Form factors for $D_s \to \phi l \nu$ semileptonic decay from 2+1-flavor lattice QCD

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Outline

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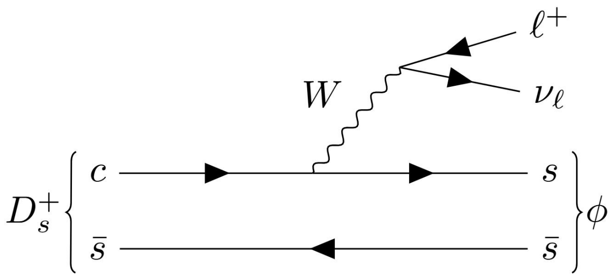
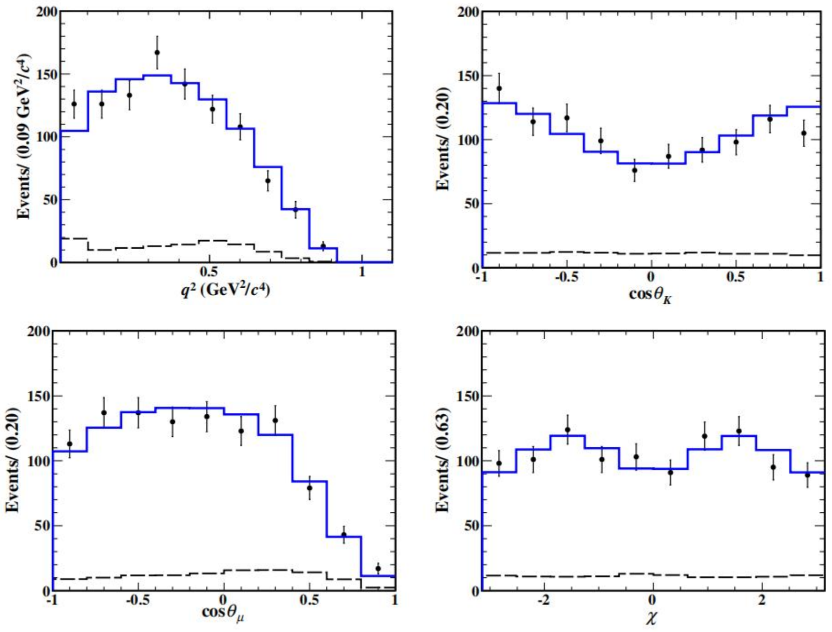
- Introduction to lattice QCD
- Lattice set up
- Methods
- Results
- Summary

Motivations



- Semi-leptonic decays offer an ideal place to deeply understand hadronic transitions in the nonperturbative region of QCD, and can help to explore the weak and strong interactions in charm sector
- Vector meson decay makes this transition notoriously difficult to model due to theoretical complexity
- Combining with the experimental data, the CKM matrix element can be extracted, and it helps to test unitarity of CKM matrix and search for new physics beyond SM
- Calculating branching fractions helps to test $\mu - e$ lepton flavor universality



[BESIII, JHEP 12, 072 (2023)]

SM parameter

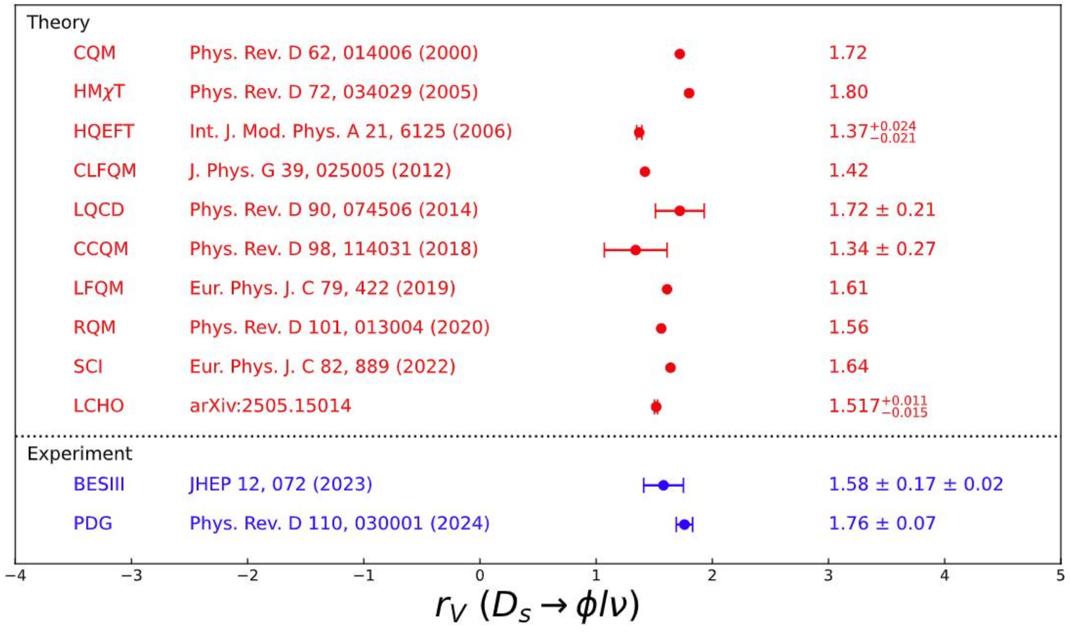
$$\frac{\mathrm{d}\Gamma(D_s \to \phi \ell \nu)}{\mathrm{d}q^2} = \frac{G_F^2 |V_{cs}|^2 |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\left(|H_+|^2 + |H_-|^2 + |H_0|^2\right) + \frac{3m_\ell^2}{2q^2}|H_t|^2\right]$$

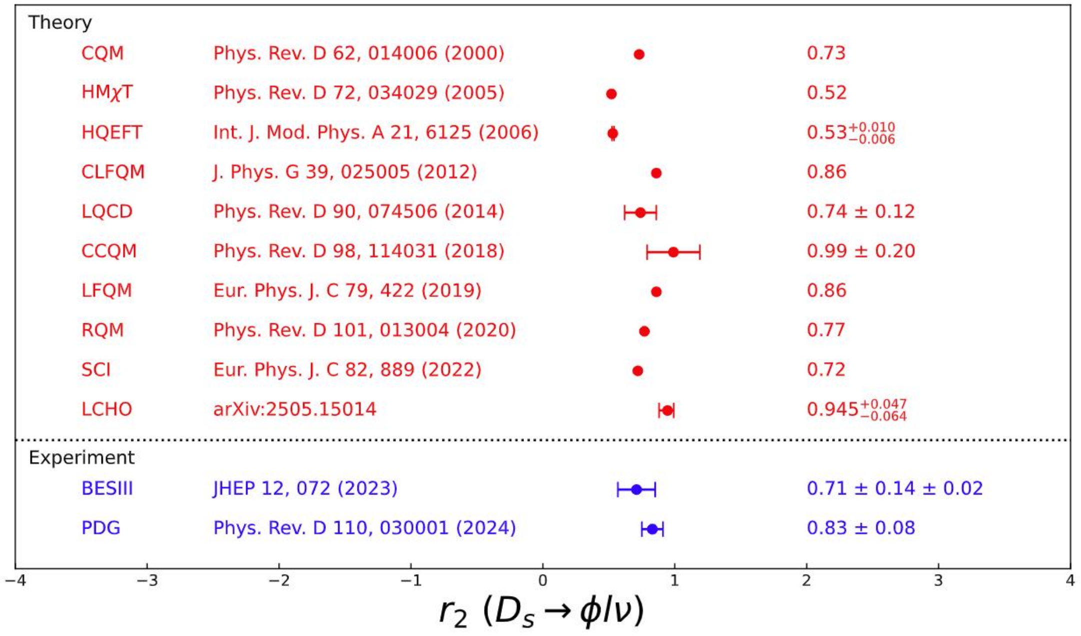
Experimental data

non-perturbative

Motivations

- Status of theoretical and experimental studies





A precise lattice calculation is important!

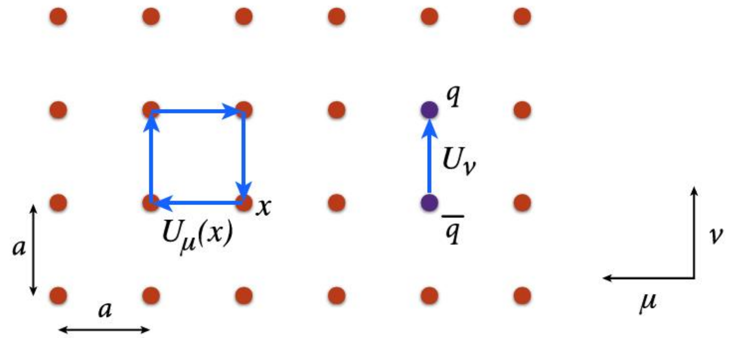
- Provide lattice QCD input to investigate the SU(3) symmetry (by combining with $D \to K^* l\nu$ calculation)

Introduction to lattice QCD

- Path integral in discrete Euclidean space

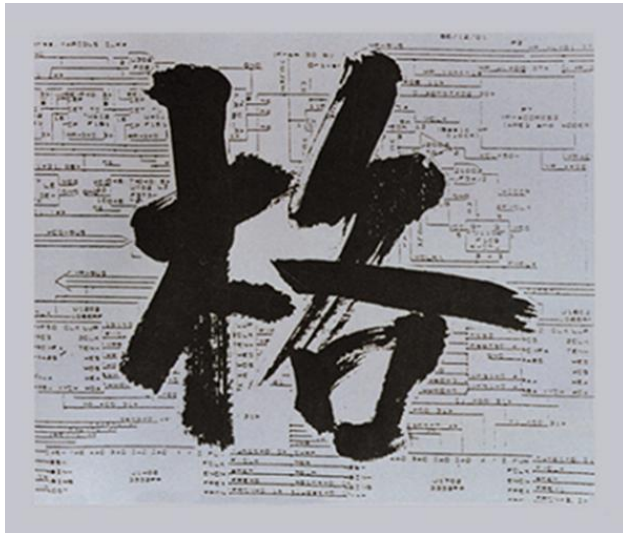
$$Z = \int [dU] \prod_f [dq_f][d\bar{q}_f] e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

$$Z = \int [dU] e^{-S_g[U]} \prod_f \det(D[U] + m_f)$$



- Expectation values of gauge-invariant operators, also known as "correlation functions"

$$\langle \mathcal{O}(U,q,\bar{q}) \rangle = (1/Z) \int [dU] \prod_f [dq_f][d\bar{q}_f] \mathcal{O}(U,q,\bar{q}) e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

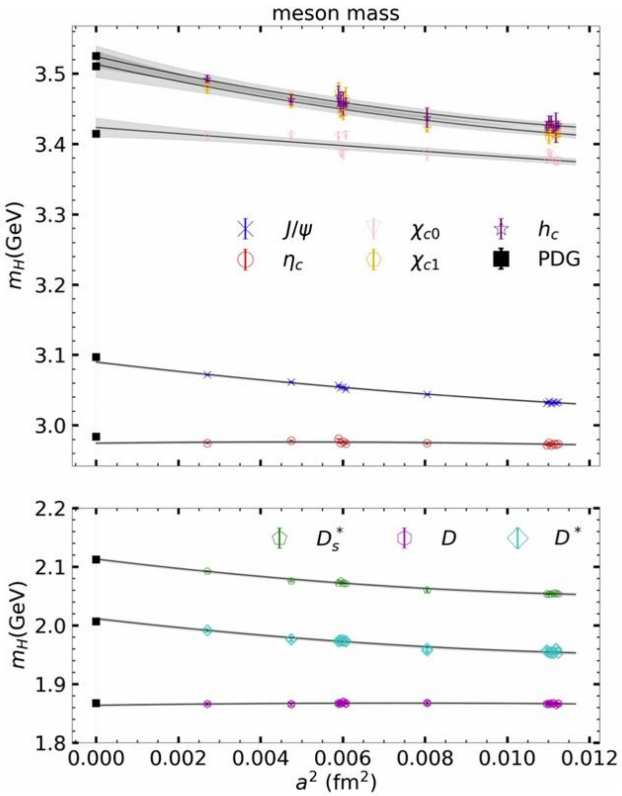


- Monte-Carlo method and data analysis

Lattice set up

- (2+1)-flavor Wilson-clover gauge ensembles [CLQCD, PRD 111, 054504 (2025)]
- Computer resources: “SongShan” supercomputer at Zhengzhou University

Ensemble	C24P29	C32P23	F32P30	F48P21	G36P29	H48P32
a (fm)	0.10524(05)(62)	0.10524(05)(62)	0.07753(03)(45)	0.07753(03)(45)	0.06887(12)(41)	0.05199(08)(31)
$\tilde{m}_s^{\mathrm{b}}$	−0.2400	−0.2400	−0.2050	−0.2050	−0.1926	−0.1700
$\tilde{m}_l^{\mathrm{b}}$	−0.2770	−0.2790	−0.2295	−0.2320	−0.2150	−0.1850
$\tilde{m}_c^{\mathrm{b}}$	0.4159(07)	0.4190(07)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
$N_{\mathrm{cfg}} \times N_{\mathrm{src}}$	450×72	200×64	180×96	150×96	200×108	150×144
m_π (MeV)	292.3(1.0)	227.9(1.2)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)
t	2 − 17	2 − 20	4 − 22	4 − 26	2 − 32	8 − 30
Z_V^s	0.85184(06)	0.85350(04)	0.86900(03)	0.86880(02)	0.87473(05)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.30566(07)	1.30673(04)	1.23990(13)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.05549(54)	1.05434(88)	1.04500(22)	1.03802(28)



Methods (scalar function)

- The parameterization for $P \to V$ semileptonic matrix element

$$\langle \phi_\sigma(\vec{p}) | J_\mu^W(0) | D_s(p') \rangle = \frac{F_0(q^2)}{Mm} \epsilon_{\mu\sigma\alpha\beta} p'^\alpha p^\beta + F_1(q^2) \delta_{\mu\sigma} + \frac{F_2(q^2)}{Mm} p_\mu p'_\sigma + \frac{F_3(q^2)}{M^2} p'_\mu p'_\sigma$$

$$\langle \phi(\varepsilon, \vec{p}) | J_\mu^W(0) | D_s(p') \rangle = \varepsilon_\nu^* \varepsilon_{\mu\nu\alpha\beta} p'_\alpha p_\beta \frac{2V}{m+M} + (M+m) \varepsilon_\mu^* A_1 + \frac{\varepsilon^* \cdot q}{M+m} (p+p')_\mu A_2 - 2m \frac{\varepsilon^* \cdot q}{Q^2} q_\mu (A_0 - A_3)$$

- Correlation functions ⟶ Scalar functions ⟶ Form factors

$\langle \phi_\sigma(\vec{p}) | J_\mu^W(0) | D_s(p') \rangle$ $\qquad \tilde{\mathcal{I}}_j \qquad$ V, A_0, A_1, A_2

- Relationship with the form factor

$$V = \frac{(m+M)}{2mM} F_0,$$
$$A_1 = \frac{F_1}{M+m},$$
$$A_2 = \frac{M+m}{2mM^2} (MF_2 + mF_3),$$
$$A_0 - A_3 = Q^2 \left(\frac{F_2}{4m^2 M} - \frac{F_3}{4mM^2} \right).$$

A_3 is not an independent form factor

$$A_3(q^2) = \frac{M+m}{2m} A_1(q^2) - \frac{M-m}{2m} A_2(q^2)$$

A_0 is then

$$A_0(q^2) = \frac{F_1}{2m} + \frac{m^2 - M^2 + Q^2}{4m^2 M} F_2 + \frac{m^2 - M^2 - Q^2}{4mM^2} F_3$$

$A_0(0) = A_3(0)$ is automatically perserved

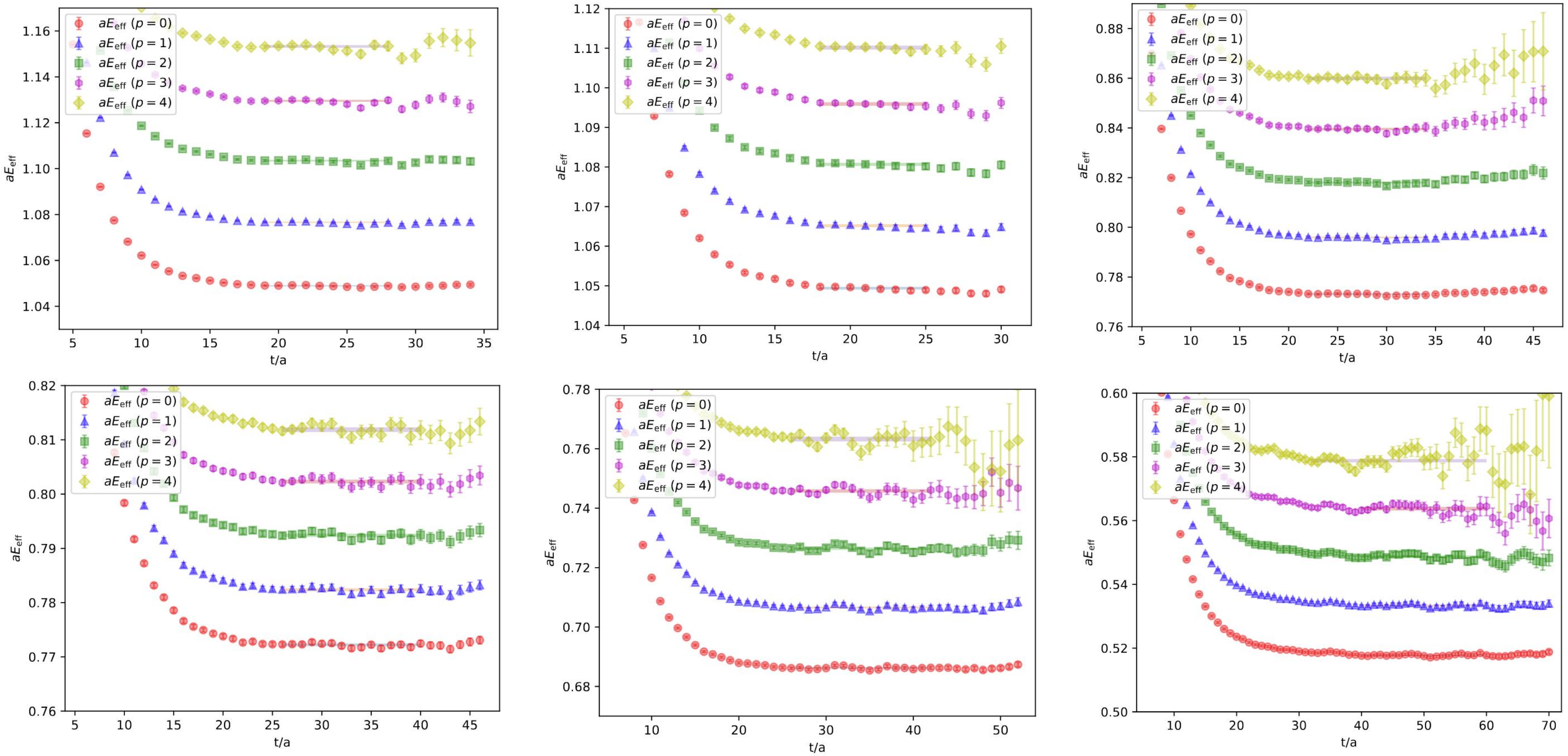
Methods (scalar function)

- A similar scalar function scheme has been used for high-precision calculation
  - $\Gamma(\eta_c \to 2\gamma) = 6.67(16)_{\mathrm{stat}}(6)_{\mathrm{syst}}\ \mathrm{keV}$ [Y. M et al, Science Bulletin 68, 1880 (2023)]
  - $\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)\ \mathrm{keV}$ [Y. M et al, PRD 109, 074511 (2024)]
  - $\mathrm{Br}(J/\psi \to D e v_e) = 1.21(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D \mu v_\mu) = 1.18(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D_s e v_e) = 1.90(8) \times 10^{-10}$
    $\mathrm{Br}(J/\psi \to D_s \mu v_\mu) = 1.84(8) \times 10^{-10}$ [Y. M et al, PRD 110, 074510 (2024)]
  - $\mathrm{Br}(J/\psi \to \gamma \eta_c) = 2.49(11)_{\mathrm{lat}}(5)_{\mathrm{exp}}\%$ [Y. M et al, PRD 111, 014508 (2025)]

Results (2-point function fitting)

$$C^{(2)}(\vec{p}, t) = \frac{Z_h^2}{2E_h}\left(e^{-E_h t} + e^{-E_h (T-t)}\right)$$

- Least χ^2 fitting considering covariance matrix between configurations and time
- There should be a plateau when meson ground states are dominant



Results (2-point function fitting)

$$C^{(2)}(\vec{p}, t) = \left(-1 - \frac{|\vec{p}|^2}{3m_h^2}\right) \frac{Z_h^2}{2E_h}\left[e^{-E_h t} + e^{-E_h(T-t)}\right]$$

- Least χ^2 fitting considering covariance matrix between configurations and time
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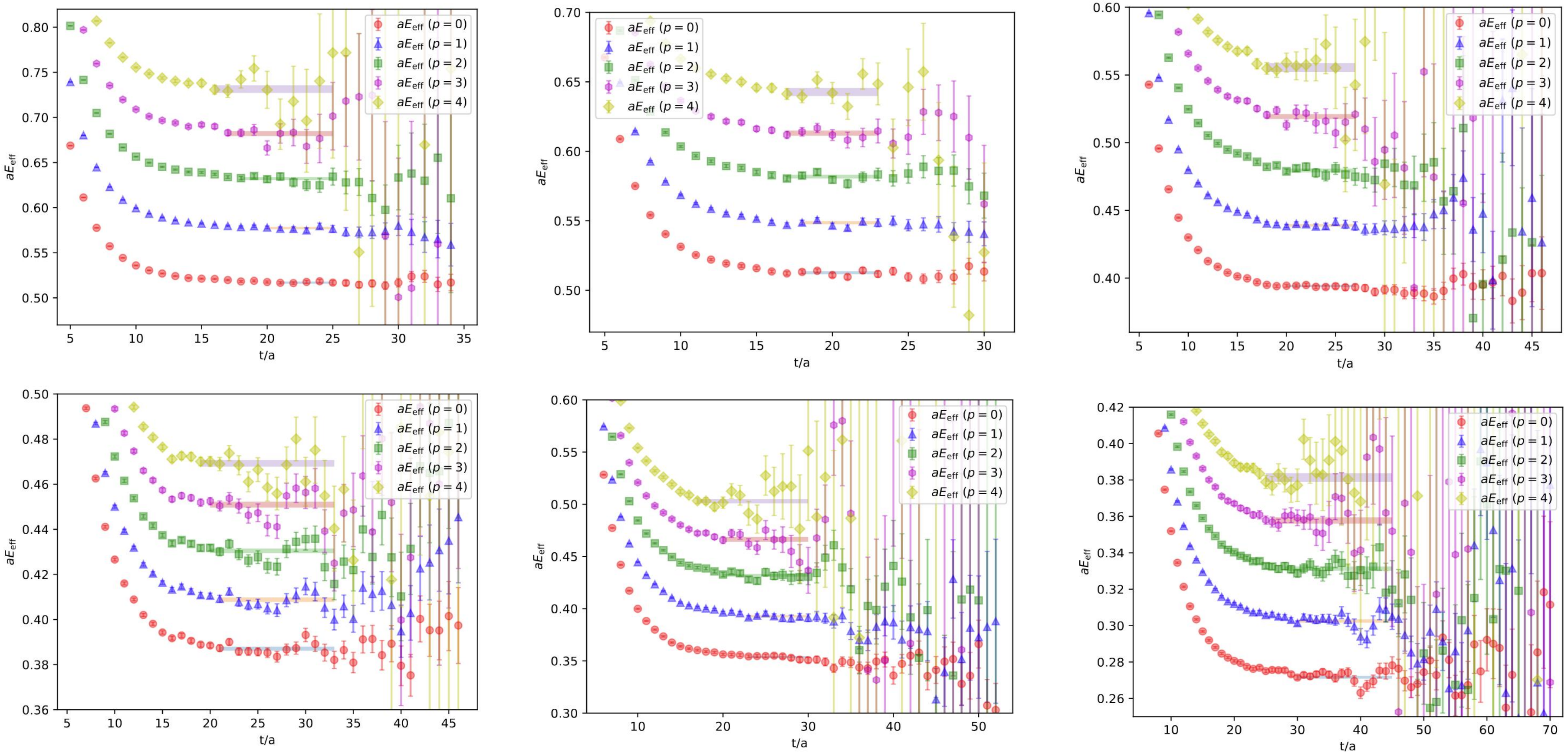
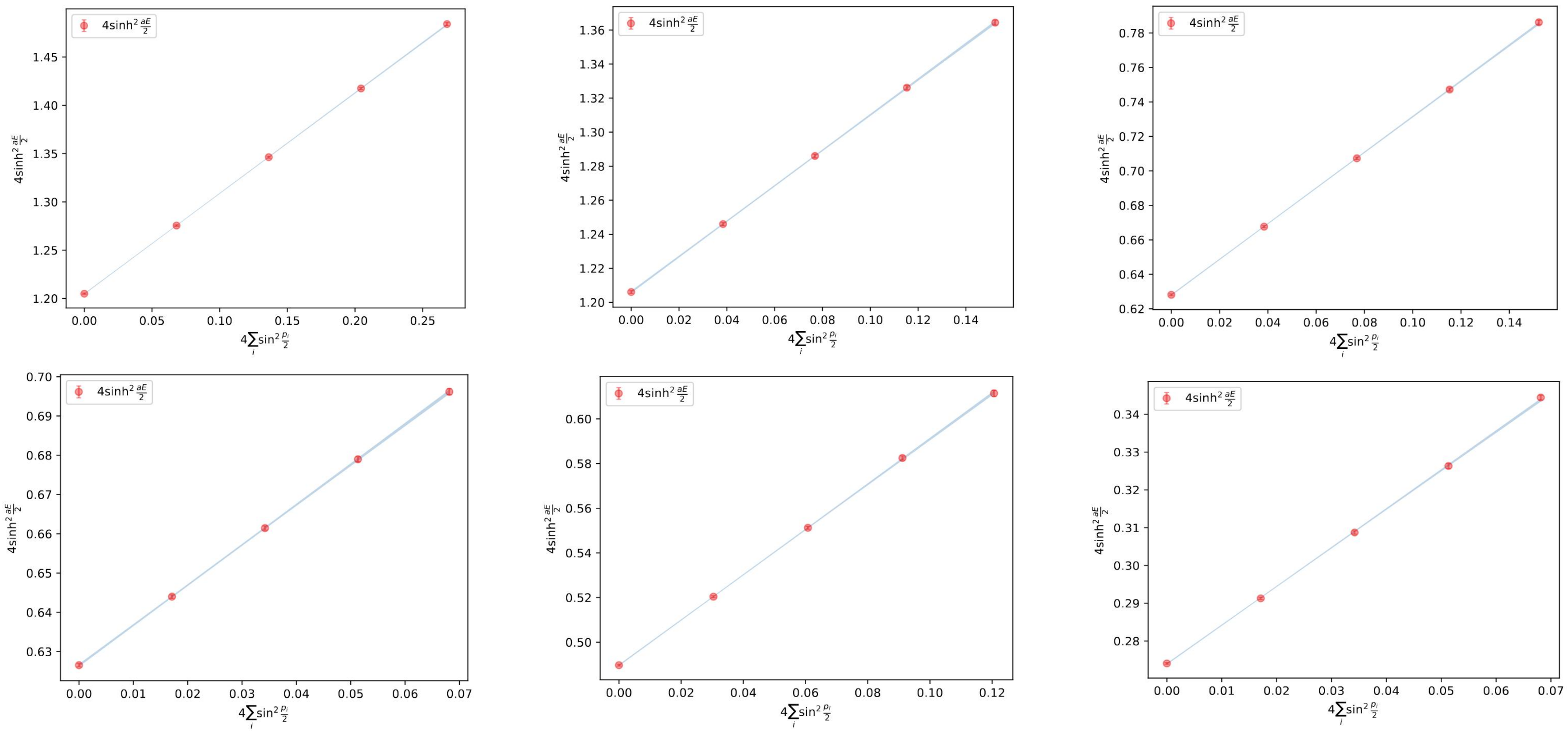


Results (dispersion relation)

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}^h_{\text{latt}} \cdot 4\sum_i \sin^2\frac{p_i}{2}$$

- We checked the dispersion relation of $\boldsymbol{D_s}$ meson at different momenta
- Use a discrete dispersion relation as the fitting function

$\mathcal{Z}_{\text{latt}}$ is 1.0402(48), 1.0405(90), 1.0347(68), 1.022(11), 1.0132(76), 1.0245(77)



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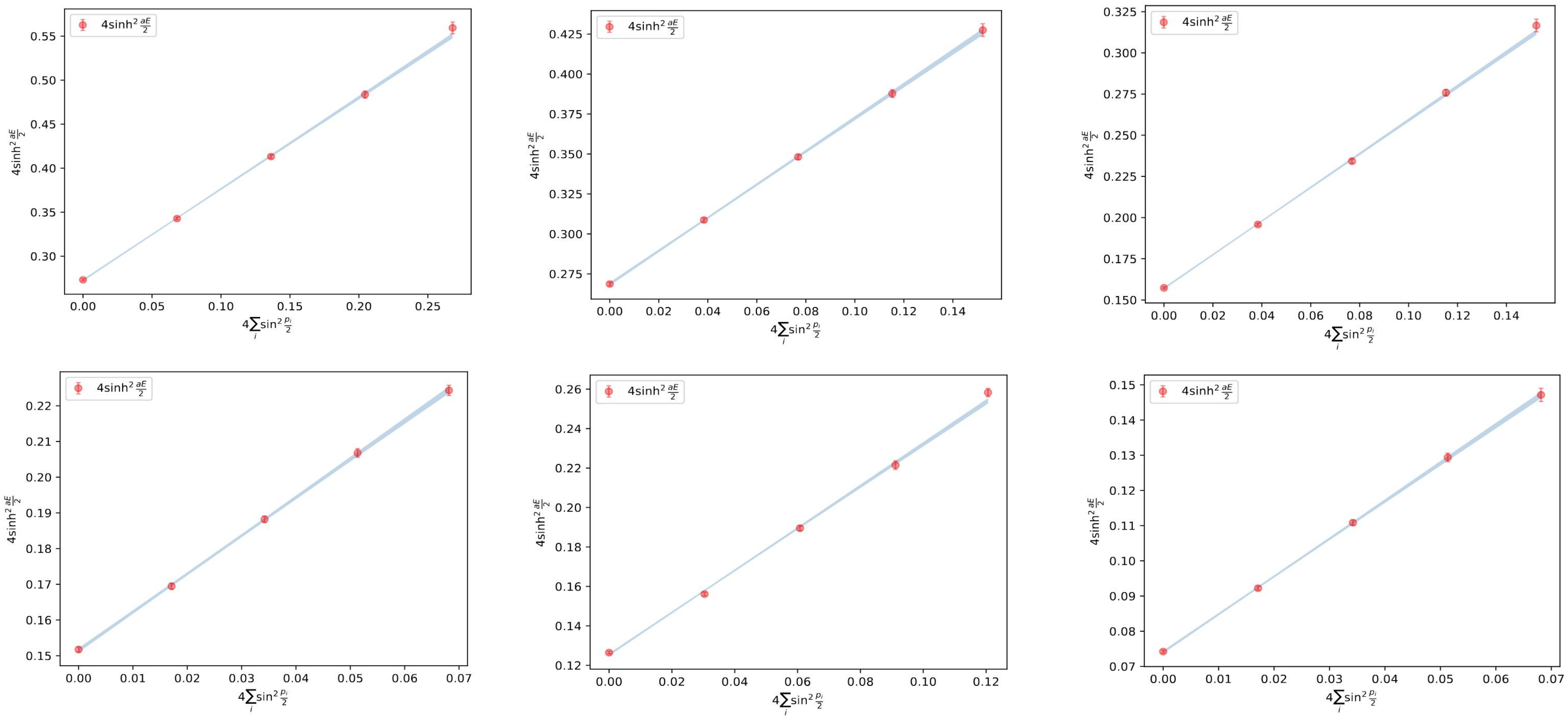
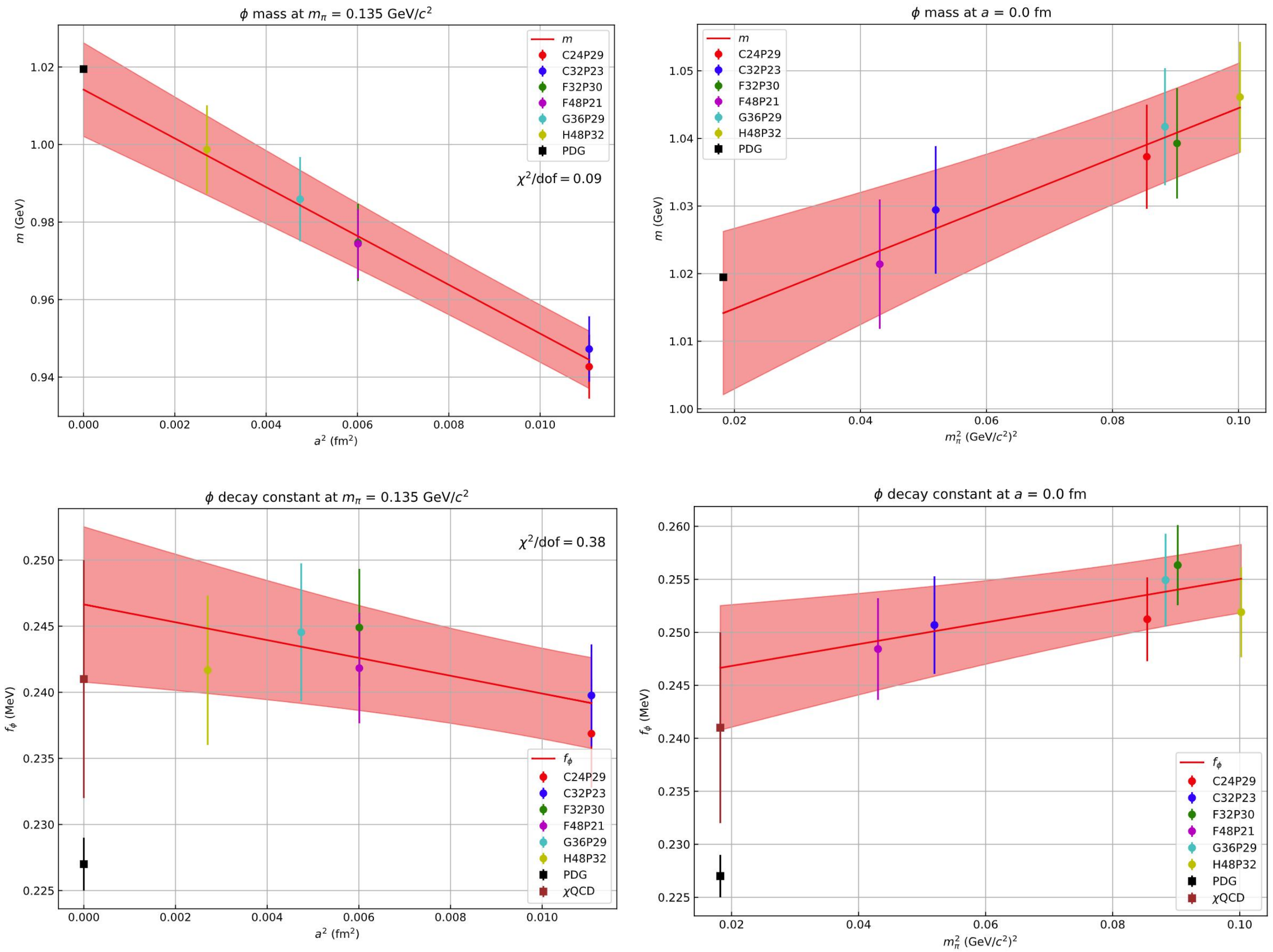
- We checked the dispersion relation of ϕ meson at different momenta
- Use a discrete dispersion relation as the fitting function

$\mathcal{Z}_{\text{latt}}$ is $1.037(13), 1.036(17), 1.021(13), 1.070(19), 1.066(14), 1.071(17)$



Results (mass and decay constant)

- We extrapolate masses and decay constants of ϕ meson to get physical results



$$m/f_\phi = c + da^2 + f\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)$$

$$m = 1.014(12)\ \mathrm{GeV}/c^2$$

$$f_\phi = 0.2466(59)\ \mathrm{GeV}$$

Results (form factor)

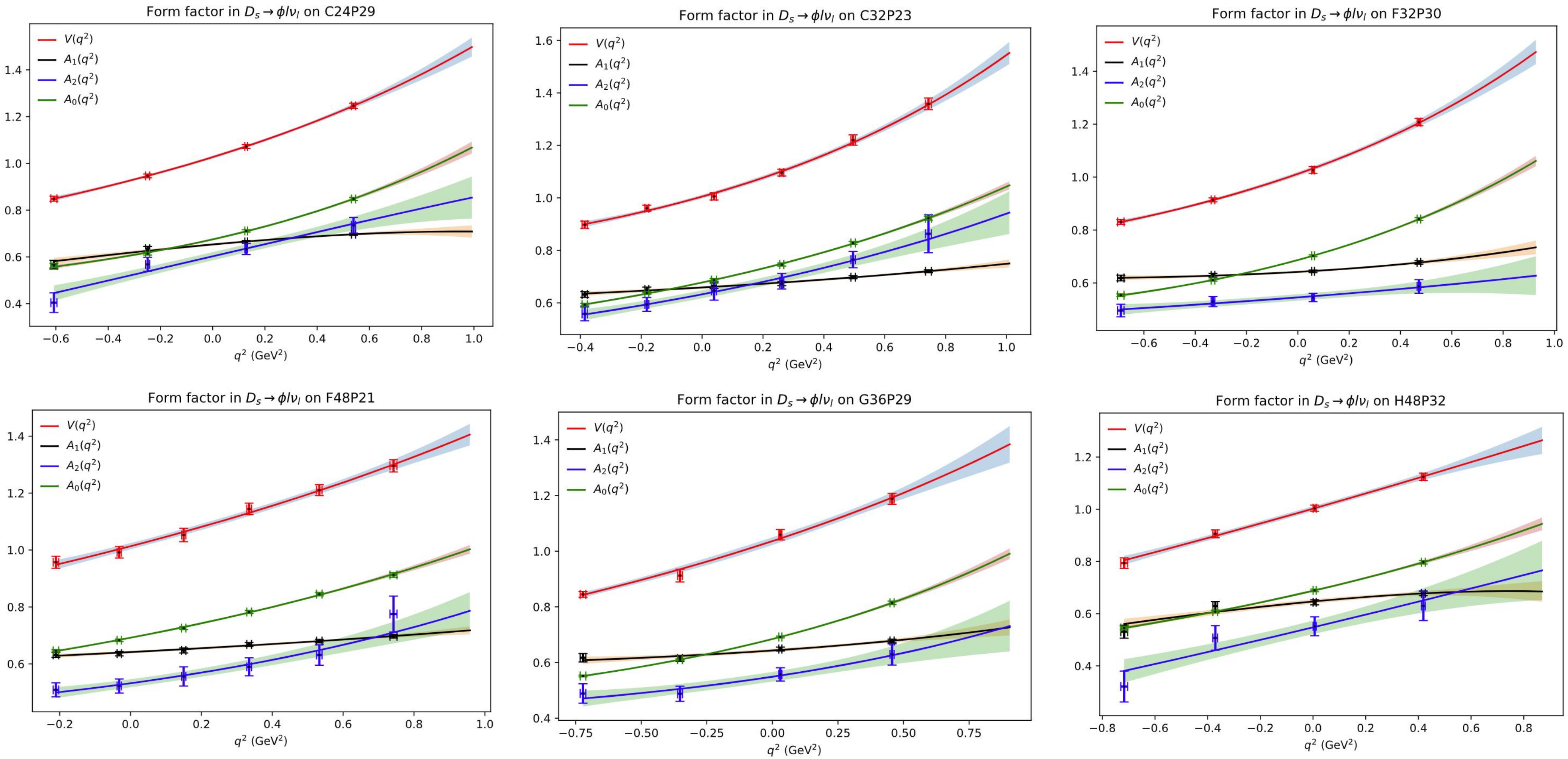
$$z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_+ = \left(m_{D_s} + m_\phi\right)^2,\ t_0 = 0$

$$V = \frac{1}{1 - q^2/m_{D_s^*}^2}\left(a_0 + a_1 z + a_2 z^2\right)$$

$$A_{0,1,2} = \frac{1}{1 - q^2/m_{D_{s1}}^2}\left(a_0 + a_1 z + a_2 z^2\right)$$

- The results have been multiplied by the renormalization constant, data point errors from jackknife analysis



Results (global fit)

- Extrapolate results to the physical pion mass and continuum limit using z-expansion

$$z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

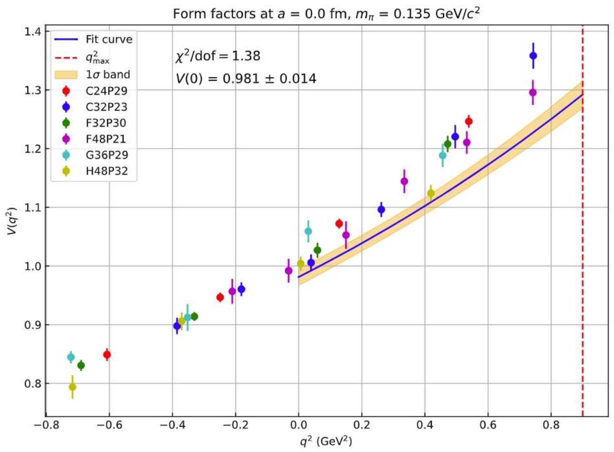
where $t_+ = \left(m_{D_s} + m_\phi\right)^2,\ t_0 = 0$

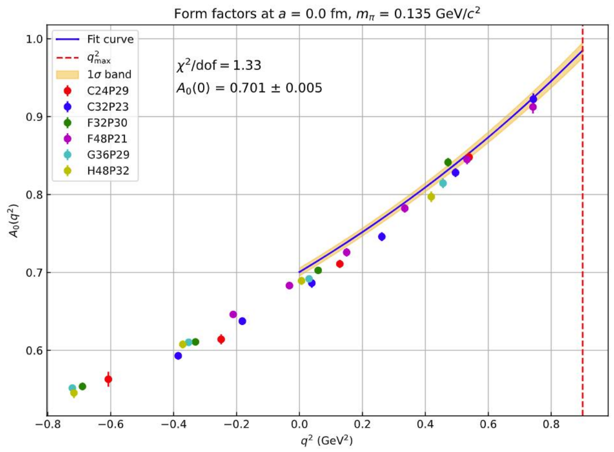
$$V\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)\right] z^i$$

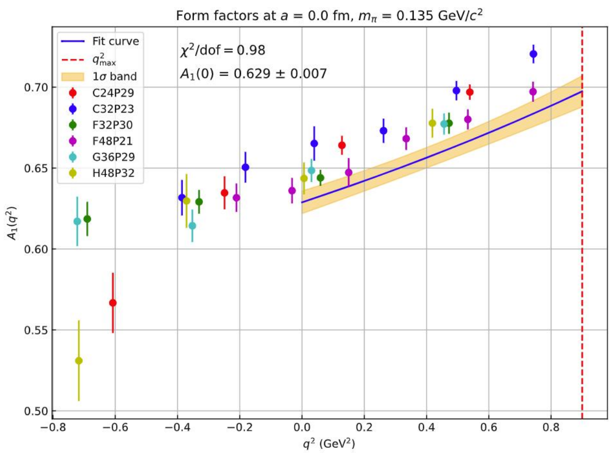
$$A_{0,1,2}\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_{s1}}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)\right] z^i$$

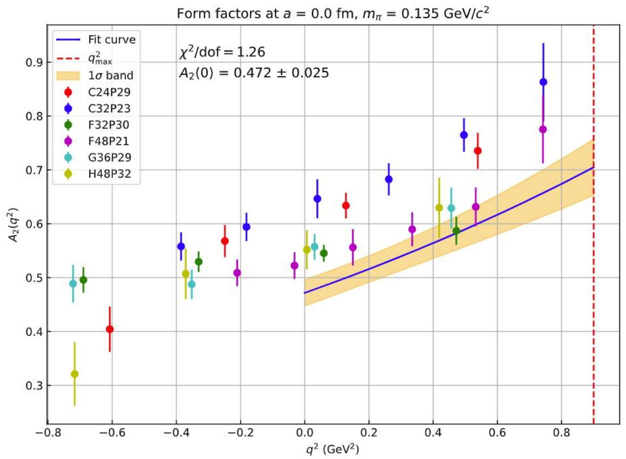
$$m_{\pi,\mathrm{phys}}^2 = 135.0\ \mathrm{MeV}/c^2,\ m_{D_s^*} = 2112.2\ \mathrm{MeV}/c^2,\ m_{D_{s1}} = 2459.5\ \mathrm{MeV}/c^2$$

- $A_3(0) - A_0(0) = 0.001(17)$, consistent with zero



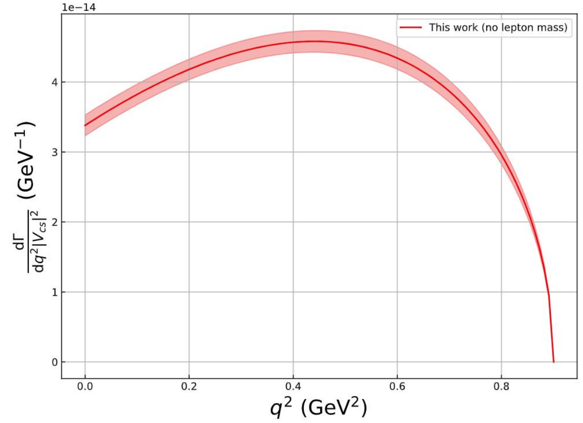


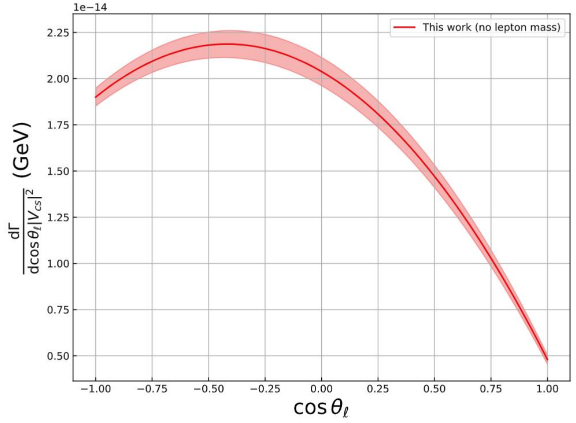


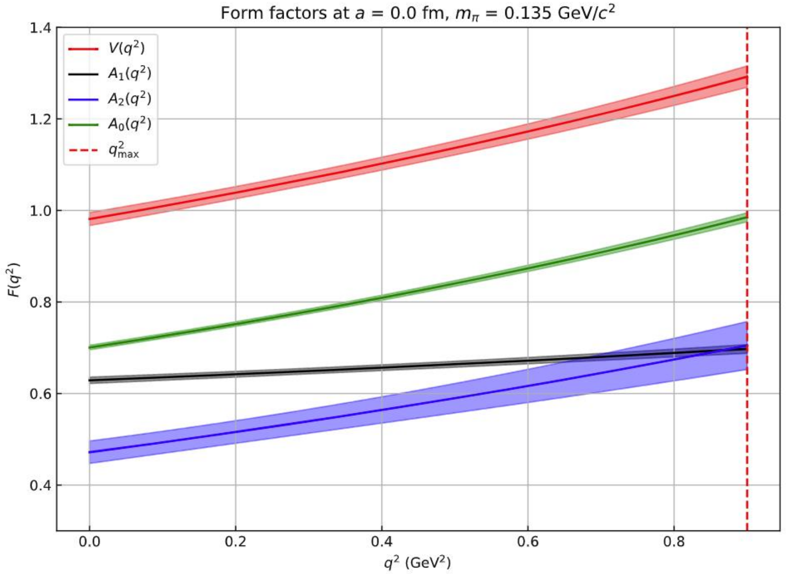


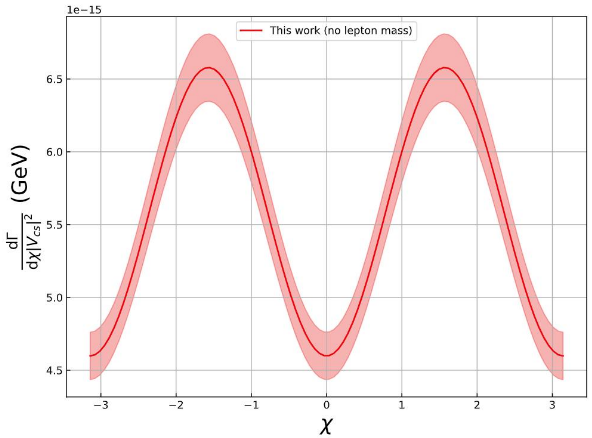
Results (differential decay width)

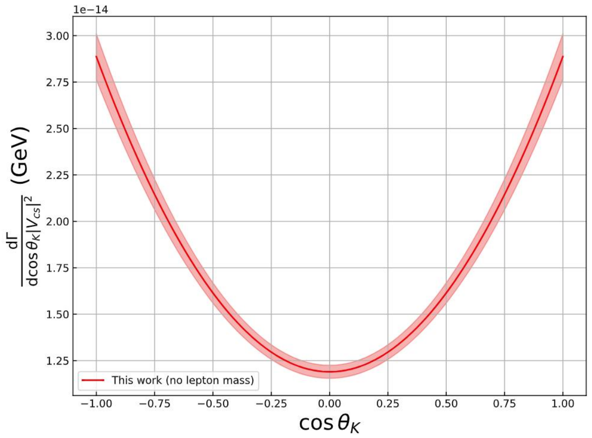
- Differential decay width using *z*-expansion, where the lepton mass is neglected [Rev. Mod. Phys 67, 893 (1995)]

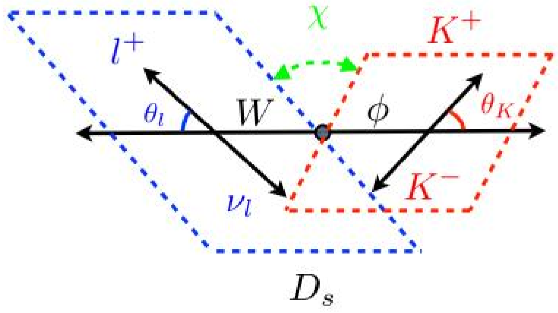












Results

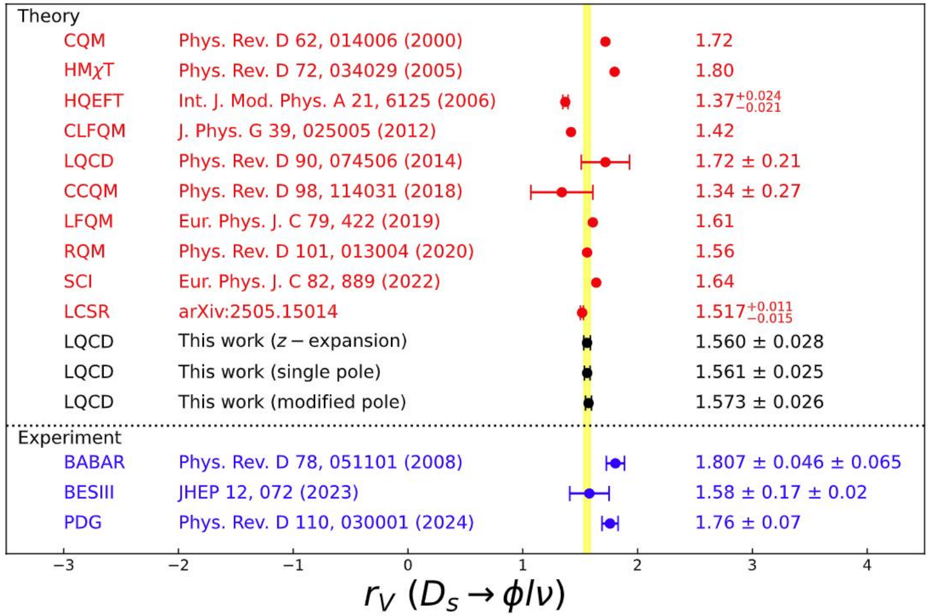
- Summary of preliminary results (form factor)

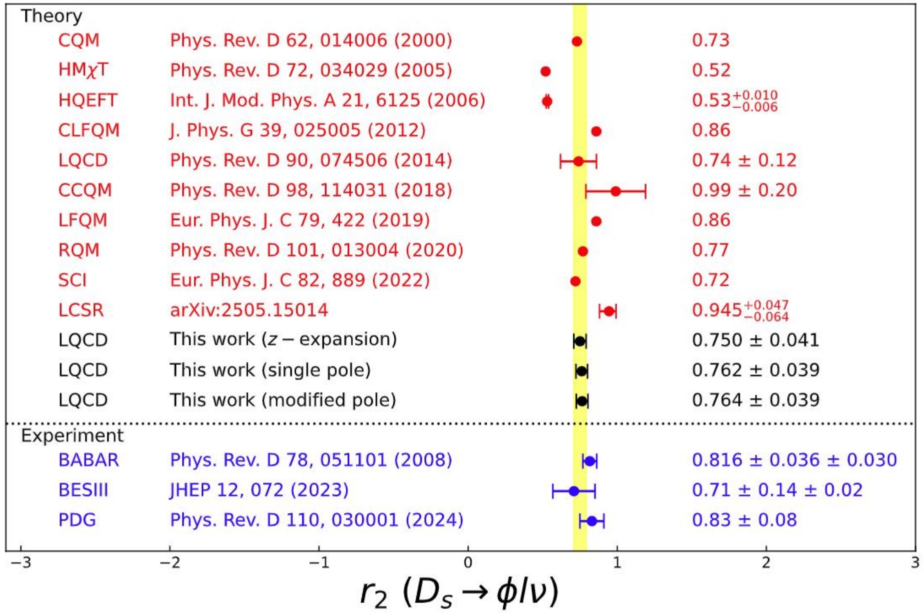
single pole

$$F\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/h^2}\left(c + da^2\right)\left[1 + f\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)\right]$$

modified pole

$$F\left(q^2, a, m_\pi\right) = \frac{1}{\left(1 - q^2/m_{\mathrm{pole}}^2\right)\left(1 - hq^2/m_{\mathrm{pole}}^2\right)}\left(c + da^2\right)\left[1 + f\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)\right]$$





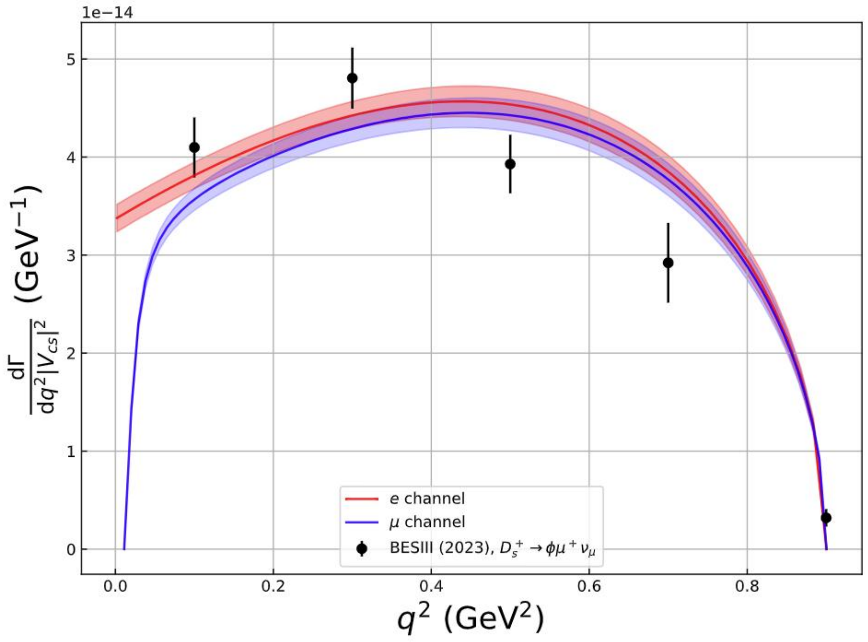
Results

- Summary of preliminary results (branching fraction)

PDG $|V_{cs}|$ as input

$$\frac{d\Gamma(D_s \to \phi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\left(|H_+|^2 + |H_-|^2 + |H_0|^2\right) + \frac{3m_\ell^2}{2q^2} |H_t|^2\right]$$

$\mathcal{B}(D_s \to \phi \ell \nu) \times 10^{-2}$	μ channel	e channel	$\mathcal{R}_{\mu/e}$
z-expansion	2.386(88)	2.530(96)	0.9433(19)
single pole	2.408(60)	2.552(66)	0.9434(15)
modified pole	2.399(61)	2.542(67)	0.9438(14)
PDG	2.24(11)	2.34(12)	0.957(68)



Summary

- Dispersion relations of D_s meson and ϕ meson are calculated
- Form factors on six lattice sets with different q^2 are calculated
- Extrapolate form factors to the physical pion mass and continuum limit
- Differential decay width and branching fraction results are calculated
- Preliminary work on $D \to K^* l\nu$ form factors are ongoing

Thank you for your attention!