Nonperturbative Effects in Energy Correlators

based on ***JHEP 10 (2023) 187*** *(2305.19311)*
+ ***Phys.Rev.Lett. 133 (2024) 23, 231902*** *(2405.19396)*

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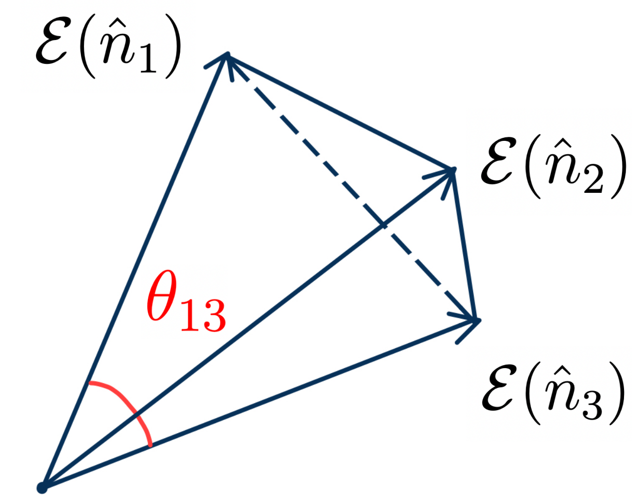
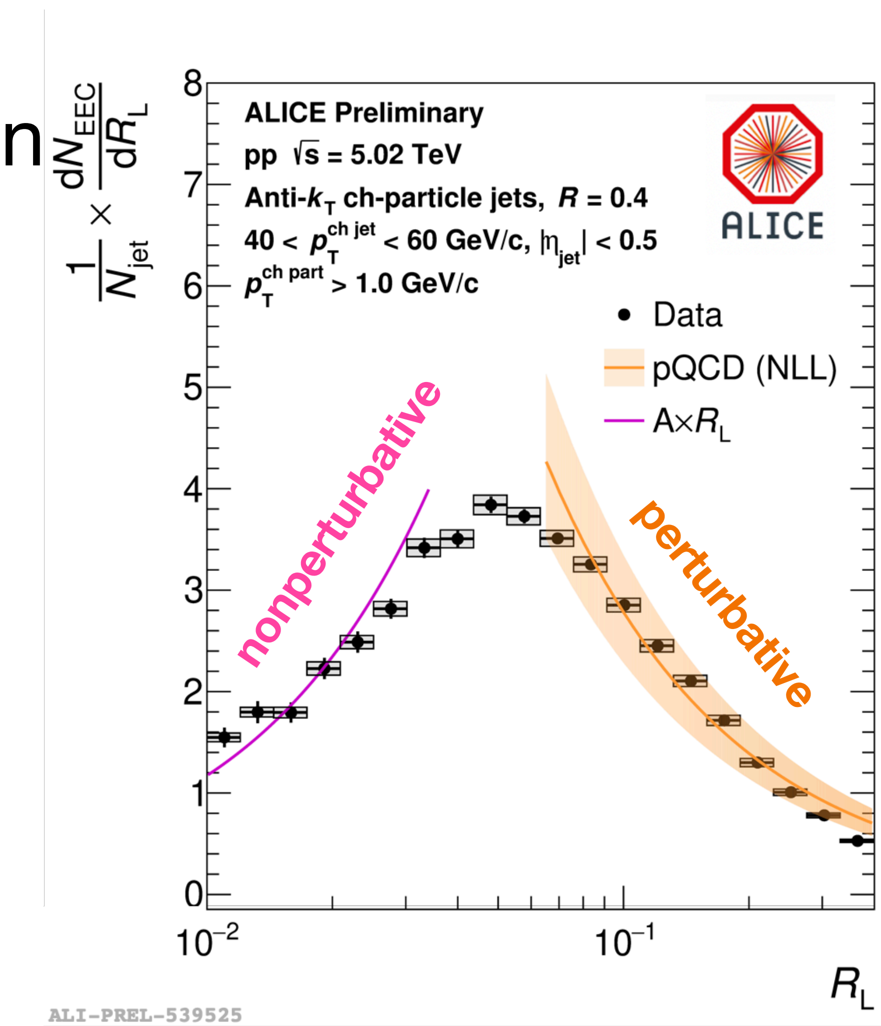
"New Opportunities in Particle and Nuclear Physics with Energy Correlators"
May 15, 2025

Outline

- Two-point energy correlator (EEC)
  - Renormalon and R-scheme
- Projected N-point energy correlator (pENC)
  - Universal nonperturbative correction
- Collinear region and small-angle resummation
  - Nonperturbative correction in the collinear limit (e^+e^-, pp)
  - Approaching the confinement transition
  - Impact on α_s extraction





Energy-energy correlator

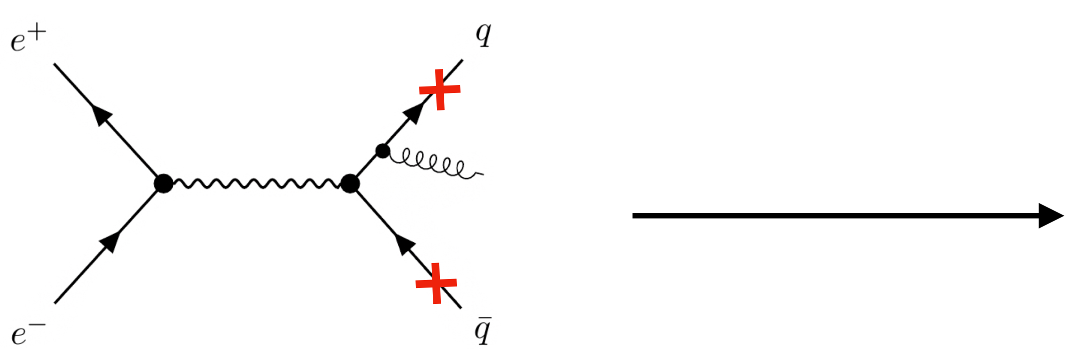
- Two-point energy correlator (EEC)

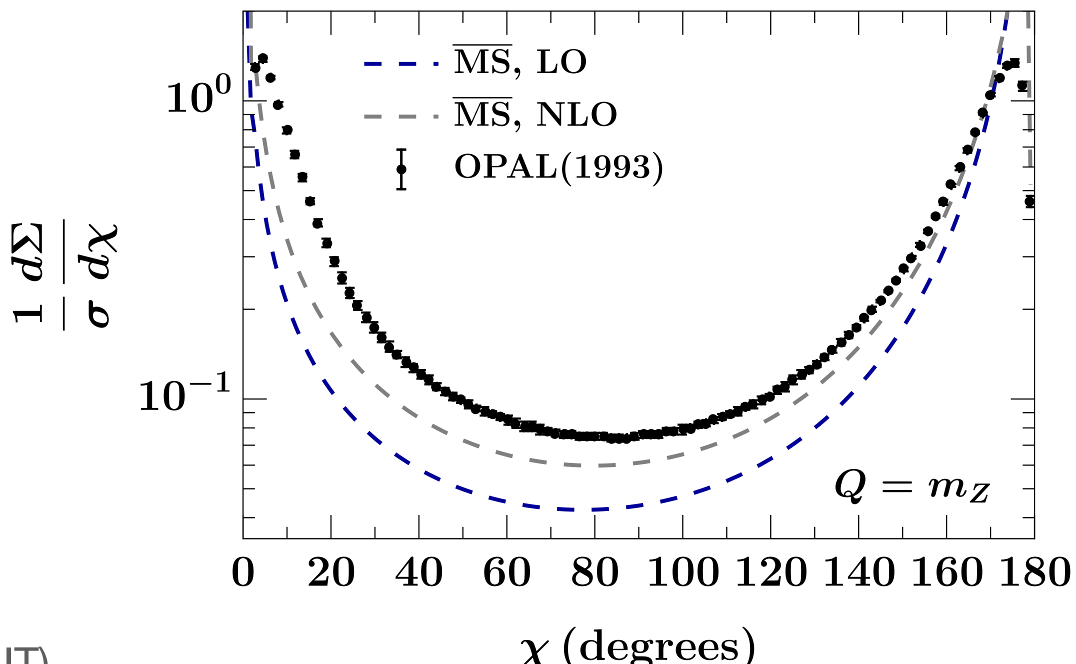
$$\frac{d\sigma^{[2]}}{dz} = \sum_X \int d\sigma_{e^+e^-\to X} \sum_{i,j\in X} \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$
$$= \int d^4x \frac{e^{iq\cdot x}}{Q^2} \int d\Omega_{\vec{n}_1} \int d\Omega_{\vec{n}_2} \delta\left(z - \frac{1-\vec{n}_1\cdot\vec{n}_2}{2}\right)$$
$$L_{\mu\nu} \times \langle 0 \left| J^{\mu\dagger}(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) J^\nu(0) \right| 0 \rangle$$

Energy flow operator

$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$$

- Perturbative calculation (analytic at NLO, numeric at NNLO)





Nonperturbative correction

- Event shape observables in e^+e^- collisions exhibit a universal leading nonperturbative correction

$$\frac{d\sigma}{de}(e) = \frac{d\hat{\sigma}}{de}\left(e - c_e \frac{\Omega_1}{Q}\right)$$

Event shape

$$\frac{d\Sigma}{dz} = \frac{d\hat{\Sigma}}{dz} + c_{\text{EEC}} \frac{\Omega_1}{Q}$$

EEC

- Nonperturbative matrix element $\bar{\Omega}_1$

C. Lee, G. F. Sterman, hep-ph/0603066, 0611061

$$\Omega_1 \equiv \frac{1}{N_c} \langle 0 | \, \mathrm{tr} \, \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \underbrace{\mathcal{E}_T(0)} Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

Transverse energy flow operator $\mathcal{E}_T(\eta) = \cosh^{-3}\eta \int d\phi \, \mathcal{E}(\vec{n})$

- c_e is analytically calculable

- Hadron mass corrections

G. P. Salam, D. Wicke, hep-ph/0102343
V. Mateu, I. Stewart, J. Thaler, 1209.3781

Renormalon and R scheme

- $\overline{\text{MS}}$ perturbative series has an **ambiguity** → renormalon

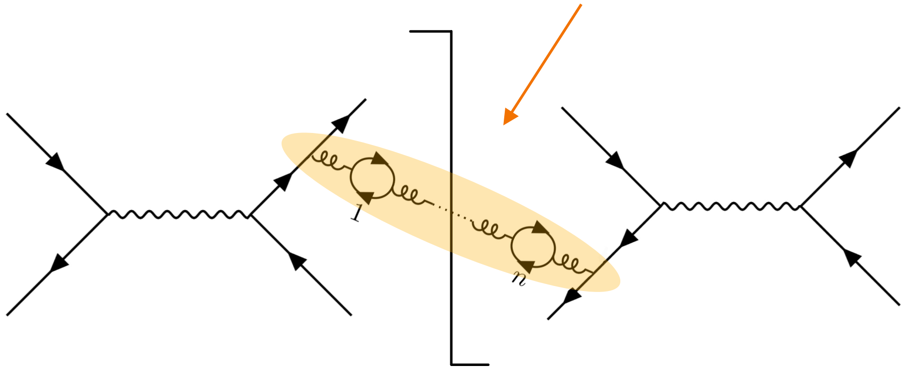
$z = \frac{1-\cos\chi}{2}$

renormalon ambiguity in EEC

$$\Delta_{1/2}\left(\frac{1}{\sigma_0}\frac{d\hat{\Sigma}}{dz}\right) = -\frac{8iC_F e^{5/6}}{\beta_0}\frac{1}{[z(1-z)]^{3/2}}\frac{\Lambda_{\text{QCD}}}{Q}$$

S. Schindler, I. Stewart, **ZS**, JHEP 10 (2023) 187

Probe with bubble sum



- Ambiguities must cancel in observable: can extract c_{EEC}

Known renormalon
A. H. Hoang, I. Stewart, 0709.3519

- $\overline{\text{MS}}$ scheme: $$\frac{1}{\sigma_0}\frac{d\Sigma}{dz} = \frac{1}{\sigma_0}\frac{d\hat{\Sigma}}{dz} + \frac{1}{2[z(1-z)]^{3/2}}\frac{\bar{\Omega}_1}{Q}$$

Renormalon we calculated

- R scheme: remove renormalon from both pert. series and $\bar{\Omega}_1$

A. H. Hoang, A. Jain, I. Scimemi, I. Stewart, 0803.4214, 0908.3189

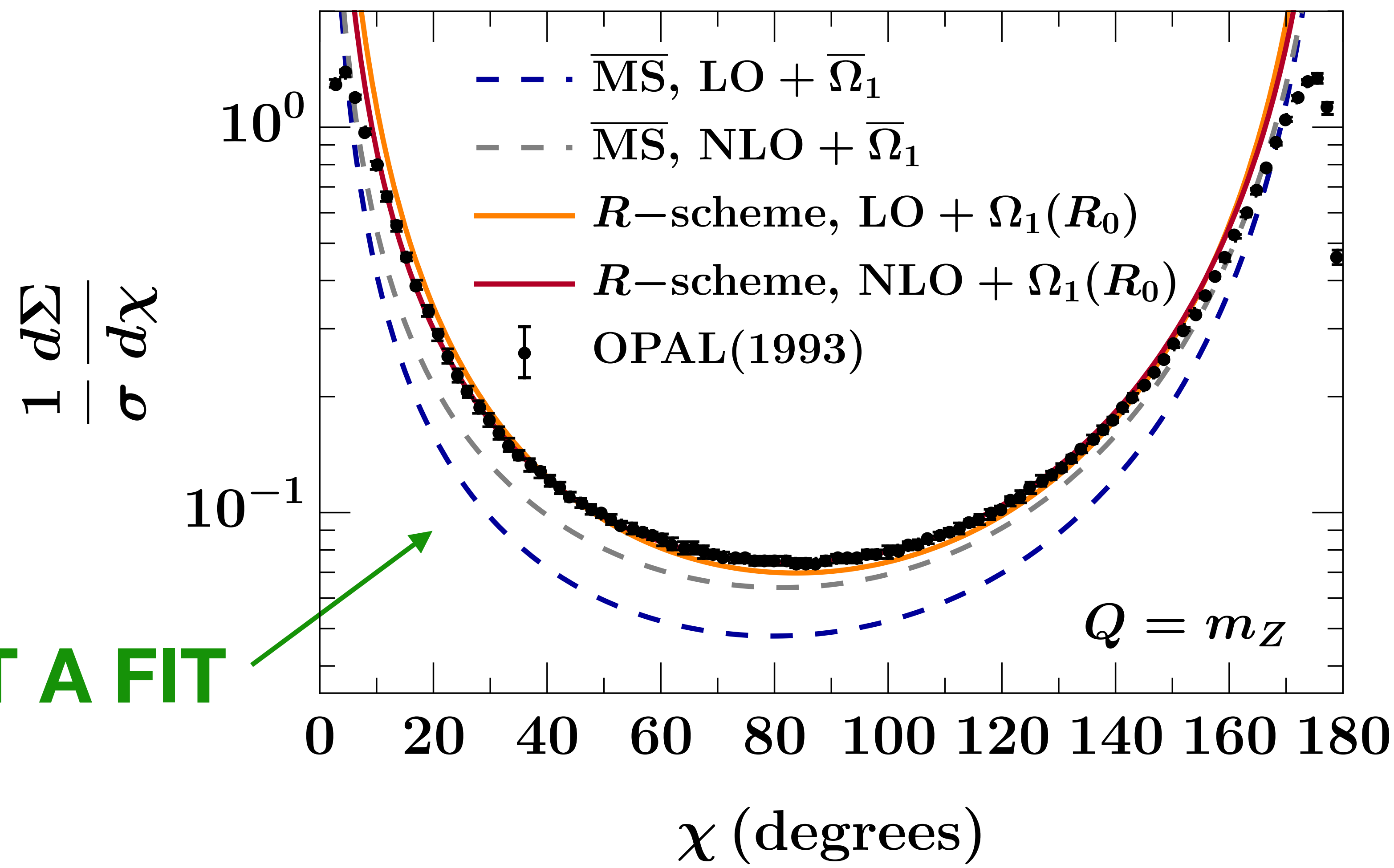
EEC w/ nonperturbative correction

S. Schindler, I. Stewart, **ZS**, JHEP 10 (2023) 187

- Construct EEC in R scheme with nonperturbative correction

$$\frac{1}{\sigma_0}\frac{d\Sigma}{dz} = \frac{1}{\sigma_0}\frac{d\hat{\Sigma}^{R-\text{scheme}}(R)}{dz} + \frac{1}{2\left[[z(1-z)]^{3/2}\right]_+}\frac{\Omega_1(R)}{Q}$$

- Test Ω_1 **universality** and improvement to convergence



NOT A FIT

Using fit value of Ω_1 and $\alpha_s(m_Z)$

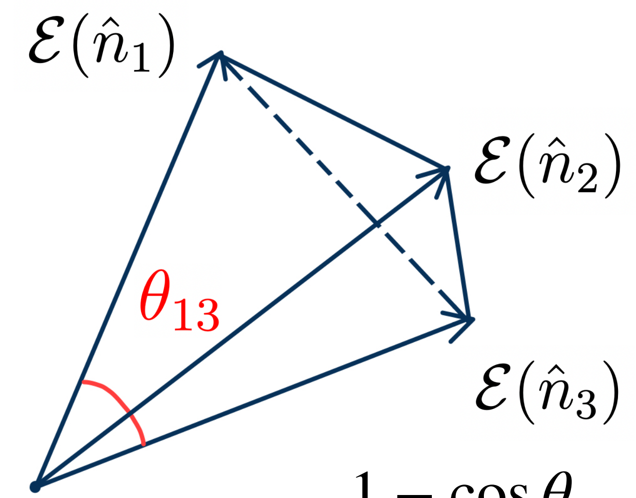
R. Abbate, M. Fickinger, A. H. Hoang, V. Mateu, I. Stewart, 1006.3080

Included hadron mass corrections

$$z = \frac{1-\cos\chi}{2}$$

Projected Energy Correlators

- N-point energy correlator:
  - Depends on $N(N-1)/2$ angles
- *Projected* N-point energy correlator (pENC):



$$z_{13} = \frac{1-\cos\theta_{13}}{2}$$

H. Chen, I. Moult, X-Y. Zhang, H-X. Zhu, 2004.11381

Integrate out information about the shape

$$\frac{d\Sigma^{[N]}}{dx_L} \equiv \sum_{i_1,\ldots,i_N} \int d\sigma \frac{\prod_{k=1}^{N} E_{i_k}}{Q^N} \delta\left(x_L - \max_{1\le l,m\le N}\{z_{lm}\}\right)$$

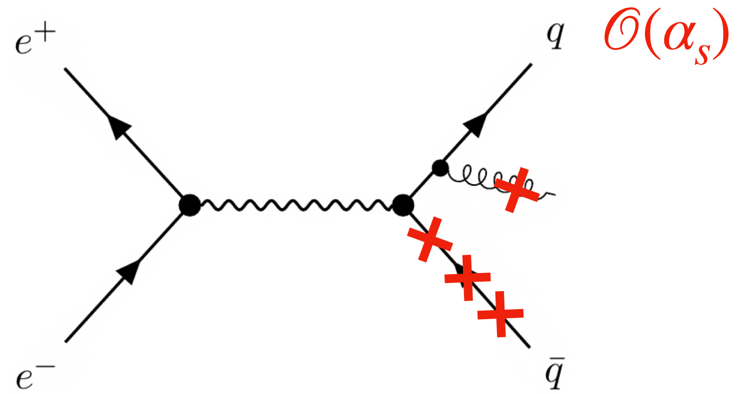
$$= \int d^4x \frac{e^{iq\cdot x}}{Q^N} \prod_{i=1}^{N} \int d\Omega_{\vec{n}_i} \delta\left(x_L - \frac{1-\min\left(\vec{n}_i\cdot\vec{n}_j\right)}{2}\right)$$
$$\times L_{\mu\nu} \left\langle 0 \left| J^{\mu\dagger}(x) \mathcal{E}\left(\vec{n}_1\right) \mathcal{E}\left(\vec{n}_2\right) \ldots \mathcal{E}\left(\vec{n}_N\right) J^\nu(0) \right| 0 \right\rangle$$

Energy flow operator $\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$

Leading NP Correction to pENC

C. Lee, G. F. Sterman, hep-ph/0603066, 0611061

- The **same** matrix element Ω_1 gives *leading* (in α_s) NP correction for pENC



$$\Omega_1 \equiv \frac{1}{N_c}\langle 0|\,\mathrm{tr}\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(0) Y_n \overline{Y}_{\bar{n}}\,|0\rangle$$

Transverse energy flow operator $\mathcal{E}_T(\eta) = \cosh^{-3}\eta \int d\phi\, \mathcal{E}(\vec{n})$

- The same calculation as for the EEC gives x_L dependence; detector combinatorics gives factor of N

K. Lee, A. Pathak, I. Stewart, **ZS**, Phys.Rev.Lett. 133 (2024) 23, 231902

$$\frac{1}{\sigma}\frac{d\sigma^{[N]}}{dx_L} = \frac{1}{\sigma}\frac{d\hat{\sigma}^{[N]}}{dx_L} + \frac{N}{2^N}\frac{\overline{\Omega}_1}{Q(x_L(1-x_L))^{3/2}}$$

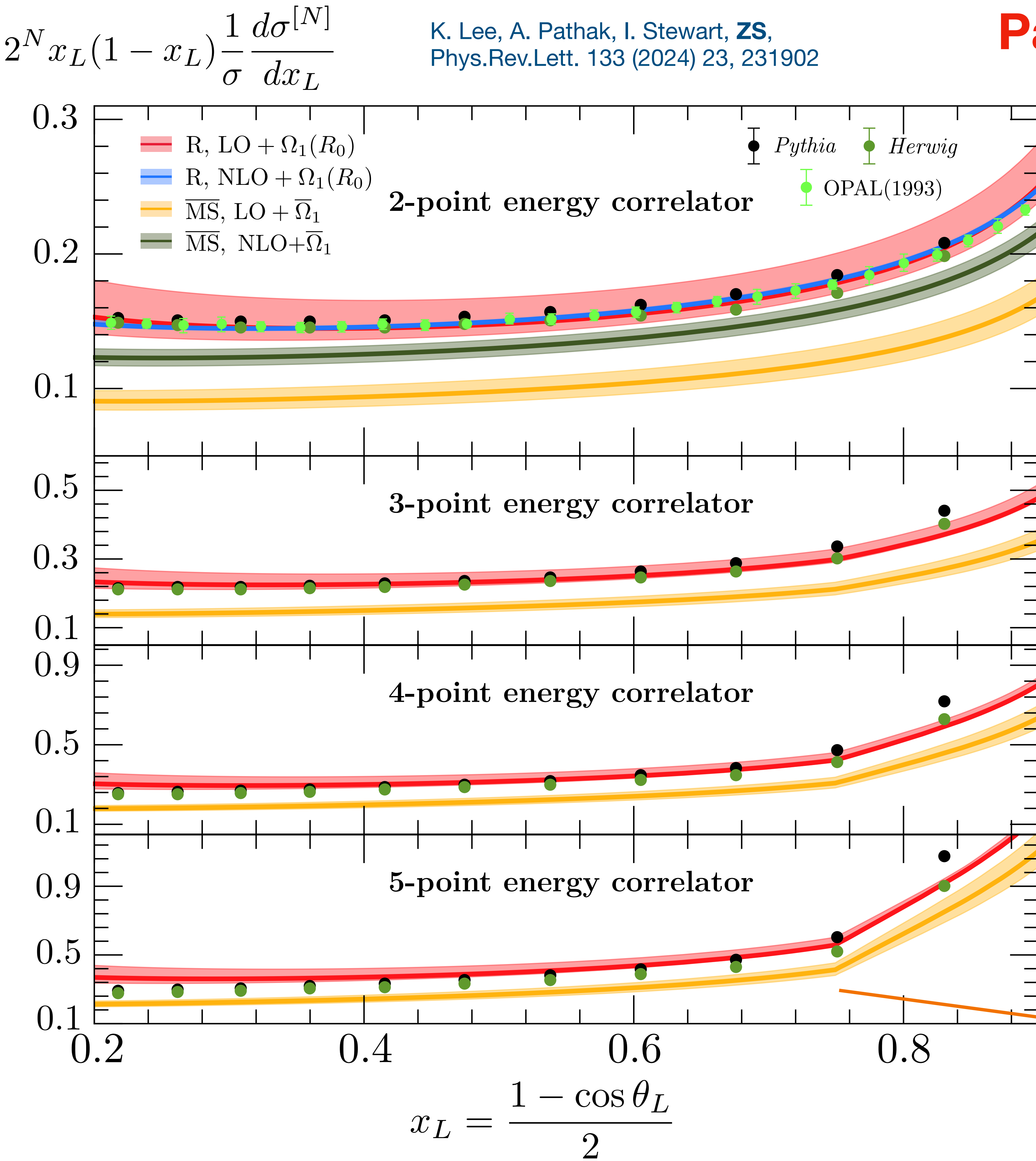
$\overline{\mathrm{MS}}$ scheme

*At higher order in α_s , the x_L and N dependence can be different

- Translate to R scheme for better convergence

Parameter free predictions!

K. Lee, A. Pathak, I. Stewart, **ZS**, Phys.Rev.Lett. 133 (2024) 23, 231902



α_s and Ω_1 values are from thrust fit

R. Abbate, M. Fickinger, A. H. Hoang, V. Mateu, I. Stewart, 1006.3080

EEC results: agree with MC and data

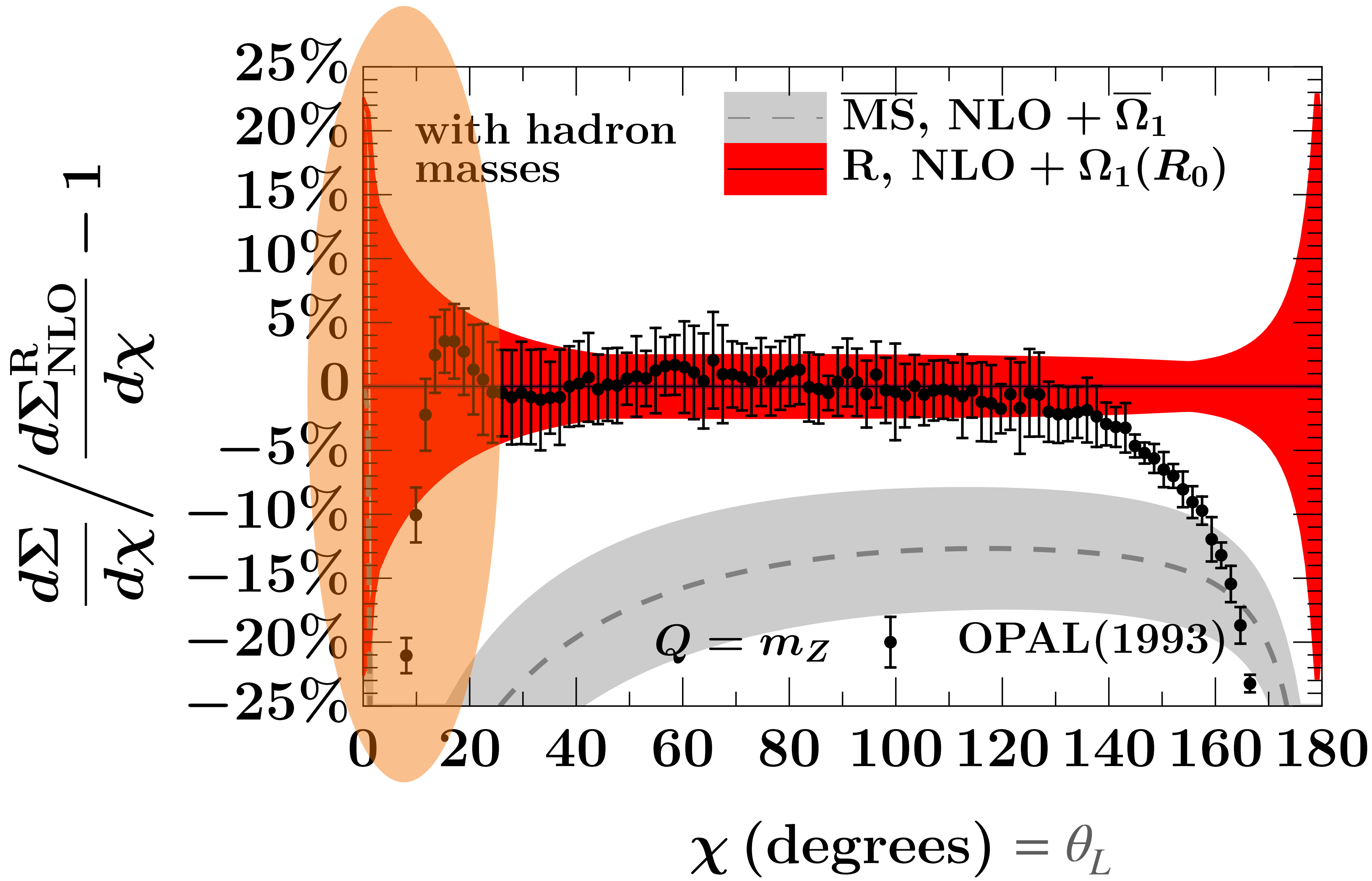
pENC results: R scheme agrees with MC better than $\overline{\text{MS}}$

Kink at $x_{L_0} \leq \frac{1-\cos(2\pi/3)}{2}$ from 3-particle contribution

Small angle region

S. Schindler, I. Stewart, **ZS**, JHEP 10 (2023) 187

- Fixed-order EEC results don't agree well with data at endpoints



- Need to resum $\log(x_L)$ in collinear region $\quad x_L = \frac{1 - \cos\theta_L}{2} \to 0$

*in back-to-back region, 3-particle contribution also gives leading NP correction, so the situation is more complicated for N>2

Collinear factorization for pENC

- Factorize into jet and hard function (true for e^+e^- and pp)

H. Chen, I. Moult, X-Y. Zhang, H-X Zhu, 2004.11381
W. Chen, J. Gao, Y-B. Li, Z. Xu, X-Y. Zhang, H-X Zhu, 2307.07510
K. Lee, B. Mecaj, I. Moult, 2205.03414

Cumulative distribution →

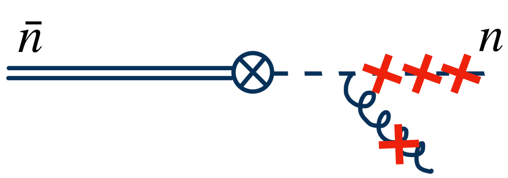
$$\Sigma^{[N]}(x_L) = \frac{1}{\sigma}\int_0^{x_L} dx'_L \frac{d\sigma^{[N]}}{dx'_L}$$
$$= \int_0^1 dx\, x^N\, \vec{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2}, \mu\right) \cdot \vec{H}\left(x, \frac{Q^2}{\mu^2}, \mu\right)$$

pENC jet function captures x_L dependence

- Leading nonperturbative correction arises in $\vec{J}^{[N]}$ and the same Ω_1 will appear

$$\sum_X \sum_{i_1 \ldots i_N \in X} \langle 0|\chi_n|X\rangle \frac{\Pi_{k=1}^N E_{i_k}}{Q^N} \Theta\big(\max_{1\le l,m\le N}\{z_{lm}\} < x_L\big) \langle X|\bar{\chi}_n|0\rangle$$

$W^\dagger(\bar{n}\cdot A_n)\xi_n$ leading NP correction $\bar{\xi}_n W(\bar{n}\cdot A_n)$



$$\langle 0|\overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \quad \mathcal{E}_T \quad Y_n \overline{Y}_{\bar{n}}|0\rangle$$

NP Correction to Jet Function

- Scale of the jet function is $Q' = \sqrt{x_L}\, x\, Q$

$$2^N J^{[N]}\left(\ln\frac{Q'}{\mu},\mu\right) = 2^N \hat{J}^{[N]}\left(\ln\frac{Q'}{\mu},\mu\right) + c^{\mathrm{LL}}\frac{\overline{\Omega}_1}{Q'} + \mathcal{O}\left(\frac{\alpha_s(Q')\Lambda_{\mathrm{QCD}}}{Q'}\right)$$

- Valid up to *leading log* accuracy

$$c^{\mathrm{LL}} = -N\, 2^{N-1}\, \hat{\mathcal{J}}^{[N-1]}\left(\ln\frac{Q'}{\mu},\mu\right)$$

additional x^{-1} from Q'

leading log coefficient

- RG consistency: $\hat{\mathcal{J}}^{[N]}$ has the same evolution structure as $\hat{J}^{[N]}$

$$\frac{\mathrm{d}\vec{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2}\right)}{\mathrm{d}\ln\mu^2} = \int_0^1 \mathrm{d}y\, y^N \vec{J}^{[N]}\left(\ln\frac{x_L y^2 Q^2}{\mu^2}\right)\cdot\hat{P}(y)$$

Splitting functions

H. Chen, I. Moult, X-Y. Zhang, H-X Zhu, 2004.11381

NP Correction to Jet Function

K. Lee, A. Pathak, I. Stewart, **ZS**,
Phys.Rev.Lett. 133 (2024) 23, 231902

- Putting everything together

$$2^N J^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right) = \underbrace{2^N \hat{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right)}_{\text{perturbative series in } \overline{\mathrm{MS}}} - \underbrace{\frac{N\overline{\Omega}_1}{\sqrt{x_L}xQ}\, 2^{N-1}\, \hat{\mathcal{J}}^{[N-1]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right)}_{\text{leading NP correction at LL accuracy}}$$

- Move to R scheme

$$\Omega_1(R) \equiv \overline{\Omega}_1 - R\sum_{n=1}^{\infty} d_n\left(\frac{\mu}{R}\right)\left[\frac{\alpha_s(\mu)}{4\pi}\right]^n$$

$$2^N \hat{J}_{\mathrm{R}}^{[N]}\left(\ln\frac{x_L Q^2}{\mu^2},\mu\right) \equiv 2^N \hat{J}^{[N]}\left(\ln\frac{x_L Q^2}{\mu^2},\mu\right) - \sum_{n=1}^{\infty}\frac{N\,R}{Q\sqrt{x_L}}\, d_n(\mu/R)\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n$$

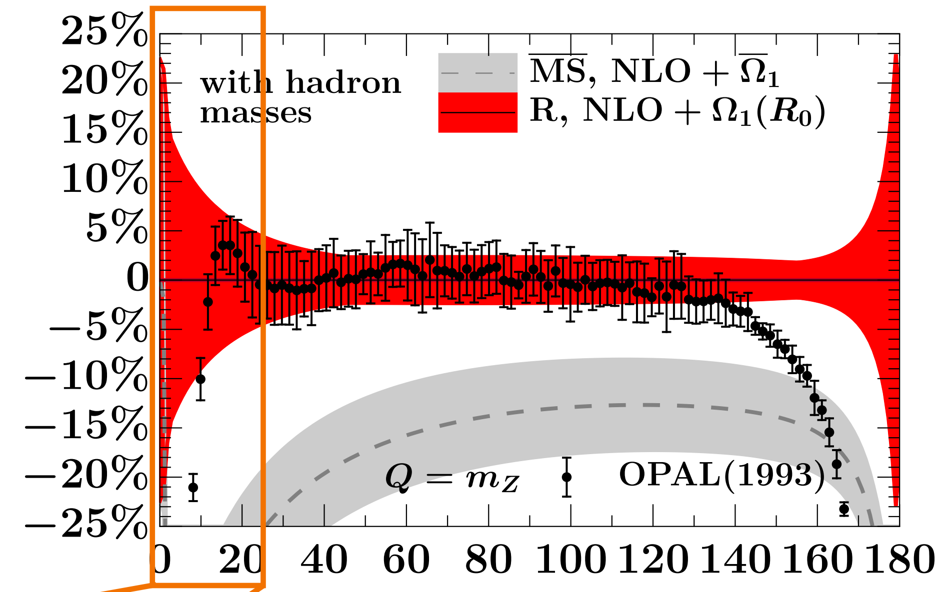
- Construct cross section *in collinear limit* with factorization

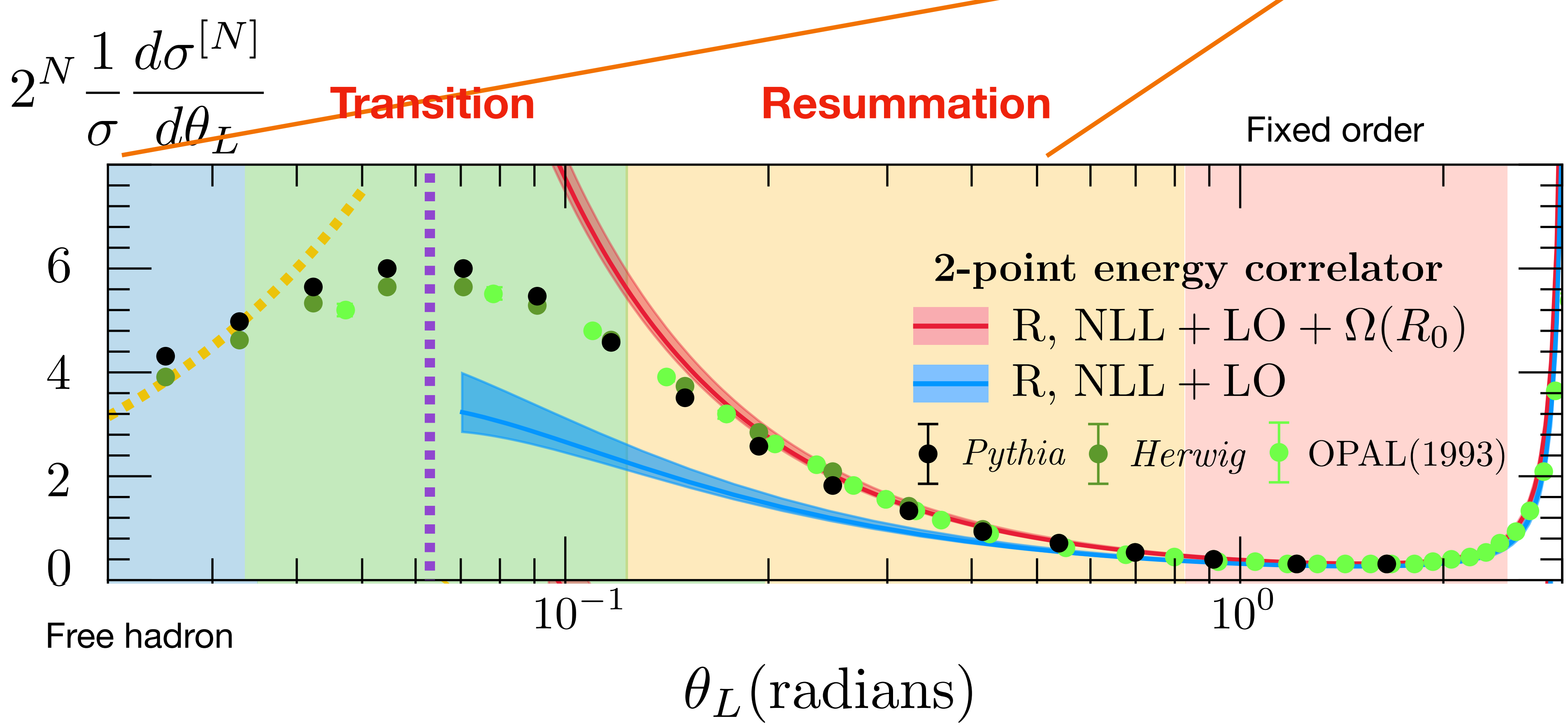
$$\Sigma^{[N]}(x_L) = \int_0^1 dx\, x^N\, \vec{J}^{[N]}\left(\ln\frac{x_L x^2 Q^2}{\mu^2},\mu\right)\cdot \vec{H}\left(x,\frac{Q^2}{\mu^2},\mu\right)$$

Confinement transition $x_L \to 0$

- Including resummation significantly improves our description of the approach to the transition region

- Nonperturbative effects are essential





Identify location of peak at $2\Omega_1/(Q\sqrt{x_{L,\,\text{peak}}}) \sim 2/3$

Confinement transition $x_L \to 0$

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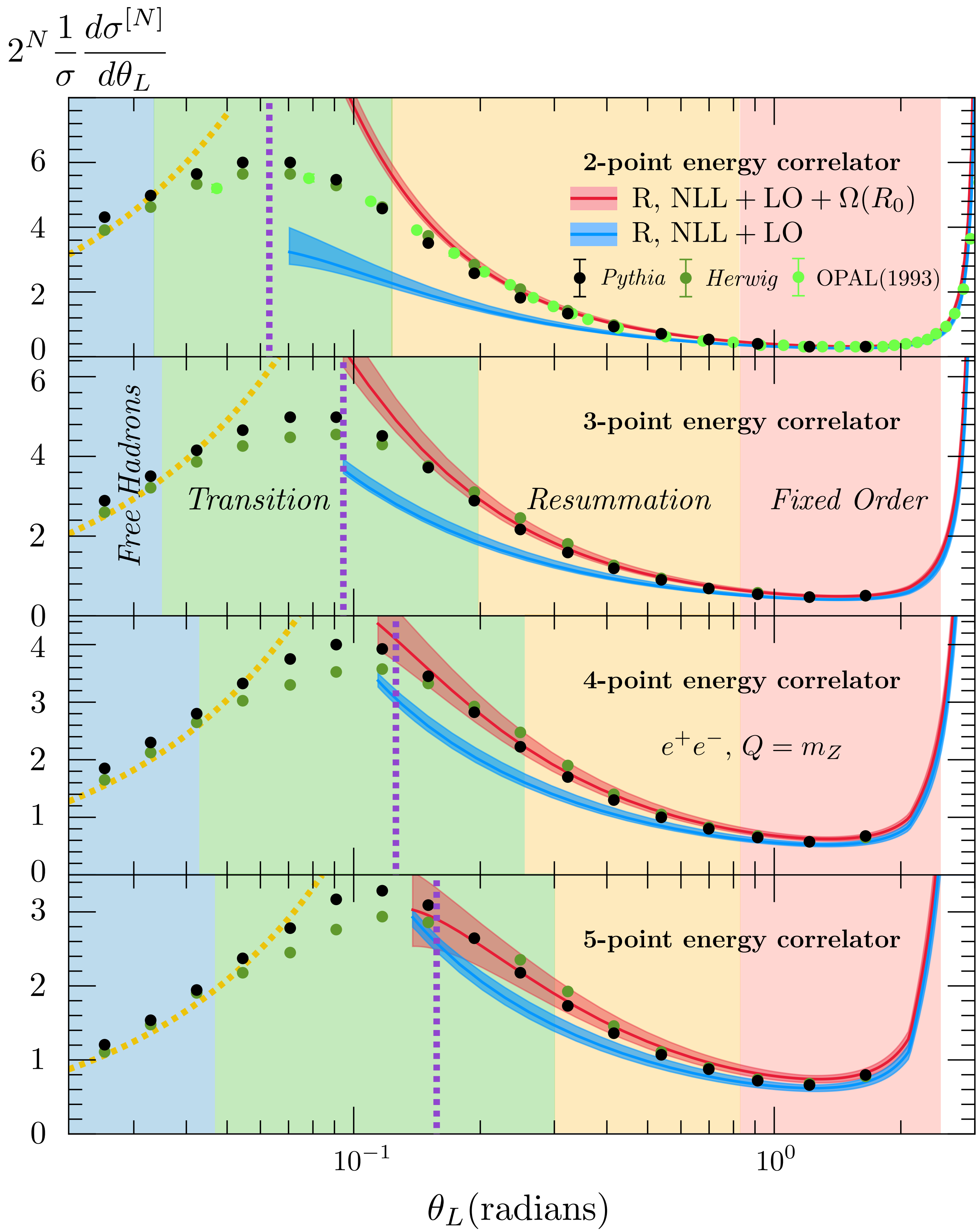
Phys.Rev.Lett. 133 (2024) 23, 231902



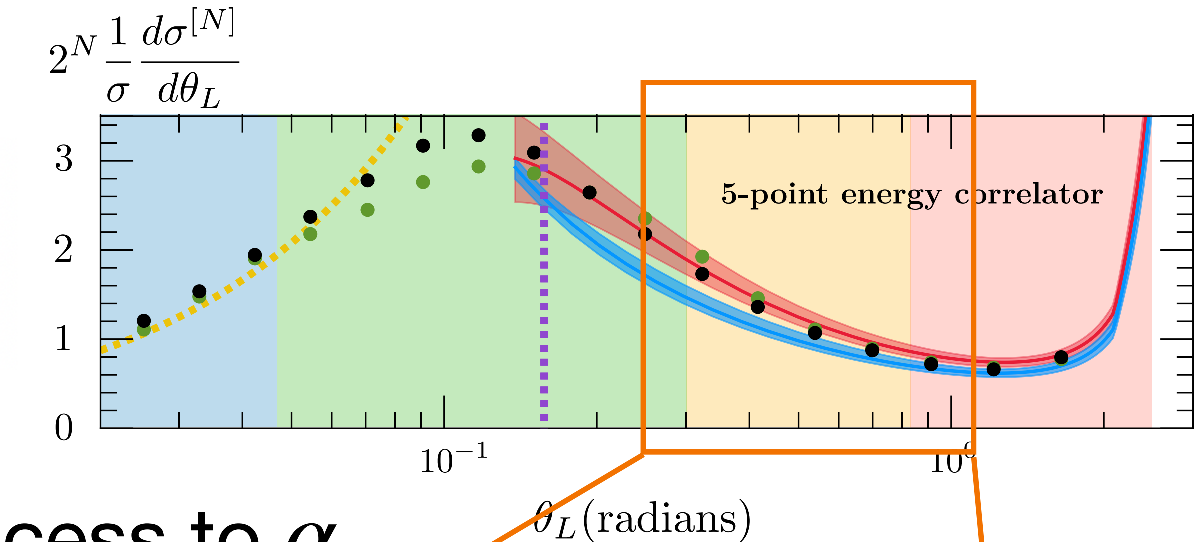
- Predict location of peak for pENC

$$N\Omega_1/(Q\sqrt{x_{L,\mathrm{peak}}}) \sim 2/3$$

- **Including NP correction is essential** to agree with hadron-level MC
- Normalizing to the total cross section (not just perturbative region!)

Ratio of pENC/EEC

- In collinear region: $\dfrac{d\hat{\sigma}^{[N]}}{dx_L} \sim x_L^{\gamma(N)-1}$



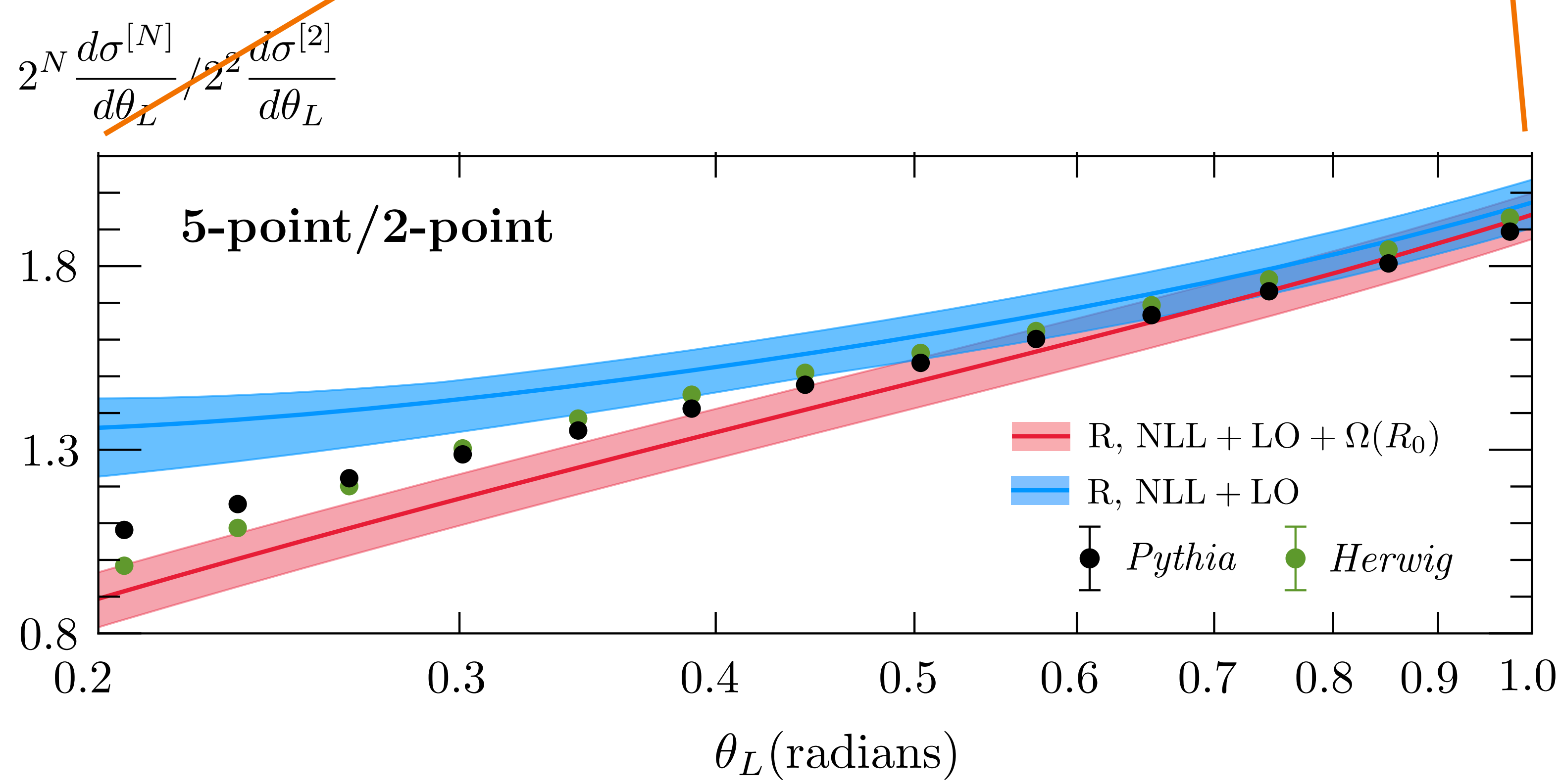
- Ratios of energy correlators give access to α_s

$$\frac{d\hat{\sigma}^{[N]}/dx_L}{d\hat{\sigma}^{[M]}/dx_L} \sim x_L^{\gamma(N)-\gamma(M)} \sim x_L^{\alpha_s[\gamma^{(0)}(N)-\gamma^{(0)}(M)]}$$

CMS 2402.13864: using pE3C/EEC

$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

- NP correction changes linear scaling behavior!



Scaling behavior

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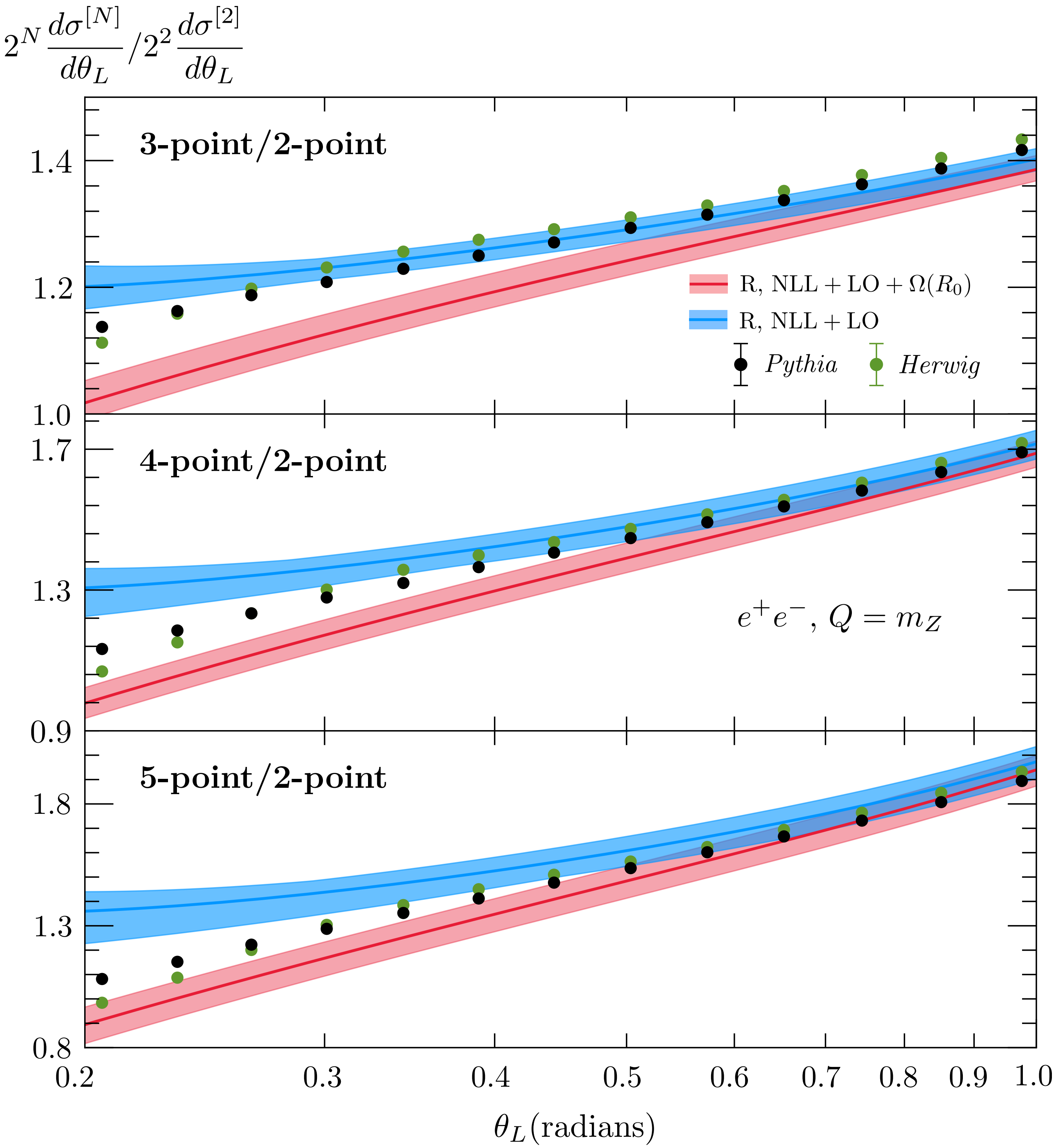

- Close to transition region, hadron-level MC deviates from linear slope
- Our model-independent NP correction captures this deviation well
- NP effects **do not completely cancel** in ratios!

Impact on α_s extraction

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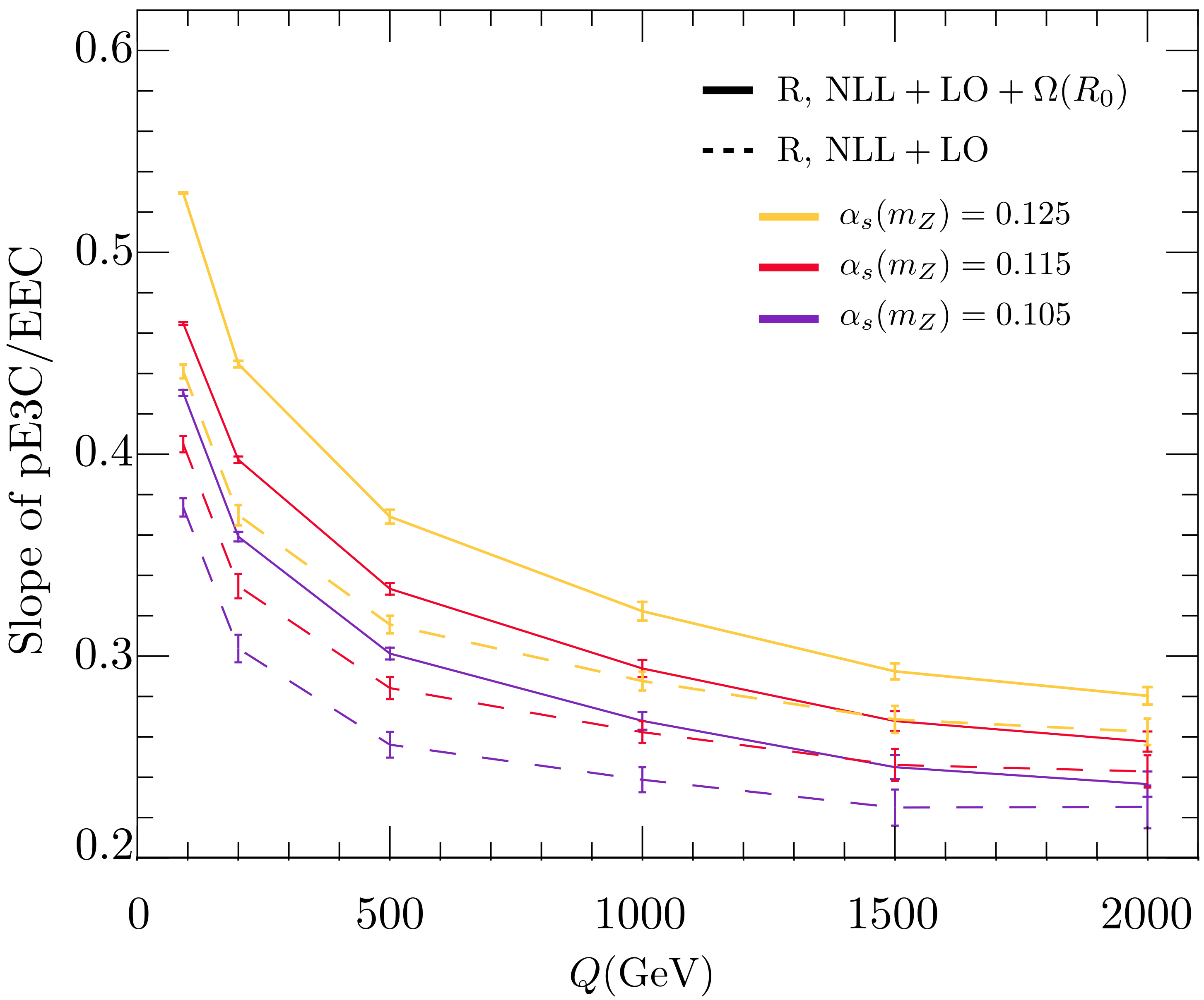
Phys.Rev.Lett. 133 (2024) 23, 231902



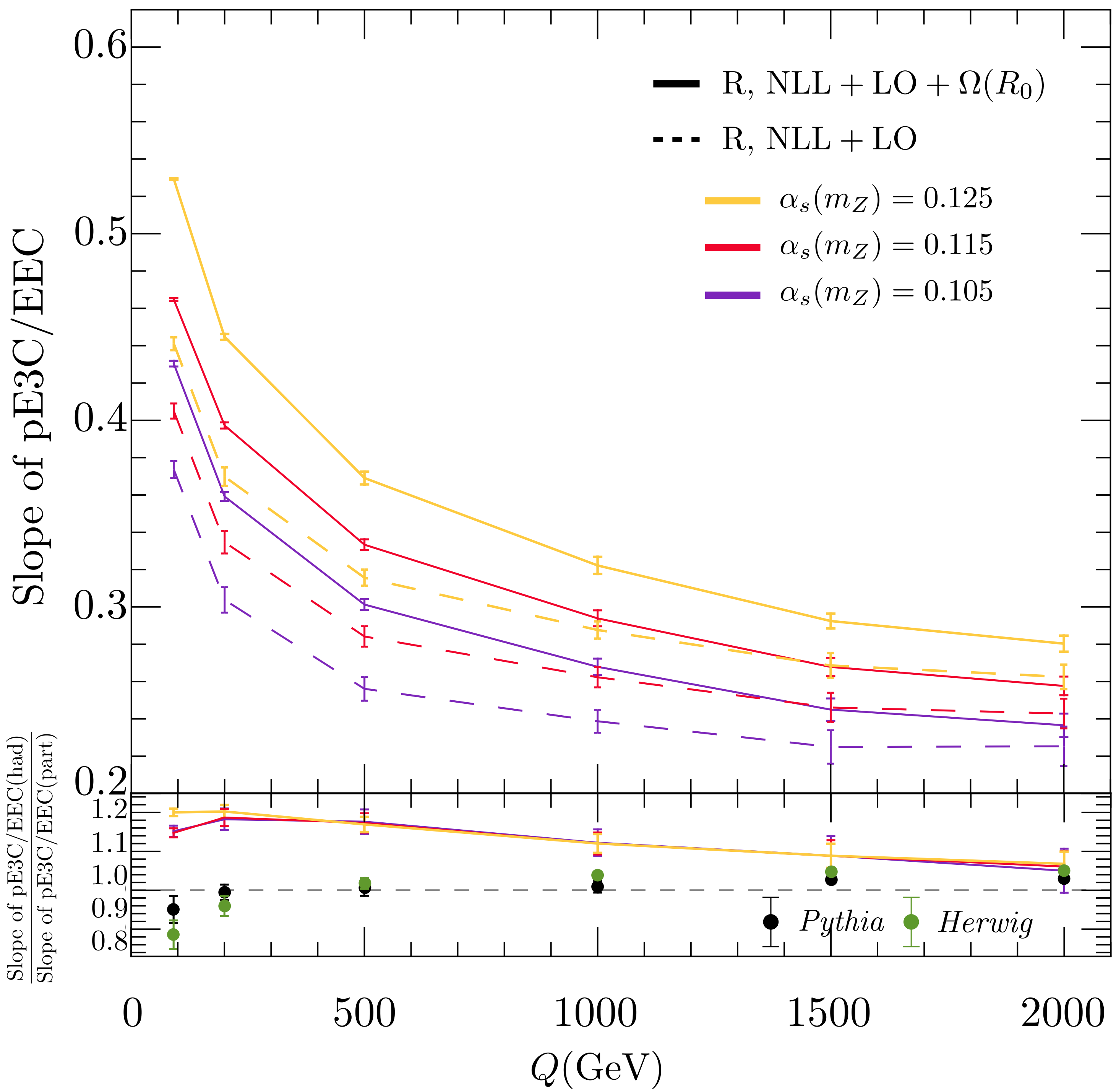
CMS 2402.13864: using pE3C/EEC

$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

- Extract slope away from transition region
- Asymptotic freedom as α_s decrease with high energy
- Including NP corrections **significantly impact the slope**!
- Degenerate effect between including NP correction and decreasing α_s

Impact on α_s extraction

K. Lee, A. Pathak, I. Stewart, **ZS**,
Phys.Rev.Lett. 133 (2024) 23, 231902



- NP effects **do not completely cancel** in ratios!
- Lower panel: difference between parton and hadron level prediction
- **Hadronization effects are larger than standard prediction from MC(had - part)!**

Summary

- We give a **model-independent** prediction of the leading nonperturbative correction to the projected energy correlator
  - Derive N and x_L dependence
  - The **universal matrix element** Ω_1 shows up for all angle cross section and collinear jet function
- We show that the x_L-resummed R scheme series with nonperturbative correction better captures the approach to **confinement transition**
- We demonstrate in a model-independent way that including nonperturbative corrections has a more **significant impact on** α_s **extraction** than predicted by MCs

BACKUP SLIDES

Renormalon Analysis

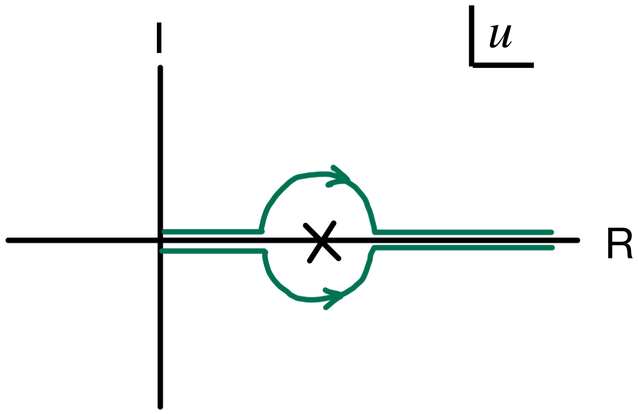
- Ambiguity in perturbative series → nonperturbative correction

$$\Delta_{1/2}\left(\frac{1}{\sigma_0}\frac{d\hat{\Sigma}}{dz}\right) = \oint_{u=1/2} du \, \exp\left[-u\frac{4\pi}{\beta_0\alpha_s(\mu)}\right]\frac{\mathrm{Res}_{1/2}}{u-1/2}$$

Inverse Borel transformation

Ambiguity in perturbative series
from u=1/2 renormalon

Residue around u=1/2 pole
in Borel plane



- Observables are physical and have no ambiguity

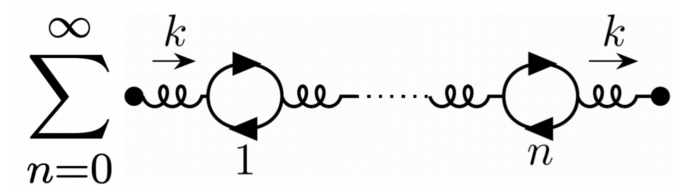
- Provide a cross check on the coefficient of Ω_1

$$\frac{d\Sigma}{dz} = \frac{d\hat{\Sigma}}{dz} + c_{\mathrm{EEC}}\frac{\Omega_1}{Q}$$

Renormalon in the EEC

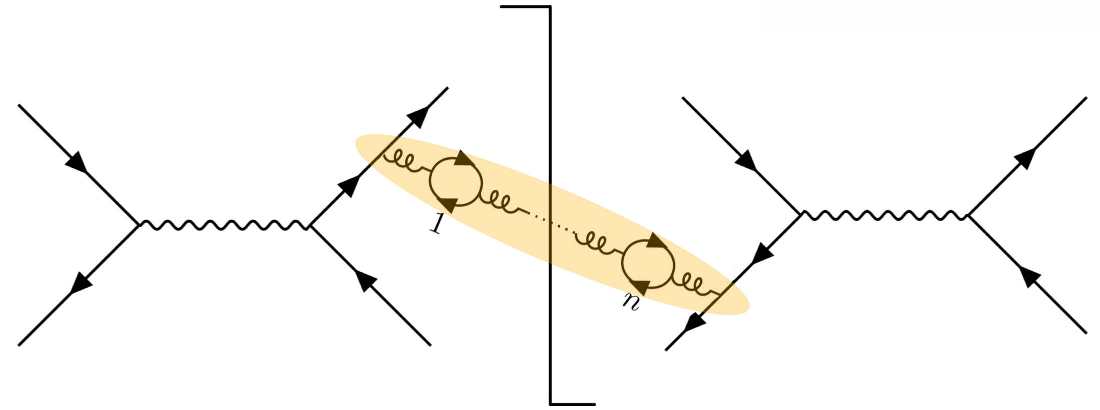
S. Schindler, I. Stewart, **ZS**, 2305.19311

- Leading renormalon: bubble sum approximation

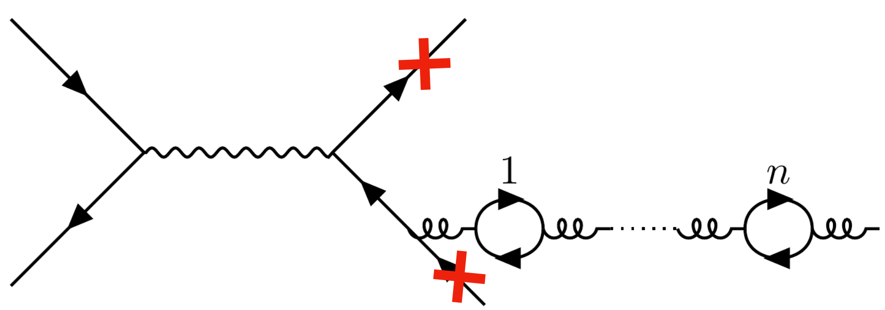


- "Probe" gluon

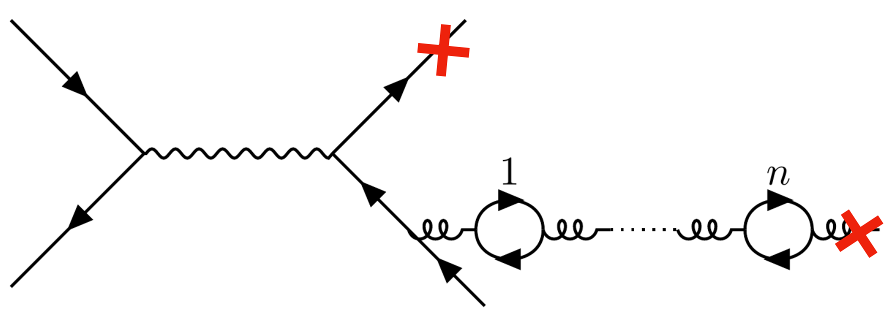
$$\frac{-1}{(-k^2 - i0)^{1+u}} \frac{4\pi}{\beta_0} (\mu^2 e^c)^u$$



- Renormalon ambiguity in the qg or $\bar{q}g$ contribution only



$u = 1/2$ residue = 0



$u = 1/2$ residue $\neq 0$

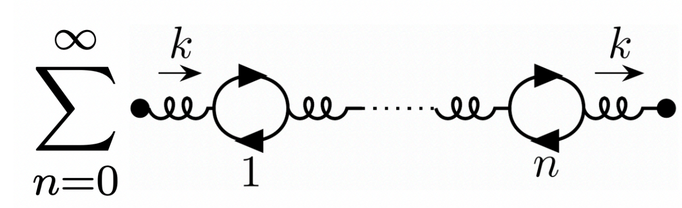
$$x = \frac{E}{Q}$$

$$\mathcal{I}_{qg} \equiv (1-z) \int_0^1 dx \, \frac{x(1-x)^{-2u}}{(1-xz)^{4-2u}} \left[1 - 4xz + x^2(1+2z+4z^2) - 2x^3 z(1+2z) + 2x^4 z^2\right]$$

Renormalon in the EEC

S. Schindler, I. Stewart, **ZS**, 2305.19311

- Leading renormalon: bubble sum approximation



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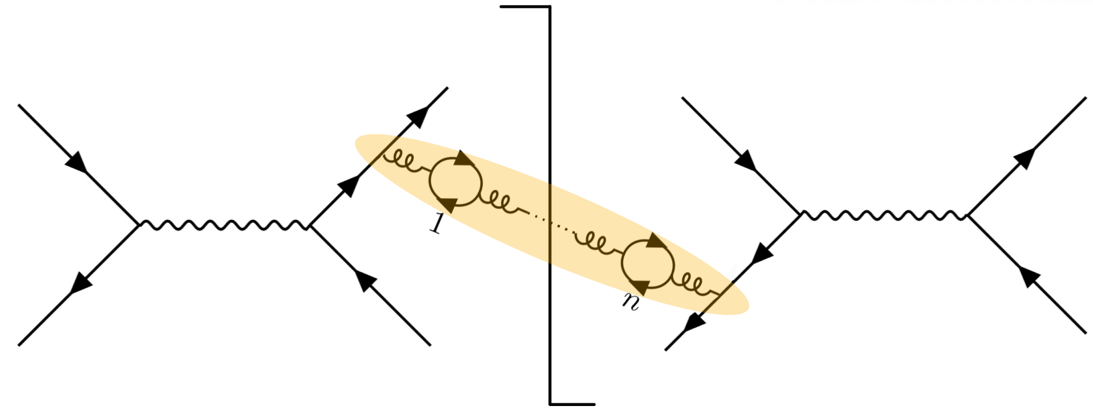

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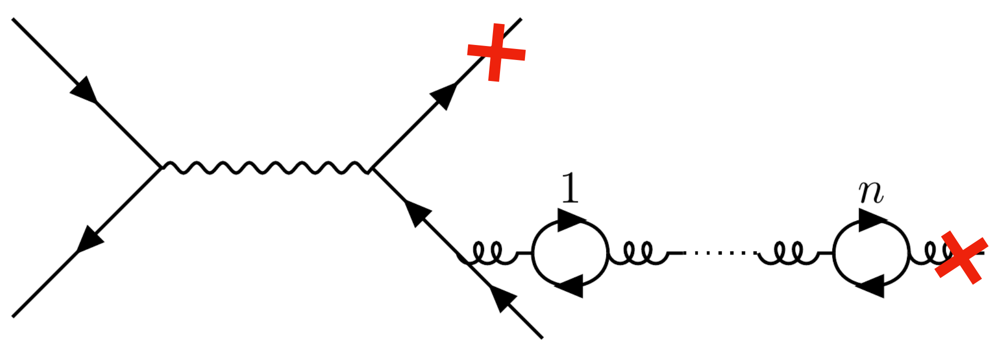
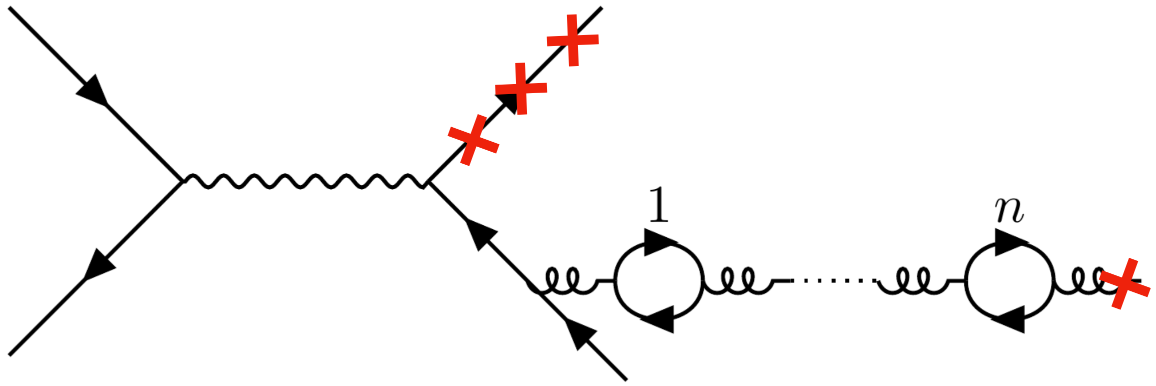


$u = 1/2$ residue $\neq 0$

$$\Delta_{1/2}\left(\frac{1}{\sigma_0}\frac{d\hat{\Sigma}}{dz}\right) = -\frac{8\mathrm{i}C_F e^{5/6}}{\beta_0} \frac{1}{[z(1-z)]^{3/2}} \frac{\Lambda_{\mathrm{QCD}}}{Q}$$

$$z = \frac{1-\cos\chi}{2}$$

Renormalon in pENC



- One "probe" gluon contribution

$$\frac{2}{Q^N}\binom{N}{1}E^{N-1}E_g$$

$$\frac{d\Sigma^{[N]}}{dx_L} \equiv \sum_{i_1,\ldots,i_N} \int d\sigma \, \frac{\prod_{k=1}^{N} E_{i_k}}{Q^N} \delta\left(x_L - \max_{1\le l,m\le N}\{z_{lm}\}\right)$$

$$z_{ij} = \frac{1-\cos\theta_{ij}}{2}$$

- Modified qg renormalon residue:

$$\mathcal{I}_{qg}^{N-pt} = \int_0^1 dx \, \frac{x^{N-1}(1-x)^{-2u}\left(2x^4z^2 - 2x^3(2z^2+z) + x^2(4z^2+2z+1) - 4xz + 1\right)}{(1-xz)^{4-2u}}$$

$$x = \frac{E}{Q}$$

$$\Delta_{1/2}\left(\frac{1}{\sigma_0}\frac{d\hat{\Sigma}^{[N]}}{dx_L}\right) = \frac{N}{2^{N-1}}\Delta_{1/2}\left(\frac{1}{\sigma_0}\frac{d\hat{\Sigma}^{[2]}}{dx_L}\right)$$

$$= -\frac{N}{2^{N-1}}\frac{8\mathrm{i}C_F e^{5/6}}{\beta_0}\frac{1}{[x_L(1-x_L)]^{3/2}}\frac{\Lambda_{\mathrm{QCD}}}{Q}$$

Same residue from integral