

Unraveling $K(1690)$ as a pseudoscalar $ud\bar{d}\bar{s}$ tetraquark state



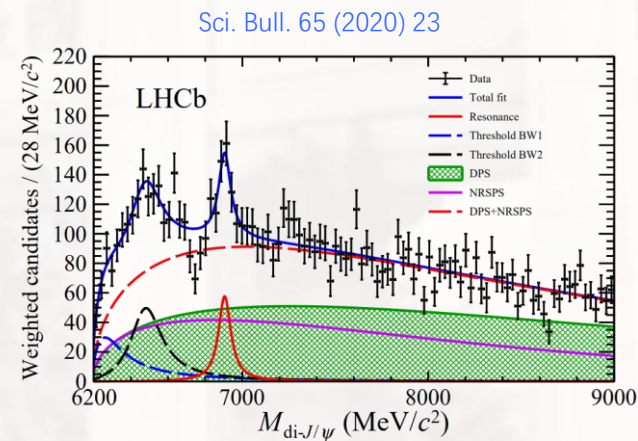
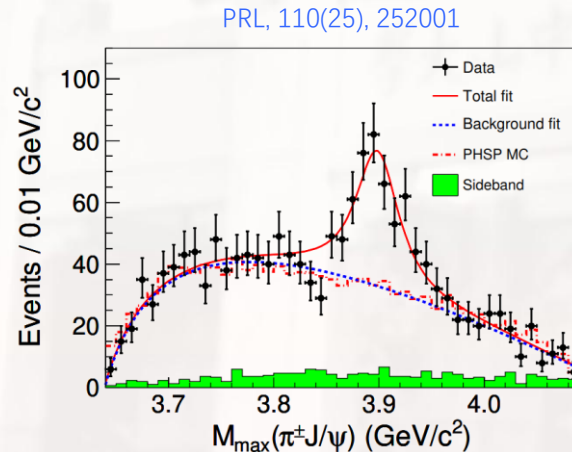
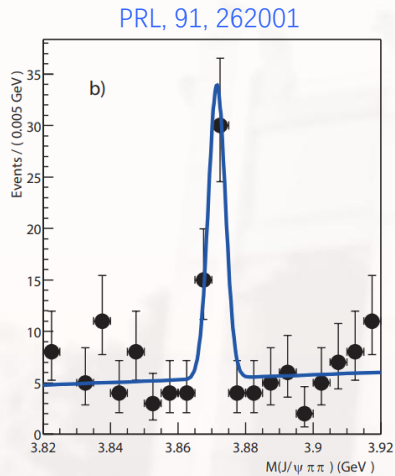
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Based on arxiv: 2507.05726
张锦鹏, 陈绪梁, 欧阳梓熙, 俞祥, 陈伟, 吴佳俊

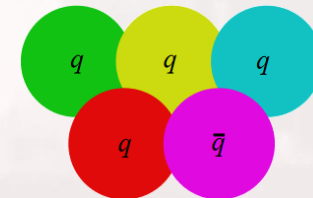
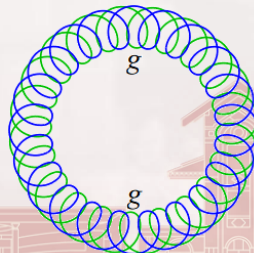
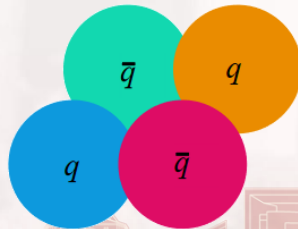
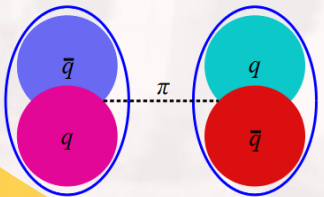
强子物理与有效场论前沿讲习班 8.22

QCD and exotic hadron states

- Hadron can be classified as mesons and baryons.
- QCD allows existence of exotic structures. Glueball, hybrid, tetraquark...
PLB, 205(2-3): 397-400. PLB, 313(1-2): 276-282. PRL, 129(19), 192002.
- Exotic quantum number ----- J^{PC} : $\pi_1(1400), \pi_1(1600), \eta_1(1855)$...



L	$S = 0$	$S = 1$
0	0^{-+}	1^{--}
1	1^{+-}	$0^{++}, 1^{++}, 2^{++}$
2	2^{-+}	$1^{--}, 2^{--}, 3^{--}$



A crypto-exotic $J^P = 0^-$ state

- Three resonance structures in P-wave ρK channel around (1.4, 1.7, 1.9) GeV.
- QM predicts only two excited K meson below 2.5 GeV
- Lightest -- $K(1430)$ Heaviest -- $K(1830)$
- Middle one -- $K(1690)$??? A candidate for exotic strange meson

Tetraquark or hybrid meson ?

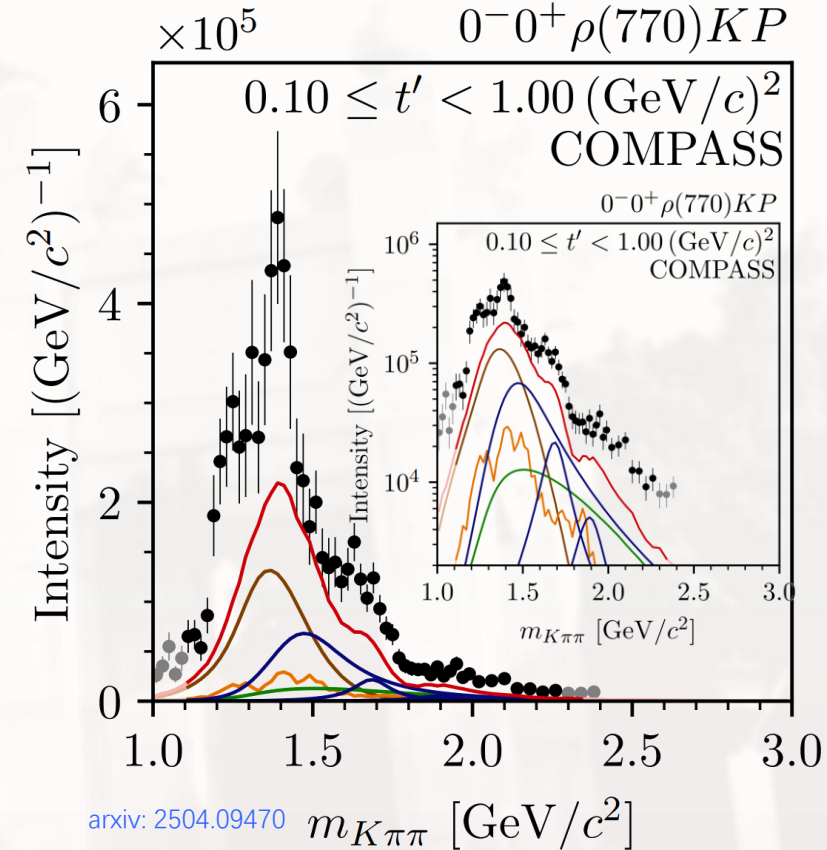
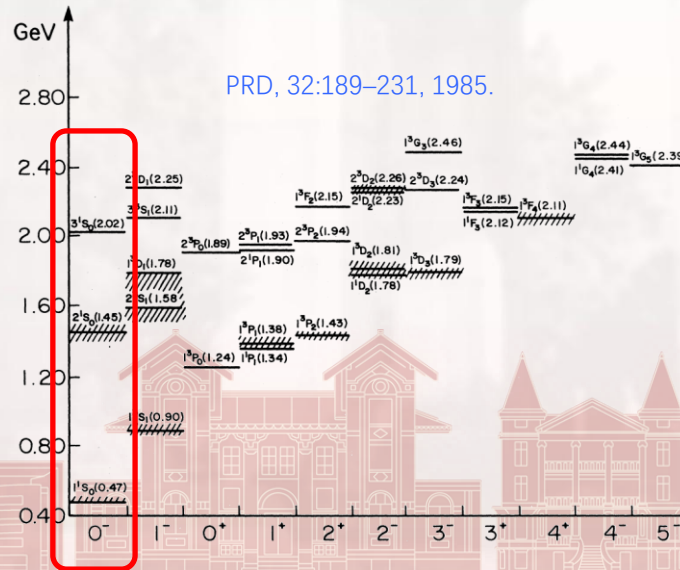
arxiv: 2504.09470



PDG

		$K(1690)/K(1630)$	$K(1830)$
m_0 [MeV/c ²]		$1687 \pm 10^{+2}_{-67}$	$1893 \pm 17^{+13}_{-39}$
Γ_0 [MeV/c ²]		$140 \pm 20^{+50}_{-50}$	$160 \pm 40^{+60}_{-80}$
m_0 [MeV/c ²]		1629 ± 7	$1874 \pm 43^{+59}_{-115}$
Γ_0 [MeV/c ²]		16^{+19}_{-16}	$168 \pm 90^{+208}_{-104}$

$J^P = ?$ of $K(1630)$



arxiv: 2504.09470

Bule-resonance components

Quark-gluon level: Operator product expansion (OPE)

$$\Pi(q^2) = \sum C_n(q^2) \langle \mathcal{O}_n \rangle$$

Start from two-point correlation function

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T[J(x)J^+(0)] | 0 \rangle$$

Hadron level:

Dispersion relation and
 $\rho(q^2) = f_X^2 \delta(q^2 - m^2) + \dots$

Mass m and decay constant f_X

Quark-hadron duality, Boral transform

$$f_X^2 e^{-m^2/M_B^2} = \int_0^{s_0} \rho(s) e^{-s/M_B^2} ds$$

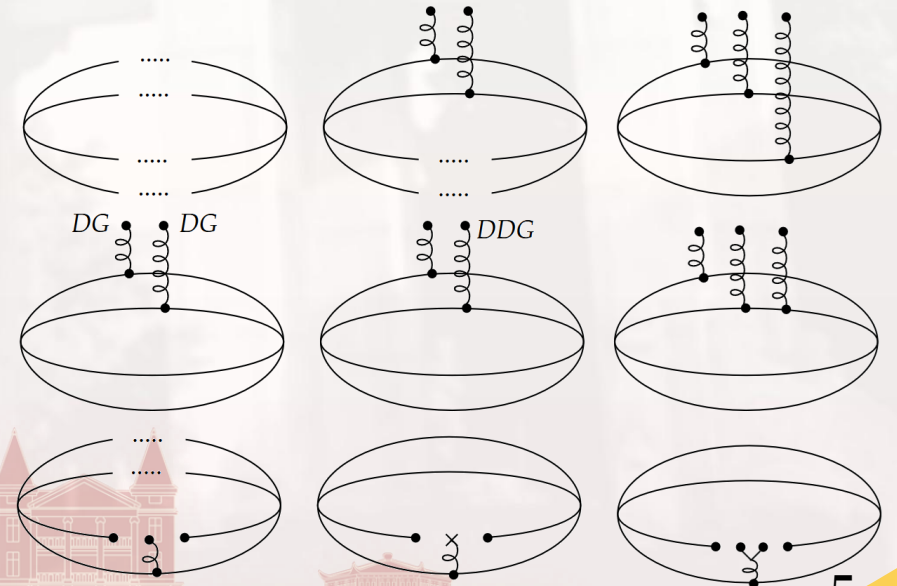
$K(1690)$ – pseudoscalar Tetraquark $uq\bar{q}\bar{s}$?

- There exist 10 $J^P = 0^-$ tetraquark currents
- Up to dimension-8: $\langle \bar{q}q \rangle, \langle g^2 G^2 \rangle, \langle \bar{q}q \rangle^2, \langle \bar{q}Gq \rangle, \langle g^3 f G^3 \rangle$
 $\langle \bar{q}GDq \rangle, \langle \bar{q}(DG)q \rangle, \langle \bar{q}q \rangle \langle g^2 G^2 \rangle, \langle \bar{q}q \rangle \langle \bar{q}Gq \rangle$

Convergence $\left| \frac{\Pi_{D=6}(M_B^2)}{\Pi_{\text{pert}}(M_B^2)} \right| \leq 25\%, \left| \frac{\Pi_{D=8}(M_B^2)}{\Pi_{\text{pert}}(M_B^2)} \right| \leq 10\%.$

Pole Contribution $\frac{L_0(s_0, M_B^2)}{L_0(\infty, M_B^2)} \geq 10\%$

$$\begin{aligned}
 6_F \otimes \bar{6}_F(S) & \begin{cases} S_6 = u_a^T C d_b (\bar{d}_a \gamma_5 C \bar{s}_b^T + \bar{d}_b \gamma_5 C \bar{s}_a^T) \\ V_6 = u_a^T C \gamma_5 d_b (\bar{d}_a C \bar{s}_b^T + \bar{d}_b C \bar{s}_a^T) \\ T_3 = u_a^T C \sigma_{\mu\nu} d_b (\bar{d}_a \sigma_{\mu\nu} \gamma_5 C \bar{s}_b^T - \bar{d}_b \sigma_{\mu\nu} \gamma_5 C \bar{s}_a^T) \end{cases} \\
 \bar{3}_F \otimes 3_F(A) & \begin{cases} S_3 = u_a^T C d_b (\bar{d}_a \gamma_5 C \bar{s}_b^T - \bar{d}_b \gamma_5 C \bar{s}_a^T) \\ V_3 = u_a^T C \gamma_5 d_b (\bar{d}_a C \bar{s}_b^T - \bar{d}_b C \bar{s}_a^T) \\ T_6 = u_a^T C \sigma_{\mu\nu} d_b (\bar{d}_a \sigma_{\mu\nu} \gamma_5 C \bar{s}_b^T + \bar{d}_b \sigma_{\mu\nu} \gamma_5 C \bar{s}_a^T) \end{cases} \\
 \bar{3}_F \otimes \bar{6}_F(M) & \begin{cases} A_6 = u_a^T C \gamma_\mu d_b (\bar{d}_a \gamma_\mu \gamma_5 C \bar{s}_b^T + \bar{d}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \\ P_3 = u_a^T C \gamma_\mu \gamma_5 d_b (\bar{d}_a \gamma_\mu C \bar{s}_b^T - \bar{d}_b \gamma_\mu C \bar{s}_a^T) \end{cases} \\
 6_F \otimes 3_F(M) & \begin{cases} P_6 = u_a^T C \gamma_\mu \gamma_5 d_b (\bar{d}_a \gamma_\mu C \bar{s}_b^T + \bar{d}_b \gamma_\mu C \bar{s}_a^T) \\ A_3 = u_a^T C \gamma_\mu d_b (\bar{d}_a \gamma_\mu \gamma_5 C \bar{s}_b^T - \bar{d}_b \gamma_\mu \gamma_5 C \bar{s}_a^T) \end{cases}
 \end{aligned}$$



$\langle g^3 f G^3 \rangle$ in QCD Sum Rules

- Vacuum condensates are important in QCDSR. $\langle \bar{q}q \rangle, \langle g^2 G^2 \rangle, \langle \bar{q}q \rangle^2, \langle \bar{q}Gq \rangle, \langle g^3 f G^3 \rangle \dots$

- Tri-gluon condensate $\langle g^3 f G^3 \rangle$ is usually neglected.

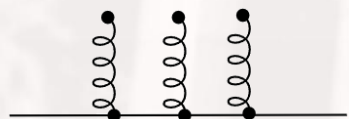
PLB 110 (1982) 476 NPB 213 (1983) 285–304 CPC 45 (9) (2021) 093103.

- Its contribution is unimportant in systems with heavy quarks.

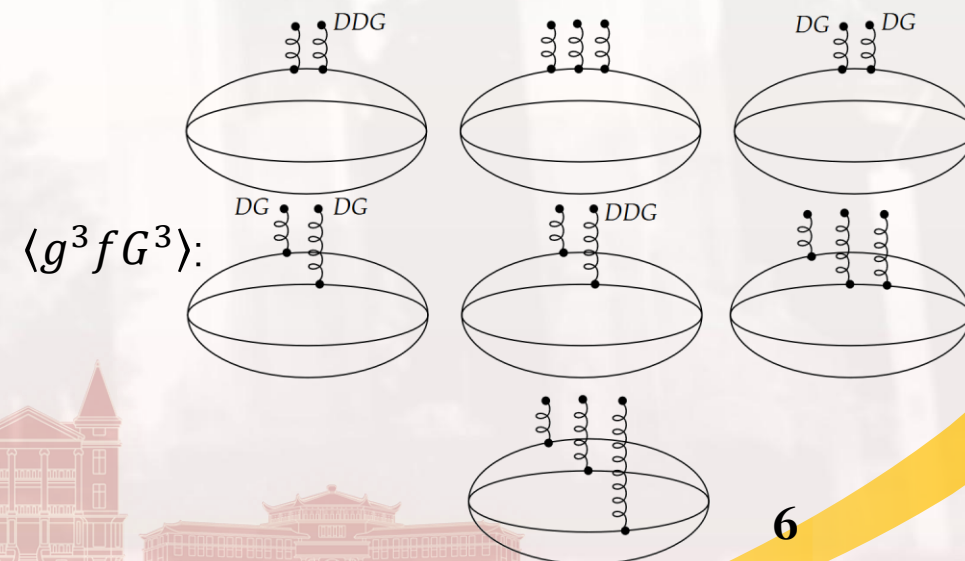
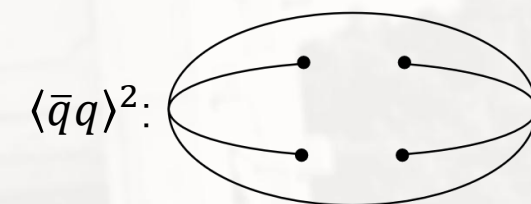
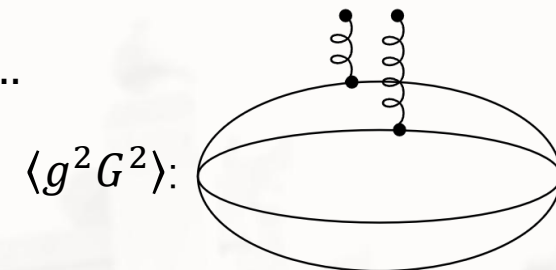
- Complexity: too many diagrams. There exist 7 diagrams in LO of $\langle g^3 f G^3 \rangle$

diagrams of $\langle g^3 f G^3 \rangle > \langle \bar{q}q \rangle + \langle g^2 G^2 \rangle + \langle \bar{q}q \rangle^2 + \langle \bar{q}Gq \rangle$

- Unavoidable IR divergence:

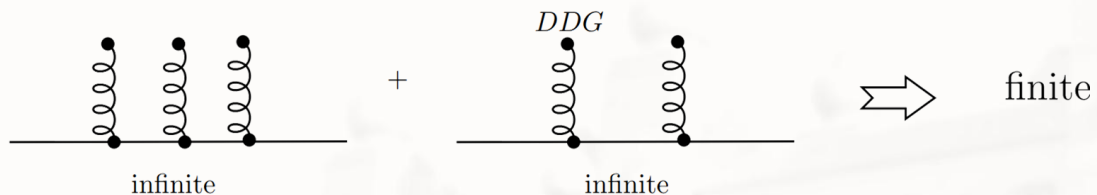


$$S_{\langle GGG \rangle}^{ij}(x) = \frac{-i\delta_{ij}\langle g^3 f G^3 \rangle \Gamma\left(\frac{d}{2} - 2\right) \not{x}}{9216d\pi^{d/2}(-x^2)^{d/2-3}}$$



IR safety of $\langle g^3 f G^3 \rangle$ in QCD Sum Rules

- IR safety for the single quark propagator. (ignore $g^4 \langle \bar{q} q \rangle^2$)



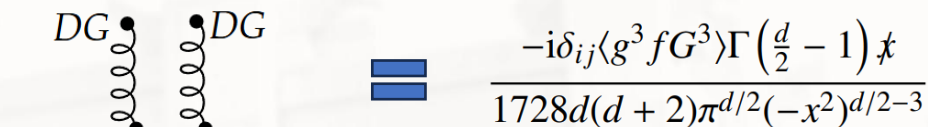
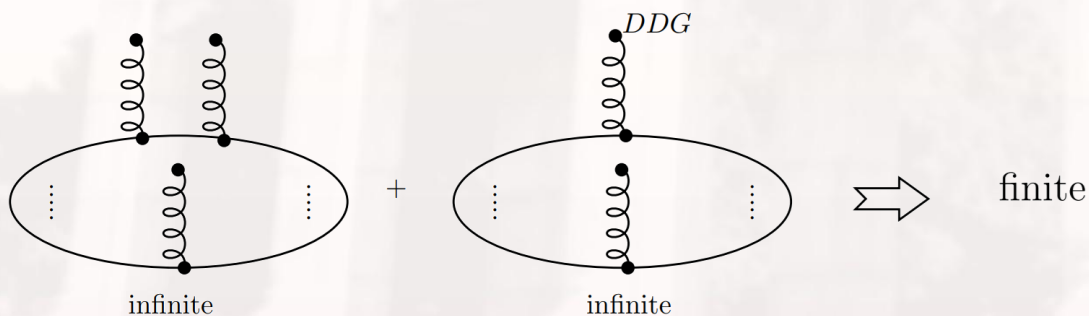


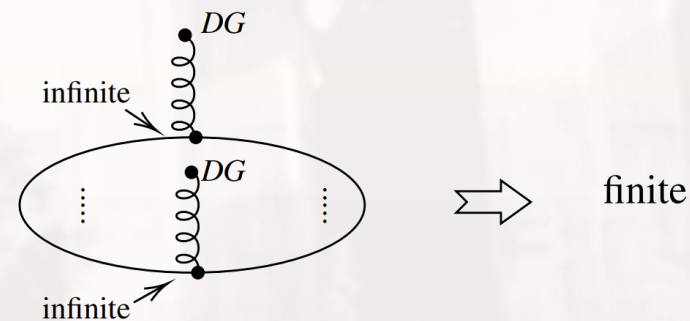
Diagram showing two wavy gluon lines attached to a horizontal line, labeled 'DG' and 'DG'. This is followed by an equals sign and a formula:

$$\frac{-i\delta_{ij}\langle g^3 f G^3 \rangle \Gamma\left(\frac{d}{2} - 1\right) \not{x}}{1728d(d+2)\pi^{d/2}(-x^2)^{d/2-3}}$$

- IR safety for the two different quark propagators.



Must be calculated in d-dimensional !
Propagator and condensate.

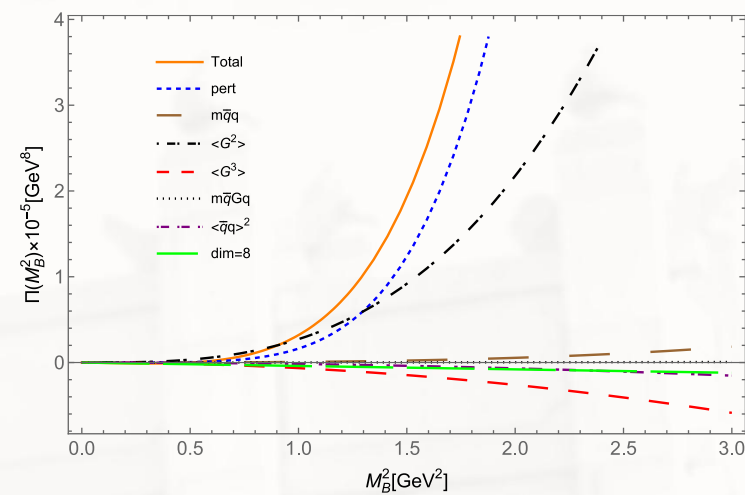


The LO of $\langle g^3 f G^3 \rangle$ can be fully calculated.

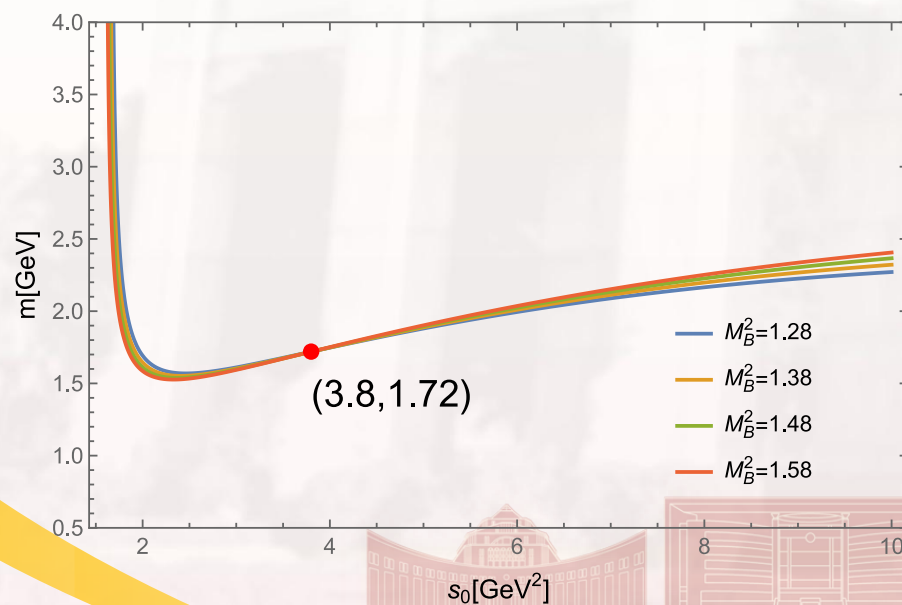
The impact of $\langle g^3 f G^3 \rangle$

- Take current P_3 as an example. Contribution

$$\Pi^{\text{pert}} > \Pi^{\langle G^2 \rangle} > \Pi^{\langle G^3 \rangle} > \Pi^{\langle \bar{q}q \rangle^2} > \dots$$



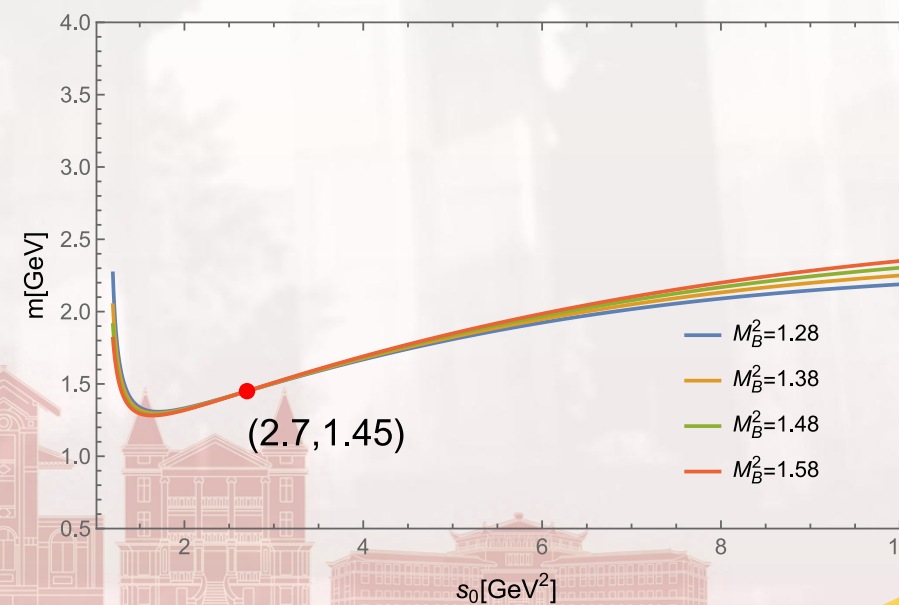
With $\langle g^3 f G^3 \rangle$



Differ by 20%!



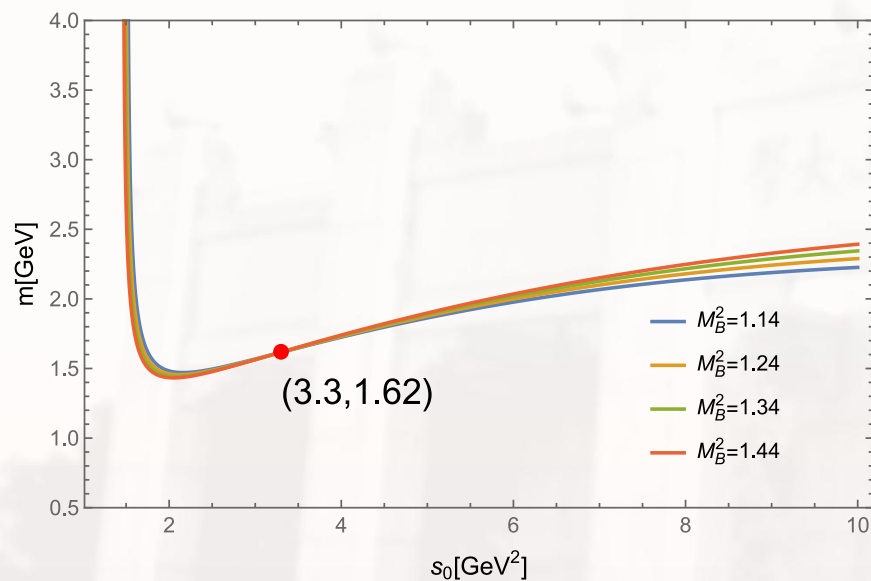
Without $\langle g^3 f G^3 \rangle$



The impact of $\langle g^3 f G^3 \rangle$

- Take current P_6 as another example.

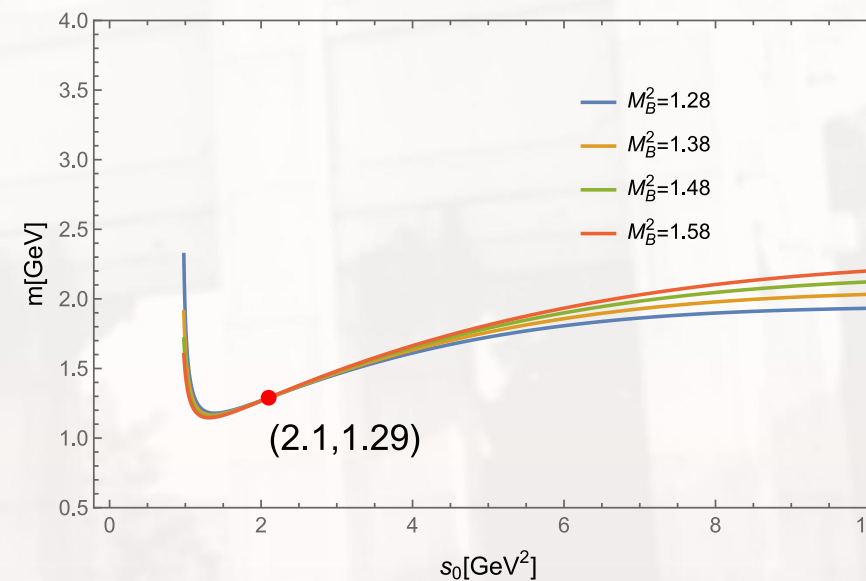
With $\langle g^3 f G^3 \rangle$



Differ by 25%!



Without $\langle g^3 f G^3 \rangle$



$\langle g^3 f G^3 \rangle$ provides significant contribution in light tetraquark systems!

Results for $ud\bar{d}\bar{s}$

- Uncertainty is from s_0 , condensates and m_s .

Currents	$s_0(\pm 0.2 \text{ GeV}^2)$	$M_B^2 (\text{GeV}^2)$	$m (\text{GeV})$	PC(%)
P_6	3.3	1.14 ~ 1.35	1.62 ± 0.10	> 15
P_3	3.8	1.28 ~ 1.50	1.72 ± 0.10	> 20
V_3	3.3	1.42 ~ 1.62	1.60 ± 0.16	> 10
A_3	2.8	0.95 ~ 1.20	1.48 ± 0.20	> 10
T_6	2.4	0.75 ~ 1.10	1.36 ± 0.05	> 15
T_3	4.7	0.90 ~ 1.35	1.92 ± 0.04	> 25

1. Masses of currents (P_6, P_3, V_3) are consistent with $K(1690)$

3. (A_3, T_6) and T_3 are close to $K(1460)$ and $K(1830)$

- Complete $\langle g^3 f G^3 \rangle$ provides significant contribution in light tetraquark systems.
- 10 currents for $0^- u d \bar{d} \bar{s}$ have been calculated by QCDSR.
- The masses of currents (P_6, P_3, V_3) are consistent with $K(1690)$

Thank you!

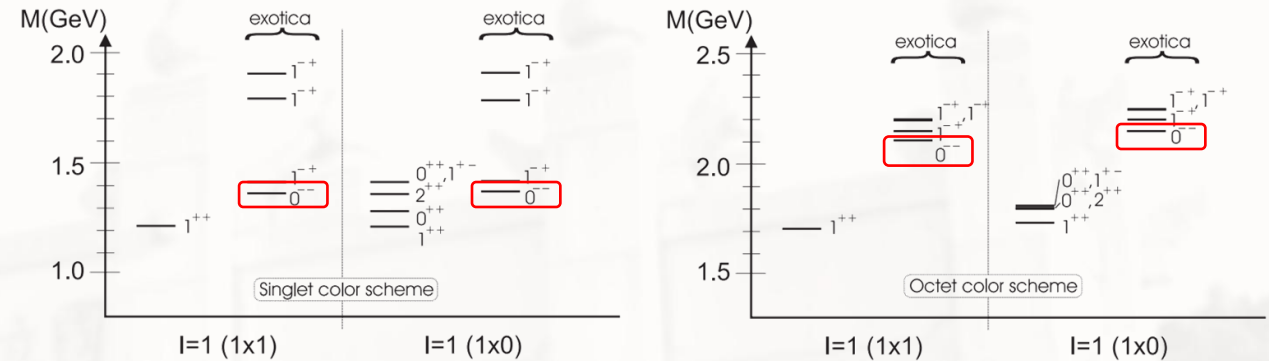


Related research

EPJA 31 (2007) 656–661 PLB 653 (2007) 216–223.

- QCD Coulomb gauge approach

$qq\bar{q}\bar{q} \quad 0^{--}: \sim 1.36\text{GeV}(\text{singlet color})$
 $\sim 2.1\text{GeV}(\text{octet color})$



- Cornell potential-based phenomenology [arXiv:2505.01195](https://arxiv.org/abs/2505.01195).

Table 1: S Wave SSSQ All units are in MeV.

State	J^P	T_{SSSQ}				T_{SQQQ}			
		$3 \otimes 3$		$6 \otimes 6$		$3 \otimes 3$		$6 \otimes 6$	
		M_{SR}	M_{NR}	M_{SR}	M_{NR}	M_{SR}	M_{NR}	M_{SR}	M_{NR}
1^1S_0	0^+	1688.83	1845.90	1161.27	899.72	1439.99	1239.35	1631.54	739.30
2^1S_0		2705.68	2797.19	2932.67	3104.81	2470.83	2790.35	2595.88	2973.53
3^1S_0		3181.37	3280.76	3879.06	4055.92	2962.87	3091.70	3749.25	3932.74
1^3S_1	1^+	1871.25	1998.89	-	-	1653.02	1804.26	-	-
2^3S_1		2754.48	2840.05	-	-	2535.26	2647.59	-	-
3^3S_1		3214.45	3306.98	-	-	3003.36	3122.71	-	-
1^5S_2	2^+	2338.73	2300.37	-	-	2070.76	2140.97	-	-
2^5S_2		2852.10	2918.98	-	-	2658.23	2742.72	-	-
3^5S_2		3277.07	3359.33	-	-	3083.88	3187.37	-	-
1^1P_1	1^-	2635.17	2710.62	2706.09	2868.47	2415.16	2513.20	2561.05	2732.62
2^1P_1		3091.39	3175.25	3692.30	3850.09	2876.64	2985.56	3552.21	3721.48
3^1P_1		3455.38	3554.63	4403.68	4580.34	3248.90	3374.29	4278.30	4463.72
1^3P_0	0^-	2314.79	2390.20	1010.72	1151.97	2082.28	2180.37	841.42	988.89
2^3P_0		2821.00	2905.99	2353.68	2501.65	2590.99	2701.27	2173.36	2327.59
3^3P_0		3206.10	3304.82	3204.07	3376.79	2982.91	3109.17	3034.28	3212.69

QCDSR

PRD 79 (2009) 114034

0^{--} currents for $qq\bar{q}\bar{q}$

All currents give results over 2GeV

PRD 95 (2017) 7, 076017

1^{+-} currents $qq\bar{q}\bar{q}$

$\sim 1.66\text{GeV}$ for 0^{--}