核子结构基础

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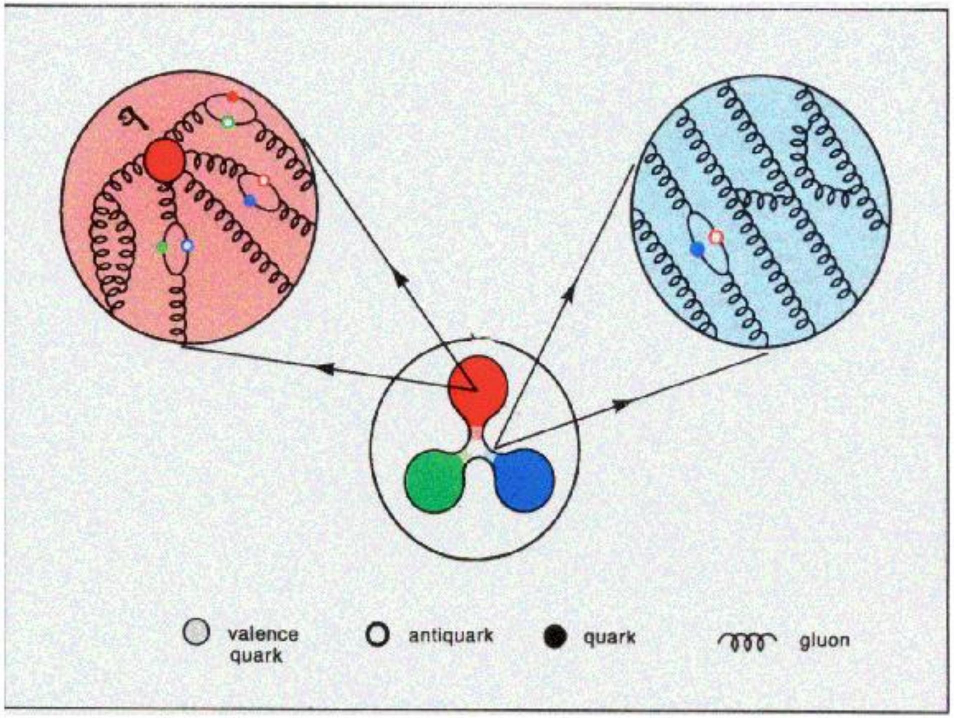
Our View of the Proton

with history

- **Point-Like** 1919
- **Finite Size with Radius** 1930s-1950s
- **Quark Model** 1960s
- **QCD: Quarks and Gluons** 1970s
- **Puzzles and Anomalies** 1980s-present

  - **Quark Sea of the Nucleon**
  - **Baryon-Meson Fluctuations**
  - **Statistical Features**
  - **……**

Nucleons: Building Block of Matter



- Nucleon anomalous magnetic moment ***(Stern, Nobel Prize 1943)***
- Electromagnetic form factor from electron scattering ***(Hofstadter, Nobel Prize 1961)***
- Deep-in-elastic scattering, quark underlying structure of the nucleon ***(Freedman, Kendell, Feldman, Nobel Prize 1990)***

Understanding the underlying nucleon structure from quantum chromodynamics is essential

Surprises & Anomalies

about the Quark Structure of Nucleon: Sea

- **Spin Structure:**

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

"puzzle": where is the proton's missing spin

- **Strange Content**

$$\Delta s \neq 0 \quad s(x) \neq \bar{s}(x)$$

Brodsky & Ma, PLB381(96)317

- **Flavor Asymmetry**

$$\bar{u} \neq \bar{d}$$

- **Isospin Symmetry Breaking**

$$\bar{u}_p \neq \bar{d}_n \quad \bar{d}_p \neq \bar{u}_n$$

Ma, PLB 274 (92) 111
Boros, Londergan, Thomas, PRL81(98)4075

The Quark Model

---Starting from 1964:   夸克模型的提出

- Gell-Mann and Zweig proposed the Quark Model

    Hadrons are baryons composed by three quarks, and mesons composed by a quark and an antiquark.

    Baryons: p=uud, n=ddu， Λ=uds, …

    Mesons: a quark and an antiquark

    The findings of c quark, b quark, and t quark

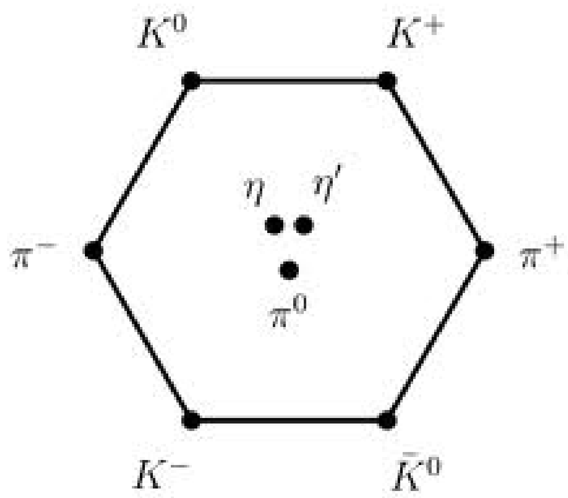
- QCD as the basic theory of strong interactions

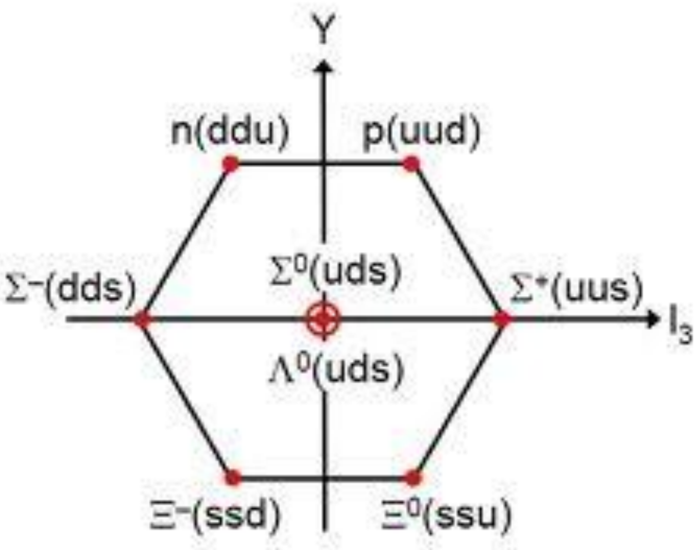
    quarks and gluons as the fundamental building blocks of hadrons or new forms of matter

Hadron States重子态

•Hadrons (Mesons and Baryons) are classified as multiplet of SU(3) flavor group

-> constructed of 3 quarks only,

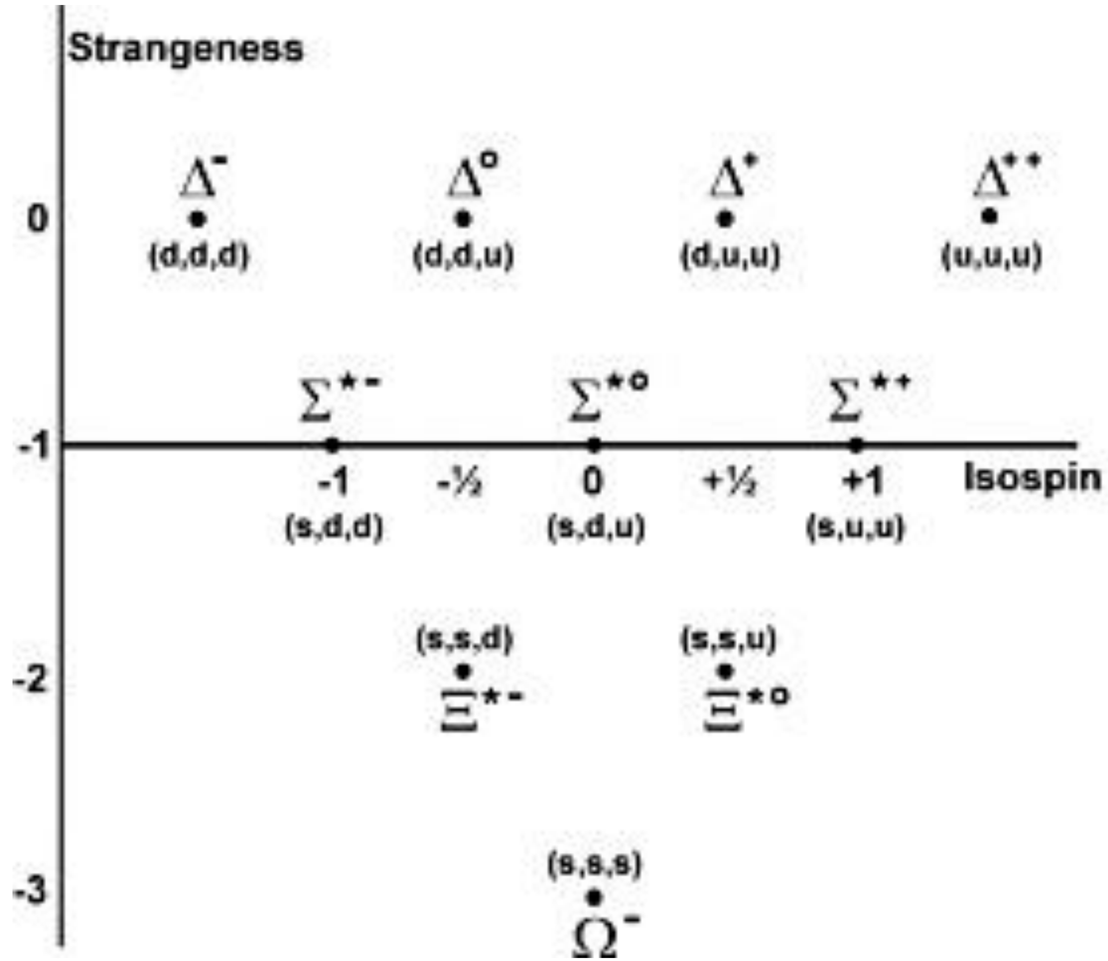




meson octet with $J^P=0^-$
八重态

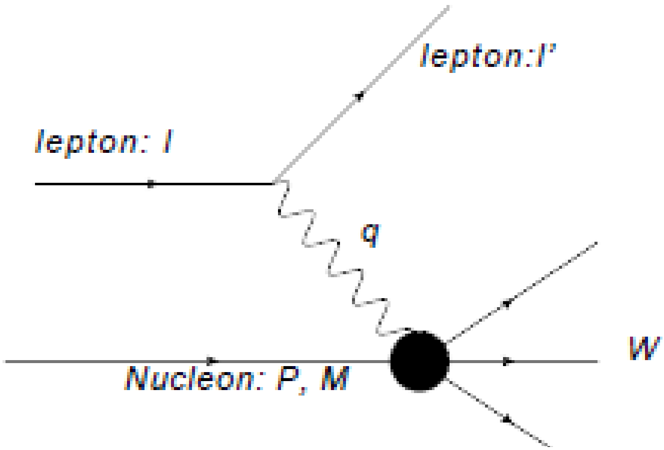
baryon octet with $J^P=\frac{1}{2}^+$
八重态

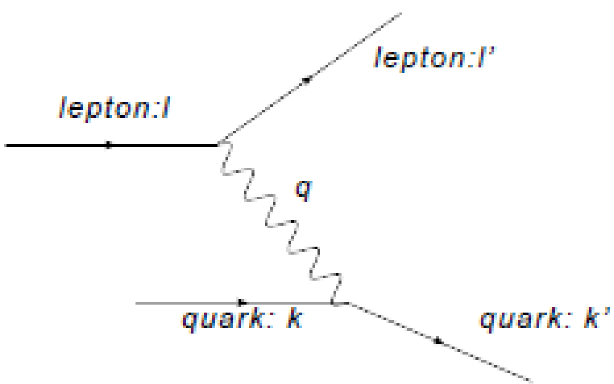
Baryon States 十重态

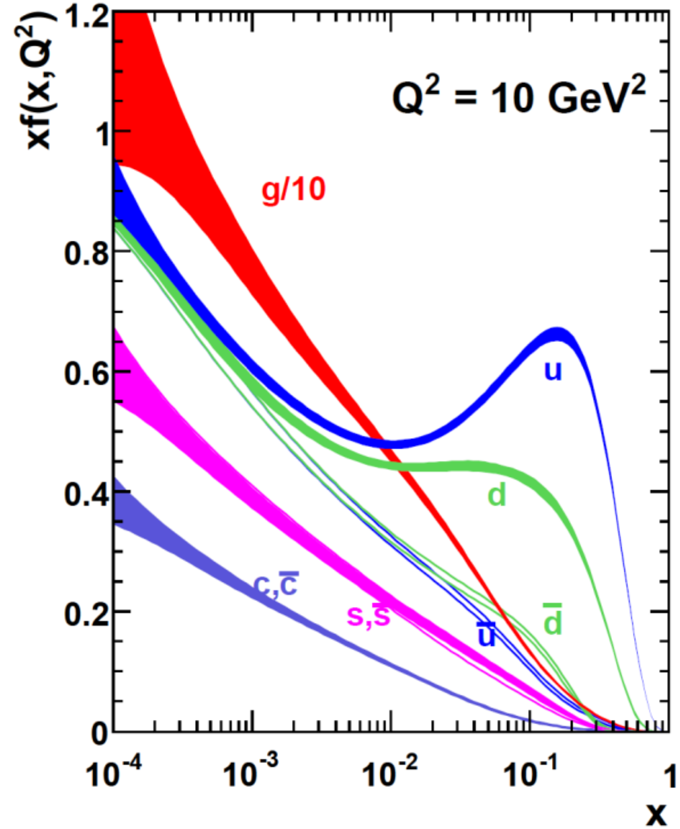


Deep Inelastic Lepton-Nucleon Scattering

----- *A powerful tool*



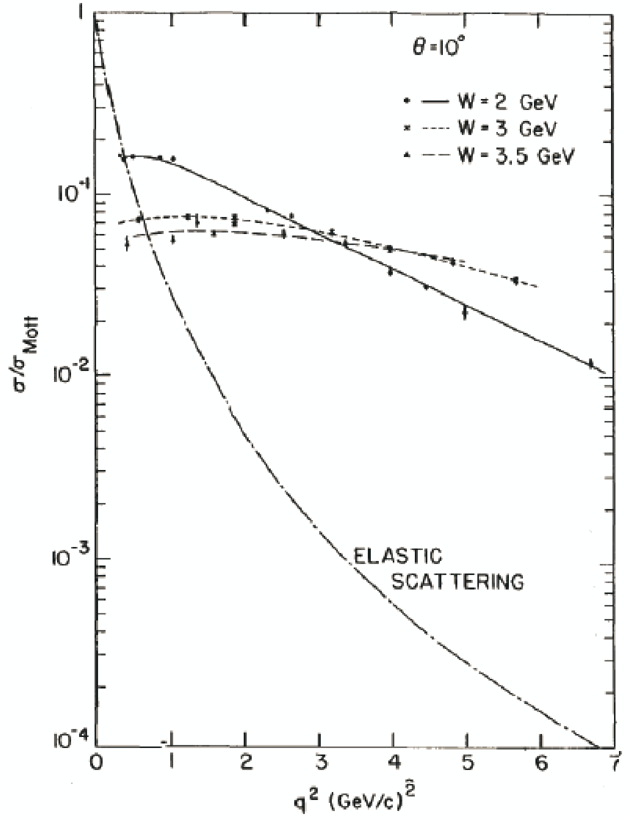




x=1维纵向动量

The Early MIT-SLAC Experiment

- - - - - *The weak Q^2 dependence*

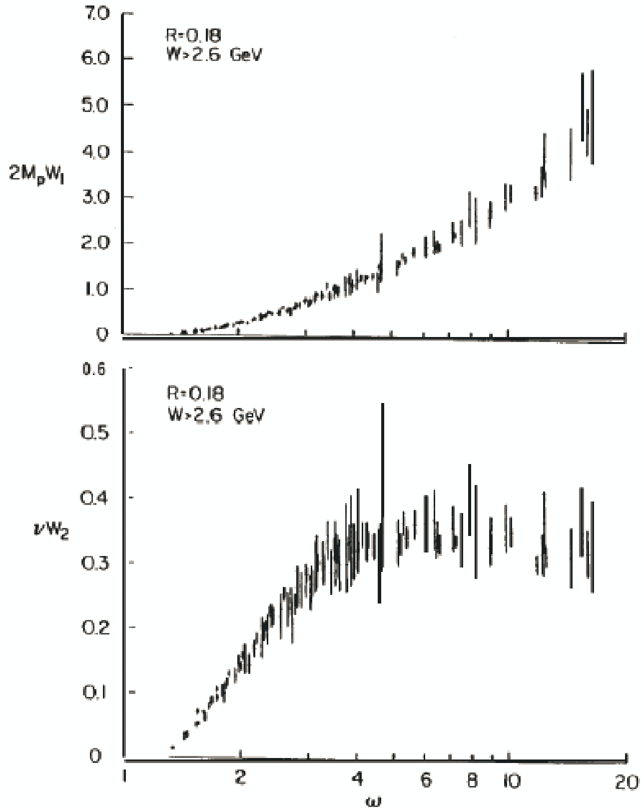


$$Q^2 = 2EE'(1 - \cos\theta)$$

The weak Q^2 dependence of the DIS cross section

The Early MIT-SLAC Experiment

----- *The scaling behaviours of structure functions*



$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\mathrm{d}E'} = \sigma_{\mathrm{mott}}[W_2(\nu,Q^2) + 2W_1(\nu,Q^2)\tan^2\frac{\theta}{2}]$$

$$2MW_1(\nu,Q^2) \to F_1(\omega),$$

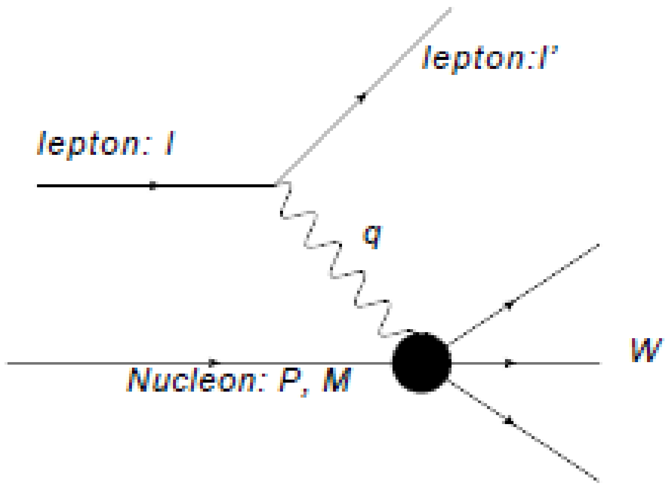
$$\nu W_2(\nu,Q^2) \to F_2(\omega).$$

$$\omega = 2M\nu/Q^2$$

The scaling behaviors of the structure functions νW_2 and $2MW_2$.

The Parton Model

- - - - - *Established in the infinite moment frame of the nucleon*



the ith parton has momentum $k_i = x_i P$

$$F_2(\nu, Q^2) = \nu W_2(\nu, Q^2)$$

$$W_2^{(i)}(\nu, Q^2) = e_i^2 \delta(\nu - Q^2/2Mx_i) = \frac{e_i^2 x_i}{\nu} \delta(x_i - Q^2/2M\nu)$$

$$F_2(x) = \sum_i e_i^2 \int_0^1 x\, f_i(x) \delta(x - Q^2/2M\nu) \mathrm{d}x$$

$$x_B = Q^2/2M\nu$$

The Parton Model

- - - - - *Identify the partons as quarks (1)*

The Callan-Gross ratio $R = \frac{W_2}{W_1}[1 + \frac{\nu^2}{Q^2}] - 1$

spin 1/2

rule out pions as constituents

quarks or bare protons

The Parton Model

----- *Identify the partons as quarks (2)*

$$\frac{1/2\int[F_2^{ep}(x)+F_2^{en}(x)]\mathrm{d}x}{1/2\int[F_2^{\nu p}(x)+F_2^{\nu n}(x)]\mathrm{d}x} = \frac{e_u^2+e_d^2}{2}$$

$18/5=3.6$

$$1/2\int[F_2^{ep}(x)+F_2^{en}(x)]\mathrm{d}x = \frac{e_u^2+e_d^2}{2}\int_0^1 x[u_p(x)+\overline{u}_p^2+d_p(x)+\overline{d}_p^2]\mathrm{d}x = 0.14\pm 0.005;$$

3.4 ± 0.7

$$1/2\int[F_2^{\nu p}(x)+F_2^{\nu n}(x)]\mathrm{d}x = \int_0^1 x[u_p(x)+\overline{u}_p^2+d_p(x)+\overline{d}_p^2]\mathrm{d}x = 0.49\pm 0.07.$$

The Parton Model

- - - - - Identify the partons as quarks (3)

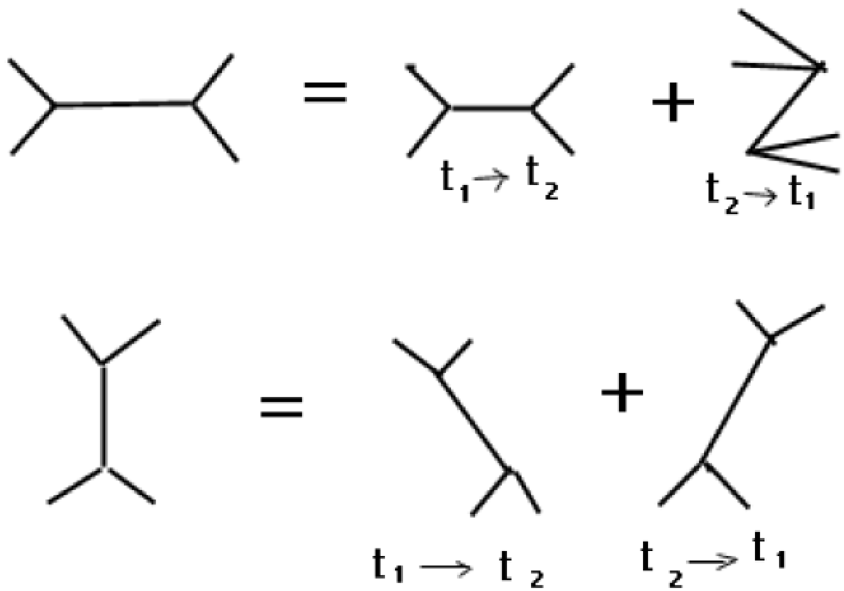
The Gross-Llewellyn Smith sum rule $S_{\mathrm{GLS}} = \frac{1}{2}\int_0^1 [F_3^{\nu N} + F_3^{\overline{\nu} N}]\mathrm{d}x = 3$

$$S_{\mathrm{GLS}} = (\text{number of quarks}) - (\text{number of antiquarks})$$

$$3.2 \pm 0.6;$$

The Theoretical Foundations of the Parton Model

- - - - - ***Infinite momentum frame technique by Weinberg in 1966***



Time-Ordered Perturbative Theory

Dynamics at infinite momentum

Steven Weinberg, Phys. Rev. 150 (1966) 1313-1318

The Theoretical Foundations of the Parton Model

----- *Infinite momentum frame technique by Weinberg in 1966*

1. 无穷大动量坐标系方法的优点

(1). 计算简化

在无穷大动量坐标系方法中，由于很多图的贡献为零，仅有数量较少的图有贡献，因而计算大大简化。

(2). 时间膨胀（Dilation）效应

在一般坐标系中去探测一个静止的强子系统时，由于强子的特征长度约为10^{-13}cm，探针的速度即使是光速，探得强子结构的时间也得有$10/c \sim 10^{-23}$秒才行。而10^{-23}秒是强子内部强相互作用的典型时间尺度，因此，这种探测很难得到强子内部细致结构的信息。

The Theoretical Foundations of the Parton Model

----- *Infinite momentum frame technique by Weinberg in 1966*

这种在一般坐标系中遇到的困难可用无穷大坐标系方法去解决。在强子以无穷大动量运动的坐标系中，由于Einstein时间膨胀效应，强子内部的相互作用可以减缓，因而其相互作用时间远远大于10^{-23}秒。此时如在强子运动的垂直方向用探针探测此强子系统，由于此方向上无时间膨胀效应，探测时间不会变化。因此，可以使探测时间远远小于强子内部的强相互作用时间，从而比较清楚地获得强子内部结构的信息。

这说明，无穷大动量坐标系方法在揭示强子结构方面，具有其它一般方法难以替代的优点。

The Theoretical Foundations of the Parton Model

----- *Infinite momentum frame technique by Weinberg in 1966*

(3). 真空效应

在一般编时微扰理论和Feynman-Dyson微扰理论中，由于允许在真空态中产生正负粒子对，因此裸真空态$|0\rangle$不是全哈密顿量的本征态。这样，原则上就不可用Fock态$a^+|0\rangle$（其中a^+是产生算符）来描述物理粒子。

在无穷大动量坐标系方法中，由于只有所有的η都大于零的图贡献，且不允许从真空中产生正负粒子对，因此，$|0\rangle$是全哈密顿量的本征态。这样，就可用Fock态来描述物理粒子。

The Theoretical Foundations of the Parton Model

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The Theoretical Foundations of the Parton Model

----- *The Light-Front Dynamics by Dirac in 1949*

P.A. M. Dirac, Forms of relativistic dynamics,
Reviews of Modern Physics 21 (3) (1949) 392-399.

Dirac的light-front dynamics（光前动力学）的原创思想来源于1949年狄拉克发表的论文《Reviews of Modern Physics》中的“Forms of Relativistic Dynamics“部分。

在该论文中，Dirac提出了将狭义相对论与动力学的哈密顿表述相结合的多种方式，讨论了三种不同的形式：瞬间形式（instant form）、点形式（point form）和前形式（front form），其中前形式即光前动力学的基础 。

The Theoretical Foundations of the Parton Model

----- *The Light-Front Dynamics by Dirac in 1949*

1. 瞬时动力学（Instant-Form Dynamics）

在以$q_\mu = (t, x, y, z)$作为时空变量时，时空平移生成元为$P_\mu = (H, \mathbf{P})$，转动生成元为$\mathbf{J}$，罗仑兹加速（Boost）生成元为$\mathbf{K}$. 令

$$J_l = -\frac{1}{2}\varepsilon_{lmn}M^{mn}, \; K_l = M_{l0}, \tag{3.30}$$

则有

$$\begin{cases} [P^\mu, P^\nu] = 0; \\ [M^{\mu\nu}, M^{\rho\sigma}] = \mathrm{i}\,(M^{\mu\rho}g^{\nu\sigma} + M^{\nu\sigma}g^{\mu\rho} - M^{\nu\rho}g^{\mu\sigma} - M^{\mu\sigma}g^{\nu\rho}); \\ [M^{\mu\nu}, P^\sigma] = -\mathrm{i}\,[P^\mu g^{\nu\sigma} - P^\nu g^{\mu\sigma}]\,, \end{cases} \tag{3.31}$$

其中生成元P^μ，$M^{\mu\nu}$构成彭加勒群的十个生成元，式(3.31)即为此群的李代数。

The Theoretical Foundations of the Parton Model

----- *The Light-Front Dynamics by Dirac in 1949*

在上述生成元中，使$t=0$超面不变子群的生成元有六个：

$$\mathbf{P},\ \mathbf{J}. \tag{3.32}$$

它们称为运动学生成元。其余生成元H, $\mathbf{K}$则使$t=0$的超面产生移动，称为动力学生成元。

对单个粒子的动力学系统，其生成元为

$$\begin{cases} P_r = p_r, & M_{rs} = q_r p_s - q_s p_r, \\ P_0 = (p_s p_s + m^2)^{1/2}, & M_{r0} = q_r (p_s p_s + m^2)^{1/2}. \end{cases} \tag{3.33}$$

对多个粒子的动力学系统，其生成元为

$$\begin{cases} P_r = \sum p_r, & M_{rs} = \sum (q_r p_s - q_s p_r), \\ P_0 = \sum (p_s p_s + m^2)^{1/2} + V, & M_{r0} = \sum q_r (p_s p_s + m^2)^{1/2} + V_r. \end{cases} \tag{3.34}$$

其中V和V_r为相互作用项，它们的选取应使生成元(3.34)满足彭加勒群李代数条件，即式(3.31)

The Theoretical Foundations of the Parton Model

----- *The Light-Front Dynamics by Dirac in 1949*

2. 点式动力学（Point-Form Dynamics）

选取使超面

$$q^{\mu}q_{\mu}=\kappa^{2}, \text{ 其中} q_{0}>0, \tag{3.35}$$

不变的子群作为运动学子群，则运动学生成元有六个，为

$$M_{\mu\nu}=\sum(q_{\mu}p_{\nu}-q_{\nu}p_{\mu}); \tag{3.36}$$

使超面(3.35)移动的动力学生成元有四个，为

$$P_{\mu}=\sum\{p_{\mu}+q_{\mu}B(p^{\sigma}p_{\sigma}-m^{2})\}+V_{\mu}, \tag{3.37}$$

其中B满足条件

$$1+2p^{\mu}q_{\mu}B+q^{\mu}q_{\mu}B(p^{\sigma}p_{\sigma}-m^{2})=0. \tag{3.38}$$

The Theoretical Foundations of the Parton Model

---- *The Light-Front Dynamics by Dirac in 1949*

3. 光前动力学（Front-Form Dynamics）

以$q^\mu = (t-z, x, y, t+z)$作为广义的时空变量，其中$\tau = t-z$为广义时间，$Z = t+z$为广义空间z分量。选取使$\tau = t - z = 0$超面不变子群作为运动学子群，则单粒子系统的生成元为

$$\text{运动学}\left\{\begin{array}{l} P_i = p_i,\ P_- = p_-, \\ M_{12} = q_1 p_2 - q_2 p_1,\ M_{i-} = q_i p_-,\ M_{+-} = q_+ p_-; \end{array}\right. \quad i = 1,2 \quad (3.39)$$

$$\text{动力学}\left\{\begin{array}{l} P_+ = (p_1^2 + p_2^2 + m^2)/p_-, \\ M_{i+} = \{q_i(p_1^2 + p_2^2 + m^2)/p_- - q_+ p_i\}. \end{array}\right. \quad i = 1,2 \quad (3.40)$$

其中$a^\pm = a_0 \pm a_3$. 多粒子系统的运动学生成元为单粒子之和，而动力学生成元为

$$\left\{\begin{array}{l} P_+ = \sum(p_1^2 + p_2^2 + m^2)/p_- + V, \\ M_{i+} = \sum \{q_i(p_1^2 + p_2^2 + m^2)/p_- - q_+ p_i\} + V_i. \end{array}\right. \quad i = 1,2 \quad (3.41)$$

我们注意到，此时有七个运动学生成元和三个动力学生成元。

The Theoretical Foundations of the Parton Model

---- *The Light-Front Dynamics by Dirac in 1949*

在选取$\tau = t + \mathbf{x}\cdot\mathbf{n}$的一般情况下，定义$a^{\pm} = a_0 \pm \mathbf{n}\cdot\mathbf{a}$，则运动学生成元为$\mathbf{P}_{\perp}$, P^{+}, J_3, K_3, $\mathbf{E} = \mathbf{K}_{\perp} + \mathbf{n}\times\mathbf{J}$, 共七个；动力学生成元为$P^{-}$, $\mathbf{F} = \mathbf{K}_{\perp} - \mathbf{n}\times\mathbf{J}$, 共三个。

这种动力学形式称为光前动力学（Front-Form Dynamics or Light-Front Dynamics）.

The Theoretical Foundations of the Parton Model

---- *The connection between IMF and Light-Front Dynamics*

一般坐标系的变量为

$$\left\{\begin{array}{l} x, y, z, t; \\ P_x, P_y, P_z, P_t. \end{array}\right. \tag{3.42}$$

无穷大动量坐标系的变量为

$$\left\{\begin{array}{l} x', y', z', t'; \\ P'_x, P'_y, P'_z, P'_t. \end{array}\right. \tag{3.43}$$

The Theoretical Foundations of the Parton Model

---- *The connection between IMF and Light-Front Dynamics*

设此无穷大动量坐标系以速度V沿一般坐标系z方向运动，则

$$\begin{cases} x' = x, \\ y' = y, \\ z' = (z + Vt)/(1 - V^2)^{1/2}, \\ t' = (t + Vz)/(1 - V^2)^{1/2}; \end{cases} \tag{3.44}$$

$$\begin{cases} P'_x = P_x, \\ P'_y = P_y, \\ P'_z = (P_z + VP_t)/(1 - V^2)^{1/2}, \\ P'_t = (P_t + VP_z)/(1 - V^2)^{1/2}. \end{cases} \tag{3.45}$$

The Theoretical Foundations of the Parton Model

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The Theoretical Foundations of the Parton Model

---- *The connection between IMF and Light-Front Dynamics*

定义$\alpha = P_z'(1-V^2)^{1/2}$，当$V \to 1$时，

$$\alpha = P_t + P_z. \tag{3.46}$$

所以，在无穷大动量坐标系中可用$k_x = P_x'$, $k_y = P_y'$, $\alpha \to P_z'$作为动量变量。其中$P_t + P_z$在一般坐标系中的物理意义为$t-z$方向上的生成元。

The Theoretical Foundations of the Parton Model

---- *The connection between IMF and Light-Front Dynamics*

	无穷大动量坐标系中生成元	一般坐标系中生成元
运动学部分	P'_x	P_x
	P'_y	P_y
	P'_z	$\alpha = P_t + P_z$
	J'_z	J_z
	J'_x	$K_x + J_y$
	J'_y	$K_y - J_x$
	K'_z	K_z
	对应$t' = 0$超面	对应$t + z = 0$超面
动力学部分	P'_t	$H = P_t - P_z$
	K'_x	$K_x - J_y$
	K'_y	$K_y + J_x$

其中无穷大动量坐标系中的运动学生成元对应于$t' = 0$的超面，而一般坐标系中的运动学生成元对应于$t + z = 0$的超面。

由以上的讨论，我们得出如下的结论：

无穷大动量坐标系的瞬时动力学等效于一般坐标系的光前动力学。

The Theoretical Foundations of the Parton Model

- - - - *Derivation of the parton model from light-cone formalism*

Asymptotic Sum Rules at Infinite Momentum

J.D. Bjorken (SLAC) (Sep, 1968)

Published in: *Phys.Rev.* 179 (1969) 1547-1553

Inelastic Electron Proton and gamma Proton Scattering, and the Structure of the Nucleon

J.D. Bjorken (SLAC), Emmanuel A. Paschos (SLAC) (Apr, 1969)

Published in: *Phys.Rev.* 185 (1969) 1975-1982

Very high-energy collisions of hadrons

Richard P. Feynman (Caltech) (1969)

Published in: *Phys.Rev.Lett.* 23 (1969) 1415-1417

Photon-hadron interactions

R.P. Feynman (Caltech, Kellogg Lab) (1973)

Modification of Impulse Approximation and Scaling Variables

Bo-Qiang Ma (Peking U.) (1986)

Published in: *Phys.Lett.B* 176 (1986) 179-184

New scaling variable from light cone perturbation theory

Bo-Qiang Ma (CCAST World Lab, Beijing and Peking U.), Ji Sun (Peking U.) (Dec 17, 1990)

Published in: *Int.J.Mod.Phys.A* 6 (1991) 345-364

The Theoretical Foundations of the Parton Model

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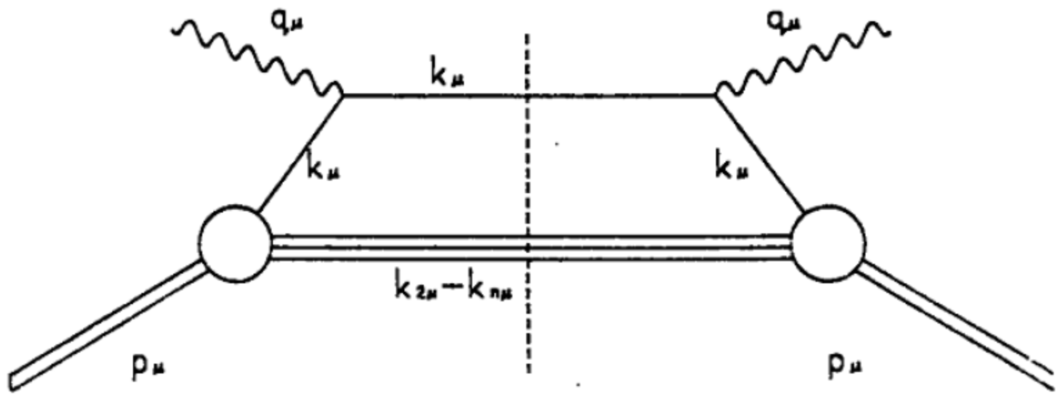


图 8 轻子核子深度非弹散射强子张量$W_{\mu\nu}$的计算

$$W_{\mu\nu}(q,p) = \frac{1}{4\pi}\sum_X \langle P|\, J_\mu(0)\,|X\rangle\,\langle X|\, J_\nu(0)\,|P\rangle\,(2\pi)^4\delta^4(p+q-p_X)$$

$$W_{\mu\nu}(q,p) = \frac{1}{4\pi}\int \mathrm{d}^4x\ \exp\left(\mathrm{i}\,q\cdot x\right)\langle P|\, J_\mu(x)J_\nu(0)\,|P\rangle$$

The Theoretical Foundations of the Parton Model

---- *Derivation of the parton model from light-cone formalism*

$$\begin{cases} p_\mu = (p^+, M^2/p^+, \mathbf{0}_\perp) \\ q_\mu = (0, 2M\nu/p^+, \mathbf{q}_\perp), \end{cases} \qquad q^2 = -Q^2,\ \ p\cdot q = M\nu$$

$$k_{i\mu} = (k_i^+, (\mathbf{k}_{i\perp}^2 + m^2)/k_i^+, \mathbf{k}_{i\perp}) = (x_i p^+, (\mathbf{k}_{i\perp}^2 + m^2)/x_i p^+, \mathbf{k}_{i\perp}).$$

$$\psi_n(\underline{p}; \underline{k}_1, \underline{k}_2 ... \underline{k}_n) 16\pi^3 \delta^2(\mathrm{p}_\perp - \sum_{i=1}^{n} \mathrm{k}_{i\perp}) \delta(1 - \sum_{i=1}^{n} x_i)$$

The Theoretical Foundations of the Parton Model

---- *Derivation of the parton model from light-cone formalism*

$$
\begin{aligned}
W_{\mu\nu} = \frac{1}{4\pi} \quad & \int 16\pi^3 \delta^2(\mathrm{p}_\perp - \mathrm{k}_\perp - \textstyle\sum_{i=2}^{n} \mathrm{k}_{i\perp})\delta(1 - x - \sum_{i=2}^{n} x_i)\frac{\mathrm{d}^2\mathbf{k}_\perp \mathrm{d}k^+}{16\pi^3 k^+} \\
& 16\pi^3 \delta^2(\mathrm{p}_\perp - \mathrm{k}''_\perp - \textstyle\sum_{i=2}^{n} \mathrm{k}_{i\perp})\delta(1 - x'' - \sum_{i=2}^{n} x_i)\frac{\mathrm{d}^2\mathbf{k}''_\perp \mathrm{d}k''^+}{16\pi^3 k''^+} \\
& \psi_n(\underline{p}; \underline{k}, \underline{k}_2 \cdots \underline{k}_n)\psi_n^*(\underline{p}; \underline{k}'', \underline{k}_2 \cdots \underline{k}_n)\frac{\mathrm{d}^2\mathbf{k}'_\perp \mathrm{d}k'^+}{16\pi^3 k'^+} \textstyle\prod_{i=2}^{n} \frac{\mathrm{d}^2\mathbf{k}_{i\perp} \mathrm{d}k_i^+}{16\pi^3 k_i^+} \\
& (2\pi)^4 \delta^4(k' + \textstyle\sum_{i=2}^{n} k_i - p - q) \langle k| J_\mu(0) |k'\rangle \langle k'| J_\nu(0) |k\rangle \, .
\end{aligned}
$$

$$
W_{\mu\nu} = \int \frac{\mathrm{d}^2\mathbf{k}_\perp \mathrm{d}k^+}{16\pi^3 k^+} \frac{\rho(\underline{k})}{x} w_{\mu\nu}(k, k')
$$

$$
\rho(\underline{k}) = \int \prod_{i=2}^{n} \frac{\mathrm{d}^2\mathbf{k}_{i\perp} \mathrm{d}k_i^+}{16\pi^3 k_i^+} 16\pi^3 \delta^2(\mathrm{p}_\perp - \mathrm{k}_\perp - \sum_{i=2}^{n} \mathrm{k}_{i\perp})\delta(1 - x - \sum_{i=2}^{n} x_i)|\psi_n(\underline{p}; \underline{k}, \underline{k}_2 \ldots \underline{k}_n)|^2
$$

$$
\begin{aligned}
w_{\mu\nu}(k, k') \quad &= \quad \frac{1}{4\pi} \int \frac{\mathrm{d}^2\mathbf{k}'_\perp \mathrm{d}k'^+}{16\pi^3 k'^+} (2\pi)^4 \delta^3(\underline{k}' - \underline{k} - \underline{q})\delta(k'^- + \textstyle\sum_{i=2}^{n} k_i^- - p^- - q^-) \\
& \quad \langle k| J_\mu(0) |k'\rangle \langle k'| J_\nu(0) |k\rangle \\
&= \quad 2e_i^2 [k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k \cdot k' - m^2)] \frac{\delta(k'^- + \sum_{i=2}^{n} k_i^- - p^- - q^-)}{k'^+}.
\end{aligned}
$$

The Theoretical Foundations of the Parton Model

---- *Derivation of the parton model from light-cone formalism*

$$x_p \to \begin{cases} \text{Weizmann变量} & x_w = \frac{Q^2+m^2}{2M\nu+M^2} & \text{忽略}\mathbf{k}_\perp, \lambda \\ \text{Bloom}-\text{Gilman变量} & x' = \frac{Q^2}{2M\nu+M^2} & \text{忽略}\mathbf{k}_\perp, \lambda, m \\ \text{Bjorken变量} & x = \frac{Q^2}{2M\nu} & \text{忽略}\mathbf{k}_\perp, \lambda, m, M \end{cases} \tag{3.96}$$

在Bjorken极限$\nu \to \infty$, $Q^2 \to \infty$和$x = Q^2/2M\nu$为$0 \to 1$ 区间的固定值情况下，可得

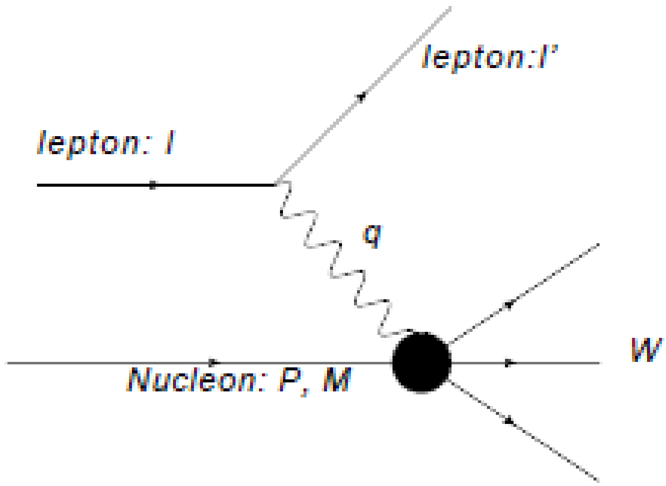
$$F_2(\nu, Q^2) = F_2(x) = \sum_n \sum_i e_i^2 x f_n(x), \tag{3.97}$$

$f_n(x)$为核子的夸克动量分布函数

$$f_n(x) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp \mathrm{d}k^+}{16\pi^3} \rho(\underline{k}).$$

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*



$$k_\mu = (E, \mathbf{k}),$$
$$k'_\mu = (E', \mathbf{k}'),$$

$$q_\mu = (\nu, \mathbf{q}) = (E' - E, \mathbf{k}' - \mathbf{k})$$

$$P_\mu = (M, \mathbf{0})$$

轻子4动量转移平方 $\rightarrow$ $Q^2 = -q^2,$

Bjorken变量 $\rightarrow$ $x = -q^2/2p \cdot q = Q^2/2M\nu,$

轻子转移给靶核的能量比 $\rightarrow$ $y = p \cdot q/p \cdot k = \nu/E,$

相对论不变量 $\rightarrow$ $S = 2p \cdot k = 2ME = Q^2/xy.$

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

带电轻子核子深度非弹散射公式:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}y} = \frac{4\pi\alpha^2 S}{Q^4}\left\{xy^2 F_1(x,Q^2) + \left[(1-y) - \frac{xyM}{2E}\right] F_2(x,Q^2)\right\}. \tag{4.5}$$

在Bjorken极限下，$E \to \infty$，而x和y都为$0 \to 1$区间上的固定值，因此项$\frac{xyM}{2E}$可以忽略。

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

两种结构函数形式间关系:

$$
\begin{aligned}
F_1(x,Q^2) &= M\,W_1(x,Q^2),\\
F_2(x,Q^2) &= \nu\,W_2(x,Q^2).
\end{aligned}
$$

核子吸收纵向光子和横向光子截面: σ_L和σ_T

$$
\begin{aligned}
W_1(x,Q^2) &= \tfrac{K}{4\pi^2\alpha}\sigma_T,\\
W_2(x,Q^2) &= \tfrac{K}{4\pi^2\alpha}(\sigma_T+\sigma_L)\tfrac{Q^2}{Q^2+\nu^2},
\end{aligned}
$$

其中K为入射光子流因子。定义R值为纵向光子与横向光子截面之比

$$
R=\frac{\sigma_L}{\sigma_L}=\frac{W_2}{W_1}(1+\frac{\nu^2}{Q^2})-1.
$$

在Bjorken极限下，实验上发现$R\to 0$, 由此可得如下关系式。

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

Callan-Gross关系式:

$$F_2(x, Q^2) = 2xF_1(x, Q^2). \tag{4.9}$$

在Bjorken极限下，此式在部分子为自旋1/2的点粒子时成立。此关系式的实验验证也是推断部分子为夸克的一个证据。对R值的精确测量可提供QCD修正和其它高阶效应的信息。

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

结构函数与夸克动量分布函数:

在Bjorken极限下

$$F_2(x) = 2xF_1(x) = \sum_i e_i^2 x f_i(x), \tag{4.10}$$

其中求和表示对所有电荷分别为e_i的部分子求和，包括夸克与反夸克。我们将选用标记$q(x) = f_q(x)$表示q类夸克在核子中的动量分布。

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

质子和中子的结构函数:

$$\frac{1}{x}F_2^p(x) = \frac{4}{9}[u^p(x)+\overline{u}^p(x)] + \frac{1}{9}[d^p(x)+\overline{d}^p(x)] + \frac{1}{9}[s^p(x)+\overline{s}^p(x)] + \cdots , \quad (4.11)$$

$$\frac{1}{x}F_2^n(x) = \frac{4}{9}[u^n(x)+\overline{u}^n(x)] + \frac{1}{9}[d^n(x)+\overline{d}^n(x)] + \frac{1}{9}[s^n(x)+\overline{s}^n(x)] + \cdots . \quad (4.12)$$

假设质子和中子存在同位旋对称性:

$$u^p = d^n = u, \quad \overline{u}^p = \overline{d}^n = \overline{u};$$

$$d^p = u^n = d, \quad \overline{d}^p = \overline{u}^n = \overline{d};$$

$$s^p = s^n = s, \quad \overline{s}^p = \overline{s}^n = \overline{s},$$

$$\frac{1}{x}F_2^p = \frac{4}{9}(u+\overline{u}) + \frac{1}{9}(d+\overline{d}) + \frac{1}{9}(s+\overline{s}) + \cdots ,$$

$$\frac{1}{x}F_2^n = \frac{4}{9}(d+\overline{d}) + \frac{1}{9}(u+\overline{u}) + \frac{1}{9}(s+\overline{s}) + \cdots .$$

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

中微子与核子的带电流交换深度非弹散射:

中微子和反中微子可以通过带电相互作用玻色子$W^{\pm}$和带电轻子的作用顶角，与核子内的夸克交换$W^{\pm}$产生深度非弹散射。其一般公式为

$$\frac{\mathrm{d}^2\sigma^{\nu,\overline{\nu}}}{\mathrm{d}x\,\mathrm{d}y} = \frac{G^2 S}{2\pi}\left\{xy^2 F_1^{\nu,\overline{\nu}}(x,Q^2) + \left[(1-y) - \frac{xyM}{2E}\right] F_2^{\nu,\overline{\nu}}(x,Q^2) \pm y(1-\frac{y}{2})F_3^{\nu,\overline{\nu}}(x,Q^2)\right\}. \tag{4.16}$$

在Bjorken极限下，结构函数可以写成

$$\begin{aligned} F_1^{\nu,\overline{\nu}}(x,Q^2) &= q(x) + \overline{q}(x), \\ F_2^{\nu,\overline{\nu}}(x,Q^2) &= 2x[q(x) + \overline{q}(x)], \\ F_3^{\nu,\overline{\nu}}(x,Q^2) &= 2[q(x) - \overline{q}(x)]. \end{aligned} \tag{4.17}$$

易证，中微子和反中微子截面公式可以写成

$$\frac{\mathrm{d}^2\sigma^{\nu N\to\mu^- X}}{\mathrm{d}x\,\mathrm{d}y} = \frac{G^2 xS}{2\pi}\left[q(x) + (1-y)^2\overline{q}(x)\right], \tag{4.18}$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} N\to\mu^+ X}}{\mathrm{d}x\,\mathrm{d}y} = \frac{G^2 xS}{2\pi}\left[\overline{q}(x) + (1-y)^2 q(x)\right]. \tag{4.19}$$

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

其中何种夸克参与反应，可通过下面的子过程加以判断：

中微子通过交换W^+与夸克作用

$$\nu\ d \to \mu^- u;\ \nu\ d \to \mu^- c;$$

$$\nu\ \overline{u} \to \mu^- \overline{d};\ \nu\ \overline{u} \to \mu^- \overline{s};$$

$$\nu\ s \to \mu^- c;\ \nu\ s \to \mu^- u,$$

反中微子通过交换W^-与夸克作用

$$\overline{\nu}\ u \to \mu^+ d;\ \overline{\nu}\ u \to \mu^+ s;$$

$$\overline{\nu}\ \overline{d} \to \mu^+ \overline{u};\ \overline{\nu}\ \overline{d} \to \mu^+ \overline{c};$$

$$\overline{\nu}\ \overline{s} \to \mu^+ \overline{c};\ \overline{\nu}\ \overline{s} \to \mu^+ \overline{u}.$$

计算上面各种贡献时，还需考虑Cabbibo耦合带来的影响。

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae*

部分子求和规则

核子中q类夸克的数目为

$$N_q = \int_0^1 dx\, q(x).$$

$$\int_0^1 dx[u(x) - \overline{u}(x)] = 2,$$

$$\int_0^1 dx[d(x) - \overline{d}(x)] = 1,$$

$$\int_0^1 dx[s(x) - \overline{s}(x)] = 0,$$

$$\int_0^1 dx[c(x) - \overline{c}(x)] = 0.$$

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae* 部分子求和规则

Gottfried求和规则:

如质子中子间同位旋对称性严格成立，且核子具有u和d夸克海的对称性

$$\overline{u}(x) = \overline{d}(x), \tag{4.27}$$

则有

$$S_G = \int_0^1 \frac{\mathrm{d}x}{x}[F_2^p(x) - F_2^n(x)] = \frac{1}{3}. \tag{4.28}$$

此求和规则由Gottfried提出。Gottfried之和S_G在早期实验中被发现接近1/3, 此结果被看作部分子是夸克的证据之一。但最近的实验测量结果为$S_G = 0.235 \pm 0.026$，此求和规则的破坏，表明核子海中u-d的不对称性或质子中子间同位旋对称性的破坏。

Deep Inelastic Lepton-Nucleon Scattering

----- *Useful formulae* 部分子求和规则

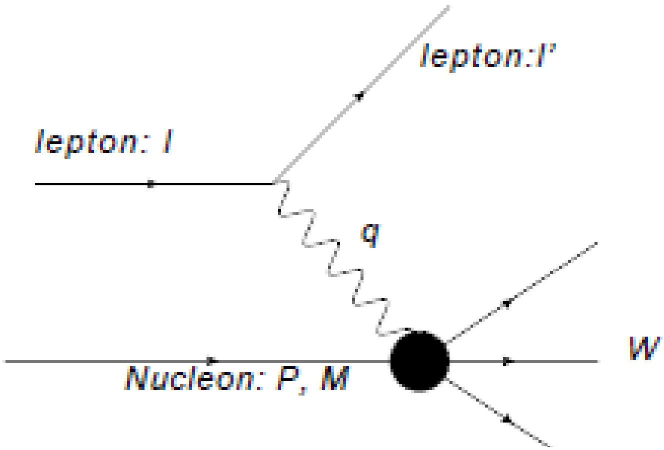
Gross-Llewellyn Smith求和规则:

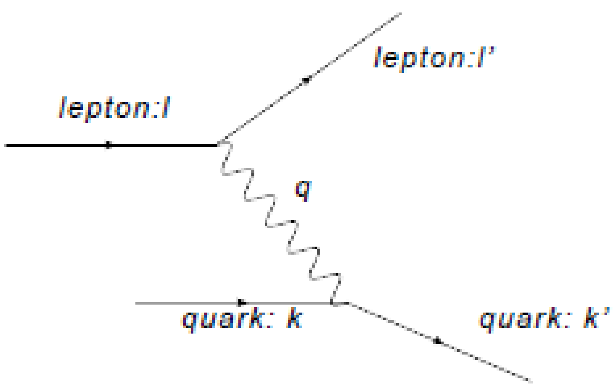
$$\frac{1}{2}\int_0^1 \mathrm{d}x[F_3^{\nu p}(x,Q^2)+F_3^{\overline{\nu}p}(x,Q^2)] \equiv 3[1-\frac{\alpha_{GLS}(Q)}{\pi}], \qquad (4.29)$$

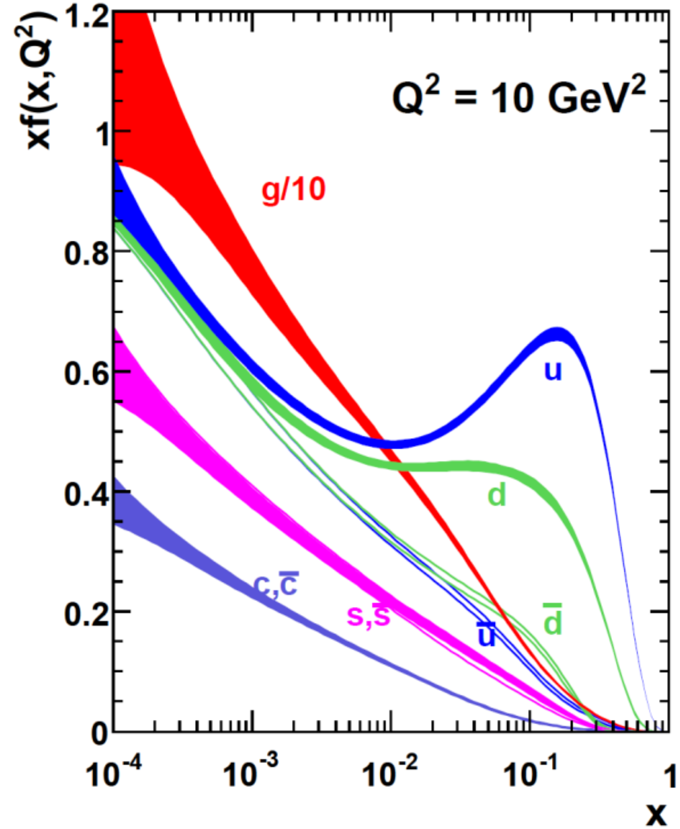
其中$\alpha_{GLS}(Q)$为QCD耦合常数，3表示核子内纯夸克数目（夸克数目减去反夸克数目）。此求和规则在早期被证明是正确的，是部分子为夸克的有力证据。它的精确测量，可对标准模型提供精确检验，并提供核子结构非微扰效应的信息（见，马伯强，高能物理与核物理，1999年23卷922页）。

Deep Inelastic Lepton-Nucleon Scattering

----- *A powerful tool*







x=1维纵向动量

Conclusion

- Parton model is established in the IMF or light-cone formalism.
- The identification of partons with quarks are supported by experimental evidence.
- A proper understanding of nucleon structure relies on the quark-parton model in IMF or light-cone formalism.


谢谢！