Theoretical study of $N(1535)$ and $a_0(980)$ in the process $\Lambda_c^+ \to \pi^+\eta n$

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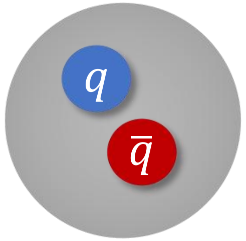
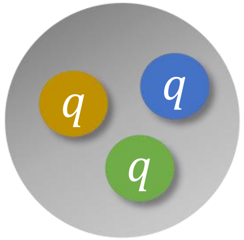
Introduction

- ✓ **Conventional hadrons**

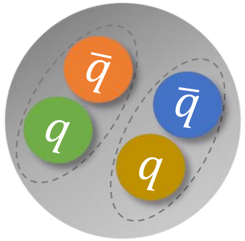


Meson

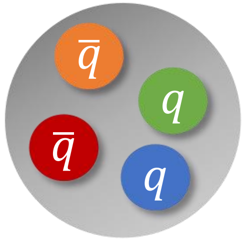


Baryon

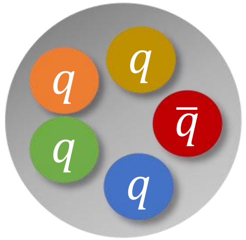
- ✓ **Exotic hadrons**



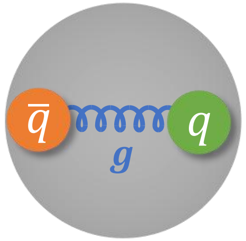
Molecule



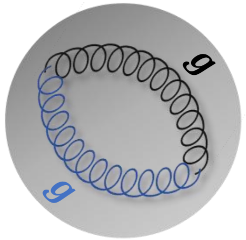
Tetraquark



Pentaquark



Hybird



Glueball

……

Introduction

$$\boldsymbol{N(1535)\ \ I(J^P) = 1/2(1/2^-)}$$

✓ Mass reverse problem

$N(1535)$ $J^P = 1/2^-$ n=1 L=1

$N(1440)$ $J^P = 1/2^+$ n=2 L=0

$N(1535)$ $J^P = 1/2^-$

95 MeV

$N(1440)$ $J^P = 1/2^+$

High mass vs. $N(1440)$

✓ Coupling to channels with strangeness

$N(1535)$ $\xrightarrow{\text{couples strongly to}}$ ηN, $K\Lambda$, $K\Sigma$

uud

Introduction

✓ PART of theoretical interpretations of the $N(1535)$

- The lowest L = 1 orbital excited uud state with a large mixing of the $[ud][us]\bar{s}$ pentaquark component:

  C. Helminen and D. O. Riska, Nucl. Phys. A699, 624 (2002)

  B. S. Zou, Eur. Phys. J. A 35, 325 (2008)

- A three-quark core:

  Z. W. Liu et al. Phys. Rev. Lett. 116, 082004 (2016)

  C. D. Abellet al. Phys. Rev. D 108, 094519 (2023)

- Dynamically generated state:

  P. C. Bruns et al. Phys. Lett. B 697, 254 (2011)

  K. P. Khemchandani et al. Phys. Rev. D 88, 114016 (2013)

  J.J.Xie and L.S.Geng, Phys.Rev.D 96 (2017) 5, 054009

  R. Pavao, S. Sakai and E. Oset, Phys.Rev.C 98 (2018) 1, 015201

  Y. Li, S. W. Liu et al. Phys. Rev. D 110 (2024) 7, 074010

Introduction

$\boldsymbol{a_0(980)\ \ I^G(J^{PC}) = 1^-(0^{++})}$

Quark	u	d	s
Mass (MeV)	2.16±0.04	4.7±0.04	93.5±0.5

$q\bar{q}$	$^3S_1$	$q\bar{q}$	$^3P_0$
$\phi(1020)$	$s\bar{s}$	$f_0(980)$	$s\bar{s}$
$K^*(892)$	$d\bar{s}$	$K_0^*(700)$	$d\bar{s}$
$\rho(770)$	$u\bar{u} - d\bar{d}$	$f_0(500)$	$u\bar{u} + d\bar{d}$
$\omega(782)$	$u\bar{u} + d\bar{d}$	$a_0(980)$	$u\bar{u} - d\bar{d}$

- ✓ **Conventional quark model** $f_0(500) \approx a_0(980) < K_0^*(700) < f_0(980)$
- ✓ **Experiment** $f_0(500) < K_0^*(700) < a_0(980) < f_0(980)$

Introduction

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✓ PART of theoretical interpretation of $a_0(980)$

- $K\bar{K}$ molecular state:

  J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48 (1982) 659
  J. R. Pelaez, Phys. Rev. Lett. 92 (2004) 102001

- Compact tetraquark state:

  J. R. Pelaez, Phys. Rev. Lett. 92 (2004) 102001
  H. J. Lee, Eur. Phys. J. A 30 (2006) 423-426
  S. Stone and L. Zhang, Phys. Rev. Lett. 111 (2013) 6, 062001

- Dynamically generated states:

  G. Janssen, B. C. Pearce et al. Phys. Rev. D 52, 2690 (1995)
  J. J. Xie, L. R. Dai and E. Oset, Phys. Lett. B 742, 363 (2015)
  R. Molina, J. J. Xie et al. Phys. Lett. B 803 (2020) 135279
  X. C. Feng, L. L. Wei et al. Phys. Lett. B 846 (2023) 138185
  M. Y. Duan, W. T. Lyu et al.Phys. Rev. D 111 (2025) 1, 016004

Introduction

✓ Charm baryon decay

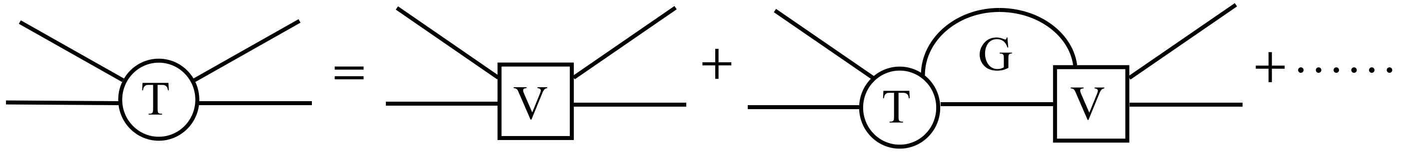
CS mode	$10^4\mathcal{B}$	CS mode	$10^4\mathcal{B}$
$\Lambda^0\pi^0K^+$	13.6 ± 2.5	$p\eta^0\eta^0$	4.46 ± 1.18
$\Lambda^0K^+\eta^0$	0.59 ± 0.24	$p\pi^+\pi^-$	46.4 ± 3.0
$\Lambda^0\pi^+K^0$	39.3 ± 7.6	pK^+K^-	4.82 ± 1.05
$\Sigma^0\pi^0K^+$	8.24 ± 1.41	$pK^0\bar{K}^0$	3.53 ± 1.68
$\Sigma^0K^+\eta^0$	$(4.29\pm1.23)\times10^{-2}$	$n\pi^+\pi^0$	25.3 ± 5.5
$\Sigma^0\pi^+K^0$	3.31 ± 0.97	$\boxed{n\pi^+\eta^0}$	$\boxed{45.2\pm12.1}$
$\Sigma^-\pi^+K^+$	3.73 ± 1.11	$nK^+\bar{K}^0$	6.54 ± 2.17
$\Xi^-K^+K^+$	$(3.33^{+4.23}_{-3.33})\times10^{-3}$	$\Sigma^+\pi^0K^0$	3.46 ± 1.01
$\Xi^0K^+K^0$	$(1.24^{+1.62}_{-1.24})\times10^{-3}$	$\Sigma^+K^0\eta^0$	$(8.38\pm2.40)\times10^{-2}$
$p\pi^0\pi^0$	39.6 ± 9.2	$\Sigma^+\pi^-K^+$	16.1 ± 3.0
$p\pi^0\eta^0$	25.8 ± 6.1		

TABLE I. The experimental data from Refs. [1–5,7–14,44–50] and reproductions for $\mathcal{B}(\mathbf{B}_c\to\mathbf{B}_nPP')$.

Channels	Data	Our fittings	Channels	Data	Our fittings
$10^2\mathcal{B}(\Lambda_c^+\to p\pi^+K^-)$	3.4 ± 0.4	3.4 ± 0.4	$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^+\pi^0\pi^0)$	1.3 ± 0.1	1.3 ± 0.1
$10^3\mathcal{B}(\Lambda_c^+\to\Lambda^0K^+\bar{K}^0)$	5.6 ± 1.1	5.9 ± 1.0	$10^4\mathcal{B}(\Lambda_c^+\to p\pi^-K^+)$	1.0 ± 0.1	1.0 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+\to\Lambda^0\pi^+\eta^0)$	1.8 ± 0.3	1.9 ± 0.3	$10^4\mathcal{B}(\Lambda_c^+\to nK^+\bar{K}^0)$	$8.6^{+3.8}_{-3.0}$	6.5 ± 2.2
$10^3\mathcal{B}(\Lambda_c^+\to\Lambda^0\pi^0K^+)$	1.5 ± 0.3	1.4 ± 0.3	$10^2\mathcal{B}(\Lambda_c^+\to p\pi^0K_S^0)$	1.9 ± 0.1	1.9 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^+\pi^+\pi^-)$	2.9 ± 0.5	2.8 ± 0.5	$10^2\mathcal{B}(\Lambda_c^+\to n\pi^+K_S^0)$	1.9 ± 0.1	1.9 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^-\pi^+\pi^+)$	1.9 ± 0.2	2.0 ± 0.2	$10^2\mathcal{B}(\Xi_c^+\to\Sigma^+\pi^+K^-)$	2.6 ± 1.2	3.9 ± 0.4
$10^2\mathcal{B}(\Lambda_c^+\to\Sigma^0\pi^+\pi^0)$	2.2 ± 0.8	1.0 ± 0.1	$10^2\mathcal{B}(\Xi_c^+\to\Xi^0\pi^+\pi^0)$	6.7 ± 3.5	1.0 ± 0.3
$10^3\mathcal{B}(\Lambda_c^+\to\Sigma^0\pi^+\eta^0)$	8.2 ± 0.9	8.3 ± 0.8	$10^3\mathcal{B}(\Xi_c^+\to\Sigma^+\pi^+\pi^-)$	14.0 ± 8.0	6.5 ± 1.6
$10^3\mathcal{B}(\Lambda_c^+\to\Sigma^+\pi^-K^+)$	2.0 ± 0.4	1.6 ± 0.3	$10^3\mathcal{B}(\Xi_c^+\to\Sigma^-\pi^+\pi^+)$	5.1 ± 3.4	6.9 ± 2.3
$10^3\mathcal{B}(\Lambda_c^+\to\Xi^-\pi^+K^+)$	3.3 ± 0.9	1.5 ± 0.5	$10^3\mathcal{B}(\Xi_c^+\to\Sigma^+K^+K^-)$	4.2 ± 2.5	0.4 ± 0.2
$10^3\mathcal{B}(\Lambda_c^+\to p\pi^+\pi^-)$	4.7 ± 0.3	4.6 ± 0.3	$10^2\mathcal{B}(\Xi_c^0\to\Lambda^0\pi^+K^-)$	1.2 ± 0.4	1.3 ± 0.3
$10^4\mathcal{B}(\Lambda_c^+\to pK^+K^-)$	5.2 ± 1.2	4.8 ± 1.0	$10^4\mathcal{B}(\Xi_c^0\to\Lambda^0K^+K^-)$	5.1 ± 1.9	4.5 ± 0.7
$10^2\mathcal{B}(\Lambda_c^+\to p\bar{K}^0\eta^0)$	0.8 ± 0.2	0.7 ± 0.1	$10^3\mathcal{B}(\Xi_c^0\to\Xi^0K^+K^-)$	0.7 ± 0.2	1.0 ± 0.1
$10^3\mathcal{B}(\Lambda_c^+\to\Xi^0\pi^0K^+)$	7.8 ± 1.6	8.0 ± 1.5	$10^2\mathcal{B}(\Xi_c^+\to\Xi^-\pi^+\pi^+)$	2.9 ± 1.3	4.0 ± 1.1

C. Q. Geng, C. W. Liu and S. L. Liu, Phys. Rev. D 109 (2024) 9, 093002

✓ Chiral unitary approach ----Coupled channel Bethe-Salpeter equation



$$T=[1-VG]^{-1}V$$

Formalism

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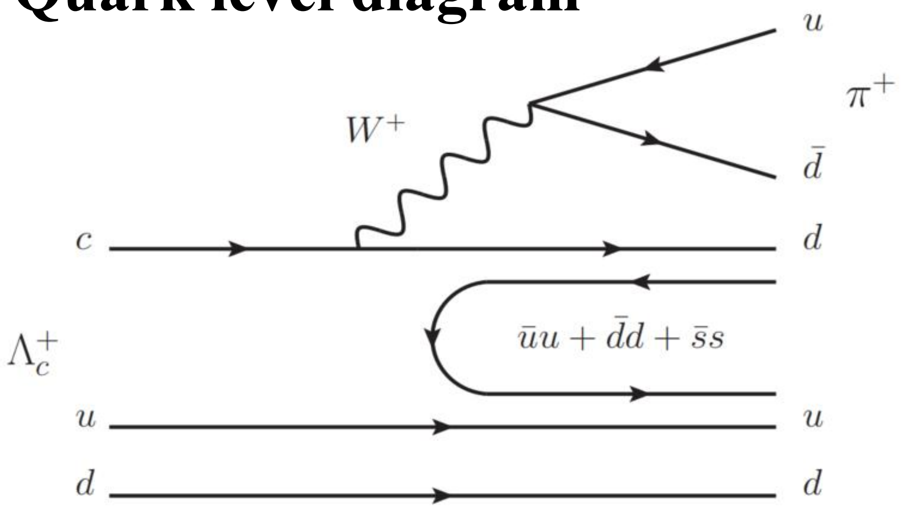
$N(1535)$——dynamically generated by the S-wave meson-baryon interaction

✓ **Quark level diagram**



$$\Lambda_c^+ = \frac{1}{\sqrt{2}} c(ud - du)$$

$$\Rightarrow \frac{1}{\sqrt{2}} \pi^+ \sum_i d(\bar{u}u + \bar{d}d + \bar{s}s)(ud - du)$$

$$= \frac{1}{\sqrt{2}} \pi^+ \sum_i M_{2i} q_i \,(ud - du)$$

✓ **SU(3) pesudoscalar meson matrix**

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \frac{\sqrt{6}\eta'}{3} \end{pmatrix}$$

✓ **Flavor-wave functions of the baryon**

$$p = \frac{u(ud - du)}{\sqrt{2}} \qquad n = \frac{d(ud - du)}{\sqrt{2}}$$

$$\Lambda = \frac{u(ds - sd) + d(us - su) - 2s(ud - du)}{\sqrt{2}}$$

Formalism

$N(1535)$—dynamically generated by the S-wave meson-baryon interaction

✓ **Components of the final states**

$$\Lambda_c^+ = \pi^+(\pi^- p - \frac{\sqrt{2}}{2}\pi^0 n + \frac{\sqrt{3}}{3}\eta n - \frac{\sqrt{6}}{3}K^0\Lambda)$$

isospin basis

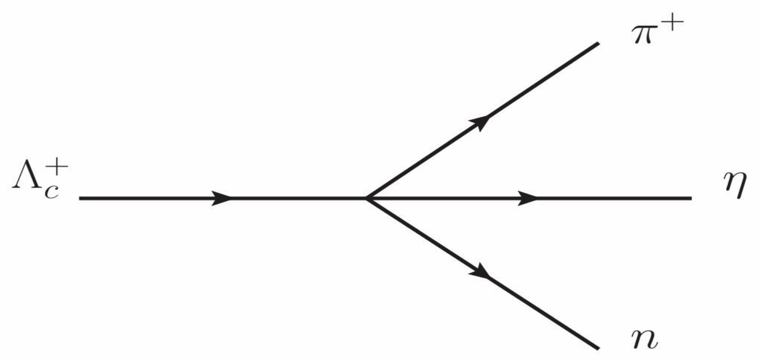
$$\Lambda_c^+ = \pi^+(-\frac{\sqrt{6}}{2}\pi N^{I=\frac{1}{2}} + \frac{\sqrt{3}}{3}\eta N^{I=\frac{1}{2}} - \frac{\sqrt{6}}{3}K\Lambda^{I=\frac{1}{2}})$$

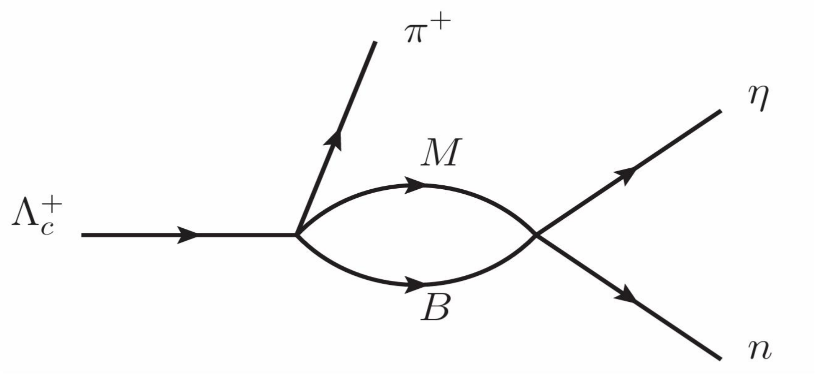
✓ **Amplitude**

$$\mathcal{M}^{Tree} = h_{\eta n}$$

$$\mathcal{M}^{N(1535)} = h_{\pi N}G_{\pi N}t_{\pi N\to\eta n} + h_{\eta n}G_{\eta N}t_{\eta N\to\eta n} + h_{K\Lambda}G_{K\Lambda}t_{K\Lambda\to\eta n}$$

✓ **Final states interaction**





Formalism

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$N(1535)$—dynamically generated by the S-wave meson-baryon interaction

$$\mathcal{M}^{N(1535)} = h_{\pi N}G_{\pi N}t_{\pi N\to\eta n} + h_{\eta n}G_{\eta N}t_{\eta N\to\eta n} + h_{K\Lambda}G_{K\Lambda}t_{K\Lambda\to\eta n}$$

✓ Loop function

$$G_i = i\int \frac{d^4q}{(2\pi)^4}\frac{2M_i}{(P-q)^2 - M_i^2 + i\varepsilon}\frac{1}{q^2 - m_i^2 + i\varepsilon}$$

$$G(s) = \frac{2M}{16\pi^2 s}\{\sigma(arctan\frac{s+\Delta}{\sigma\lambda_1} + arctan\frac{s-\Delta}{\sigma\lambda_2}) - [(s+\Delta)ln\frac{q_{max}(1+\lambda_1)}{m_1} + [(s-\Delta)ln\frac{q_{max}(1+\lambda_2)}{m_2}]\}$$

q_{max} =1150 MeV

✓ Bethe-Salpetere quation

$$T = [1 - VG]^{-1}V$$

$$V_{ij} = -C_{ij}\frac{1}{4f^2}(2\sqrt{s} - M_i - M_j) \times (\frac{M_i + E_i}{2M_i})^{1/2}(\frac{M_j + E_j}{2M_j})^{1/2}$$

TABLE I. The S-wave meson-baryon scattering coefficients [54].

	πN	ηn	$K\Lambda$	$K\Sigma$
πN	2	0	3/2	−1/2
ηn		0	−3/2	−3/2
$K\Lambda$			0	0
$K\Sigma$				2

Y. Li, S. W. Liu et al. Phys. Rev. D 110 (2024) 7, 074010

Formalism

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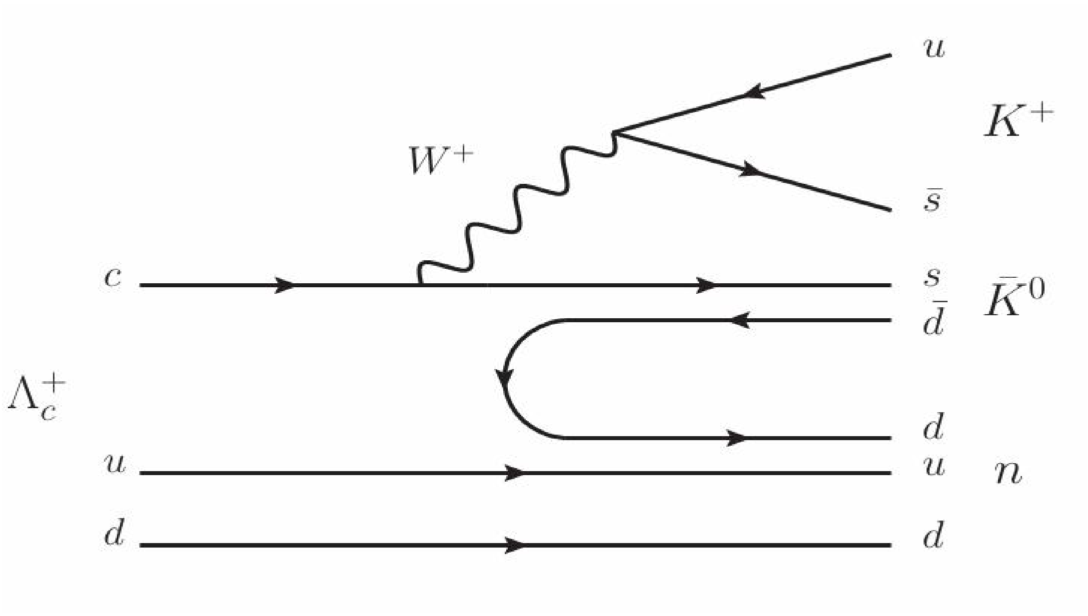
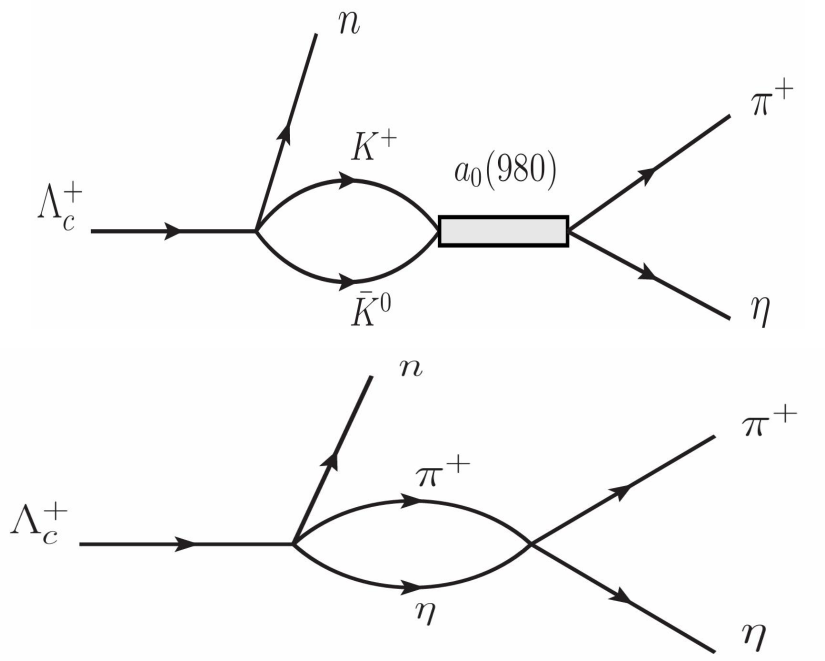
$a_0(980)$--dynamically generated by the S-wave meson-meson interaction

✓ **Quark level diagram**



$$\boxed{\Lambda_c^+ \Longrightarrow \pi^+\left(-\frac{\sqrt{6}}{2}\pi N + \frac{\sqrt{3}}{3}\eta N - \frac{\sqrt{6}}{3}K\Lambda\right)}$$

✓ **Final states interaction**



✓ **Amplitude**

$$\mathcal{M}^{a_0(980)} = h_{\pi^+\eta}\, G_{\pi^+\eta}\, t_{\pi^+\eta\to\pi^+\eta} + h_{K^+\bar{K}^0} G_{K^+\bar{K}^0} t_{K^+\bar{K}^0\to\pi^+\eta}$$

Formalism

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$a_0(980)$--dynamically generated by the S-wave meson-meson interaction

$$\mathcal{M}^{a_0(980)} = h_{\pi^+\eta} G_{\pi^+\eta} t_{\pi^+\eta\to\pi^+\eta} + h_{K^+\bar{K}^0} G_{K^+\bar{K}^0} t_{K^+\bar{K}^0\to\pi^+\eta}$$

✓ **Loop function**

$$G = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_1^2 + i\varepsilon} \frac{1}{q^2 - m_2^2 + i\varepsilon}$$

$$= \int_0^{q_{max}} \frac{|\vec{q}|^2 d|\vec{q}|}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1\omega_2[s - (\omega_1+\omega_2)^2 + i\varepsilon]}$$

q_{max} =600 MeV

✓ **Bethe-Salpetere quation**

$$T = [1 - VG]^{-1}V$$

$$V_{K^+\bar{K}^0\to K^+\bar{K}^0} = -\frac{s}{4f^2}$$

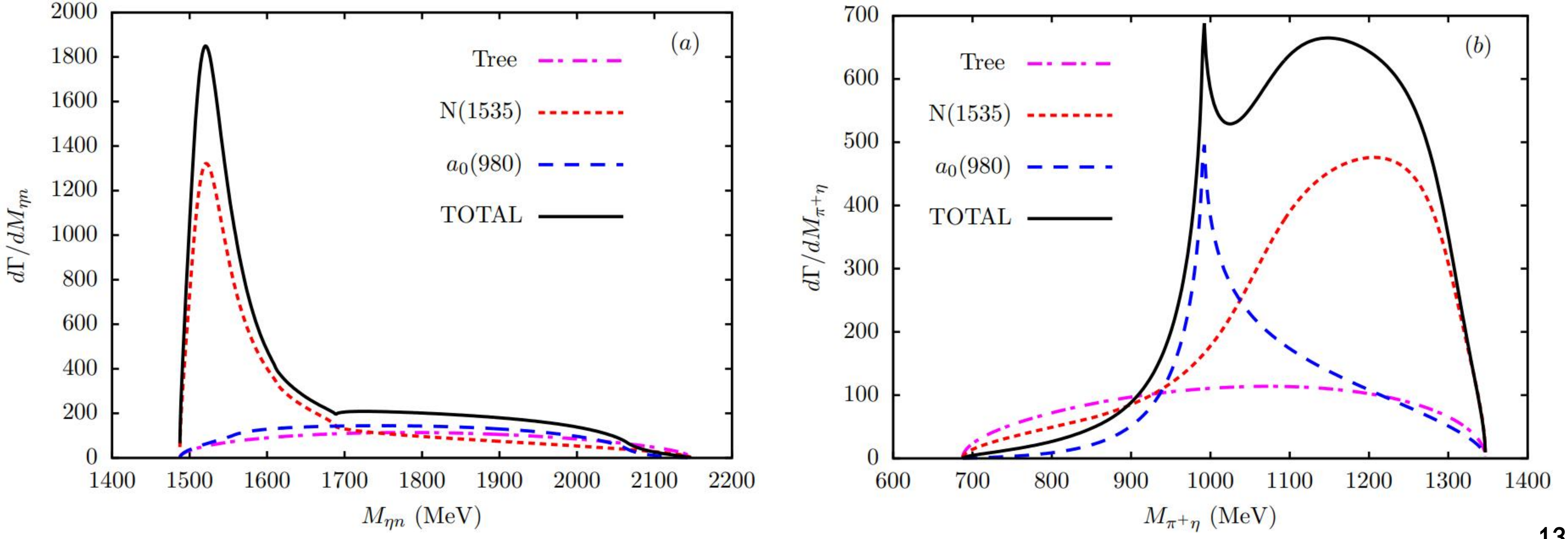
$$V_{K^+\bar{K}^0\to\pi^+\eta} = -\frac{3s - 2m_K^2 - m_\eta^2}{3\sqrt{3}f^2}$$

$$V_{\pi^+\eta\to\pi^+\eta} = -\frac{2m_K^2}{3f^2} \quad (f = 93\text{ MeV})$$

J. J. Xie, L. R. Dai and E. Oset, Phys. Lett. B 742, 363 (2015)

Results

- ✓ **Total amplitude** $\mathcal{M} = \mathcal{M}^{Tree} + \mathcal{M}^{N(1535)} + \mathcal{M}^{a_0(980)}$
- ✓ **Double differential width** $\frac{d^2\Gamma}{d^2M_{\eta n}\, d^2M_{\pi^+\eta}} = \frac{1}{(2\pi)^3}\frac{4M_{\Lambda_c^+}M_n}{32M^3_{\Lambda_c^+}}|\mathcal{M}|^2$



Results

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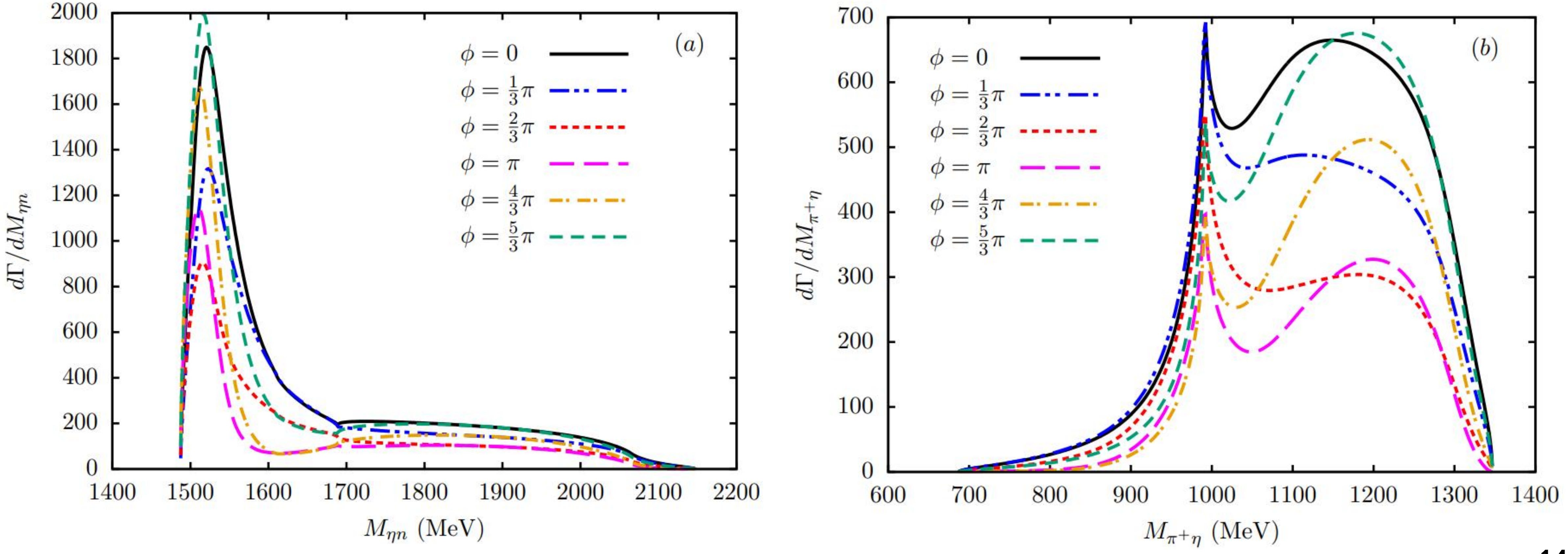
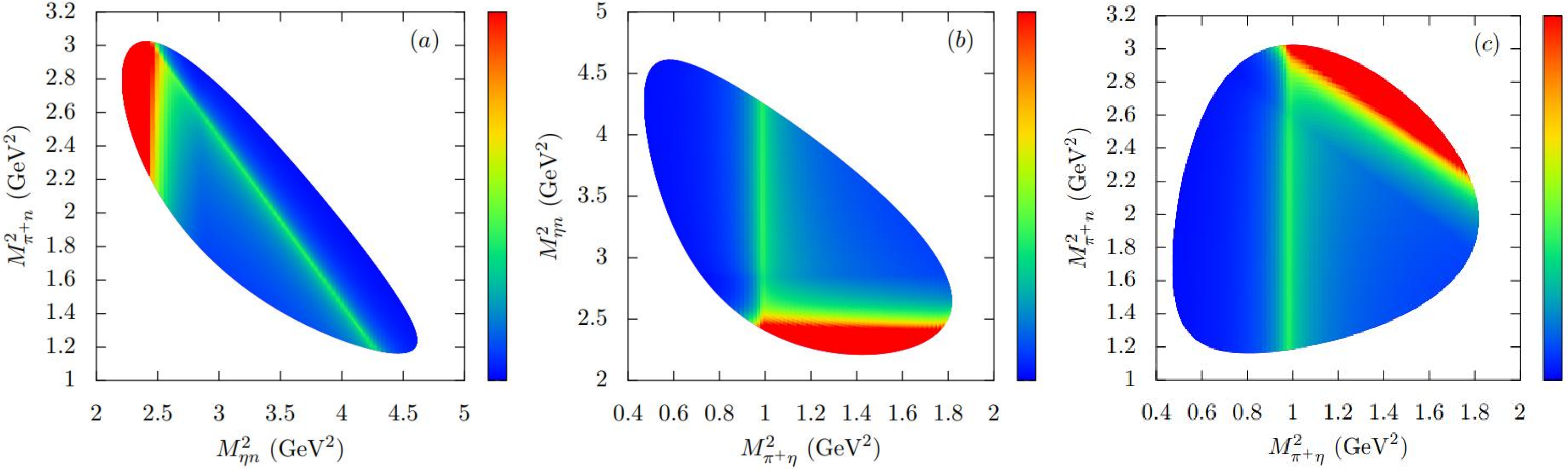
✓ The invariant mass distributions with phase interference

$$\mathcal{M} = \mathcal{M}^{Tree} + \mathcal{M}^{N(1535)} + \mathcal{M}^{a_0(980)} e^{i\phi}$$



Results

✓ The Dalitz plots



$M^2_{\eta n}$ vs $M^2_{\pi^+ n}$ $M^2_{\pi^+ \eta}$ vs $M^2_{\eta n}$ $M^2_{\pi^+ \eta}$ vs $M^2_{\pi^+ n}$

Summary

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- ✓ **The significant peak structure in the ηn invariant mass distribution of the decay $\Lambda_c^+ \to \pi^+\eta n$ should be related to $N(1535)$.**
- ✓ **A cusp structure in the $\pi^+\eta$ invariant mass distribution of the decay $\Lambda_c^+ \to \pi^+\eta n$ should be related to $a_0(980)$.**
- ✓ **More precise measurement results from future experiments will help us to better understand the nature of $N(1535)$ and $a_0(980)$.**

Thanks for your attention !