Energy pattern for the odderon: tracks, spin, and target fragmentation

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May. 16th

New Opportunities in Particle and Nuclear Physics with Energy Correlators

Based on: Heikki Mantysaari, Yossathorn Tawabutr, X.B. Tong, *arXiv: 2503.20157*
Kao-bao Chen, Jing-ping Ma, X.B.Tong, *JHEP* 08 (2024) 227

HELSINKI INSTITUTE OF PHYSICS

UNIVERSITY OF JYVÄSKYLÄ

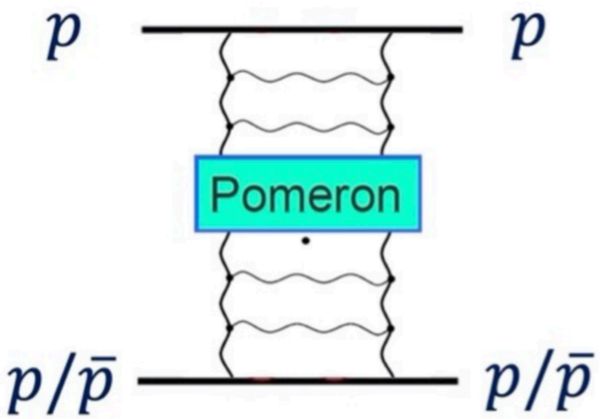
Centre of Excellence in Quark Matter

Odderon: brother of pomeron

- Propagators to govern soft-momentum exchanges in high-energy scatterings

Pomeranchuk 1961

C-even



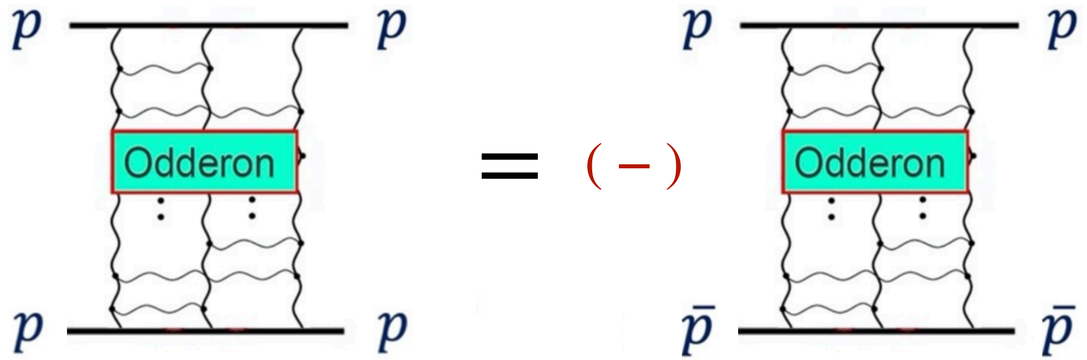
$\sigma_{\rm tot} \propto$ Pomeron

⇩

Known for decade

Lukaszuk & Nicolescu 1973

C-odd



Encode the charge asymmetry in nucleon

$\sigma_{tot}^{pp} - \sigma_{tot}^{p\bar{p}} \propto$ Odderon

Typically small, require precise comparisons

⇩

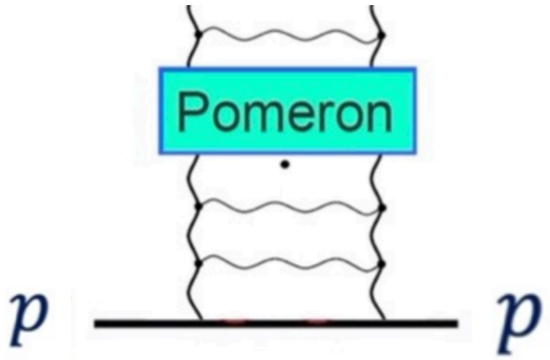
Elusive for decades!

Modern theory of Odderon in QCD

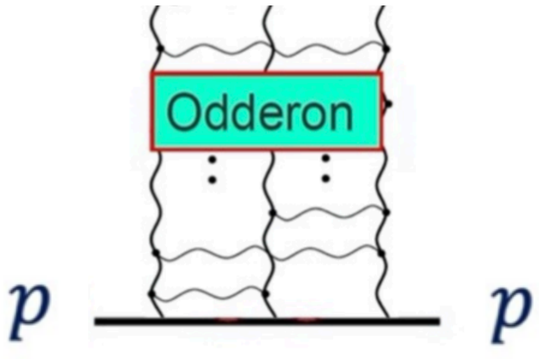
- Colorless small-x gluon correlations in the high-energy nucleon

- At the lowest order of pQCD

C-even



C-odd



- Two-gluon correlations

$$\left\langle A^{a,+} A^{a,+} \right\rangle$$

- Three-gluon correlations (color-symmetric)

$$\left\langle d^{abc} A^{a,+} A^{b,+} A^{c,+} \right\rangle$$

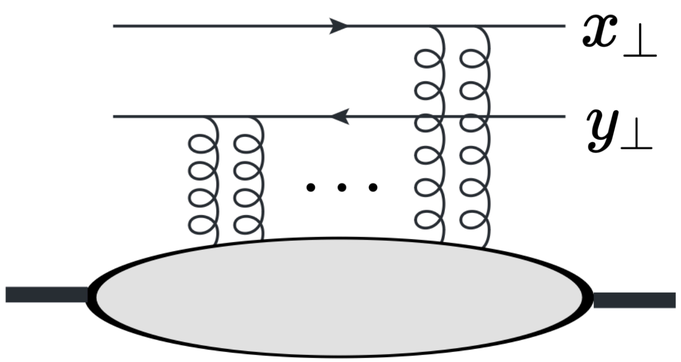
- change signs under charge conjugation $A^+ \to -A_T^+$

Modern theory of Odderon in QCD

- Odderon from Multiple gluon exchanges:

➡ the imaginary part of the Dipole-proton S-matrix [Hatta, Iancu, Itakura, McLerran, 2005]

$$S(x_\perp, y_\perp) = \frac{1}{N_c}\langle \mathrm{tr}\left[U(x_\perp)U^\dagger(y_\perp)\right]\rangle = P(x_\perp, y_\perp) + iO(x_\perp, y_\perp)$$



Pomeron ⇩ C-even, well understood

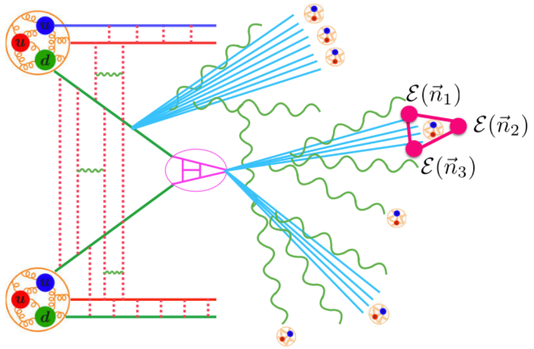
Odderon ⇩ C-odd, flips sign under charge conjugation $x_\perp \leftrightarrow y_\perp$

$$S^\dagger(x_\perp, y_\perp) = S(y_\perp, x_\perp)$$
$$\Downarrow$$
$$O(x_\perp, y_\perp) = -O(y_\perp, x_\perp)$$

Badly constraint!

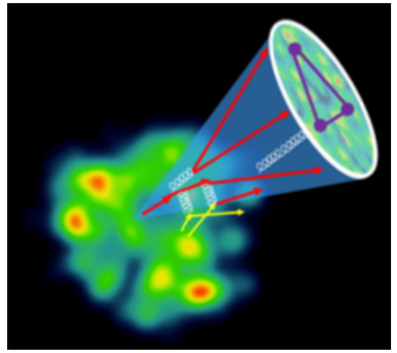
Don’t even know the normalization or overall signs…

New opportunities for QCD dynamics



Jet substructure

[Chen, Moult, Zhang, Zhu, 20]



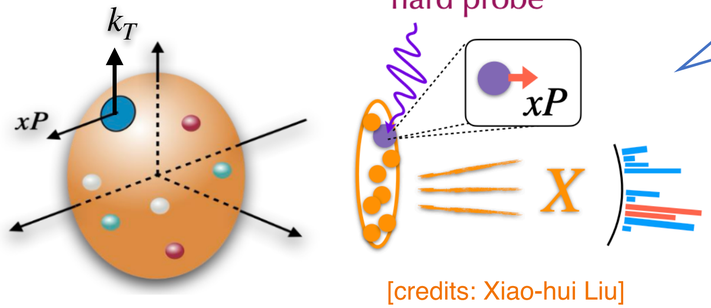
Quark-gluon plasma

[Andres, Dominguez, Elayavalli, Holguin, Marquet, Moult 22; Yang, He, Moult, Wangm, 23;….]

Energy correlator observables

$$\left\langle \mathcal{E}\left(\vec{n}_1\right)\mathcal{E}\left(\vec{n}_2\right)\cdots\mathcal{E}\left(\vec{n}_k\right)\right\rangle$$

[See more in Ian Moult's review]

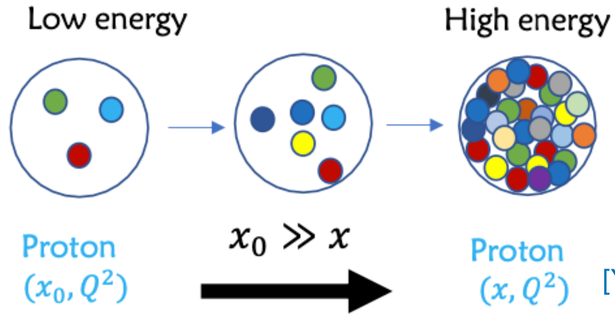


[credits: Xiao-hui Liu]

Nucleon structure

TMDs: [Li-Makris-Vitev 21; Kang, Lee, Shao, Fan 24]

Nucleon energy correlators (target fragmentation): **[Liu-Zhu, 23]**



[Yu shi]

Small-x physics (pomeron & gluon saturation)

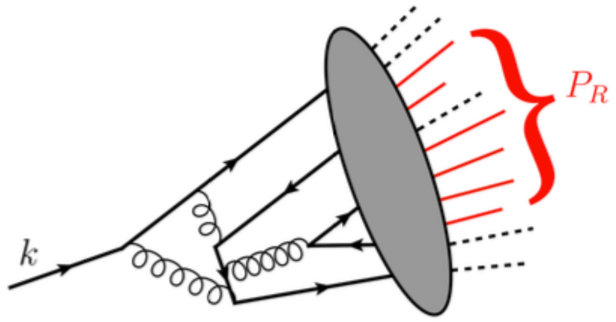
[**Liu-Liu-Pan-Yuan-Zhu, 23;** Kang, Penttala, Zhao, Zhou, 23;]

➡ **Odderons?**

Energy correlators on tracks

[See also YiBei, Max, Anjali, JingYu's talks]

- Track-based measurement



Charged hadrons

$(\pi^{\pm}, K^{\pm}, p, \bar{p}\ldots)$

- Tracking detectors: superior angular resolution
- Charged hadron EECs: simply related to the EECs at the partonic level

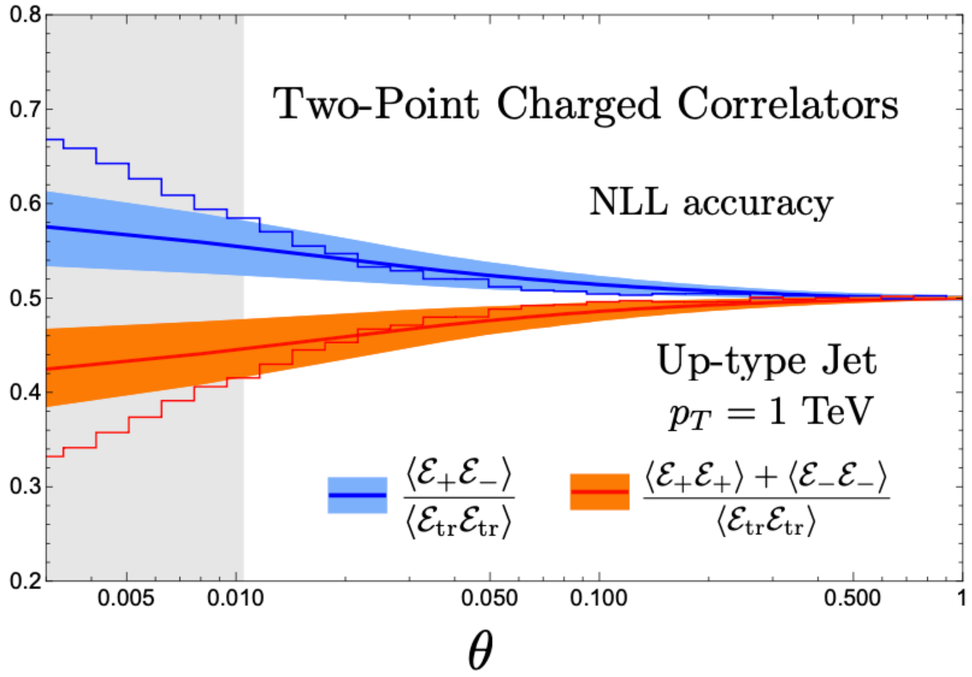
[Chen, Moult, Zhang, Zhu, 20; Jaarsma, Li, Moult, Waalewijna, Zhu,22,23,24;]

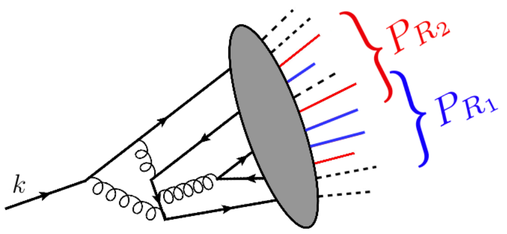
$$\langle \mathcal{E}_R(n_1)\mathcal{E}_R(n_2)\rangle = \sum_{a_1,a_2} T_{a_1}(1)T_{a_2}(1)\langle \mathcal{E}_{a_1}(\vec{n}_1)\mathcal{E}_{a_2}(\vec{n}_2)\rangle + \sum_a T_a(2)\langle \mathcal{E}_a^{(1,1)}(\vec{n}_1)\rangle \delta(\vec{n}_1-\vec{n}_2)$$

NP number

- $\langle \mathcal{E}_+\mathcal{E}_-\rangle, \langle \mathcal{E}_+\mathcal{E}_+\rangle$: positively or negatively charged hadrons

[Lee, Moult, 2308.00746]





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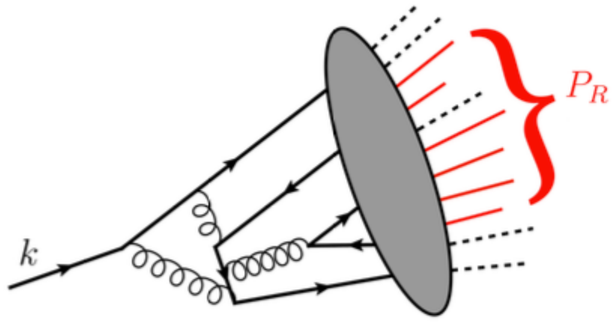

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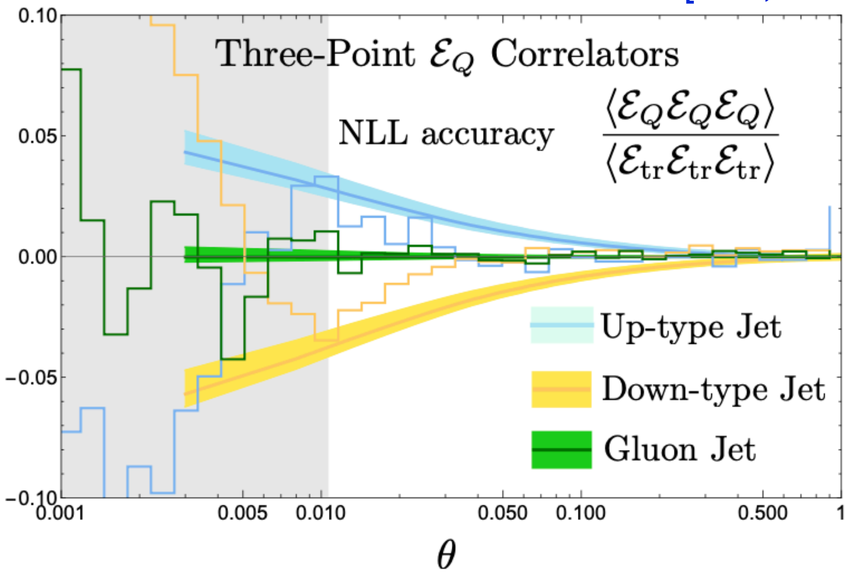
- Charge-weighted EEC

$$\mathcal{E}_{\mathbb{Q}}(\vec{n}_1)|k\rangle = E_k Q_k \delta(\hat{n}_1 - \hat{k})|k\rangle$$

[Lee, Moult, 2308.00746]



Sensitive to the charge of partons initiating the jet

Can we use these correlators to explore C-odd effects from the initial-state nucleon?

Energy correlators on tracks [See also YiBei, Max, Anjali, JingYu's talks]

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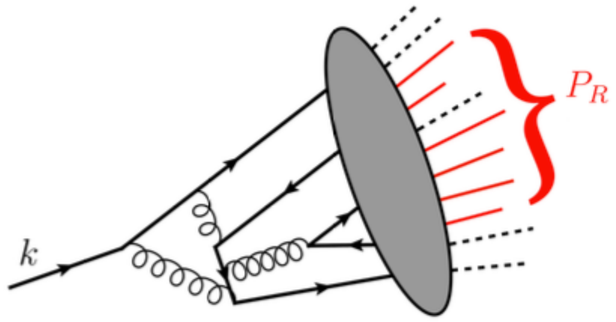

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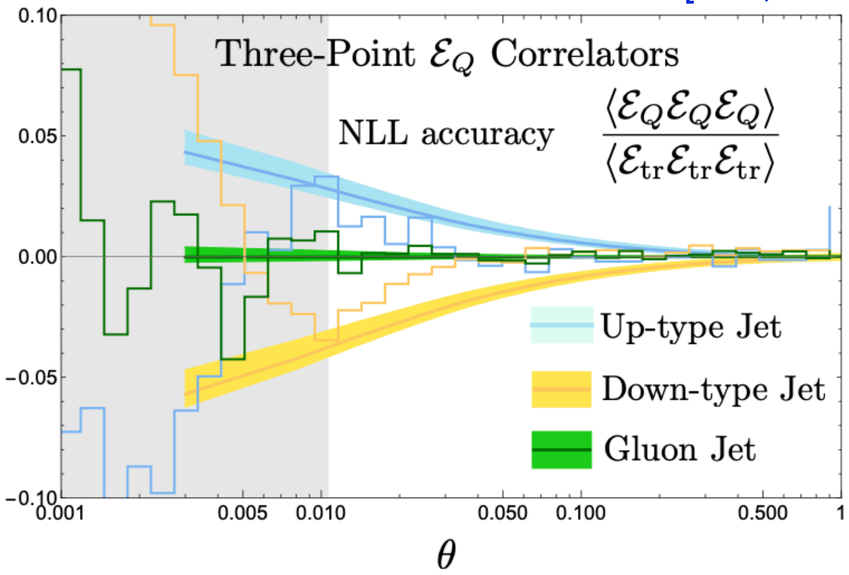


Sensitive to the charge of partons initiating the jet

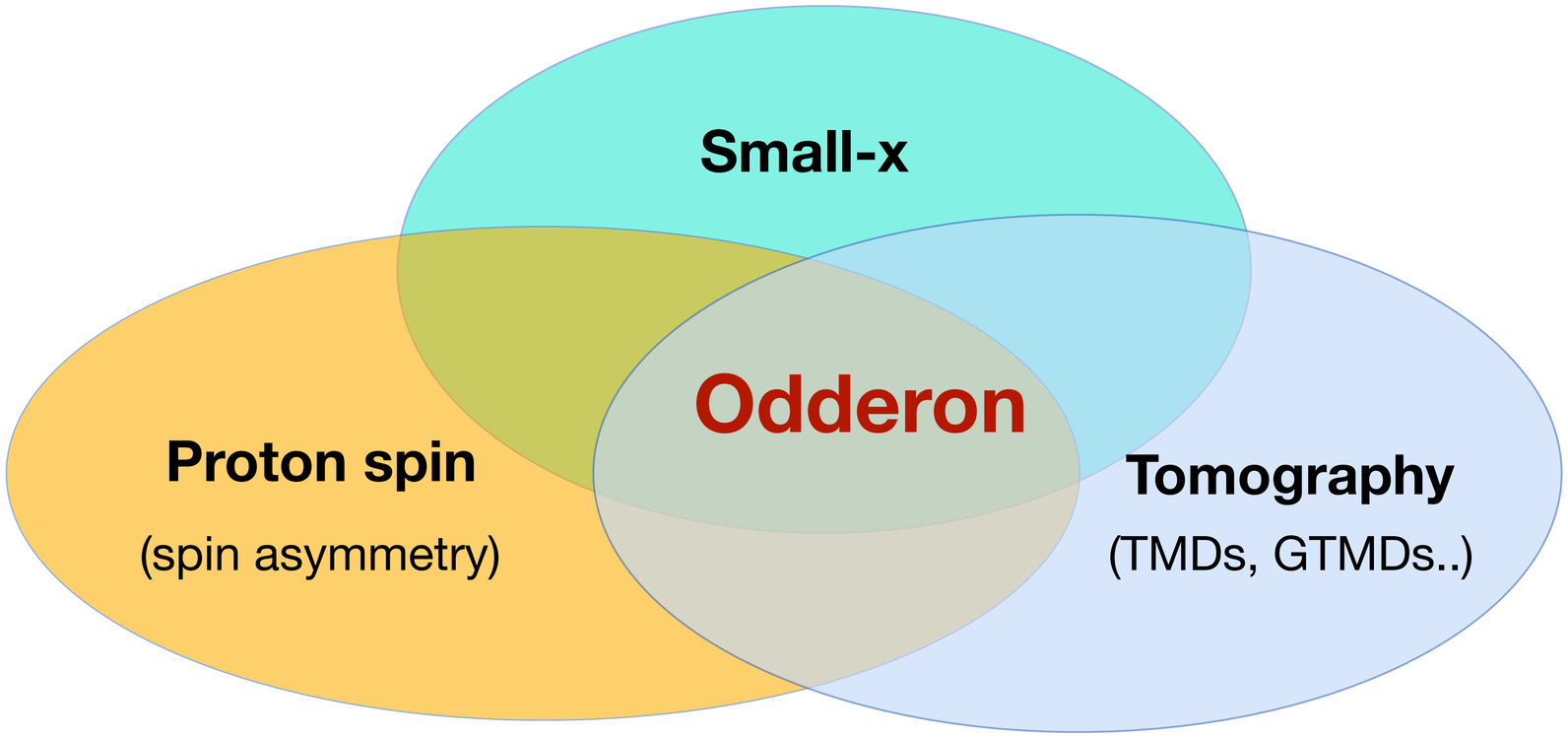
Can we use these correlators to explore C-odd effects from the initial-state nucleon?

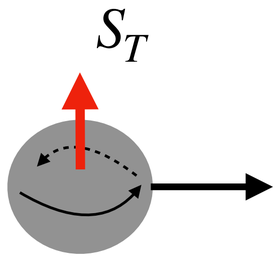
Energy pattern on tracks: $\langle \mathcal{E}_{+/-}(\hat{n})\rangle_{\rm DIS}$ & $\langle \mathcal{E}_{\mathbb{Q}}(\hat{n})\rangle_{\rm DIS}$ ➡ **Probing the Odderon in the DIS**

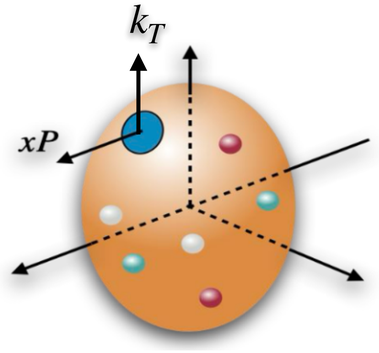
[Mantysaari, Tawabutr, Tong, *2503.20157*]

Spin-dependent odderon

- Energy pattern on tracks can remarkably improves the probe for the spin-dependent odderon
- But what is the spin-dependent odderon?
- What are the limitations of the previous proposals?

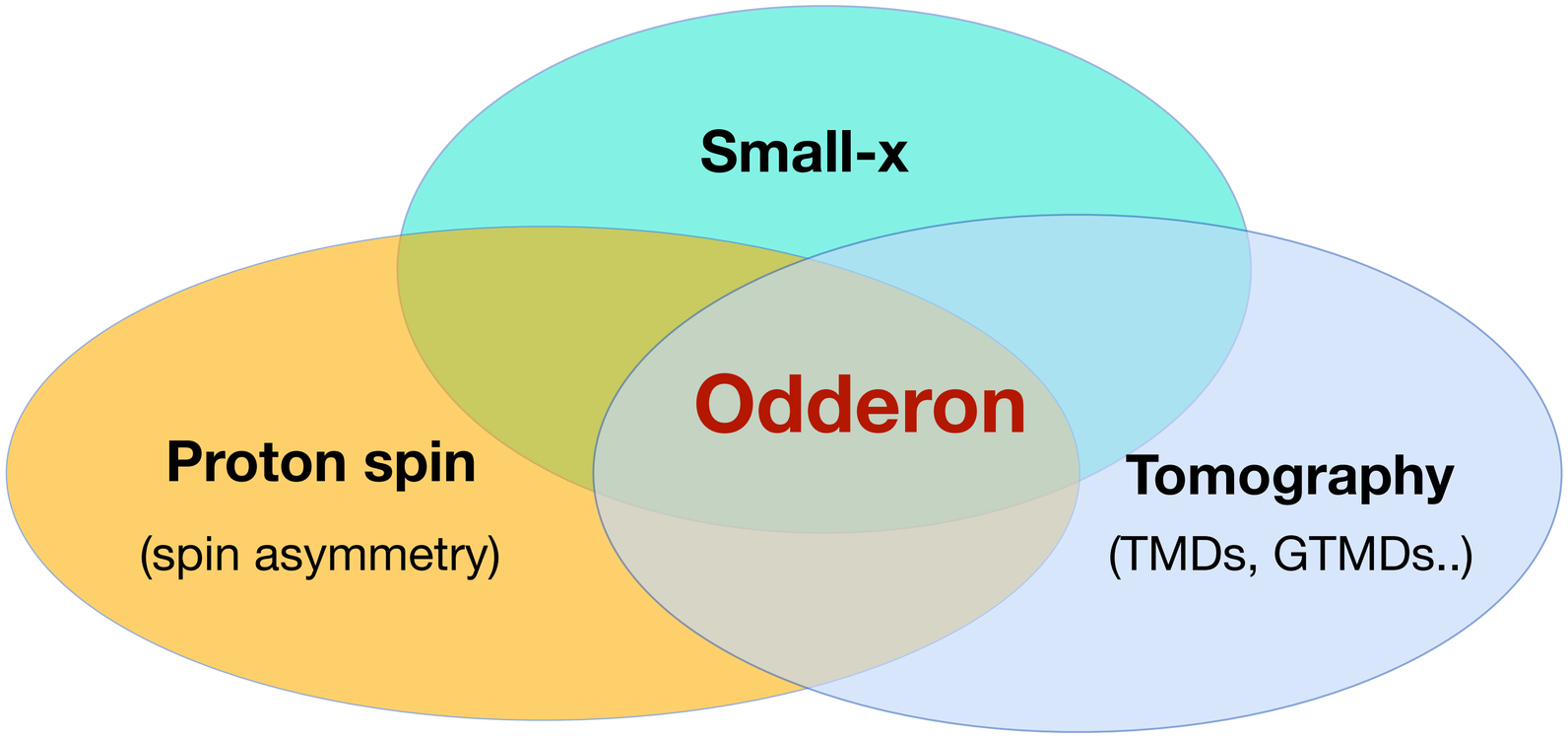


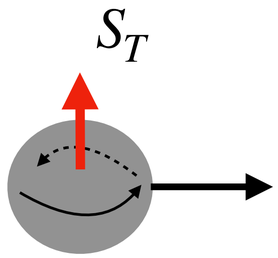


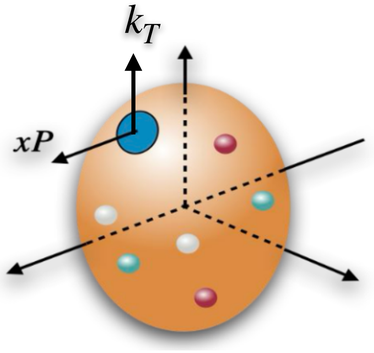


Spin-dependent odderon [Jian Zhou 2013]

- Not just C-odd gluon exchanges
- Connected to the three pillars of the EIC physics.



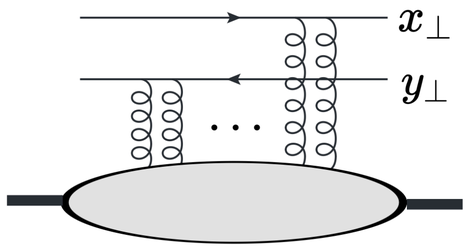




Conventional odderon V.S. Spin-dependent odderon

- C-odd: $O(x_\perp, y_\perp) = -O(y_\perp, x_\perp)$

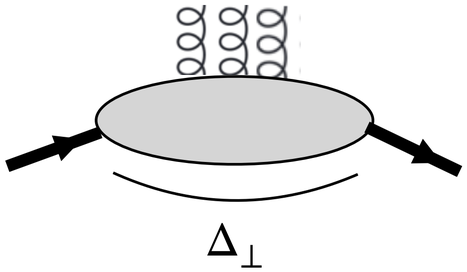
  proportional to the vector $(\vec{x} - \vec{y})_\perp$



- Target recoil momentum

$$O \propto (x - y)_\perp \cdot \Delta_\perp$$

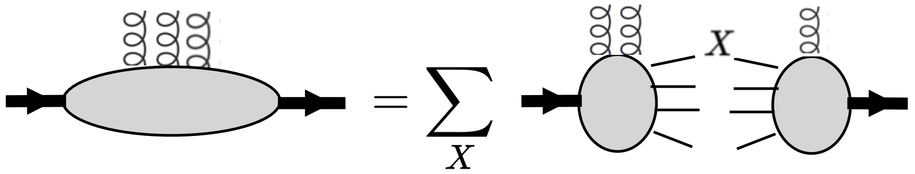
Exclusive process: target unbroken



★ Transversely polarized proton spin $S_\perp$

$$O_T \propto (x - y)_\perp \times S_\perp$$

Can be measured in an inclusive process:

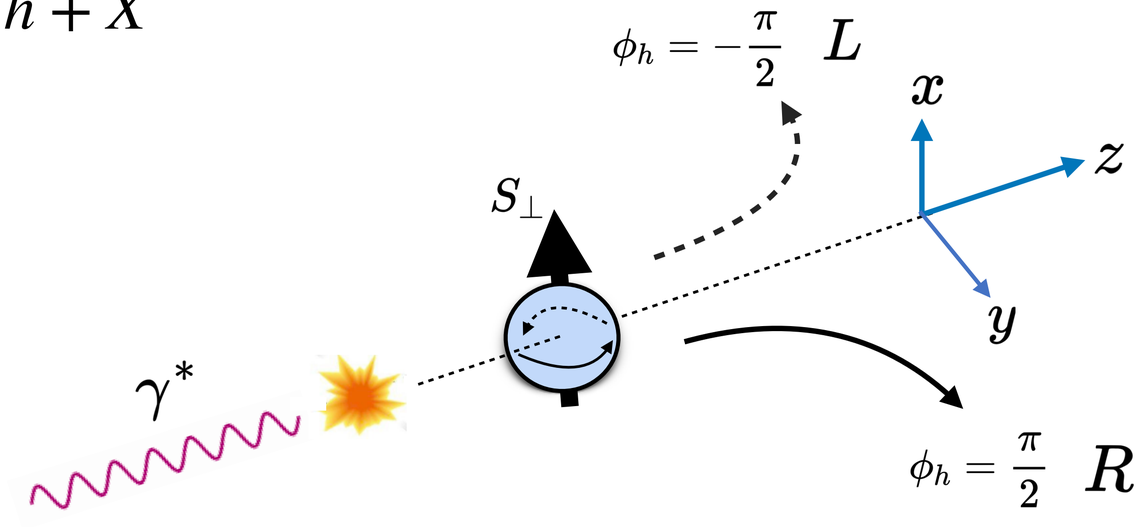


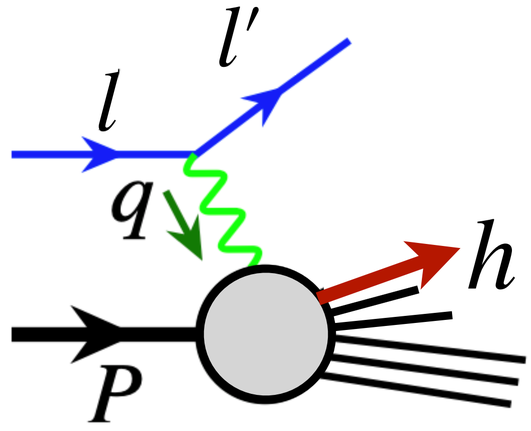
Typical observables: single spin asymmetry in SIDIS

✓Semi-inclusive DIS:

[Yao, Hagiwara, Hatta 2019]
[Dong, Zheng, Zhou 2019]
[Zhu, Zheng, Zhang 2024]

$$\gamma^* + p^\uparrow \to h + X$$





$$d\sigma \propto S_\perp \times P_{h\perp} \propto \sin(\phi_h - \phi_S) \Longrightarrow A_{UT} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}$$

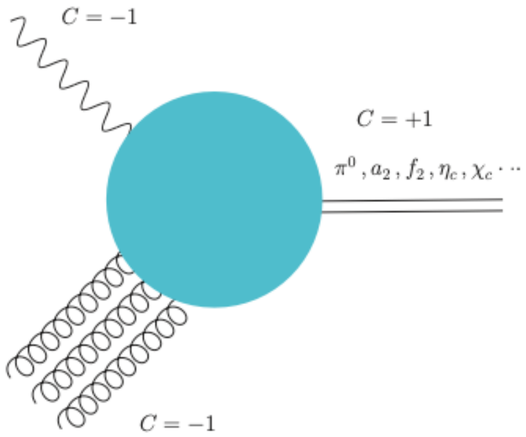
- $O_T \propto k_{g\perp} \times S_\perp$: hadron production is left-right asymmetric!
- However, the odderon ($\sim d^{abc} A^{+,a} A^{+,b} A^{+,c}$) is not the only possible origin!
  - C-even gluonic correlations can also contribute, e.g., $f^{abc} A^{+,a} A^{+,b} A^{+,c}$

[Kang, Qiu PRD 78, 034005 (2008)]
[Schafer,Zhou 1308.4961]
[Chen, Ma, Tong 2108.13582]

Lack of C-odd constraints in SIDIS

- Conventional odderon $O \propto (x-y)_\perp \cdot \Delta_\perp$

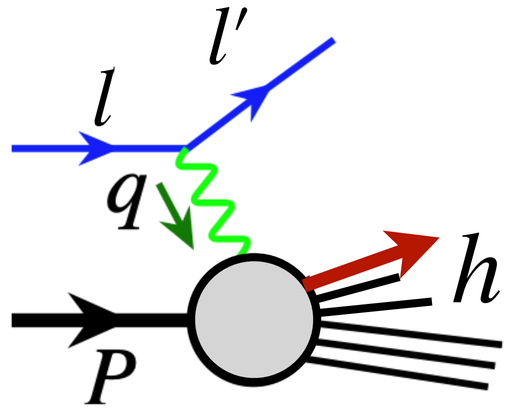
✓Exclusive ep scattering: $\gamma^* + p \to p' + \pi^0$



- C-odd probe by measuring the C-even hadron

★ Spin-dependent odderon $O_T \propto (x-y)_\perp \times S_\perp$

✓SIDIS: $\gamma^* + p^\uparrow \to h + X$



- Not measuring everything
- Lose the information of C-parity conservation
- Potential contamination from C-even gluonic contributions: $f^{abc} A^{+,a} A^{+,b} A^{+,c}$

➡ How to make a C-odd tag in the inclusive measurement ?

More questions

★ Spin-dependent odderon

$$O_T \propto (x-y)_\perp \times S_\perp$$

- How to make a C-odd tag in the inclusive measurement?

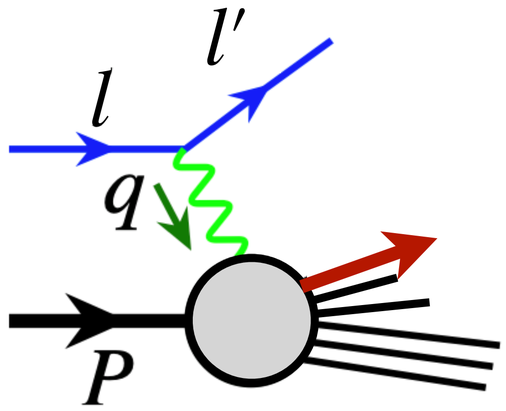
- Require identifying hadrons, involving convolution of fragmentation functions.

  ➡ Is there a way to minimize non-perturbative effects?

- Focus on open charm ($D^0, \ldots$)

  ➡ Why not charged hadrons ($\pi^\pm, K^\pm, p, \bar{p}$) ?

✓Semi-inclusive DIS: $\gamma^* + p^\uparrow \to h + X$

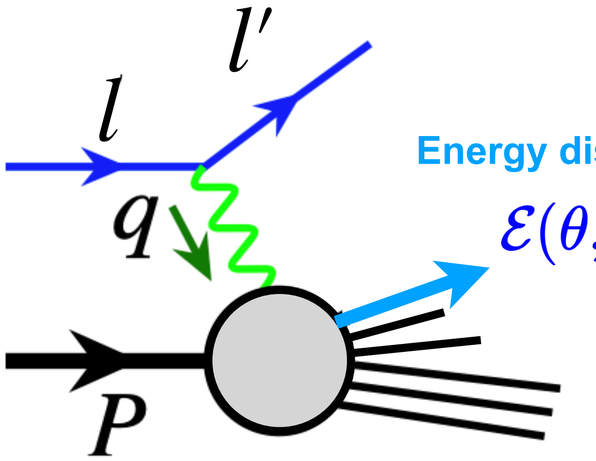


[Yao, Hagiwara, Hatta 2019]

[Dong, Zheng, Zhou 2019]

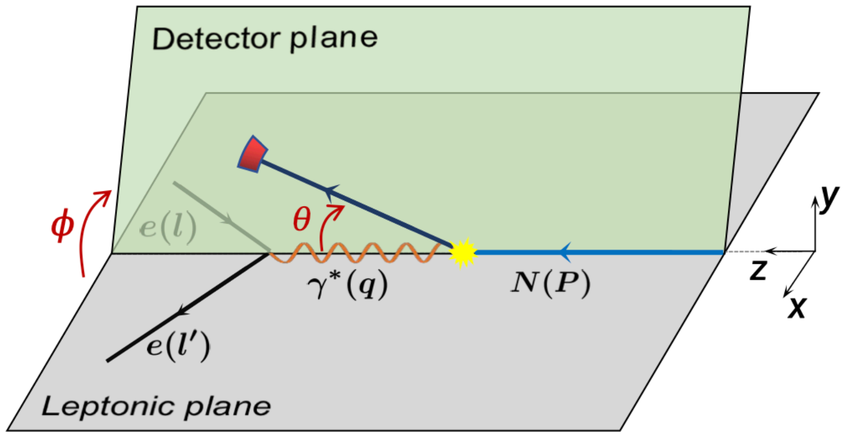
[Zhu, Zheng, Zhang 2024]

The DIS energy pattern



Energy distribution

$\mathcal{E}(\theta,\phi)$



$$\Sigma(\theta,\phi)_{\rm DIS} = \sum_h \int d\sigma^{e+N\to e+h+X} \frac{E_h}{E_N} \delta\left(\theta^2-\theta_h^2\right)\delta(\phi-\phi_h)$$

Reduce collinear singularities

Reduce soft singularities

[Meng-Olness-Soper NPB 371 (1992) 79]

[Li-Makris-Vitev PRD 103 (2021) 094005]

[Liu-Zhu, PRL130 (2023) 091901]

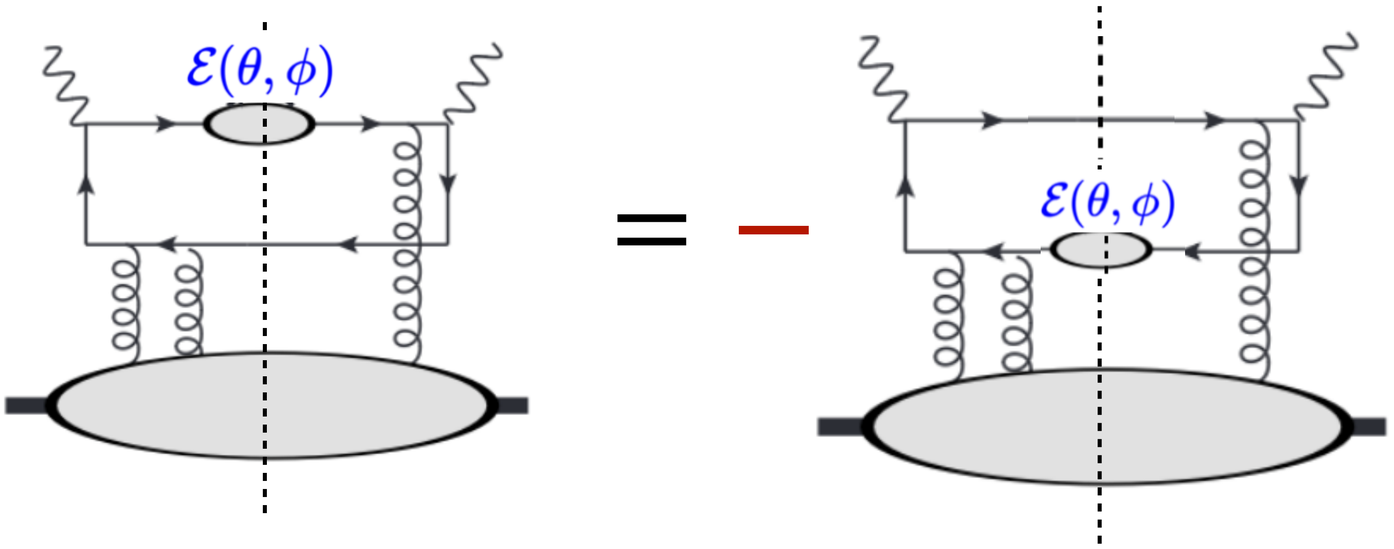
[Kang-Lee-Shao-Fan JHEP 03 (2024) 153]

[Chen-Ma-Tong, JHEP 08 (2024) 227]

- An extension of the energy pattern in e^+e^- annihilation: [Basham-Brown-Ellis-Love, PRD 17 (1978) 2298]
  - One-point energy correlator
  - Cleaner probe ; reducing non-perturbative effects in the final states, e.g. $\sum_h \int_0^1 dz\, z D_{h/a}(z) = 1$
- Single spin asymmetry : a left-right asymmetry of hadronic energy distributions

The need for the track information

- The inclusive energy pattern is infrared-collinear safe
- However, the odderon contributions to the inclusive energy-pattern SSA vanish, due to the C-odd nature



the quark jet channel the anti-quark jet channel

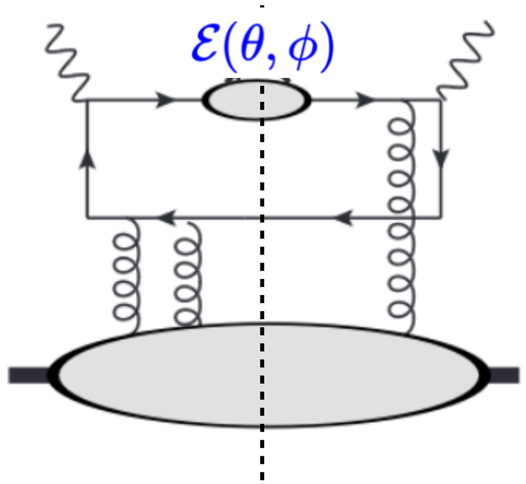
➡ Restricting the measurement to charged hadrons , $\mathbb{S} = \{h^+\}, \{h^-\}$

✓ The charged-hadron energy flux have different sensitivities to the quark and anti-quark fragmentation, preventing cancelations

Incorporating track information

★ **Measuring the DIS energy pattern on positively or negatively charged hadrons:**

$$\Sigma_{\mathbb{S}}(\theta,\phi)_{\mathrm{DIS}} = \sum_{h\in\mathbb{S}} \int d\sigma^{e+N\to e+h+X} \frac{E_h}{E_N} \delta\left(\theta^2-\theta_h^2\right)\delta\left(\phi-\phi_h\right)$$



- This again introduce fragmentation effects. $\mathbb{S} = \{h^+\},\{h^-\}$
- However, the energy weighting results in a simple factorization form

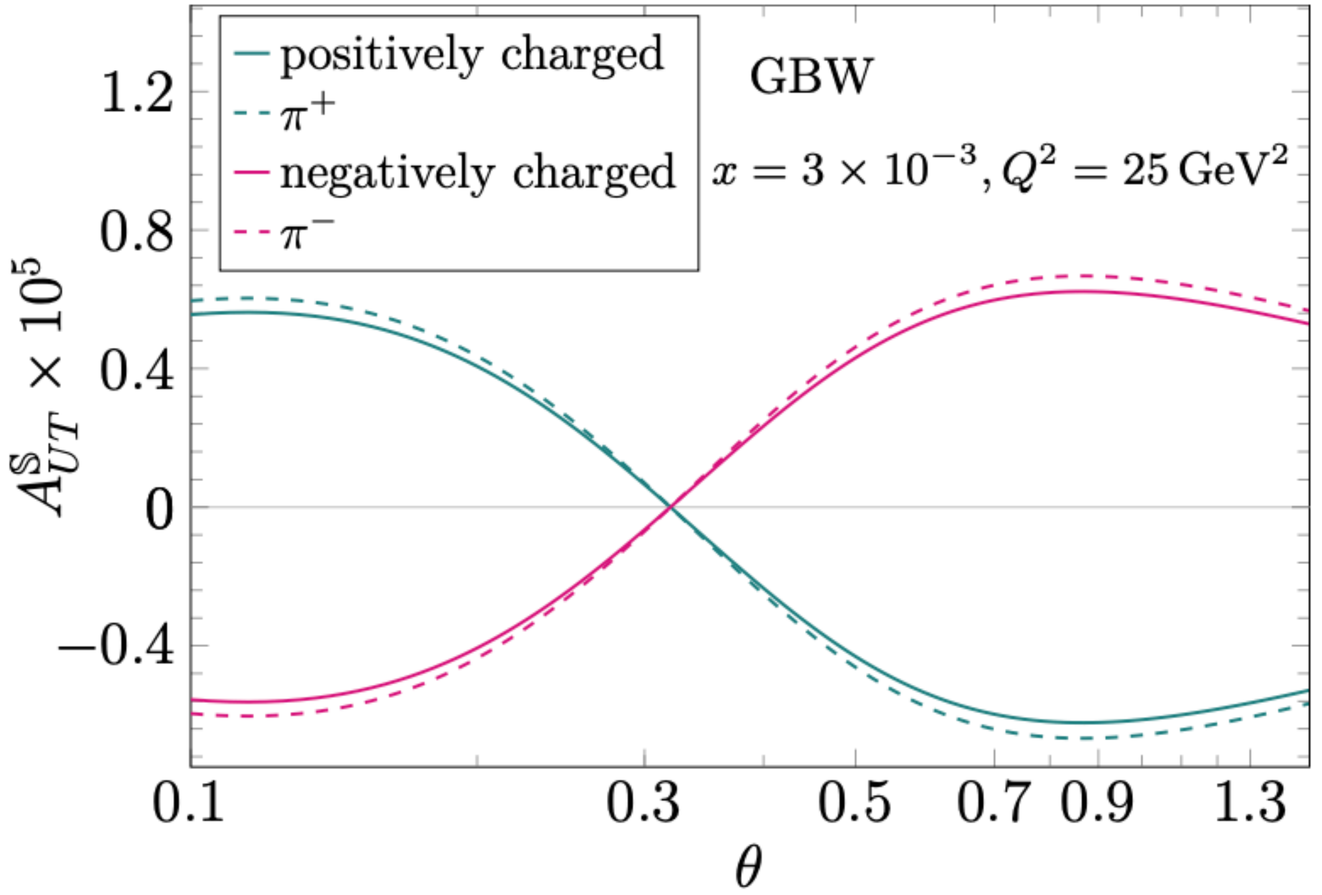
$$\Sigma_{\mathbb{S}}(\theta,\phi) \propto \left(\sum_{h\in\mathbb{S}} \int_0^1 dz\, z\left[D_{h/q}(z) - D_{h/\bar{q}}(z)\right]\right) \times \left(\int d^3k_q \frac{E_q}{E_N}\delta(\theta^2-\theta_q^2)\delta(\phi-\phi_q)\mathcal{O}_q(z_q,k_{q\perp})\right)$$

First moment of FFs

Σ_q, the energy pattern of the quark producing from the odderon and γ^* scattering

➡ Reducing the non-perturbative effects, similar to [Jaarsma, Li, Moult, Waalewijna, Zhu,22,23,24;]

A unique prediction from the odderon domination



- Signature: the energy-pattern SSAs change sign from $\mathbb{S} = \{h^+\}$ to $\mathbb{S} = \{h^-\}$
- However, may be obscured by the C-even contributions

Incorporating charge

- A C-odd tag can be set by weighting the hadronic charge Similar to [Lee, Moult, 2308.00746]

$$\Sigma_{\mathbb{Q}}(\theta,\phi)_{\rm DIS} = \sum_h \int d\sigma^{e+N\to e+h+X} \frac{E_h Q_h}{E_N} \delta\left(\theta^2-\theta_h^2\right)\delta\left(\phi-\phi_h\right)$$

- The charge-weighted energy pattern (charge pattern)

$$\Sigma_{\mathbb{Q}}(\theta,\phi) \propto \left(\sum_h \int_0^1 dz\, zQ_h\left[D_{h/q}(z)-D_{h/\bar{q}}(z)\right]\right) \times \left(\int d^3k_q \frac{E_q}{E_N}\delta(\theta^2-\theta_q^2)\delta(\phi-\phi_q)\mathcal{O}_q(z_q,k_{q\perp})\right)$$

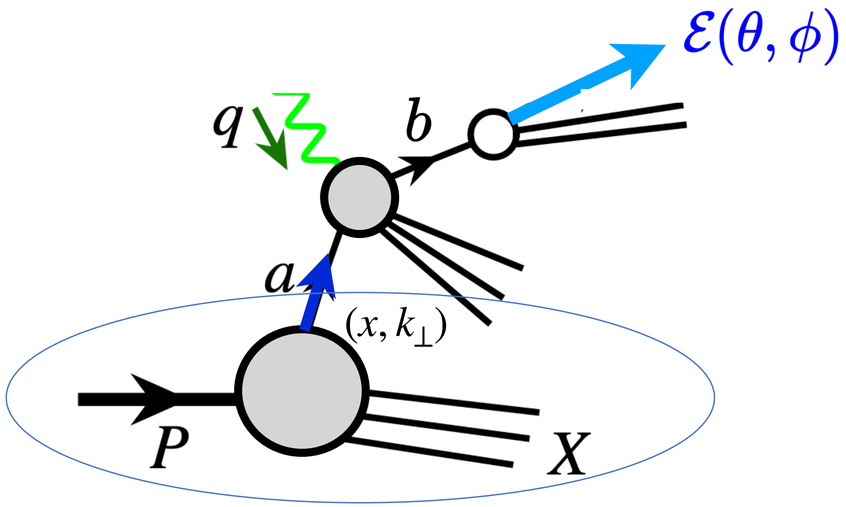
- A C-odd prism: by charge conservation, the total charge from q and $\bar{q}$ much have opposite sign

$$\sum_h \int_0^1 dz z Q_h D_{h/q}(z) = -\sum_h \int_0^1 dz z Q_h D_{h/\bar{q}}(z)$$

- the SSA: free from the C-even contribution, only sensitive to the spin-dependent odderon !

Two kinematic regions of the DIS energy pattern

Current fragmentation region



- TMDs

  ➡ the probability of finding a parton with $(x, k_\perp)$ in the broken target

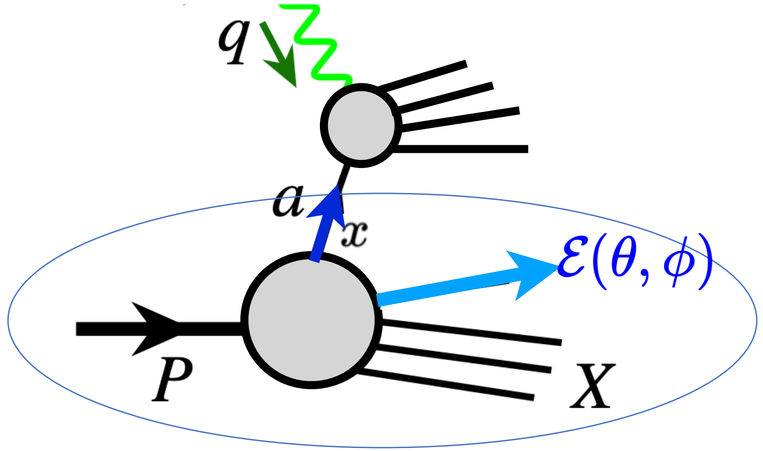
  Factorization: [Li-Makris-Vitev PRD 103 (2021) 094005]

  ➡ Spin-dependent odderon: common origins of T-odd quark/gluon TMDs at small-x

  [Boer-Echevarria-Mulders-Zhou, PRL. 116 , 122001, 2016]

  [Dong- Zheng-Zhou PLB 788 401, 2019]

Target fragmentation region



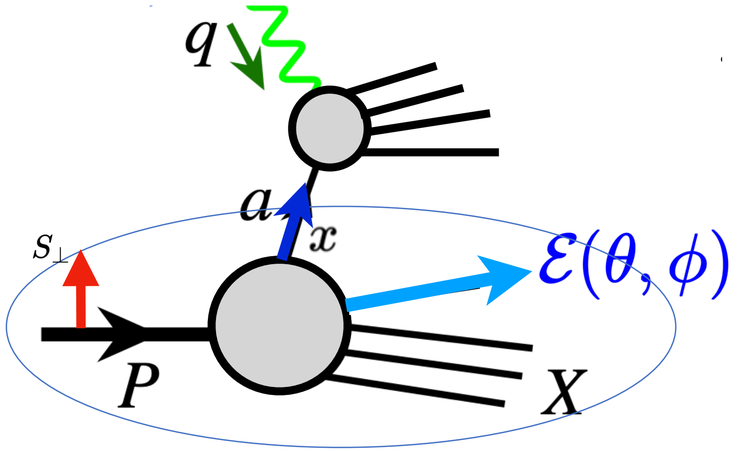
- Nucleon energy correlator [Liu-Zhu PRL 130, 2023]

  ➡ the probability of finding a parton while observing a energy flow from target remnants

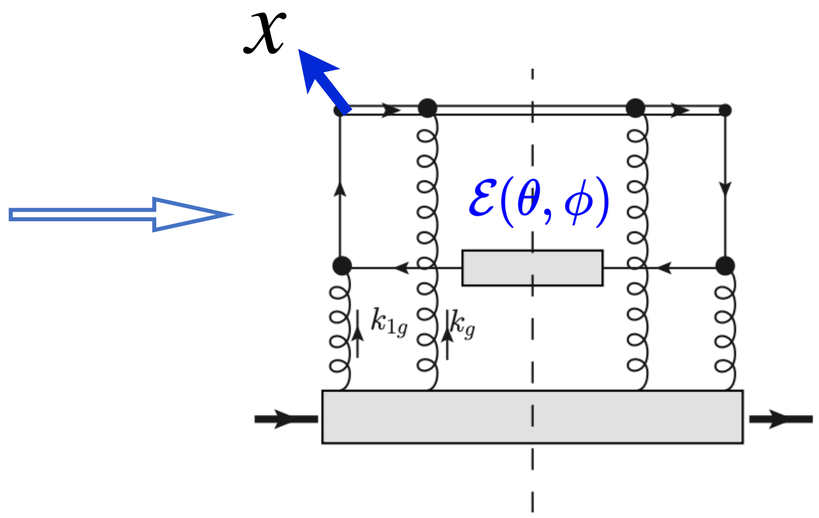
  ➡What is connection between the spin-dependent odderon and NEC?

Nucleon energy correlator at small-x

- At small-x, the energy flow and the parton from the splitting from small-x gluons



$$f_T^{t,q}(x,\theta)\ \sin(\phi-\phi_S) \propto \int \frac{d\eta}{4\pi} e^{-ixP^+\eta^-} \langle PS_\perp|\bar{\psi}(\eta^-)\mathcal{L}_n^\dagger(\eta^-)\gamma^+\hat{\mathcal{E}}(\theta,\phi)\mathcal{L}_n(0)\psi(0)|PS_\perp\rangle$$



- Quark Sivers NEC at small x:

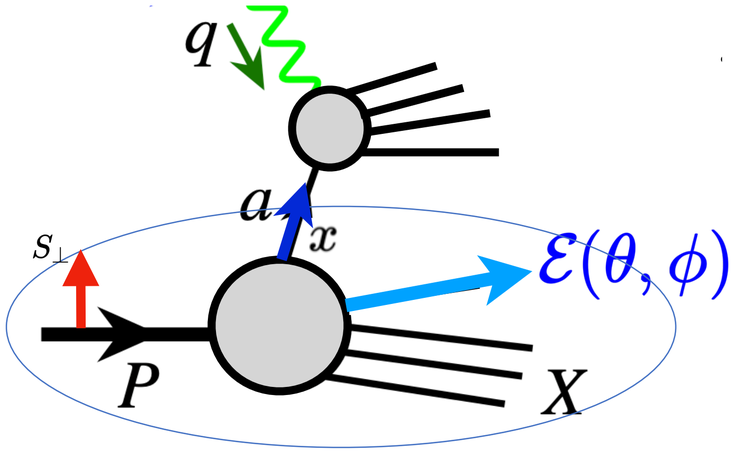
$$f_T^{t,q}(x,\theta) = \frac{N_c}{\theta^2(2\pi)^4}\int_0^{1-x}\frac{d\xi}{\xi}(\boldsymbol{k}_\perp^2)^2\int d^2\boldsymbol{k}_{g\perp}\left[\frac{\boldsymbol{k}_{g\perp}+\boldsymbol{k}_\perp}{\epsilon_f^2+(\boldsymbol{k}_{g\perp}+\boldsymbol{k}_\perp)^2}-\frac{\boldsymbol{k}_\perp}{\epsilon_f^2+\boldsymbol{k}_\perp^2}\right]^2\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{k}_{g\perp}}{|\boldsymbol{k}_\perp|}O_{1T,x_g}^\perp(\boldsymbol{k}_{g\perp}^2)$$

Spin-dependent odderon

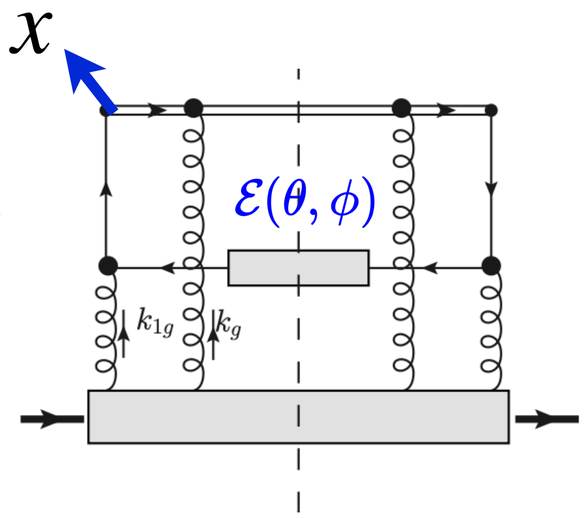
[See the pomeron case in Liu et al PRL. 130 (2023) 181901]

Nucleon energy correlator on tracks

- At small-x, the energy flow and the parton from the splitting from small-x gluons



$$f_T^{t,q}(x,\theta)\ \sin(\phi-\phi_S) \propto \int \frac{d\eta}{4\pi} e^{-ixP^+\eta^-} \langle PS_\perp|\bar{\psi}(\eta^-)\mathcal{L}_n^\dagger(\eta^-)\gamma^+\hat{\mathcal{E}}(\theta,\phi)\mathcal{L}_n(0)\psi(0)|PS_\perp\rangle$$



- The original inclusive measurement is not feasible, due to C-odd nature of the odderon: $\sum_{a=q,\bar{q}} e_a^2 f_T^{t,a}(x,\theta) = 0$

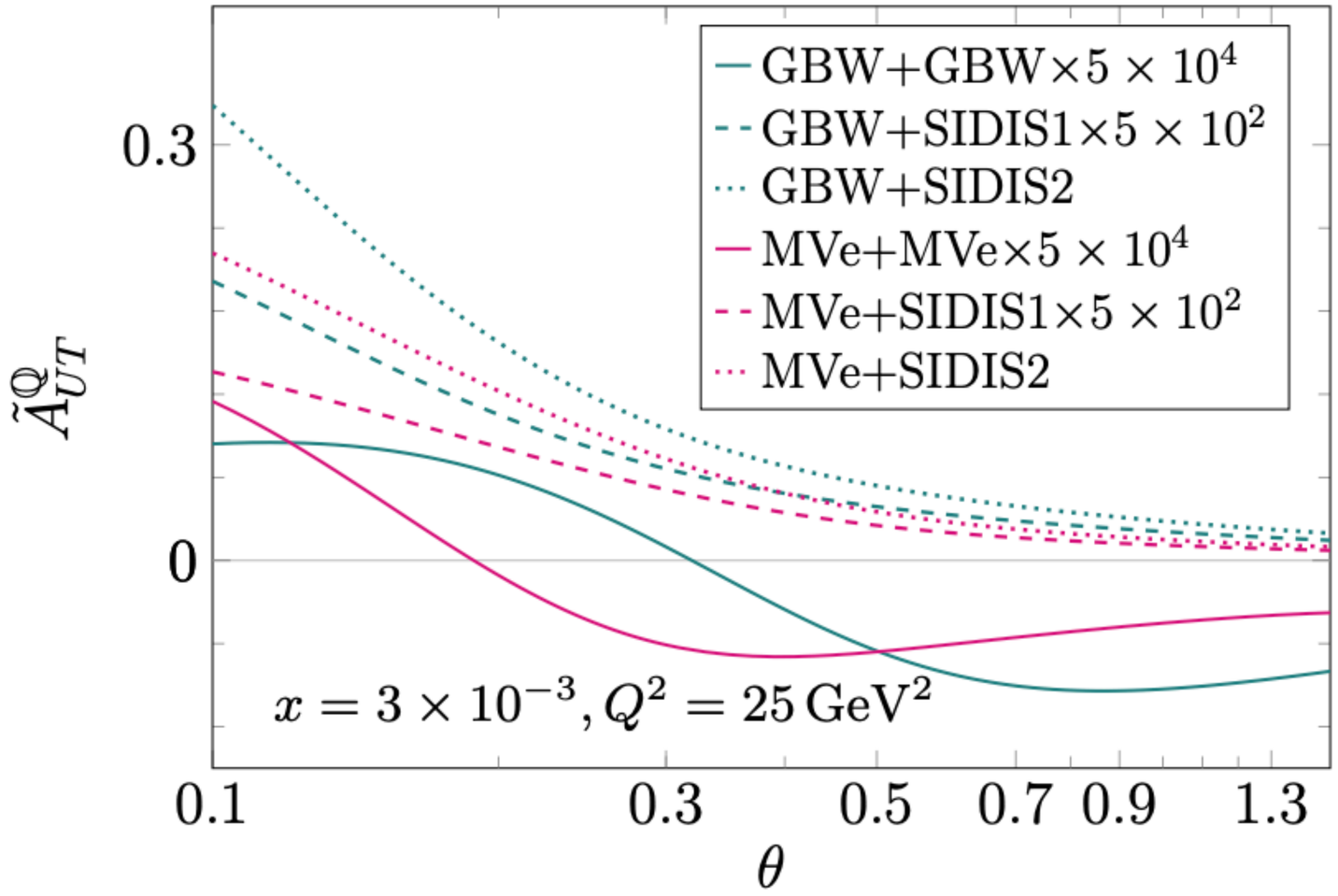
✦ NEC from charged hadrons

$$f_{T,\mathbb{S}}^{t,q}(x,\theta) = \left(\sum_{h\in\mathbb{S}} \int_0^1 z d_{h/\bar{q}}(z) dz\right) f_T^{t,q}(x,\theta)$$

✦ The charge-weighted NEC

$$f_{T,\mathbb{Q}}^{t,q}(x,\theta) = \left(\sum_{h\in\mathbb{S}} \int_0^1 z Q_h d_{h/\bar{q}}(z) dz\right) f_T^{t,q}(x,\theta)$$

Numerics for the charge-weighed energy pattern in the TFR



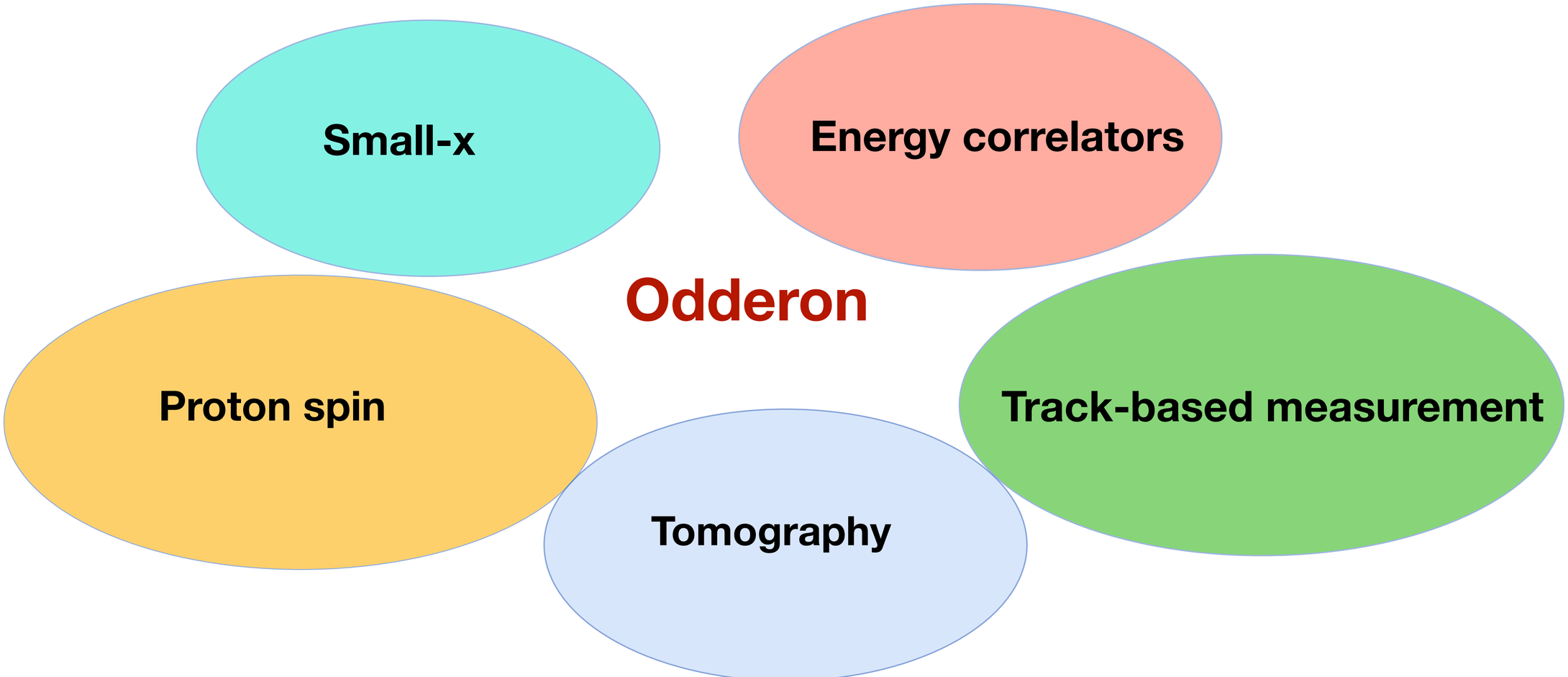
➡Huge differences of SSA between different model of the spin-depdent odderon

➡A sensitive probe

New ways for imaging the odderon:
The energy pattern on tracks

H. Mantysaari, Y. Tawabutr, X.B. Tong, arXiv:2503.20157

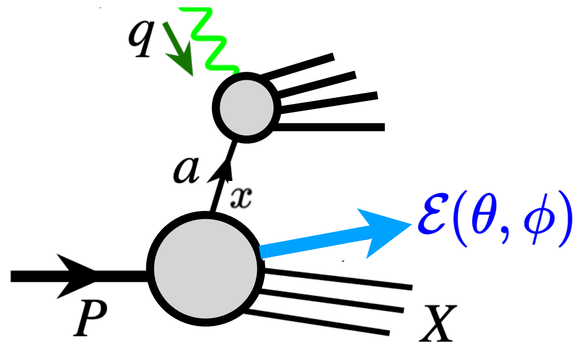
➡ A unique way to address the limitations of the probe for the spin-dependent odderon



Summary

- DIS energy pattern for the spin-dependent odderon

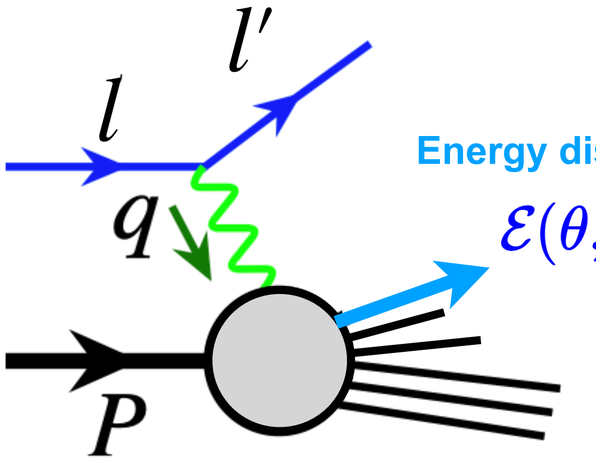
  - Incorporating track information ➡ From NP function to NP number
  - Incorporating charge weight ➡ A C-odd tag
  - Nucleon energy correlator ➡ Connection to TFR



➡ Provide a sensitive probe for the spin-dependent odderon

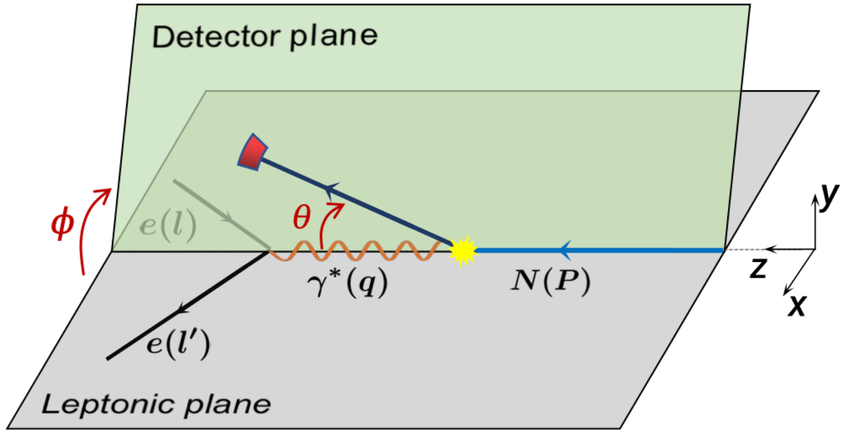
➡ Connection between T-odd gluon NECs and spin-dependent odderon? Are they reduce to the same origin at small-x as TMDs?

The DIS energy pattern for the odderon



Energy distribution

$\mathcal{E}(\theta,\phi)$



★ **Measuring the energy pattern from a subset of hadrons** (positively or negatively charged hadrons)

$$\Sigma_{\mathbb{S}}(\theta,\phi)_{\mathrm{DIS}} = \sum_{h\in\mathbb{S}} \int d\sigma^{e+N\to e+h+X} \frac{E_h}{E_N} \delta\left(\theta^2-\theta_h^2\right)\delta\left(\phi-\phi_h\right) \qquad \mathbb{S}=\left\{h^+\right\},\left\{h^-\right\}$$

★ **Measuring the charge pattern—— a C-odd tag**

$$\Sigma_{\mathbb{Q}}(\theta,\phi)_{\mathrm{DIS}} = \sum_{h} \int d\sigma^{e+N\to e+h+X} \frac{E_h Q_h}{E_N} \delta\left(\theta^2-\theta_h^2\right)\delta\left(\phi-\phi_h\right)$$