



中国科学院大学
University of Chinese Academy of Sciences



中国科学院高能物理研究所
Institute of High Energy Physics, Chinese Academy of Sciences

Combined study of the isospin-violating decay $D_s^* \rightarrow D_s \pi^0$ and radiative decay $D_s^* \rightarrow D_s \gamma$ with intermediate meson loops

Wang and Zhao *Phys. Rev. D* **111**, 096007

Jun Wang (王俊)¹

In collaboration with Prof. Qiang Zhao (赵强)

Institute of High Energy Physics, Chinese Academy of Sciences

University of Chinese Academy of Sciences

2025 强子物理和有效场论前沿讲习班

2025-08-22, 中国郑州

¹junwang@ihep.ac.cn

Contents



① Introduction

② Formalism

③ Results and Discussions

④ Summary and Outlook



1 Introduction



In 2023, BESIII reported the first determination of the quantum numbers of the D_s^* meson.
 Ablikim et al. Phys. Lett. B 846, 138245

$$D_s^{*\pm}$$

$$I(J^P) = 0(1^-)$$

$J^P = 1^-$ established by ABLIKIM 23AZ.

$D_s^{*\pm}$ MASS

The fit includes D^\pm , D^0 , D_s^\pm , $D^{*\pm}$, D^{*0} , $D_s^{*\pm}$, $D_1(2420)^0$, $D_2^*(2460)^0$,
 and $D_{s1}(2536)^\pm$ mass and mass difference measurements.

<u>VALUE (MeV)</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>
2112.2±0.4 OUR FIT			
2106.6±2.1±2.7	¹ BLAYLOCK	87	MRK3 $e^+e^- \rightarrow D_s^\pm \gamma X$

¹ Assuming D_s^\pm mass = 1968.7 ± 0.9 MeV.



The width of D_s^* is still not determined!

$D_s^{*\pm}$ WIDTH

<u>VALUE (MeV)</u>	<u>CL%</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>
< 1.9	90	GRONBERG 95	CLE2	$e^+ e^-$
< 4.5	90	ALBRECHT 88	ARG	$E_{\text{cm}}^{ee} = 10.2 \text{ GeV}$
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
< 4.9	90	BROWN 94	CLE2	$e^+ e^-$
< 22	90	BLAYLOCK 87	MRK3	$e^+ e^- \rightarrow D_s^\pm \gamma X$



The dominant decay modes of the D_s^* are the radiative decay $D_s^* \rightarrow D_s \gamma$ ($M1$ transition) and the isospin-violating decay $D_s^* \rightarrow D_s \pi^0$.

D_s^{*+} DECAY MODES

D_s^{*-} modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D_s^+ \gamma$	(93.6 \pm 0.4) %
Γ_2	$D_s^+ \pi^0$	(5.77 \pm 0.35) %
Γ_3	$D_s^+ e^+ e^-$	(6.7 \pm 1.6) $\times 10^{-3}$
Γ_4	$e^+ \nu_e$	(2.1 $^{+1.2}_{-0.9}$) $\times 10^{-5}$

BESIII has provided the most precise measurement of branching fraction of $\mathcal{B}(D_s^{*+} \rightarrow D_s^+ \pi^0)$ relative to $\mathcal{B}(D_s^{*+} \rightarrow D_s^+ \gamma)$

$$\mathcal{B}(D_s^{*+} \rightarrow D_s^+ \pi^0)/\mathcal{B}(D_s^{*+} \rightarrow D_s^+ \gamma) = (6.16 \pm 0.43 \pm 0.18)\%. \quad (1)$$

Ablikim et al. Phys. Rev. D 107, 032011

The radiative decay $D_s^* \rightarrow D_s \gamma$ has been well defined in the constituent quark model and there have been a lot of studies of the radiative transition.

Aliiev, Iltan, and Pak Phys. Lett. B 334, 169–174

Yu, Li, and Wang Eur. Phys. J. C 75, 243

Deng, Chen, and Deng Chin. Phys. C 38, 013103

Donald et al. Phys. Rev. Lett. 112, 212002

Fayyazuddin and Mobarek Phys. Rev. D 48, 1220–1224

Goity and Roberts Phys. Rev. D 64, 094007

Kamal and Xu Phys. Lett. B 284, 421–426

Wang et al. Phys. Rev. D 100, 016019

Cheng et al. Phys. Rev. D 49, 5857–5881

Choi J. Korean Phys. Soc. 53, 1205

Tran et al. Chin. Phys. C 48, 023103

Cheung and Hwang Eur. Phys. J. C 76, 19

Meng et al. Phys. Rev. D 109, 074511

The isospin-violating decay $D_s^* \rightarrow D_s \pi^0$ has not yet been broadly investigated. Several works have been dedicated to this issue based on the $\eta - \pi^0$ mixing.

Cho and Wise Phys. Rev. D 49, 6228–6231

Ivanov arXiv hep-ph/9805347,

Terasaki arXiv 1511.05249,

Cheung and Hwang Eur. Phys. J. C 76, 19

Recently, a heavy meson chiral perturbation theory calculation was presented with the $\mathcal{O}(p^3)$ loop corrections, and it was found that the $\mathcal{O}(p^3)$ corrections may actually be significant. [Yang et al. Phys. Rev. D 101, 054019.](#)



Our Work

The isospin-violating decay $D_s^{*+} \rightarrow D_s^+ \pi^0$ and the radiative decay $D_s^{*+} \rightarrow D_s^+ \gamma$ are studied simultaneously with the intermediate meson loops.

The isospin-violating decay $D_s^{*+} \rightarrow D_s^+ \pi^0$

$\eta - \pi^0$ mixing as the leading order and $\mathcal{D}^{(*)} \mathcal{K}^{(*)}$ rescatterings with or without the $\eta - \pi^0$ mixing as the next-to-leading order are considered.

The radiative decay $D_s^{*+} \rightarrow D_s^+ \gamma$

In the same frame work of the isospin-violating decay $D_s^{*+} \rightarrow D_s^+ \pi^0$ with the vector-meson-dominance(VMD) model.



2 Formalism



Isospin-violating through $\eta - \pi^0$ mixing

For the process $D_s^{*+} \rightarrow D_s^+ \eta$, the corresponding effective Lagrangian is

$$\mathcal{L}_{D_s^* D_s \eta} = i g_{\mathcal{D}^* \mathcal{D} \mathcal{P}} \sin \alpha_P D_s (D_s^*)_\mu \partial^\mu \eta. \quad (2)$$

where $\alpha_P = 40.6^\circ$ is the $\eta - \eta'$ mixing angle in the SU(3) flavor basis.

For the $\eta \rightarrow \pi^0$ process, we use the $\eta - \pi^0$ mixing angle $\theta_{\eta\pi^0}$ which is given by the leading order chiral expansion

$$\tan(2\theta_{\eta\pi^0}) = \frac{\sqrt{3} m_d - m_u}{2 m_s - \hat{m}}. \quad (3)$$

where $\hat{m} = (m_u + m_d)/2$. [Gasser and Leutwyler Nucl. Phys. B 250, 465–516](#)

Since $\theta_{\eta\pi^0}$ is very small, we take

$$\theta_{\eta\pi^0} \simeq \frac{\sqrt{3} m_d - m_u}{4 m_s - \hat{m}}. \quad (4)$$



Effective Lagrangians

$$\begin{aligned}
 \mathcal{L} = & -ig_{\mathcal{D}^*\mathcal{D}\mathcal{P}}(\mathcal{D}^i\partial^\mu\mathcal{P}_{ij}\mathcal{D}_\mu^{*j\dagger} - \mathcal{D}_\mu^{*i}\partial^\mu\mathcal{P}_{ij}\mathcal{D}^{j\dagger}) + \frac{1}{2}g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}}\epsilon_{\mu\nu\alpha\beta}\mathcal{D}_i^{*\mu}\partial^\nu\mathcal{P}^{ij}\overleftrightarrow{\partial}^\alpha\mathcal{D}_j^{*\beta\dagger} \\
 & -ig_{\mathcal{D}\mathcal{D}\mathcal{V}}\mathcal{D}_i^\dagger\overleftrightarrow{\partial}^\mu\mathcal{D}^j(\mathcal{V}^\mu)_j^i - 2f_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathcal{V}^\nu)_j^i(\mathcal{D}_i^\dagger\overleftrightarrow{\partial}^\alpha\mathcal{D}^{*\beta j} - \mathcal{D}_i^{*\beta\dagger}\overleftrightarrow{\partial}^\alpha\mathcal{D}^j) \\
 & +ig_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_i^{*\nu\dagger}\overleftrightarrow{\partial}^\mu\mathcal{D}_\nu^{*j}(\mathcal{V}^\mu)_j^i + 4if_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_{i\mu}^{*\dagger}(\partial^\mu\mathcal{V}^\nu - \partial^\nu\mathcal{V}^\mu)_j^i\mathcal{D}_\nu^{*j},
 \end{aligned} \tag{5}$$

where \mathcal{D} and \mathcal{D}^* represent the pseudoscalar and vector charm meson fields, respectively, i.e.

$$\mathcal{D} = (D^0, D^+, D_s^+), \quad \mathcal{D}^* = (D^{*0}, D^{*+}, D_s^{*+}), \tag{6}$$

\mathcal{P} and \mathcal{V} are 3×3 matrices representing the pseudoscalar nonet and vector nonet meson fields

$$\mathcal{P} = \begin{pmatrix} \frac{\sin\alpha_P\eta' + \cos\alpha_P\eta + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\sin\alpha_P\eta' + \cos\alpha_P\eta - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \cos\alpha_P\eta' - \sin\alpha_P\eta \end{pmatrix} \mathcal{V} = \begin{pmatrix} \frac{\rho^0 + \omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \tag{7}$$

Casalbuoni et al. Phys. Rept. 281, 145–238 Cheng, Chua, and Soni Phys. Rev. D 71, 014030



For the light hadron vertices, we adopt the following effective Lagrangian:

$$\begin{aligned}
 \mathcal{L}_{VPP} &= ig_{VPP} \text{Tr}[(\mathcal{P}\partial_\mu\mathcal{P} - \partial_\mu\mathcal{P}\mathcal{P})\mathcal{V}^\mu], \\
 \mathcal{L}_{VVP} &= g_{VVP}\varepsilon_{\alpha\beta\mu\nu} \text{Tr}[\partial^\alpha\mathcal{V}^\mu\partial^\beta\mathcal{V}^\nu\mathcal{P}], \\
 \mathcal{L}_{VVV} &= ig_{VVV} \text{Tr}[(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu].
 \end{aligned} \tag{8}$$

The following coupling constants are adopted:

$$\begin{aligned}
 g_{D^*D^*\pi} &= \frac{g_{D^*D\pi}}{\sqrt{m_D m_{D^*}}} = \frac{2g}{f_\pi}, \quad g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}}, \quad f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}, \\
 g_{D^*D_s K} &= \sqrt{\frac{m_{D_s}}{m_D}} g_{D^*D\pi}, \quad g_{D_s DV} = \sqrt{\frac{m_{D_s}}{m_D}} g_{DDV}, \quad \frac{g_{D_s^* D_s \eta}}{\sqrt{m_{D_s^*} m_{D_s}}} = \frac{g_{D^*D\pi} \sin \alpha_P}{\sqrt{m_{D^*} m_D}} \\
 g_{D_s^* DK} &= \sqrt{\frac{m_{D_s^*}}{m_{D^*}}} g_{D^*D\pi}, \quad g_V = \frac{m_\rho}{f_\pi}, \quad g_{D^*D^*K} = \frac{g_{D^*DK}}{\sqrt{m_D m_{D^*}}} = \frac{2g}{f_K}, \\
 g_{D^*D_s^* K^*} &= \sqrt{\frac{m_{D_s^*}}{m_{D^*}}} g_{D^*D^*V}, \quad f_{D^*D_s^* K^*} = \sqrt{\frac{m_{D_s^*}}{m_{D^*}}} f_{D^*D^*V}, \cdot
 \end{aligned} \tag{9}$$

where $g = 0.59$, $\beta = 0.9$, $f_\pi = 132$ MeV, $f_K = 155$ MeV and $\lambda = 0.56$ GeV⁻¹.

Cheng, Chua, and Soni Phys. Rev. D 71, 014030 Zhang, Wang, Li, and Zhao Phys. Rev. D 85, 074015
 Li, and Zhao Phys. Rev. Lett. 102, 172001 Isola et al. Phys. Rev. D 68, 114001



The relative strengths and phases of the coupling constants for vector and scalar mesons can be determined by $SU(3)$ flavor symmetry relations, and expressed by overall coupling constants g_{VVP} and g_{VPP} [Cao and Zhao Phys. Rev. D 109, 093005](#),

$$\begin{aligned}
 g_{K^{*+}K^{*-}\pi^0} &= -g_{K^{*0}\bar{K}^{*0}\pi^0} = \frac{1}{\sqrt{2}}g_{VVP}, \\
 g_{K^{*0}\bar{K}^0\pi^0} &= -g_{K^{*0}\pi^0\bar{K}^0} = g_{\bar{K}^{*0}\pi^0 K^0} = -g_{\bar{K}^{*0}K^0\pi^0} = \frac{1}{\sqrt{2}}g_{VPP}, \\
 g_{K^{*+}\pi^0 K^-} &= -g_{K^{*+}K^-\pi^0} = g_{K^{*-}K^+\pi^0} = -g_{K^{*-}\pi^0 K^+} = \frac{1}{\sqrt{2}}g_{VPP}.
 \end{aligned} \tag{10}$$

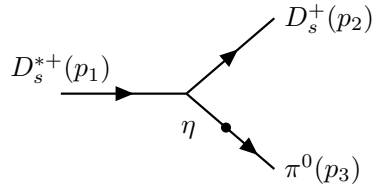


Tree and loop transition amplitudes of $D_s^{*+} \rightarrow D_s^+ \pi^0$

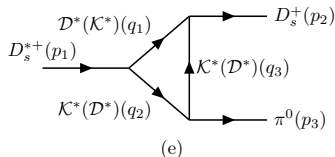
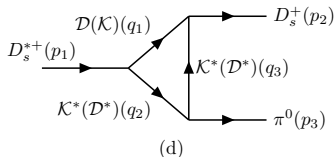
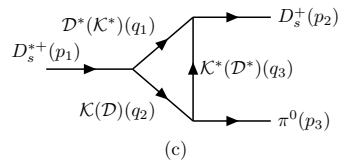
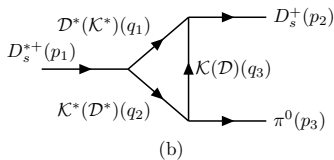
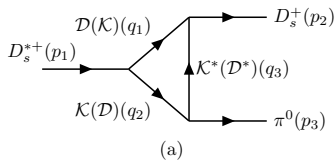
The tree-level amplitude is given by

$$i\mathcal{M}_{\text{tree}} = ig_{D_s^* D_s \eta} \varepsilon_{D_s^*} \cdot p_3 \theta_{\eta \pi^0} = ig_{\text{tree}} \varepsilon_{D_s^*} \cdot (p_2 - p_3),$$

where $g_{\text{tree}} \equiv g_{D_s^* D_s \eta} \theta_{\eta \pi^0} / 2$.



The loop diagrams at one-loop level without $\eta - \pi^0$ mixing:

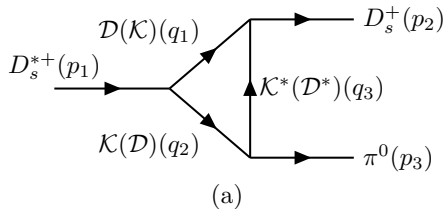




For example , for the first diagram, the transition amplitudes are given below, with a UV cutoff

$$\mathcal{F}(q_i^2) = \prod_i \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q_i^2} \right), \text{ and } \Lambda_i \equiv m_i + \alpha \Lambda_{\text{QCD}},$$

$$\Lambda_{\text{QCD}} = 220 \text{MeV}.$$

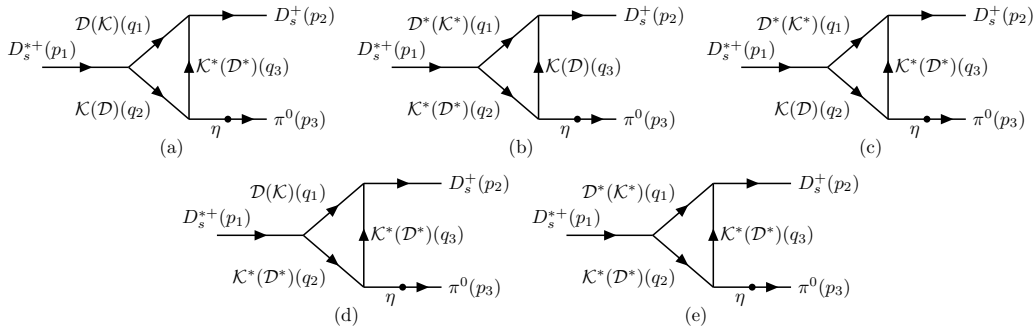


$$i\mathcal{M}_a(\mathcal{D}, \mathcal{K}, \mathcal{K}^*) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{D}_s^* \mathcal{D} \mathcal{K} q_2} \cdot \varepsilon^s g_{\mathcal{D}_s \mathcal{D} \mathcal{K}^*} (q_1 + p_2)_\alpha \left(g^{\alpha\beta} - \frac{q_3^\alpha q_3^\beta}{m_3^2} \right) g_{\mathcal{K}^* \mathcal{K} \pi} (p_3 + q_2)_\beta}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \mathcal{F}(q_i^2), \quad (11)$$

$$i\mathcal{M}_a(\mathcal{K}, \mathcal{D}, \mathcal{D}^*) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{D}_s^* \mathcal{D} \mathcal{K} q_1} g_{\mathcal{D}_s \mathcal{D}^* \mathcal{K} q_2} g_{\mathcal{D}^* \mathcal{D} \pi} q_1 \cdot \varepsilon^s q_1^\mu \left(g_{\mu\nu} - \frac{q_{3\mu} q_{3\nu}}{m_3^2} \right) p_3^\nu}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \mathcal{F}(q_i^2). \quad (12)$$



The loop diagrams at one-loop level with $\eta - \pi^0$ mixing:



$$\begin{aligned}
 i\mathcal{M}_a(\mathcal{D}, \mathcal{K}, \mathcal{K}^*, \eta) &= i\mathcal{M}_a(\mathcal{D}, \mathcal{K}, \mathcal{K}^*) \cdot \frac{g_{\mathcal{K}\mathcal{K}^*\eta}\theta_{\eta\pi^0}}{g_{\mathcal{K}\mathcal{K}^*\pi}}, \\
 i\mathcal{M}_a(\mathcal{K}, \mathcal{D}, \mathcal{D}^*, \eta) &= i\mathcal{M}_a(\mathcal{K}, \mathcal{D}, \mathcal{D}^*) \cdot \frac{g_{\mathcal{D}^*\mathcal{D}\eta}\theta_{\eta\pi^0}}{g_{\mathcal{D}^*\mathcal{D}\pi}}.
 \end{aligned}
 \tag{13}$$



Power counting of the loop amplitudes:

Assuming the intermediate mesons are close to being on-shell, the propagator reads

$$1/(t - m_3^2) \simeq -1/m_3^2 \quad (14)$$

The integrand counts:

$$(v^5/v^4) \times p_\pi \cdot p_{D_s} / m_3^2 \simeq v E_{D_s} E_\pi / m_3^2 \quad (15)$$

For the two loop amplitudes which cancel each other, we have

$$v E_{D_s} E_\pi (1/m_{K^{*\pm}}^2 - 1/m_{K^{*0}}^2) \simeq v E_\pi \delta_{K^*} / m_{K^*}^2 \simeq v \underbrace{(m_\pi / m_{K^*})}_{\text{loop power suppression}} \underbrace{(\delta_{K^*} / m_{K^*})}_{\text{isospin breaking term}}, \quad (16)$$

with $\delta_{K^*} \equiv m_{K^{*0}} - m_{K^{*\pm}}$. The loop amplitude can be expressed as

$$i\mathcal{M}_{\text{loop}} = ig_{\text{loop}} \varepsilon_{D_s^{*+}} \cdot (p_2 - p_3). \quad (17)$$

The total amplitude can be written as

$$i\mathcal{M}_{D_s^{*+} \rightarrow D_s^+ \pi^0} = i(g_{\text{tree}} + g_{\text{loop}}) \varepsilon_{D_s^{*+}} \cdot (p_{D_s^+} - p_{\pi^0}) \equiv ig_{\text{total}} \varepsilon_{D_s^{*+}} \cdot (p_{D_s^+} - p_{\pi^0}). \quad (18)$$

Tree and loop amplitudes of $D_s^{*+} \rightarrow D_s^+ \gamma$ in the VMD model



The radiative decay of D_s^{*+} and its loop correction can be evaluated in the same framework

$$i\mathcal{M}_{\text{tree}}^\gamma = ig_{\text{tree}}^\gamma(\gamma)\epsilon_{\mu\nu\alpha\beta}p_1^\mu p_3^\nu \epsilon_1^\alpha \epsilon_3^\beta, \quad (19)$$

where g_{tree}^γ can be calculated using the VMD model,

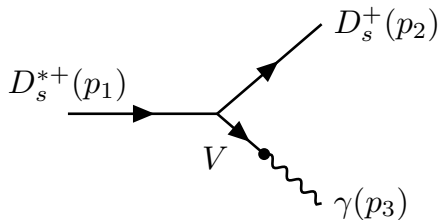
$$g_{\text{tree}}^\gamma = ig_{D_s^* D_s V} \frac{em_V^2}{f_V} G_V \quad (20)$$

with

$$G_V \equiv \frac{-i}{p_\gamma^2 - m_V^2 + im_V \Gamma_V} = \frac{-i}{-m_V^2 + im_V \Gamma_V}. \quad (21)$$

where $V = \rho, \omega, \phi$, and e/f_V can be determined by experimental data of $\Gamma_{V \rightarrow e^+e^-}$:

$$\frac{e}{f_V} = \left[\frac{3\Gamma_{V \rightarrow e^+e^-}}{2\alpha_e |\mathbf{p}_e|} \right]^{\frac{1}{2}} \quad (22)$$





To match the coupling constants in HQEFT, we have

$$g_{D_s^* D_s V} = 4f_{D_s^* D_s V}. \quad (23)$$

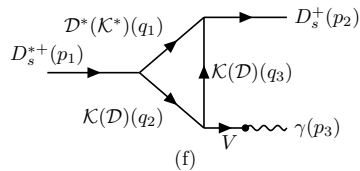
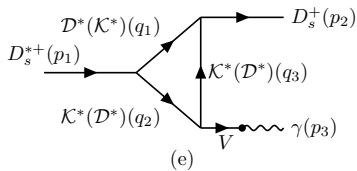
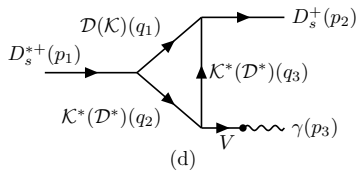
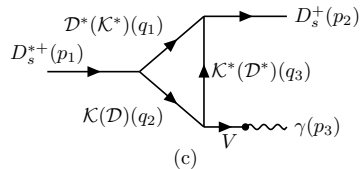
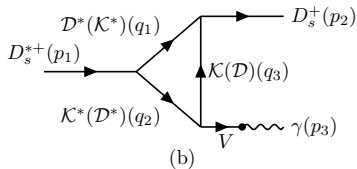
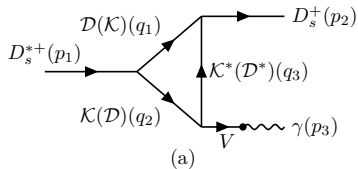
For the tree level $V = \phi, J/\psi$, the coupling constant can be extracted:

$$g_{D_s^* D_s \gamma} = i \left(g_{D_s^* D_s \phi} \frac{em_\phi^2}{f_\phi} G_\phi + g_{D_s^* D_s \psi} \frac{em_\psi^2}{f_\psi} G_\psi \right), \quad (24)$$

with $g_{D_s^* D_s \psi} = 2g_{J\psi D\bar{D}}/\tilde{M}$, $\tilde{M} = \sqrt{M_{D_s^*} M_{D_s}}$
 Zhang, Li, and Zhao *Phys. Rev. Lett.* **102**, 172001.



Loop amplitudes based on the VMD:

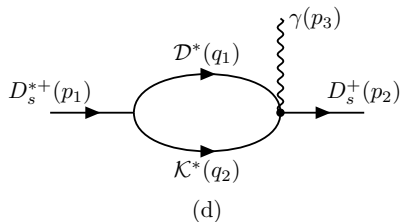
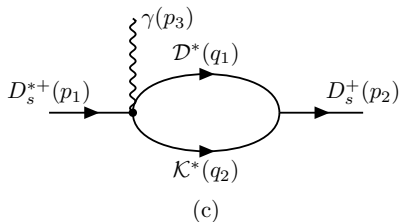
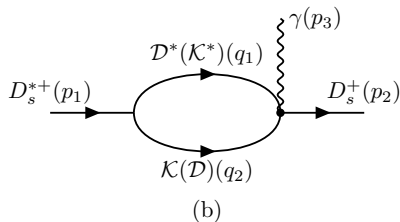
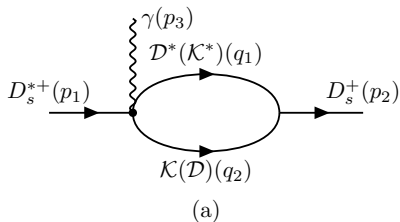


$$i\mathcal{M}_a(\mathcal{D}, \mathcal{K}, \mathcal{K}^*, \gamma) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{K}\mathcal{K}^*\gamma} g_{\mathcal{D}_s^* \mathcal{D}\mathcal{K}} g_{\mathcal{D}_s \mathcal{D}\mathcal{K}^*} q_2 \cdot \varepsilon^s (q_1 + p_2)_\alpha \left(g^{\alpha\beta} - \frac{q_3^\alpha q_3^\beta}{m_3^2} \right) \varepsilon_{\beta\sigma\mu\nu} \varepsilon_3^\sigma q_3^\mu p_3^\nu}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \mathcal{F}(q_i^2),$$

$$i\mathcal{M}_a(\mathcal{K}, \mathcal{D}, \mathcal{D}^*, \gamma) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{D}_s^* \mathcal{D}\mathcal{K}} g_{\mathcal{D}^* \mathcal{D}_s \mathcal{K}} g_{\mathcal{D}^* \mathcal{D}\gamma} q_1 \cdot \varepsilon^s q_1^\mu \left(g_{\mu\nu} - \frac{q_{3\mu} q_{3\nu}}{m_3^2} \right) \varepsilon_{\nu\sigma\alpha\beta} \varepsilon_3^\sigma q_3^\alpha p_3^\beta}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \mathcal{F}(q_i^2).$$



Contact diagrams induced by the requirement of gauge invariance:



(a) ✓, (b) ×, (c) ×, (d) ×



Meson-photon coupling constants in the VMD model:

$$\begin{aligned}
 g_{K^+K^+\gamma} &= i(g_{\rho^0 K^+K^-} \frac{em_{\rho^0}^2}{f_{\rho^0}} G_{\rho^0} + g_{\omega K^+K^-} \frac{em_{\omega}^2}{f_{\omega}} G_{\omega} + g_{\phi K^+K^-} \frac{em_{\phi}^2}{f_{\phi}} RG_{\phi}) , \\
 g_{K^0K^0\gamma} &= i(g_{\rho^0 K^0\bar{K}^0} \frac{em_{\rho^0}^2}{f_{\rho^0}} G_{\rho^0} + g_{\omega K^0\bar{K}^0} \frac{em_{\omega}^2}{f_{\omega}} G_{\omega} + g_{\phi K^0\bar{K}^0} \frac{em_{\phi}^2}{f_{\phi}} RG_{\phi}) , \\
 g_{D^+D^+\gamma} &= i(g_{\rho^0 D^+D^-} \frac{em_{\rho^0}^2}{f_{\rho^0}} G_{\rho^0} + g_{\omega D^+D^-} \frac{em_{\omega}^2}{f_{\omega}} G_{\omega} + g_{\phi D^+D^-} \frac{em_{\phi}^2}{f_{\phi}} RG_{\phi}) , \\
 g_{D^0D^0\gamma} &= i(g_{\rho^0 D^0\bar{D}^0} \frac{em_{\rho^0}^2}{f_{\rho^0}} G_{\rho^0} + g_{\omega D^0\bar{D}^0} \frac{em_{\omega}^2}{f_{\omega}} G_{\omega} + g_{\phi D^0\bar{D}^0} \frac{em_{\phi}^2}{f_{\phi}} RG_{\phi}) .
 \end{aligned} \tag{25}$$

where $R = 0.8$ is the $SU(3)$ flavor-symmetry-breaking parameter for process involving the strange quark pair.

The total radiative decay amplitude can be expressed as

$$i\mathcal{M}_{D_s^{*+} \rightarrow D_s^+ \gamma} = i(g_{\text{tree}}^{\gamma} + g_{\text{loop}}^{\gamma}) \varepsilon_{\mu\nu\alpha\beta} p_1^{\mu} p_3^{\nu} \varepsilon_1^{\alpha} \varepsilon_3^{\beta} \equiv ig_{\text{total}}^{\gamma} \varepsilon_{D_s^{*+}} \cdot (p_{D_s^+} - p_{\pi^0}) . \tag{26}$$



3 Results and Discussions



The numerical values of the coupling constants

Table: The values of the VPP coupling constants.

Coupling Constant	$g_{D_s^{*+}D^0K}$	$g_{D_s^+D^0K^*}$	$g_{D^{*0}D^0\pi}$	$g_{D^{*-}D^+\pi}$	$g_{D_s^{*+}D^+K}$	$g_{D^{*0}D_s^+K}$	$g_{D_s^+D^+K^*}$	$g_{D^{*+}D_s^+K}$	g_{VPP}
Numerical Value	18.40	3.84	17.29	-17.33	18.42	17.77	3.84	17.78	4.18

Table: The values of the VVP coupling constants.

Coupling Constant	$g_{D_s^{*+}D^{*0}K}$	$f_{D^{*0}D_s^+K^*}$	$f_{D_s^*D^0K^*}$	$g_{D^{*0}\bar{D}^{*0}\pi^0}$	$g_{D^{*+}D^{*-}\pi^0}$	g_{VVP}
Numerical Value(GeV^{-1})	7.81	2.38	2.47	8.94	-8.94	7.93



Table: The values of the VVV coupling constants.

Coupling Constant	g_{VVV}	$g_{D_s^{*+}D^{*0}K^*}$	$f_{D_s^*D^{*0}K^*}$
Numerical Value	4.47	3.83	4.79

Table: The values of the electromagnetic coupling constants in VMD model.

Coupling Constant	$g_{K^{*+}K^+\gamma}$	$g_{K^{*0}K^0\gamma}$	$g_{D^{*0}D^0\gamma}$	$g_{D^{*+}D^+\gamma}$
Numerical Value(GeV^{-1})	$-0.288 + 0.063i$	$0.369 + 0.062i$	$-0.383 - 0.082i$	$0.492 + 0.082i$
Coupling Constant	$g_{K^{*+}K^{*+}\gamma}$	$g_{K^{*0}K^{*0}\gamma}$	$g_{D^{*0}D^{*0}\gamma}(f_{D^{*0}D^{*0}\gamma})$	$g_{D^{*+}D^{*+}\gamma}(f_{D^{*+}D^{*+}\gamma})$
Numerical Value	$-0.162 - 0.035i$	$0.208 + 0.034i$	$-0.139 - 0.031i(-0.695 - 0.152i)$	$0.178 + 0.030i(0.892 + 0.150i)$
Coupling Constant	$g_{K^+K^+\gamma}$	$g_{K^0K^0\gamma}$	$g_{D^0D^0\gamma}$	$g_{D^+D^+\gamma}$
Numerical Value	$-0.288 + 0.063i$	$0.369 + 0.062i$	$-0.383 - 0.082i$	$0.492 + 0.082i$



Numerical results

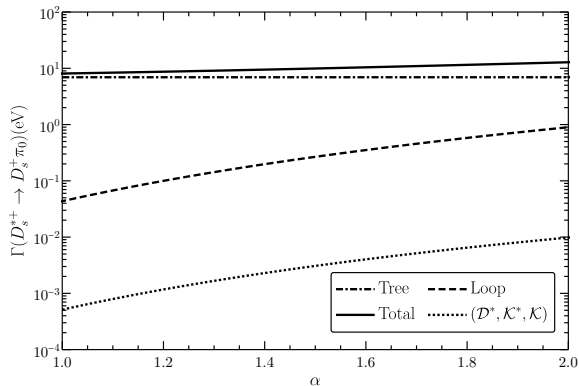


Table: Comparisons of the partial decay width of $D_s^{*+} \rightarrow D_s^+ \pi^0$. The decay widths are in the units of eV. The uncertainties of our result are given by $\alpha = 1.5 \pm 0.15$.

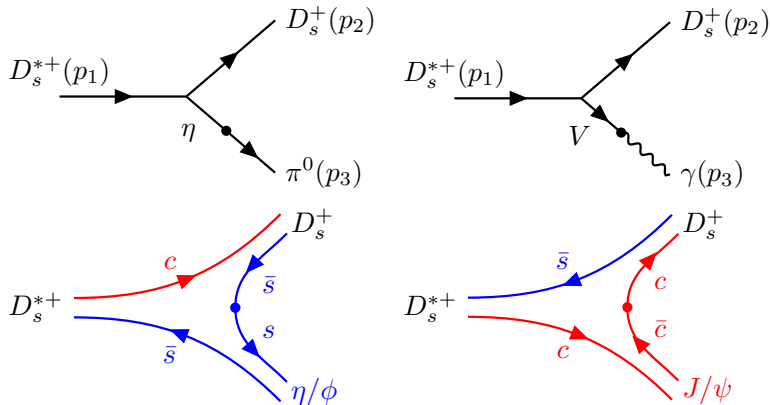
	CM ^a	χ PT ^b	Our work
$\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)$	277^{+28}_{-26}	$8.1^{+3.0}_{-2.6}$	$9.92^{+0.76}_{-0.66}$

^aCheung and Hwang Eur. Phys. J. C 76, 19

^bYang et al. Phys. Rev. D 101, 054019



Connection and difference between $D_s^{*+} \rightarrow D_s^+ \pi^0$ and $D_s^{*+} \rightarrow D_s^+ \gamma$



Cancellation occurs between the ϕ and J/ψ term.

The main uncertainties come from the $g_{J/\psi D \bar{D}}$ coupling with $g_{J/\psi D \bar{D}} = 7.0 \sim 7.5$. Precise measurement of D_s^{*+} width can provide a strong constraint on this coupling.



Table: Experimentally measured branching ratios and relative branching ratios of $D_s^{*+} \rightarrow D_s^+ \pi^0$ and $D_s^{*+} \rightarrow D_s^+ \gamma$.

	$\text{BR}(D_s^{*+} \rightarrow D_s^+ \gamma)$	$\text{BR}(D_s^{*+} \rightarrow D_s^+ \pi^0)$	$\text{BR}(D_s^{*+} \rightarrow D_s^+ \pi^0)/\text{BR}(D_s^{*+} \rightarrow D_s^+ \gamma)$
PDG ²	$(93.6 \pm 0.4)\%$	$(5.77 \pm 0.35)\%$	$(6.2 \pm 0.4)\%$
BESIII ³	$(93.54 \pm 0.38 \pm 0.22)\%$	$(5.76 \pm 0.38 \pm 0.16)\%$	$(6.16 \pm 0.43 \pm 0.18)\%$

Table: Contributions of the tree diagram, loop diagrams, and the combination of tree and loop diagrams to the partial decay width of $D_s^{*+} \rightarrow D_s^+ \gamma$ with $\alpha = 1.0, 1.35, 1.5, 1.65$ and 2.0 in unit of eV and $g_{J/\psi D \bar{D}} = 7.23$.

α	1.0	1.35	1.5	1.65	2.0
Γ_{tree}	153.89	153.89	153.89	153.89	153.89
Γ_{loop}	0.54	2.22	3.59	5.55	13.14
Γ_{total}	155.30	157.84	159.69	162.16	171.15

²Navas et al. Phys. Rev. D 110, 030001

³Ablikim et al. Phys. Rev. D 107, 032011



Table: The partial decay widths of $D_s^{*+} \rightarrow D_s^+ \pi^0$ and $D_s^{*+} \rightarrow D_s^+ \gamma$, as well as the total width of D_s^{*+} as the sum of both, for $\alpha = 1.5 \pm 0.15$ with $g_{J/\psi D \bar{D}}$ fixed at 7.23, and for $g_{J/\psi D \bar{D}} = 7.23 \pm 0.06$.

	$\Gamma(D_s^{*+} \rightarrow D_s^+ \pi^0)$	$\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)$	$\Gamma_{\text{total}}(D_s^{*+})$
$\alpha = 1.5 \pm 0.15, g_{J/\psi D \bar{D}} = 7.23$	$9.92^{+0.76}_{-0.66}$	$159.7^{+2.5}_{-1.8}$	$169.6^{+2.6}_{-2.0}$
$\alpha = 1.5 \pm 0.15, g_{J/\psi D \bar{D}} = 7.23 \pm 0.06$	$9.92^{+0.76}_{-0.66}$	160^{+13}_{-12}	170^{+13}_{-12}

Table: The partial decay width of $D_s^{*+} \rightarrow D_s^+ \gamma$ and the total width of D_s^{*+} obtained using the experimentally measured relative branching ratio and the calculated width of $D_s^{*+} \rightarrow D_s^+ \pi^0$ with $\alpha = 1.5$.

	$\Gamma(D_s^{*+} \rightarrow D_s^+ \gamma)$	$\Gamma_{\text{total}}(D_s^{*+})$
PDG ⁴	160^{+10}_{-10} eV	172^{+10}_{-10} eV
BESIII ⁵	161^{+16}_{-16} eV	172^{+16}_{-16} eV

⁴Navas et al. Phys. Rev. D 110, 030001

⁵Ablikim et al. Phys. Rev. D 107, 032011



4 Summary and Outlook



Summary and Outlook

- Isospin-violating decay $D_s^* \rightarrow D_s \pi^0$ and radiative decay $D_s^* \rightarrow D_s \gamma$ are studied in the same framework.
- Loop corrections are crucial for the $D_s^* \rightarrow D_s \pi^0$ decay, while they are negligible in the radiative decay.
- With the experimental data for the branching ratio fraction, we obtain a better constraint on the coupling $g_{J/\psi D \bar{D}}$.
- These mechanisms can also help us understand other isospin-violating decay processes⁶.
- The decay $D_s^* \rightarrow D_s \pi_0$ is near the threshold of D_s^* , understanding this decay channel will aid in comprehending other near-threshold dynamics.
- Future precise measurement of the total width of D_s^* at BESIII is strongly recommended.

⁶Study of $B_c(1P)^+ \rightarrow B_c^{(*)+} \pi^0$ [Wang and Zhao arXiv hep-ph, 2507.19952](#)



Thanks