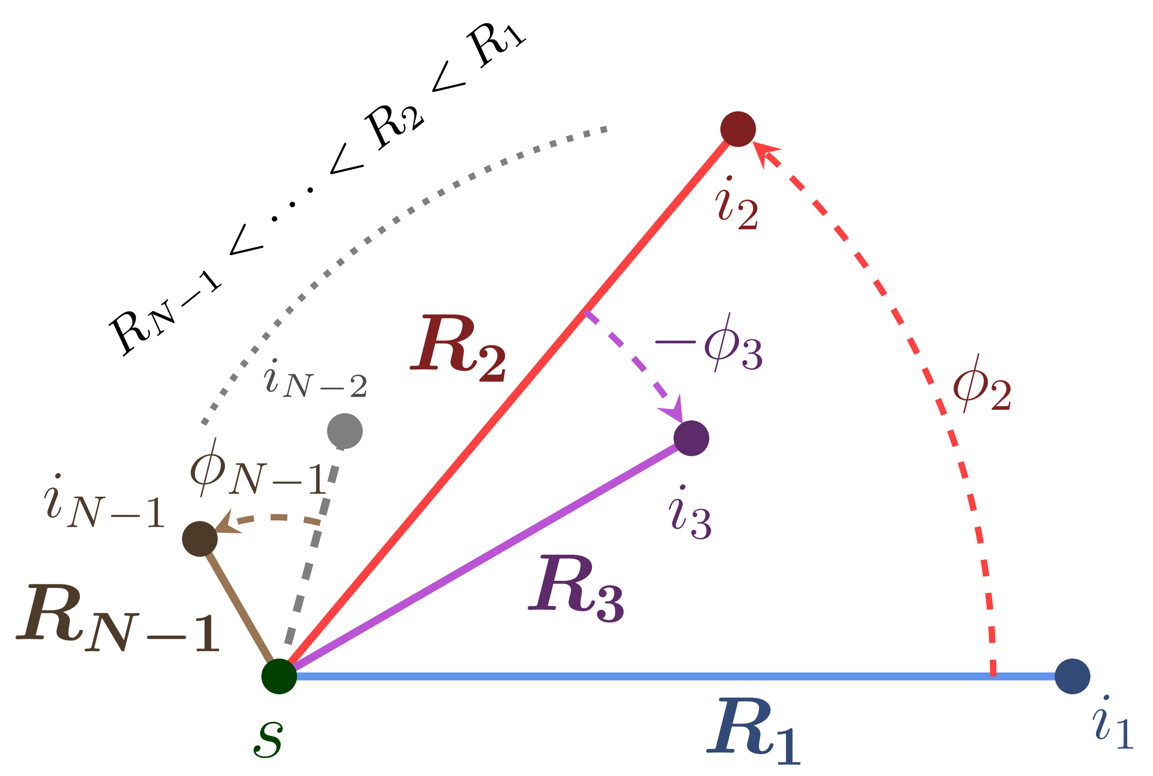

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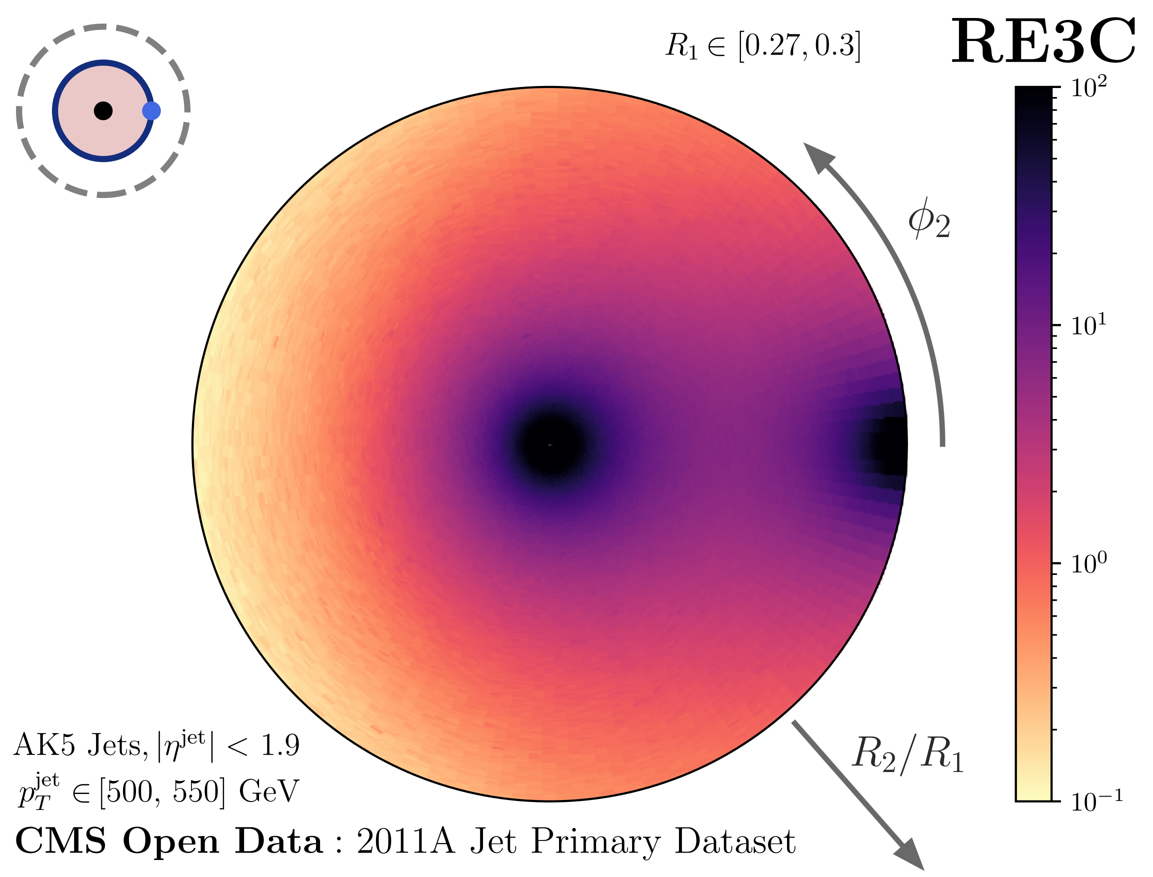


Energy-Energy Correlators (EEC)

- Energy-energy correlators measure correlation between energy depositions in two detector elements

$$\frac{d\sigma}{d\theta} = \int d\sigma \sum_{i,j} \frac{E_i E_j}{Q^2} \delta(\theta - \theta_{ij})$$

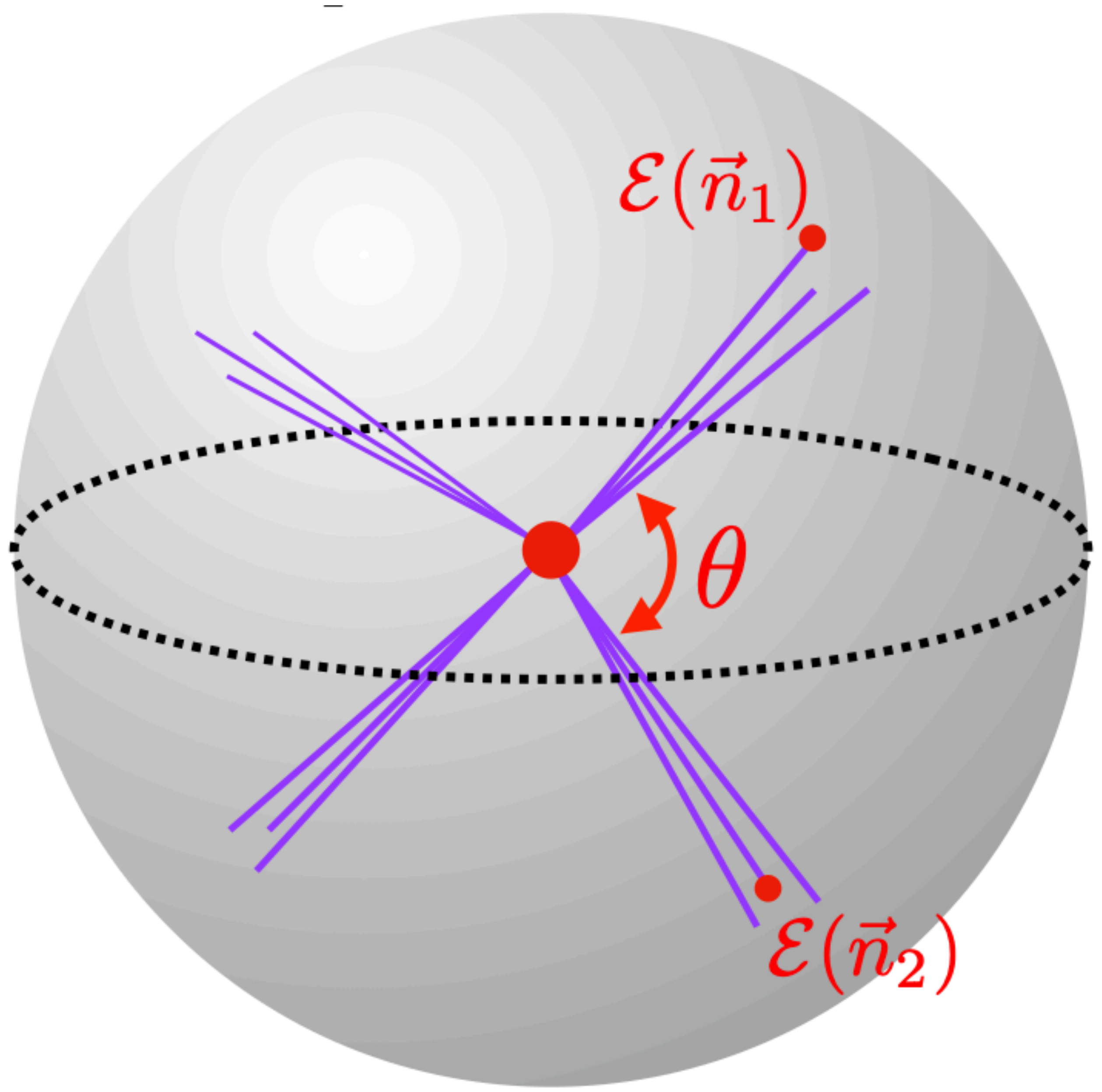
- Can be expressed in terms of expectation value of energy flow operators

energy flow operator : $\mathcal{E}(\hat{n}) = \lim_{r\to\infty} \int_0^\infty \mathrm{d}t\, r^2 n^i T_{0i}(t, r\hat{n})$

energy flow direction

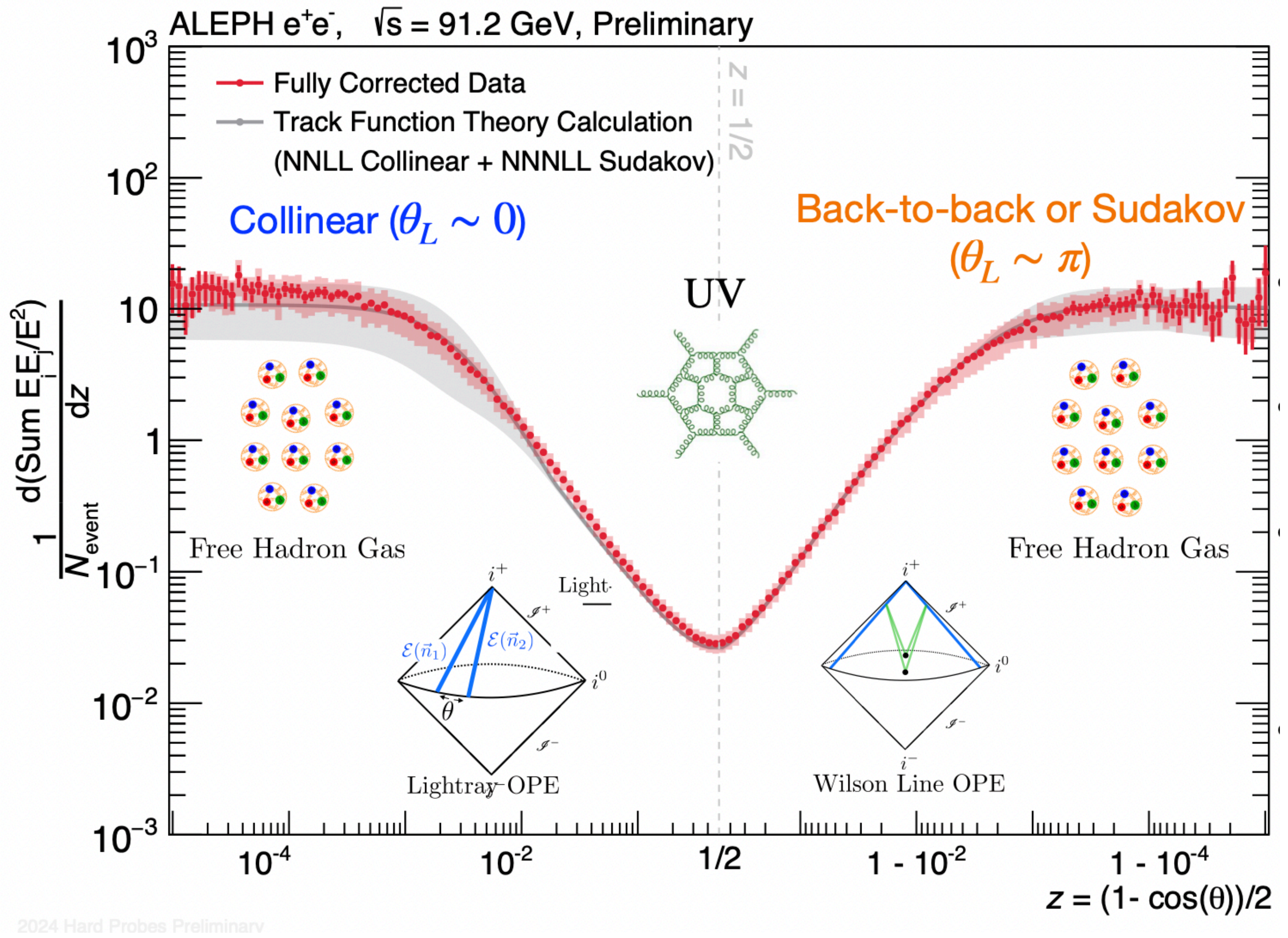
- EEC's are correlation functions of these operators $\langle O'(-q)|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)|O(q)\rangle$

Basham, Brown, Ellis and Love, 1978



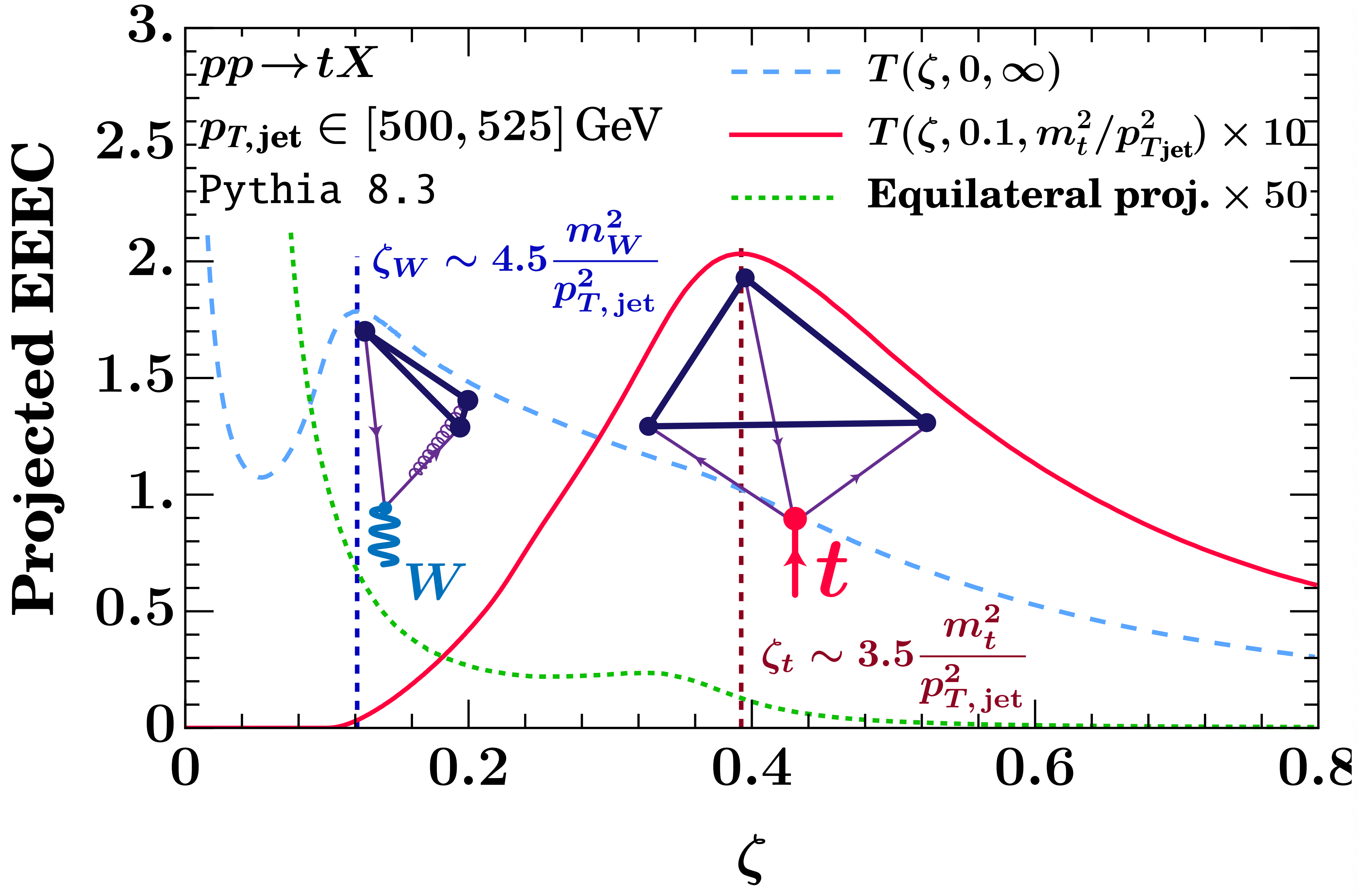
Motivation

Better theory control, calculable on tracks	Potential for precision top mass determination	Promising observables to reveal intrinsic scales in HI jets



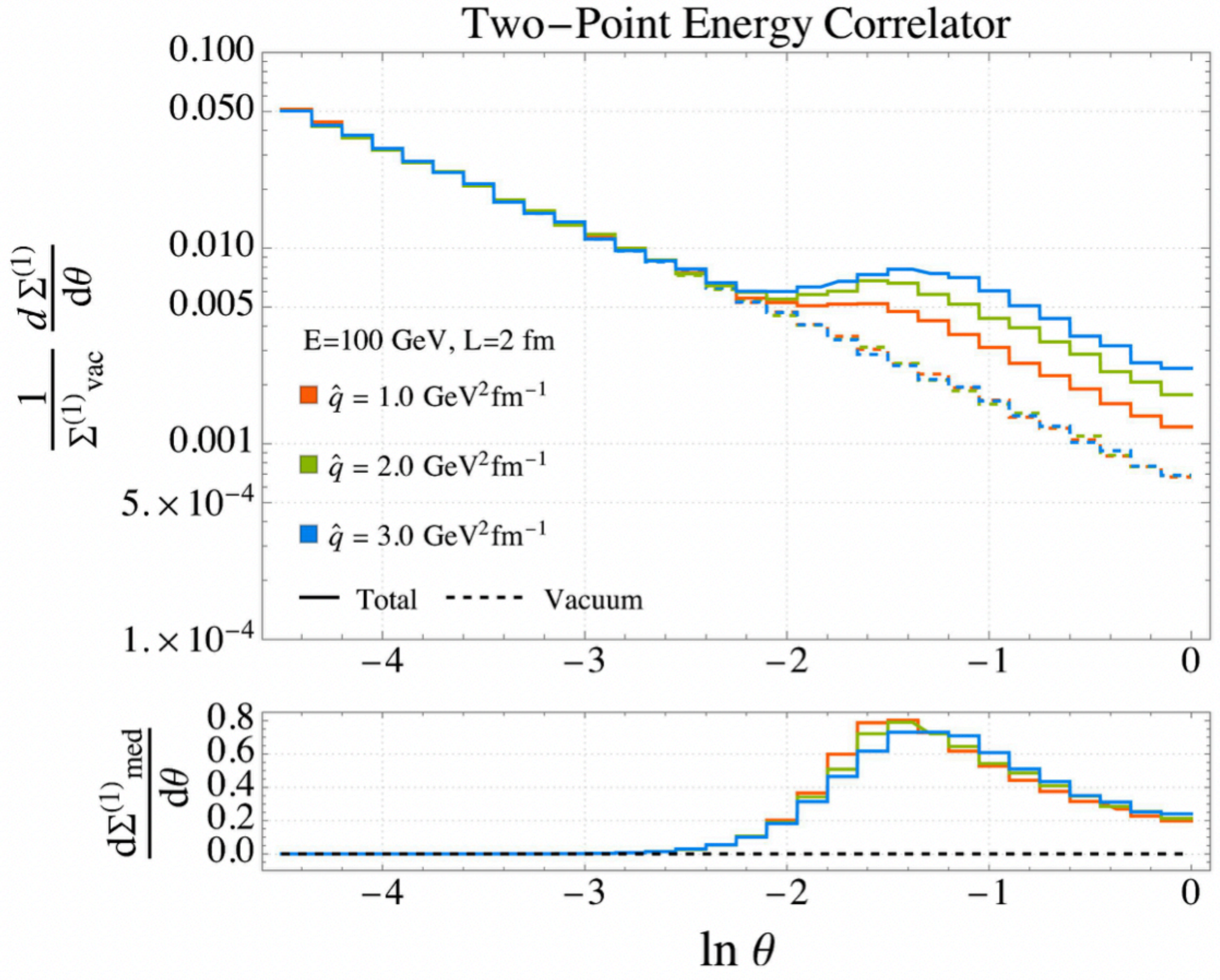
See Max and Yibei's talk

Jaarsma, Li, Moult, Waalewijn, Zhu



See Aditya's talk

Holguin, Moult, Pathak, Procura 2023



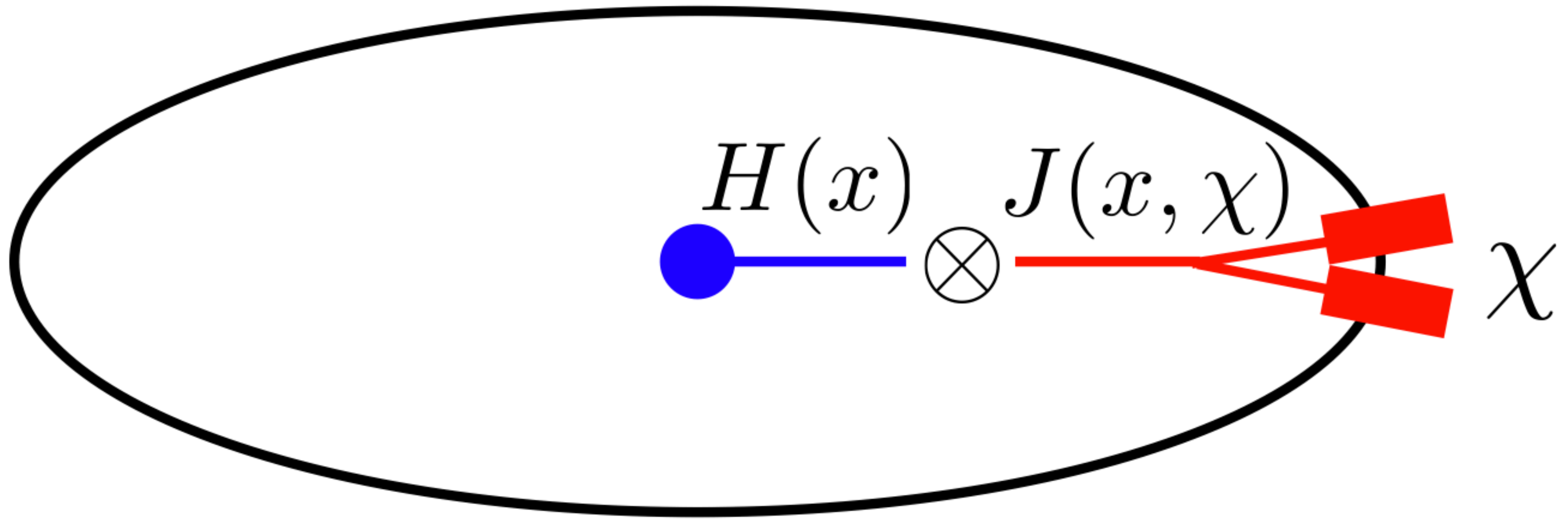
Increasing θ

See Andrey, Wen-Jing, Varun, Balbeer and João's talk

Andres, Dominguez, Holguin, Marquet, Moult 2023

Projected energy correlators and analytic continuation

- EEC exhibits collinear universality described by the factorization formula



$$\Sigma\left(\chi, \log\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 \mathrm{d}x\, x^2\, \vec{J}\left(\log\frac{\chi\, x\, Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

Dixon, Moult, Zhu 2019

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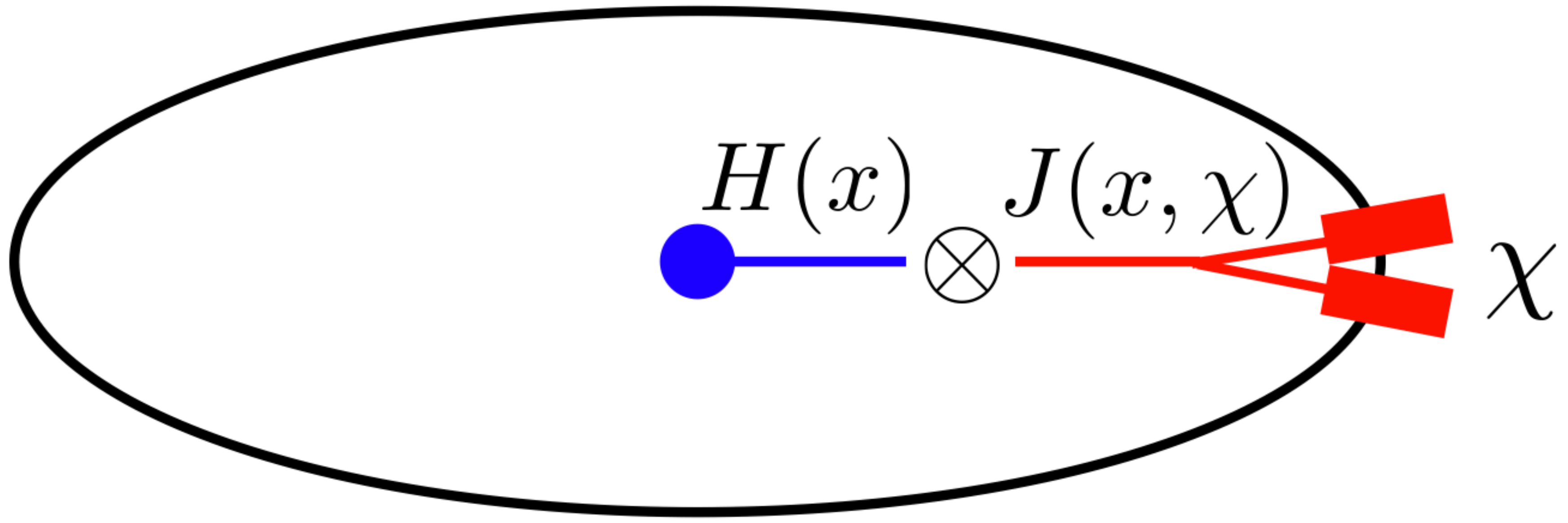

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Dixon, Moult, Zhu 2019

- Multi-point generalization retains dependence on all $N(N-1)/2$ angles
- Projected N-point energy correlators (PENC) obtained by tracking largest separation and integrating out the shape information

$$\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_L} = \int \mathrm{d}\sigma \sum_{i_1,\ldots,i_N} \frac{p_{T,i_1}\cdots p_{T,i_N}}{P_T^N}\, \delta(R_L - \max\{R_{i_j i_k}\})$$

similar factorization formula :

$$\Sigma^{[N]}\left(R_L, \log\frac{Q^2}{\mu^2}, \mu\right) = \int_0^1 \mathrm{d}x\, x^N\, \vec{J}^{[N]}\left(\log\frac{R_L\, x\, Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

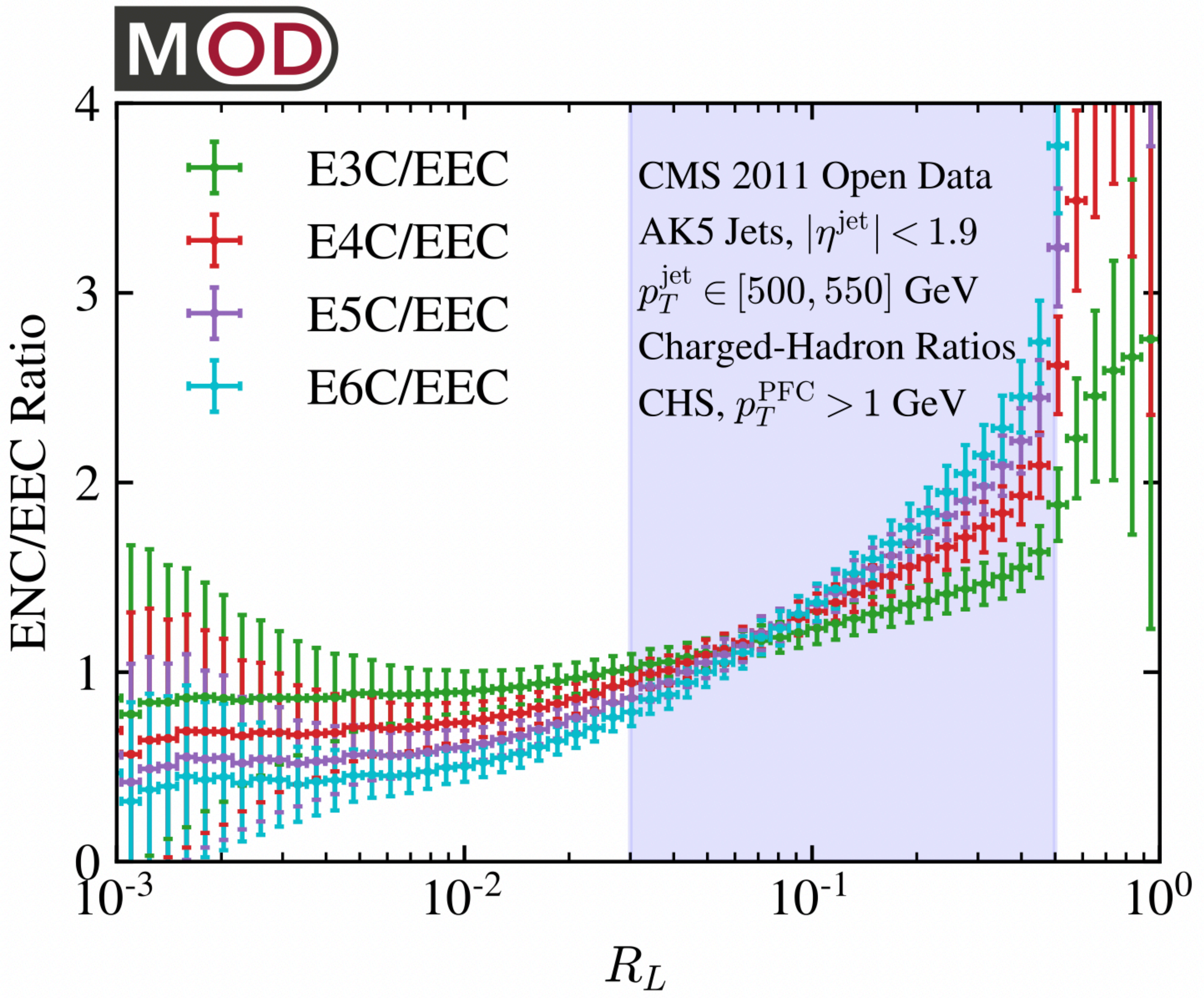
Projected energy correlators and analytic continuation

- In the perturbative region, PENC show a power law scaling $\sim R_L^{\gamma(N)}$ with

$$\gamma(N) \sim \int_0^1 \mathrm{d}x\, x^N P(x)$$

with N-th moment of DGLAP splitting functions $P(x)$.

Collinear PENC evaluated on CMS data



Komiske, Moult, Thaler, Zhu 2023

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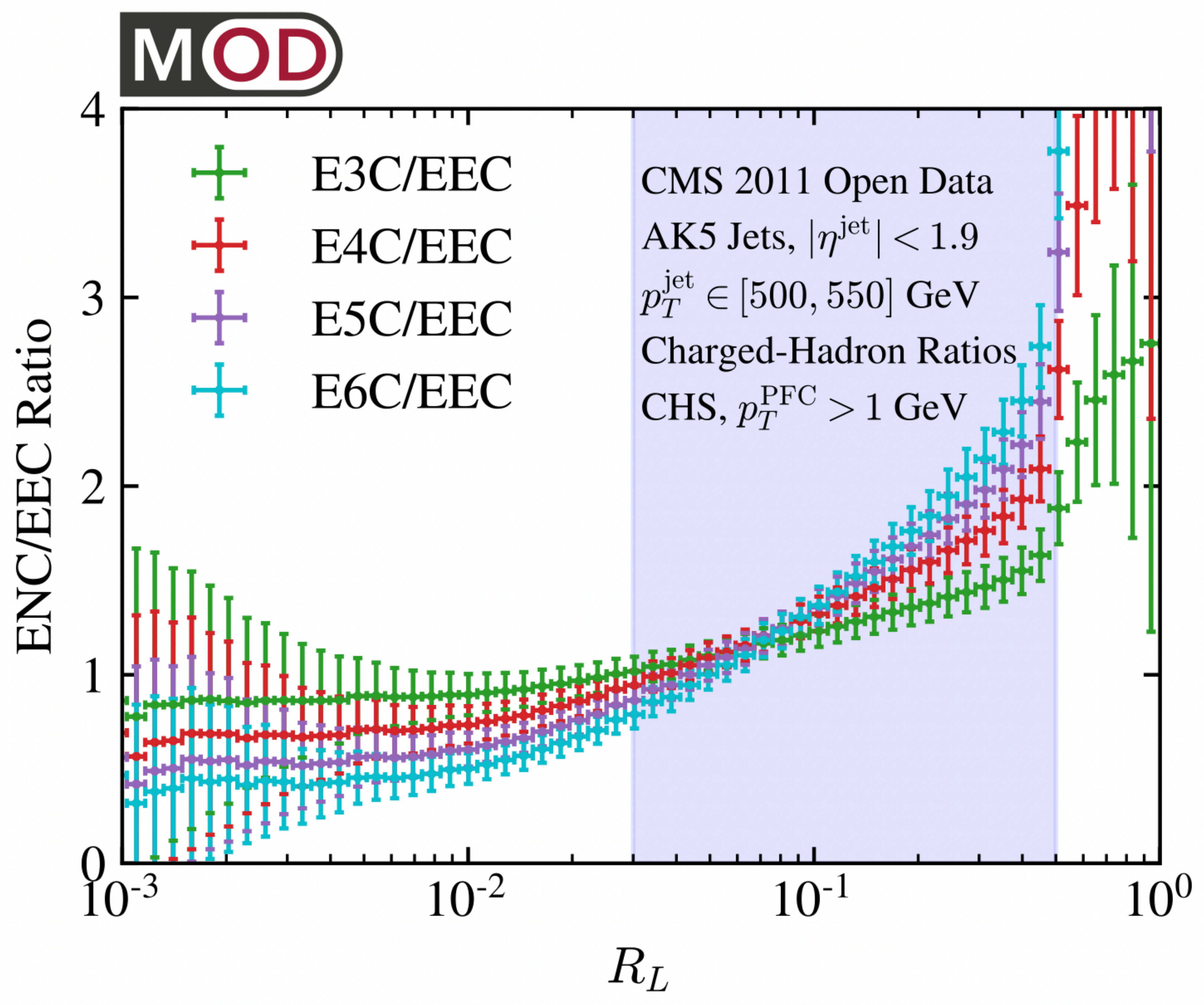
- Analytic continuation in N allows to test different moments of splitting functions.

- For $N \to 0$, we can study small-x physics using jets !

Collinear PENC evaluated on CMS data



Komiske, Moult, Thaler, Zhu 2023

Analytic Continuation in N

- Rewrite the projected correlator as

$$\frac{d\sigma^{[N]}}{dR_L} = \sum_X \int d\sigma_X \sum_{S\subset X} \mathcal{W}^{[N]}(S)\,\delta(R_L - \max\{R_{ij}\}_{i,j\in S})\,,$$

$$\mathcal{W}^{[N]}(\emptyset) = 0\,, \qquad \mathcal{W}^{[N]}(S) = \left(\sum_{i\in S} z_i\right)^N - \sum_{S'\subsetneq S} \mathcal{W}^{[N]}(S')\,.$$

- For eg., for two particles the above weights simplify to

$$\boldsymbol{\mathcal{W}^{[1]} = z_1 + z_2}$$
$$\mathcal{W}^{[2]} = (z_1+z_2)^2 - z_1^2 - z_2^2 = 2z_1z_2$$
$$\mathcal{W}^{[3]} = (z_1+z_2)^3 - z_1^3 - z_2^3 = 3z_1^2z_2 + 3z_1z_2^2$$

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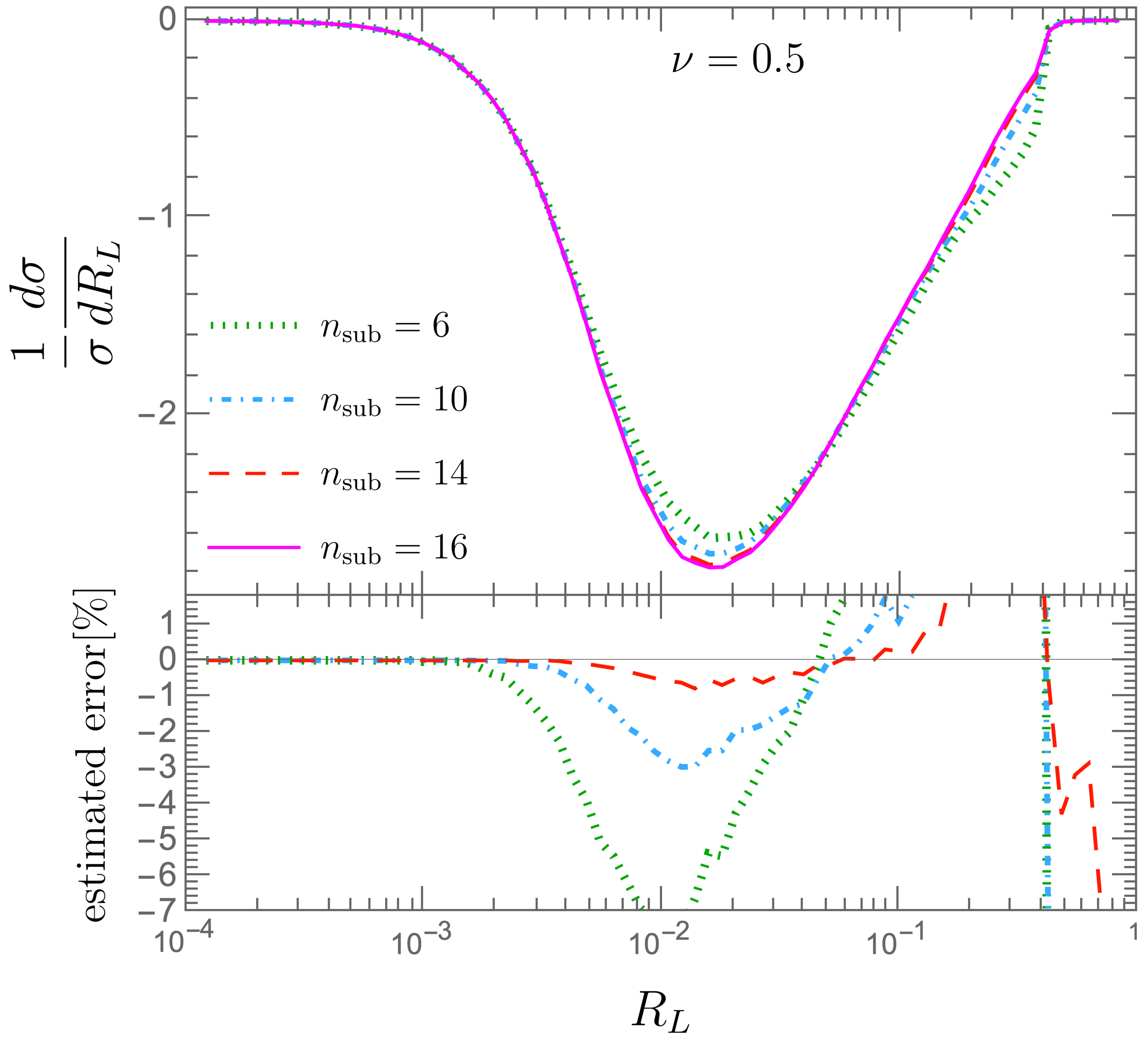
- This form can be analytically continued in N.
- For real world jets with M particles, computationally **prohibitive** $\mathcal{O}(2^{2M})$.

An approximate approach

A.B, Chen, Waalewijn 2024

- Correlations at scales R_L are insensitive to details at much smaller scales $\Rightarrow$ Approximate particles by jets thereby reducing M.

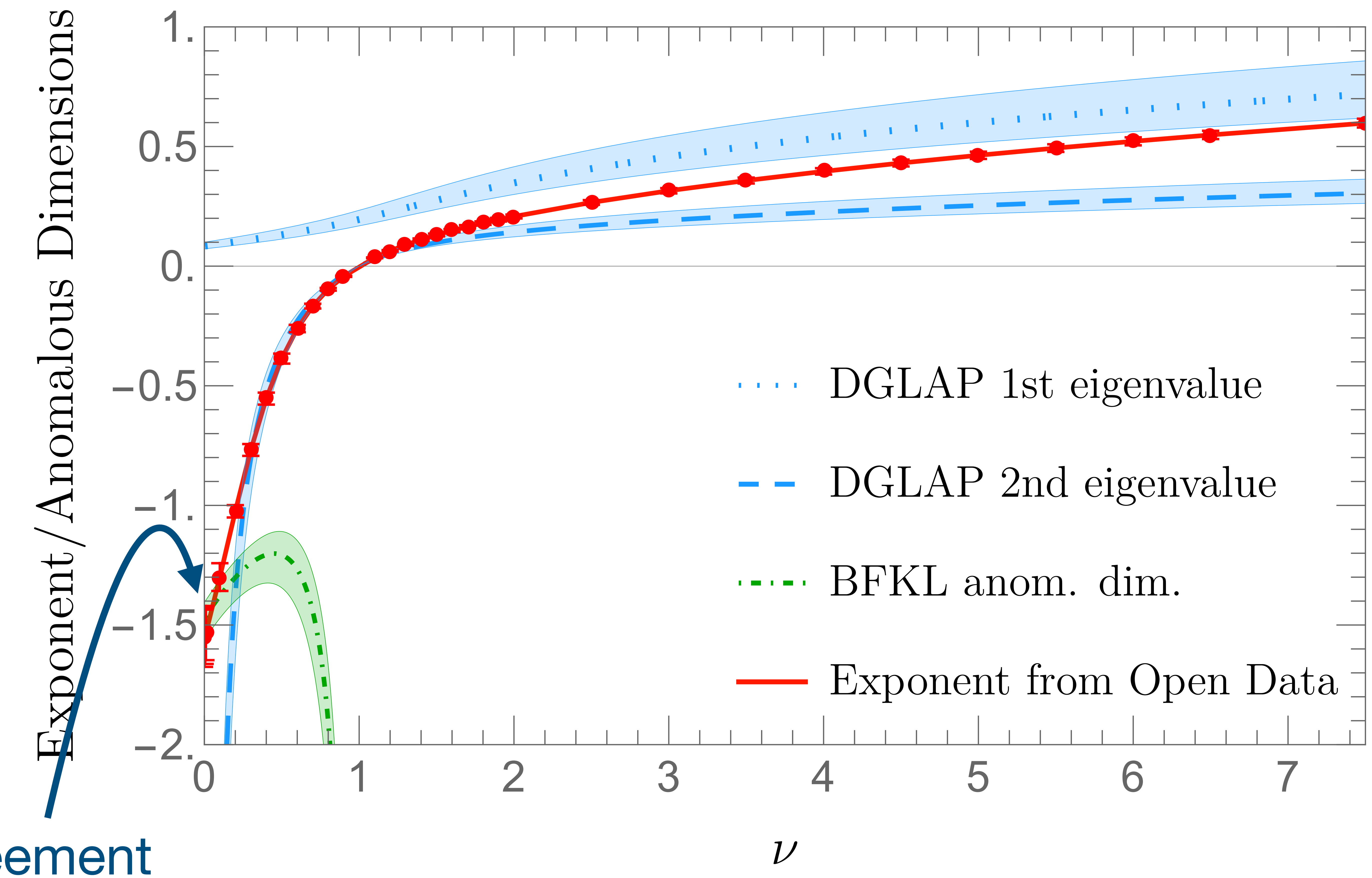
- Computational cost $\mathcal{O}(2^{2M}) \rightarrow \mathcal{O}(M\,2^M)$



Power law from CMS data

- Fit to a power law $\sim aR_L^b$.
- Fit exponent well within eigenvalues of DGLAP anomalous dimension
- For small-N, fit exponent saturates while DGLAP eigenvalue diverges irrespective of quark/gluon mixing fractions.
- Interestingly, agreement with BFKL.

See also Hao's talk



agreement
with BFKL

A.B, Chen, Waalewijn 2024

Problems with traditional parametrization

- Computationally time intensive: $\mathcal{O}(M^N)$ for integer N or $\mathcal{O}(2^{2M})$ for non-integer N.
- Previous approach improves this cost but still has an exponential scaling.
- Moreover, parameterization in terms of all distances is **redundant**:

$$\binom{N}{2} > 2N - 3 \qquad \text{for} \quad N \geq 3$$

A new parametrization

- Simpler non-redundant parametrizaton : Isolate a special point s and **only** consider the distance to it:

$$\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_1} = \int \mathrm{d}\sigma \sum_{s} z_s \sum_{i,j,k,\dots} z_j z_k \cdots \delta(R_1 - \max\{R_{si}, R_{sj}, \cdots\})$$

- From triangle inequality $R_L/2 \leq R_1 \leq R_L$, so R_1 is a good measure of overall scale.

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- From triangle inequality $R_1 \leq R_L \leq 2R_1$, so R_1 is a good measure of overall scale.

- Looking at the cumulative distribution:

$$\Sigma^{[N]}(R_1) = \int^{R_1} \mathrm{d}R_1' \frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_1'} = \int \mathrm{d}\sigma \sum_{s} z_s [z_{\mathrm{disk}}(s, R)]^{N-1}$$

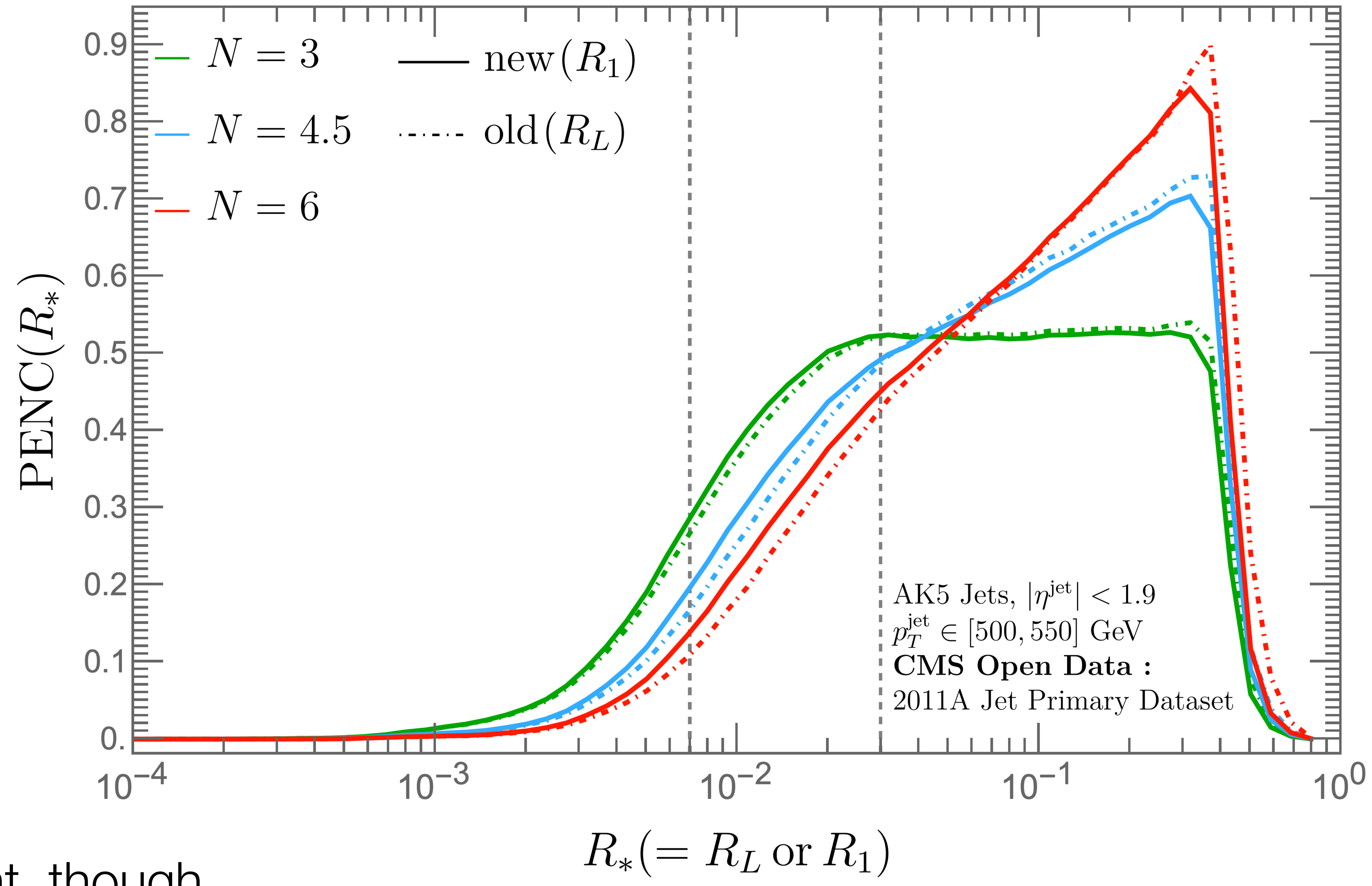
- Sorting takes $\mathcal{O}(M \ln M)$ and for each special particle we get another factor $M \rightarrow \mathcal{O}(M^2 \log M)$ for all N!

Comparing old and new parametrization

- Differences in collinear region are small.
- Mostly visible in the **transition region**
- Same factorization formula carries over

$$\Sigma^{[N]}(R_1, Q) = \int_0^1 \mathrm{d}x\, x^{[N]} \sum_i J_i^{[N]}\left(\frac{x\, Q^2 R_1^2}{\mu^2}\right) \cdot H_i\left(x, \frac{Q^2}{\mu^2}\right)$$

- H_i independent of specific parametrization of $\mathrm{PENC}(R_*)$
- RGE for J_i also parametrization independent, though functional form is not.



Alipour-fard, A.B, Thaler, Waalewijn 2024

Perturbative agreement upto NLL

- For two particles,
  $R_1 = R_L \Rightarrow J(R_L) = J(R_1) + \mathcal{O}(\alpha_s^2)$

- By triangle inequality $R_1 \leq R_L \leq 2R_1$, one expects $R_L \sim (1+c)R_1$

NLL equivalence implies $c \sim \mathcal{O}(\alpha_s)$

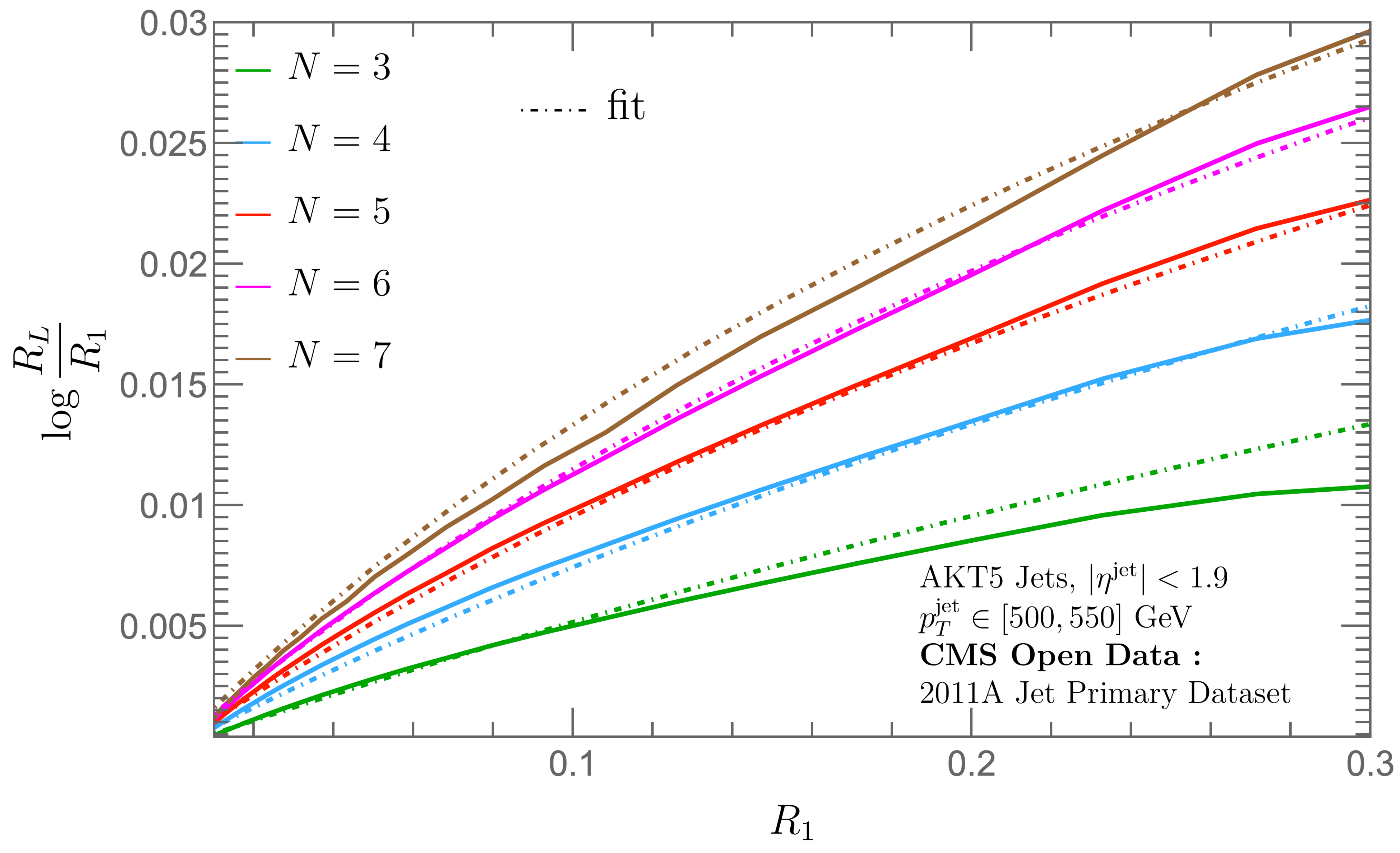
- Test numerically by constructing **quantiles**

$$\Sigma_{\mathrm{new}}^{[N]}(R_1) = \Sigma_{\mathrm{old}}^{[N]}(R_L(R_1))$$

- We find a fit form

$$\log(R_L/R_1) \sim \frac{\alpha_s}{\pi} \log[1 + (N-1)R_L]$$

Alipour-fard, A.B, Thaler, Waalewijn 2024



A look into non-perturbative effects

- Leading non-perturbative contribution by considering one energy flow detector along the direction of soft hadron

A.B, Pels, Waalewijn

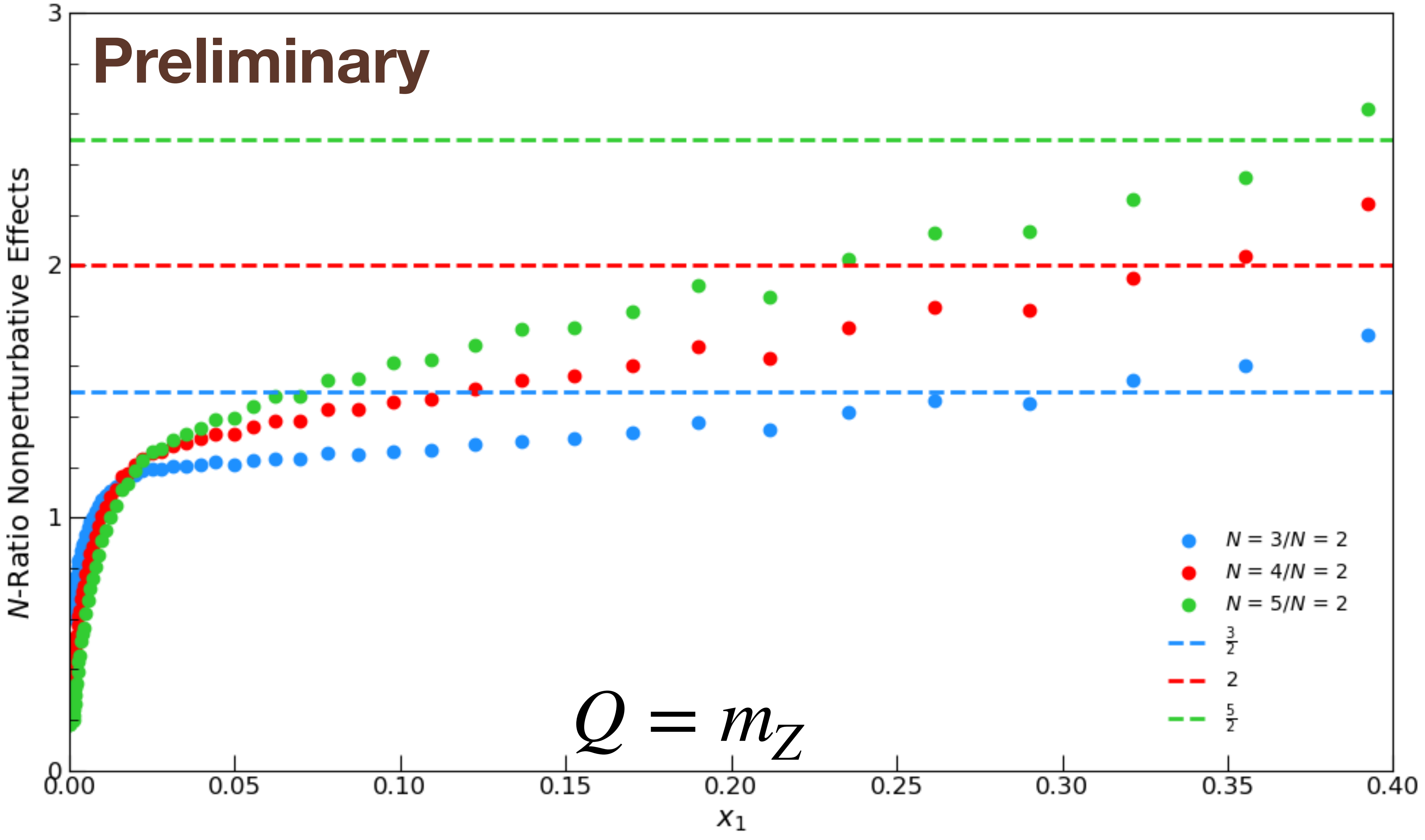
$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma^{[N]}}{\mathrm{d}R_1} = \frac{1}{\sigma}\frac{\mathrm{d}\sigma^{[N]}_{\mathrm{pert}}}{\mathrm{d}R_1} + N\frac{\Lambda_{\mathrm{QCD}}}{Q\,R_1^{3/2}}$$

- For $N > 1$, leading structure same as proposed by Lee et.al.

Schindler, Stewart, Sun 2023
Chen, Monni, Xu, Zhu 2024
Lee, Pathak, Stewart, Sun 2024

- Extract in MC by $\dfrac{(\mathrm{had}-\mathrm{part})(N)}{(\mathrm{had}-\mathrm{part})(2)}$

- RG effects visible in the shape.



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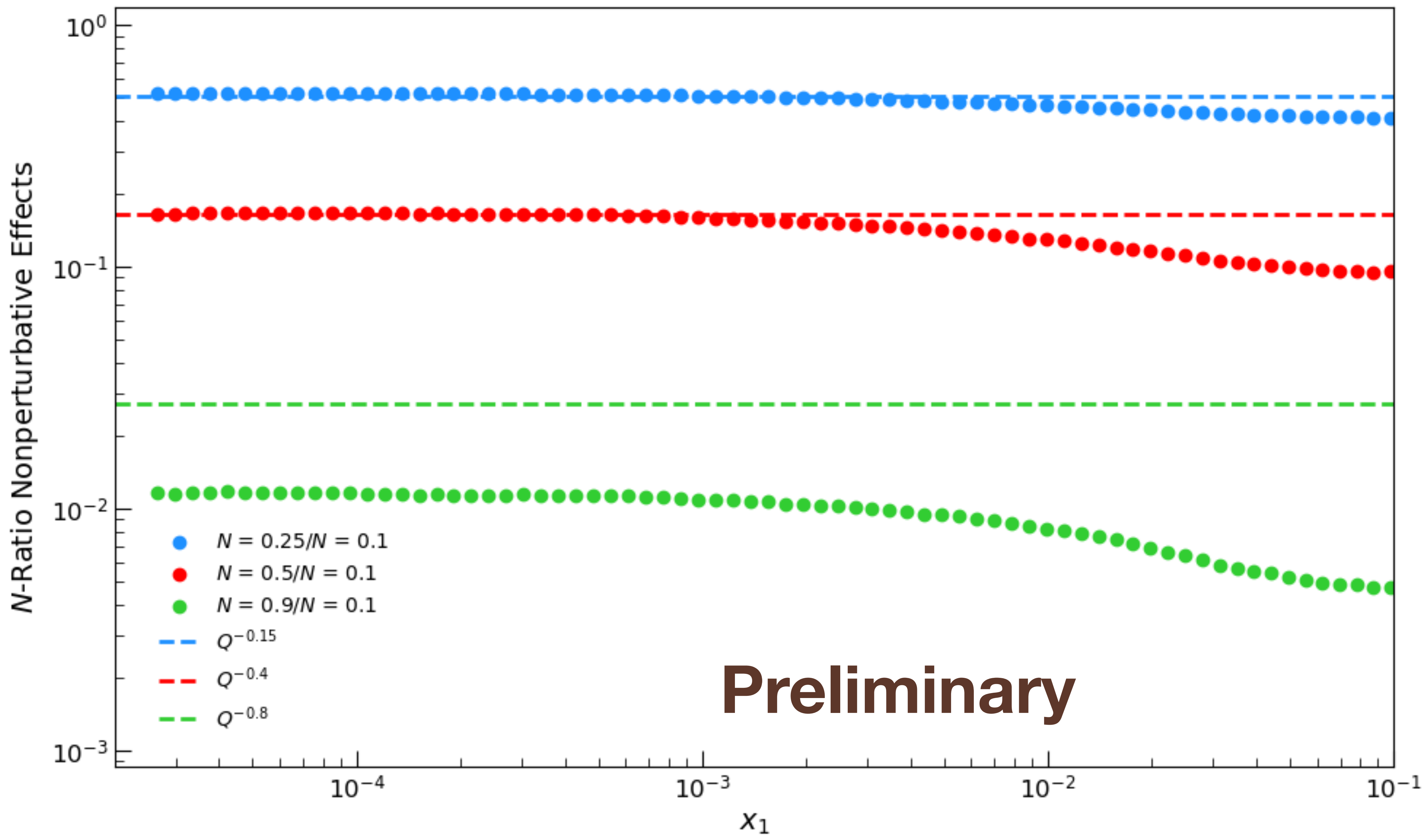
Relevant when N is small

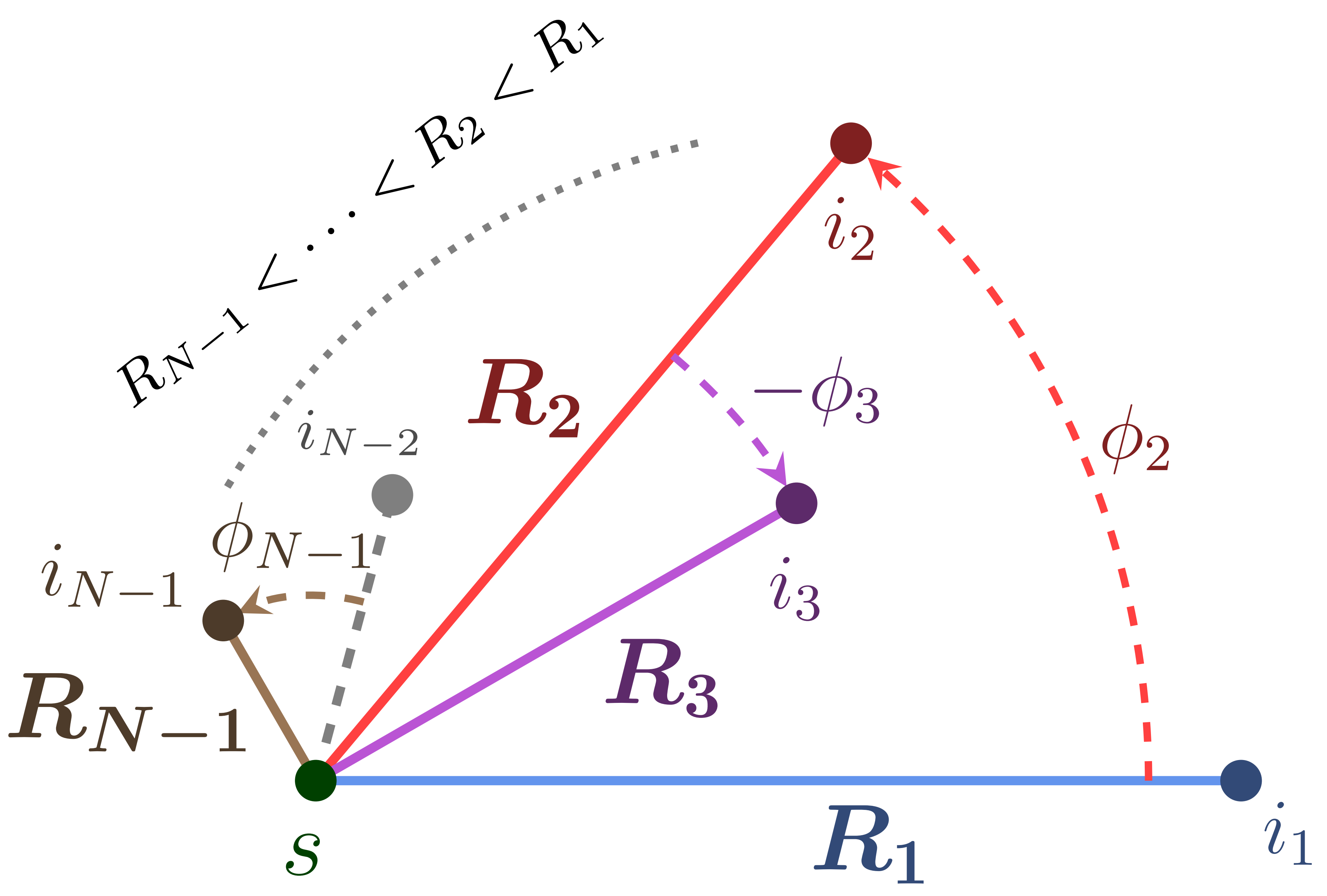
- For $N < 1$, **new result** that goes like $\Lambda_{\mathrm{QCD}}^N \Rightarrow$ overall size of NP corrections stronger for $N < 1$.

- Test this in MC by $\dfrac{(\mathrm{had}-\mathrm{part})(N)}{(\mathrm{had}-\mathrm{part})(0.1)}$

- Well governed by the predicted scaling for sufficiently small-N



Looking at energy distribution inside jets



Alipour-fard, A.B, Thaler, Waalewijn '24

$$\mathrm{RENC}(R_1, R_2, \phi_2, R_3, \phi_3, \ldots) = \int d\sigma \sum_{s} z_s \sum_{i_1 \geq \ldots \geq i_{N-1}} z_{i_1} \ldots z_{i_{N-1}}$$

$$\times \binom{N}{n_1\, n_2 \ldots} \, \delta(R_1 - R_{s,i_1})\, \delta(R_2 - R_{s,i_2})$$

$$\times\, \delta(\phi_2 - \phi_{i_1,i_2}) \;\ldots$$

due to **ordering** of distances $R_1 > R_2 > \ldots$

- A natural generalization $\Rightarrow$ Resolved energy correlators
- Use polar coordinates around the special particle.
- Non-redundant parameterization.
- Preserves orientation information.

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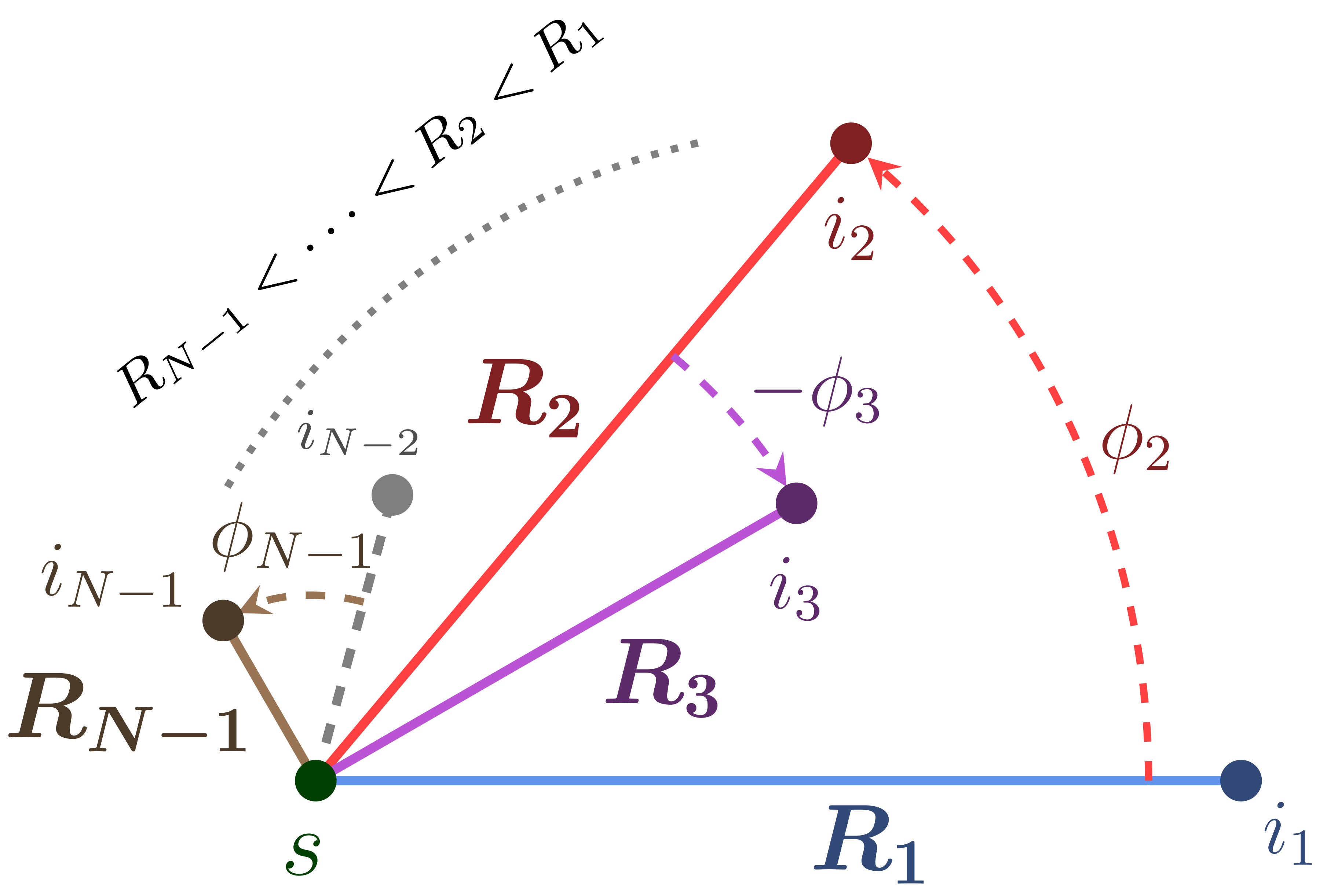
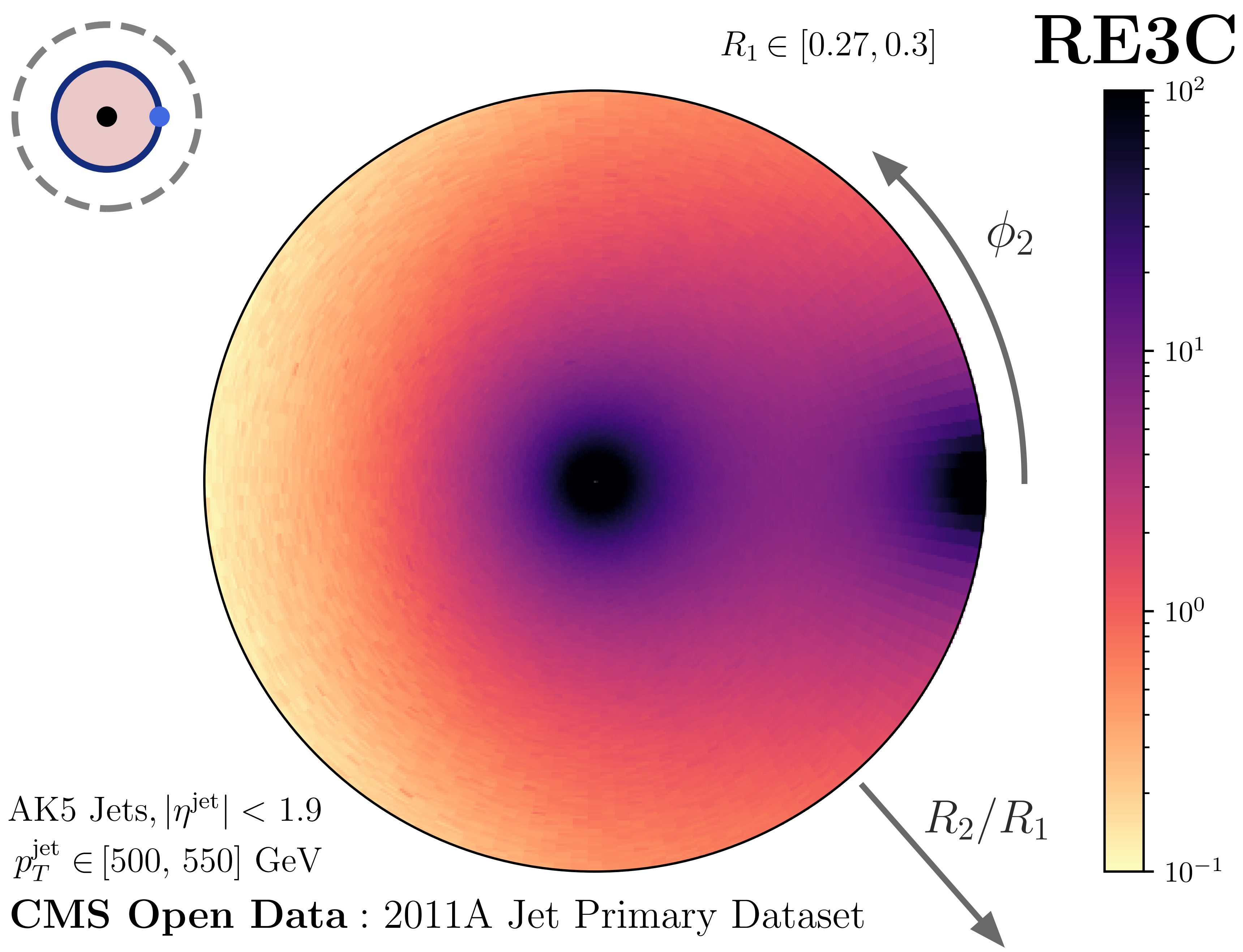


Alipour-fard, A.B, Thaler, Waalewijn '24



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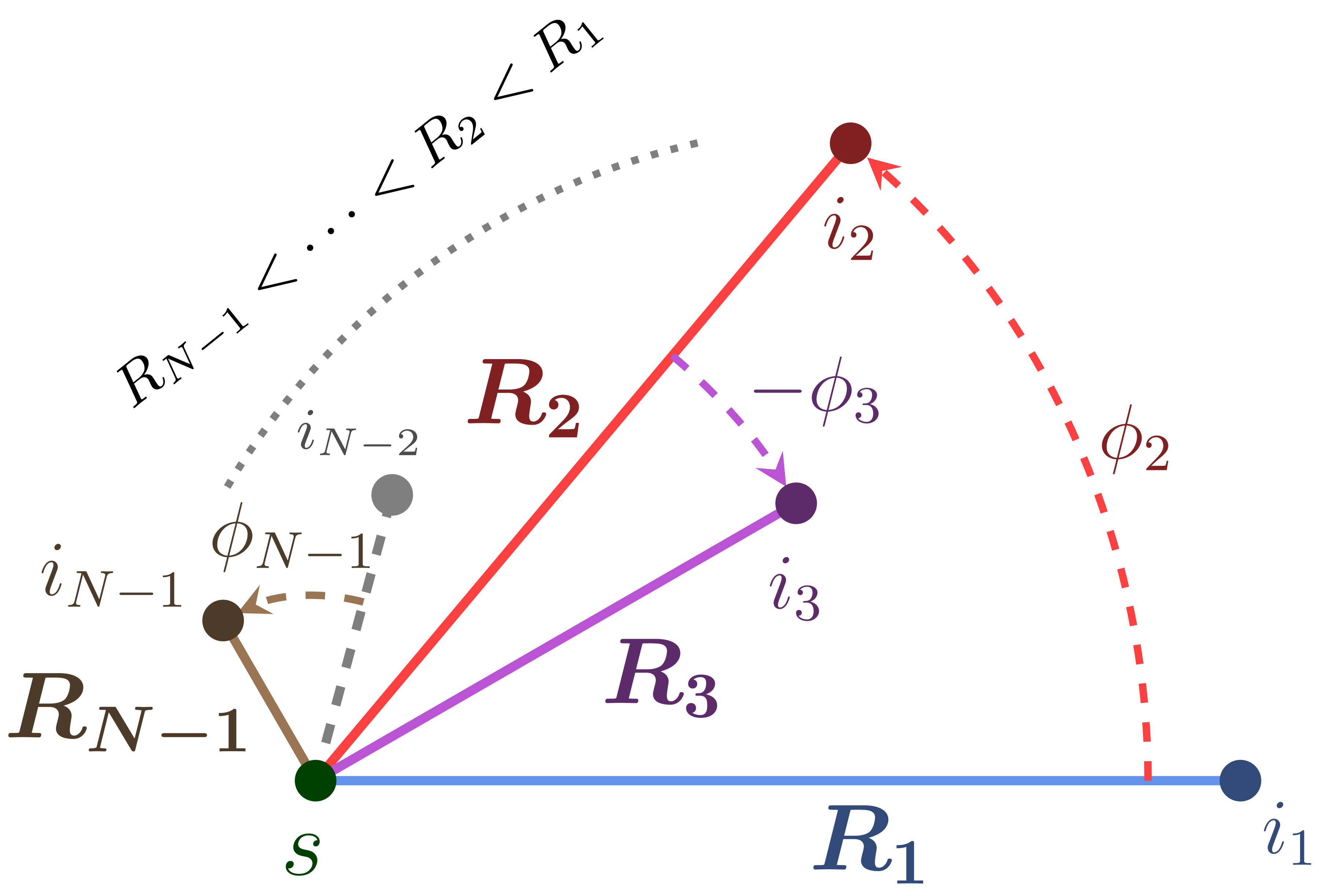

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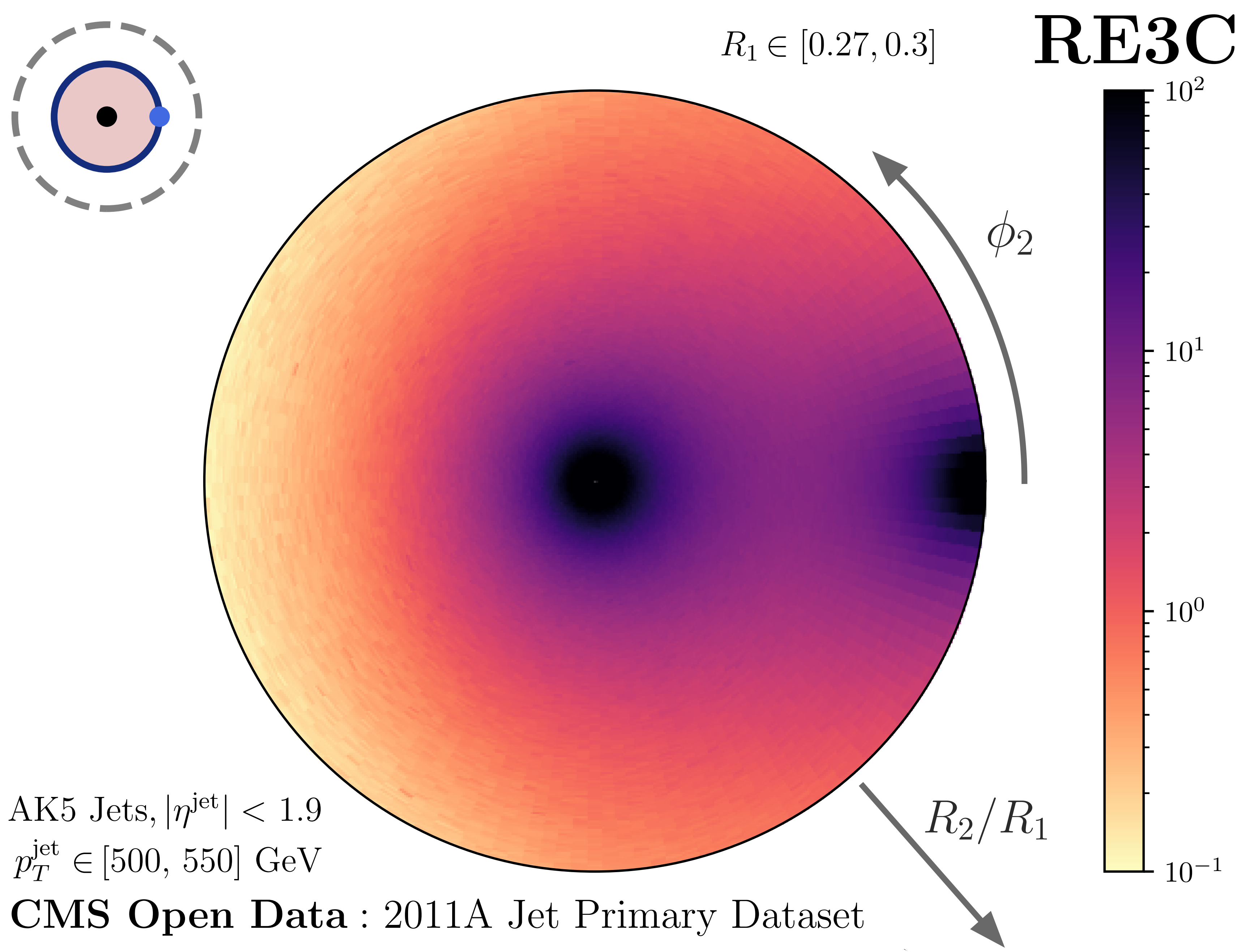

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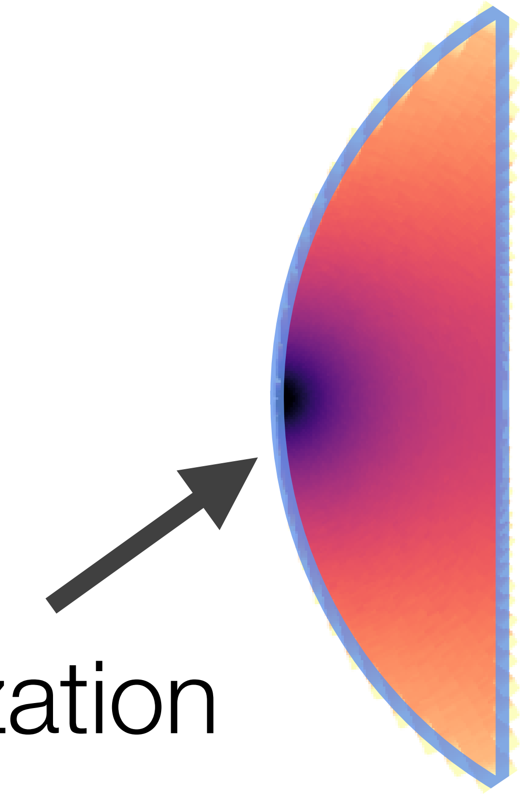
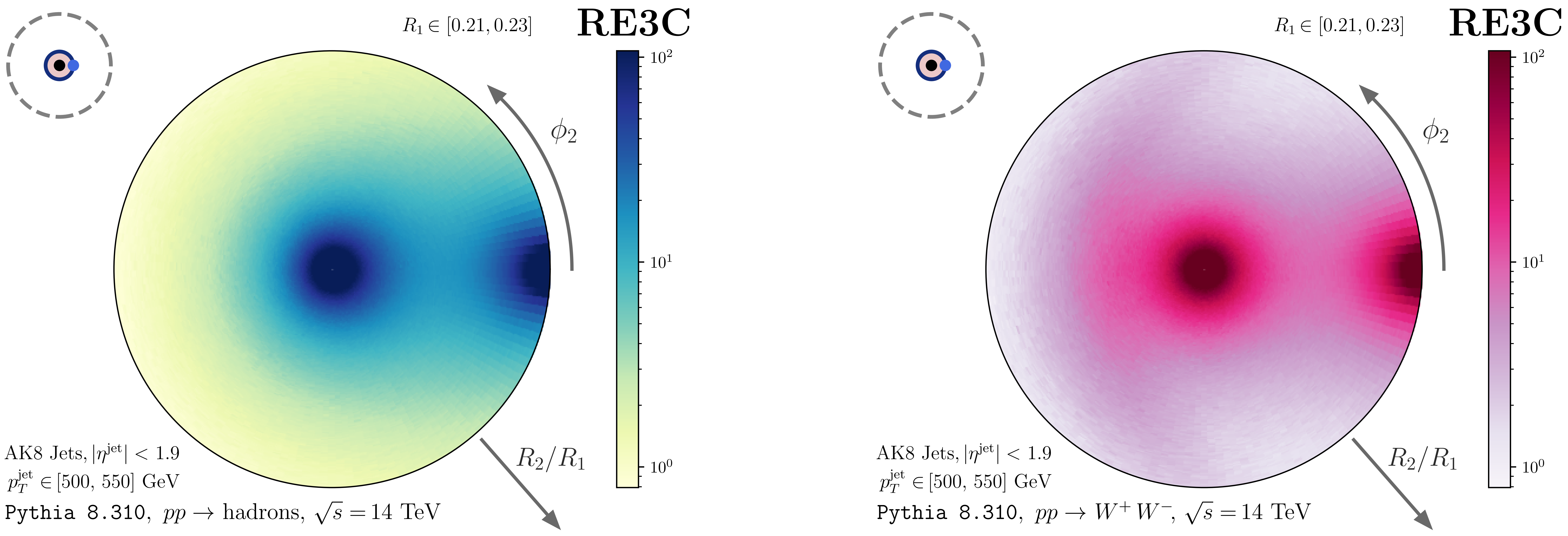


Old parametrization

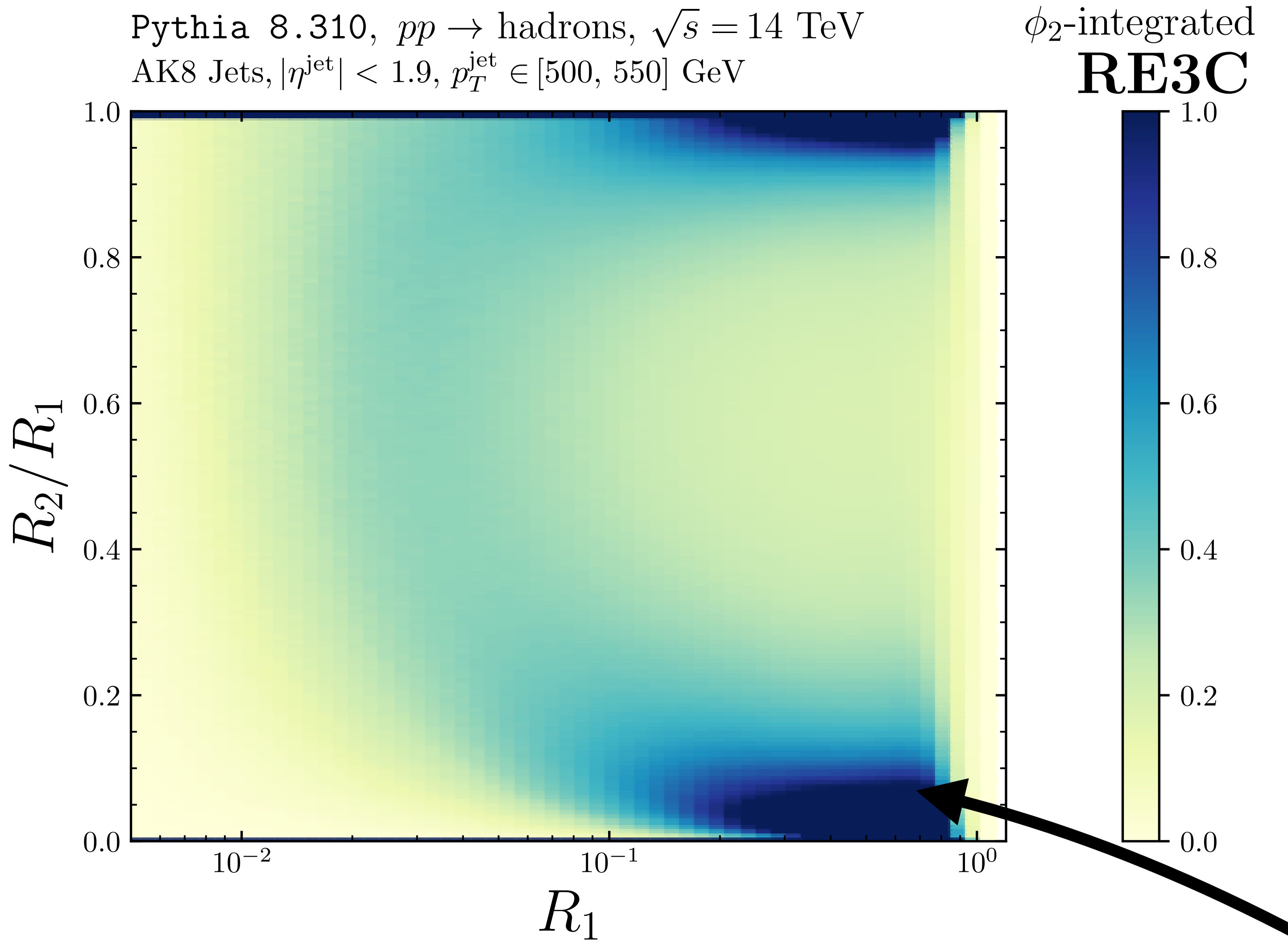
Distinguishing jet samples

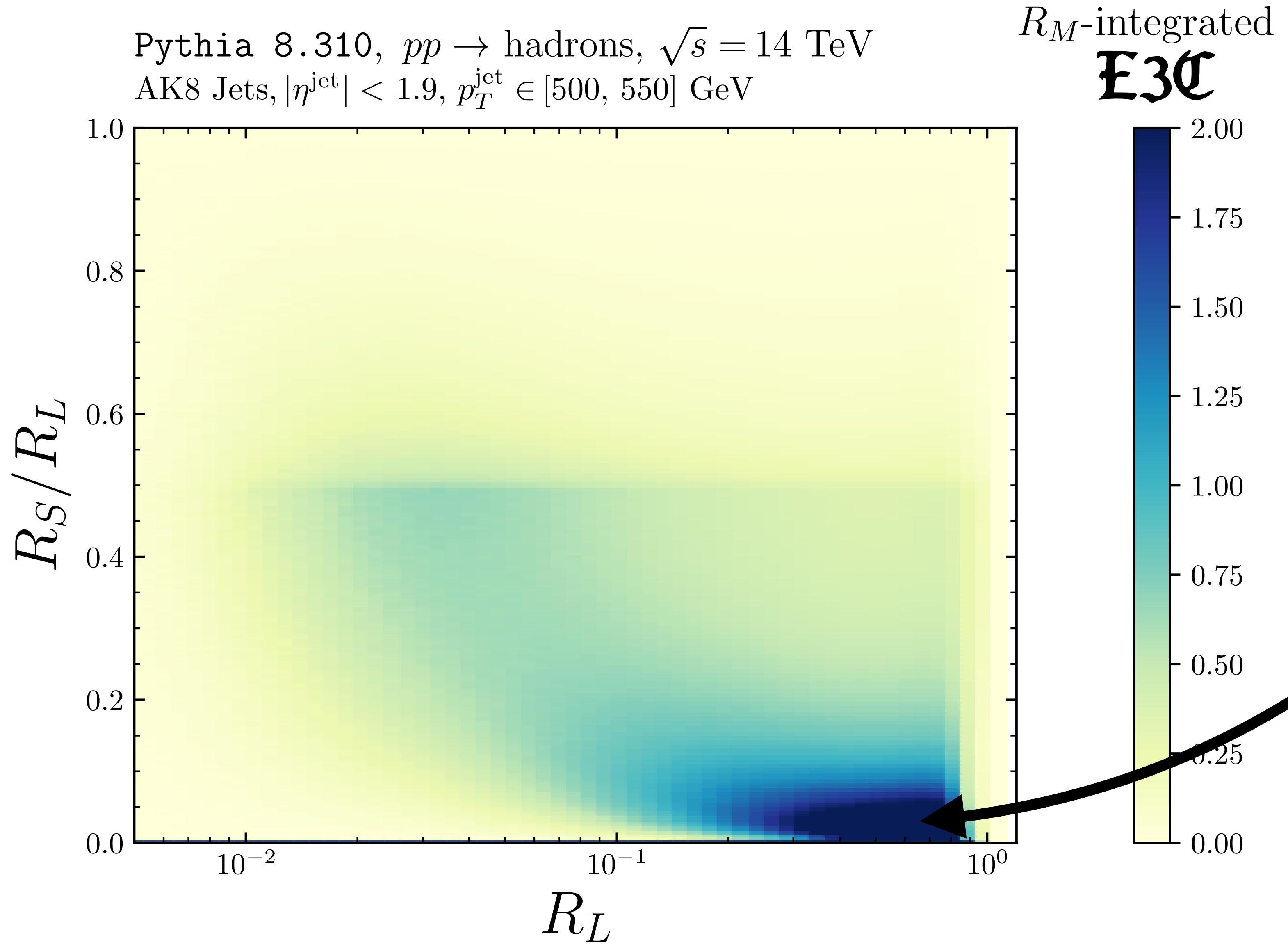


Alipour-fard, A.B, Thaler, Waalewijn '24

- Qualitative differences between QCD jets and W-boson initiated jets.
- Not visible in old parametrization as orientation information is lost.

Radial distribution for different jets





Old and new agree in the OPE limit.

Alipour-fard, A.B, Thaler, Waalewijn ‘24

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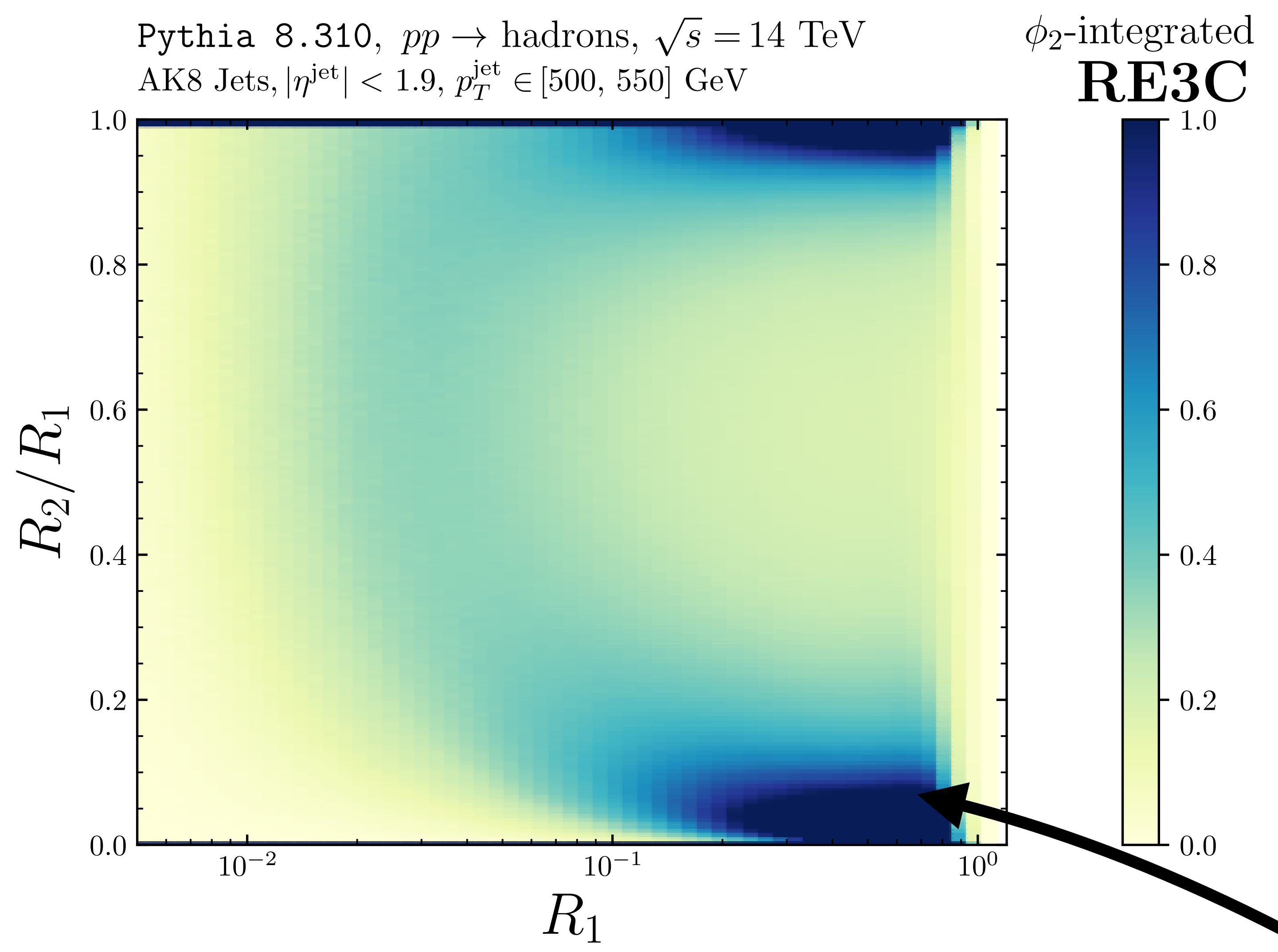
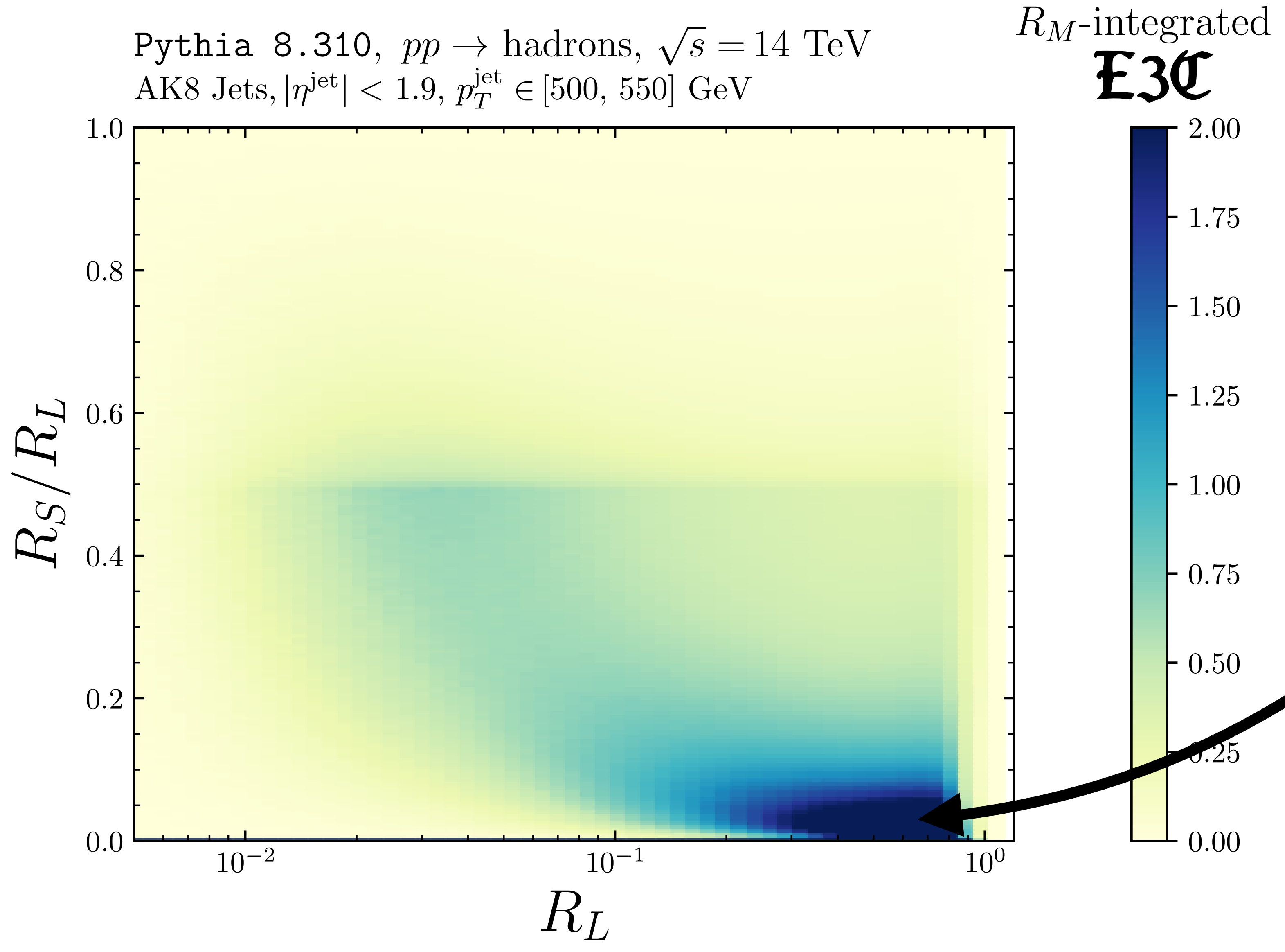
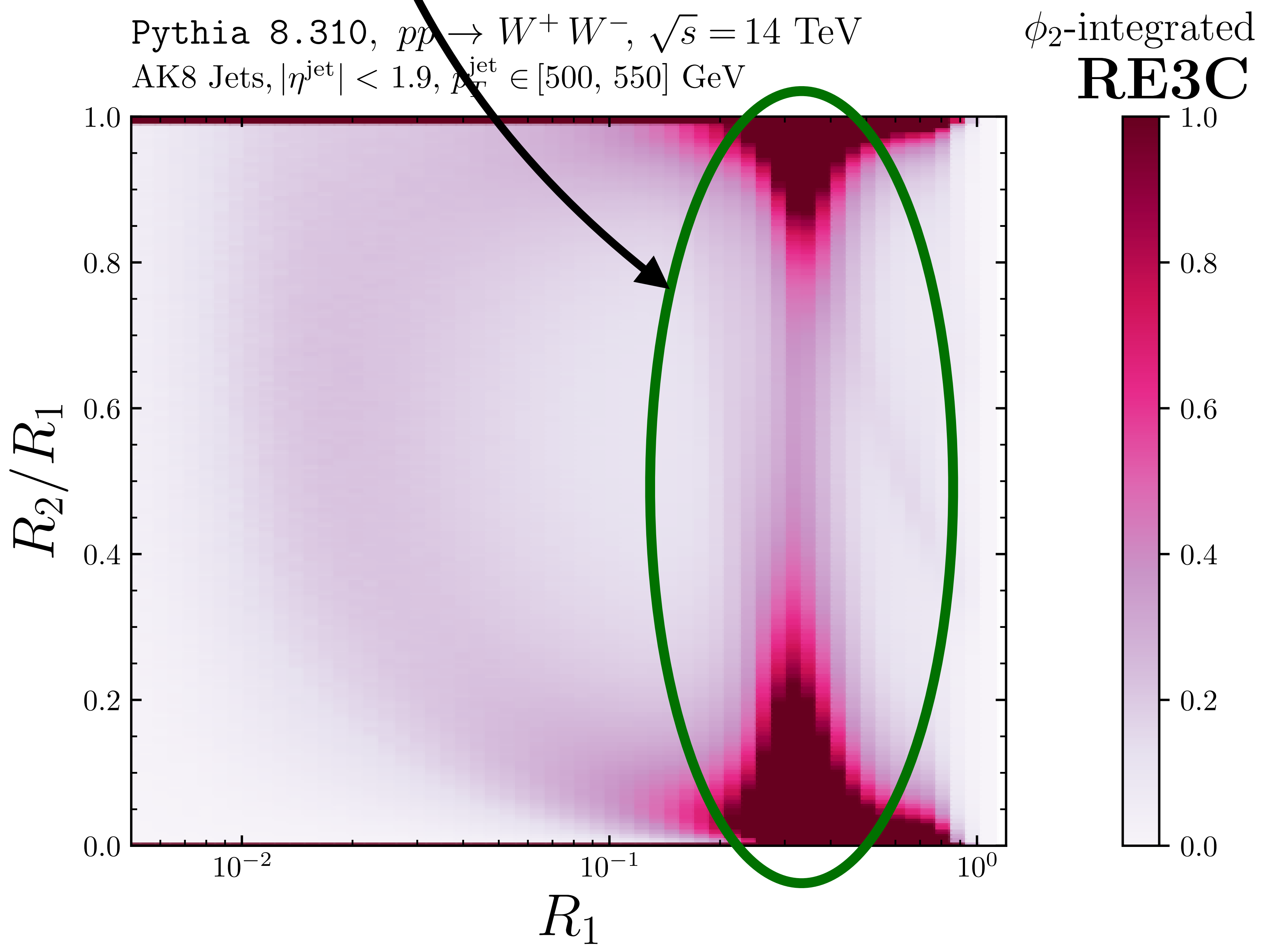




Old and new agree in the OPE limit.

W-boson mass imprinted



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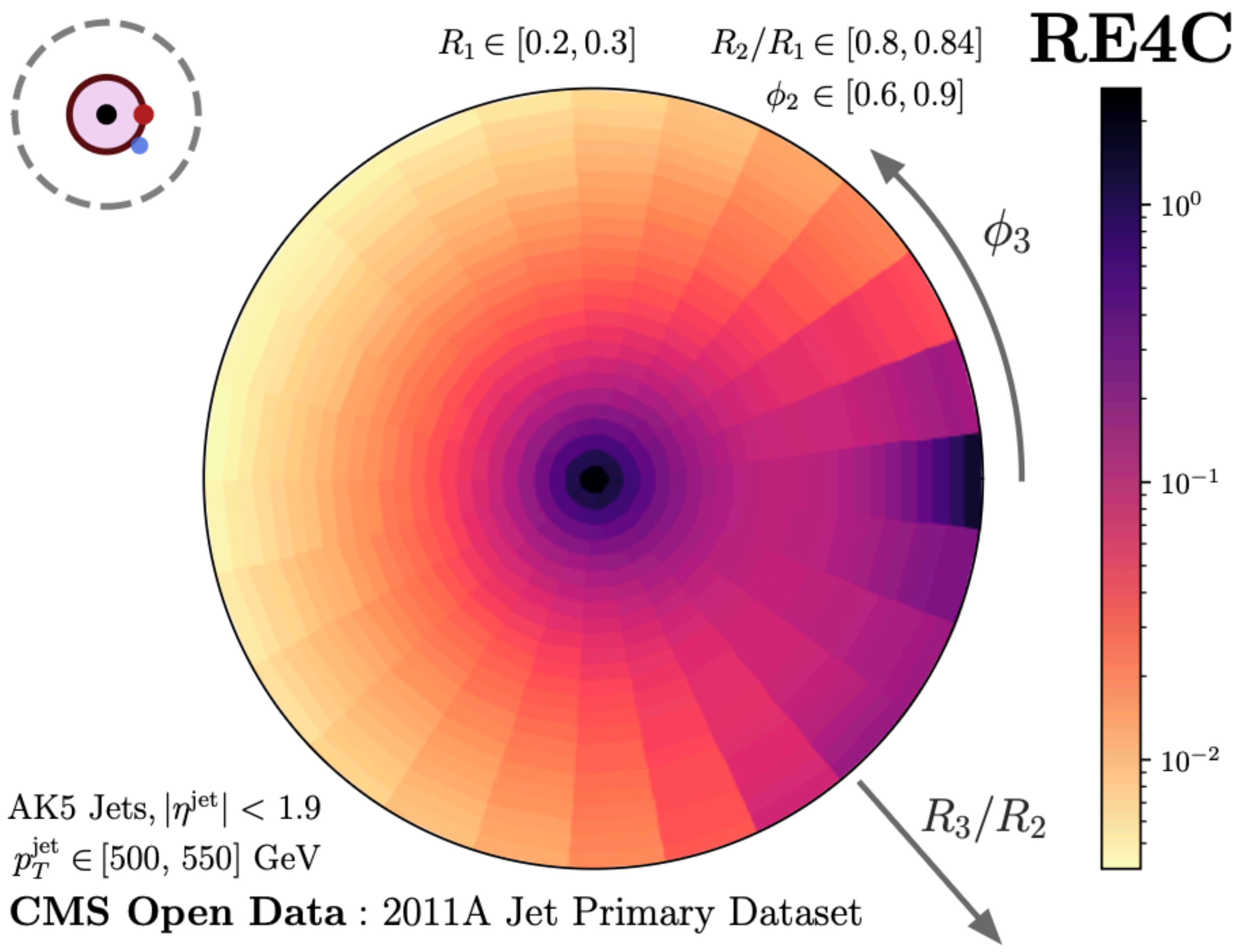
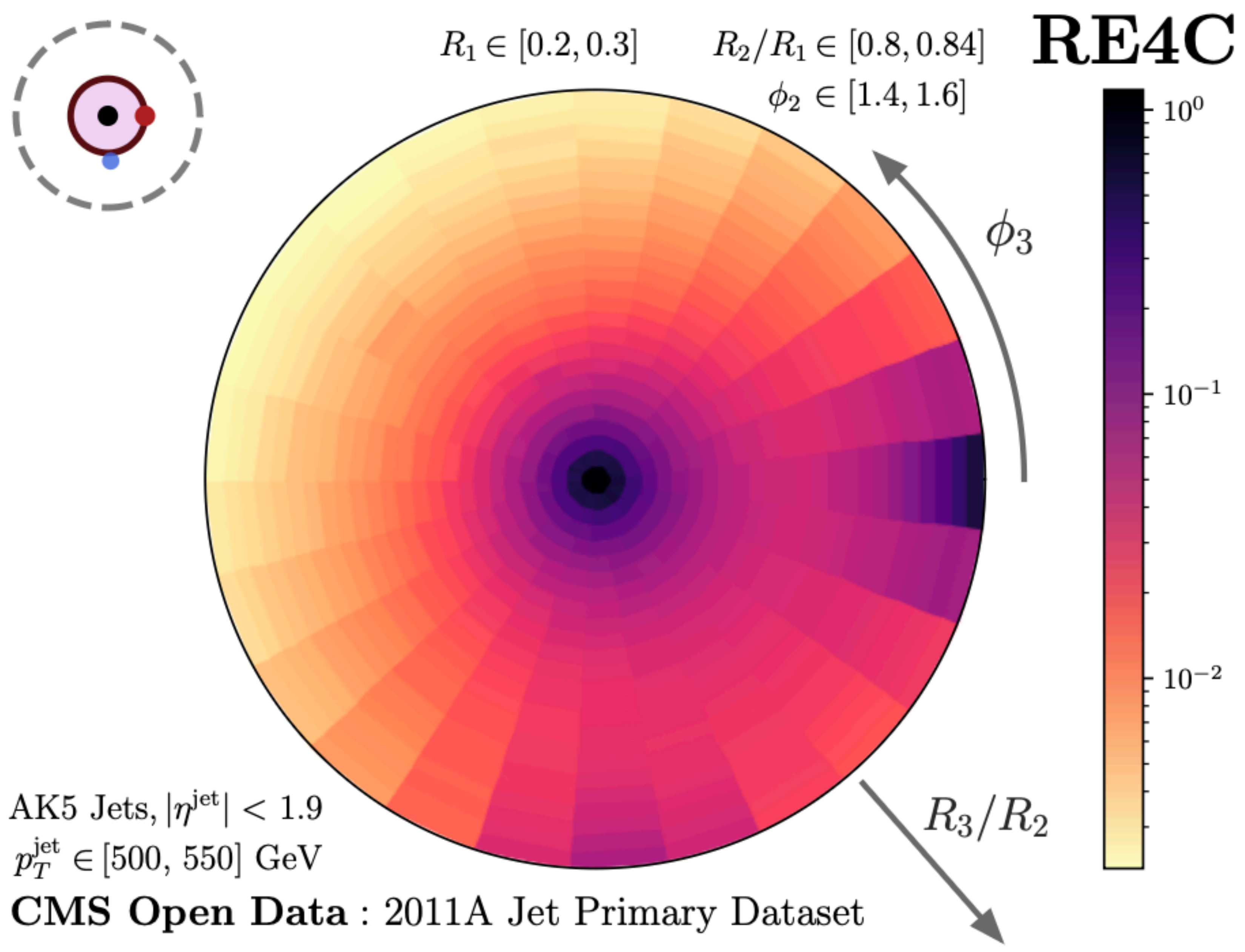
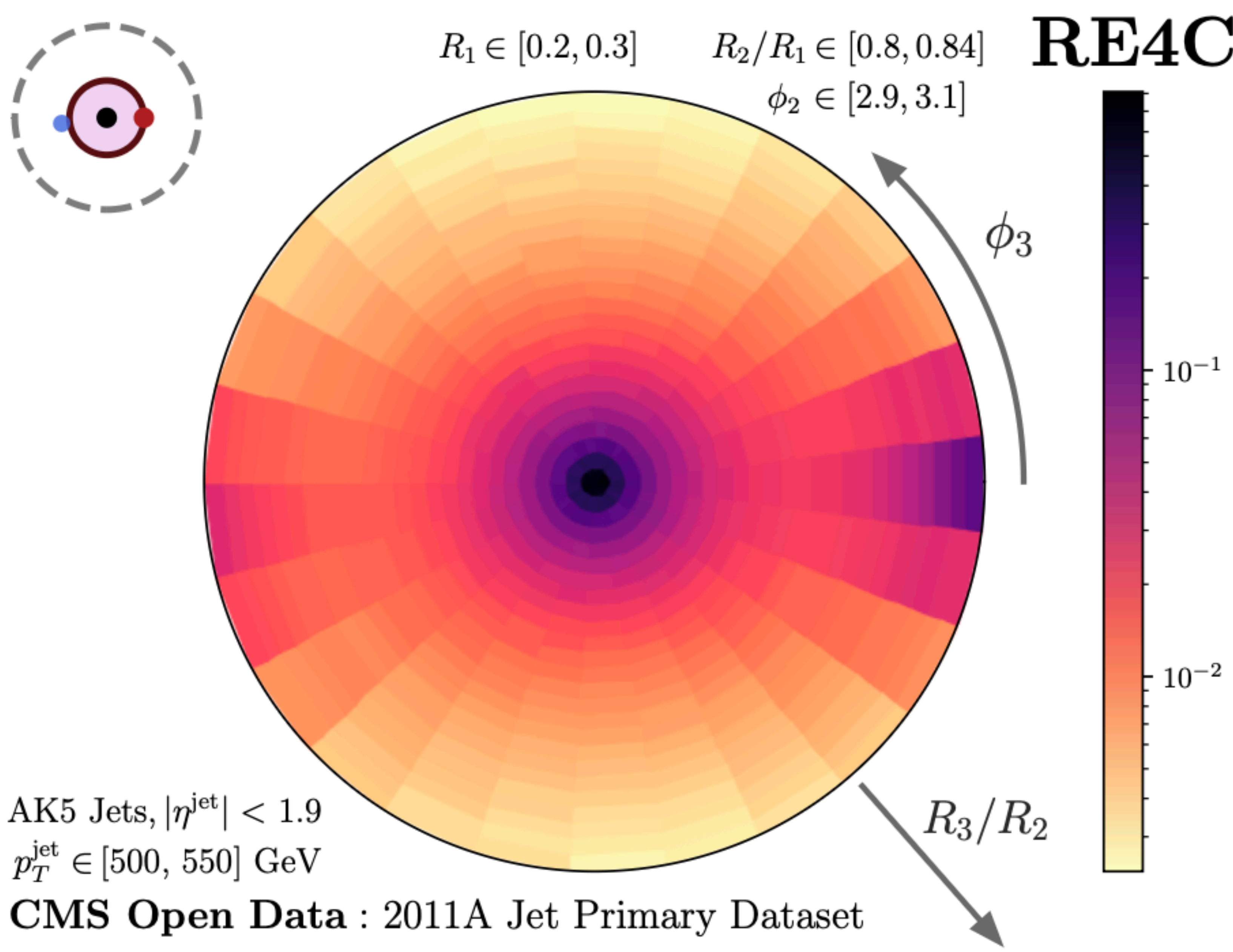
Just for fun (RE4C)







Summary & Outlook

- We developed an approximate approach to evaluate arbitrary ν-correlators on real jet samples and used it to extract QCD anomalous dimension from CMS Open Data.

- We proposed a new **non-redundant** parametrization that also preserves orientation.

- In perturbative region, differences are small $\sim \mathcal{O}(1\%)$. Leading non-perturbative corrections compared with MC data.

- Provides practical computational costs that can allow for potential future explorations. Theoretically nothing is lost, phenomenologically and experimentally everything is gained.

- Potential for future explorations with generalizations to more than one special particle as well high multiplicity environments of heavy ion collisions.

Thanks for your attention !

Backup

NLL equivalence

$$\sum_{n,k} d_{n,k}\alpha_s^{n+k}\log^n[R_1(1+c)] = \sum_{n,k} d_{n,k}\alpha_s^{n+k}[\log R_1 + \log(1+c)]^n$$

$$= \sum_{m,k,k'} \binom{m+k'}{m} d_{m+k',k}\alpha_s^{m+k+k'}\log^m R_1 \log^{k'}(1+c)\,,$$

$n = m+k'$