QCD: from Lagrangian to applications

1. QCD basis (第17章) QED第6章.
2. renormalization. (第15章+17.5.17.6,17.7,17.8)
3. RGE and its solution. (第19章)

   composite operator renormalization

4. 共线因子化

   ① Infrared divergences in $e^+e^- \to q\bar{q}$ (20.1节)

   ② OPE in $e^+e^- \to q\bar{q}$ (20.2节)

   ③ OPE in DIS (20.3节)

   ④ factorization in DIS (20.4节)

   ⑤ OPE and 4-quark operator (HQET book, 第一章)

   ⑥ Drell-Yan

5. TMD因子化.

   ①

第一部份 QCD basis

1. QCD Lagrangian

2 quantization

3. Feynman Rules

4. parameters in QCD

5. QCD process at tree level and perturbation theory.

$e^+e^- \to \mu^+\mu^-$ — — — $u\bar{u} \to d\bar{d}$

$e^-\mu^- \to e^-\mu^-$ — — — $ud \to ud$

$e^+e^- \to \gamma\gamma$ — — — $u\bar{u} \to gg$

— — —

1. QCD Lagrangian

SU(3): gauge group $\quad SU(N_c)$

$$\mathcal{L}_{QCD} = -\frac{1}{2} tr\left[F^{\mu\nu} F_{\mu\nu}\right] + \sum_{i=1}^{N_f} \bar{q}_i (i\not{D} - m_i) q_i$$
$$= -\frac{1}{4} F^{a,\mu\nu} F^{a}_{\mu\nu} + \sum_{i=1}^{N_f} \bar{q}_i (i\not{D} - m_i) q_i$$

$q_i(x)$ quark field

$A^{a,\mu}(x)$ gauge field

$A^{\mu} = T^{a} A^{a,\mu}(x) \quad a = 1 \cdots N_c^2 - 1 \quad N_c \times N_c$ 矩阵

$D^{\mu} = \partial^{\mu} - i g_s T^{a} A^{a,\mu}$ $\quad \boxed{D^{\mu} = \partial^{\mu} \underline{\underline{\pm}} i g_s \underline{\underline{T^{a} A^{a,\mu}}}}$ 另一种约定

$$F^{\mu\nu} = [D^{\mu}, D^{\nu}]$$
$$= \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} - i g_s [A^{\mu}, A^{\nu}];$$

$$F^{a,\mu\nu} = \partial^{\mu} A^{a\nu} - \partial^{\nu} A^{a,\mu} + g_s f^{abc} A^{b}_{\mu} A^{c}_{\nu}$$

gauge transformation: $W(x)$ element $SU(N_c)$

$$q(x) \rightarrow \underline{\underline{W(x)}}\, q(x) = \underline{\underline{e}}^{i T^{a} \theta^{a}} q(x)$$

$$\Rightarrow A^{\mu} \rightarrow W(x) A^{\mu}(x) W^{+}(x) + \frac{i}{g_s} W(x) \partial^{\mu} W^{+}(x) \qquad W^{+} = \frac{1}{W}$$

$$D^{\mu} q(x) \longrightarrow W(x)\, \underline{D^{\mu} q(x)} \qquad \underline{W^{+} W = 1}$$

$$(\partial^{\mu} - i g_s A^{\mu}) q(x) \rightarrow \left(\partial^{\mu} - i g_s \left(\underline{W A^{\mu} W^{+}} + \underline{\frac{i}{g_s}} W \partial^{\mu} W^{+}\right)\right) W(x) q(x)$$
$$= \partial^{\mu} (W(x) q(x)) - i g_s W A^{\mu} q(x) + \underline{W(\partial^{\mu} W^{+})} W(x) q(x)$$
$$= (\partial^{\mu} W(x)) q(x) + W(x) \partial^{\mu} q(x) - i g_s W(x) A^{\mu}(x) q(x) - \underline{(\partial^{\mu} W)} q(x)$$
$$= W(x) D^{\mu} q(x)$$

$\mathcal{L}_{QCD}$ is invariant under gauge transformation.

2. quantization:

ⓐ canonical quantization.

QM

1. generalized coordinate

momentum

$\underline{q_i} \quad \underline{p_j}$

2. $[q_i\ p_j] = i\hbar\delta_{ij}$

$[q_i, q_j] = [p_i, p_j] = 0$

$\longrightarrow$

QFT

1. coordinate. $\leftarrow \phi(x)$

$$\phi_i(t) = \frac{1}{\Delta V_i}\int_{\Delta V_i} d^3x\ \phi(x)$$

$$L = \int \underline{d^3x}\ \mathcal{L} = \sum_{i=1}^{\infty} \Delta V_i\ \bar{\mathcal{L}}_i$$

$$\dot{\phi}_i(x) = \frac{1}{\Delta V_i}\int_{\Delta V_i} d^3x\ \frac{\partial}{\partial t}\phi(x)$$

$$p_i = \frac{\partial L}{\partial\dot{\phi}_i} = \Delta V_i \frac{\partial\bar{L}_i}{\partial\dot{\phi}_i} = \Delta V_i\ \pi_i$$

2. $[\phi_i\ p_j] = \underline{i\hbar}\,\delta_{ij}$

$[\phi_i\ \phi_j] = [p_i\ p_j] = 0$

$[\phi_i\ \pi_j] = i\hbar\,\underline{\delta_{ij}}\,\frac{1}{\underline{\Delta V_i}}$

$[\phi(\vec{x}), \pi(\vec{y})] = i\hbar\,\delta^3(\vec{x}-\vec{y})$

$\pi(\vec{y}) = \frac{\partial\mathcal{L}}{\partial\dot{\phi}(y)}$

3. plane-wave expansion

$$\underline{\phi(x)} = \int\frac{d^3k}{(2\pi)^3 2E}\left(a(k)e^{-ik\cdot x} + a^{+}(k)e^{ik\cdot x}\right)$$

$$\pi(y) = \dot{\phi}(x)$$

gauge field: (EM)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\pi_\mu = \frac{\partial \mathcal{L}}{\partial \dot{A}^\mu} = -F_{0\mu}$$

无法对 A_0 量子化. 起源于非物理自由度.

不定度规量子化:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 \qquad \text{取 } \alpha = 1$$

$$\pi_\mu = -F_{0\mu} - g_{0\mu}(\partial\cdot A)$$

为了回到原始拉氏量, 选择 $\langle\psi|\partial A|\psi\rangle$ 投影出物理状态.

$|\psi\rangle$ 中横向极化的矢量粒子自动满足 $\langle\psi|\partial\cdot A|\psi\rangle = 0$ 即为物理态

但标量与纵向光子需满足:

$$[a_0(k) - a_3(k)]|\phi\rangle = 0$$

将 $|\phi\rangle$ 按净标量和纵向光子数展开:

$$|\phi\rangle = c_0|\phi_0\rangle + c_1|\phi_1\rangle + \cdots$$

则: $\langle\phi_n|N'|\phi_n\rangle = n\langle\phi_n|\phi_n\rangle$

$$= \langle\phi_n|\int d\tilde{k}\,[a_3^\dagger(k)a_3(k) - a_0^\dagger(k)a_0(k)]|\phi_n\rangle = 0$$

此即 $\langle\phi_n|\phi_n\rangle = \delta_{n0}$ 仅有 $n=0$ 时 粒子态才非零

可以记 $|\phi_0\rangle \equiv |0\rangle$, 物理量不受影响. (但仍存自由度)

电磁场传播子:

$$\langle 0|T[A_\mu(x)A_\nu(y)]|0\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{-i}{k^2+i\varepsilon}\left[g_{\mu\nu}-(1-\alpha)\frac{k_\mu k_\nu}{k^2+i\varepsilon}\right]$$

⑥ path integral:

$$Z[J] = \int [dA] \exp\left[i \int d^4x \left(\mathcal{L} + AJ \right) \right]$$

$$[dA] = \prod_{\mu, a} [d A_\mu^a].$$

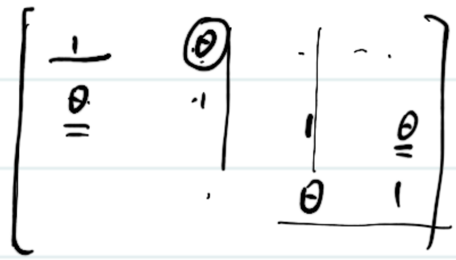
$$A_\mu^a \to A_\mu'^a(x) = \underline{A_\mu^a(x) + f^{abc} \theta^b A_\mu^c(x) - \frac{1}{g} \partial_\mu \theta^a}.$$

θ 无穷小参量

$$[dA] \to [dA'] = [dA] \cdot \det\left(\frac{\partial A'^a}{\partial A^b} \right)$$

$$= [dA] \det\left(\delta^{ab} - \underline{\underline{f^{abc} \theta^c}} \right)$$

$$= [dA] \left(1 + o(\theta^2) \right)$$



$[dA]$ 包含非物理自由度.

$A_\mu^a \to A_\mu'^a(\theta)$

$$[dA] \Rightarrow \underbrace{[d\theta^a]}_{\text{非物理自由度}} \cdot \underbrace{\left[\frac{dA}{d\theta^a} \right]}_{\text{物理}}$$

考察某规范固定条件: $\underline{G^\mu A_\mu^a = B^a}$ $\quad a = 1 \cdots 8$

$\downarrow$

$$G^\mu A_\mu^{(\theta)a} = B^a$$

引入

$$\underline{\Delta_G(A)} \int [d\theta^a] \, \underline{\delta\left(G^\mu A_\mu^{(\theta)a} - B^a \right)} = 1$$

$$Z[0] = \int [dA] \, e^{i \int d^4x \, \mathcal{L}(x)} = \int [d\theta^a] \underline{\left[\frac{dA}{d\theta^a} \right] \cdot e^{i \int d^4x \, \mathcal{L}}}$$

$$= \int [dA] \, \Delta_G(A) \int [d\theta^a] \, \delta\left(G^\mu A_\mu^{(\theta)a} - B^a \right) e^{i \int d^4x \, \mathcal{L}}$$

$$= \int [d\theta^a] \underbrace{\int [dA] \, \delta\left(G^\mu A_\mu^{(\theta)a} - B^a \right) \Delta_G(A) \, e^{i \int d^4x \, \mathcal{L}}}_{\delta\left(G^\mu A_\mu^a - B^a = 0 \right)}$$

$$[d\theta] = \prod_a [d\theta^a]$$

$\Delta_G(A)$:
$$\left(M_G(x, y) \right)^a_{\ b} = \frac{\delta\left(G^\mu A_\mu^{(\theta)a}(x) \right)}{\delta \theta^b(y)}$$

$$\Delta_G(A) = \det(M_G)$$

去掉unphysical自由度:

$$Z[0] = \int [dA]\, \delta(G^\mu A_\mu^{(\theta)a} - B^a)\, \Delta_G(A)\, e^{i\int d^4x \mathcal{L}}$$

$$= \int [dA] \det(M_G)\, \delta(G^\mu A_\mu^{(\theta)a} - B^a)\, e^{i\int d^4x \mathcal{L}}$$

$$\Delta_G(A) \int [d\theta^b]\, \delta(G^\mu A_\mu^{(\theta)a} - B^a) = 1,$$

$$\Delta_G \propto \int [d\theta]\, \delta(\theta^a - \theta^{0a}) \cdot \frac{1}{\frac{\partial (G^\mu A^a_\mu)}{\partial \theta}} = 1$$

$$\int dB^a(x) \exp\left[-\frac{i}{2\alpha}\int d^4x (B^a(x))^2\right], \Delta ?$$

按照一定权重:

$$Z_N[0] = \int Z_0[0] \cdot \exp\left[-\frac{i}{2\alpha}\int d^4x (B^a(x))^2\right] [dB^a]$$

$$= \int [dA] \det M_G\ e^{i\int d^4x \left(\mathcal{L} - \frac{1}{2\alpha}(G^\mu A_\mu^a)^2\right)}$$

$\longrightarrow \mathcal{L}_{GF}$

$$\det(M_G) = \int [dc][dc^*] \exp\left[-i\int d^4x\, d^4y\, C^{a*}(x)\, M_G(x,y)^{ab}\, C^b(y)\right]$$

$$\int dx\, e^{-ax^2} = \sqrt{\frac{\pi}{a}}$$

$$\int dx\, dy\, e^{-a(x^2+y^2)} = \frac{\pi}{a}$$

$\doteq \det$

$$G^\mu A_\mu^a - B^a = \partial_\mu A_\mu^{(\theta)a} - B^a = 0$$

$$\det(M_G) = \frac{\delta(G^\mu A_\mu^{(\theta)a})}{\delta \theta^b} = -\frac{1}{g} G^\mu(\delta^{ab}\partial_\mu - g f^{abc} A_\mu^c)\, \delta^4(x-y)$$

$$= -\frac{1}{g}(\delta^{ab}\Box - g f^{abc}\partial^\mu A_\mu^c)\, \delta^4(x-y)$$

$\mathcal{L}_{eff} = \mathcal{L}_G + \mathcal{L}_{GF} + \mathcal{L}_{FP}$ $\quad F^{\mu\nu}$ $\quad \underline{F^{0i} = E^i}$ $\quad \underline{B^i = \frac{1}{2}\varepsilon^{ijk} F^{jk}}$

$\mathcal{L}_{GF}$ gauge fixing term

covariant gauge $\quad \mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial_\mu A^{a,\mu})^2$

$\left| \int [D\underline{A}] = \int [\underline{d\alpha}] [\underline{\underline{\frac{DA}{d\alpha}}}] \right.$

$\mathcal{L}_{FP}$ Faddeev-Popov term ghost field

$\rightarrow$ Feynman rule $\quad$ Feynman diagram

$\xi = 1$ Feynman gauge

other useful gauge $\quad n \cdot A = 0$ $\quad$ axial gauge

$n \cdot A = 0 \quad n^2 = 0$ $\quad$ light cone gauge

physical gauge $\quad$ no ghost needed

unitary gauge

路径积分下完整的生成泛函：

$$Z[J,\xi,\xi^*,\eta,\bar{\eta}] = \int [dA][d\psi][d\bar{\psi}][dc][dc^*]$$
$$\times \exp\left\{ i\int d^4x \left(\mathcal{L}_{eff} + AJ + c^*\xi + \xi^* c + \bar{\psi}\eta + \bar{\eta}\psi \right) \right\}$$

其中

$$\mathcal{L}_{eff} = \mathcal{L}^0_{eff} + \mathcal{L}^I_{eff}$$

$$\mathcal{L}^0_{eff} = \mathcal{L}^0_G + \mathcal{L}^0_F + \mathcal{L}^0_{FP}$$

$$\mathcal{L}^0_G = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) - \frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2$$

$$\mathcal{L}^0_F = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$\mathcal{L}^0_{FP} = (\partial^\mu c^{a*})(\partial_\mu c^a)$$

$$\begin{aligned}
\mathcal{L}^I_{eff} = & -\frac{g}{2} f^{abc}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) A^{b\mu} A^{c\nu} \qquad = \mathcal{L}^I_{eff}[A^a, c^a, c^{a*}, \psi, \bar{\psi}] \\
& - \frac{g^2}{4} f^{abc} f^{cde} A^a_\mu A^b_\nu A^{c\mu} A^{d\nu} \\
& + g\bar{\psi} T^a \gamma^\mu A^a_\mu \psi \\
& - g f^{abc}(\partial^\mu c^{a*} c^b A^c_\mu)
\end{aligned}$$

自由场生成泛函可以写为：

$$Z_0[J,\xi,\xi^*,\eta,\bar{\eta}] = \int [dA][d\psi][d\bar{\psi}][dc][dc^*]$$
$$\times \exp\left\{ i\int d^4x \left(\mathcal{L}^0_{eff} + AJ + c^*\xi + \xi^* c + \bar{\psi}\eta + \bar{\eta}\psi \right) \right\}$$
$$= Z^G_0[J]\, Z^{FP}_0[\xi,\xi^*]\, Z^F_0[\eta,\bar{\eta}]$$

此时完整生成泛函可以形式地记为：

$$Z[J,\xi,\xi^*,\eta,\bar{\eta}] = \exp\left[i\int d^4x\, \mathcal{L}^I_{eff}\left[\frac{\delta}{i\delta J^{a\mu}}, \frac{\delta}{i\delta \xi^{a*}}, \frac{\delta}{i\delta(-\xi^a)}, \frac{\delta}{i\delta\bar{\eta}}, \frac{\delta}{i\delta(-\eta)} \right] \right]$$
$$\times Z_0[J,\xi,\xi^*,\eta,\bar{\eta}]$$

对于 $Z_0^G[J]$，可以写成

$$Z_0^G[J]=\int[dA]\exp\left[i\int d^4x\,\frac{1}{2}\left[A^{a\mu}K_{\mu\nu}^{ab}A^{b\nu}+AJ\right]\right]$$

其中 $K_{\mu\nu}^{ab}=\delta^{ab}\left[g_{\mu\nu}\Box-\left(1-\frac{1}{\alpha}\right)\partial_\mu\partial_\nu\right]$

定义 $K_{\mu\nu}^{ab}$ 的逆函数 $D_{\mu\nu}^{ab}$，满足：

$$\int d^4z\;K_{\mu\lambda}^{ac}(x-z)\cdot g^{\lambda\rho}\,D_{\rho\nu}^{cb}(z-y)=\delta^{ab}g_{\mu\nu}\delta^4(x-y)$$ 加个 i 更符合约定

由此可解得

$$D_{\mu\nu}^{ab}(x)=\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{k^2+i\varepsilon}\delta_{ab}\left[-g_{\mu\nu}+(1-\alpha)\frac{k_\mu k_\nu}{k^2+i\varepsilon}\right]$$ 加个 i

动量空间 $D_{\mu\nu}^{ab}(k)=\frac{1}{k^2+i\varepsilon}\delta_{ab}\left(-g_{\mu\nu}+(1-\alpha)\frac{k_\mu k_\nu}{k^2+i\varepsilon}\right)$ 加个 i

将 A 积分出来可得：

$$Z_0^G[J]=\exp\left\{-\frac{i}{2}\int d^4x\,d^4y\;J^{a\mu}(x)\,D_{\mu\nu}^{ab}(x-y)\,J^{b\nu}(y)\right\}$$

两点关联函数可记为：

$$\langle 0|T[A_\mu^a(x)\,A_\nu^b(y)]|0\rangle=\frac{(-i)^2}{Z_0^G[0]}\frac{\delta^2 Z_0^G[J]}{\delta J^{a\mu}(x)\,\delta J^{b\nu}(y)}\bigg|_{J=0}=i\,D_{\mu\nu}^{ab}(x-y)$$ 如果定义中加了 i 此处就可省

由此可以推得传播子.

类似地 可以给出鬼场和旋量场传播子：

$$\langle 0|T[C^a(x)\,C^{*b}(y)]|0\rangle=\frac{(-i)^2}{Z_0^{FP}(0,0)}\frac{\delta^2 Z_0^{FP}[\xi,\xi^*]}{\delta\xi^{a*}\,\delta(-\xi^b)}\bigg|_{\xi,\xi^*=0}=i\,D^{ab}(x,y)$$

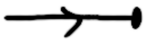
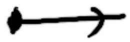
$$\langle 0|T[\psi(x)\,\bar\psi(y)]|0\rangle=\frac{(-i)^2}{Z_0^F(0,0)}\frac{\delta^2 Z_0^F[\eta,\bar\eta]}{\delta\bar\eta\,\delta(-\eta)}\bigg|_{\eta,\bar\eta=0}=i\,S_F(x-y)$$

$$D^{ab}(x-y)=\delta^{ab}\int\frac{d^4k}{(2\pi)^4}e^{-ik(x-y)}\frac{1}{k^2+i\varepsilon}$$

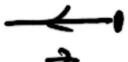
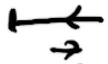
$$S(x-y)=\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}\frac{1}{\not p-m+i\varepsilon}$$

3. Feynman rule:

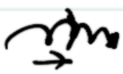
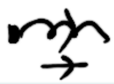
External lines

入射  $u(p)$  $\bar{u}(p)$ 出射

p p

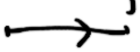
 $\bar{v}(p)$  $v(p)$

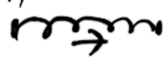
$\vec{p}$ $\vec{p}$

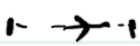
$a, \mu,$  $\varepsilon^{\mu}(p)$  $\varepsilon^{*\mu}(p)$

p

Propagators:

i  j $\dfrac{i}{\not{p}-m+i\varepsilon}\delta_{ij}$

a, μ  b, ν $\dfrac{-i g^{\mu\nu}\delta^{ab}}{p^2+i\varepsilon}$; $\alpha=1$

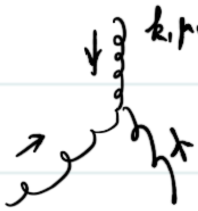
a  b $\dfrac{i}{p^2+i\varepsilon}\delta_{ab}$

Vertex: p' i  k, μ, a $i g_s \gamma^{\mu} (T^a)_{ij}$

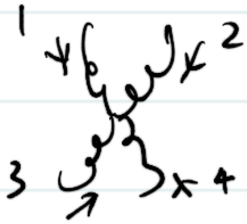
p j

p, a  k, μ, b $-g_s f^{abc} p^{\mu}$

p', c

 $k_1 \mu_1 a_1$ $k_2 \mu_2 a_2$ $k_3 \mu_3 a_3$

$$g_s f^{a_1 a_2 a_3}\left[(k_1-k_2)_{\mu_3} g_{\mu_1\mu_2} + (k_2-k_3)_{\mu_1} g_{\mu_2\mu_3} + (k_3-k_1)_{\mu_2} g_{\mu_1\mu_3}\right]$$



$$-ig_s^2\left[f^{a_1a_2b}f^{a_3a_4b}\left(g_{\mu_2\mu_3}g_{\mu_1\mu_4}-g_{\mu_1\mu_3}g_{\mu_2\mu_4}\right) + \text{permutation}\right]$$

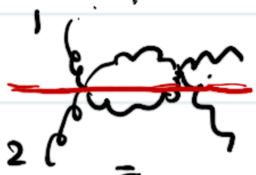
$(1234) \to \underline{(2341) \to (4123)}$

额外费曼规则:

$\int \frac{d^4p}{(2\pi)^4}$ 圈

 $-\mathrm{Tr}[\ ]$

 $-$



symmetry factor $\left(\frac{1}{2}\right)$ 沿红线对折不影响结果

注意是对折内圈、不动外线粒子

4. parameters in QCD:

m_i mass of quark $\qquad\qquad M_\pi^2 = \underset{\Delta}{B_0}\,(\underline{m_u + m_d})$

$m_u \sim m_d$ a few MeV $\qquad m_s \sim 100\,\text{MeV}$

$m_c \sim 1\,\text{GeV} \qquad m_b \sim 4\,\text{GeV} \qquad m_t \sim 170\,\text{GeV}$

g_s coupling constant of $SU(3)$

Def: $\alpha_s = \dfrac{g_s^2}{4\pi}$ $\qquad\qquad$ Λ 强子化标度

dimension trans<u>mutasim</u>

$$\underline{\alpha_s(\mu)} = \frac{1}{\beta_0}\frac{1}{\ln\frac{\mu^2}{\Lambda^2}}$$

$$\underline{\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = \beta(\alpha_s)}$$

$$\beta(\alpha_s) = \beta_0 \alpha_s^2 + \beta_1 \alpha_s^3 + \beta_2 \alpha_s^4 + \cdots$$

$$\beta_0 = -\frac{1}{4\pi}\left(11 - \frac{2}{3}N_f\right)$$

$$\beta_1 = -\frac{1}{8\pi^2}\left(51 - \frac{19}{3}N_f\right)$$

$$\beta_2 = -\frac{1}{128\pi^3}\left(2857 - \frac{5033}{9}N_f + \frac{325}{27}N_f^2\right)$$

$\underline{\beta < 0}$ $\quad \mu^2 \nearrow \infty . \quad \alpha_s(\mu) \to 0$

Asymptotic freedom

θ term. $\qquad \theta \cdot F_{\mu\nu}\tilde{F}^{\mu\nu} = \theta\, \varepsilon_{\mu\nu\alpha\beta}\, F^{\mu\nu,ab} F^{\alpha\beta\, ab}$

5 Perturbation theory and elementary process.

massless limit $\quad \mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi$

QED基本过程

- $e^+e^- \to \mu^+\mu^-$: $\quad \overline{|M|}^2 = \frac{1}{4}\sum_s |M|^2 = \frac{8e^4}{\hat{s}^2}\frac{\hat{t}^2+\hat{u}^2}{4}$

$\hat{s} = (p_1+p_2)^2 \qquad$ 加^为了表示是部分子层次

$\hat{t} = -\frac{\hat{s}}{2}(1-\cos\theta)$

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2\hat{s}}\frac{1}{16\pi}\frac{8e^4}{\hat{s}^2}\frac{\hat{t}^2+\hat{u}^2}{4}$$

$$\frac{d\sigma}{d\hat{t}} = \frac{2\pi\alpha^2}{\hat{s}^2}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$$

- $e^-\mu^- \to e^-\mu^-$: $\quad \frac{d\sigma}{d\hat{t}} = \frac{2\pi\alpha^2}{\hat{s}^2}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$

QCD基本过程:

- $ud \to ud$: $\quad \frac{d\sigma}{d\hat{t}} = \frac{4\pi\alpha_s^2}{9\hat{s}^2}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \qquad$ t道

- $u\bar{u} \to d\bar{d}$ $\quad \frac{d\sigma}{d\hat{t}} = \frac{4\pi\alpha_s^2}{9\hat{s}^2}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} \qquad$ s道

- $u\bar{u} \to u\bar{u}$ $\quad \frac{d\sigma}{d\hat{t}} = \frac{4\pi\alpha_s^2}{9\hat{s}^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} + \frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2} - \frac{2}{3}\frac{\hat{u}^2}{\hat{s}\hat{t}}\right]$

注意干涉项. 不能认为无自旋!

- $uu \to uu$ $\quad \frac{d\sigma}{d\hat{t}} = \frac{4\pi\alpha_s^2}{9\hat{s}^2}\left[\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2} + \frac{\hat{t}^2+\hat{s}^2}{\hat{u}^2} - \frac{2}{3}\frac{\hat{s}^2}{\hat{u}\hat{t}}\right]$

- $q\bar{q} \to gg$: $\quad \frac{d\sigma}{d\hat{t}} = \frac{32\pi\alpha_s^2}{27\hat{s}^2}\left[\frac{\hat{u}}{\hat{s}} + \frac{\hat{t}}{\hat{u}} - \frac{9}{4}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)\right]$

- $gg \to q\bar{q}$: $$\frac{d\sigma}{d\hat{t}} = \frac{\pi \alpha_s^2}{6\hat{s}^2}\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{9}{4}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)\right]$$

- $qg \to qg$: $$\frac{d\sigma}{d\hat{t}} = \frac{4\pi \alpha_s^2}{9\hat{s}^2}\left[-\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}} + \frac{9}{4}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

- $gg \to gg$: $$\frac{d\sigma}{d\hat{t}} = \frac{9\pi \alpha_s^2}{2\hat{s}^2}\left[3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right]$$

作业：1 习题17.1

2. 计算gcd的各种基本树过程