重整化群

1. RGE   Callan-Symanzik Eq.

other schemes

2. RGE: a formal discussion.   QED UV ☆

3. solution to RGE: 直接给出解.

running coupling constant.

formal discussion

4. Running coupling constant and fixed point

5 Composite operator renormalization:

ⓐ 2-quark operator ☆   ⓑ 4-quark operator

6. PDF & LCDA RGE: ☆

7. Baryon LCDA evolution

8. HQET LCDA evolution

9. coordinate space evolution

不讲

复习 QED重整化

$$\mathcal{L} = \bar\psi_0(i\not\partial - m_0)\psi_0 - \bar\psi_0 e_0 \not A_0 \psi_0 - \frac{1}{4} F_0^{\mu\nu} F_{0\mu\nu} \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\psi_0 = Z_2^{1/2}\psi \qquad \delta_2 = Z_2 - 1$$

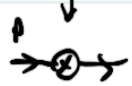
$$m_0 = Z_m m \qquad \delta_m = m_0 Z_2 - m$$

$$e_0 = Z_e e \qquad \delta_3 = Z_3 - 1$$

$$A_0^\mu = Z_3^{1/2} A^\mu \qquad \delta_1 = \frac{e_0}{e} Z_2 Z_3^{1/2} - 1$$

$m=0$ 只考虑电子

$$\mathcal{L} = \bar\psi(i\not\partial - m)\psi - \bar\psi e \not A \psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + (Z_2 - 1)\bar\psi i\not\partial\psi + \cdots$$



$$i(Z_2 - 1)\not p = i\delta_2 \not p$$

电子自能:(m=0为例)

$$\frac{i}{\not p} + \frac{i}{\not p}(-i\Sigma_2^{l})\frac{i}{\not p} + \frac{i}{\not p}(-i\Sigma_2^{ct})\frac{i}{\not p} + \cdots$$

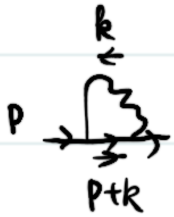
$$\frac{i}{1-x} = i(1+x)$$

$$S(p) = \frac{i}{\not p - \Sigma_2^{l} - \Sigma_2^{ct}} + \cdots = \frac{i}{\not p\left[1 - \frac{\Sigma_2^{l}}{\not p} - \frac{\Sigma_2^{ct}}{\not p}\right]}$$

$$= \frac{i}{\not p} + \frac{i}{\not p}\left(\frac{\Sigma_2^{l}}{\not p} + \frac{\Sigma_2^{ct}}{\not p}\right)$$

$$= \frac{i}{\not p} + \frac{i}{\not p}(-i\Sigma_2^{l})\frac{i}{\not p} + \frac{i}{\not p}(-i\Sigma_2^{ct})\frac{i}{\not p}$$

$$-i\Sigma_2^{ct} = i\delta_2 \not p$$



取 $p^2<0$

$$-i\Sigma_2^1(p) = \int \frac{d^dk}{(2\pi)^d} \cdot (-ie\gamma_\mu) \frac{i(\not p+\not k)}{(p+k)^2} \cdot (-ie\gamma_\nu) \times \frac{-ig^{\mu\nu}}{k^2}$$

$$= -e^2 \int \frac{d^dk}{(2\pi)^d} \frac{\gamma_\mu(\not p+\not k)\gamma^\mu}{(k+p)^2 k^2}$$

$$= -e^2 \int \frac{d^dk}{(2\pi)^d} \frac{(2-d)(\not k+\not p)}{(k+p)^2 k^2}$$

$$\underline{\gamma_\mu \gamma^\nu} \gamma^\mu = (2-d)\gamma^\nu$$

$$\Downarrow$$

$$(2g_\mu^\nu - \gamma^\nu\gamma_\mu)$$

$$= -e^2 \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{(2-d)(\not k+\not p)}{[x(k+p)^2+(1-x)k^2]^2}$$

$$\gamma_\mu\gamma^\mu = \underbrace{\gamma_0\gamma^0}_{1} + \underbrace{\gamma_1\gamma^1}_{1} + \cdots \underbrace{\gamma_i\gamma^i}_{1}$$

$$= -e^2 \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{(2-d)(\not k+\not p)}{[(k+xp)^2 - x^2p^2 + xp^2]^2}$$

$$= -e^2 \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{(2-d)(\not p+\not k - x\not p)}{[k^2 - (-x(1-x)p^2)]^2}$$

$$k_0\gamma^0 - k^1\gamma^1 - k^2\gamma^2 - k^3\gamma^3$$

$$= -e^2 \int_0^1 dx \int \frac{d^dk}{(2\pi)^d} \frac{(2-d)(1-x)\not p}{[k^2 - (-x(1-x)p^2)]^2} \quad \triangle$$

$$\Downarrow$$

$$\underline{= -e^2 \int_0^1 dx \quad \frac{(-1)^2 i}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \left[\frac{1}{-x(1-x)p^2}\right]^{2-\frac{d}{2}}} \quad (2-d)(1-x)\not p$$

$d=4-2\varepsilon$

$$= -ie^2 \frac{1}{(4\pi)^2} \int_0^1 dx\, (1+\varepsilon\ln 4\pi)\left(\frac{1}{\varepsilon} - \gamma_E\right) \cdot \left(1 - \varepsilon \ln(-x(1-x)p^2)\right)$$

$$\times (-2+2\varepsilon)(1-x)\not p$$

$$= -i\frac{e^2}{(4\pi)^2}\left[\int_0^1 dx \ \frac{1}{\varepsilon}(-2)(1-x)\not p\right.$$

$$\boxed{\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E}$$

$$+ \int_0^1 dx \left[(-\gamma_E)(-2)(1-x)\not p\right.$$

$$x^\varepsilon = 1 + \varepsilon \cdot \ln x$$

$$+ \ln 4\pi \cdot (-2)(1-x)\not p$$

$$- \ln\left(x(1-x)p^2\right) \cdot (-2)(1-x)\not{p}$$

$$+ 2\cdot(1-x)\not{p} \Big]$$

$$= -i\frac{e^2}{(4\pi)^2}\left[-\frac{1}{\varepsilon}\not{p} + \gamma_E\not{p} - \ln(4\pi)\not{p} + (-2+\ln(-p^2))\not{p} + \not{p}\right]$$

$$-i\Sigma_2^l = +i\frac{\alpha}{4\pi}\left(\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi + 1 - \ln(-p^2)\right)\not{p}$$

$$-i\Sigma(p) = -i\Sigma_2^l - i\Sigma_2^{ct}$$

重整化条件 不一定是 $\Sigma(\not{p})\big|_{\not{p}=m} = 0$ $\frac{d}{d\not{p}}\Sigma(\not{p})\big|_{\not{p}=m} = 0$，只要有限即可

重整化方案:

(1) onshell renormalization: (此为之前采用方法) 对QCD，实现不了这种方案

$$S(R) = \frac{i}{\not{p} - \Sigma_2^l - \Sigma_2^{ct}}$$

- 在 $\not{p} = m$ 时 $S(R) = \frac{i}{\not{p} - m}$
- $m = 0$. 当 $\not{p} = 0$ $S(R) = \frac{i}{\not{p}}$ $\Rightarrow$ $\Sigma_2^l + \Sigma_2^{ct}\Big|_{\not{p}=0} = 0$

(2) offshell renormalization. (momentum-space subtraction scheme. MOM)

当 $p^2 = -M^2$ 时 要求 $S(R)\Big|_{p^2=-M^2} = \frac{i}{\not{p}}$

$$\Rightarrow \Sigma_2^l + \Sigma_2^{ct}\Big|_{p^2=-M^2} = 0$$

$$-\frac{\alpha}{4\pi}\left(\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi + 1 + \ln(M^2)\right) + (-\delta_2) = 0$$

$$\Rightarrow \delta_2 = -\frac{\alpha}{4\pi}\left(\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi + 1 + \ln M^2\right)$$

$$S(R) = \frac{i}{\not{p} - \Sigma_2^l(p) - \Sigma_2^{ct}} = \frac{i}{\not{p}\left[1 - \frac{\alpha}{4\pi}\ln\frac{-p^2}{M^2}\right]}$$

(3) Minimal subtraction (MS)

$$\delta_2 = -\frac{\alpha}{4\pi}\frac{1}{\varepsilon}$$

$$S(p) = \frac{i}{\not{p}\left(1-\frac{\alpha}{4\pi}\left(\gamma_E - 1 + \ln\frac{-p^2}{4\pi\mu^2}\right)\right)}$$

(4). Modified MS: ($\overline{MS}$) $\qquad T(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E$

$$\delta_2 = -\frac{\alpha}{4\pi}\left(\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi\right)$$

$$S(p) = \frac{i}{\not{p}\left[1-\frac{\alpha}{4\pi}\left(-1 + \ln\frac{-p^2}{\mu^2}\right)\right]}$$

在一个确定的重整化方案下，仍然有对重整化标度的依赖。采用不同标度进行重整化得到的物理量结果要一致，参量存在关联。

重整化群:

$$J_r = Z_3^{1/2} J_0 \quad \Big| \quad A^{\mu r} = Z_3^{-1/2} A^{\mu 0}$$

$$e_r = Z_e^{-1} e_0$$

$$m_r = Z_m m_0$$

M^2 $(J_r,\ e_r\ \ m_r)$

M'^2: $(J_r'\ \ e_r'\ \ m_r')$

物理量: $W_r(J_r, e_r\ m_r, M) = W_r'(J_r'\ e_r'\ m_r'\ M')$

小写 小写 小写

有限重整化: $J_r' = z_3^{1/2} J_r \qquad e_r' = z_e e_r \qquad m_r' = z_m m_r$

$$z_3 = Z_3' / Z_3 \qquad z_e = Z_e / Z_e' \qquad z_m = Z_m / Z_m'$$

例: $z_e(M', M) = Z_e(M) / Z_e(M')$

① 封闭性: $M.\ M'\ M''$

$$Z_e(M) \quad Z_e(M') \quad Z_e(M'')$$

$$z_e(M', M'') = Z_e(M') / Z_e(M'')$$

$$z_e(M, M') = Z_e(M) / Z_e(M')$$

$$z_e(M', M'')\, z_e(M, M') = Z_e(M) / Z_e(M'') = z_e(M, M'')$$

③ 恒元

$$Z_e(M, M) = Z_e(M) / Z_e(M) = 1$$

④ 逆元：定义 $Z_e^{-1}(M, M') = Z_e(M') / Z_e(M)$

此即重整化群.

$\overline{MS}$ 下重整化群方程：(RGE)

标量场 $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \underset{= g\mu^\varepsilon\phi^4}{\frac{\lambda}{4!}\phi^4}$ $\lambda \equiv g$

a. $[g] = \varepsilon \rightarrow \mu^\varepsilon g$

b. $\phi_0 = Z_3^{1/2}\phi$

$m_0 = Z_m m$

$g_0\mu_0^\varepsilon = Z_g g \mu^\varepsilon$

对质量，耦合常数 RGE：

质量： $\frac{dm_0}{d\mu} = 0 \Rightarrow \frac{d}{d\mu}(Z_m m) = 0$

$$\Rightarrow \frac{dZ_m}{d\mu}m + Z_m\frac{dm}{d\mu} = 0$$

$$\Rightarrow \mu\frac{dm}{d} = -\mu\frac{m}{Z_m}\frac{dZ_m}{d\mu}$$

$$\Rightarrow \frac{dm}{\frac{d\mu}{\mu}} = -\frac{m}{Z_m}\frac{dZ_m}{\frac{d\mu}{\mu}}$$

$$\Rightarrow \frac{dm}{d\ln\mu} = -\frac{m}{Z_m}\frac{dZ_m}{d\ln\mu} \equiv -m\gamma_m$$

$\gamma_m = \frac{1}{Z_m}\frac{dZ_m}{d\ln\mu}$ …… 质量反常量纲

耦合常数：

$$\frac{d(g_0\mu_0^\varepsilon)}{d\ln\mu} = 0$$

$$g = \left(\frac{\mu_0}{\mu}\right)^\varepsilon Z_g^{-1} g_0$$

$$\frac{dg}{d\ln\mu} = -\varepsilon g - \frac{\mu}{Z_g}\frac{dZ_g}{d\mu}g \equiv \beta$$

反常量纲 $\beta \equiv \beta(g)$

$\gamma_m = \gamma_m(g)$

$\overline{MS}$

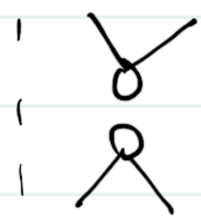
$$Z_g = 1 + g^2 \frac{A_{11}}{\varepsilon} + g^4 \left(\frac{A_{22}}{\varepsilon^2} + \frac{A_{21}}{\varepsilon}\right) + \cdots$$

$$Z_m = 1 + g^2 \frac{B_{11}}{\varepsilon} + g^4 \left(\frac{B_{22}}{\varepsilon^2} + \frac{B_{21}}{\varepsilon}\right) +$$

A_{ij} B_{ij} 为常数. 没有 $\ln\mu$ 之类的.

关联函数:

$$G_n^{(0)} = \langle\Omega|T[\phi_0(x_1)\phi_0(x_2)\cdots\phi_0(x_n)]|\Omega\rangle$$
$$= Z_3^{\frac{n}{2}}\langle\Omega|T[\phi(x_1)\cdots\phi(x_n)]|\Omega\rangle$$
$$= Z_3^{n/2}\, G_n(g, m, \mu)$$



$$\frac{dG_n^{(0)}}{d\ln\mu} = 0 \Rightarrow \frac{d}{d\ln\mu}\left(Z_3^{\frac{n}{2}} G_n\right) = 0$$

$$\frac{dG_n}{d\ln\mu} = -n\cdot\frac{\mu}{2Z_3}\frac{dZ_3}{d\mu} G_n \equiv -n\gamma G_n$$

$$\gamma = \frac{\mu}{2Z_3}\frac{dZ_3}{d\mu}$$ --- 场的反常量纲

$$Z_3 = 1 + \frac{\alpha}{4\pi}\frac{1}{\varepsilon}\cdot A$$

$$\mu \frac{dG_n}{d\mu} = \mu \frac{dG_n(p, \mu, g(\mu, m), m(\mu, g))}{d\mu}$$

$$= \mu \frac{\partial G_n}{\partial \mu} + \mu \frac{\partial g}{\partial \mu} \frac{\partial G_n}{\partial g} + \mu \frac{\partial m}{\partial \mu} \frac{\partial G_n}{\partial m}$$

$$= \mu \frac{\partial G_n}{\partial \mu} + \beta(g) \frac{\partial G_n}{\partial g} + \gamma_m m \frac{\partial G_n}{\partial m}$$

$$\boxed{\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_m m \frac{\partial}{\partial m} + n\gamma \right) G_n = 0}$$

---- 'tHooft - Weinberg RGE

质量 onshell. 其它 offshell	Gell-man- Low RGE
所有 onshell.	Callan- Symanzik RGE（感觉与 Peskin 定义不同）
所有 offshell.	Georgi- Politzer RGE

名字很混乱！很多书上有问题（不确定是否黄主的量子场论导论是否有问题）

2. Formal understanding of RG & RGE

生成泛函：标量场

$$Z[J] = \int D\phi \, e^{i\int d^4x\,(\mathcal{L} + J\phi)}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

动量空间截断：$|k| < \Lambda$ （某个截断：Λ）

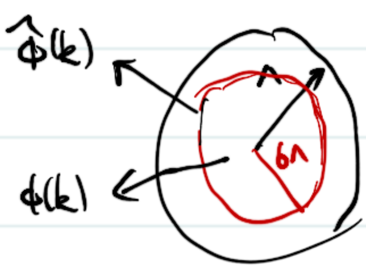
欧氏空间讨论 RG：（类似于格点QCD）

$$Z = \int [D\phi]_\Lambda \exp\left[-\int d^dx\left[\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right]\right]$$

$$[D\phi]_\Lambda = \prod_{|k|<\Lambda} d\phi(k)$$

①：把 $\phi(k)$ 分为两个部分 $|k| < b\Lambda$ ， $b\Lambda < |k| < \Lambda$

$$\hat{\phi}(k) = \begin{cases} \phi(k) & \underline{b\Lambda < |k| < \Lambda} \\ 0 & |k| < b\Lambda \end{cases} \qquad \phi(k) = \begin{cases} 0 & b\Lambda < |k| < \Lambda \\ \underline{\underline{\phi(k)}} & |k| < b\Lambda \end{cases}$$



$$\underset{\text{原来}}{\phi(k)} = \phi(k) + \hat{\phi}(k)$$

$$Z[\phi] = \int \underline{[D\phi]_{b\Lambda<|k|<\Lambda}} \int [D\phi]_{|k|<b\Lambda} \exp\Big[-\int d^dx\Big[\frac{1}{2}(\underline{\underline{\partial_\mu\phi}} + \partial_\mu\hat{\phi})(\underline{\underline{\partial^\mu\phi}} + \partial^\mu\hat{\phi})$$
$$+\frac{1}{2}m^2(\underline{\underline{\phi}} + \hat{\phi})^2 + \frac{\lambda}{4!}(\underline{\underline{\phi}} + \hat{\phi})^4\Big]$$

$$= \int [D\phi]_{b\Lambda} \exp\left[-\int d^dx\, \mathcal{L}(\phi)\right] \int D\hat{\phi} \exp\Big[-\int d^dx \Big[\tfrac{1}{2}\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi} + m\phi\hat{\phi}$$
$$+ \tfrac{1}{2}m^2\hat{\phi}^2 + \lambda\left(\phi^3\hat{\phi} + \tfrac{1}{4}\phi^2\hat{\phi}^2 + \tfrac{1}{6}\phi\hat{\phi}^3 + \tfrac{\lambda}{4!}\hat{\phi}^4\right)\Big]\Big]$$

$$\equiv \int [D\phi]_{b\Lambda} \exp\left[-\int d^dx\, \mathcal{L}_{eff}\right]$$

$\mathcal{L}_{eff}(\phi)$ 形式：

② $$\mathcal{L}_0 = \int d^dx \left(\tfrac{1}{2}\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi}\right) = \frac{1}{2}\int_{b\Lambda<|k|<\Lambda} \frac{d^dk}{(2\pi)^d}\, \hat{\phi}^*(k)\, k^2\, \hat{\phi}(k)$$

$$\overbracket{\hat{\phi}(k)\,\hat{\phi}(p)} = \frac{\int D\hat{\phi}\, e^{-\mathcal{L}_0}\,\hat{\phi}(k)\,\hat{\phi}(p)}{\int D\hat{\phi}\, e^{-\mathcal{L}_0}} = \underline{\frac{1}{k^2}(2\pi)^d\,\delta^d(k+p)\,\Theta(k)}$$

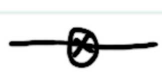
$$\langle\overbracket{\phi(x_1)\,\phi(x_2)} = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x_1-x_2)}\, \frac{i}{p^2-m^2+i\varepsilon}$$

$$\Theta(k) = \begin{cases} 1 & b\Lambda<|k|<\Lambda \\ 0 & |k|<b\Lambda \end{cases}$$

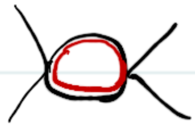
③ 对低能影响

$$-\int d^dx\, \frac{\lambda}{4!}\phi^2\hat{\phi}\hat{\phi} = -\int d^dx\, \frac{\lambda}{4!}\phi^2(x)\,\overbracket{\hat{\phi}(x)\,\hat{\phi}(x)}$$
$$= -\frac{1}{2}\int \frac{d^dk_1}{(2\pi)^d}\,\mu\,\phi(k)\,\phi(-k)$$

$$\mu = \frac{\lambda}{2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2} = \frac{\lambda}{(4\pi)^{d/2}}\,\frac{1}{\Gamma(d/2)}\,\frac{1-b^{d-2}}{d-2}\,\Lambda^{d-2}$$

 $\Rightarrow$ 

④ 展开 λ^2:

 $= -\frac{1}{4}\int d^dx\, \delta\phi^4$ 

$$\delta = -4!\,\frac{2}{2!}\left(\frac{\lambda}{4}\right)^2\int_{b\Lambda<|k|<\Lambda}\frac{d^dk}{(2\pi)^d}\left(\frac{1}{k^2}\right)^2 = \frac{-3\lambda^2}{(4\pi)^{d/2}\Gamma(d/2)}\,\frac{1-b^{d-4}}{d-4}\,\Lambda^{d-4}$$

$$\xrightarrow{d\to 4} -\frac{3\lambda^2}{16\pi^2}\ln\frac{1}{b}$$

⑤ 汇总:

$$\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \cdots$$

$$= \frac{1}{2}[1+\Delta Z]\,\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}(m^2+\Delta m^2)\phi^2 + \frac{\lambda}{4!}(\lambda+\Delta\lambda)\phi^4 + \Delta C(\partial_\mu\phi\partial^\mu\phi)^2 + \Delta D\phi^6 + \cdots$$

$$\underset{|k|<b\Lambda}{\int d^dx\,\mathcal{L}_{eff}(x)} = \int d^dx'\, b^{-d}\left[\frac{1}{2}(1+\Delta D)\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}(m^2+\Delta m^2)\phi^2 + \frac{1}{4}(\lambda+\Delta\lambda)\phi^4 + \cdots\right]$$

$x' = xb$

$$= \int d^dx'\left[\frac{1}{2}\partial'_\mu\phi'\partial'^\mu\phi' + \frac{1}{2}m'^2\phi'^2 + \frac{\lambda'}{4!}\phi'^4 + \cdots\right]$$

$$m'^2 = (m^2+\Delta m^2)(1+\Delta Z)^{-1}b^{-2}$$

$$\lambda' = (\lambda+\Delta\lambda)(1+\Delta Z)^{-2}b^{d-4}$$

$$C' = (C+\Delta C)(1+\Delta Z)^{-2}b^{d}$$

$$D' = (D+\Delta D)(1+\Delta Z)^{-3}b^{2d-6}$$

$$\phi' = b^{2-d}(1+\Delta Z)^{1/2}\phi$$

Previously

ⓐ $$Z[J]\Big|_{J=0} = \int [D\phi]_\Lambda \exp\Big[-\int d^dx \Big[\underbrace{\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}m^2\phi^2 + \tfrac{\lambda}{4!}\phi^4}_{\mathcal{L}}\Big]$$

$$[D\phi]_\Lambda = \prod_{|k|<\Lambda} d\phi(k)$$

ⓑ

$$\hat{\phi}(k) = \begin{cases} \phi(k) & k \in [b\Lambda, \Lambda] \\ 0 & k \in [0, b\Lambda] \end{cases}$$

$$\phi(k) = \begin{cases} 0 & k \in [b\Lambda, \Lambda] \\ \phi(k) & k \in [0, b\Lambda] \end{cases}$$

ⓒ

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}[\partial_\mu\phi + \partial_\mu\hat{\phi}][\partial^\mu\phi + \partial^\mu\hat{\phi}] + \tfrac{1}{2}m^2(\phi+\hat{\phi})^2 + \tfrac{\lambda}{4!}(\phi+\hat{\phi})^4 \\ &= \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \underline{\underline{\tfrac{1}{2}m^2\phi^2}} + \tfrac{\lambda}{4!}\phi^4 \\ &\quad + \tfrac{1}{2}\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi} + \tfrac{1}{2}m^2\hat{\phi}^2 + \tfrac{\lambda}{4!}\hat{\phi}^4 \\ &\quad + \underline{\lambda\left(\tfrac{1}{6}\phi^3\hat{\phi} + \tfrac{1}{4}\phi^2\hat{\phi}^2 + \tfrac{1}{6}\phi\hat{\phi}^3\right)} + \cdots\end{aligned}$$

ⓓ 积分 integrating out

$$\langle\Omega|T[\Phi(x)\,\Phi(y)]|\Omega\rangle = \frac{\langle 0|T[\phi_I(x)\,\phi_I(y)\,e^{i\int d^4x\,\mathcal{H}_I}]|0\rangle}{\langle 0|T[e^{i\int d^4x\,\mathcal{H}_I}]|0\rangle}$$

$$\langle 0|\, \hat\phi(k)\hat\phi(p)\,|0\rangle = \hat\phi(k)\hat\phi(p)$$

$$= \int d^4x\, e^{ik\cdot x}\, \hat\phi(x) \int d^4y\, e^{ip\cdot y}\, \hat\phi(y)$$

$$= \int d^4x\, e^{ik\cdot x} \int d^4y\, e^{ip\cdot y} \underline{\int \frac{d^4p'}{(2\pi)^4}\, e^{-ip'\cdot(x-y)}\, \frac{i}{p'^2-m^2}}$$

$$= \int d^4x\, e^{ik\cdot x}\, e^{-ip'x} \int \frac{d^4p'}{(2\pi)^4}\, \frac{i}{p'^2}\, (2\pi)^4\, \delta^4(p+p')$$

$$= \int d^4x\, e^{ik\cdot x}\, e^{ipx}\, \frac{i}{p^2}$$

$$= (2\pi)^4\, \delta^4(k+p)\, \frac{i}{k^2} \qquad \left(|k| \in [b\Lambda, \Lambda]\right)$$

② $\mathcal{L}_I = \frac{\lambda}{4}\phi^2\hat\phi^2$

$$S = -\int d^dx\, \mathcal{L}_I = -\int d^dx\, \frac{\lambda}{4!}\phi^2\hat\phi^2(x)$$

$$= -\int d^dx\, \frac{\lambda}{4!}\phi^2 \int \frac{d^dp}{(2\pi)^d}\, e^{-ip\cdot(x-x)}\, \frac{i}{p^2}$$

$$= -\int d^dx\, \frac{\lambda}{4!}\phi^2 \int \frac{d^dp}{(2\pi)^d}\, \frac{i}{p^2}\Big|_{p\in[b\Lambda,\Lambda]}$$

$$= -\int d^dx\, \frac{\lambda}{4!}\phi^2(x)\, \frac{\int d\Omega_d\, p^{d-1}\, dp}{(2\pi)^d}\, \frac{i}{p^2}$$

$$= -\int d^dx\, \frac{\lambda}{4!}\phi^2(x)\, \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})(2\pi)^d} \int \frac{p^{d-1}dp}{p^2}\Big|_{p\in[b\Lambda,\Lambda]} \qquad \int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

$$= -\int d^dx \, \frac{\lambda}{4!}\phi^2(x) \, \frac{2\pi^{\frac{d}{2}}}{(2\pi)^d\,\Gamma(\frac{d}{2})} \int \frac{\frac{1}{2}(p^2)^{\frac{d-2}{2}} \cdot dp^2}{p^2}$$

$$= -\int d^dx \, \frac{\lambda}{4!}\phi^2(x) \, \frac{2\pi^{\frac{d}{2}}}{(2\pi)^d\,\Gamma(\frac{d}{2})} \, \frac{1}{2}\int (p^2)^{\frac{d-4}{2}} dp^2$$

$$= -\int d^dx \, \frac{\lambda}{4!}\phi^2(x) \, \frac{\pi^{\frac{d}{2}}}{(2\pi)^d\,\Gamma(\frac{d}{2})} \, \frac{1}{\frac{d-4}{2}+1}\,(p^2)^{\frac{d-4}{2}+1}\Big|_{p=b\Lambda}^{p=\Lambda}$$

$$= -\int d^dx \quad \phi^2 \, \frac{1}{(4\pi)^{\frac{d}{2}}\Gamma(\frac{d}{2})} \times \frac{\lambda}{4!} \quad \frac{2}{d-2}\left[(\Lambda^2)^{\frac{d-2}{2}} - (b^2\Lambda^2)^{\frac{d-2}{2}}\right]$$

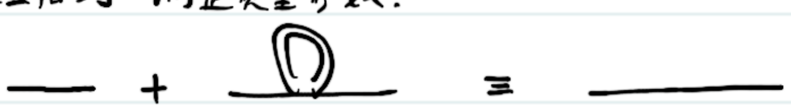
$$= -\int d^dx \, \phi^2(x) \, \underbrace{\frac{1}{(4\pi)^{\frac{d}{2}}\Gamma(\frac{d}{2})} \, \frac{\lambda}{4!} \, \frac{2\Lambda^{d-2}}{d-2}\left[1-b^{d-2}\right]}_{m_0}$$

$$= -\int d^dx \, m_0^2\phi^2(x)$$

(f):

$m^2 \to m^2 + m_0^2$

实际上相当于：调整质量参数.



重整化

$m^2 \to m^2 + m_0^2$

(g): $\phi^2\tilde{\phi}^2$

$$\int d^4x \, \phi^2(x)\tilde{\phi}^2(x) \int d^4y \, \phi^2(y)\tilde{\phi}^2(y) \to \int d^4x \, \lambda_0 \phi^4(x)$$

$\lambda \to \lambda + \lambda_0$

相当于调整耦合常数.

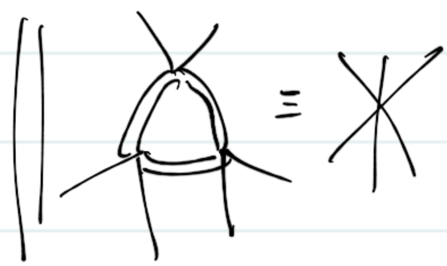
ⓑ 总结

$$\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \cdots$$

$$= \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\,(1+\Delta Z) + \frac{1}{2}(m^2 + \underline{\underline{\Delta m^2}})\phi^2$$

$$+ \frac{1}{4!}(\lambda+\Delta\lambda)\phi^4 + (\Delta C)(\partial_\mu\phi)^4 + \Delta D\,\phi^6$$

$k \in [0, b\Lambda]$



$x' = x\,b$

$$\int d^dx\,\mathcal{L}_{eff} = \int d^dx'\,b^{-d}$$

$$\times\Big[\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\,(1+\Delta Z)$$

$$+\frac{1}{2}(m^2+\Delta m^2)\,\phi^2(x)$$

$$+\cdots\Big]$$

$$= \int d^dx'\,b^{-d}\Big[b^2\,\frac{1}{2}\partial'_\mu\phi(x')\,\partial'^\mu\phi(x')\,(1+\Delta Z) + \cdots$$

重新定义算符: $\phi' = \left[b^{2-d}(1+\Delta Z)\right]^{1/2}\phi(x) = \underline{\underline{b}}^{\frac{2-d}{2}}\sqrt{(1+\Delta Z)}\,\phi(x)$

$$= \int d^dx'\,\Big[\frac{1}{2}\partial'_\mu\phi'\,\partial'^\mu\phi'(x') + \cdots\Big]$$

$[\phi(x)] = \frac{d-2}{2}$

$$\int \underset{-d}{d^d x}\, \frac{1}{2} \underset{1}{\partial_\mu} \phi \underset{1}{\partial^\mu} \phi$$

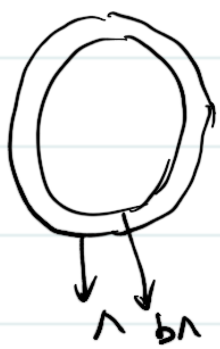
$m'^2 = (m^2+\Delta m^2)(1+\Delta Z)^{-1} b^{-2}$ $\nearrow$ relevant

$\lambda' = (\lambda+\Delta\lambda)(1+\Delta Z)^{-2} b^{d-4}$ $\rightarrow$ marginal

$\underline{\underline{C'}} = (C+\Delta C)(1+\Delta Z)^{-2} b^{d}$ $\searrow$ irrelevant

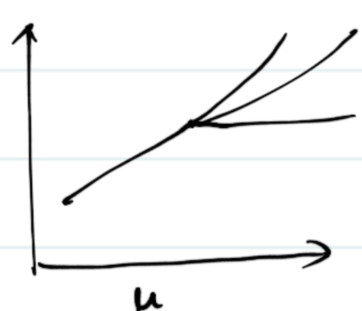
$\underline{\underline{D'}} = (D+\Delta D)(1+\Delta Z)^{-3} b^{2d-6}$ $\searrow$

$\underline{b \to 0}$



几点说明：(1) 高能 → 低能 $b \to 0$ $\underline{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4}$

$\underline{\text{低能} \to \text{高能}}$



重整化群为半群

(2) 两种微扰论

使用 $\mathcal{L}_0$，高能部分通过圈图展现，如果紫外发散，修正很大，可能破坏微扰论

使用 $\mathcal{L}_{eff}$，高能部分通过有效参数整体展现，通过逐步加入修正，不破坏微扰论

(3) 假设忽略 Δm^2，$\delta\lambda$，ΔZ，则

$$\underbrace{m'^2 = m^2 b^{-2}}_{\text{relevant}},\quad \underbrace{\lambda' = \lambda b^{d-4}}_{\text{marginal}},\quad \underbrace{C' = C b^{d},\quad D' = D b^{2d-6}}_{\text{irrelevant}}$$

与可重整性可直接对应

⊗ how to calculate β function? 直接计算顶角图即可，也可以通过不同的关联函数抽取

→ scalar　　　　可以参见 Peskin 书 P412 页，但采用方案不同.

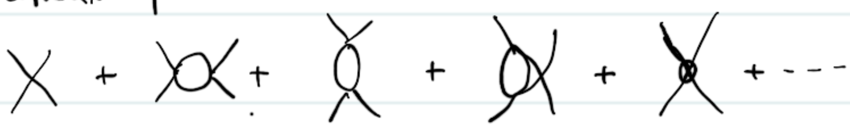
→ QED　　　　Peskin 采用 offshell，这里采用 $\overline{MS}$.

→ QCD

下面先以标量场说明两种方法给出一样结果.

— Scalar:

⊗ four-point correlation function:

$G^{(4)} =$  $+ \cdots$

RGE: $\left(\mu\frac{\partial}{\partial\mu} + \beta(\lambda)\frac{\partial}{\partial\lambda} + 4\gamma(\lambda)\right) G^{(4)}(p_1,p_2,p_3,p_4) = 0$

perturbation theory: $G^{(4)} = \left[-i\lambda + \underline{(-i\lambda)^2\left(iV(s) + iV(t) + iV(u)\right)} - i\delta\lambda\right]\prod_i \frac{i}{p_i^2}$

In $\overline{MS}$: $\delta_\lambda = \frac{3\hat{\lambda}^2}{2(4\pi)^2}\left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi\right)$　　这里需要确定 $\lambda = \hat{\lambda}\mu^{2\varepsilon}$?

是否树图贡献单独提出来?

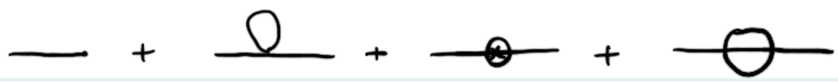
$$(-i\lambda)^2\left(iV(s) + iV(t) + iV(u)\right) = i\frac{3\hat{\lambda}^2}{2(4\pi)^2}\left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi + \ln\mu^2 + \cdots\right)$$

$$\mu\frac{\partial}{\partial\mu}G^{(4)} = \frac{\partial G^{(4)}}{\partial\ln\mu} = \frac{i\,3\hat{\lambda}^2}{(4\pi)^2}\prod_i\frac{i}{p_i^2} \Rightarrow \beta(\lambda) = \frac{3\hat{\lambda}^2}{(4\pi)^2}$$

问题　$\frac{\partial\delta\lambda}{\partial\ln\mu} = 0$?　　其实　$\frac{\partial\hat{\lambda}}{\partial\ln\mu} = \mu\frac{\partial\hat{\lambda}}{\partial\mu} = \mu\frac{\partial\lambda/\mu^{2\varepsilon}}{\partial\mu} = -2\varepsilon\hat{\lambda}$

所以　$\frac{\partial\delta\lambda}{\partial\ln\mu} = \frac{3\hat{\lambda}^2}{2(4\pi)^2}\cdot(-2\varepsilon)\cdot\left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi\right) = -\frac{3\hat{\lambda}^2}{(4\pi)^2}$　　这里只考虑了一个 $\hat{\lambda}$ 贡献

- ⊗ 2-point correlation:

$G^{(2)}(p) =$ 

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(\lambda)\frac{\partial}{\partial\lambda} + 2\gamma(\lambda)\right) G^{(2)}(p) = 0$$

$G^{(2)}$ does not receive corrections at 1-loop. and no correction from λ

$\rightarrow \quad \gamma = 0$ at one-loop order

⊗ 2-point generic form:

$G^{(2)}(p) =$ 

$$= \frac{i}{p^2} + \frac{i}{p^2}\left(A_\lambda^{p^2}\left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi + \ln\mu^2\right)\right)\frac{i}{p^2} + \frac{i}{p^2}(ip^2\delta_Z)\frac{i}{p^2}$$

δ_Z will exactly cancel the $\left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi\right)$ term, then

$$\mu\frac{\partial G^{(2)}}{\partial\mu} = 2A \cdot G^{(2)}$$

substituting into RGE and neglecting β, we have

$$\gamma = \frac{1}{2}\frac{\partial\delta_Z}{\partial\ln\mu} = -A$$

⊗ 一般情况

$$G^{(n)} = (\text{tree-level}) + \begin{pmatrix}\text{1PI loop}\\ \text{diagrams}\end{pmatrix} + (\text{Vertex c.t.}) + (\text{external leg corrections})$$

$$= \prod_i\left(\frac{i}{p_i^2}\right)\left[-ig - iB\left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi + \ln\mu^2\right) - i\delta g + (-ig)\sum_i\left(A_i\left(\underbrace{\frac{1}{\varepsilon} - \gamma + \ln 4\pi}_{\text{相同依赖}} + \ln\mu^2 - \delta_{Z_i}\right)\right)\right.$$

$$\mu\frac{\partial}{\partial\mu}G^{(n)} = \left(-2iB - 2ig\sum A_i\right)\prod_i\frac{i}{p_i^2}$$

on the other hand $\quad \left(\mu\frac{\partial}{\partial\mu} + \beta_0\frac{\partial}{\partial g} + \sum_i\gamma_i\right)G^{(n)} = 0$

$$\gamma_i = \frac{1}{2}\frac{\partial\delta_{Z_i}}{\partial\ln\mu} = -A_i$$

$$-2iB - 2ig\sum_i A_i + \beta\,\frac{\partial \overset{\overset{-i}{\parallel}}{G^{(n)}}}{\partial g} + \sum_i (-A_i)\cdot(-ig) = 0 \qquad G^{(n)}$$

$$\boxed{\beta = -2B - \sum_i A_i}$$

× 另一种理解方式:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$\delta_\lambda = 1 + B\times\left(\frac{2}{\epsilon} - \gamma + \ln 4\pi\right)$$

$$\delta_{Z_\phi} = 1 + A\left(\frac{2}{\epsilon} - \gamma + \ln 4\pi\right)$$

$$= (Z_\phi^2 Z_\lambda - 1)\cdot(-i\lambda) \equiv (-i\delta_\lambda)$$

$$\hookrightarrow Z_\lambda = \underline{\left(1 + \frac{\delta_\lambda}{\lambda}\right)} \Big/ Z_\phi^2 = \frac{1}{\lambda}(\lambda + \delta_\lambda)\cdot(1 - 2\delta_{Z_\phi})$$

此即从顶点图减掉自能部分.

$$\beta = \frac{\partial\lambda}{\partial\ln\mu} = \frac{\partial\,\lambda_0/Z_\lambda}{\partial\ln\mu} = -\lambda\frac{\partial Z_\lambda}{\partial\ln\mu} = -2B - \sum_{i=1}^{4} A_i$$

QED: β function 计算:

⊗ 任意规范下光子传播子:

$$D^{\mu\nu}(q) = D(q)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \frac{-i}{q^2}\frac{q^\mu q^\nu}{q^2}$$

D(q) 是可被重整的两点关联函数

⊗

$$\delta_1 = \delta_2 = -\frac{e^2}{(4\pi)^2}\Gamma(2-\tfrac{d}{2}) + \cdots$$

$$\delta_3 = -\frac{e^2}{(4\pi)^2}\frac{4}{3}\Gamma(2-\tfrac{d}{2}) + \cdots$$

(务必完成这个计算) ☆☆

$$\gamma_2 = \frac{1}{2}\frac{\partial\delta_2}{\partial\ln\mu} = +\frac{e^2}{(4\pi)^2} \qquad \gamma_3 = \frac{1}{2}\frac{\partial\delta_3}{\partial\ln\mu} = \frac{4}{3}\frac{e^2}{(4\pi)^2}$$

$$\frac{\partial e}{\partial\ln\mu} = \mu\frac{\partial(e_0/\mu^\varepsilon)}{\partial\mu} = e_0\mu\frac{\partial\mu^{-\varepsilon}}{\partial\mu} = -\varepsilon e_0\mu^{-\varepsilon} = -\varepsilon e$$

$$\frac{e_0}{e}Z_2 Z_3^{1/2} = Z_1 = 1+\delta_1 \quad \to e = e_0\cdot\frac{Z_2 Z_3^{1/2}}{Z_1} = e_0\left[1+\delta_2+\tfrac{1}{2}\delta_3-\delta_1\right]$$

$$\beta(e) = \frac{\partial e}{\partial\ln\mu}$$ (注意 这里重点关注单圈!)

$$= e_0\mu\frac{\partial}{\partial\mu}\left[\delta_2+\tfrac{1}{2}\delta_3-\delta_1\right]$$ (注意 $e_0 = e$ at this order!)

$$= e\mu\frac{\partial}{\partial\mu}\left[\delta_2+\tfrac{1}{2}\delta_3-\delta_1\right] = \frac{e^3}{12\pi^2}$$

RGE solution:

MS / $\overline{MS}$

* n-point correlation. for scalar field

$$G_n^{(0)} = \langle\Omega| T[\phi_0(x_1) \cdots \phi_0(x_n)] |\Omega\rangle$$

$$G_n = \langle\Omega| T[\phi(x_1) \cdots \phi(x_n)] |\Omega\rangle$$
$$= Z^{-\frac{n}{2}} \langle\Omega| T[\phi_0(x_1) \cdots \phi_0(x_n)] |\Omega\rangle$$

$$\mu \frac{d}{d\mu} G_n = \mu \frac{d}{d\mu} \left[Z^{-\frac{n}{2}} \, G_n^{(0)} \right]$$
$$= \left(\mu \frac{d}{d\mu} Z^{-\frac{n}{2}}\right) G_n^{(0)} = \underline{\underline{-n\gamma G_n}}$$

$$\underline{\gamma = \frac{1}{Z^{1/2}} \mu \frac{d}{d\mu} Z^{1/2}}$$

$$G_n(p, \mu, \alpha(\mu), m(\mu), \cdots) \equiv \underline{\underline{G_n(p, \mu, \alpha(\mu))}}$$

* 提供

$$\boxed{G_n(p, \mu, \alpha(\mu)) = \exp\left[-n \int_{g(\mu_0)}^{g(\mu)} \frac{dg' \, \gamma(g')}{\beta(g')}\right] G_n(p, \mu_0, \alpha(\mu_0))}$$

(该形式有什么不同呢？)

$$\mu \frac{dG_n}{d\mu} = \mu \frac{d}{d\mu} \left(\underline{-n \int_{g(\mu_0)}^{g(\mu)} \frac{dg' \, \gamma(g')}{\beta(g')}} \right) G_n$$

$$= -n \underline{\underline{\mu \frac{d}{d\mu} g(\mu)}} \cdot \frac{\gamma(g)}{\beta(g)} G_n$$

$$= -n \gamma G_n$$

重整化标度改了.

$$(*) \quad \boxed{G(ap, \overset{g}{\lambda}, \mu) = \frac{i}{a^2 p^2} \exp\left[2 \int_0^t \gamma(\bar{\lambda}(t', \lambda))\, dt' \right] \bar{G}_{as}(\bar{\lambda}(t, \lambda))}$$

$\underline{t = \ln a}$

* 注意 $\bar{G}$ 只依赖于 $\bar{\lambda}$, 这里相当于 RG improved 微扰理论.

详细求解见下面.

之前方法

RGE solution：详细求解.

$$\mu\frac{d}{d\mu}G_n = -n\gamma G_n$$

标量场 $m=0$ $\lambda\equiv g$ $\frac{g^2}{4\pi}=\alpha$

$G_n(p,\lambda,\mu)$

$$\mu\frac{d}{d\mu}G_n = \mu\frac{\partial}{\partial\mu}G_n + \underbrace{\mu\frac{\partial\lambda}{\partial\mu}}_{\beta(\lambda)}\frac{\partial G}{\partial\lambda}$$

$$\Rightarrow\left(\mu\frac{\partial}{\partial\mu}+\beta(\lambda)\frac{\partial}{\partial\lambda}\right)G_n = -n\gamma G_n$$

$$\rightarrow\left(\mu\frac{\partial}{\partial\mu}+\beta(\lambda)\frac{\partial}{\partial\lambda}+n\gamma\right)G_n(p,\lambda,\mu)=0$$

以 $n=2$ 为例：按照量纲分析 修改求解求程

① 直接求解 g.

② 不引入 α，直接用 $t=\ln\frac{p}{\mu}$

$$G_2(p,\lambda,\mu)=\frac{i}{p^2}\,g\left(\frac{p}{\mu},\lambda\right)$$

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(\lambda)\frac{\partial}{\partial\lambda}+2\gamma\right)g\left(\frac{p}{\mu},\lambda\right)=0$$

由于 g 是 $\left(\frac{p}{\mu}\text{ 和 }\lambda(\mu)\right)$ 的函数. 定义 $t=\ln\frac{p}{\mu}$. 则

$$\left(\frac{\partial}{\partial t}-\beta\frac{\partial}{\partial\lambda}-2\gamma\right)g(e^t,\lambda)=0$$

重新定义 $g(e^t,\lambda)=\exp\left[-2\int_0^\lambda\frac{\gamma(\lambda')}{\beta(\lambda')}d\lambda'\right]\bar{g}(e^t,\lambda)$

某一下限，由匹配确定. 暂选为0，或取 λ_0 也行 ☆

则 $\left(\frac{\partial}{\partial\ln(p/\mu)}-\beta\frac{\partial}{\partial\lambda}\right)\bar{g}\left(\frac{p}{\mu},\lambda\right)=0$

定义跑动耦合常数： $\lambda(\mu)$ 是重整化标度为 μ 时的耦合常数.

$$\frac{d}{d\ln(p/\mu)}\bar{\lambda}(p,\lambda)=\beta(\bar{\lambda}),\quad \text{满足 } \bar{\lambda}(p=\mu,\lambda)=\lambda(\mu)$$

则可以证明 $\bar{g}\left(\frac{p}{\mu},\lambda\right)=\bar{g}(\bar{\lambda}(p,\lambda))$

证明：从跑动耦合常数定义出发

$$\frac{d\bar{\lambda}(p,\lambda)}{\beta(\bar{\lambda})} = d\ln(p/\mu)$$

积分 $$\int_{\lambda}^{\bar{\lambda}} \frac{d\bar{\lambda}(p,\lambda)}{\beta(\bar{\lambda})} = \int_{p'=\mu}^{p'=p} d\ln(p'/\mu) = \ln\frac{p}{\mu}$$

左右对λ求导：

$$\frac{\partial\bar{\lambda}}{\partial\lambda}\frac{1}{\beta(\bar{\lambda})} - \frac{1}{\beta(\lambda)} = 0 \to \beta(\lambda)\frac{\partial\bar{\lambda}}{\partial\lambda} - \beta(\bar{\lambda}) = 0$$

$$\beta(\lambda)\frac{\partial\bar{\lambda}}{\partial\lambda} - \frac{\partial\bar{\lambda}}{\partial\ln(p/\mu)}$$

即 $$\left(\frac{\partial}{\partial\ln(p/\mu)} - \beta(\lambda)\frac{\partial}{\partial\lambda}\right)\bar{\lambda} = 0$$

如何确定？

则 $$g\left(\frac{p}{\mu},\lambda\right) = \exp\left[-2\int_0^{\lambda} d\lambda' \frac{\gamma(\lambda')}{\beta(\lambda')}\right]\cdot \bar{g}(\bar{\lambda}(p,\lambda))$$

$$= \exp\left[-2\int_{\bar{\lambda}}^{\lambda} d\lambda' \frac{\gamma(\lambda')}{\beta(\lambda')} - 2\int_0^{\bar{\lambda}} d\lambda' \frac{\gamma(\lambda')}{\beta(\lambda')}\right] \bar{g}(\bar{\lambda}(p,\lambda))$$

$$= \exp\left[-2\int_{\bar{\lambda}}^{\lambda} d\lambda' \frac{\gamma(\lambda')}{\beta(\lambda')}\right] \bar{g}'(\bar{\lambda}(p,\lambda))$$

重新定义

$$= \exp\left[-2\int_{p'=p}^{p'=\mu} d\ln(p'/\mu)\ \gamma(\bar{\lambda}(p',\lambda))\right] \bar{g}'(\bar{\lambda}(p,\lambda))$$

$$= \exp\left[2\int_{p'=\mu}^{p'=p} d\ln\left(\frac{p'}{\mu}\right) \gamma(\bar{\lambda}(p',\lambda))\right] \bar{g}'(\bar{\lambda}(p,\lambda))$$

原来2pt为：

$$G^{(2)}(p,\lambda,\mu) = \frac{i}{p^2} \exp\left[2\int_{p'=\mu}^{p'=p} d\ln\frac{p'}{\mu}\ \gamma(\bar{\lambda}(p',\lambda))\right] \bar{g}'(\bar{\lambda}(p,\lambda))$$

逐项分析：

(1) $G^{(2)}(p,\lambda,\mu)$ 是外动量为p，重整化标度为μ，重整化后耦合常数为λ的振幅

一般形式为 $$\frac{i}{p^2}\left(1+\frac{\alpha_s(\mu)}{\pi}\left(A\ln\frac{p}{\mu}+\cdots\right)\right)$$

如果p与μ相差很大 则微扰论可能失效

(2) $\bar{g}'(\bar{\lambda}(p,\lambda))$ 是当 $\mu'=p$ 时的振幅（去掉 $\frac{i}{p^2}$ 因子），此时耦合常数为 $\bar{\lambda}$

没有log.　$\bar{\lambda}$ 中的宗量为μ时 $\bar{\lambda}\to\lambda$

根据 $G^{(2)}$ 的表达式为知. $\bar{g}(\bar{\lambda}(p,\lambda)) = 1 + O(\bar{\lambda}^2)$

(3). 指数因子为从能标 μ 到动量 p 的演化. 其中求和了大的对数.

展开之后包含无穷圈

(4) 跑动耦合常数

$$\frac{d}{d\ln(p/\mu)}\bar{\lambda}(p,\lambda) = \frac{3\bar{\lambda}^2}{16\pi^2}. \qquad \bar{\lambda}(p=\mu,\lambda) = \lambda$$

则 $$\frac{3}{16\pi^2}\left[\frac{1}{\lambda} - \frac{1}{\bar{\lambda}}\right] = \log\frac{p}{\mu}$$

$$\bar{\lambda}(p,\lambda) = \frac{\lambda}{1 - \frac{3\lambda}{16\pi^2}\ln(p/\mu)}$$

展开之后包含 $\lambda^{n+1}\ln^n\left(\frac{p}{\mu}\right)$, 来自于高圈修正中的对数项

RGE in QED:

potential in momentum space. $V(q^2)$

RGE: $\left(\mu \frac{\partial}{\partial \mu} + \beta(e) \frac{\partial}{\partial e}\right) V(q, e, \mu) = 0$ $\qquad \boxed{\gamma = 0}$ ☆

$$\left(q \frac{\partial}{\partial q} - \beta(e) \frac{\partial}{\partial e} + 2\right) V(q, e, \mu) = 0$$

求解 $V(q, e, \mu) = \frac{1}{q^2} \bar{V}(\bar{e}(q, e))$

At tree level $V = \frac{e^2}{q^2}$

then $V(q, e, \mu) = \frac{\bar{e}^2(q, e)}{q^2}$

$$\bar{e}^2(q) = \frac{e^2}{1 - \frac{e^2}{6\pi^2} \ln\left(\frac{q}{\mu}\right)}$$

* 跑动耦合常数：

- $$\frac{d}{d\ln(p/\mu)}\bar{\lambda}(p,\lambda)=\beta(\bar{\lambda})$$

$$\bar{\lambda}(p=\mu,\lambda)=\lambda$$

$$\bar{\lambda}(p;\lambda)=\frac{\lambda}{1-\frac{3\lambda}{16\pi^2}\ln\frac{p}{\mu}}$$

- $$\frac{d}{d\ln(p/\mu)}\bar{e}(p,e)=\beta(\bar{e}) \qquad \beta(\bar{e})=\frac{\bar{e}^3}{12\pi^2}$$

$$\frac{12\pi^2}{2}\left(\frac{1}{e^2}-\frac{1}{\bar{e}^2}\right)=\ln\frac{p}{\mu}$$

$$\bar{e}^2(q)=\frac{e^2}{1-\frac{e^2}{6\pi^2}\ln\frac{p}{\mu}}$$

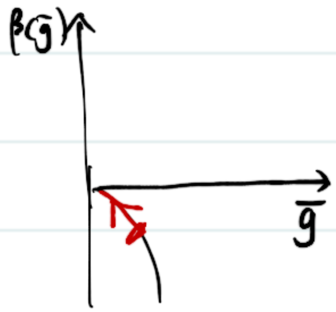
由 $\alpha=\frac{e^2}{4\pi}$ 可定义：

$$\bar{\alpha}(q)=\frac{\alpha}{1-\frac{\alpha}{3\pi}\ln\left(-\frac{q^2}{Am^2}\right)}$$

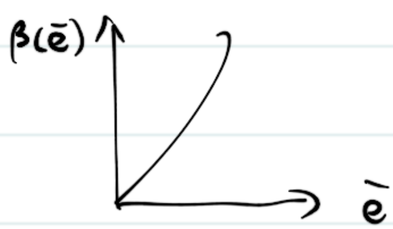
这两个例子均显示，β 是正，而 $\bar{\lambda}(p)$，$\bar{\alpha}(p)$ 中增大 p，耦合常数变大

固定点：反常量纲的物理意义

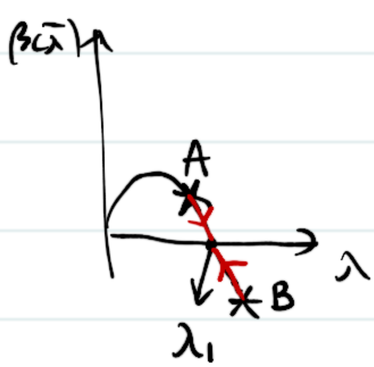
QCD： $\beta(\bar{g}) = -\beta_0 \bar{g}^3 + O(\bar{g}^5)$ β_0 常数

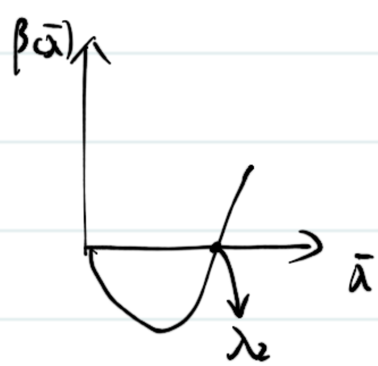


QED： $\beta(\bar{e}) = \beta_0' \bar{e}^3 + O(\bar{e}^5)$ β_0 常数



固定点：



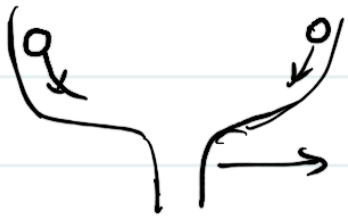


(a) 紫外 $\frac{\partial}{\partial t}\bar{\lambda}(t,\lambda) = \beta(\bar{\lambda})$ 用 $t = \ln\frac{p}{\mu}$. t 增大 即 p 增大.

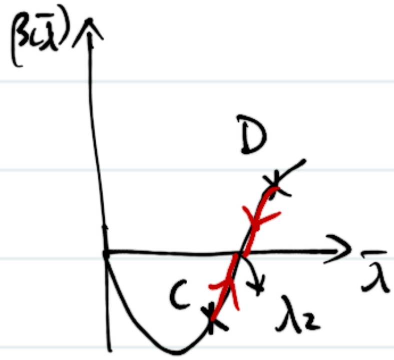
A处 t 增加 $dt > 0$. $\beta(\bar{\lambda}) > 0$ $d\bar{\lambda}(t,\lambda) > 0$ $\bar{\lambda}$ 增加。

B处：t 增加 $dt > 0$ $\beta(\bar{\lambda}) < 0$ $d\bar{\lambda}(t,\lambda) < 0$ $\bar{\lambda}$ 减小.

不管初值如何，只要增大动量，就一定会到固定点 λ_1 (紫外)



固定点　　紫外固定点



(*) 红外

$$\frac{\partial}{\partial t}\bar\lambda(t,\lambda)=\beta(\bar\lambda)$$

C处：　t减小：$dt<0$　　$\beta(\bar\lambda)<0$，　$d\bar\lambda>0$　　$\bar\lambda$增大

D处：　t减小　$dt<0$　　$\beta(\bar\lambda)>0$　　$d\bar\lambda<0$　　$\bar\lambda$减小

红外固定点.

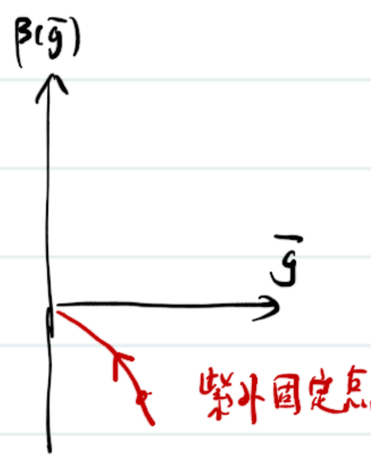
跑动耦合常数：

QCD：
$$\beta(\bar g)=-\beta_0\bar g^3$$
$$\bar g(t=0,g)=g$$

$$\frac{\partial\bar g(t,g)}{\partial t}=\beta(\bar g)=-\beta_0\bar g^3$$

$$\bar g(t)^2=\frac{g^2}{1+2\beta_0 g^2 t}$$

t增加，　$t\to+\infty$　　$\bar g(t)=0$



(*) 反常量纲，在固定点附近，可以近似

$$\beta \approx -B(\bar{\lambda}-\lambda_*) \qquad (\text{紫外固定点})$$

$$\frac{d\bar{\lambda}}{d\ln\frac{p}{\mu}} = -B(\bar{\lambda}-\lambda_*)$$

$$\bar{\lambda}(p) = \lambda_* + C\left(\frac{\mu}{p}\right)^B$$

不用放在这里

2PF：

$$G(p) \approx G(\lambda_*)\exp\left[-\ln\frac{p}{\mu}\cdot 2(1-\gamma(\lambda_*))\right]$$

$$\approx \frac{1}{p^2}\,G'(\lambda_*)\,\exp\left[2\ln\frac{p}{\mu}\,\gamma(\lambda_*)\right]$$

$$\approx C\left(\frac{1}{p^2}\right)^{1-\gamma(\lambda_*)}$$

反常量纲.

composite operator renormalization: 复合算符重整化

$\psi \quad A_\mu \quad g_s \quad \alpha$

QCD: UV divergences in amplitudes can be absorbed in $Z_2, Z_3 \quad Z_g \quad Z_\alpha$

but new operators: induce new UV divergences and new addition renorm.

bilinear $\quad O = \bar{q}\Gamma q \qquad \Gamma = \gamma_\mu, \gamma_\mu\gamma_5, I, \gamma_5, \sigma_{\mu\nu}$

矢量 轴矢量 标量 赝标量 张量

→ 标量算符: $O_s = \bar{q} I q = \bar{q} q$

$$q^{(0)} = Z_2^{1/2} q$$

$$O_s^{(0)} = \bar{q}^{(0)} q^{(0)} = Z_s O_s$$

$$O_s = \frac{O_s^{(0)}}{Z_s} = \frac{\bar{q}^{(0)} q^{(0)}}{Z_s} = \frac{Z_2 \bar{q} q}{Z_s} \approx \left(\frac{Z_2}{Z_s} - 1\right)\bar{q} q + \bar{q} q$$

$\boxed{\text{重整化条件: } \langle O_s \rangle \text{矩阵元有限}}$

(*) $\langle \bar{q} q \rangle$ 矩阵元: 微扰论可算

树图 单图

树图 $\quad iM = \bar{u} u$ (动量空间)

$$\int \frac{d^dk}{(2\pi)^4}\, \bar u(p)\, ig_s\gamma_\mu T^a \frac{i(\not p+\not k)}{(p+k)^2}\, \overset{I}{\underset{\downarrow}{}}\Gamma\, \frac{i(\not p+\not k)}{(p+k)^2} \cdot ig_s\gamma^\mu T^a u(p)$$

$$\times \frac{-i}{k^2}$$

$$= -ig_s^2 C_F \int \frac{d^dk}{(2\pi)^4}\, \bar u(p)\gamma^\lambda \frac{\not k}{k^2}\, I \cdot \frac{\not k}{k^2}\gamma_\lambda u(p) \cdot \frac{1}{k^2}$$

$$= -ig_s^2 C_F\, \bar u(p)\, u(p) \cdot \int \frac{d^4k}{(2\pi)^4} \frac{4}{(k^2)^2}$$

$$= -ig_s^2 C_F\, 4\bar u(p)\, u(p)\ \frac{(-1)^2 i}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \cdot \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}}$$

$$= \frac{g_s^2}{(4\pi)^2}\, 4C_F\, \bar u(p)\, u(p)\ \left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi - \ln\Delta\right)$$

$$= \frac{\alpha_s}{4\pi}\, 4C_F \left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi - \ln\Delta\right) \bar u(p)\, u(p) \ 。$$

(*) 重整化条件 $\left(\frac{Z_2}{Z_S}-1\right)\bar u u + \frac{\alpha_s}{4\pi} 4C_F \left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi - \ln\Delta\right)\bar u u \rightarrow$ 有限

$\langle O_s \rangle$

考虑到 $Z_2 = Z_4 = 1 - \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi - \ln\Delta\right)$

可以得到 $Z_S = 1 + 3\frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\varepsilon} - \gamma + \ln 4\pi\right)$ (in $\overline{MS}$)

(*) 利用反常量纲定义式:

$$\gamma_S = \frac{1}{Z_S}\, \mu \frac{\partial Z_S}{\partial \mu}$$

加上 $\alpha_s^0 = \alpha_s \mu^{2\varepsilon}$ 即 $\mu \frac{d\alpha_s^0}{d\mu} = \mu \frac{\partial \alpha_s}{\partial\mu} \mu^{2\varepsilon} + 2\varepsilon \cdot \alpha_s \mu^{2\varepsilon}$

$$\Rightarrow \mu \frac{\partial \alpha_s}{\partial \mu} = -2\varepsilon \alpha_s$$

得到 $\gamma_S = -\frac{\alpha_s}{4\pi} C_F \times 6$

(*) 张量流

$$\int \frac{d^dk}{(2\pi)^d} \bar{u}(p)\, ig\underline{\gamma_\alpha} T^a \frac{i(\not p+\not k)}{(p+k)^2}\, a_{\mu\nu}\, \frac{i(\not p+\not k)}{(p+k)^2}\, ig\underline{\gamma^\alpha} T^a u(p)$$

$$\propto \int \frac{d^dk}{(2\pi)^4} \bar{u}(p)\, \underline{\gamma_\alpha} \frac{(\not k\, a_{\mu\nu}\, \not k)}{k^2 k^2} \underline{\gamma^\alpha} u(p) \frac{1}{k^2} = 0$$

$$k_\alpha \gamma^\alpha \quad k_\beta \gamma^\beta \to g_{\alpha\beta}$$

(*) trace formula:

$$C^{00}_{\frac{1}{2}s_1\frac{1}{2}s_2} \times \left[u^{s_1}(k_1)\ \Gamma\ u^{s_2}(k_2)\right] \times u^{s_3}(k_3)$$

✓ ① with spinor + Clebsch Gordon

② without CG/spin, $\mathrm{tr}\left[\underline{\not p\gamma_5 C}\ \Gamma\right]$

一样

$$\mathrm{tr}\left[\underline{\not p\gamma_5 C} \cdots\right] = \underline{C^{00}_{\frac{1}{2}s_1\frac{1}{2}s_2}} \left[u^{s_1} \cdots\right]$$

Peskin书用关联函数讨论复合算符重整化，请注意差别

复合算符重整化：4夸克有效算符

复合算符重整化：光锥坐标系

$$n^\mu = (1,0,0,-1)/\sqrt{2}$$

$$\bar{n}^\mu = (1,0,0,1)/\sqrt{2}$$

选择动量沿 $\bar{n}^\mu$ 方向. $p^\mu = (p^0,0,0,p^z) \sim p^0(1,0,0,1) = \sqrt{2}p^0\,\bar{n}^\mu \equiv p^+\bar{n}^\mu$

$$p^+ = n\cdot p = (p^0+p^z)/\sqrt{2} \sim \sqrt{2}p^0$$

$$\psi = \left(\frac{\not{n}\bar{\not{n}}}{2} + \frac{\bar{\not{n}}\not{n}}{2}\right)\psi$$

$$A^\mu = n\cdot p\,\bar{n}^\mu + \bar{n}\cdot p\,n^\mu + A_\perp^\mu$$

大小分量 与 Wilson线

(*) PDF & LCDA. → $\boxed{\text{务必完成作业}}$

DGLAP 与 Brodsky-Lepage.

(*) 以PDF为例. 定域与非定域复合算符的关系 Peskin书 635页

并演示 LCDA 的 Gegenbauer矩展开.

(*) HQET LCDA:

① 定义: $\langle H| O_v^P(t n_+)|0\rangle = -i \tilde{f}_H m_H n_+\cdot v \int_0^\infty d\omega\, e^{i\omega t n_+ v} \varphi_+(\omega,\mu)$

$$O_v^P(t n_+) = \bar{h}_v(0) \not{n}_+ \gamma_5 W(0, t n_+) q_s(t n_+)$$

或者:

$$\int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0|\bar{q}(\tau n_+) W_c(\tau n_+, 0) \not{n}_+ \gamma_5 h_v(0)|\bar{B}(m_B v)\rangle = \tilde{f}_B m_B \varphi_+(\omega,\mu)$$

② HQET 简介: $\mathcal{L}_h = \bar{h}_v (i v\cdot D) h_v = \bar{h}_v (i v\cdot\partial + g_s t^a v\cdot A^a) h_v$

$$D_\mu = \partial_\mu - i g_s t^a A_\mu^a$$

传播子为: $\frac{i}{v\cdot q}$

$$\frac{i(\not{p}+m_Q)}{p^2-m_Q^2} = \frac{i(m_Q \not{v} + \not{k} + m_Q)}{(m_Q v+k)^2 - m_Q^2} = \frac{i m_Q (1+\not{v})}{2 m_Q v\cdot k} = \frac{1+\not{v}}{2}\frac{i}{v\cdot k}$$

耦合顶点: $i g_s t^a v_\mu$

② 定域算符重整化：

$$O_\Gamma = \bar q \Gamma h_v$$

$$O_\Gamma^{(0)} = \bar q^{(0)} \Gamma h_v^{(0)} = \sqrt{Z_q}\sqrt{Z_h}\,\bar q \Gamma h_v$$

$$O_\Gamma = \frac{1}{Z_O} O_\Gamma^{(0)} = \frac{\sqrt{Z_q}\sqrt{Z_h}}{Z_O}\,\bar q\Gamma h_v = \bar q \Gamma h_v + ct.$$

单圈图修正：

$$iM = \int \frac{d^dq}{(2\pi)^d}\, ig_s\mu^\varepsilon t^a \gamma^\lambda \frac{i\not q}{q^2}\Gamma \frac{i}{v\cdot q}\, ig_s \mu^\varepsilon t^a v_\lambda \frac{-i}{q^2}$$

$$= -ig_s^2 C_F \mu^{2\varepsilon}\int \frac{d^dq}{(2\pi)^d}\frac{\not v \not q\,\Gamma}{q^4\, v\cdot q}$$

$$= -ig_s^2 C_F \mu^{2\varepsilon}\int \frac{d^dq}{(2\pi)^d}\frac{\not v \not q\,\Gamma}{[q^2-m^2]^2\, v\cdot q}\Big|_{m\to 0}$$

参数化 $\frac{1}{a^r b^s} = 2^s \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}\int_0^\infty d\lambda \frac{\lambda^{s-1}}{(a+2b\lambda)^{r+s}}$

$$\frac{1}{(q^2-m^2)^2\, v\cdot q} = 2\frac{\Gamma(3)}{\Gamma(2)}\int_0^\infty d\lambda \frac{1}{[q^2-m^2+2\lambda v\cdot q]^3}$$

$$= 4\int_0^\infty d\lambda \frac{1}{[(q+\lambda v)^2 - m^2-\lambda^2 v^2]^3}$$

$$iM = -ig_s^2 C_F\mu^{2\varepsilon}\int\frac{d^dq}{(2\pi)^d}\int_0^\infty d\lambda\, 4\frac{\not v(\not q-\lambda\not v)\Gamma}{(q^2-m^2-\lambda^2v^2)^3}$$

$$= -4ig_s^2C_F\mu^{2\varepsilon}\int_0^\infty d\lambda\int\frac{d^dq}{(2\pi)^d}\frac{-\lambda\Gamma}{(q^2-m^2-\lambda^2)^3}$$

$$= 4ig_s^2 C_F\mu^{2\varepsilon}\int_0^\infty d\lambda\,\lambda\, \frac{(-1)^3 i}{(4\pi)^{d/2}}\frac{\Gamma(3-\frac d2)}{\Gamma(3)}\frac{1}{(m^2+\lambda^2)^{3-\frac d2}}$$

$$= \frac{2g_s^2C_F\,\Gamma(1+\varepsilon)\mu^{2\varepsilon}}{(4\pi)^{2-\varepsilon}}\,\underline{\frac12\int_0^\infty d\lambda^2 \frac{1}{(m^2+\lambda^2)^{1+\varepsilon}}}$$

$$\frac{(-1)}{\varepsilon}(m^2+\lambda^2)^{-\varepsilon}\Big|_0^\infty$$

$$= \frac{g_s^2 C_F\, \Gamma(1+\varepsilon)\mu^{2\varepsilon}}{(4\pi)^{2-\varepsilon}} \quad \frac{1}{\varepsilon}(m^2)^{-\varepsilon}$$

$$\text{uv div.} = \frac{g_s^2 C_F}{(4\pi)^2}\frac{1}{\varepsilon} = \frac{g_s^2}{12\pi^2\varepsilon}$$

由于 $\quad Z_h = 1 + \frac{g_s^2}{6\pi^2\varepsilon} \qquad \sqrt{Z_h} = 1 + \frac{g_s^2}{12\pi^2\varepsilon}$

$\quad Z_q = 1 - \frac{g_s^2}{12\pi^2\varepsilon} \qquad \sqrt{Z_q} = 1 - \frac{g_s^2}{24\pi^2\varepsilon}$

可得: $\quad Z_0 = 1 + \frac{g_s^2}{8\pi^2\varepsilon}$

$$= 1 + \frac{g_s^2}{12\pi^2\varepsilon} + \frac{g_s^2}{12\pi^2\varepsilon} - \frac{g_s^2}{24\pi^2\varepsilon}$$

$$\gamma_0 = \frac{d\ln Z_0}{d\ln\mu} = \frac{1}{8\pi^2\varepsilon}\frac{dg_s^2}{d\ln\mu} = \frac{1}{8\pi^2\varepsilon}\cdot(-2\varepsilon)g_s^2 = -\frac{g_s^2}{4\pi^2}$$

$$\frac{dg_s^2}{d\ln\mu} = \mu\cdot\frac{d(g_s^0)^2\mu^{-2\varepsilon}}{d\mu} = \mu\,(g_s^0)^2(-2\varepsilon)\,\mu^{-2\varepsilon-1} = (-2\varepsilon)g_s^2$$

重夸克自能图:

$$i\mathcal{M} = \int\frac{d^dq}{(2\pi)^d}\, i g_s\mu^\varepsilon t^a v_\mu \frac{i}{v\cdot(p+q)}\, i g_s\mu^\varepsilon t^a v^\mu \frac{-i}{q^2-m^2} \qquad \text{gluon mass}$$

$$= -g_s^2\mu^{2\varepsilon}C_F\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2-m^2)\,v\cdot(p+q)}$$

$$= -g_s^2\mu^{2\varepsilon}C_F\int_0^\infty d\lambda\int\frac{d^dq}{(2\pi)^d}\frac{1}{[q^2-m^2+2\lambda v\cdot(p+q)]^2}$$

$$\overset{q\to q-\lambda v}{=} -g_s^2\mu^{2\varepsilon}C_F\int_0^\infty d\lambda\int\frac{d^dq}{(2\pi)^d}\frac{1}{[q^2-m^2-\lambda^2+2\lambda v\cdot p]^2}$$

$$= -g_s^2\mu^{2\varepsilon}C_F\int_0^\infty d\lambda\,\frac{(-1)^2 i}{(4\pi)^{d/2}}\frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)}\frac{1}{(m^2+\lambda^2-2\lambda v\cdot p)^{2-\frac{d}{2}\to\varepsilon}}$$

需要注意的是:  相应的抵消项为 $i(Z_h-1)\,v\cdot p$

一种方法为对 iM 求 $v\cdot p$ 微分:

$$\frac{d\, iM}{d\, v\cdot p} = -i g_s^2 C_F \mu^{2\varepsilon}\Gamma(\varepsilon)\frac{1}{(4\pi)^{2-\varepsilon}}\int_0^\infty d\lambda\ (-\varepsilon)(-2\lambda)\cdot\left(m^2+\lambda^2-2\lambda v\cdot p\right)^{-1-\varepsilon}$$

Baryon LCDA:

$$\int \frac{d t_1 p}{(2\pi)} \int \frac{d(t_2 p)}{(2\pi)} e^{i x_1 p^+ t_1 + i x_2 p^+ t_2} \underbrace{\varepsilon_{ijk} \langle 0 | U_i^T(t_1 n) \Gamma D_j(t_2 n) \, S_k(0) | \Lambda \rangle}_{O_\Lambda} = \underline{\underline{f_\Lambda}} \, u_\Lambda(p) \cdot \underline{\underline{\Phi(x_1 x_2)}}$$

目标: $\mu \dfrac{d\Phi(x_1 x_2, \mu)}{d\mu} = \int dy_1 dy_2 \, \underline{\underline{V(x_1 x_2, y_1 y_2, \mu)}} \cdot \Phi(y_1 y_2, \mu)$

leading twist $\Gamma = C\gamma_5 \not{n}$ $\qquad \underline{\not{p} \sim \not{\bar{n}} \cdot n \cdot p} = \not{\bar{n}} p^+$

$$O_\Lambda^{(0)} = \underline{Z_\Lambda} \, O_\Lambda$$

$$O_\Lambda = \frac{1}{Z_\Lambda} O_\Lambda^{(0)} = \frac{Z_2^{3/2}}{Z_\Lambda} \left(\varepsilon_{ijk} \, U_i^T(t_1 n) \Gamma D_j(t_2 n) \, S_k(0) \right)$$

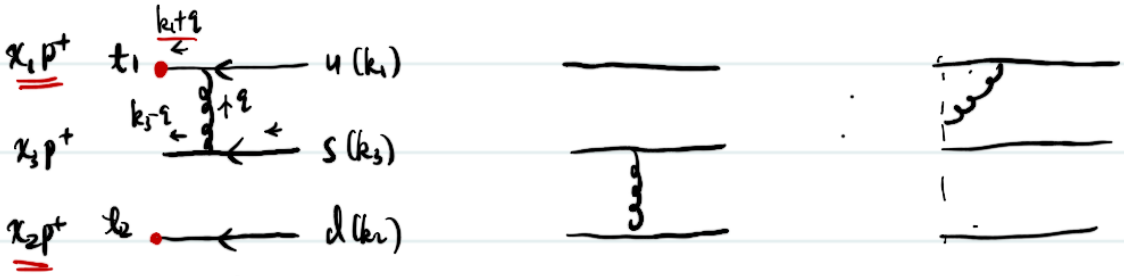
(1)

		动量	分数	颜色$(\frac{1}{6}\varepsilon_{abc})$ (自旋即投影)
树图结果 x_1	$u(k_1)$	k_1	x_1^0	a
x_3	$s(k_3)$	k_3	x_3^0	c
x_2	$d(k_2)$	k_2	x_2^0	b

$$iM^0 = \delta(x_1 - x_1^0)\,\delta(x_2 - x_2^0) \; \underbrace{\frac{1}{6} \varepsilon_{ijk} \varepsilon_{abc} \delta_{ia} \delta_{jb} \delta_{kc}}_{1} \cdot \left(u^T(x_1 p) C\gamma_5 \not{n} \, u(x_2 p) \right) u(x_3 p)$$

$$= \delta(x_1 - x_1^0)\,\delta(x_2 - x_2^0) \left[u^T(x_1 p) \, C\gamma_5 \not{n} \, u(x_2 p) \right] u(x_3 p)$$

This has to be understood as a summation over a nontrivial Clebsch-Gordon Coefficient



(a)

别忘系数

$$M^{t} = \int \frac{dt_1 p^+}{(2\pi)} \frac{dt_2 p^+}{(2\pi)} e^{i x_1 p^+ t_1 + i x_2 p^+ t_2} u^{S}(k_1)\,\Gamma\, u^{S}(k_2)\, u^{s_3}(k_3) \cdot e^{-ik_1^+ t_1 - i k_2^+ t_2} \cdot \varepsilon_{ijk} \cdot \frac{\varepsilon_{abc}}{6} \cdot \delta_{ai}\delta_{jb}\delta_{kc}$$

$$= \underline{u^{s_1}(k_1)\,\Gamma\, u^{s_2}(k_2)}\; u^{s}(k_3) \cdot \delta(x_1 - x_1^0)\,\delta(x_2 - x_{2}^0) \qquad \left(x_1^0 = \frac{k_1^+}{p^+} \quad x_2^0 = \frac{k_2^+}{p^+}\right)$$

$$\left[u^{S}(k_1)\,\Gamma\, u^{S_2}(k_2)\right]\Big|_{s_1 s_2} u(k_3) = \frac{1}{2}\,\mathrm{tr}\left[\not{p}\, C \gamma^5\, \Gamma\right] u(k_3) \qquad \text{trace formula}$$

$$= 2p^+\, u(k_3) \equiv S^{(0)}$$

$$M^{(a)} = i g_s^2 \left(-\frac{C_F}{2}\right)(p^+)^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 + i\varepsilon} \frac{1}{(k_3-q)^2} \frac{1}{(k_1+q)^2}\, \delta(x_1 p^+ - q^+ - k_1^+)\, \delta(x_2 p^+ - k_2^+)$$

$$\times \left[u^{s_1}(k_1)^T \gamma^\mu (\not{q} + \not{k}_1)\, \Gamma\, u^{s_2}(k_2)\right] (\not{k}_3 - \not{q})\, \gamma_\mu\, u^{s_3}(k_3)$$

$s_1\ s_2\ s_3$ spin / helicity

$$= i g_s^2 \left(-\frac{C_F}{2}\right)(p^+)^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 (k_3-q)^2} \frac{1}{(k_1+q)^2}\, \delta(x_1 p^+ - q^+ - k_1^+)\, \delta(x_2 p^+ - k_2^+)$$

$$\times \frac{1}{2}(-1)\,\mathrm{tr}\left[\not{p}\, \gamma_5\, \gamma^\mu (\not{q} + \not{k}_1)\, \gamma^5\, \not{n}\right] \left[\not{k}_3 - \not{q}\right] \gamma_\mu\, u(k_3)$$

$$\left[-q_\perp^2\right] \cdot u^{s_1 T}(k_1)\,\Gamma\, u^{s_2}(k_2)\, u^{s_3}(k_3)$$

$\Gamma = C \gamma_5 \not{n}$ $\qquad C = i\gamma^2\gamma^0$

$\not{p} = n \cdot p\, \not{\bar{n}}$ $\qquad \not{k}_1 = n \cdot k_1 \not{\bar{n}}$

旋量结构分析：

$$\mathrm{tr}[\not{p}\gamma_5\gamma^\mu(\not{q}+\not{k}_1)\gamma^5\not{n}] \qquad [\not{k}_3-\not{q}]\gamma_\mu u(k_3)$$

γ^μ 拆分为 $\quad \gamma^\mu = \underbrace{\not{n}\bar{n}^\mu + \not{\bar{n}}n^\mu}_{\text{都消失}} + \gamma_\perp^\mu$

此时：$\quad \mathrm{tr}[\not{p}\gamma_5\ \gamma_\perp^\mu\ \not{q}_\perp\ \underline{\gamma^5}\underline{\not{n}}] = p^+\ \mathrm{tr}[\not{n}\not{\bar{n}}\ \gamma_\perp^\mu\not{q}_\perp] = 4p^+\underbrace{n\cdot\bar{n}}_{1}\ q_\perp^\mu = 4p^+ q_\perp^\mu$

$$-\frac{1}{2}\mathrm{tr}[\quad][\not{k}_3-\not{q}]\gamma_\mu u(k_3) = -\frac{1}{2}4p^+(-\not{q})\not{q}_\perp u(k_3)$$

$$= 2p^+(-q_\perp^2)u(k_3) = (-q_\perp^2)S^{(1)}$$

注意 这里 $q_\perp^2$ 是欧式空间的结果 $\qquad S^{(1)} = 2p^+u(k_3)$

于是原积分变为:

$$M^{(a)} = i g_s^2 \left(-\frac{C_F}{2}\right)(p^+)^2 \int \frac{dq^+ dq^- d^{d-2}q_\perp}{(2\pi)^d} \frac{1}{q^2}\frac{1}{(k_3-q)^2}\frac{1}{(k_1+q)^2}\delta(x_1p^+-q^+-k_1^+)\,\delta(x_2p^+-k_2^+)$$

$$\times(-q_\perp^2)\, S^{(0)}$$

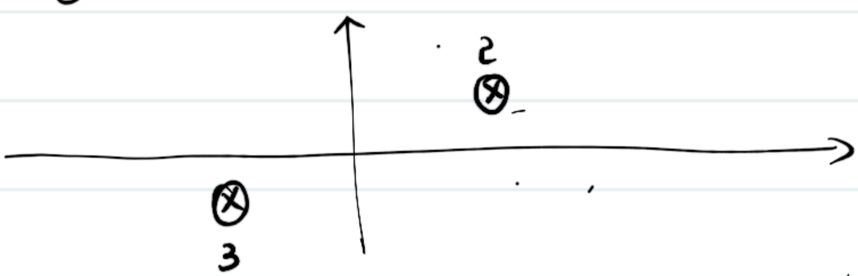
对于 q^- 积分 可以选择留数定理　　　此处 $q_\perp^2$ 为欧氏空间定义

$$q^- = \frac{+q_\perp^2 - i\varepsilon}{2q^+} \qquad q^- = \overset{p}{\underline{k_3^-}} + \frac{+q_\perp^2-i\varepsilon}{2(q^+-k_3^+)} \qquad q^- = -k_1^- + \frac{+q_\perp^2 - i\varepsilon}{2(q^+ + k_1^+)}$$

将 $q^+ = x_1p^+ - k_1^+$ 代入:

$$q^- = \frac{+q_\perp^2-i\varepsilon}{2(x_1p^+-k_1^+)} \qquad q^- = \frac{+q_\perp^2-i\varepsilon}{2(\underbrace{x_1p^+-k_1^+-k_3^+}_{<0})} \qquad q^-=\frac{+q_\perp^2-i\varepsilon}{2x_1p^+}$$

①　　　　②　　　　③



三个奇点中 ②由于 $x_1p^+<k_1^++k_3^+$,始终位于实轴上方.　③始终位于实轴下方

当 $x_1p^+<k_1^+=x_1^0p^+$时,①位于实轴上方,此时选择奇点③

$$M_1^{(a)} = i g_s^2 \frac{C_F}{2}(p^+)^2 S^{(0)}\int\frac{dq^+d^{d-2}q_\perp}{(2\pi)^d}\,\delta(x_1p^+-q^+-k_1^+)\,\delta(x_2p^+-k_2^+)\times q_\perp^2$$

$$\times(-2\pi i)\frac{1}{\left[2(x_1p^+-k_1^+)\times\frac{q_\perp^2}{2x_1p^+}-q_\perp^2\right]\left[2(x_1p^+-k_1^+-k_3^+)\frac{q_\perp^2}{2x_1p^+}-q_\perp^2\right]\cdot 2x_1p^+}$$

$$= i g_s^2\frac{C_F}{2}S^{(0)}\int\frac{d^{d-2}q_\perp}{(2\pi)^d}\cdot(-2\pi i)\,\delta(x_2-x_2^0)$$

$$\times\frac{x_1}{2(-x_1^0)(-x_1^0-x_3^0)\,q_\perp^2}$$

对于横向动量积分 $\displaystyle\int\frac{d^{d-2}q_\perp}{(2\pi)^{d-2}}\frac{1}{q_\perp^2} = \frac{1}{(4\pi)^{\frac{d-2}{2}}}\frac{\Gamma(1-\frac{d-2}{2})}{\Gamma(1)}\left(\frac{1}{\Delta}\right)^\varepsilon$

结果为：$M_1^{(a)} = \frac{1}{2} g_s^2 C_F S^{(0)} \frac{1}{2\pi} \cdot \frac{1}{4\pi} \delta(x_2 - x_2^0)\, \theta(x_1^0 - x_1) \frac{x_1}{2x_1^0} \frac{1}{x_1 + x_3}$

$$= \frac{\alpha_s}{8\pi} C_F S^{(0)} \delta(x_2 - x_2^0) \frac{x_1}{x_1^0} \frac{1}{x_1 + x_3}$$

n^μ

$$= \frac{\alpha_s C_F}{8\pi} \delta(x_2 - x_2^0) \left[\frac{x_3 \theta(x_1 - x_1^0)}{(1-x_2) x_3^0} + \frac{x_1 \theta(x_1^0 - x_1)}{(1-x_2) x_1^0} \right] \left[\frac{1}{\varepsilon_{uv}} - \frac{1}{\varepsilon_{IR}} \right] \cdot S^{(0)}$$

重子LCDA的演化:

$$\Phi(x_1 x_2) = \frac{1}{f} \int \frac{dt_1 np}{2\pi} \frac{dt_2 np}{2\pi} e^{i t_1 np} e^{i t_2 np} \times \varepsilon_{ijk}$$
$$\times \langle 0 | u_i^T(t_1 n) \Gamma d_j(t_2 n) \; S_k(0) | \Lambda(p) \rangle$$

f is the decay constant

$$f = \varepsilon_{ijk} \langle 0 | u_i^T \Gamma d_j S_k(0) | \Lambda(p) \rangle,$$

换成含量态:

$$|\Lambda\rangle = \frac{1}{6} \varepsilon_{abc} \; |u_a(x_{10}) u_b(x_{20}) S_c(x_{30})\rangle$$

tree-level:

$$\hat{\Phi}^{(0)} = \delta(x_1 - x_{10}) \; \delta(x_2 - x_{20})$$

At 1-loop: $\hat{\Phi}^{(1)} = \frac{\alpha_s C_F}{8\pi} \frac{1}{\varepsilon} \cdot f(x_1 \, x_2 \, x_{10} \, x_{20}) +$ finite term

based on composite operator renormalization constant. one has

$$Z_\Phi = \delta(x_1 - x_{10}) \, \delta(x_2 - x_{20}) + \frac{\alpha_s C_F}{8\pi} \frac{1}{\varepsilon} f(x_1 \, x_2 \, x_{10} \, x_{20})$$

$$\frac{d \ln \alpha_s}{d \ln \mu} = -2\varepsilon \alpha_s$$

$$\rightarrow \frac{d \ln Z_\Phi}{d \ln \mu} = -\frac{\alpha_s C_F}{4\pi} f(x_1 \, x_2 \, x_{10} \, x_{20})$$

The μ dependence of Z_Φ and Φ are opposite

$$\hat{\Phi}^{(0)} = Z_\Phi \cdot \Phi$$

One has the partonic-level evolution:

$$\rightarrow \quad \frac{d\hat{\Phi}(x_1\, x_2\, x_{10}\, x_{20})}{d\ln\mu} = \frac{\alpha_s C_F}{4\pi} \int dy_1\, dy_2\; f(x_1\, x_2\, y_1\, y_2)\, \frac{\delta(y_1 - x_{10})\,\delta(y_2 - x_{20})}{\hat{\Phi}(y_1\, y_2\, x_{10}\, x_{20})}$$

$$\rightarrow \quad \frac{d\Phi(x_1 x_2)}{d\ln\mu} = \frac{\alpha_s C_F}{4\pi} \int dy_1\, dy_2\; f(x_1 x_2\, y_1 y_2)\; \Phi(y_1\, y_2)$$

LSZ 约化公式

$$\langle k_1 k_2\ \text{out} | p_1 p_2\ \text{in}\rangle = \langle k_1 k_2 | S | p_1 p_2 \rangle$$

$$= \langle k_1 k_2 | T \exp\left[i\int d^4x\, \mathcal{H}_I(x)\right] | p_1 p_2 \rangle$$

$$= \langle k_1 k_2 | T\left\{1 + i\int d^4x \frac{\lambda}{4!}\phi^4(x) + \frac{1}{2!}\left(i\int d^4x \frac{\lambda}{4!}\phi^4(x)\right)^2 + \cdots\right\} | p_1 p_2 \rangle$$

$$= \quad - \quad - \quad - \quad e^{-ip_1x}\, e^{-ip_2x}$$

$$\langle 0 | \phi(x) | p_1 \rangle = e^{-ip_1x} \qquad ?$$

包含相互作用，不一定一样.

LSZ约化公式:

$|a\ in(p)\rangle = |a_{out}(p)\rangle = |a(p)\rangle$ 可认为无相互作用.

$$|p_1 p_2\ in\rangle = a^\dagger_{in}(p_1)\ a^\dagger_{in}(p_2)|0\rangle$$

$$\phi_{in}(x) = \int \widetilde{dp}\ [a_{in}(p) e^{-ip\cdot x} + a^\dagger_{in}(p) e^{ip\cdot x}]$$

$$[a_{in}(p), a^\dagger_{in}(p')] = (2\pi)^3 2E_p \delta^3(p-p')$$

$$a_{in}(p) = i\int d^3x\, e^{ip\cdot x} \overleftrightarrow{\partial_0} \hat{\phi}_{in}(x) \Big|_{x_0=-\infty}$$

$$\phi(x) \xrightarrow[x_0\to\pm\infty]{} \sqrt{Z}\,\hat{\phi}^{out}_{in}(x).$$

形式上 $a \xrightarrow[x_0\to\pm\infty]{} \sqrt{Z}\, a^{out}_{in}(p)$

$$\lim_{x_0\to\pm\infty} \langle\alpha|\phi(x)|\beta\rangle \to \sqrt{Z}\langle\alpha|\hat{\phi}^{out}_{in}(x)|\beta\rangle$$

(特别要注意 Z 与重整化常数的差别,它不是吸收 UV 发散的常数!)

S矩阵:

$$\langle k_1 k_2\ out|p_1 p_2\ in\rangle = \langle k_1\ out|a_{out}(k_2)|p_1 p_2\ in\rangle$$

$$= i\int d^3x\, e^{ik_2 x} \overleftrightarrow{\partial_0} \langle k_1|\hat{\phi}_{out}(x)|p_1 p_2\ in\rangle\Big|_{x_0=+\infty}$$

$$= i\int d^3x\, e^{ik_2 x} \overleftrightarrow{\partial_0} \langle k_1|\hat{\phi}_{in}(x)|p_1 p_2\ in\rangle\Big|_{x_0=-\infty}$$

$$+\frac{1}{\sqrt{Z}}\, i\int_{-\infty}^{+\infty} dx_0\, \partial_0 \left[\int d^3x\, e^{ik_2 x} \overleftrightarrow{\partial_0} \langle k_1|\phi(x)|p_1 p_2\ in\rangle\right]$$

$$= \langle k_1 k_2\ in|p_1 p_2\ in\rangle + \cdots$$

利用分步积分式

$$\partial_0 (f \overleftrightarrow{\partial_0} g) = f \overleftrightarrow{\partial_0}^2 g\ ,\quad \partial_0^2 e^{ik_2 x} = (\nabla^2 - m^2)\, e^{ik_2 x}$$

$$= \langle k_1 k_2\ in|p_1 p_2\ in\rangle + i\frac{1}{\sqrt{Z}}\int d^4x\, e^{ik_2 x} (\Box + m^2)\langle k_1|\hat{\phi}(x)|p_1 p_2\ in\rangle$$

$$= \langle k_1 k_2\, in | p_1 p_2\, in\rangle + \left(\frac{1}{\sqrt{Z}}\right)^4 i^4 \int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4$$

$$\times\, e^{ik_1x_1+ik_2x_2-ip_1y_1-ip_2y_2}$$

$$\times\,(\Box_{x_1}+m^2)(\Box_{x_2}+m^2)(\Box_{y_1}+m^2)(\Box_{y_2}+m^2)$$

$$\times\,\langle 0|T[\phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)]|0\rangle$$

$$\rightarrow\ \langle k_1 k_2|(S-1)|p_1 p_2\rangle = \int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4\ e^{ik_1x_1+ik_2x_2-ip_1y_1-ip_2y_2}$$

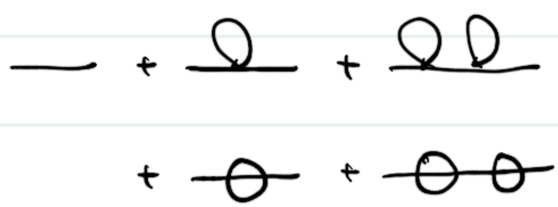
$$\times\, \frac{i(\Box_{x_1}+m^2)}{Z}\ \frac{i(\Box_{x_2}+m^2)}{Z}\ \frac{i(\Box_{y_1}+m^2)}{Z}\ \frac{i(\Box_{y_2}+m^2)}{Z}$$

$$\times\,(\sqrt{Z})^4\ \langle 0|T[\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)]|0\rangle$$

$$= \frac{k_1^2-m^2}{iZ}\ \frac{k_2^2-m^2}{iZ}\ \frac{p_1^2-m^2}{iZ}\ \frac{p_2^2-m^2}{iZ}\cdot(\sqrt{Z})^4$$

$$\left.\begin{aligned}&\times \int d^4x_1\, d^4x_2\, d^4y_1\, d^4y_2\ e^{ik_1x_1+ik_2x_2-ip_1y_1-ip_2y_2}\\ &\times \underbrace{\langle 0|T[\phi(x_1)\phi(x_2)\phi(y_1)\phi(y_2)]|0\rangle}_{G_n(x_1\ x_2\ y_1,\ y_2)}\end{aligned}\right\}\ \text{动量空间格林函数}$$

(*) $\langle 0|T[\phi(x)\phi(y)]|0\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\ \frac{iZ}{p^2-m^2}$ 动量空间 2pt



(*) 将 $\frac{k_1^2-m^2}{iZ}$ 作用到动量空间上n点格林函数上相当于做截腿.

定义 $G_4(k_1 k_2 p_1 p_2)\,(2\pi)^4(k_1+k_2-p_1-p_2)$

$$= \int d^4x_1\, d^4x_2\, d^4y_1\, d^4y_2\ \ \langle 0|T[\phi(x_1)\,\phi(x_2)\,\phi(y_1)\,\phi(y_2)]|0\rangle$$

相等，变为 $G_2(p_1)$

$$G_2(p_1,p_2)\,(2\pi)^4\delta^4(p_1-p_2) = \int d^4x\, d^4y\, e^{ip_1x} e^{-ipy} \langle 0|T[\phi(x)\,\phi(y)]|0\rangle$$

$$G_2(p_1) = \frac{iZ}{p_1^2-m^2}$$

(*) $$\langle k_1k_2|(S-1)|p_1p_2\rangle = (\sqrt{Z})^4\,(2\pi)^4\delta^4(k_1+k_2-p_1-p_2)$$

$$\cdot\, \underbrace{G_2(p_1)\,G_2(p_2)\,G_2(k_1)\,G_2(k_2)\cdot G_4(k_1,k_2\,p_1\,p_2)}_{\substack{\text{III} \\ G_4^t(k_1k_2p_1p_2)}}$$

截腿格林函数

(*) 推广至费米子和规范玻色子.

费米子: $\psi(x) = \int \widetilde{dp} \sum_\lambda [b_\lambda(p,t)\, e^{-ip\cdot x} u_\lambda(p) + d_\lambda^+(\vec{p},t)\, e^{ip\cdot x} v_\lambda(p)]$

$$b_\lambda(\vec{p},t) = \int d^3x\, \bar{u}_\lambda(p)\, e^{ip\cdot x}\, \gamma^0\, \psi(x)$$

$$d_\lambda(p,t) = \int d^3x\, \bar{\psi}(x)\, \gamma^0\, v_\lambda(p)\, e^{ip\cdot x}$$

$$\frac{\partial}{\partial t} b_\lambda(p,t) = \int d^3x\, \bar{u}_\lambda(p)\, e^{ip\cdot x} (ip^0\gamma^0)\psi(x)$$

$$+ \int d^3x\, \bar{u}_\lambda(p)\, e^{ip\cdot x}\, \gamma^0\, i\partial_t \psi(x)$$

$$= i\int d^3x\, e^{ip\cdot x}\, \bar{u}_\lambda(p)\, (i\gamma_\mu \partial^\mu - m)\psi(x)$$

类似地 $\frac{\partial}{\partial t} d_\lambda(p,t) = i\int d^3x\, \bar{\psi}(x)\, (i\gamma_\mu \overleftarrow{\partial}^\mu - m)\, v_\lambda(p)$

弱收敛条件 $\lim\limits_{x_0\to\pm\infty} \langle\beta|\, b_\lambda(p,t)\,|\alpha\rangle = \sqrt{Z_2}\, \langle\beta|\, b^{in}_{out}\,|\alpha\rangle$

$$\lim_{x_0\to\pm\infty} \langle\beta|\, d_\lambda(p,t)\,|\alpha\rangle = \sqrt{Z_2}\, \langle\beta|\, d^{in}_{out}\,|\alpha\rangle$$

$$b_{out} - b_{in} = \frac{i}{\sqrt{Z_2}} \int d^4x\, e^{ip\cdot x}\, \bar{u}_\lambda(p)\, (i\gamma^\mu \partial_\mu - m)\, \psi(x)$$

如果考虑S矩阵元，则

$$\langle k_1 a_1\, k_2 a_2\ \mathrm{out} \,|\, p_1\lambda_1\, p_2\lambda_2\ \mathrm{in}\rangle = \langle k_1 a_1\, k_2 a_2\ \mathrm{in} \,|\, p_1\lambda_1\, p_2\lambda_2\ \mathrm{in}\rangle$$

$$+\, i^4 \int d^4x_1\, d^4x_2\, d^4y_1\, d^4y_2\, e^{ik_1x_1 + ik_2x_2 - ip_1y_1 - ip_2y_2}$$

$$\times\ \bar{u}^{\alpha_1}_{a_1}(k_1)\, \bar{u}^{\alpha_2}_{a_2}(k_2)\, \frac{(-i\not\partial_{x_1} + m)_{\alpha_1\alpha_1'}}{\sqrt{Z_2}}\, \frac{(-i\not\partial_{x_2} + m)_{\alpha_2\alpha_2'}}{\sqrt{Z_2}}$$

$$\times\ \langle 0|T[\psi^{\alpha_1'}(x_1)\, \psi^{\alpha_2'}(x_2)\, \bar{\psi}^{\beta_1'}(y_1)\, \bar{\psi}^{\beta_2'}(y_2)]|0\rangle$$

$$\times\ \frac{(i\overleftarrow{\not\partial}_{y_1} + m)_{\beta_1'\beta_1}}{\sqrt{Z_2}}\, \frac{(i\overleftarrow{\not\partial}_{y_2} + m)_{\beta_2'\beta_2}}{\sqrt{Z_2}}\, u^{\beta_1}_{\lambda_1}(p_1)\, u^{\beta_2}_{\lambda_2}(p_2)$$

$$= -i(\sqrt{Z_2})^4\, \bar{u}^{\alpha_1}_{a_1}(k_1)\, \bar{u}^{\alpha_2}_{a_2}(k_2)\, G^{tc}_{4\,\alpha_1\alpha_2\beta_1\beta_2}(-k_1, p_1, p_2)\, u^{\beta_1}_{\lambda_1}(p_1)\, u^{\beta_2}_{\lambda_2}(p_2)$$

Wilson line and covariant derivative

- a consistent result

$$D_\mu = \partial_\mu - i g_s t^a A^a_\mu$$

$$U(z, y) = P\exp\left[i g_s \int_0^1 ds \frac{dx^\mu}{ds} A^a_\mu(x(s)) t^a \right] \qquad y \longrightarrow z$$

$$x(s=0) = y, \quad x(s=1) = z .$$

- a consistent understanding:

We shall use an infinitesimal Wilson line

$$U(x+\varepsilon, x) = \exp\left[i g_s \varepsilon n^\mu A^a_\mu t^a \right]$$ n^μ为 Wilson line 方向

$$D_\mu \psi(x) = \frac{\psi(x+\varepsilon) - U(x+\varepsilon, x)\psi(x)}{\varepsilon n^\mu}$$ (限定 n^μ 方向)!

$$= \frac{\psi(x+\varepsilon) - \left[1 + i g_s \varepsilon n^\nu A^a_\nu t^a\right]\psi(x)}{\varepsilon n^\mu}$$

$$= \partial_\mu \psi(x) - i g_s A^a_\mu t^a \psi(x)$$

$$= (\partial_\mu - i g_s A^a_\mu t^a)\psi(x)$$ 定义自洽

Heavy meson DA (B-meson) DA

Def: $\int d\omega \frac{d\tau}{2\pi} e^{i\omega\tau} \langle \quad \rangle = \int d\omega \phi_B^+(\omega) \tilde{f}_B m_B$

$$\int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0 | \bar{q}(\tau n_+) W_c(\tau n_+, 0) \not{n}_+ \gamma_5 h_v | \bar{B}(m_B v) \rangle = \tilde{f}_B m_B \phi_B^+(\omega)$$

$\phi_B^+(\omega)$：量纲为 -1 $\quad \int_0^\infty d\omega \phi_B^+(\omega) = 1$

$$\rightarrow \frac{d}{d\ln\mu} \phi_B^+(\omega,\mu) = -\int_0^\infty d\omega' \gamma_+(\omega,\omega',\mu) \phi_B^+(\omega',\mu)$$

h_v: HQET quark field.

HQET:

$$\mathcal{L}_{QCD} = \bar{Q}(i\not{D} - m_Q) Q$$

$$Q(x) = e^{-im_Q v\cdot x} [Q_v + \mathcal{Q}_v(x)]$$

$$Q_v = e^{im_Q v\cdot x} \frac{1+\not{v}}{2} Q(x)$$

$$\mathcal{Q}_v(x) = e^{im_Q v\cdot x} \frac{1-\not{v}}{2} Q(x)$$

① $e^{-im_Q v\cdot x}$:

$$Q(x) \sim \int \frac{d^3p}{(2\pi)^3 2E_p} [b(p) e^{-ip\cdot x} + d^\dagger(p) e^{ip\cdot x}]$$

只描述正夸克

$$p^\mu = m_Q v^\mu + k^\mu$$

大动量，Λ_{QCD}

② $\frac{1+\not{v}}{2}$, $\frac{1-\not{v}}{2}$ $\qquad \not{P} Q = m_Q Q \rightarrow m_Q \not{v} Q = m_Q Q$

领头阶 $\qquad \not{v} Q = Q$

$$\not v \frac{1+\not v}{2} = \frac{\not v + \not v \not v}{2} = \frac{\not v + v^2}{2} = \frac{\not v + 1}{2} = \frac{1+\not v}{2}$$

$$\mathcal{L}_{HQET} = \bar Q_v (i v \cdot D) Q_v = \boxed{\bar Q_v (i v \cdot \partial + g t^a v \cdot A^a) Q_v}$$

$$D_\mu = \partial_\mu - i g t^a A^a_\mu$$

$$\mathcal{L}_{QCD} = \bar Q (i \not D - m_Q) Q$$

$$= e^{i m_Q v \cdot x} (\bar Q_v + \bar q_v)(i \not D - m_Q) e^{-i m_Q v \cdot x} (Q_v + q_v(x))$$

$$= e^{i m_Q v \cdot x} \bar Q_v (i \not D - m_Q) e^{-i m_Q v \cdot x} Q_v$$

$$= e^{i m_Q v \cdot x} \bar Q_v \Big(\underbrace{(m_Q \not v - m_Q)}_{0} e^{-i m_Q v \cdot x} + e^{-i m_Q v \cdot x} (i \not D) \Big) Q_v$$

$$= \bar Q_v (i \not D) Q_v = \bar Q_v \frac{1+\not v}{2} i \not D \frac{1+\not v}{2} Q_v = \bar Q_v \, i v \cdot D \frac{1+\not v}{2} Q_v = \bar Q_v \, i v \cdot D \, Q_v$$

$$\frac{1+\not v}{2} i \not D \frac{1+\not v}{2} = \frac{1}{2} (i \not D + 2 i v \cdot D - i \not D \cdot \not v) \frac{1+\not v}{2} = i v \cdot D \frac{1+\not v}{2}$$

⊗ quark propagator

$$\frac{i(\not p + m_Q)}{p^2 - m_Q^2} \rightarrow \frac{i}{v \cdot k}$$

$$\frac{i(m_Q \not v + \not k + m_Q)}{(m_Q v + k)^2 - m_Q^2} = \frac{i m_Q (1+\not v)}{m_Q^2 + 2 m_Q v \cdot k + k^2 - m_Q^2} = \frac{i m_Q (1+\not v)}{2 m_Q v \cdot k} = \frac{1+\not v}{2} \frac{i}{v \cdot k}$$

⊗ 頂点　$\mathcal{L}_I = \bar Q_v g t^a v \cdot A^a Q_v \Rightarrow i g t^a v_\mu$

$$Q_v \Leftrightarrow h_v$$

⊗ HQET. Decay constant. local operator

$$O = \bar q \Gamma Q_v \qquad \underline{\bar q \Gamma h_v} \qquad \Gamma \to \underline{\gamma^\mu \gamma_5}$$

$$O^{(0)} = Z_O O$$

$$O = \frac{O^{(0)}}{Z_O} = \frac{1}{Z_O}\bar q^{(0)}\Gamma Q_v^{(0)} = \frac{\sqrt{Z_q Z_h}}{Z_O}\,\underline{\bar q \Gamma Q_v} = \bar q \Gamma Q_v + \left(\frac{\sqrt{Z_q Z_h}}{Z_O} - 1\right)\bar q \Gamma Q_v$$

 +  + $\left(\frac{\sqrt{Z_q Z_h}}{Z_O} - 1\right) \times$  = 有限

$\langle \bar q \Gamma Q_v \rangle$

$$iM = \int \frac{d^dq}{(2\pi)^d}\, \bar u \cdot ig t^a \mu^\varepsilon \gamma^\lambda \frac{i\not q}{q^2}\Gamma \frac{i}{v\cdot q}\, ig t^a \mu^\varepsilon v_\lambda\, u_{hv} \times \frac{-i}{q^2}$$

$$= -ig^2\mu^{2\varepsilon} C_F \int \bar u \not v \frac{\not q}{q^2}\Gamma \frac{1}{v\cdot q} u_{hv} \cdot \frac{1}{q^2}\frac{d^dq}{(2\pi)^d}$$

$\bar u$ $\qquad u_{hv}$

$$= -ig^2\mu^{2\varepsilon} C_F \int \frac{d^dq}{(2\pi)^d}\frac{\not v \not q \Gamma}{(q^2)^2 v\cdot q}$$

$$\frac{1}{a^r b^s} = 2^s \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)}\int_0^\infty d\lambda \frac{\lambda^{s-1}}{(a+2b\lambda)^{r+s}}$$ $\underline{\lambda}$ 量纲为1

$$\frac{1}{(q^2)^2 v\cdot q} = 2\frac{\Gamma(3)}{\Gamma(2)\Gamma(1)}\int_0^\infty d\lambda \frac{1}{[q^2+2\lambda v\cdot q]^3} \overset{q \neq 0\, m}{=} 4\int_0^\infty d\lambda \frac{1}{[q^2-m^2+2\lambda v\cdot q]^3}$$

$$= 4\int_0^\infty d\lambda \frac{1}{[(q+\lambda v)^2-\lambda^2-m^2]^3}$$

$$iM = -ig^2\mu^{2\varepsilon}C_F\int\frac{d^dq}{(2\pi)^d}\, 4\int_0^\infty d\lambda \frac{\not v(\not q-\lambda\not v)\Gamma}{[q^2-\lambda^2-m^2]^3}$$

$$= -i4g^2\mu^{2\varepsilon}C_F\int_0^\infty d\lambda\,(-\lambda)\,\frac{(-1)^3 i}{(4\pi)^{d/2}}\,\frac{\Gamma(3-\frac{d}{2})}{\Gamma(3)}\,\frac{1}{(\lambda^2+m^2)^{3-\frac{d}{2}}}\,\underline{\bar u\Gamma u_{hv}}$$

$$= \frac{4g^2\mu^{2\varepsilon}}{(4\pi)^{d/2}}C_F\int_0^\infty d\lambda\,\lambda\,\frac{\Gamma(1+\varepsilon)}{2}\,\frac{1}{(\lambda^2+m^2)^{1+\varepsilon}}\,\bar u\Gamma u_{hv}$$

只留$\frac{1}{\varepsilon}$
$$= \frac{g^2C_F}{(4\pi)^2}\cdot\frac{4}{2}\,\frac{1}{2\varepsilon}\,(\bar u\Gamma u_{hv})$$

$$\int_0^\infty d\lambda\,\lambda\,\frac{1}{(\lambda^2)^{1+\varepsilon}} = \int_0^\infty d\lambda\,\lambda^{-1-2\varepsilon} = -\frac{1}{2\varepsilon}\lambda^{-2\varepsilon}\Big|_0^\infty$$

$$= -\frac{1}{2\varepsilon}\left(0-(m^2)^{-2\varepsilon}\right) = \frac{1}{2\varepsilon}\cdot(1+\varepsilon\cdot\log)$$

$$= \underline{\frac{g^2C_F}{16\pi^2}\frac{1}{\varepsilon}(\bar u\quad)}.$$

$$\underline{Z_q = 1-\frac{g^2}{12\pi^2\varepsilon}} \qquad Z_h = 1+\frac{g^2}{6\pi^2\varepsilon}$$

 +  $+\left(\sqrt{\frac{Z_qZ_h}{Z_0}}-1\right)$  = 有限

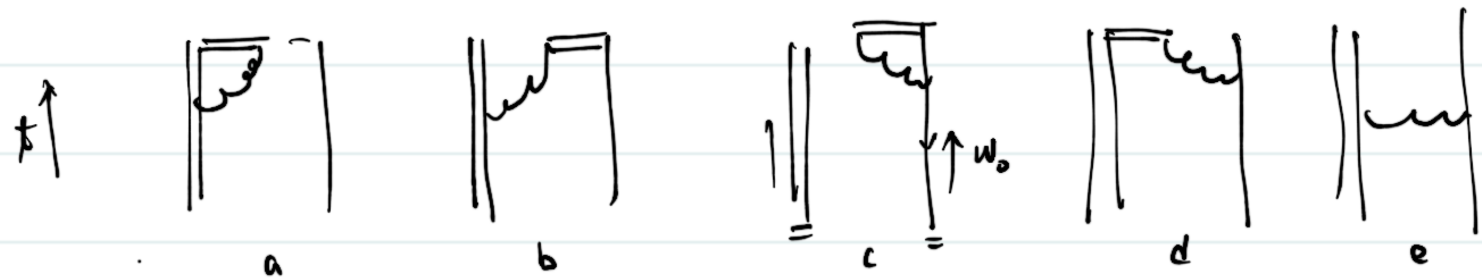
$$Z_0 = 1+\frac{g^2}{12\pi^2}\frac{1}{\varepsilon}+\frac{1}{2}\left(-\frac{g^2}{12\pi^2\varepsilon}\right)+\frac{1}{2}\left(\frac{g^2}{6\pi^2\varepsilon}\right)$$

$$= 1+\frac{g^2}{6\pi^2\varepsilon}-\frac{g^2}{24\pi^2\varepsilon} \qquad = 1+\frac{g^2}{8\pi^2\varepsilon}$$

$$\gamma_0 = \frac{d\ln Z_0}{d\ln\mu} = -\frac{g^2}{4\pi^2}$$

$$\rightarrow \langle 0|\underline{\bar q\,\gamma_+\gamma_5 h_v}|\bar B\rangle = f_B m_B$$

② HQET LCDA:

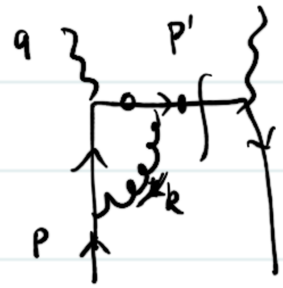


Wilson line $\qquad$ QCD $\qquad q(w_0)$

$$\langle \Phi \rangle^{(c)} = \int \frac{d\tau}{(2\pi)} e^{i\omega\tau} \langle 0 | \bar{q}(\tau n_+) \left(+ ig \int_{\infty}^{0} ds\, n_+ A^a(\tau n_+ + s n_+) t^a \right) \not{n}_+ \gamma_5 h_v(0) \cdot \int d^4 z_0\, \bar{q}\, ig\, t^a A^a q(z) \, | \bar{q} h \rangle$$

$$= \int \frac{d\tau}{(2\pi)} e^{i\omega\tau} \int d^4 z_1\, ig \int_{\infty}^{0} ds\, C_F$$

$$\times \langle 0 | \bar{q}\, g t^a A^a q(z) \times \bar{q}(\tau n_+) \quad n_+ A^a(\tau n_+ + s n_+) t^a \cdot \not{n}_+ \gamma_5 h_v \, | \bar{q} h \rangle$$



$$\frac{i(\not{p}'+\not{k})}{(p'+k)^2+i\varepsilon} = \frac{i(\cancel{n_- p'}\,\not{n}_+)}{n_+ k\; \cancel{n_- p'}+i\varepsilon} = \frac{i\,\not{n}_+}{n_+ k+i\varepsilon}$$

HQET LCDA:

$b.\bar{u}$

Def: $\qquad i\tilde{f}_B m_B \phi_B^+(\omega) = \int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0| \bar{q}(\tau n_+) W_c(\tau n_+, 0) \not{n}_+ \gamma_5 h_v(0) |\bar{B}(p_B)\rangle$

- rest frame
- $v^\mu = (1,0,0,0) \qquad n_+ v = \frac{1}{\sqrt{2}} \qquad n_+^\mu = \frac{1}{\sqrt{2}}(1,0,0,1)$

树图: $|\bar{B}\rangle \to |\bar{u}(w_0)\, b(p_b)\rangle$

$$iM^{(0)} = \int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0| \bar{q}(\tau n_+) \not{n}_+ \gamma_5 h_v(0) |\bar{u}(w_0)\, b(p_b)\rangle$$
$$= \int \frac{d\tau}{2\pi} e^{i\omega\tau}\, \bar{v}(w_0) e^{-i\tau n_+ w_0} \not{n}_+ \gamma_5 u_v(p_b)$$
$$= \delta(\omega - n_+ \cdot w_0)\, \bar{v}(w_0) \not{n}_+ \gamma_5 u_v$$
$$= i\tilde{f}_B^{(0)} m_B \phi_B^{+(0)}(\omega)$$

$h_v(0) = \int \frac{d^3k}{(2\pi)^3 2E} [b\, u e^{-ip\cdot x} + \cdots]$

$\bar{q} = \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} [b^+ \bar{u} e^{ip\cdot x} + d\, \bar{v} e^{-ip\cdot x}]$

约定: $\phi_B^{+(0)}(\omega) = \delta(\omega - n_+ w_0)$

$$W_c(\tau n_+, 0) = W_c^+(\infty, \tau n_+)\, W_c(\infty, 0)$$
$$W_c(\infty, 0) = P \exp\left[i g_s \int_0^\infty ds\, n_+ \cdot A(s n_+)\right]$$

1-loop

$$W_c^\dagger(\infty, \tau n_+)$$

$$\langle \Phi^{(1)} \rangle = \int \frac{d\tau}{2\pi} e^{i\omega\tau} \langle 0 | \bar q(\tau n_+)\, P\exp\left[-ig_s\int_0^\infty ds\, n_+ A(\tau n_+ + s n_+)\right] \slashed{n}_+ \gamma_5 h_v$$

$$\int \frac{d^4 z_1}{(2\pi)^4}\, i g_s\, \bar q\, t^a A^a q(z_1)\, | \bar u(\omega_0)\, b(p_b)\rangle$$

$$\longrightarrow D_\mu = \partial_\mu \overset{?}{\underset{\pm}{-}} i g\, t^a A^a_\mu$$

$$S = T\exp\left[-i\int d^4x\, \mathcal{H}_I\right]$$

$$= T\exp\left[i\int d^4x\, \mathcal{L}_I\right]$$

$$\mathcal{L} = \bar\psi(i\slashed{\partial} - m)\psi$$

$$= \bar\psi(i\slashed{\partial} - m + g\, t^a A^a)\psi$$

$$\mathcal{L}_I = \bar\psi\, g\, t^a A^a \psi$$

$$= \int\frac{d\tau}{2\pi} e^{i\omega\tau}\int\frac{d^4z_1}{(2\pi)^4}(-ig_s)\int_0^\infty ds\, n_+^\mu\, i g_s\, t^a t^b$$

$$\times \int\frac{d^4k}{(2\pi)^4} e^{-ik\cdot(z_1-\tau n_+)}$$

$$\times \langle 0|\bar q(z_1)\, A^a_\nu(z_1)\,\gamma^\nu \frac{i\slashed k}{k^2} \times A^b_\mu(\tau n_+ + s n_+)$$

$$\times\, \slashed n_+\gamma_5 h_v\, | \bar u(\omega_0)\, b(p_b)\rangle$$

$$= \int\frac{d\tau}{2\pi}e^{i\omega\tau}\int\frac{d^4z_1}{(2\pi)^4}(-ig_s)\int_0^\infty ds\, n_+^\mu\, t^a t^b \cdot i g_s$$

$$\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(z_1-\tau n_+)}\int\frac{d^4q}{(2\pi)^4}e^{-iq(z_1-\tau n_+ - s n_+)}$$

$$\times\ \frac{-i g_{\mu\nu}}{q^2}\,\delta^{ab}$$

$$\times\langle 0|\bar q(z_1)\,\gamma^\nu\,\frac{i\slashed k}{k^2}\,\slashed n_+\gamma_5 h_v\,|\bar u(\omega_0)\, b(p_b)\rangle$$

$$= \int\frac{d\tau}{2\pi}e^{i\omega\tau}\int\frac{d^4z_1}{(2\pi)^4}(-ig_s)\int_0^\infty ds\, n_+^\mu\, t^a t^a \times ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(z_1-\tau n_+)}\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(z_1-\tau n_+-sn_+)}$$

$$\times\ \frac{-ig_{\mu\nu}}{q^2}\ \bar v(\omega_0)\,e^{-i\omega_0\cdot z_1}\,\gamma^\nu\,\frac{i\slashed k}{k^2}\,\slashed n_+\gamma_5 u_v$$

$$\int\frac{d^4q}{(2\pi)^4}$$

$$= \int\frac{d\tau}{2\pi}e^{i\omega\tau}(-ig_s)\int_0^\infty ds\, n_+^\mu\, t^a t^a\times ig_s\int\frac{d^4k}{(2\pi)^4}(2\pi)^4\delta^4(k+q+\omega_0)\, e^{i\tau n_+ k}\, e^{i(q n_+)(\tau+s)}$$

$$\times\ \frac{-ig_{\mu\nu}}{q^2}\ \bar v(\omega_0)\,\gamma^\nu\,\frac{i\slashed k}{k^2}\,\slashed n_+\gamma_5 u_v$$

$$= \int\frac{d\tau}{2\pi}e^{i\omega\tau}(-ig_s)\int_0^\infty ds\, n_+^\mu\, t^at^a\, ig_s\int\frac{d^4q}{(2\pi)^4}e^{-i\tau(n_+q+n_+\omega_0)}\,e^{i(n_+q)(\tau+s)}$$

$$\times\ \frac{-ig_{\mu\nu}}{q^2}\ \bar v(\omega_0)\,\gamma^\nu\,\frac{-i(\slashed q+\slashed\omega_0)}{(q+\omega_0)^2}\,\slashed n_+\gamma_5 u_v$$

$$= \delta(\omega - n_+\omega_0)\,(-ig_s)\int_0^\infty ds\, n_+^\mu\, t^at^a\, ig_s\int\frac{d^4q}{(2\pi)^4}e^{-in_+ q s}$$

$$\times\ \frac{-ig_{\mu\nu}}{q^2}\ \bar v(\omega_0)\,\gamma^\nu\,\frac{-i(\slashed q+\slashed\omega_0)}{(q+\omega_0)^2}\,\slashed n_+\gamma_5 u_v$$

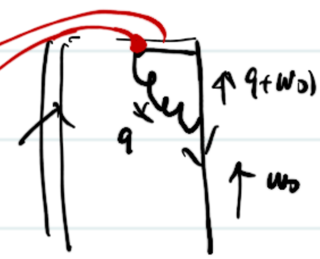
$$= \delta(\omega - n_+\omega_0)\,(-ig_s)\, n_+^\mu t^a t^a\, ig_s \int \frac{d^4q}{(2\pi)^4}\, \frac{1}{-in_+q}\, \underline{e^{-in_+q s}\Big|_0^\infty}^{\,-1}$$

$$\times \frac{-ig_{\mu\nu}}{q^2}\, \bar{v}(\omega_0)\, \gamma^\nu\, \frac{-i(\not{q}+\not{\omega}_0)}{(q+\omega_0)^2}\, \not{n}_+\gamma_5 u_v$$

$$= \delta(\omega - n_+\omega_0) + ig_s\, n_+^\mu t^a t^a\, ig_s \int \frac{d^4q}{(2\pi)^4}\, \frac{i}{n_+q}$$

$$\times \frac{-ig_{\mu\nu}}{q^2}\, \bar{v}(\omega_0)\, \gamma^\nu\, \frac{-i(\not{q}+\not{\omega}_0)}{(q+\omega_0)^2}\, \not{n}_+\gamma_5 u_v$$

$$= \delta(\omega - n_+\omega_0) \int \frac{d^4q}{(2\pi)^4} \times \bar{v}(\omega_0)\,(ig_s t^a \gamma^\nu)\, \frac{-i(\not{q}+\not{\omega}_0)}{(q+\omega_0)^2} \times \frac{i}{n_+q}\, ig_s t^a n_+^\mu \cdot \not{n}_+\gamma_5 u_v \times \frac{-ig_{\mu\nu}}{q^2}$$



$$iM = \delta(\omega - n_+\omega_0) \int \frac{d^4q}{(2\pi)^4} \times \frac{-ig_{\mu\nu}}{q^2}$$

$$\times \bar{v}(\omega_0)\cdot ig_s t^a \gamma^\nu\, \frac{-i(\not{q}+\not{\omega}_0)}{(q+\omega_0)^2} \times \frac{i}{n_+q}\, ig_s t^a n_+^\mu\, \not{n}_+\gamma_5 u_v$$

$$= ig_s^2 C_F\, \delta(\omega - \underbrace{\omega_0 n_+}_{\omega_0^+}) \int \frac{dq^+ dq^- d^2q_\perp}{(2\pi)^4}\, \frac{1}{q^+}\, \frac{1}{2q^+q^- - q_\perp^2}\, \frac{1}{2q^-(q^+ + \omega_0^+) - q_\perp^2}$$

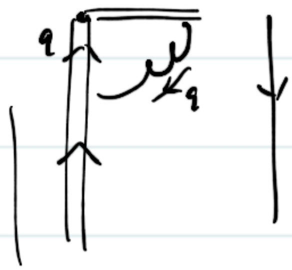
$$\times (-2)(q^+ + \omega_0^+)\, \bar{v}(\omega_0)\, \not{n}_+\gamma_5 u_v$$

极点: $\quad q^- = \dfrac{q_\perp^2 - i\varepsilon}{2q^+} \qquad q^- = \dfrac{q_\perp^2 - i\varepsilon}{2(q^+ + \omega_0^+)}$

$$\underline{-\omega_0^+ < q^+ < 0}$$

$$= -\frac{\alpha_s C_F}{2\pi}\, \delta(\omega - \omega_0^+)\, \frac{1}{\varepsilon} \int_0^1 dy\, \frac{1-y}{y}\, \bar{v}(\omega_0)\, \not{n}_+\gamma_5 u_v$$

$\langle \Phi^{(a)} \rangle$



$$= \delta(\omega-\omega_0^+)\, C_F \int \frac{d^4q}{(2\pi)^4}$$

$$\times\ \bar{v}(\omega_0)\, \not{n}_+ \gamma_5\ i g_s n_{+\mu}\ \frac{i}{n_+q}\ \frac{i}{v\cdot q}\ i g_s v^\mu \times \frac{-i}{q^2}$$

$$= -i g_s^2 C_F\, \delta(\omega-\omega_0^+) \underbrace{\int \frac{d^4q}{(2\pi)^4}\ \frac{n_+v}{n_+q}\ \frac{1}{v\cdot q}\ \frac{1}{q^2}}_{= I}\ \bar{u}(\omega_0)\, \not{n}_+ \gamma_5\, u_v$$

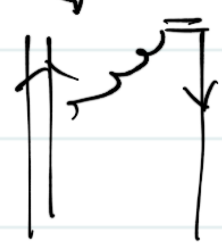
$$I = \int \frac{d^dq}{(2\pi)^d}\ \frac{1}{\sqrt{2}}\ \frac{1}{q^+}\ \frac{1}{\frac{1}{\sqrt{2}}(q^+ + q^-)}\ \frac{1}{2q^+q^- - q_\perp^2} \qquad \Big|\ q^- = -q^+ - i\varepsilon \quad . \quad q^- = \frac{q_\perp^2 - i\varepsilon}{2q^+}$$

$$= \frac{1}{(2\pi)^d} \int dq^+ d^{d-2}q_\perp\, (2\pi i)\ \frac{1}{q^+}\ \frac{2q^+}{2(2q^+)^2 - q_\perp^2}\ \frac{1}{2q^+}\ \theta(-q^+)$$

$$= -\frac{i}{8\pi}(4\pi)^\varepsilon\, \Gamma(\varepsilon) \int_0^\infty dq^+\ \frac{1}{q^+} \left(\frac{1}{2(q^+)^2}\right)^\varepsilon$$

$$= -\frac{i}{8\pi}(4\pi)^\varepsilon\, \Gamma(\varepsilon) \left[\int_0^w dq^+\ \frac{1}{q^+}\left(\frac{1}{2(q^+)^2}\right)^\varepsilon + \int_w^\infty dq^+\ \frac{1}{q^+}\ \frac{1}{(2(q^+)^2)^\varepsilon}\right]$$

$$= -\frac{i}{8\pi}(4\pi)^\varepsilon\, \Gamma(\varepsilon) \left[\int_0^w \frac{dt}{w-t} + \frac{w^{-2\varepsilon}}{2\varepsilon}\right]$$



$$iM \sim \Gamma(\varepsilon)\cdot\frac{1}{\varepsilon} \sim \frac{1}{\varepsilon^2}$$

$\underline{v^2}$. $\underline{n_+^2=0}$ $\underline{n_+v}$

① $n_+^2=0$ $\underline{n_+v \neq 0}$ cusp divergence

② PDF. LCDA: moments

no moments for HQET LCDA. $\quad \Big|\ \int dx\, \phi(x)\cdot \underset{x}{\underline{C_n^{3/2}(2x-1)}}$

③ quasi DA → QCD LCDA → HQET LCDA

operator. $\langle 0|\bar{q}(w)\gamma_+\gamma_5 h|\bar{B}\rangle$

$\langle \bar{B}|\bar{h}(w)\gamma_+ h(w)|\bar{B}\rangle$