How far can we see back in time in high-energy collisions using charm hadrons?

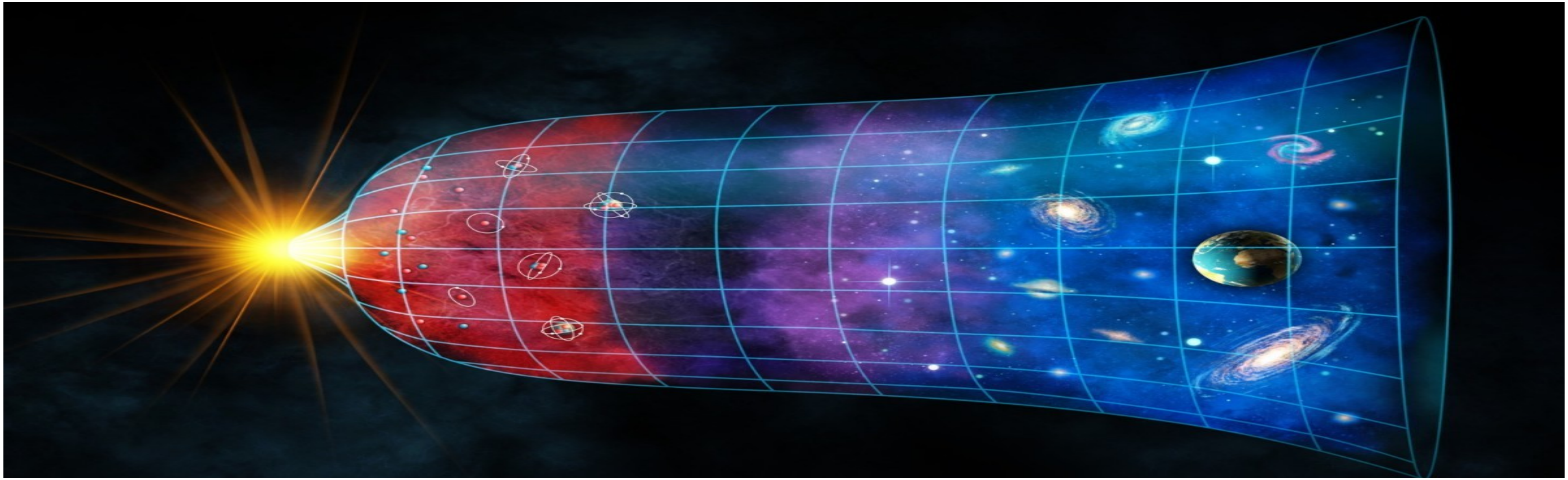
G.G. Barnaföldi, L. Gyulai, G. Bíró, R, Vértesi

Support: *Hungarian NKFIH grants FK13979, ,2021-4.1.2-NEMZ KI-2024-00031, 2024-1.2.5-TÉT-2024-00022, Wigner Scientific Computing Laboratory*

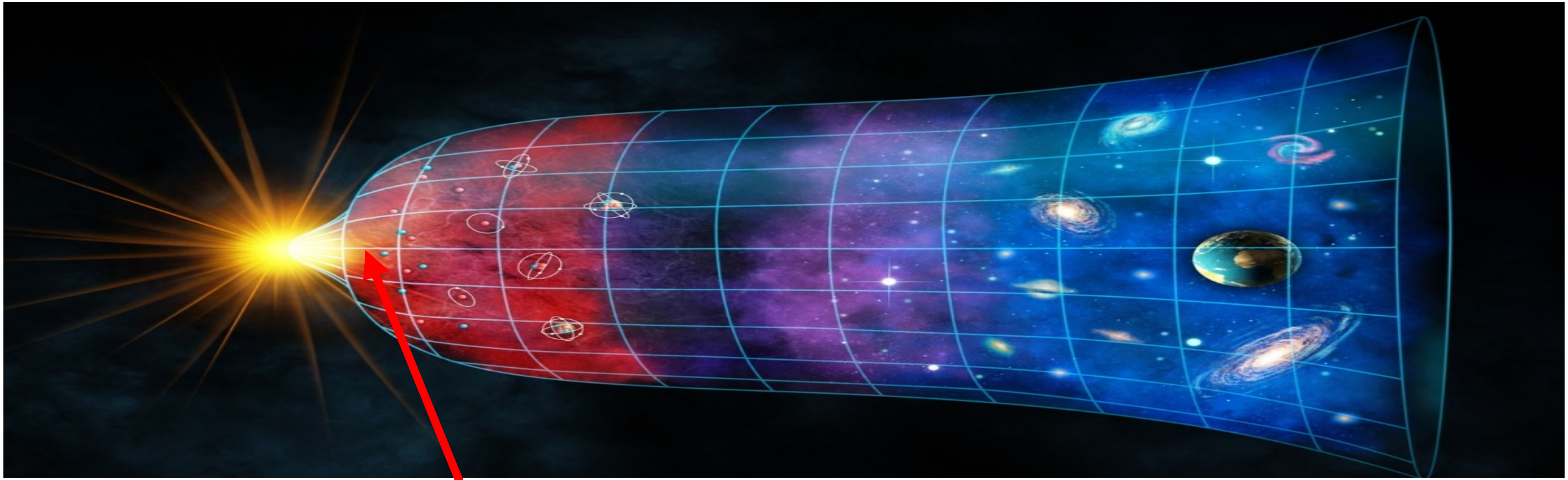
Refs: J.Phys.G 51 (2024) 8, 085103, IJMPA (arXiv 2409.01085)

New Opportunities in Particle and Nuclear Physics with Energy Correlators,
C3NT, CCNU, Wuhan, Hubei, China, 13th May 2025

How far can we see back in time in HIC?



How far can we see back in time with charm?

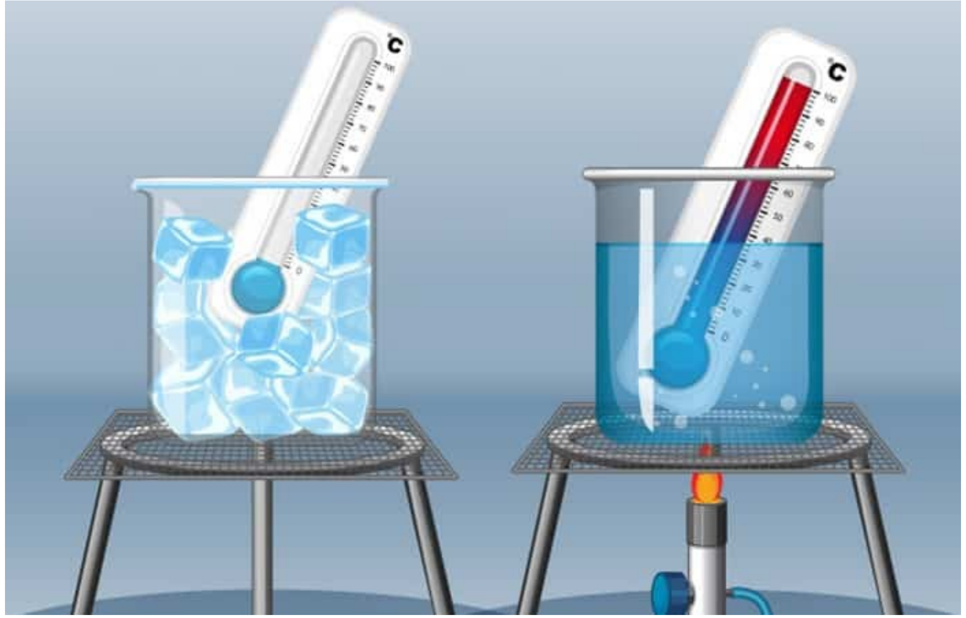


Let's focus on "charmly" to the other side of the history….

Motivation for the talk...

Our aims here are:

- define a thermometer
- check the feasibility to define a scale

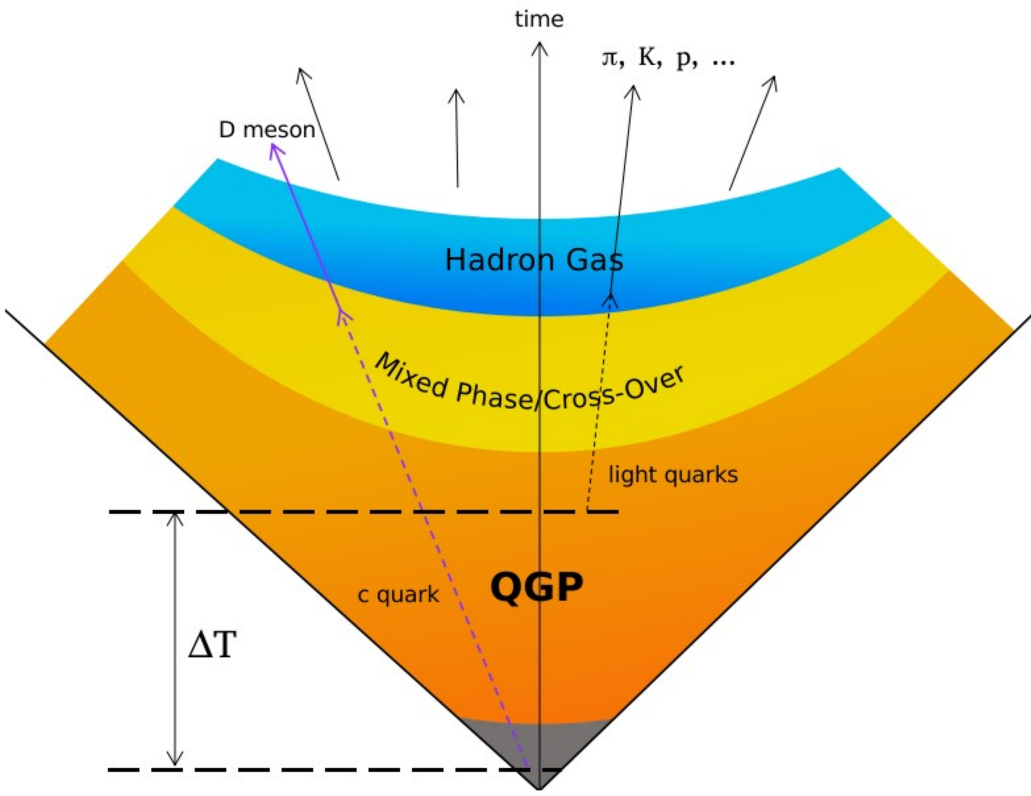


Motivation for the talk…

Our aims here are:

- define a thermometer
- check the feasibility to define a scale
- find similarities between light and heavy flavours
- find traces of different production mechanisms & timelines

All within the non-extensive statistical framework



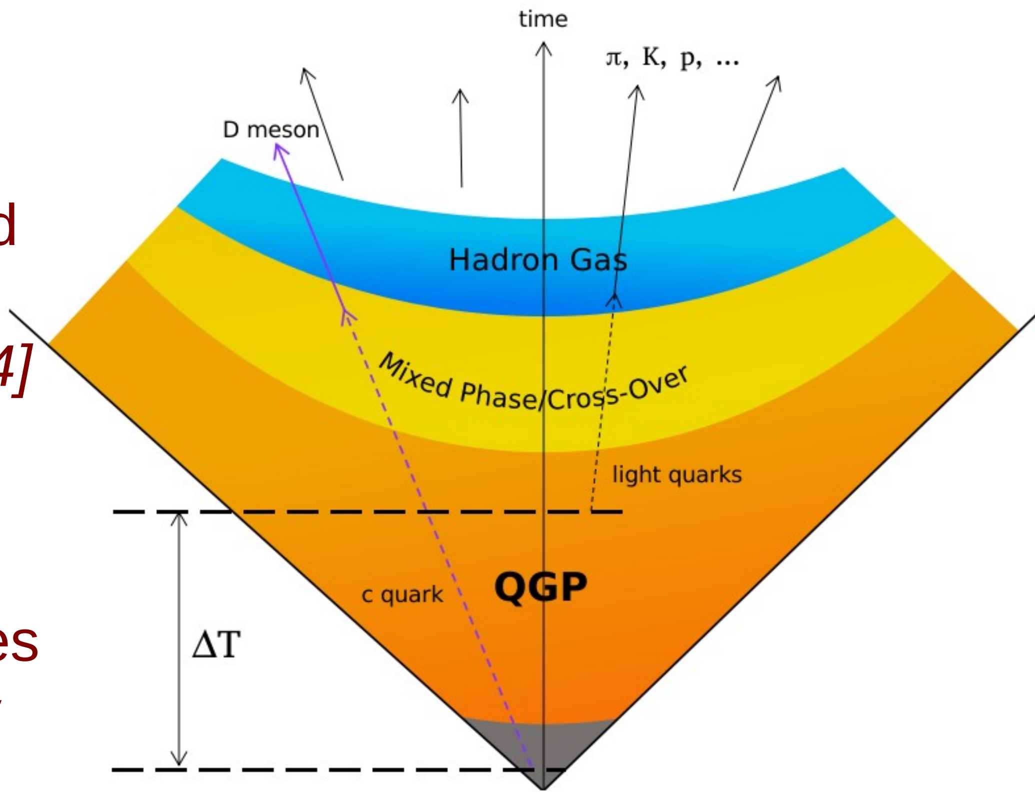
Related works

Previous studies **(K Shen, G Bíró, TS Biró, AN Mishra, GGB)**

Light-flavoured hadrons (K, π, p, Λ, Φ, Σ, Ξ, Ω) have already been studied in the non-extensive statistical framework in the broad range of collision systems and multiplicities
[*JPG 47 (2020) 10, 105002, JPG 50 (2023) 9, 095004*]

Recent works **(L Gyulai, R. Vértesi, G. Bíró, G. Paic, GGB)**

In our study we expand the list of investigated particles with D mesons (containing c quark), which are mostly produced in hard interactions early in the collisions
[*JPG 51 (2024) 8, 085103, IJMPA (arXiv:2409.01085)*]

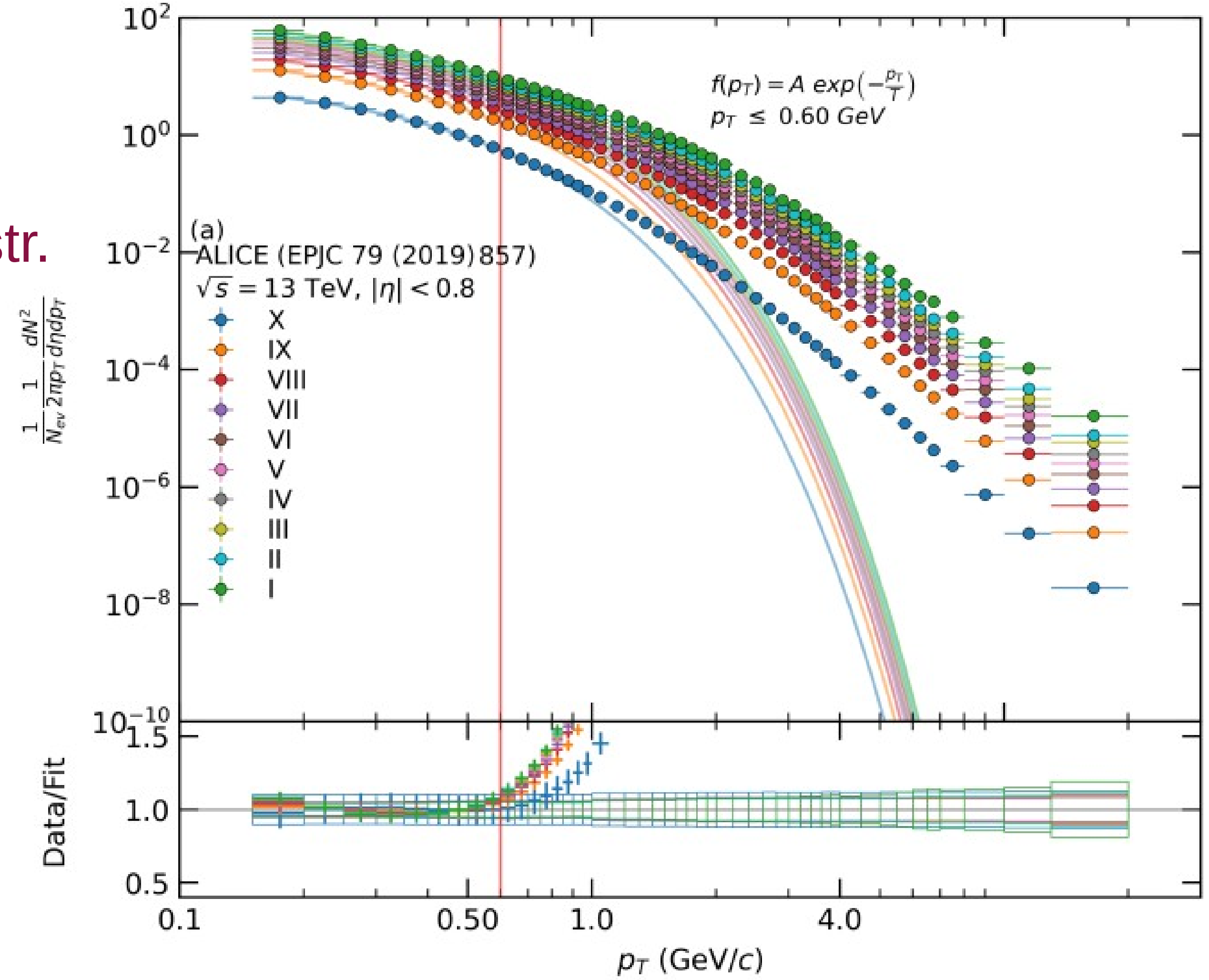


Hadron spectra vs. extensive statistics

Identified particle spectrum:

- Low-p_T part:
  - soft particle production
  - exponential-like (Boltzmann-Gibbs) distr.
  - stemming from a thermal equilibrium

G Biró, L. Serkin, G. Paic GGB: arXiv:2403.07512



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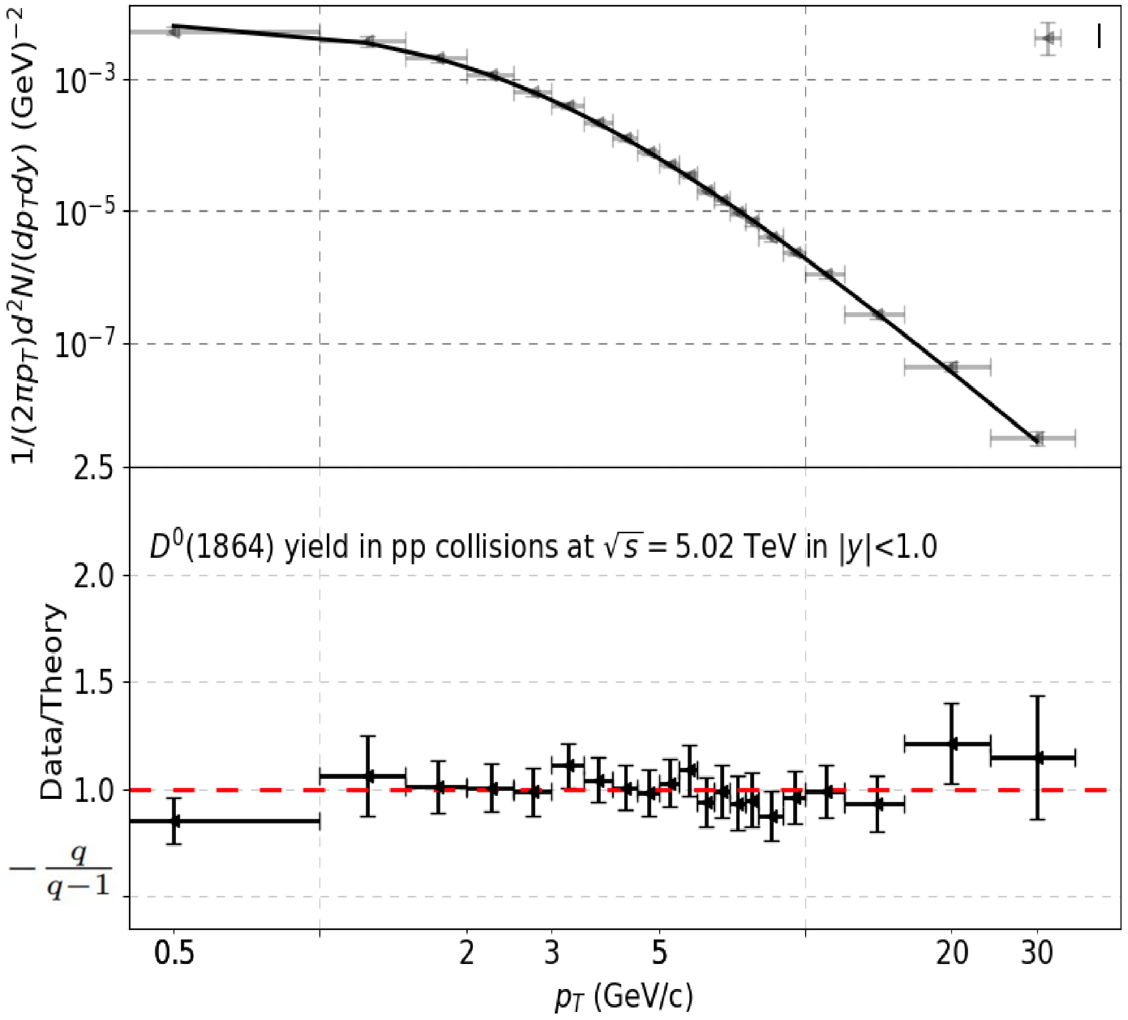
- High-p_T part:
  - jet-like origin
  - power-law tail distribution
  - described by the perturbative QCD

Tsallis-Pareto distribution smoothly connects both:

$$\left.\frac{\mathrm{d}^2 N}{2\pi p_T \mathrm{d}p_T \mathrm{d}y}\right|_{y\approx 0} = A m_T \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-\frac{q}{q-1}}$$



Quantify and compare LF hadron spectra data

- **Precise spectra description**
  - from low- to high-p_T

$$f(m_T) = A \cdot \left[1 + \frac{q-1}{T_s}(m_T - m)\right]^{-\frac{1}{q-1}}$$

  - in multiplicity classes (pp, pA, AA)

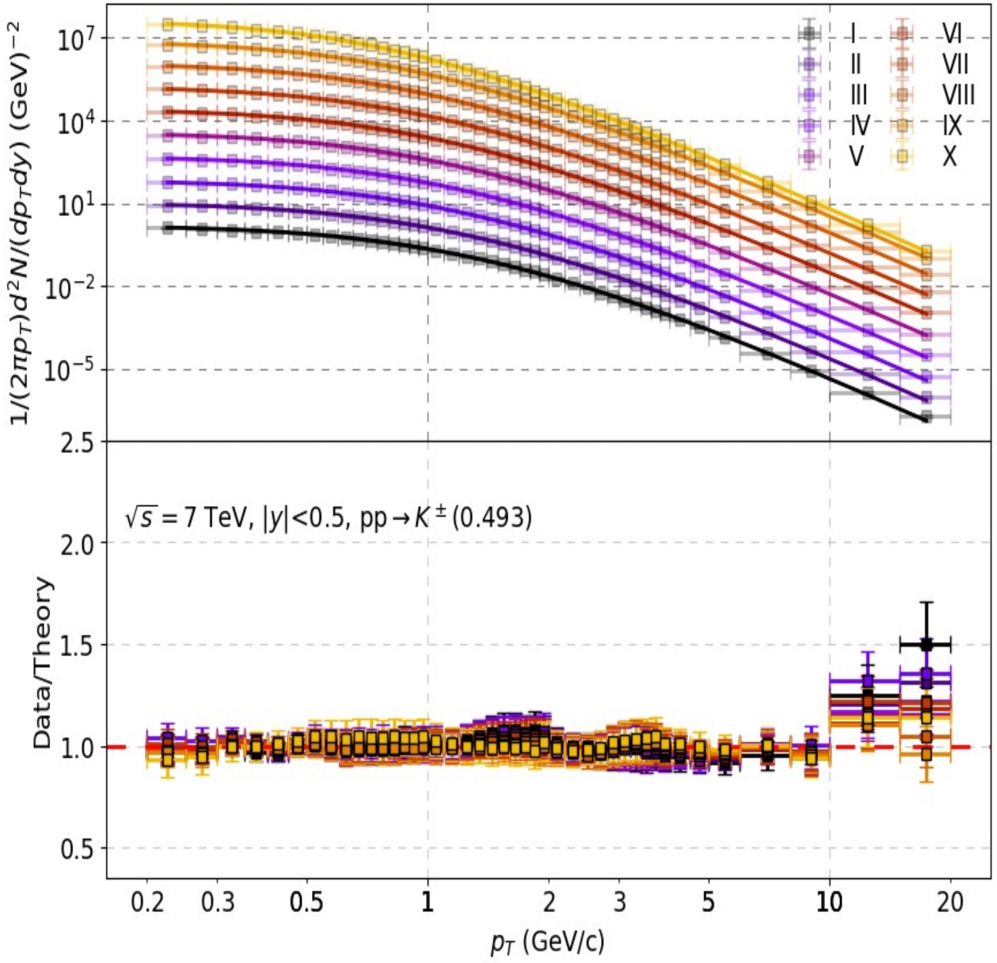
$$\left.\frac{\mathrm{dN_{ch}}}{\mathrm{dy}}\right|_{y=0} = 2\pi A T_s \left[\frac{(2-q)m^2+2mT_s+2T_s^2}{(2-q)(3-2q)}\right] \times \left[1+\frac{q-1}{T_s}m\right]^{-\frac{1}{q-1}}$$

  - With PID:

$$\pi^{\pm}, K^{\pm}, K^0_s, K^{*0}, p(\bar{p}), \Phi, \Lambda, \Xi^{\pm}, \Sigma^{\pm}, \Xi^0, \Omega$$

  - Wide range:

	pp	pA	AA
CM energy (GeV)	7000, 13000	5020	130-5020
Multiplicity range	2.2-25.7	4.3-45	13.4-2047



Identifying scaling in light flavour hadron spectra

- **QCD-inherited scaling properties**

$$f(m_T) = A \cdot \left[1 + \frac{q-1}{T_s}(m_T - m)\right]^{-\frac{1}{q-1}}$$

  - Parameter scaling with √s & multiplicity

$$\mathrm{A}(\sqrt{s_{NN}}, \langle N_{ch}/\eta \rangle, m) = A_0 + A_1 \ln \frac{\sqrt{s_{NN}}}{m} + A_2 \langle N_{ch}/\eta \rangle$$

$$\mathrm{T}(\sqrt{s_{NN}}, \langle N_{ch}/\eta \rangle, m) = T_0 + T_1 \ln \frac{\sqrt{s_{NN}}}{m} + T_2 \ln\ln \langle N_{ch}/\eta \rangle,$$

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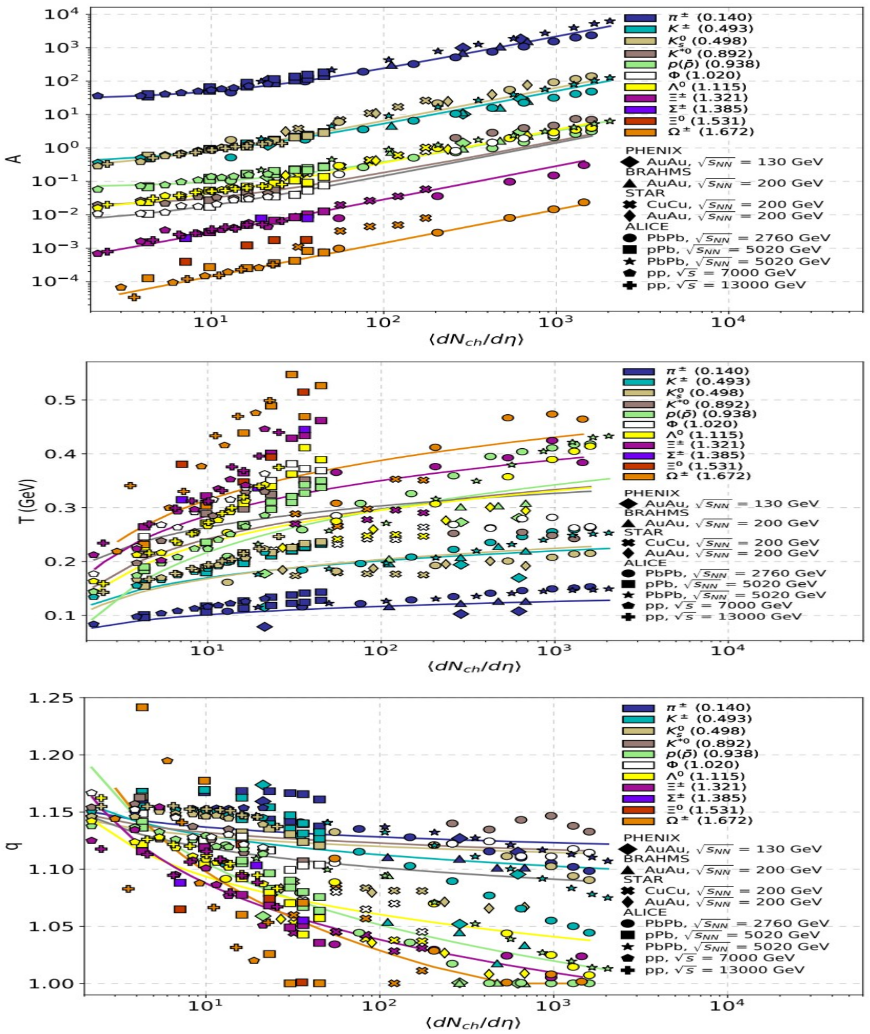
  - Details:

    G. Biró *et al: J.Phys.G 47 (2020) 10, 105002*

    K. Shen *et al Eur.Phys.J.A 55 (2019) 8, 126*



Introducing the Tsallis-thermometer

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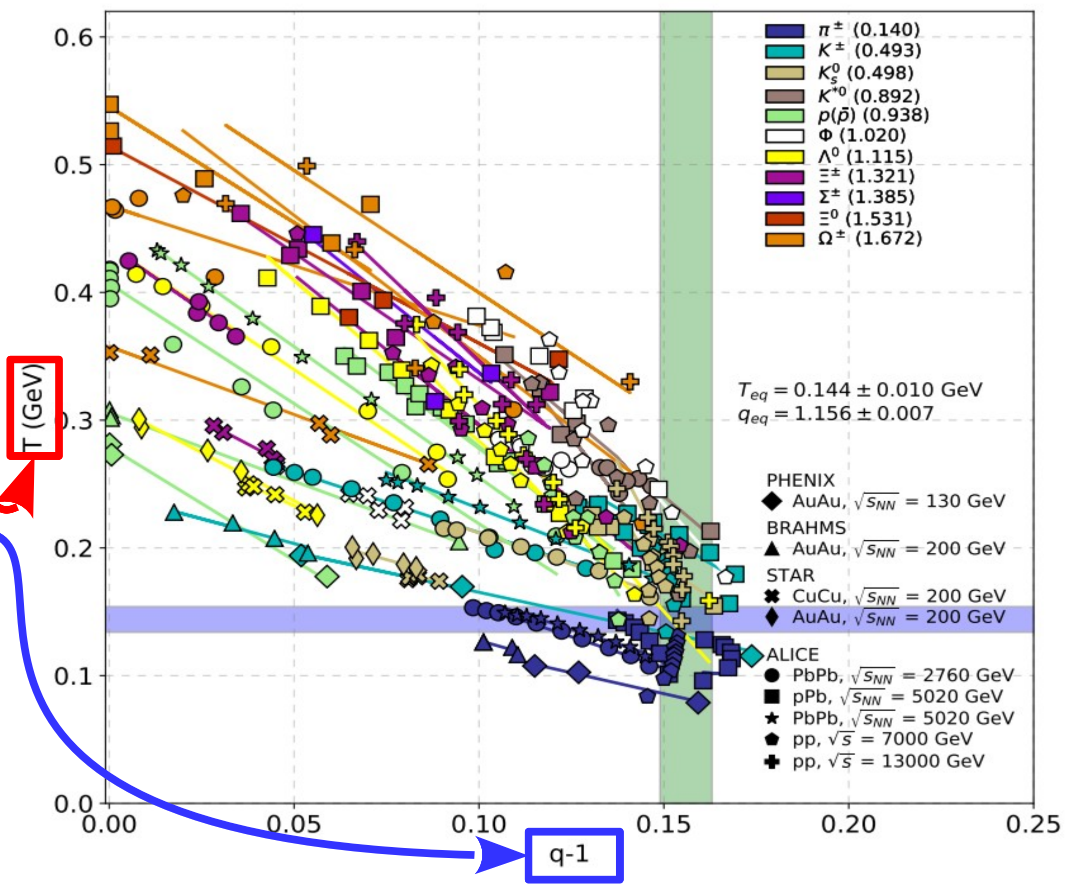

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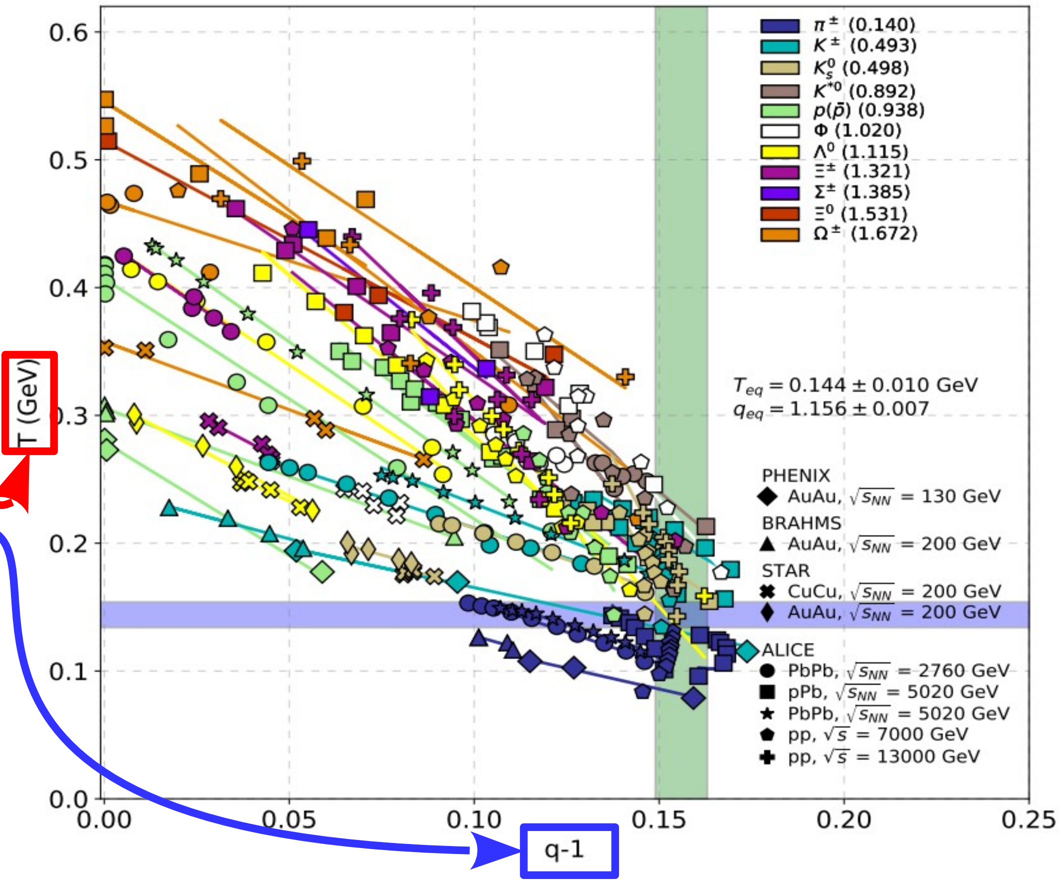
- **Light Flavour (LF)**
  - Strong dependence on event multiplicity
  - Mass hierarchy presents for light flavour

  - **LF grouping: T_{eq}≈0.14 GeV and q_{eq}≈1.15**



A connection between Tsallis parameters

Non-extensive entropy does not need thermal equilibrium: $S(E_1+E_2) \neq S(E_1)+S(E_2)$

$$\frac{1}{T} = \langle S'(E) \rangle = \langle \beta \rangle$$

$$q = 1 - \frac{1}{C} + \frac{\Delta\beta^2}{\langle\beta\rangle^2}.$$

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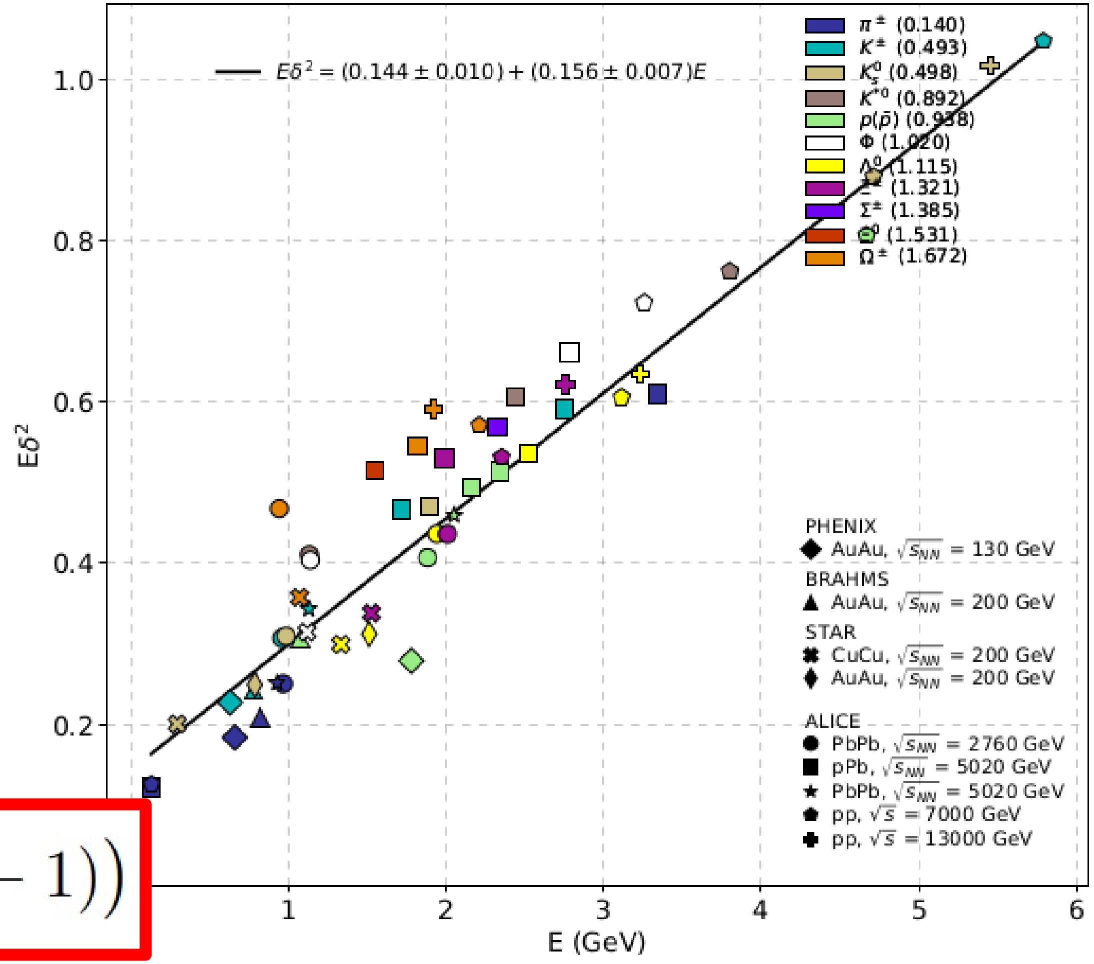

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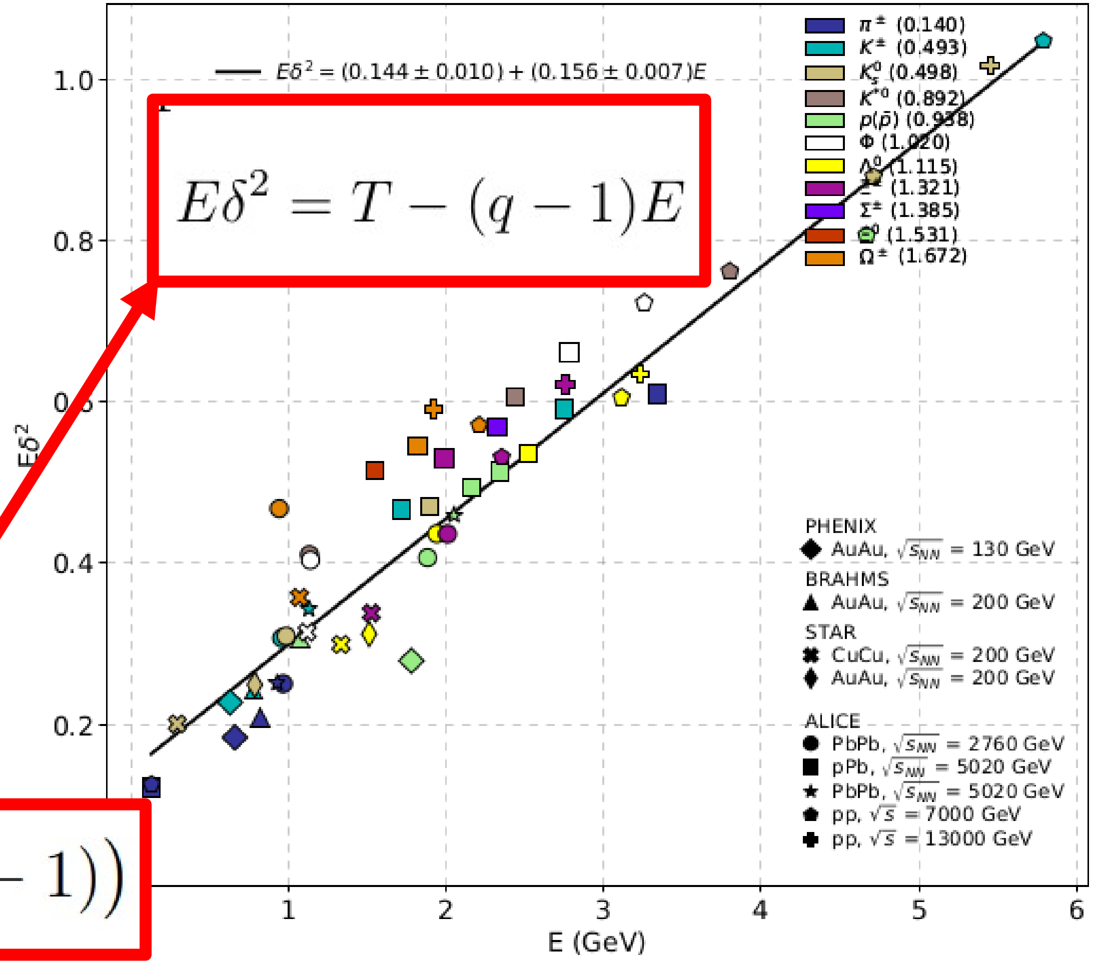
$$T = E\left(\delta^2 - (q-1)\right)$$

$$E\delta^2 = T - (q-1)E$$



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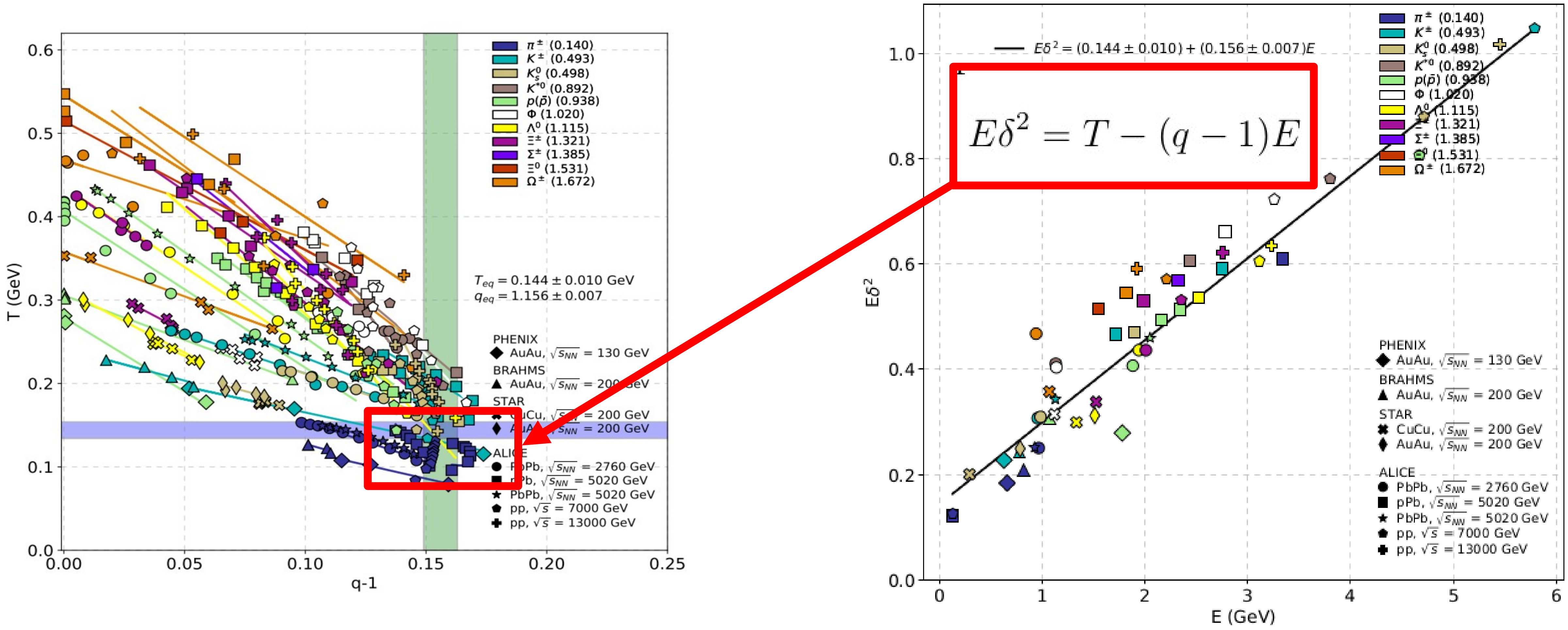

Transforming the Tsallis-thermometer and fitting the *E-Eδ²* points with a line defines the (linearized) equilibrium values for the: *T* (offset) and *q* (slope) parameters.

Applying the Tsallis thermometer

Tsallis-thermometer of light flavours

Light Flavour (LF)

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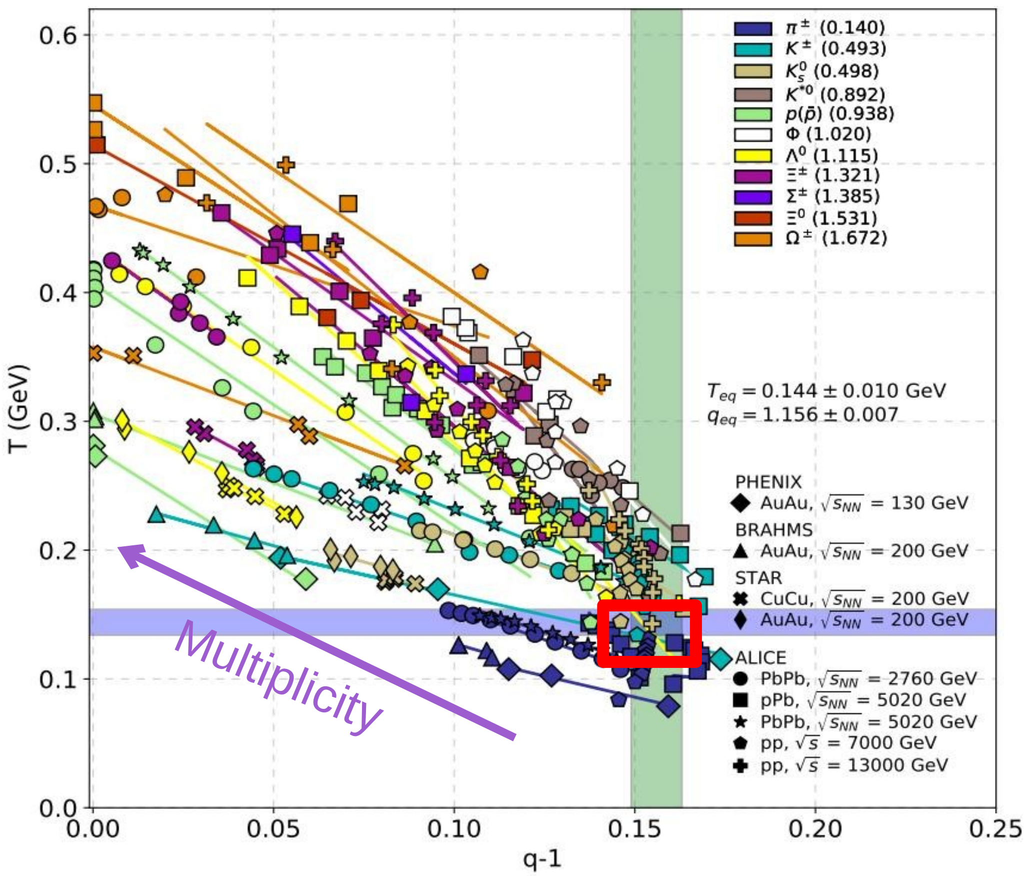

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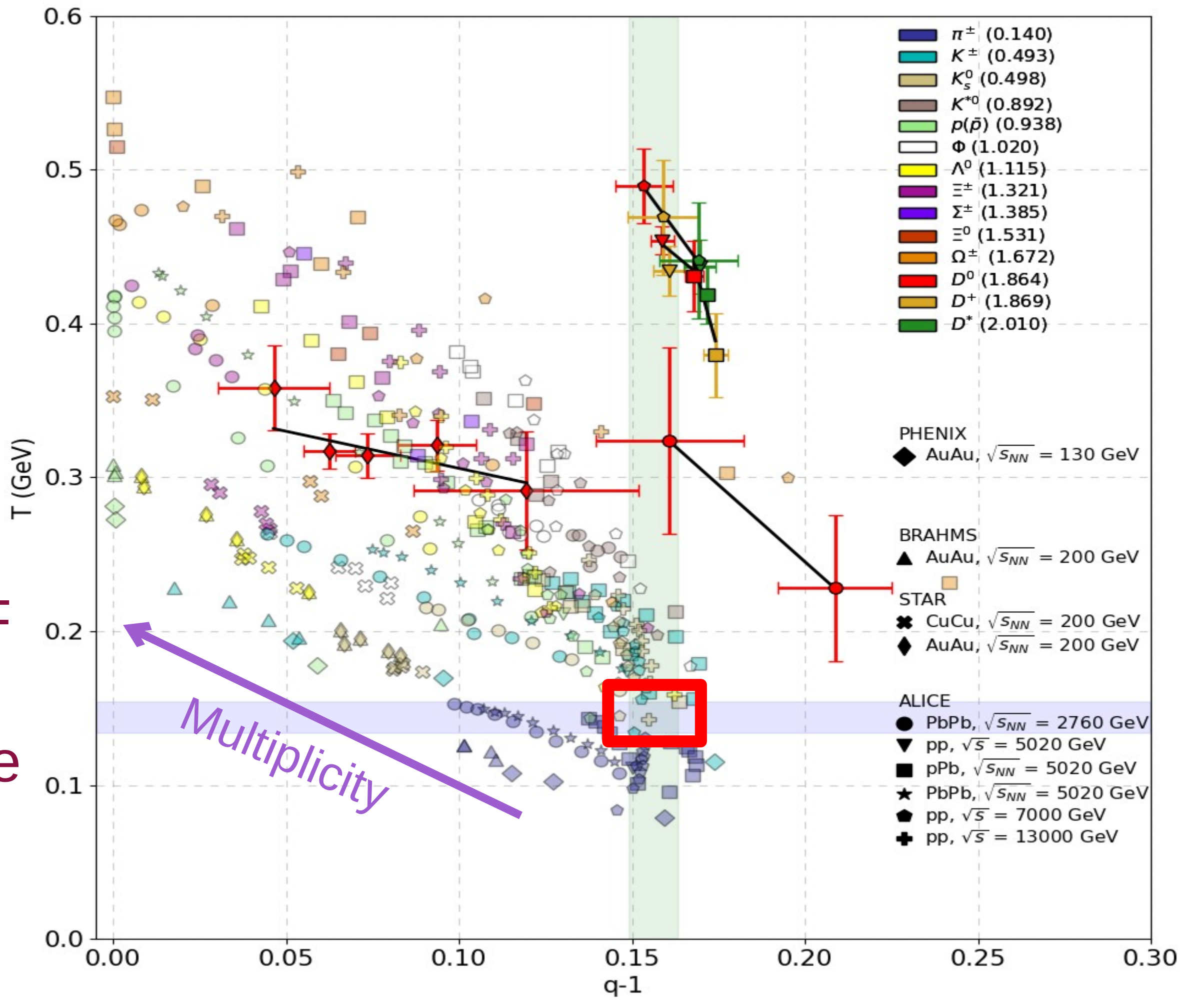
D mesons (HF)

- Dependence on the collision energy for HF is more prominent, than for LF
- A HF grouping is also present, however the “center” is shifted compared to the LF
- **HF grouping: T_{eq}≈0.25 GeV and q_{eq}≈1.21**



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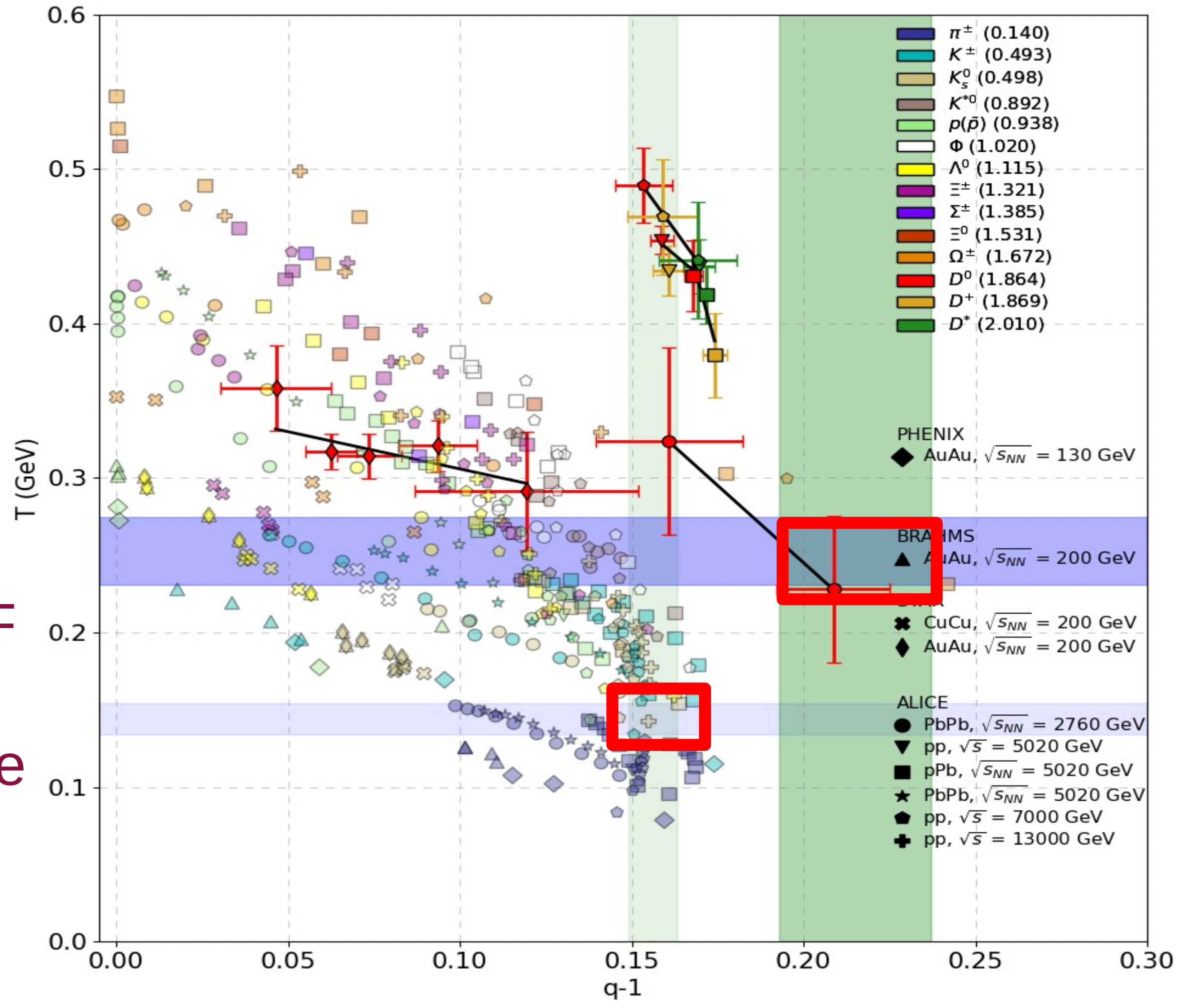

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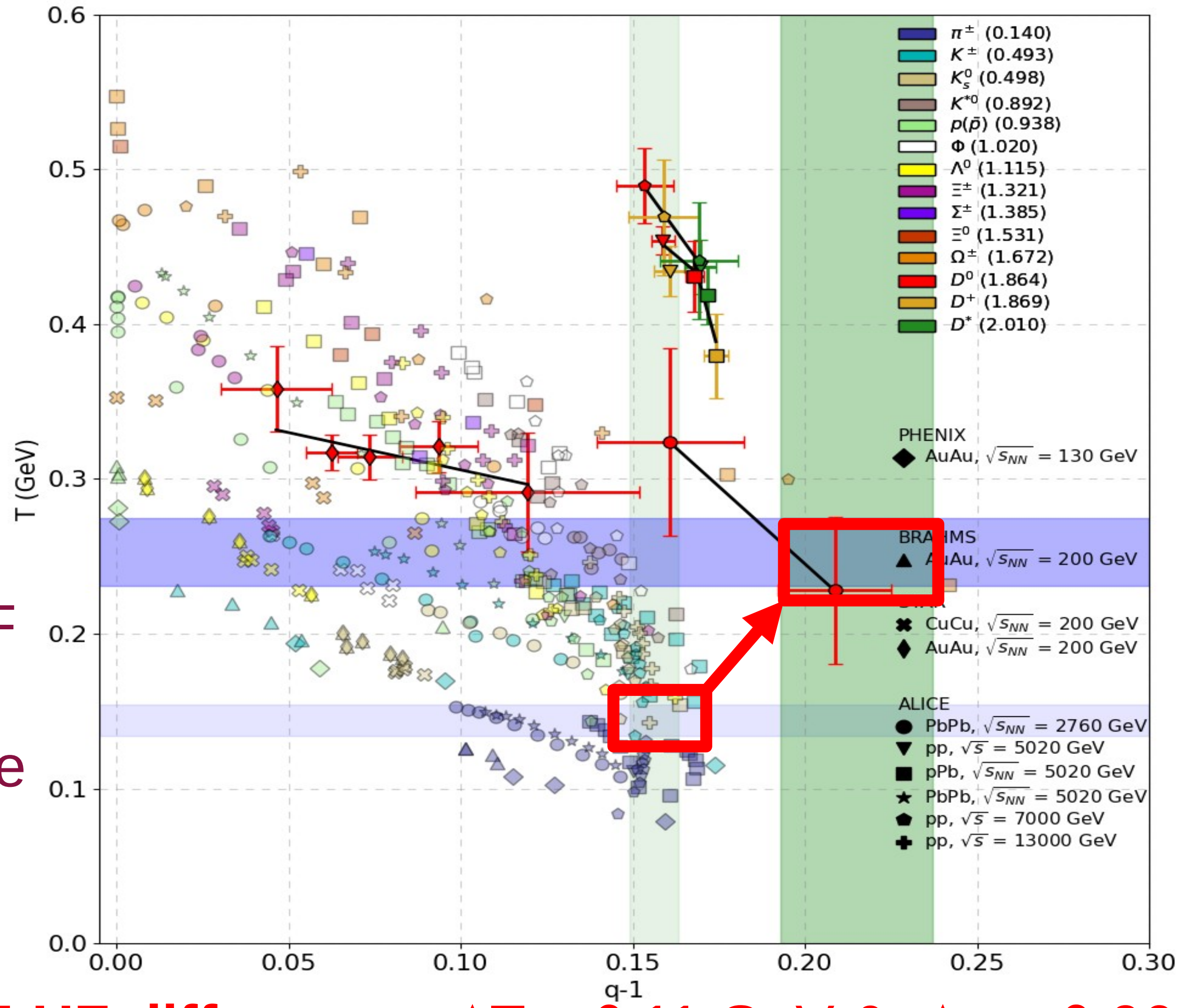

LF-HF difference: ΔT_{eq}≈0.11 GeV & Δq_{eq}≈0.06

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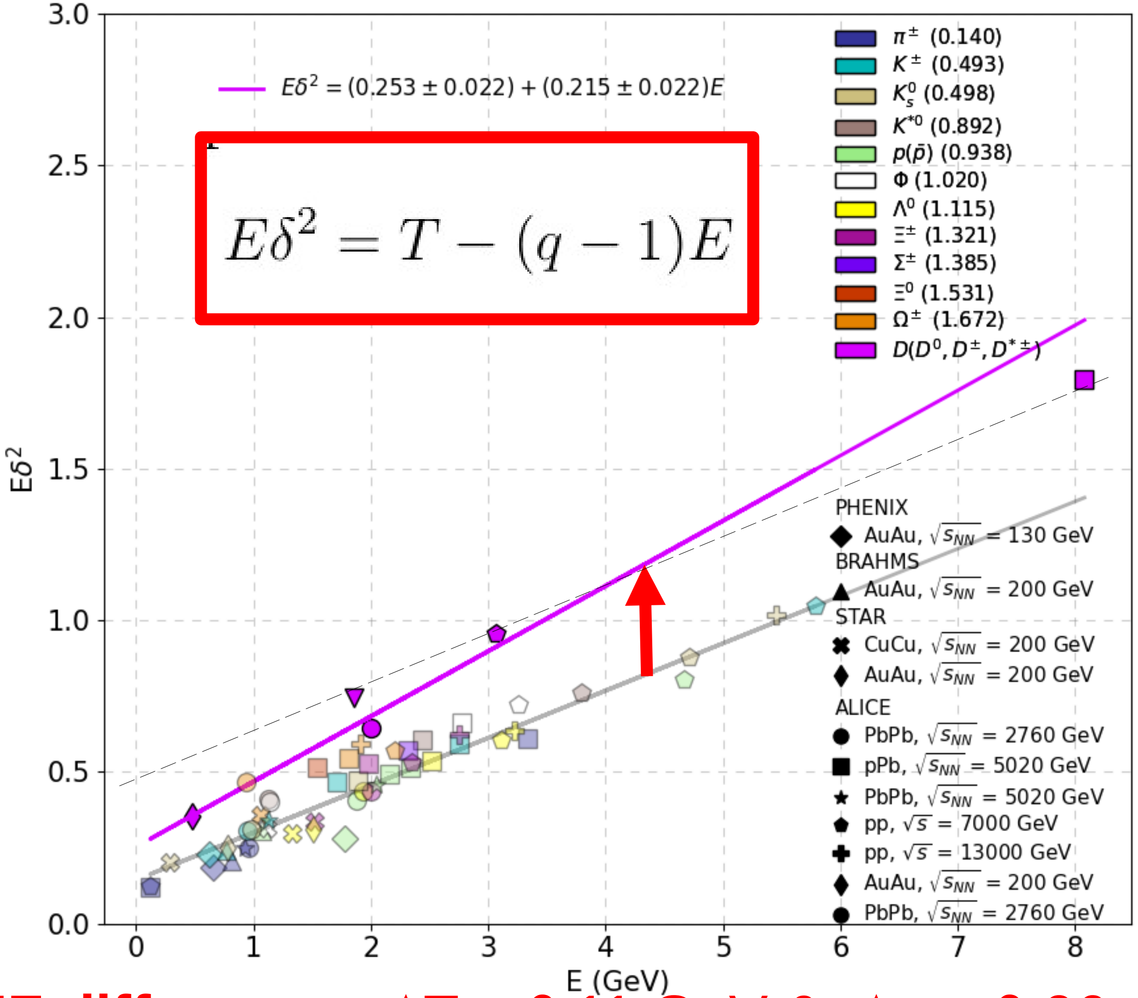

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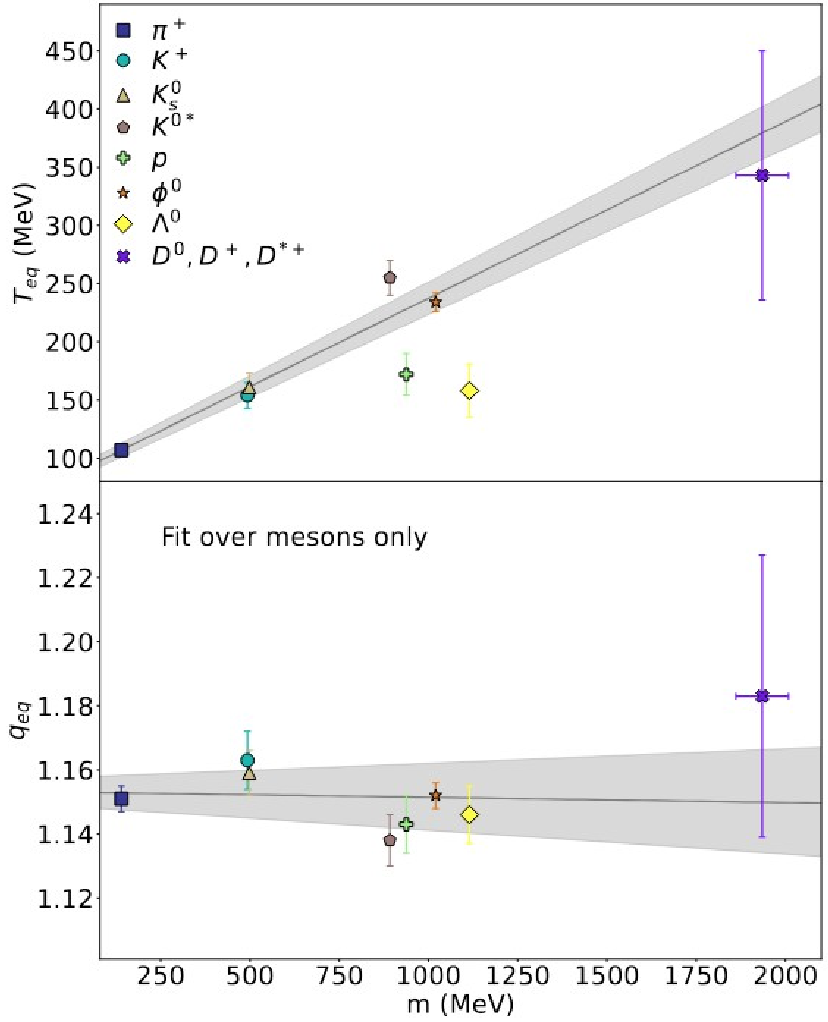
Difference in HF-LF formation time

Further properties of the fix point

- Temperature (T_{eq}) of the common fix points for mesons are linearly increase with the hadron masses.

- Temperature, T_{eq} is smaller for baryons than the same mass mesons.

- Non-extensivity parameter, q_{eq} does not present significant mass dependence



Difference in HF-LF formation time

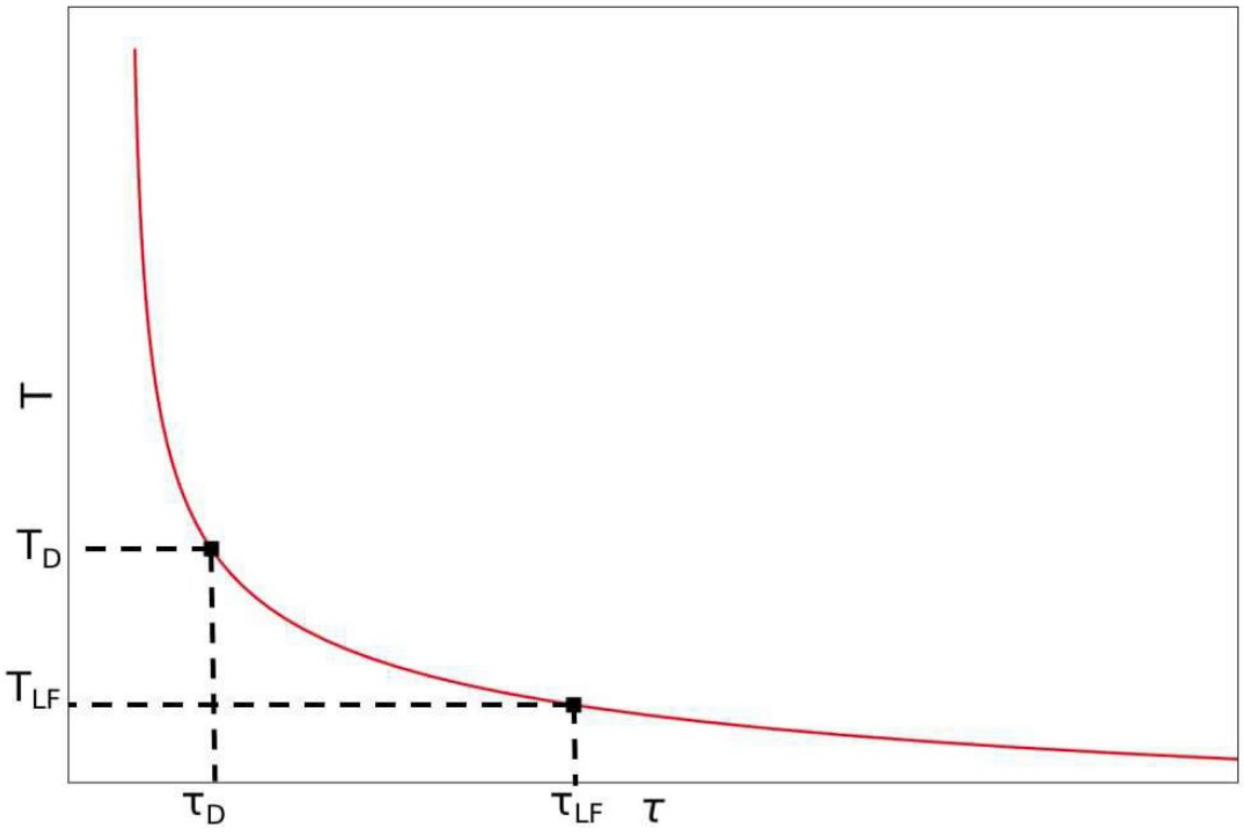
- Bjorken-model DOES NOT say anything on the thermodynamical description
  → **temperature scales can be connected**

$$\tau = \tau_0 \left(\frac{T_0}{T} \right)^3$$

- Once we know the temperature values, we could turn this to measure the time scales, using the approximated fix point value: T_{eq}

$$\tau_{\mathrm{D}} = \tau_{\mathrm{LF}} \left(\frac{T_{\mathrm{LF}}}{T_{\mathrm{D}}} \right)^3$$

- Taking all light flavours as reference,
  →**D-meson formation relative to all LF**



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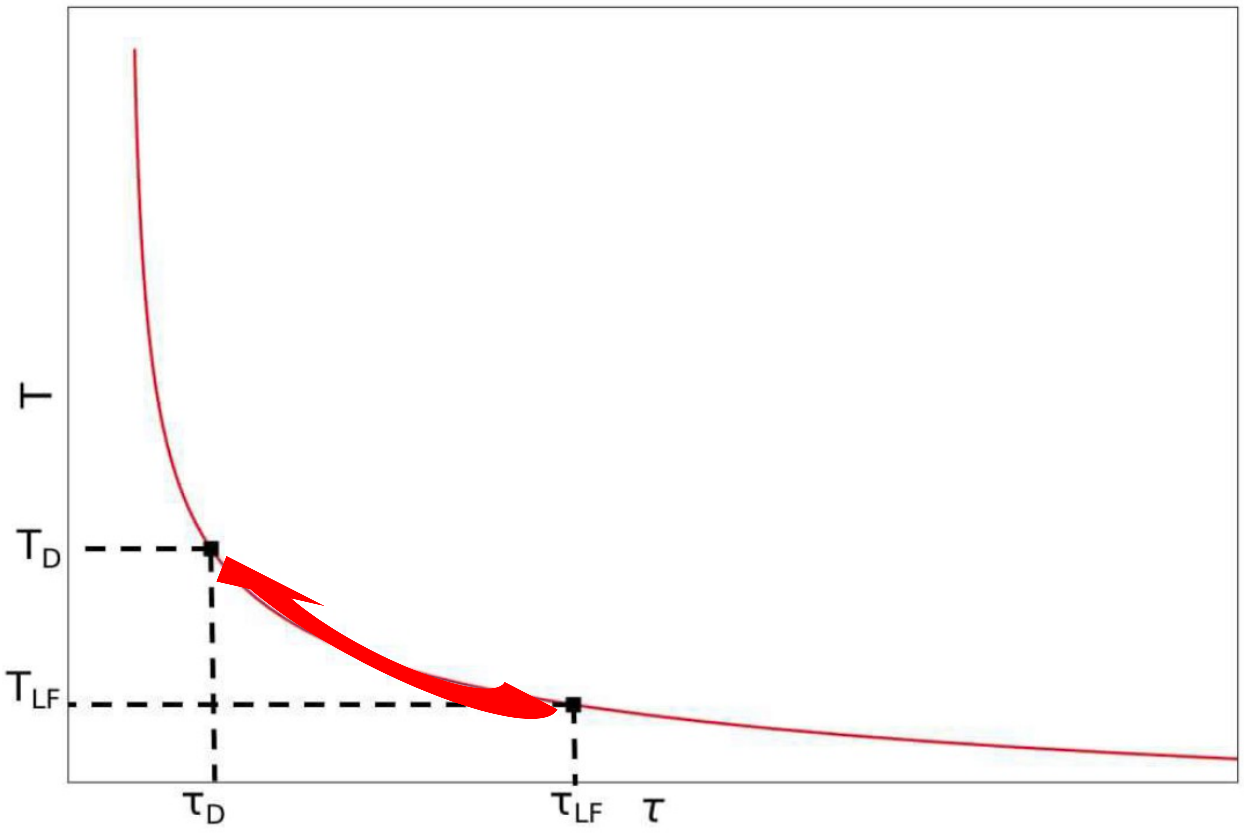
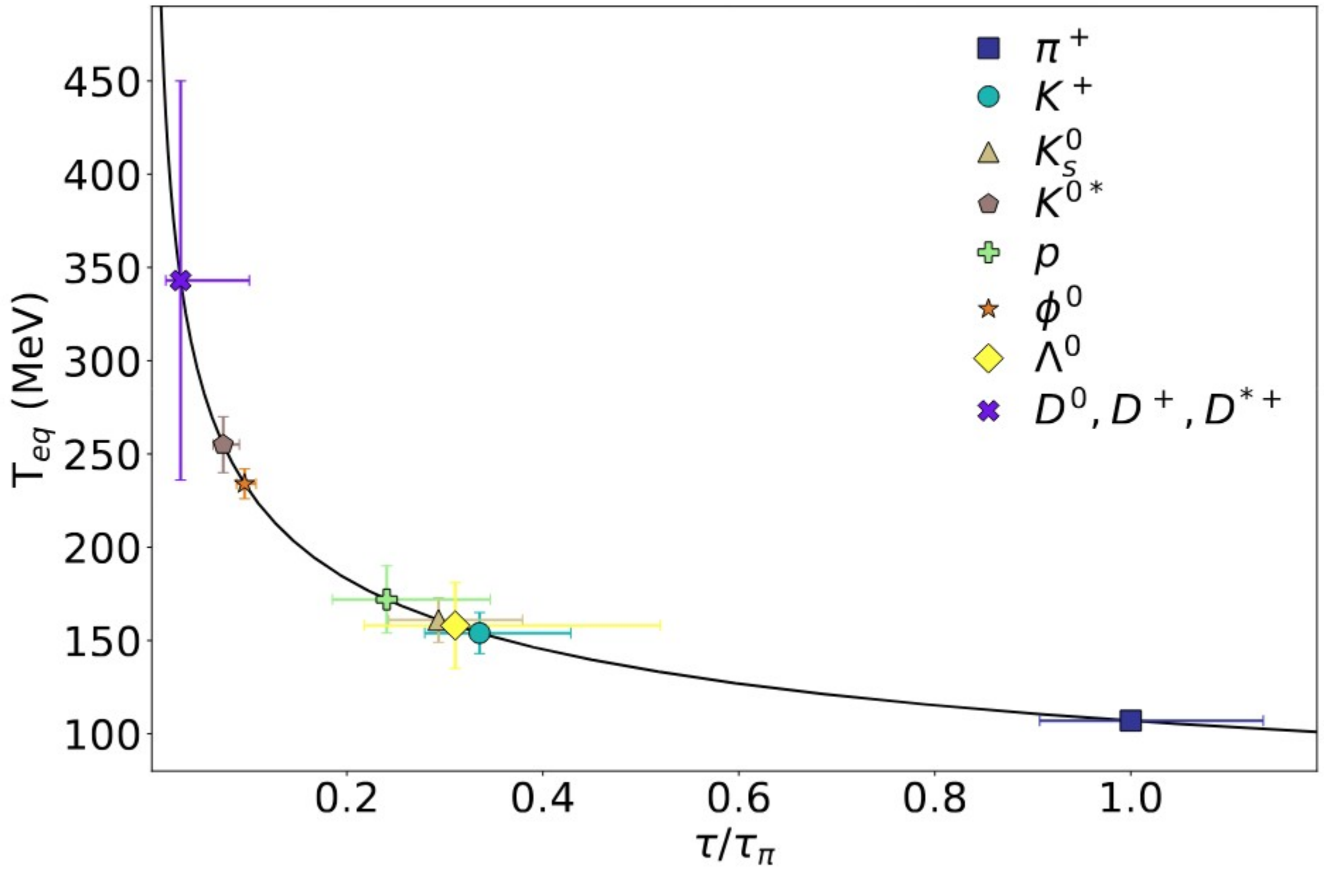


Difference in HF-LF formation time

Adding more identified hadrons

- Pion formation is the latest one
- Formation time has mass order: the lighter the hadron is, it forms later.
- Heavier baryons forms later than other mesons with the same mass
- Taking all PID & D-mesons (here only at LHC energies) → **D-meson formation relative to π is 30x earlier...**



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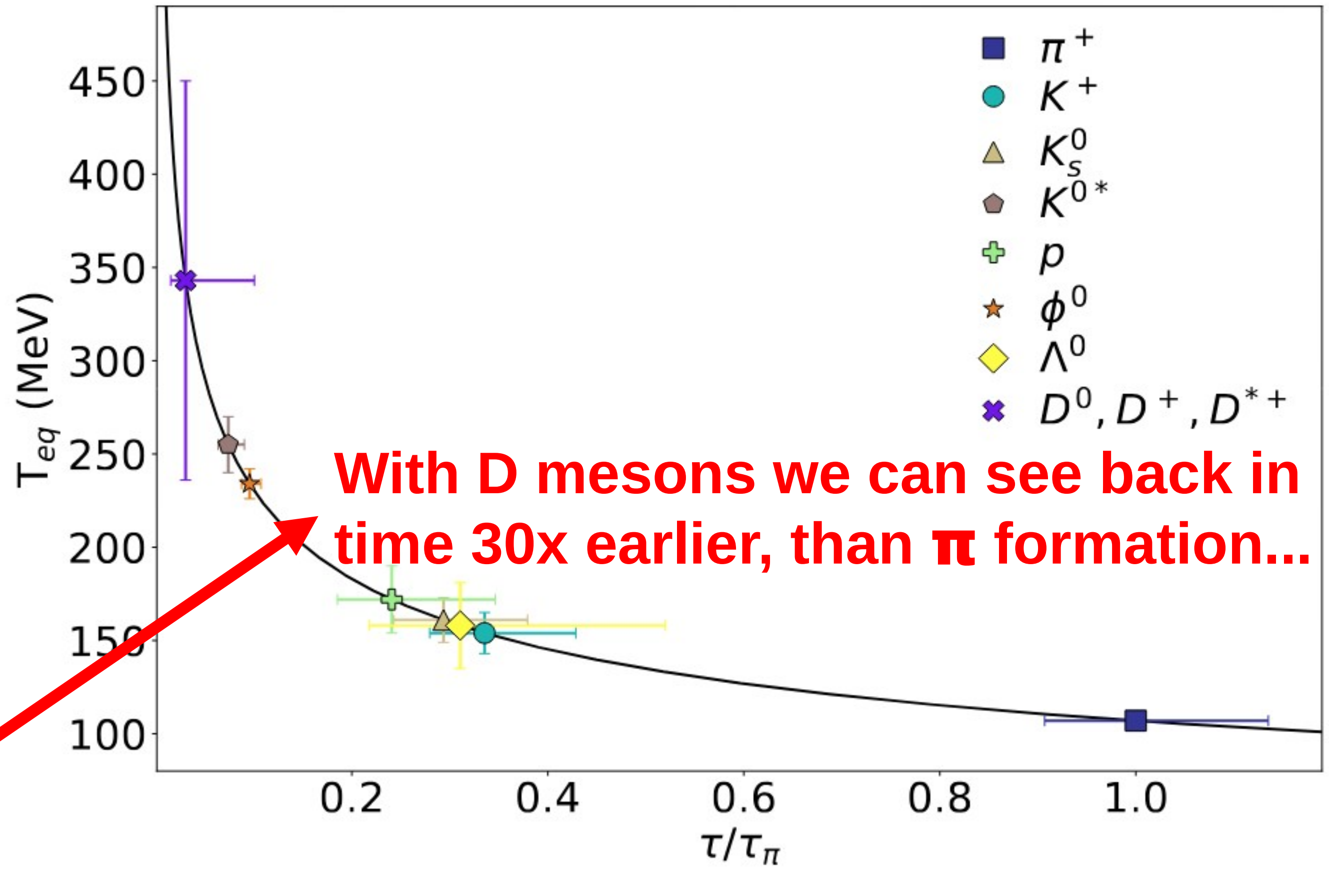
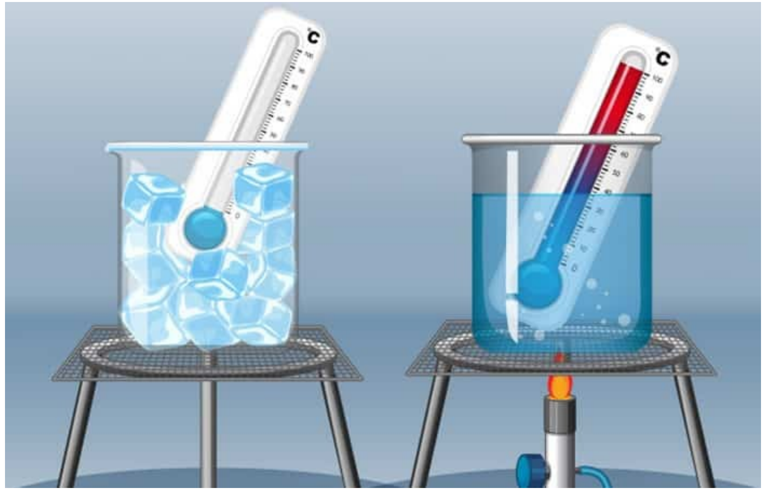


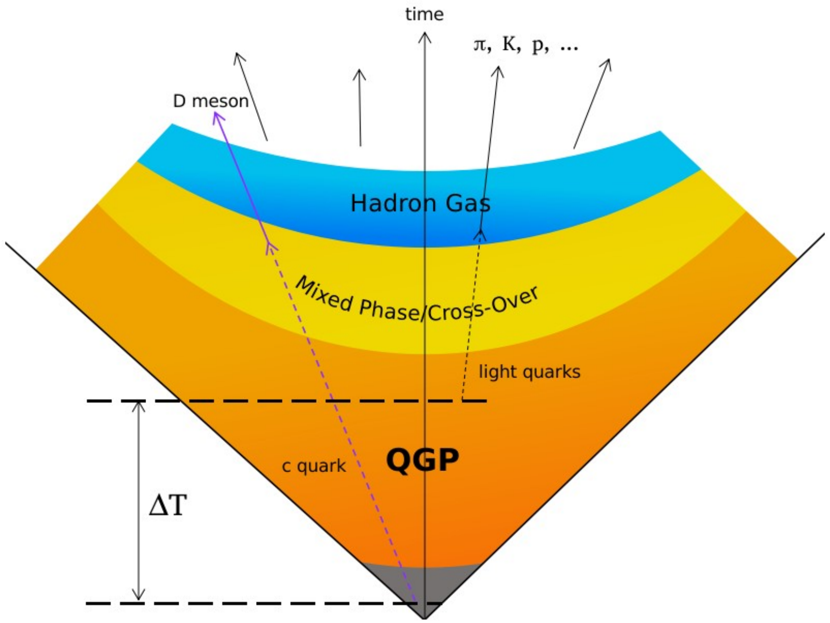
With D mesons we can see back in time 30x earlier, than π formation...

Conclusions

- **Non-extensive statistical framework**
  - Based on the data, our model is working for both LF and D-meson production
  - Works from RHIC to LHC energies at the highest p_T
  - Tsallis-Pareto fits well in all multiplicities
- **Comparing LF & HF via Tsallis-thermometer**
  - Tsallis-thermometer present similar trends, but scales are different between LF and HF.
  - Mass hierarchy is present and stronger for HF
  - Overall grouping is different between mesons & baryons, and between LF & HF

→ To take away… Bjorken model is compatible with the Tsallis-thermometer, and relative formation time can be estimated.





Backups

Thermodynamical consistency?

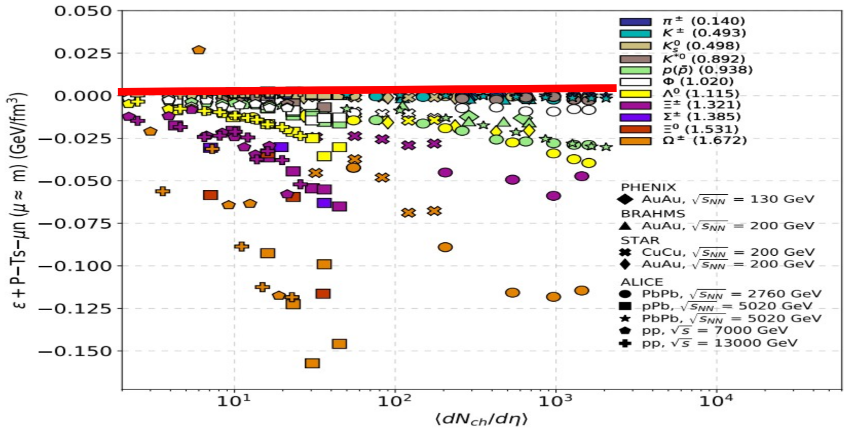
Thermodynamical consistency: fulfilled up to a high degree

$$\mathrm{P} \;= \mathrm{g} \int \frac{d^3p}{(2\pi)^3} T f,$$

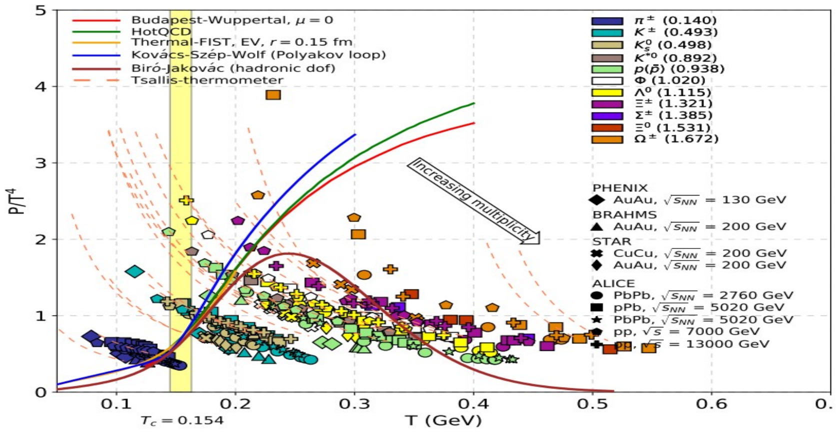
$$\mathrm{N} \;= \mathrm{nV} = \mathrm{gV} \int \frac{d^3p}{(2\pi)^3} f^q,$$

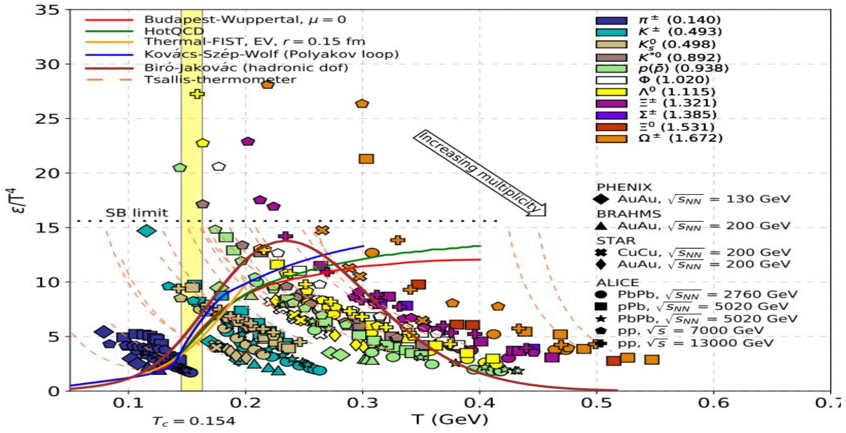
$$\mathrm{s} \;= \mathrm{g} \int \frac{d^3p}{(2\pi)^3} \left[\frac{E-\mu}{T} f^q + f \right],$$

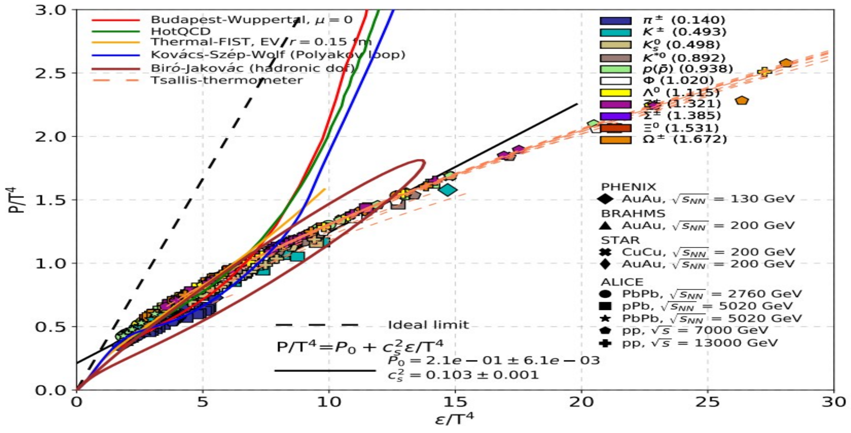
$$\varepsilon = g \int \frac{d^3p}{(2\pi)^3} E f$$



Compare EoS to data: Lattice QCD (parton) & Biró-Jakovác parton-hadron







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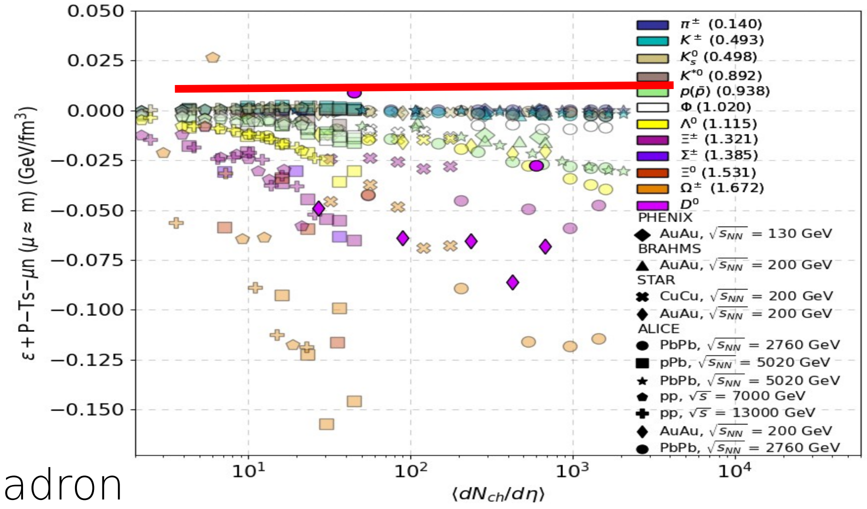
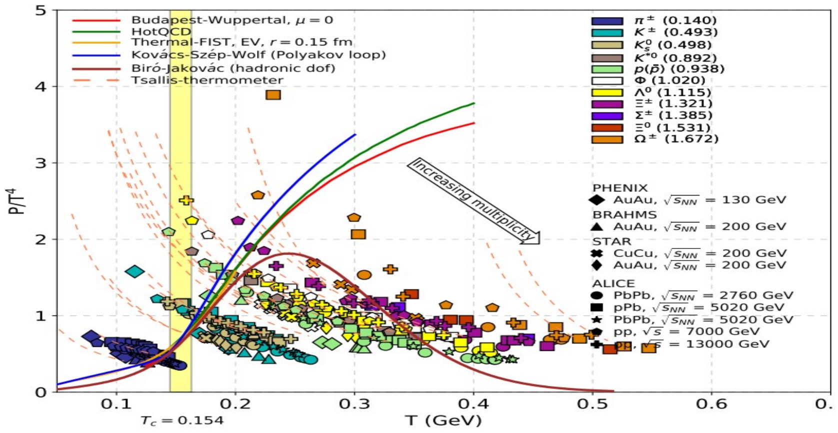
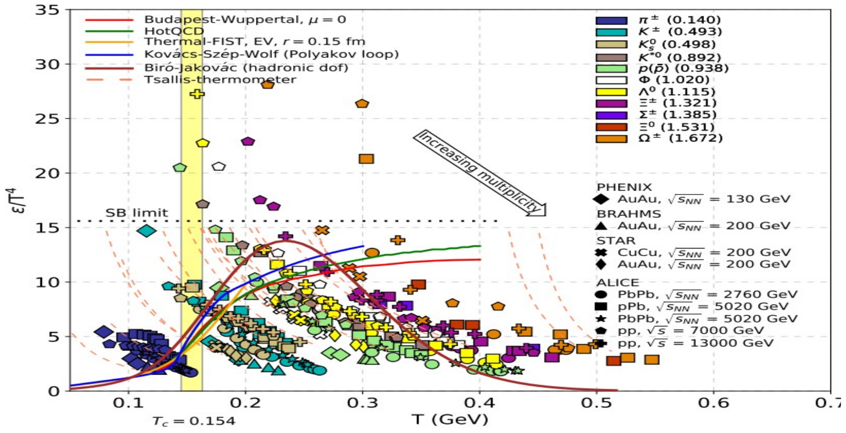
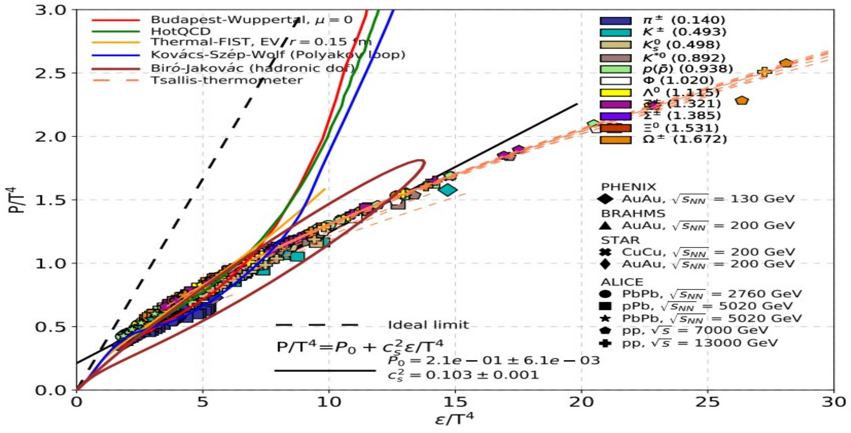


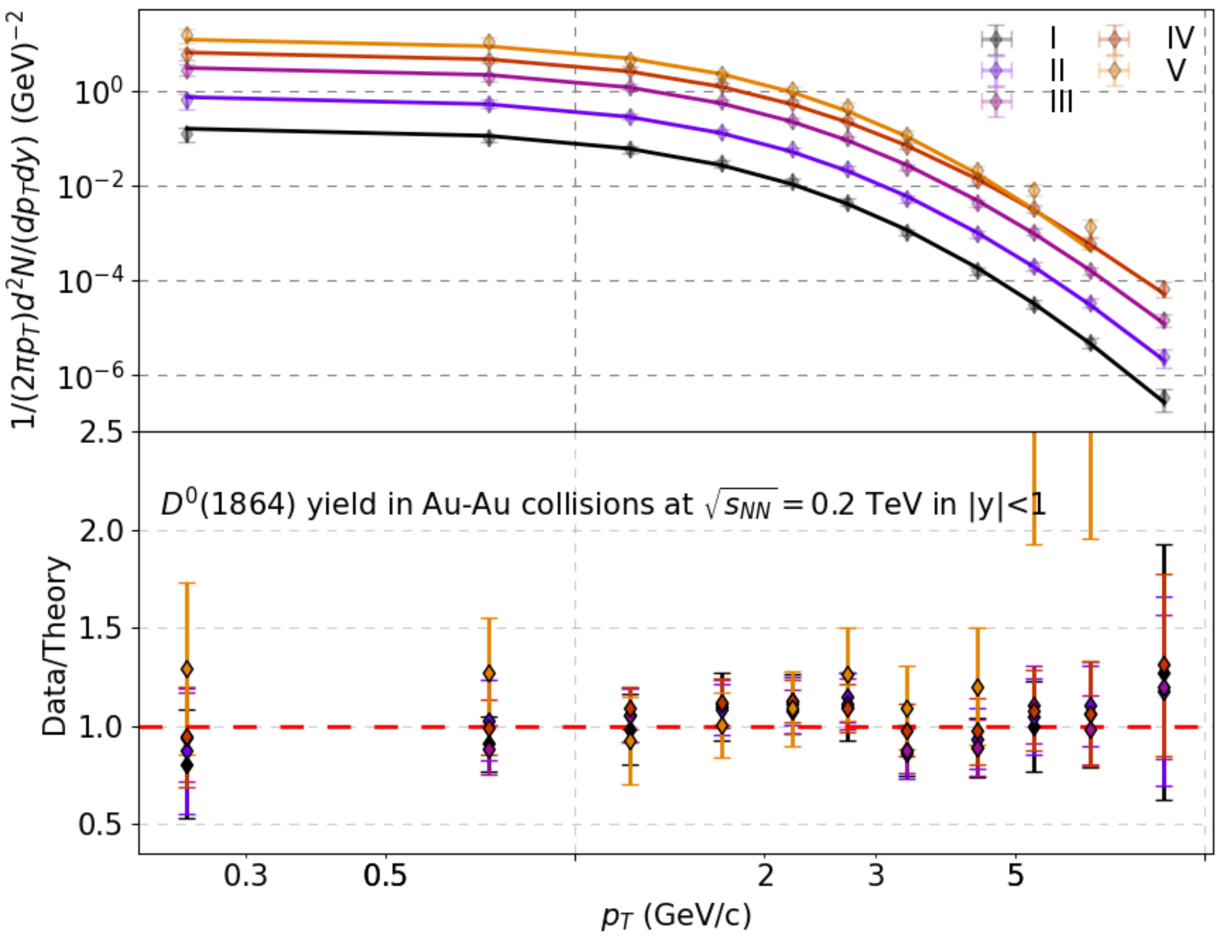
Compare EoS to data: Lattice QCD (parton) & Biró-Jakovác parton-hadron

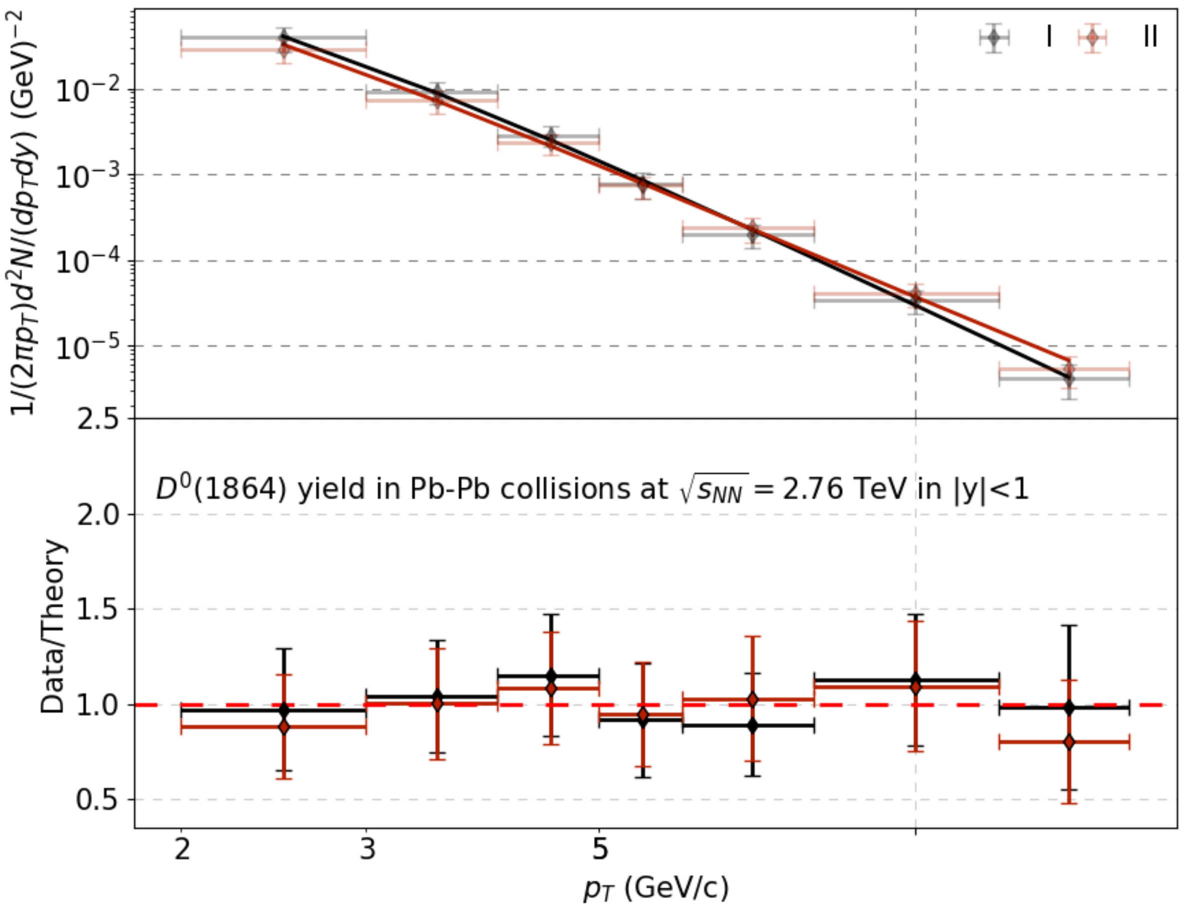




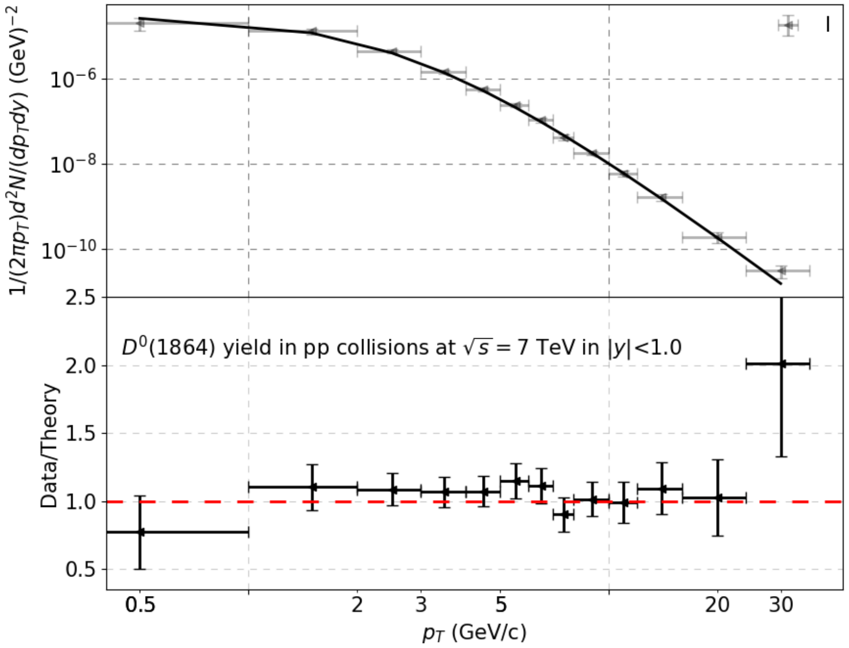


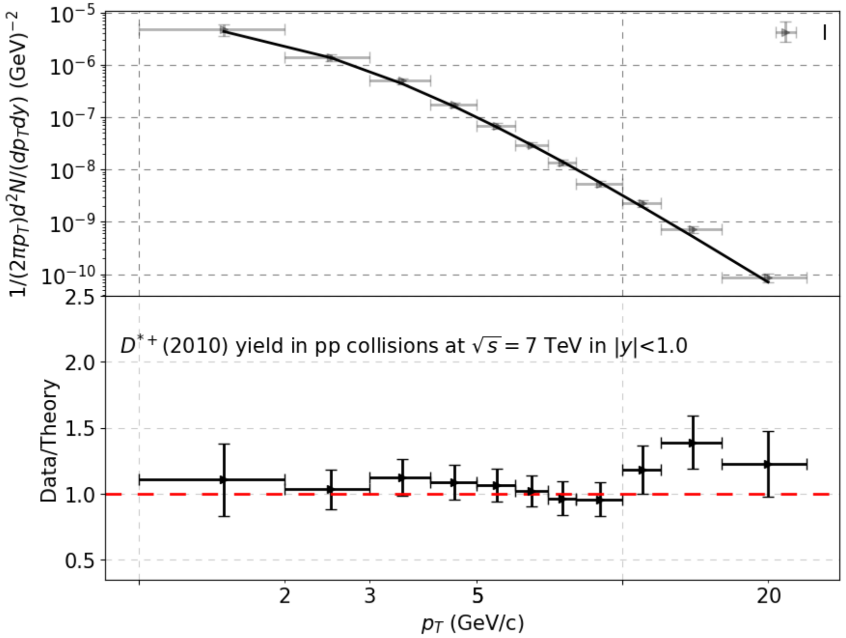
HF hadron spectra

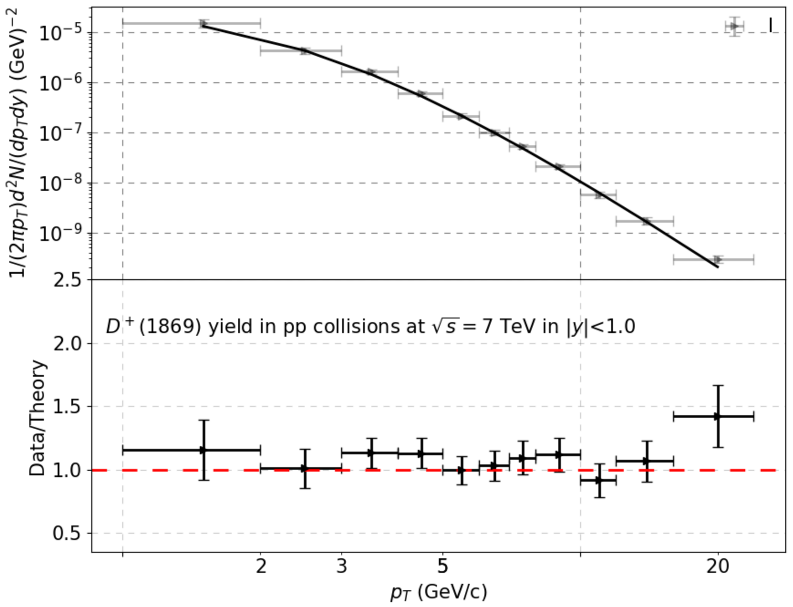




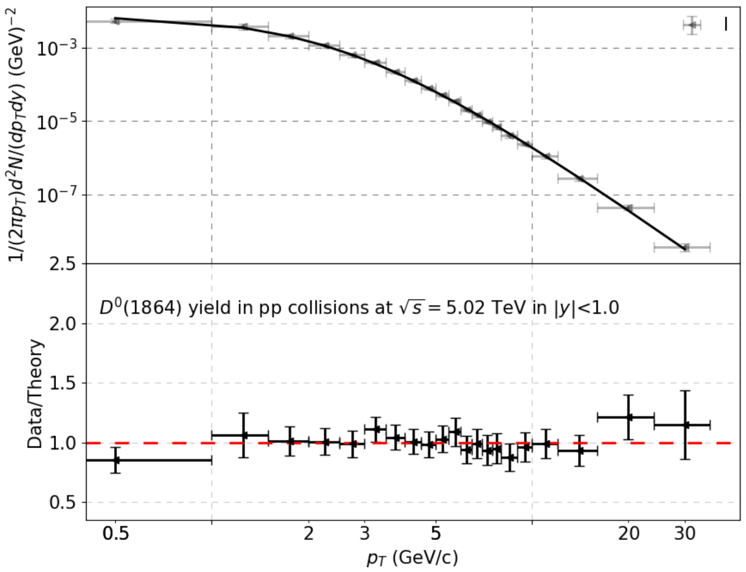
HF hadron spectra

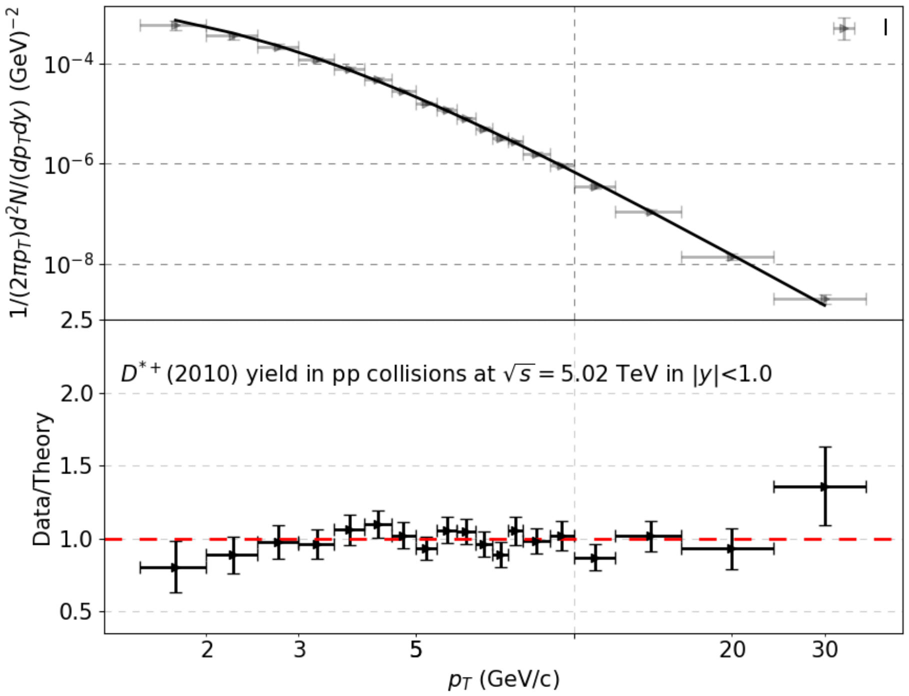


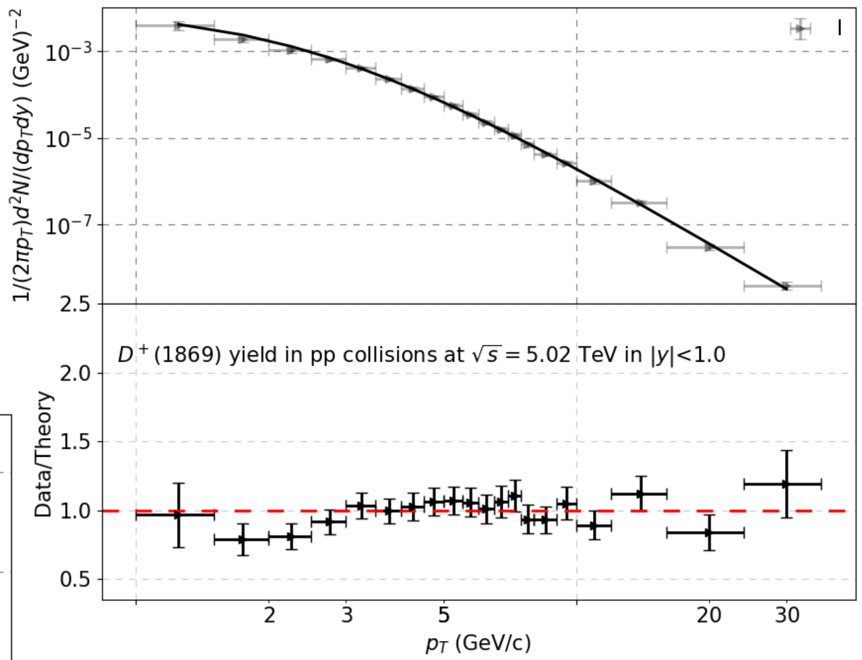




HF hadron spectra







HF hadron spectra