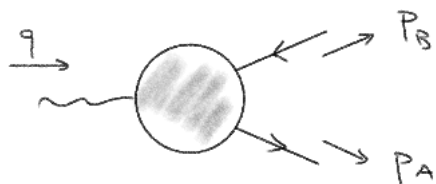


Lecture note for 2025 summer school at ZZU

Outline :

- 1) PSS & reduced graphs
- 2) Power counting
- 3) DIS & structure functions
- 4) Parton model & PDF
- 5) Leading-power regions for DIS
- 6) Renormalization of PDF
- 7) Factorization formula for DIS
- 8) NLO correction

1. Pinch-singular surface & reduced graphs

子过程 $\gamma^* \rightarrow q\bar{q}$ 考虑 $\gamma^* \rightarrow q\bar{q}$ 的 Green 函数可出现于 e^+e^- 湮灭的子图

$$\Gamma^\mu \stackrel{\text{def}}{=} \int d^4x d^4y e^{iP_A \cdot x} e^{iP_B \cdot y} \langle 0 | T \{ \psi(x) \bar{\psi}(y) j^\mu(0) \} | 0 \rangle$$

$$= \langle \Gamma^\mu_{\text{irred}} \times \text{full external propagators} \rangle$$

$$q = P_A + P_B, \quad Q = \sqrt{q^2}$$

目标: $Q^2 \gg P_A^2, P_B^2$ 的渐近行为

在 fixed order perturbation theory (FOPT)

 P_A, P_B on-shell $\Rightarrow$ IR 发散色禁闭会阻止 P_A, P_B on-shell, 并截断 IR 发散但这里的目的是: 联系大 Q^2 行为与 massless 理论中的发散

下面具体分析

$$\text{Frame: } q = Q(1, 0, 0, 0)$$

$$P_A = \frac{Q}{2} \left(1, 0, 0, \sqrt{1 - \frac{4P_A^2}{Q^2}} \right)$$

$$P_B = \frac{Q}{2} \left(1, 0, 0, -\sqrt{1 - \frac{4P_B^2}{Q^2}} \right)$$

Scaling in unit of Q

对于 G_{irred} 中 L -loop 1PI 图

$$G = g^{2L} \int d^n k \, I(k, P_A, P_B; m) + UVCT$$

g : 耦合常数, k : 圈动量, n 维时空

考虑情形: 内动量 k 分量 $\sim Q$, 离壳度 $\sim Q^2$

取重整化标度 $\sim Q$

在大 Q 渐近, massless 近似的修正 $\sim \frac{m}{Q}, \frac{P_A^2}{Q^2}, \frac{P_B^2}{Q^2}$

将远小于高阶 $\alpha_s(Q)$ 修正

上面对于内动量的假定一般而言是错误的,

但是提供了处理一般情形的有用标准

若 k 分量 $\gg Q$, 则由 UV 重整化处理

于是, 只需关注 k 分量 $\lesssim Q$ 的区域

以及离壳度很小的情况: $|l^2| \ll Q^2$

此时, $\frac{m}{Q}$ 的幂次展开失效, 传播子接近在壳 $\Rightarrow$ 对积分可有大贡献

下面系统分析 low virtuality 区域, 引入 scaled momenta:

$$\tilde{P}_A = \frac{P_A}{Q} \longrightarrow \frac{1}{2} (1, 0, 0, 1)$$

$$\tilde{P}_B = \frac{P_B}{Q} \longrightarrow \frac{1}{2} (1, 0, 0, -1)$$

$$\tilde{q} = \frac{q}{Q} \longrightarrow (1, 0, 0, 0)$$

" $\longrightarrow$ ": 代表 $Q \rightarrow \infty$ 极限

类似的, 定义 scaled loop-momentum 和 mass

$$\tilde{k} = \frac{k}{Q}, \quad \tilde{m} = \frac{m}{Q} \rightarrow 0$$

于是 L -loop 图:

$$G = Q^{D(G)} g^{2L} \int d^{nL} \tilde{k} I(\tilde{k}, \tilde{p}_A, \tilde{p}_B; \tilde{m}) + UVCT$$

$D(G)$: 积分的量纲

$$D(G) = nL + \dim I = \dim G - 2L \dim g = -2L \in$$

$$\text{其中 } n = 4 - 2\epsilon, \quad [g] = \epsilon$$

结论: 固定 m , $Q \rightarrow \infty$ 极限 $\Rightarrow$ 固定 Q , $m \rightarrow 0$ 极限

如果 $m \rightarrow 0$ 非奇异, 则可取 $p_A^2 = p_B^2 = m^2 = 0$, 得到大 Q 行为

对于可重整理论, $[g]_{n=4} = 0 \Rightarrow D(G) = 0$

$\Rightarrow$ 所有图 $\sim Q^{D(G)}$ 同样的幂次

Importance of PSS in massless limit

对于 $m \rightarrow 0$ 极限失效情形: 即内线 virtuality $\ll Q^2$

如果积分围道可以变形避开相应极点, 则与 virtuality $\sim Q^2$ 相同

如果积分围道无法变形避开零质量相限极点, 也就是 PSS 情况

若 k 相应传播子不在 $m \rightarrow 0$ 的 PSS

$\Rightarrow$ 积分或变形积分, 分母有非 0 最小值

$\Rightarrow$ 相应 k 积分, 分母最小值 $\sim Q^2$, 前面的 $m \rightarrow 0$ 极限可用

若 k 在 $m \rightarrow 0$ 的 PSS 附近

此时, 分母最小值 $\sim m^2, mQ$, 其随 Q 增长慢于 Q^2

$\Rightarrow$ 相应 $\tilde{\Gamma}$ 的积分, 即 $Q \rightarrow \infty$ 或 $m \rightarrow 0$, 分母最小值达到 0, PSS

$\Rightarrow$ 对于真实情况: 有限 m , 有限 Q

积分动量会靠近 PSS, 且随 Q 增大而趋近

结论: $m \rightarrow 0$ 时的这些 PSS, 给出非 UV 区圈动量积分重要贡献

Location of PSS : Landau criterion

PSS: 出现在在壳传播子动量对应于经典允许的散射过程

在壳粒子可以长程传播, 可以借用时空坐标分析

reduced graphs:

1) 将非 pinched 传播子收缩到一点

2) 每个点标记为一个时空点

$\Rightarrow$ 连接不同时空点的都是 pinched lines, “经典”粒子传播

在 massless 极限下分析 reduced graph 变得简单

此时所有 pinched line 为 lightlike 或 zero momentum

由四动量守恒 $\Rightarrow$ lightlike 动量必然与一个外动量平行

如下归纳 massless on-shell momentum 在 reduced vertex 组合

1) Adding zero momentum 线不改变其余动量

$\Rightarrow$ zero momentum 线可以在任何点.

2) 同向 non zero lightlike momenta 组合成平行的 lightlike momenta

$\Rightarrow$ lightlike momentum 可以 split, recombine 而总动量固定.

3) 不同向 lightlike momenta 组合成 off-shell momentum

$\Rightarrow$ off-shell momentum 线为 reduced vertex 内线

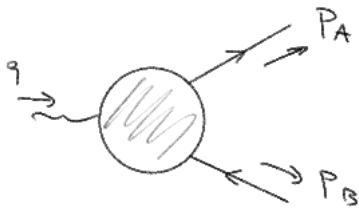
或者过程外线, 如 γ^*

4) 一个 reduced vertex 可以对应于大角度散射

$\Rightarrow$ 对经典' 极限, 末态 lightlike 线远离 reduced vertex

不发生重散射

Example 1: EM current j^μ vertex

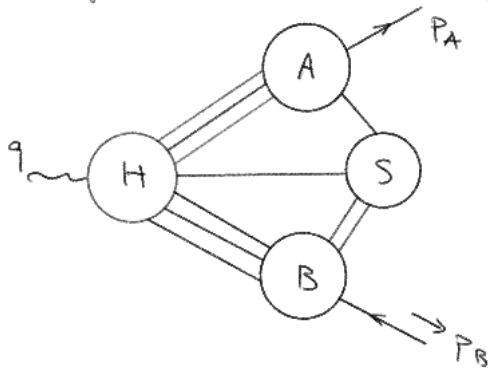


在 $Q \rightarrow \infty$ 或 $m \rightarrow 0$ 极限

$$P_{A,\infty} = \frac{Q}{2} (1, 0, 0, 1)$$

$$P_{B,\infty} = \frac{Q}{2} (1, 0, 0, -1)$$

A typical reduced graph



H 是一个硬散射的 reduced vertex

与 H 相关的有两组 collinear 子图

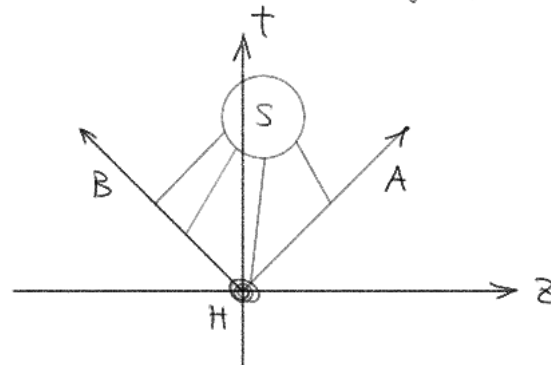
A: all lines 平行于 $P_{A,\infty}$

B: all lines 平行于 $P_{B,\infty}$

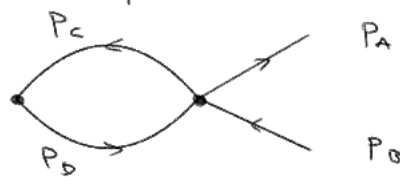
在 A, B 子图内 lightlike 线可以 split 或 recombine

S: soft 子图, 与其相关的都是 zero momenta

在时空图上反映上面 reduced graph 代表的物理散射



关于 rescatter

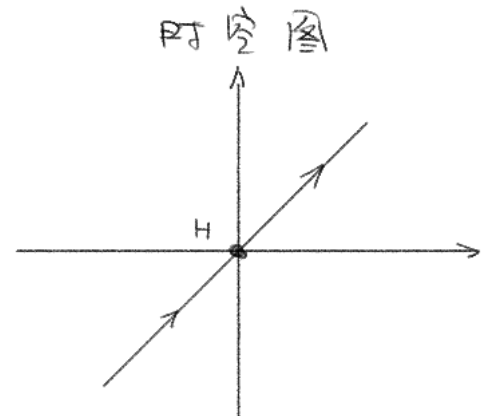
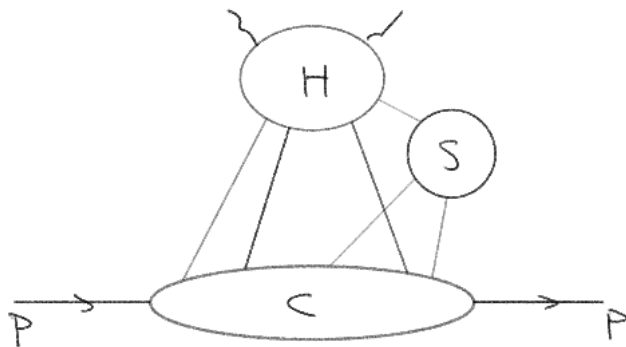


在时空图上容易看出, 相互分离的 P_C, P_D 若要 rescatter

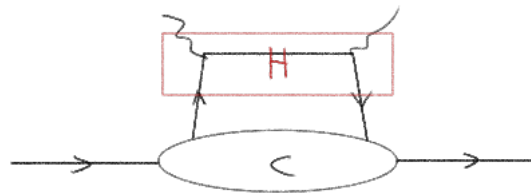
在“经典”意义下要在同一时空点, 因而包含在 H 内

Example 2: DIS

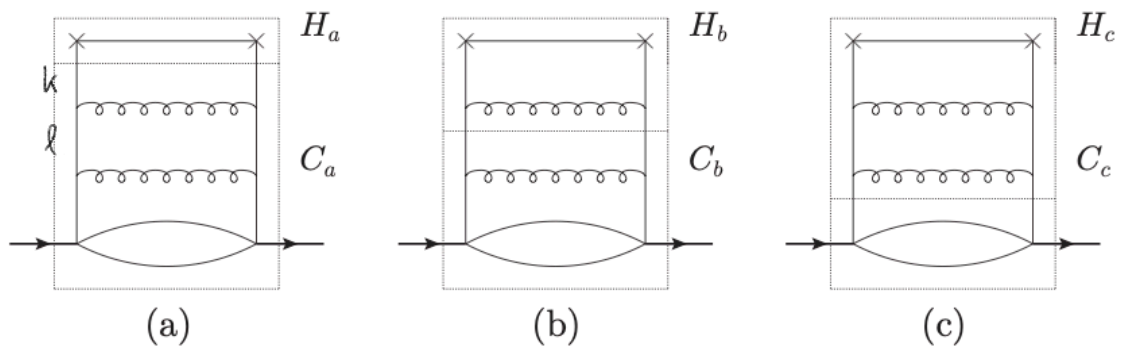
reduced graph (uncut)



parton model:

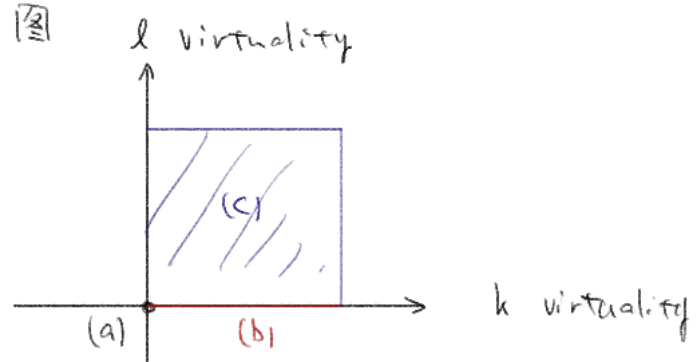


High order correction



一个 Feynman 图可以有不同个 reduce 方式

它们对应的 PSS 如下图



Example 3: Drell-Yan

$$P_A + P_B \longrightarrow (V^* \rightarrow l^+ l^-) + X$$

$$P_A = (P_A^+, \frac{m_A^2}{2P_A^+}, \vec{0}_\perp)$$

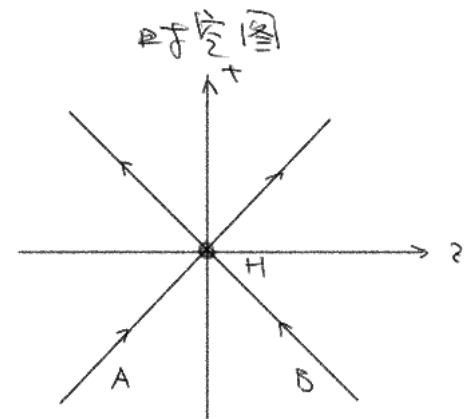
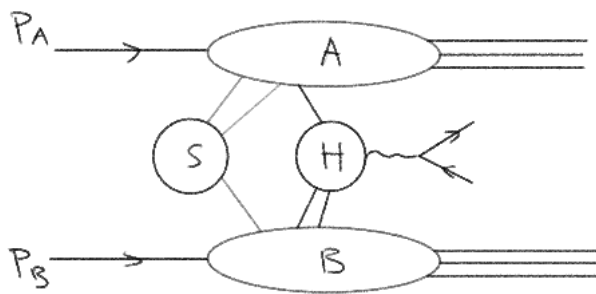
$$P_B = (\frac{m_B^2}{2P_B^-}, P_B^-, \vec{0}_\perp)$$

$$q = (x_A P_A^+ \sqrt{1 + \frac{q_T^2}{Q^2}}, x_B P_B^- \sqrt{1 + \frac{q_T^2}{Q^2}}, \vec{q}_T)$$

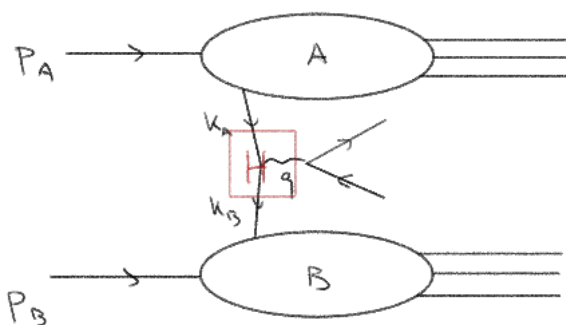
$$\text{scaling variable: } x_A = \frac{Q}{\sqrt{s}} e^y, \quad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

$$y = \frac{1}{2} \ln \frac{q^+ P_B^-}{q^- P_A^+} \quad \text{c.m. frame rapidity of } V^* (l^+ l^-)$$

Amplitude reduced graph:



parton model:



小结: Simplest reduced graph $\Rightarrow$ parton model.

with space-time interpretation

Leading power region

每个反应过程存在 minimum # collinear lines entering hard scatter

e.g. DIS, Drell-Tan

$\Rightarrow$ # collinear line entering H = # external hadrons

在此基础上

1) extra collinear lines to H (except scalar polarized gluon)

2) soft line to H

3) soft subgraph to collinear subgraph (except gluons)

$\Rightarrow$ power suppression

于是, leading power region:

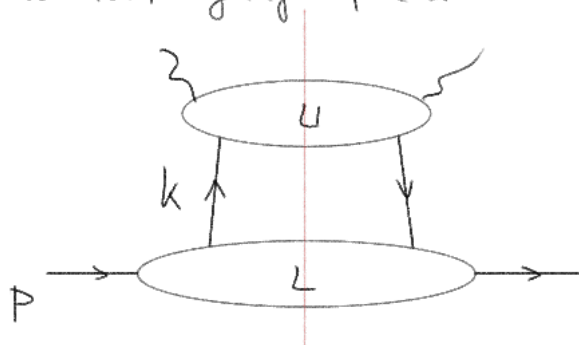
1) soft subgraph only connects to collinear subgraph
and only by gluons

2) collinear subgraph connects to hard subgraph by
minimum number of lines (consistent with desired reaction)

3) Arbitrarily many scalar polarization gluons may
connect collinear subgraph to hard subgraph

leading power region for DIS

without gauge field

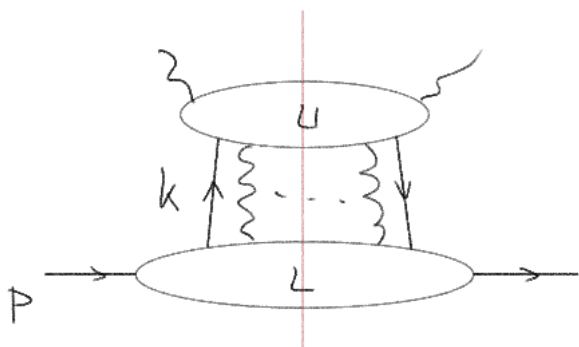


L : k collinear to P

U : hard momentum order Q

$\neq$ final state lines

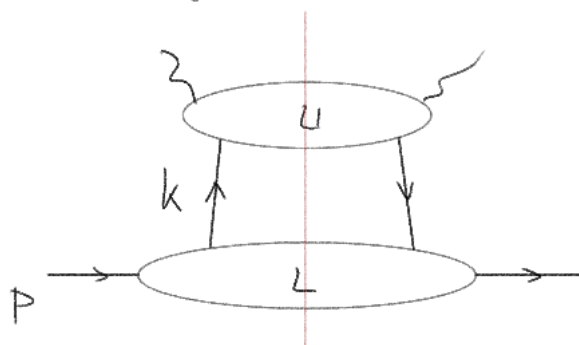
with gauge field



arbitrary many collinear gluons

Elementary treatment of factorization

由 leading power reduced graph 的朴素解释给出因子化形式



假设可将内线动量明确分为两类

Hard momenta 归为 U -subgraph, Virtuality $\sim Q^2$

Collinear momenta 归入 L -subgraph, 包括传播子 k

$(k^+, k^-, k_T) \sim (Q, \frac{m^2}{Q}, m)$ Virtuality not increase with Q

m : hadronic scale

基于这个精神, 将结构函数下

$$F = \sum_{\text{leading region } R} U(R) \otimes L(R) + \text{power suppressed term}$$

$\otimes$ Convolution: k -integral, summation of color, spin, flavor

$$U = U(k, w, w'; q), \quad L = L(k, w, w'; P)$$

w, w' : color, spin, flavor

$$U \otimes L = \sum_{w, w'} \int \frac{d^4 k}{(2\pi)^4} U(k, w, w'; q) L(k, w, w'; P)$$

同一个图可有多几个 leading regions, 求和.

w, w' 中 color, flavor 是相同的 (或对应的)

并且 对于不同 color 的分量是相等的

$$F = \sum_{j, \alpha, \alpha'} \int \frac{d^4 k}{(2\pi)^4} U(k, j, \alpha, \alpha'; q) L(k, j, \alpha, \alpha'; P) + p.s.$$

j : flavor, L 对 color 求和, U 对 color 平均

α, α' : Dirac 自旋指标.

parton approximation:

1) U 中的 k , 忽略 k^-, k_T 小分量, $k \rightarrow \hat{k} = (k^+, 0, \vec{0}_\perp)$

$$\text{记号 } U \stackrel{\leftarrow}{T}$$

2) Dirac indices, project onto leading power

记号 VL , V 代表对 Dirac 自旋投影

$$U \stackrel{\leftarrow}{T} | VL = C_{\text{region}} \otimes f_{\text{region}}$$

$$= \sum_j \int \frac{d\zeta}{\zeta} C_{j, \text{region}}(x, \zeta) f_{j, \text{region}}(\zeta)$$

$$\zeta = \frac{k^+}{P^+} \text{ 无量纲}$$

$\stackrel{\leftarrow}{T} | V$: parton approximation

$$F = \sum_{j, \alpha, \alpha'} \int_x \frac{d\zeta}{\zeta} U(\hat{k}, j, \alpha, \alpha'; q)$$

$$\cdot \sum_c \int \frac{dk^- d^2 k_T}{(2\pi)^4} \zeta P^+ L(k, j, \alpha, \alpha'; P) + p.s.$$

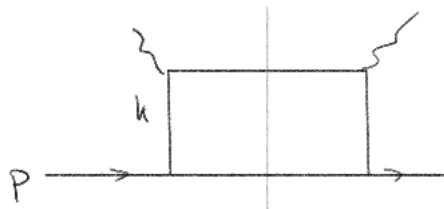
加入自旋投影 (非极化夸克)

$$= \sum_j \int \frac{d\zeta}{\zeta} \text{Tr} \left[U(\hat{k}, j; q) \frac{\hat{k}}{2} \right]$$

$$\sum_c \int \frac{dk^- d^2 k_T}{(2\pi)^4} \text{Tr} \left[\frac{\gamma^+}{2} L(k, j, c; P) \right] + p.s.$$

这样得到了 Γ 的因子化的形式.

存在问题: 能量 region 向 u.l 的分解并不明确



$k^\pm$ 积分由末态 on-shell δ 函数处理. 剩下 k_T 积分

leading power region: $k_T \sim m$

if $k_T \sim Q$. 则上面整个图为 hard graph.

问题: $m \ll k_T \ll Q$? intermediate region

$$\text{其贡献} \sim \int_{m^2}^{Q^2} \frac{dk_T^2}{k_T^2} + \mathcal{O}\left(\frac{m}{k_T}, \frac{k_T}{Q}\right)$$

不是 $\frac{m}{Q}$ power suppressed

Renormalization of PDF

(unpolarized) PDF :

$$f_j = \int \frac{d\bar{k} d^2 k_T}{(2\pi)^4} \text{Tr} \left[\frac{\gamma^+}{2} L(k, j; P) \right]$$

$$= \text{Tr} \frac{\gamma^+}{2} \int \frac{d\bar{k} d^2 k_T}{(2\pi)^4} \quad \text{Diagram: A horizontal line with momentum } P \text{ entering from the left and exiting to the right. A vertical line with momentum } k \text{ enters from the top and exits to the bottom. The two lines meet at a vertex represented by an oval.$$

$$= \int \frac{dW^-}{2\pi} e^{-i\frac{1}{2}P^+ W^-} \langle P | \bar{\psi}_j(0, W^-, \vec{0}_T) \frac{\gamma^+}{2} \psi(0) | P \rangle$$

例如考虑 quark in quark

LO:

NLO:

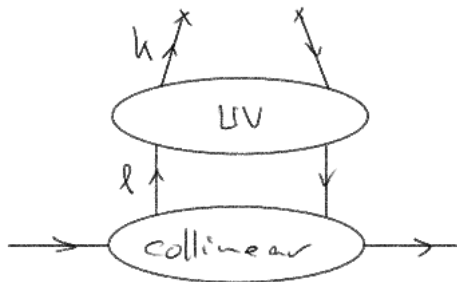
UV divergent.

定义 renormalized PDF

$$f_j(\xi) = \sum_{j'} \int_{\xi}^1 \frac{dz}{z} Z_{jj'}(z) f_{(0)j'}\left(\frac{\xi}{z}\right)$$

Lagrangian 中的 counter term 已经移除了常规的 UV div.

剩下的 UV div 来自于含 PDF 定义顶点的圈积分



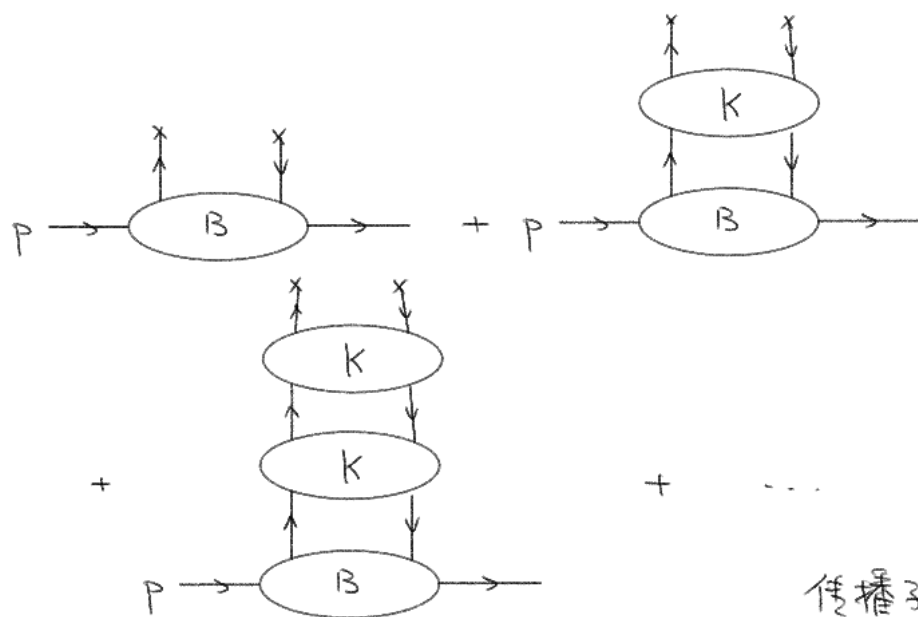
结构式为

$$\int \frac{d\ell^+}{\ell^+} H(\ell^+, k^+) \int d\bar{\ell} d^{2-2\epsilon} \ell_T L(\ell, P)$$

$Z_{jj'}$ 的动量依赖可用参数法

$$\frac{1}{(P-k)^2 - M^2}$$

Collinear $\pm$ UV subgraph 由 2 条线连接, 将其作 ladder 分解



B, K: "2PI"

传播子归入下方的 subgraph

于是 bare PDF 可形式写成

$$f_{(0)j} = z_j V \sum_{n=0}^{\infty} K^n B = z_j V \frac{1}{1-K} B \quad \text{这里乘积以卷积理解}$$

ladder 最上端会于定义 PDF 的顶点 V 收缩

$$V = \delta(k^+ - \frac{3}{2}p^+) \Gamma_{\alpha\alpha'} \delta_{jj'} \delta_{cc'} \quad \Gamma = \frac{\gamma^+}{2} \text{ for unpol}$$

K, B 均由 renormalized field 定义

z_j 来自 bare PDF 中 $\bar{\psi}_{(0)}$ 和 $\psi_{(0)}$ 的重归化

下面做 UV subtraction

- 先做波函数重归化, $\Rightarrow VB$ UV 有限

- VKB UV div 来自于 VK

subtraction, e.g. dim regularization $\epsilon=0$ 的 pole

记号 $VK \overleftarrow{P}$, pole part 不依赖 $\bar{l}, l_T$

$$\Rightarrow VK(1 - \overleftarrow{P})B \text{ UV 有限}$$

- VKB , first subtract UV div in VK

$$\Rightarrow VK(1 - \overleftarrow{P})KB, \text{ UV div.}$$

~~再~~ subtract remaining UV div

$$\Rightarrow VK(1 - \overleftarrow{P})K(1 - \overleftarrow{P})B \quad \text{UV 有限}$$

...

$$f_j = V [1 + K(1 - \overleftarrow{P}) + K(1 - \overleftarrow{P})K(1 - \overleftarrow{P}) + \dots] B$$

$$= V \sum_{n=0}^{\infty} [K(1 - \overleftarrow{P})]^n B$$

$$= V \sum_{n=0}^{\infty} K^n B - V \sum_{n=1}^{\infty} \sum_{n_1=1}^n [K(1 - \overleftarrow{P})]^{n_1-1} K \overleftarrow{P} K^{n-n_1} B$$

$$= V \frac{1}{1-K} B - V \sum_{n_1=1}^{\infty} [K(1 - \overleftarrow{P})]^{n_1-1} K \overleftarrow{P} \sum_{n_2=0}^{\infty} K^{n_2} B$$

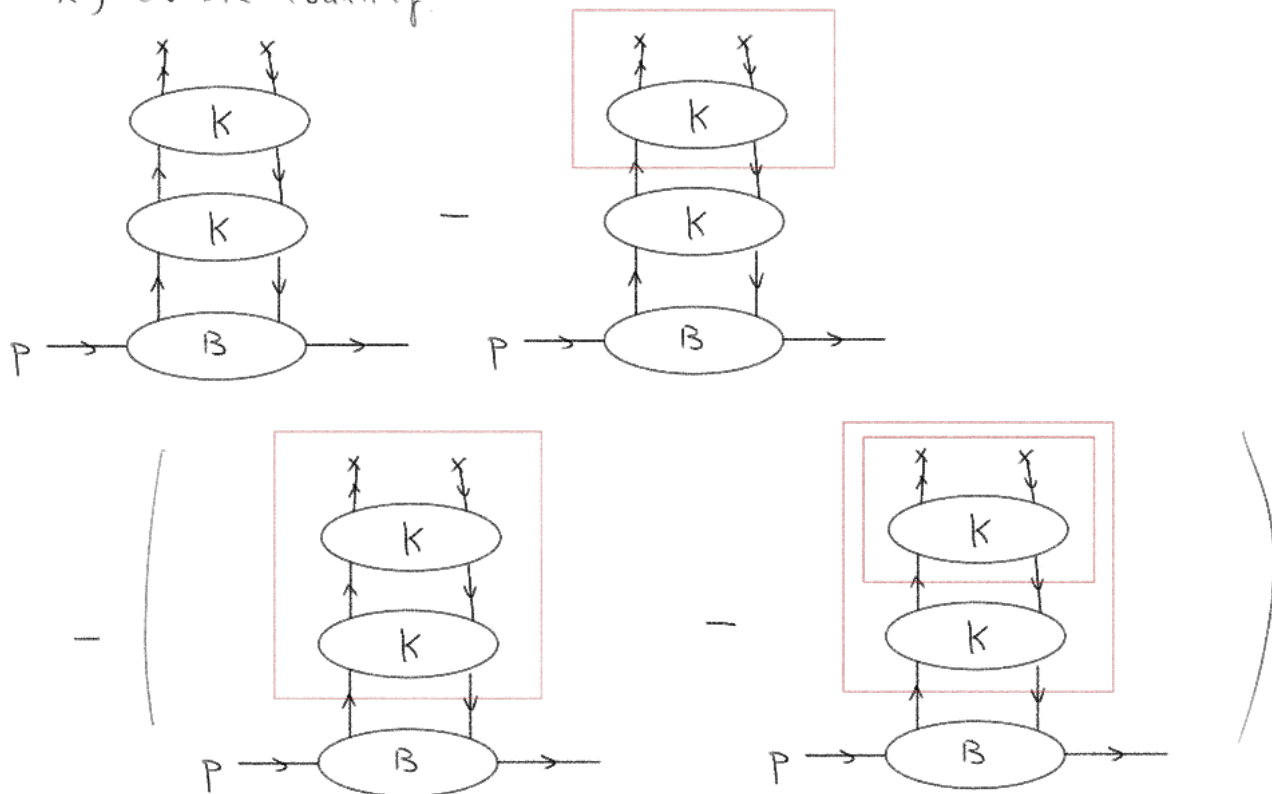
$$= V \frac{1}{1-K} B - V \sum_{n=0}^{\infty} [K(1 - \overleftarrow{P})]^n K \overleftarrow{P} \frac{1}{1-K} B$$

§ bare PDF 重整 $f_{(0)j} = Z_j V \frac{1}{1-K} B$

UK B PDF 重整化 $f_j = Z_{jj'} f_{(0)j'}$ $\frac{dZ}{Z}$ 重整

$$\Rightarrow Z_{jj'} V = \frac{1}{Z_j} [\delta_{jj'} \delta(1-z) - V \sum_{n=0}^{\infty} [K(1 - \overleftarrow{P})]^n K \overleftarrow{P}]$$

关于 double counting.



$$\begin{aligned}
 &= V K^2 B - V K \overleftarrow{P} K B - V K^2 \overleftarrow{P} B + V K \overleftarrow{P} K \overleftarrow{P} B \\
 &= V K (1 - \overleftarrow{P}) K (1 - \overleftarrow{P}) B
 \end{aligned}$$

Renormalization group & DGLAP equations

Renormalized PDF depends on renormalization scale μ

$$\frac{d}{d \ln \mu^2} f_j(\xi, \mu) = \sum_{j'} \int \frac{dz}{z} P_{jj'}(z) f_{j'}\left(\frac{\xi}{z}, \mu\right)$$

$$f_j = Z_{jj'} \otimes f_{(0)j'} \text{ at } \lambda, \quad f_{(0)j} \propto \mu \bar{\alpha} \bar{\pi}$$

$$\Rightarrow \frac{d}{d \ln \mu^2} Z_{jk}(z) = \sum_{j'} \int \frac{dz'}{z'} P_{jj'}(z') Z_{j'k}\left(\frac{z}{z'}\right)$$

$$\Rightarrow P_{jj'} = \frac{d}{d \ln \mu^2} \ln Z_{jj'}$$

Factorized formula

DIS hadronic tensor $W^{\mu\nu}$

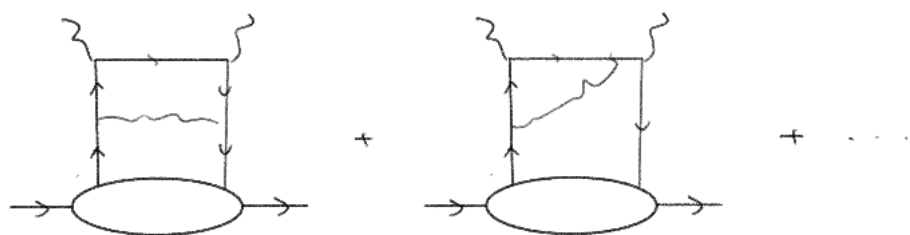
$$W^{\mu\nu} = W_{(LO)}^{\mu\nu} + (W^{\mu\nu} - W_{(LO)}^{\mu\nu})$$

$$= \text{[Diagram: LO parton approximation]} + \left(\text{[Diagram: NLO corrections]} - \text{[Diagram: renormalized PDF]} \right)$$

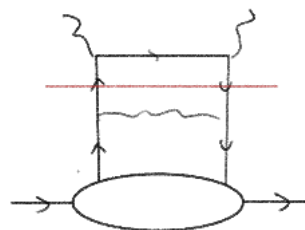
$\overline{\text{PDF}}$ parton approximation
 renormalized PDF

第二项在 LO 不贡献于 leading power region

NLO 的图



Subtract LO 中包含的



$$W_{(NLO)}^{\mu\nu} = \text{[Diagram: NLO corrections]} + \text{[Diagram: NLO corrections]} - \text{[Diagram: LO subtraction]}$$

$$W^{\mu\nu} = W_{(LO)}^{\mu\nu} + W_{(NLO)}^{\mu\nu} + (W^{\mu\nu} - W_{(LO)}^{\mu\nu} - W_{(NLO)}^{\mu\nu})$$

利用 ladder 分解

$$\begin{aligned}
 W = & \text{Diagram 1} + \text{Diagram 2} \\
 & + \text{Diagram 3} + \text{Diagram 4} + \dots \\
 = & A \frac{1}{1-K} B + D
 \end{aligned}$$

↪ power suppressed

Do a parton approximation 和 pole subtraction for PDF

- A 下方的线为 collinear

$$W_{\text{ELO}} = A \overleftarrow{T|V} \frac{1}{1-K(1-\overleftarrow{P})} B$$

- 在 $W - W_{\text{ELO}}$ 中 leading region 考虑 AK 下方线为 collinear

$$W_{\text{ENLO}} = [A(1-\overleftarrow{T|V})K + \text{c.t.}] \overleftarrow{T|V} \frac{1}{1-K(1-\overleftarrow{P})} B$$

如此进行下去, 剩余部分

$$r = W - W_{\text{ELO}} - W_{\text{ENLO}} - \dots$$

$$= \sum_{n=0}^{\infty} A(1-\overleftarrow{T|V}) [K(1-\overleftarrow{T|V})]^n B + D$$

$$= A(1-\overleftarrow{T|V}) \frac{1}{1-K(1-\overleftarrow{T|V})} B + D$$

power suppressed

于是 w 的 leading power 贡献可表示为

$$w-r = A \left[\frac{1}{1-k} - (1-\frac{\epsilon}{T|V}) \frac{1}{1-k(1-\frac{\epsilon}{T|V})} \right] B$$

$$= A \frac{1}{1-(1-\frac{\epsilon}{T|V})k} \underbrace{\frac{\epsilon}{T|V} \frac{1}{1-k}}_{\text{bare PDF}} B$$

$$= C_{(0)} \otimes f_{(0)}$$

$$f = z \otimes f_{(0)}$$

$$= C \otimes f,$$

$$C = C_{(0)} \otimes z^{-1}$$

$$= \sum_j \int_x \frac{d\vec{z}}{z} C_j \left(\frac{Q}{\mu}, \vec{z}, x \right) f_j(\vec{z}, \mu)$$