中国科学院近代物理所
INSTITUTE OF MODERN PHYSICS

Studying the $K\Lambda$ strong interaction using femtoscopic correlation functions

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arXiv: 2503.22453

2025年郑州大学强子物理与有效场论前沿讲习班
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CONTENTS

Background: Hadronic Interactions

Quantum Chromodynamics (QCD)

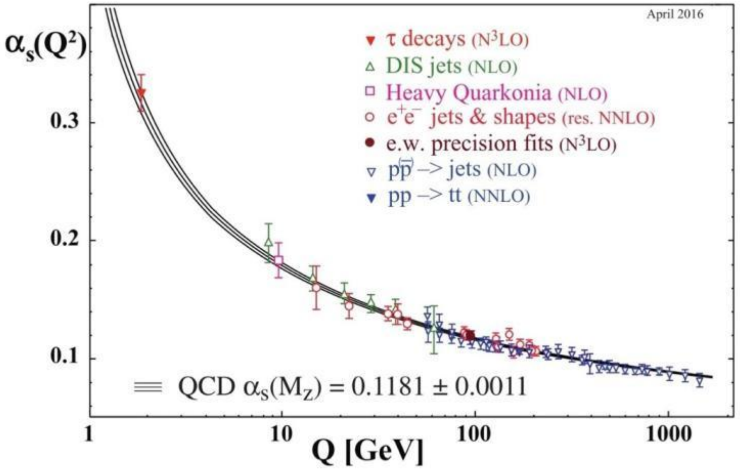
- Non-perturbation in low-energy region
- Quark confinement
- ……

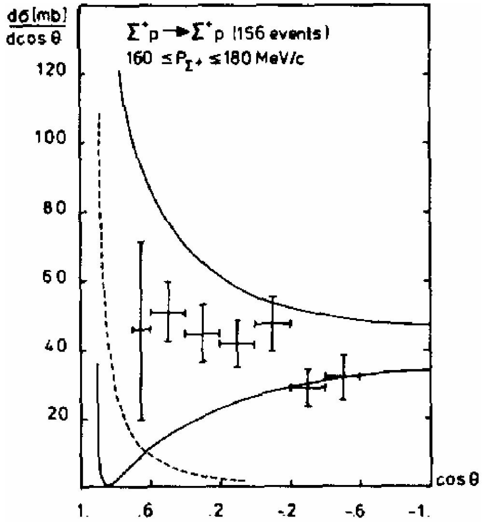
Hadronic Interaction

- Using Lattice QCD and Effective Field Theory
- Hadron molecule

Scattering Experiment

- Lots of hadrons are short-lived
- Scarce and low precision data about hyperons

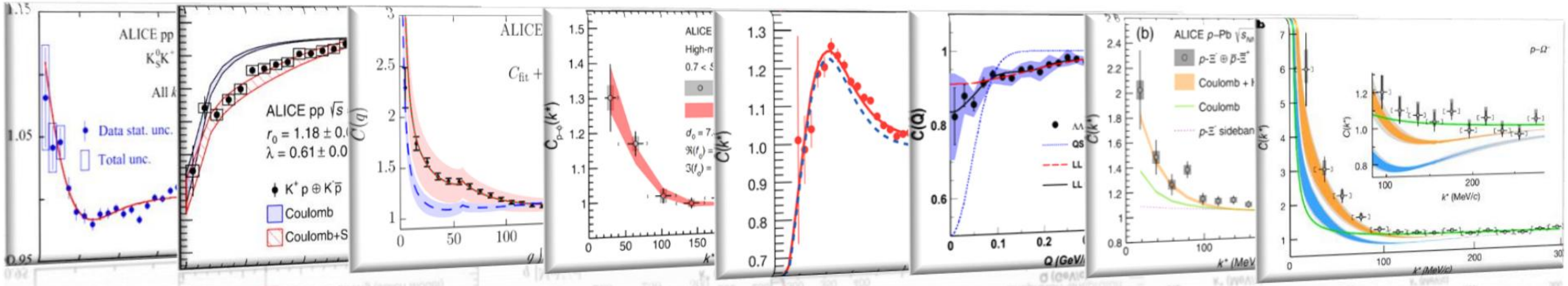




New approach is accquired !!!

Background: Progress of correlation function

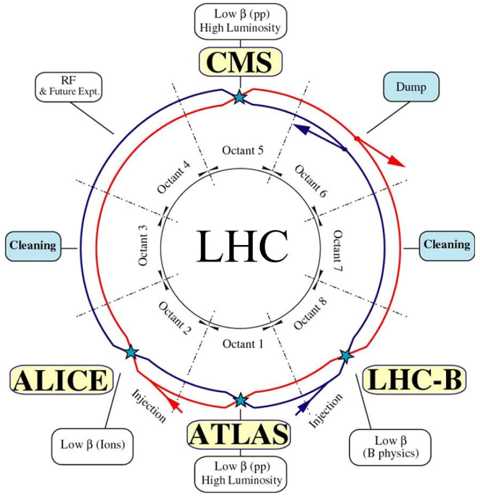
$K_S^0K^\pm$ $K^\pm p$ K^-p ϕp $\bar{p}\bar{p}$ $\Lambda\Lambda$ Ξ^-p Ω^-p

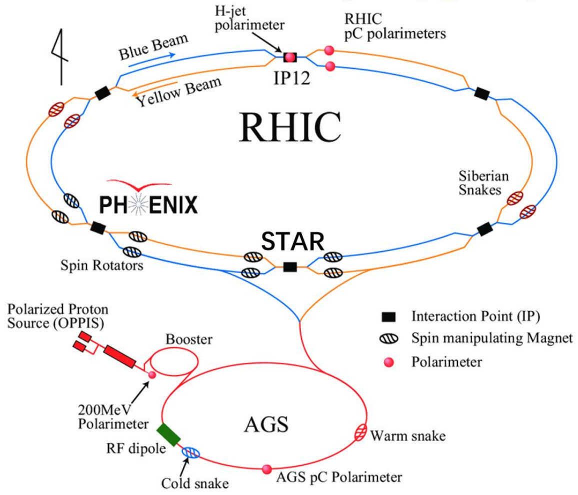


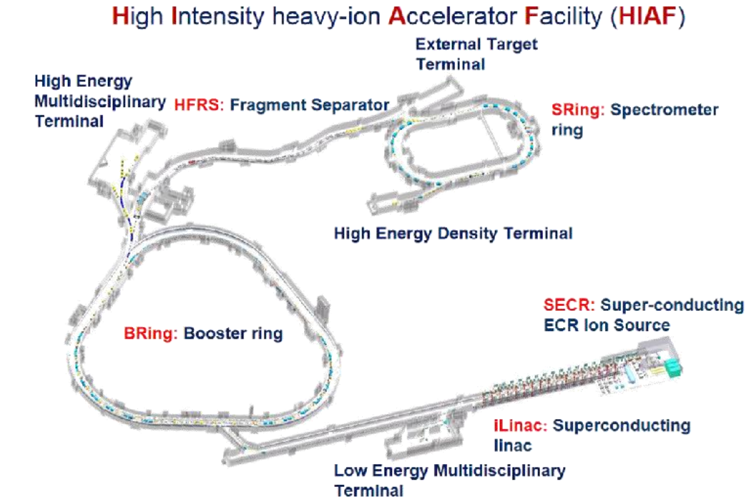
K^-d
Σ^0p
$\Lambda\Xi$
$\Xi\Xi$
$\Omega\Omega$
pD
ppp
$pp\Lambda$

ALICE Collaboration, Phys. Lett. B **790** (2019) 22
ALICE Collaboration, Phys. Rev. Lett. **124** (2020) 092301
Y. Kamiya and et al., Phys. Rev. Lett. **124** (2020) 132501
ALICE Collaboration, Phys. Rev. Lett. **127** (2021) 172301
STAR Collaboration, Nature **527** (2015) 345
STAR Collaboration, Phys. Rev. Lett. **114** (2015) 022301
ALICE Collaboration, Phys. Rev. Lett. **123** (2019) 112002
ALICE Collaboration, Nature **588** (2020) 232
L. Fabbietti, Ann. Rev. Nucl. Part. Sci. **71** (2021) 377

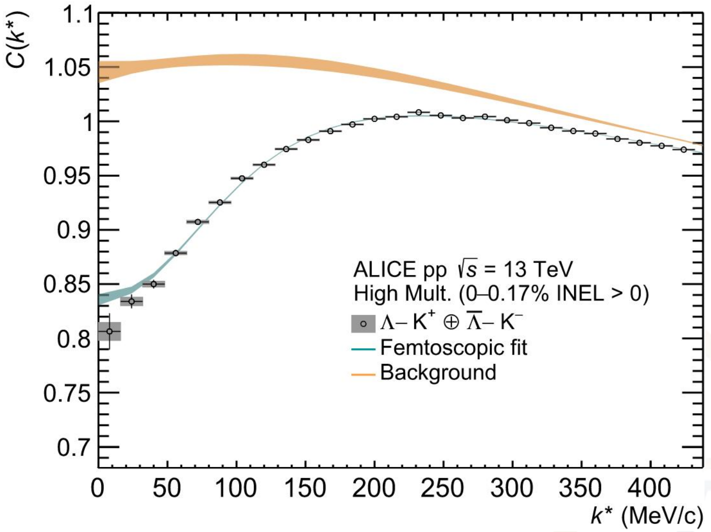
From the slide of Zhi-Wei Liu

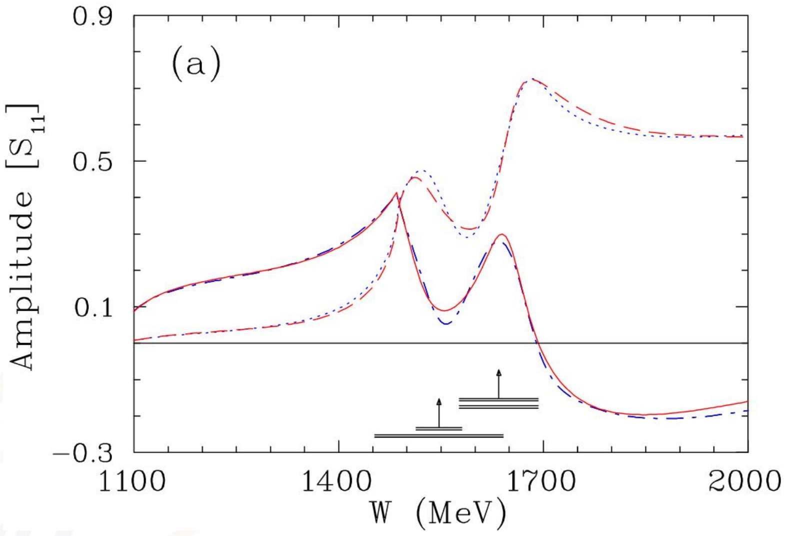


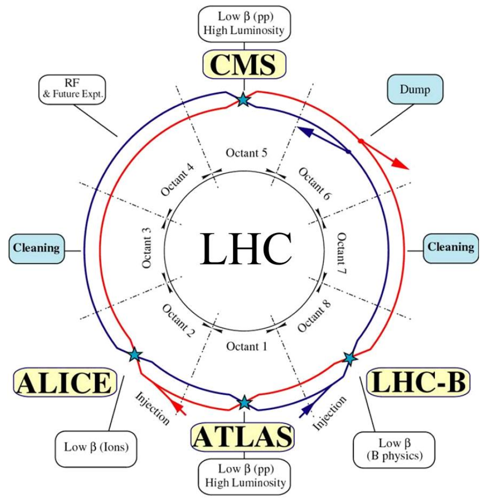


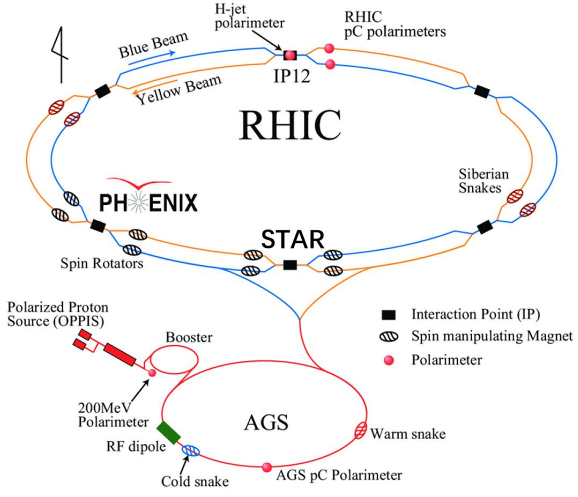


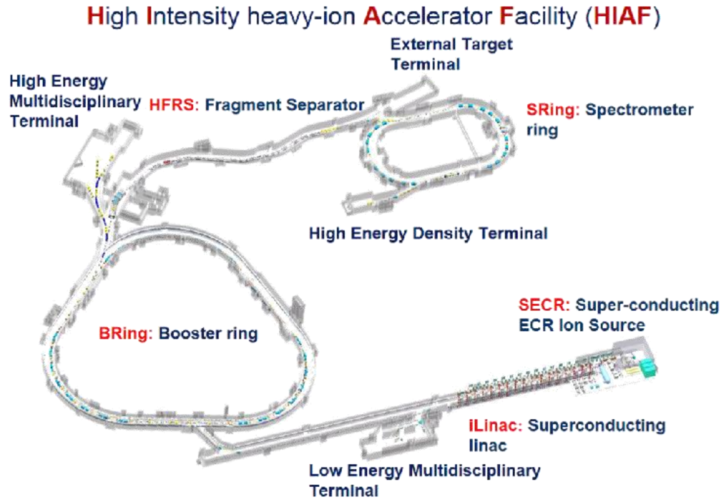
Background: Progress of correlation function



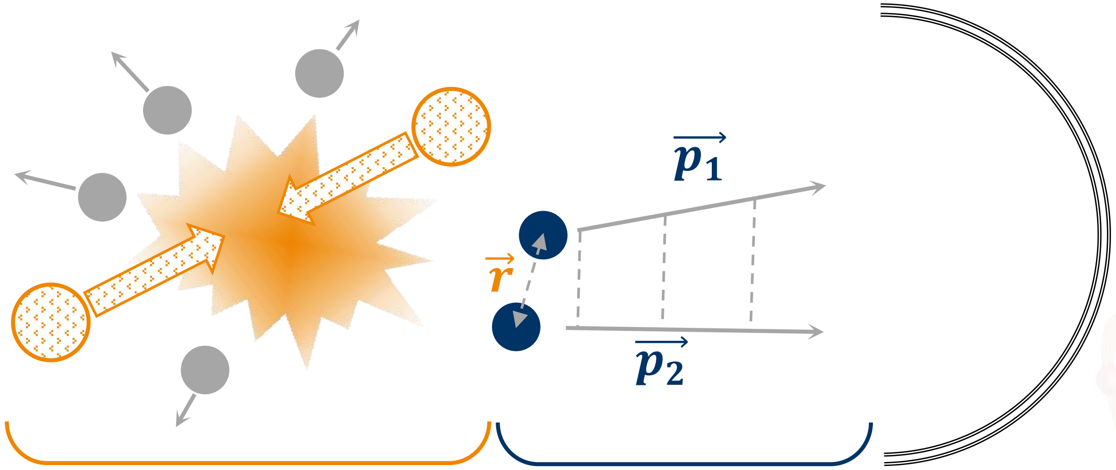








Methods: Correlation function

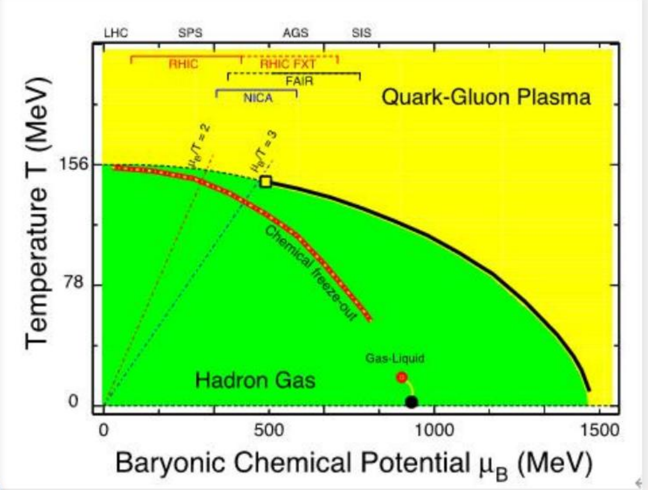


Correlation function

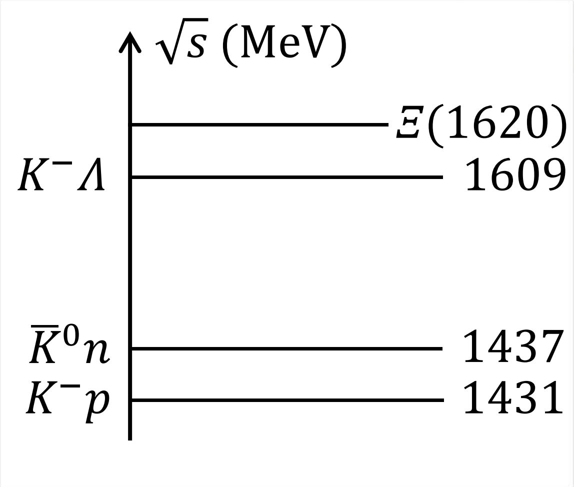
joint probability density

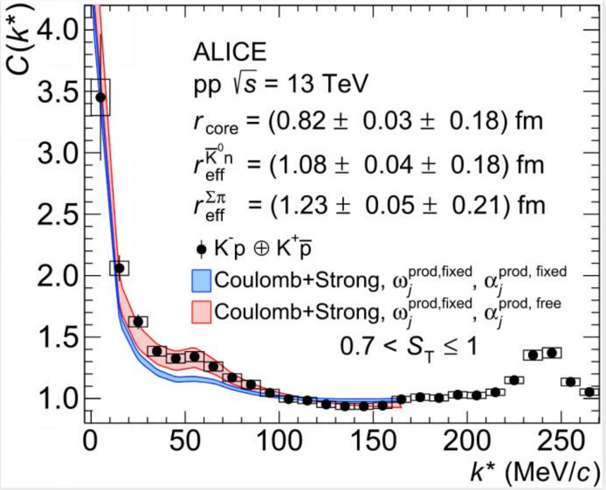
$$C(\vec{p}) = \frac{P(\vec{p_1},\vec{p_2})}{P(\vec{p_1})\cdot P(\vec{p_2})} \begin{cases} = 1, & \text{no interaction} \\ > 1, & \text{attractive} \\ < 1, & \text{repulsive} \end{cases}$$

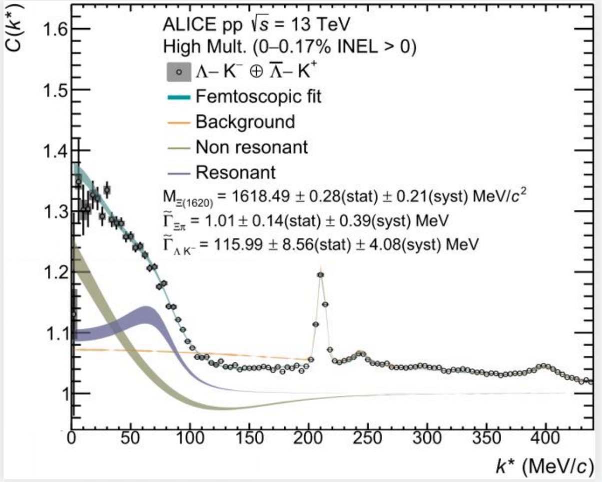
QCD phase transition



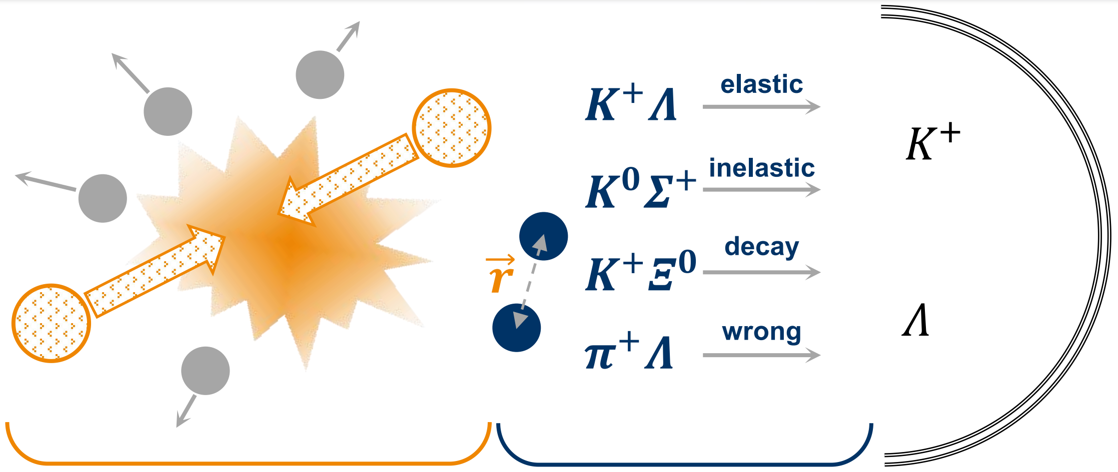
Advantages







Methods: Correlation function



Emitting Source $S(\vec{r})$ | **Wave Function** $\Psi(\vec{r},\vec{p})$ | **Detector** $\xi(\vec{p})$

$$C(\vec{p}) = \int d^3\vec{r}\, S(\vec{r})\, |\Psi(\vec{r})|^2$$

Emitting Source

$$S(\vec{r}) = \frac{1}{(4\pi R^2)^{3/2}} e^{-\frac{|\vec{r}|^2}{4R^2}}$$

long-lived resonances

$$S_{eff}(\vec{r}) = \omega_S S(\vec{r}, R_1) + (1-\omega_S) S(\vec{r}, R_2)$$

(ALICE Collaboration), Phys.Lett.B 811 (2020) 135849
(ALICE Collaboration), Phys.Lett.B 845 (2023) 138245

Correlation Analysis Tool using the Schödinger equation (CATS) Formalism

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(\vec{r})\right]\psi(\vec{r}) = E\psi(\vec{r})$$

Lednický–Lyuboshits (LL) Formalism

$$\Psi(\vec{r}) = e^{-ikr\cos\theta} + f(k)\frac{e^{-ikr}}{r}, \qquad f(k) = \frac{1}{a_0^{-1} + 0.5 r_0 k^2 - ik}$$

KooninPratt (KP) Formalism

$$T = V + VGT, \qquad \Psi = \phi + GT\phi$$

Methods: Chiral unitary approach

Chiral lagrangian

$$\mathcal{L}_1^{(B)} = \frac{1}{4f^2}\langle \bar{B} i\gamma^\mu \left[[\phi, \partial_\mu \phi], B\right]\rangle$$

$$V_{ij} = -\frac{C_{ij}}{4 f_i f_j}(k_i + k_j)\sqrt{\frac{(M_i + E_i)(M_j + E_j)}{4 M_i M_j}}$$

Lippmann-Schwinger equation

$$T = V + VGT, \qquad \Psi = \phi + GT\phi$$

$$G = \int_0^\infty \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_j^2 + i\epsilon)((P-q)^2 - m_{j2}^2 + i\epsilon)}$$

Correlation function in coupled channel

$$\Psi_{ij}(\vec{r},\vec{p}) = \delta_{ij} e^{i\vec{p}\vec{r}} + T_{ij}(\vec{r},\vec{p}) \int_0^\infty \frac{d^4 q}{(2\pi)^4} \frac{e^{i\vec{p}\vec{r}}}{(q^2 - m_j^2 + i\epsilon)((P-q)^2 - m_{j2}^2 + i\epsilon)}$$

$$= \delta_{ij} e^{i\vec{p}\vec{r}} + T_{ij}(\vec{r},\vec{p}) \widetilde{G}_j(\vec{r},\vec{p})$$

$$C_{theory}(\vec{p}) = 1 + \int_0^\infty d^3\vec{r}\, S(\vec{r}) \left(2\mathrm{Re}\left[T_{ii}(\vec{r},\vec{p}) \widetilde{G}_i(\vec{r},\vec{p}) j_0(|\vec{p}||\vec{r}|)\right] + \sum_j \left|T_{ij}(\vec{r},\vec{p}) \widetilde{G}_j(\vec{r},\vec{p})\right|^2 \right)$$

Methods: Properties

Correlation function

$$C_{theory}(\vec{p}) = 1 + \int d^3\vec{r}\, S(\vec{r}) \left(2\mathrm{Re}\left[T_{ii}(\vec{r},\vec{p})\widetilde{G}_i(\vec{r},\vec{p}) j_0(|\vec{p}||\vec{r}|)\right] + \sum_j \left|T_{ij}(\vec{r},\vec{p})\widetilde{G}_j(\vec{r},\vec{p})\right|^2 \right)$$

$$C_{fit} = N[\lambda C_{theory} + (1-\lambda)] \times C_{background}$$

R. Molina et al., Eur. Phys. J. C 84 (2024) 328

Observables

scattering length; effective range

$$T = -\frac{8\pi\sqrt{s}}{2M} \frac{1}{-a_0^{-1} + 0.5 r_0 k^2 - ik} \rightarrow a_{0,j} = \frac{2M_j T_{jj}}{8\pi\sqrt{s}}\Big|_{s=s_{th,j}}, \quad r_{0,j} = 2\frac{\partial}{\partial k^2}\left[-\frac{8\pi\sqrt{s}}{2M_j T_{jj}} + ik_j\right]\Big|_{s=s_{th,j}}$$

R. Molina et al., Phys.Rev.D 109 (2024) 054002

Properties of $N^*(1535)$

pole; coupling; decay width

$$\mathrm{Det}[1 - VG]|_{s=s_{\mathrm{pole}}} = 0 \rightarrow T = \frac{g_i g_j}{\sqrt{s} - \sqrt{s_{\mathrm{pole}}}} \rightarrow \Gamma_j = \frac{|g_j|^2}{8\pi} \frac{p_j (E_j + M_j)}{M_{N^*(1535)}}$$

Results: Coupled channel effects

$$C_{theory}(\vec{p}) = 1 + \int_0^\infty d^3\vec{r}\, S(\vec{r}) \left(2\mathrm{Re}\left[T_{ii}(\vec{r},\vec{p})\widetilde{G}_i(\vec{r},\vec{p})j_0(|\vec{p}||\vec{r}|)\right] + \sum_j \left|T_{ij}(\vec{r},\vec{p})\widetilde{G_j}(\vec{r},\vec{p})\right|^2 \right)$$

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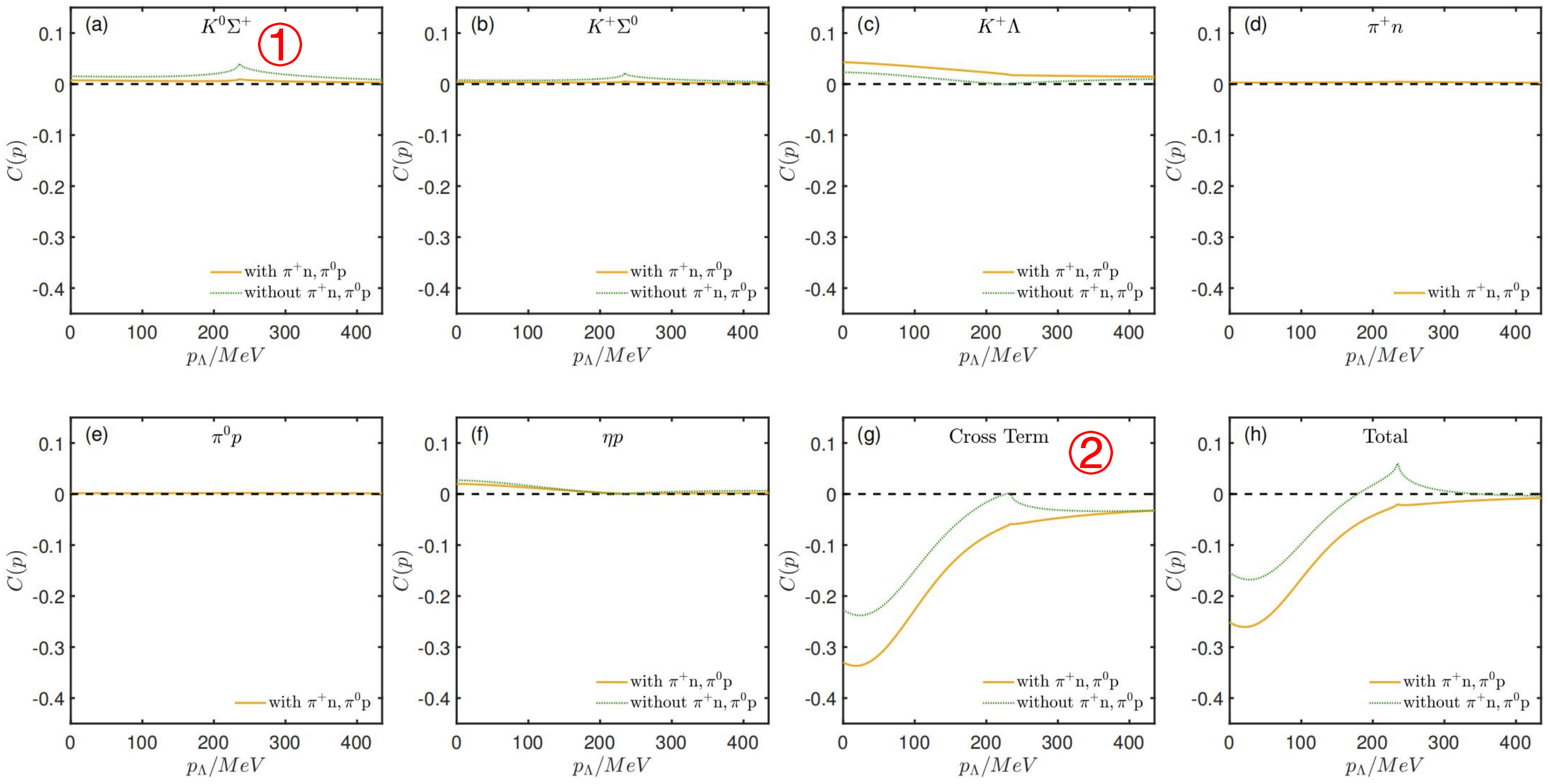


①

a peak associated higher energy channel

②

the main interaction introduced by the cross term $\mathrm{Re}\left[T_{ii}\widetilde{G}_i j_0(|\vec{p}||\vec{r}|)\right]$

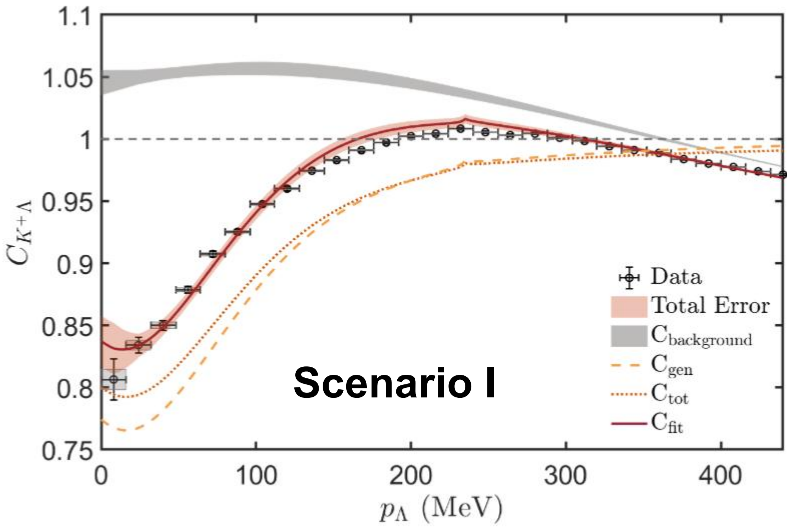
Results: Fitting results

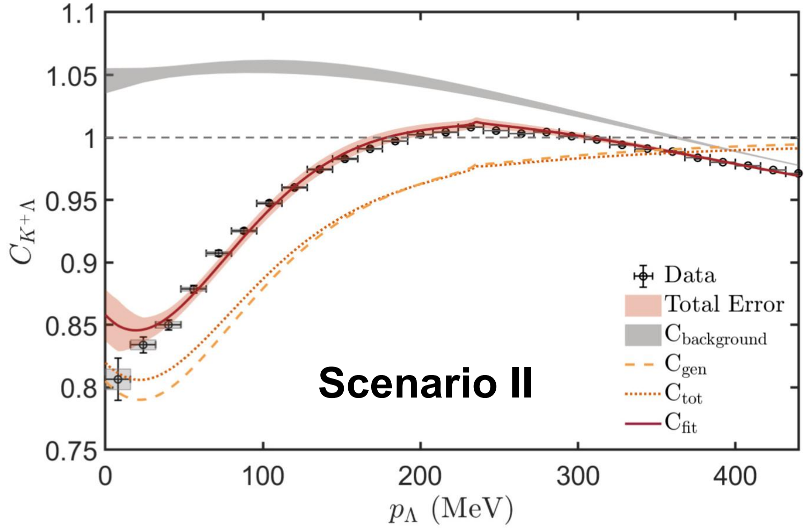
Scenario I: α_i are fixed as in [1], N and λ are free parameters

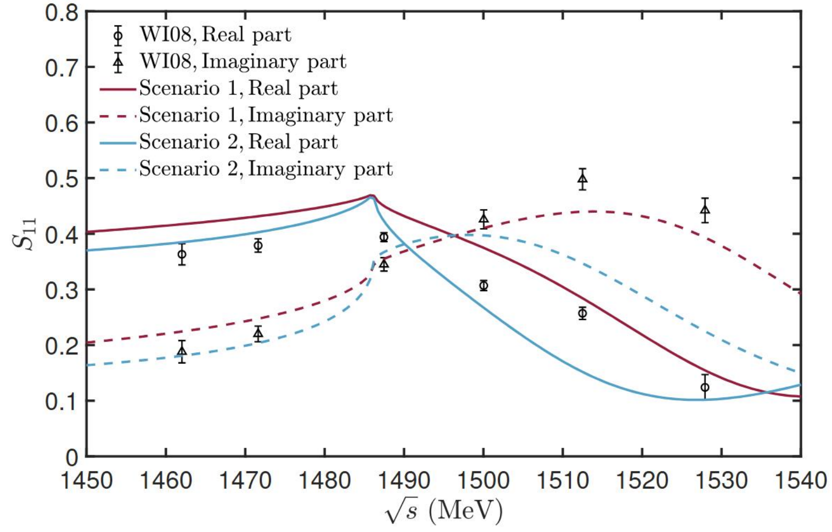
Scenario II: α_i, N, and λ are free parameters

Scenario	I	II
N	0.99554 ± 0.00045	0.99633 ± 0.00032
λ	0.87 ± 0.01	0.91 ± 0.02
$\alpha_{K\Sigma}$	-2.8	−3.17 ± 0.06
$\alpha_{K\Lambda}$	1.6	1.86 ± 0.06
$\alpha_{\eta N}$	0.2	0.76 ± 0.09
$\alpha_{\pi N}$	2.0	1.91 ± 0.04
χ^2/d.o.f.	1.9	1.2

[1]. *T. Inoue et al., Phys.Rev.C 65 (2002) 035204*







Results: Fitting results

Scenario		I	II
$\sqrt{s_{\text{pole}}}$ (MeV)		$1531 - i\,36.5$	$1512 - i\,39.1$
Coupling	$g_{K^0\Sigma^+}$	$2.21 - i\,0.18$	$2.14 - i\,0.34$
	$g_{K^+\Sigma^0}$	$1.56 - i\,0.13$	$1.51 - i\,0.24$
	$g_{K^+\Lambda}$	$1.37 - i\,0.10$	$1.13 - i\,0.16$
	$g_{\pi^+ n}$	$0.55 + i\,0.32$	$0.62 + i\,0.30$
	$g_{\pi^0 p}$	$0.40 + i\,0.22$	$0.44 + i\,0.20$
	$g_{\eta p}$	$-1.47 + i\,0.43$	$-1.63 + i\,0.58$
Partial Width (MeV)	$\Gamma_{\pi^+ n}$	19.7	21.9
	$\Gamma_{\pi^0 p}$	9.9	11.1
	$\Gamma_{\eta p}$	41.2	39.6
Scattering Length (MeV^{-1})	$a_{K^0\Sigma^+}$	$0.28 - i\,0.14$	$0.26 - i\,0.12$
	$a_{K^+\Sigma^0}$	$0.19 - i\,0.13$	$0.18 - i\,0.11$
	$a_{K^+\Lambda}$	$0.33 - i\,0.14$	$0.29 - i\,0.17$
	$a_{\pi^+ n}$	-0.08	-0.08
	$a_{\pi^0 p}$	0.02	0.02
	$a_{\eta p}$	$-0.25 - i\,0.17$	$-0.54 - i\,0.38$
Effective Range (MeV^{-1})	$r_{K^0\Sigma^+}$	$1.05 - i\,1.55$	$0.83 - i\,1.35$
	$r_{K^+\Sigma^0}$	$-1.46 - i\,3.69$	$-1.41 - i\,3.01$
	$r_{K^+\Lambda}$	$-0.52 - i\,0.38$	$0.47 - i\,0.89$
	$r_{\pi^+ n}$	$-13.39 - i\,18.04$	$-13.39 - i\,17.29$
	$r_{\pi^0 p}$	30.34	21.26
	$r_{\eta p}$	$-8.60 + i\,6.77$	$-4.66 + i\,3.02$

1. The mass and width of $N^*(1535)$

	Mass (MeV)	Width (MeV)
PDG (Pole)	1500~1520	80~130
PDG (BW)	1515~1545	125~175

2. The decay widths of $N^*(1535)$

	$\Gamma(N(1535)\to\eta N)/\Gamma tot$	$\Gamma(N(1535)\to\pi N)/\Gamma tot$	$\Gamma 1/\Gamma 2$
PDG	0.30~0.55	0.32~0.52	0.99±0.05±0.19
Scenario 1	0.56	0.41	1.4
Scenario 2	0.51	0.42	1.2

3. The scattering length of $K^+\Lambda$

	[1]	Scenario 1	Scenario 2
$a_{K^+\Lambda}$ (MeV^{-1})	0.61±0.03±0.03	0.33-0.14i	0.29-0.17i

Summary

- We successfully calculated the theoretical $K^+\Lambda$ correlation function based on Koonin-Pratt formalism

- From the analysis of coupled channels, two conclusions can be drawn:
  1. High-energy coupled channels will produce a peak at their thresholds;
  2. The interference term between $K^+\Lambda$ and plane wave contributes the main interaction.

- The correlation function and the scattering experimental data can be mutually verified in general trends, but it is difficult to precisely satisfy both sets of data simultaneously at this stage.

- We obtained the mass, width, and couplings of the $N(1535)$, which can well explain the experimental data under the picture of hadronic molecule.

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Thanks for your listening!