中国科学院高能物理研究所
Institute of High Energy Physics, Chinese Academy of Sciences

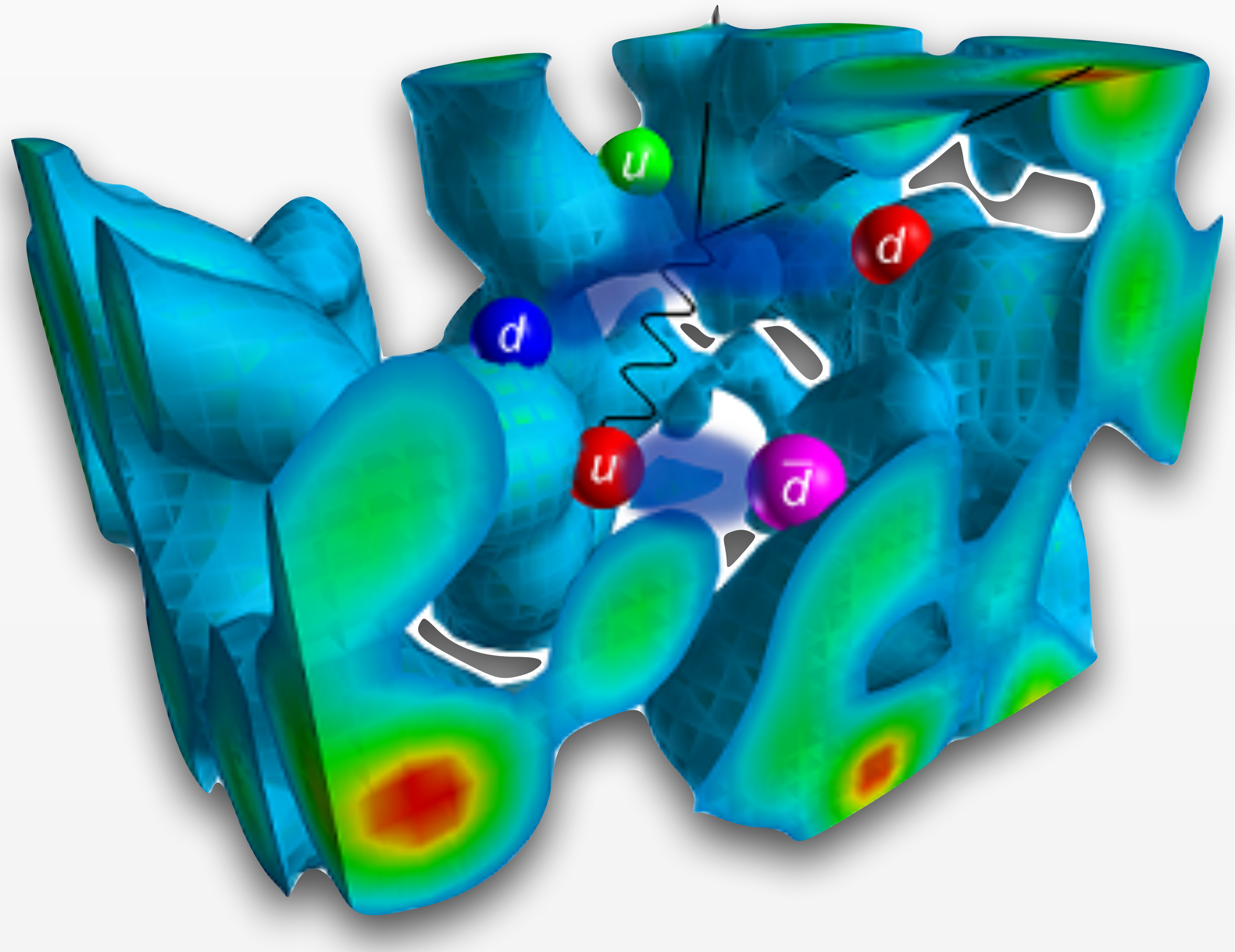
Lattice Parton Collaboration

Profiling Heavy Meson Structure from First Principles

🗣 **Xue-Ying Han**

🏫 **Institute of High Energy Physics**

📅 Aug 6, 2025 @ BESIII粲强子物理研讨会

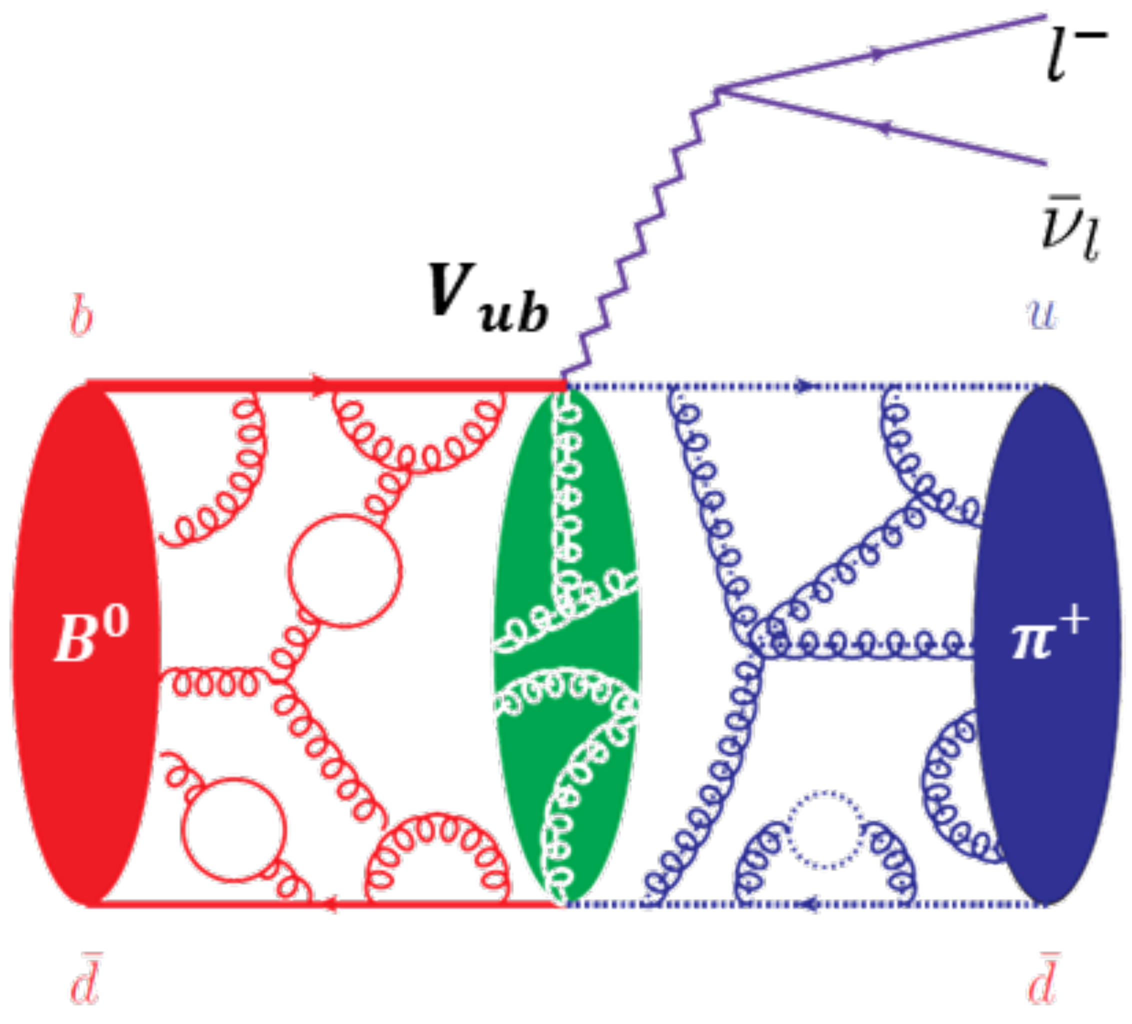


Motivation & Overview

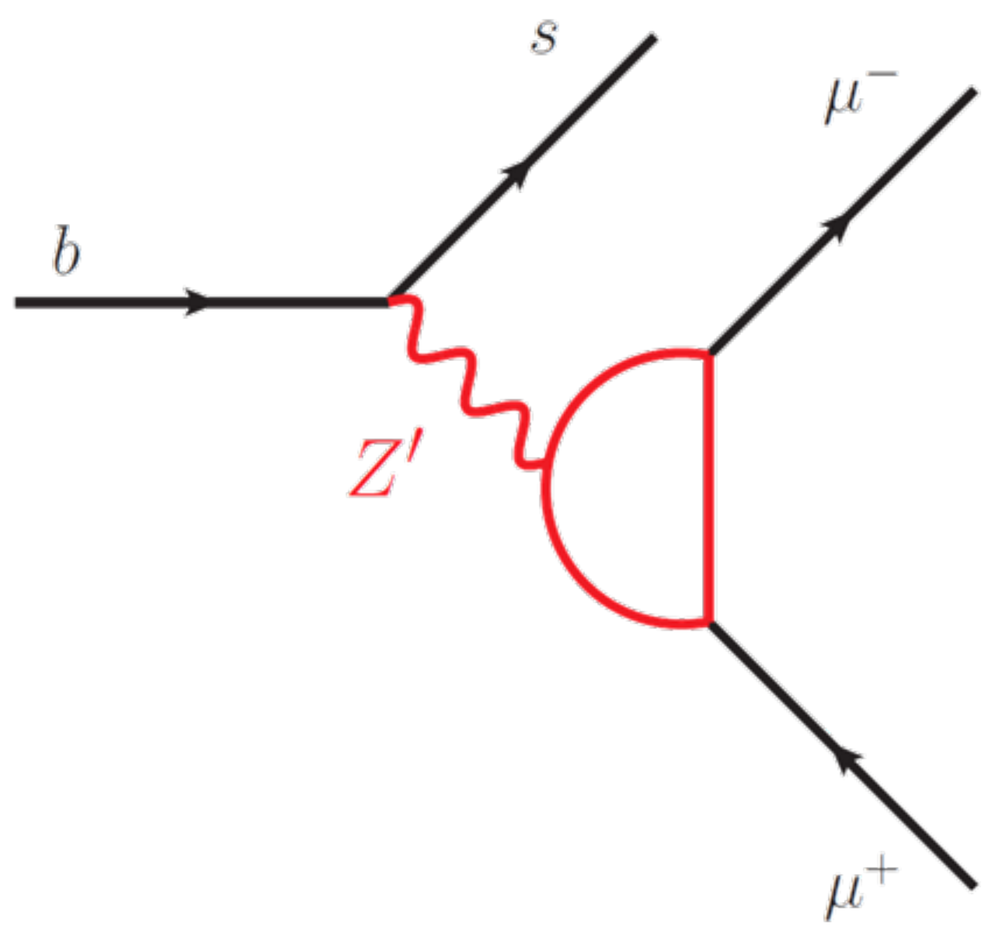
Why heavy meson structure matters?

🤔 Why heavy meson structure matters?

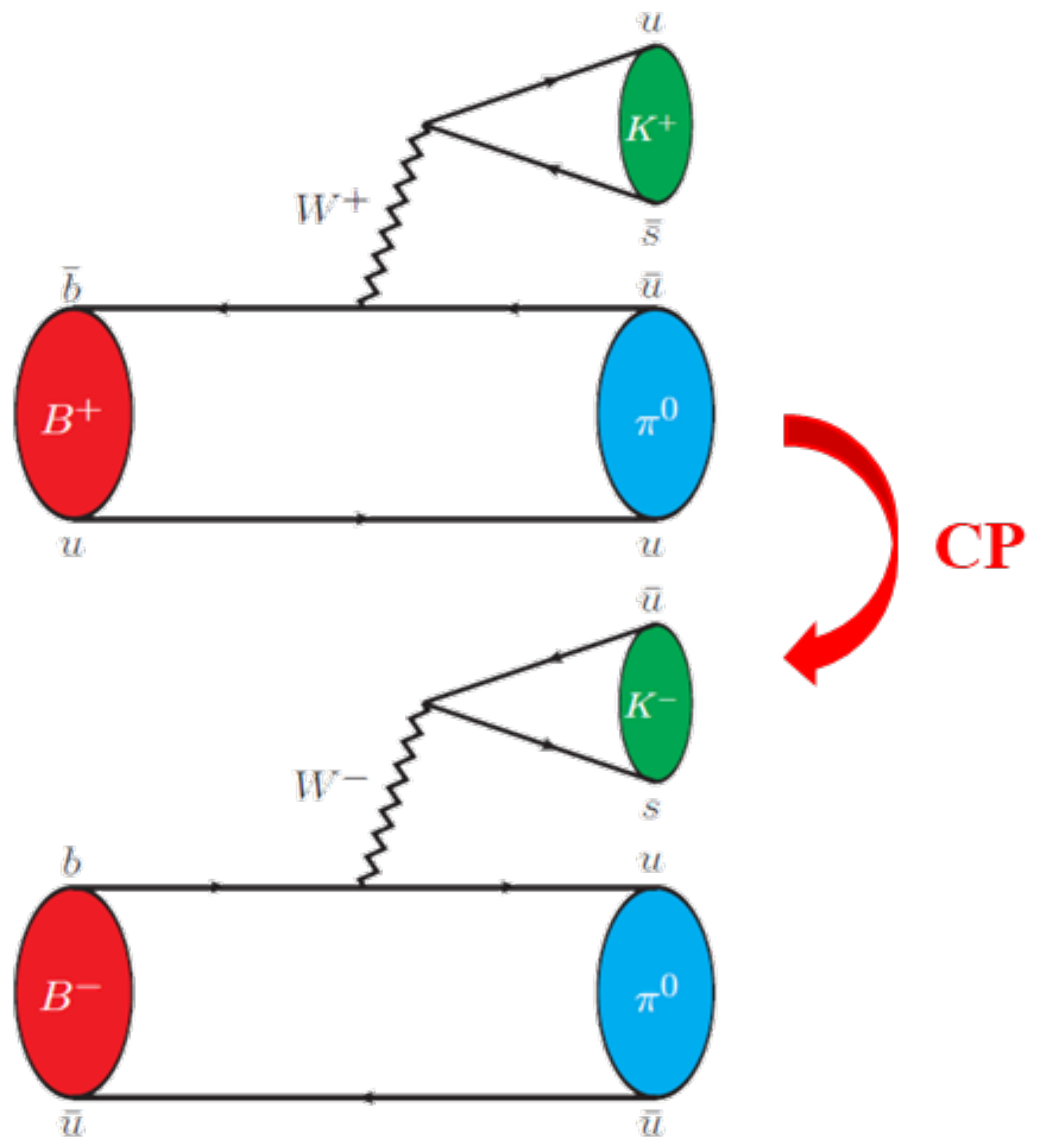
Heavy flavor physics is one of the frontier topics in particle physics



Precise test of Standard Model



Indirect searching for new physics



Study on CP violation

🤔 Why heavy meson structure matters?

V_{cb}, V_{ub} Puzzle

- $|V_{xb}|$ can be measured in semileptonic B decays with inclusive or exclusive processes
- "$|V_{xb}|$ Puzzle
  - From inclusive B decays

$$|V_{ub}^{\text{Incl.}}| = (4.19 \pm 0.17) \times 10^{-3}$$

$$|V_{cb}^{\text{Incl.}}| = (42.19 \pm 0.78) \times 10^{-3}$$

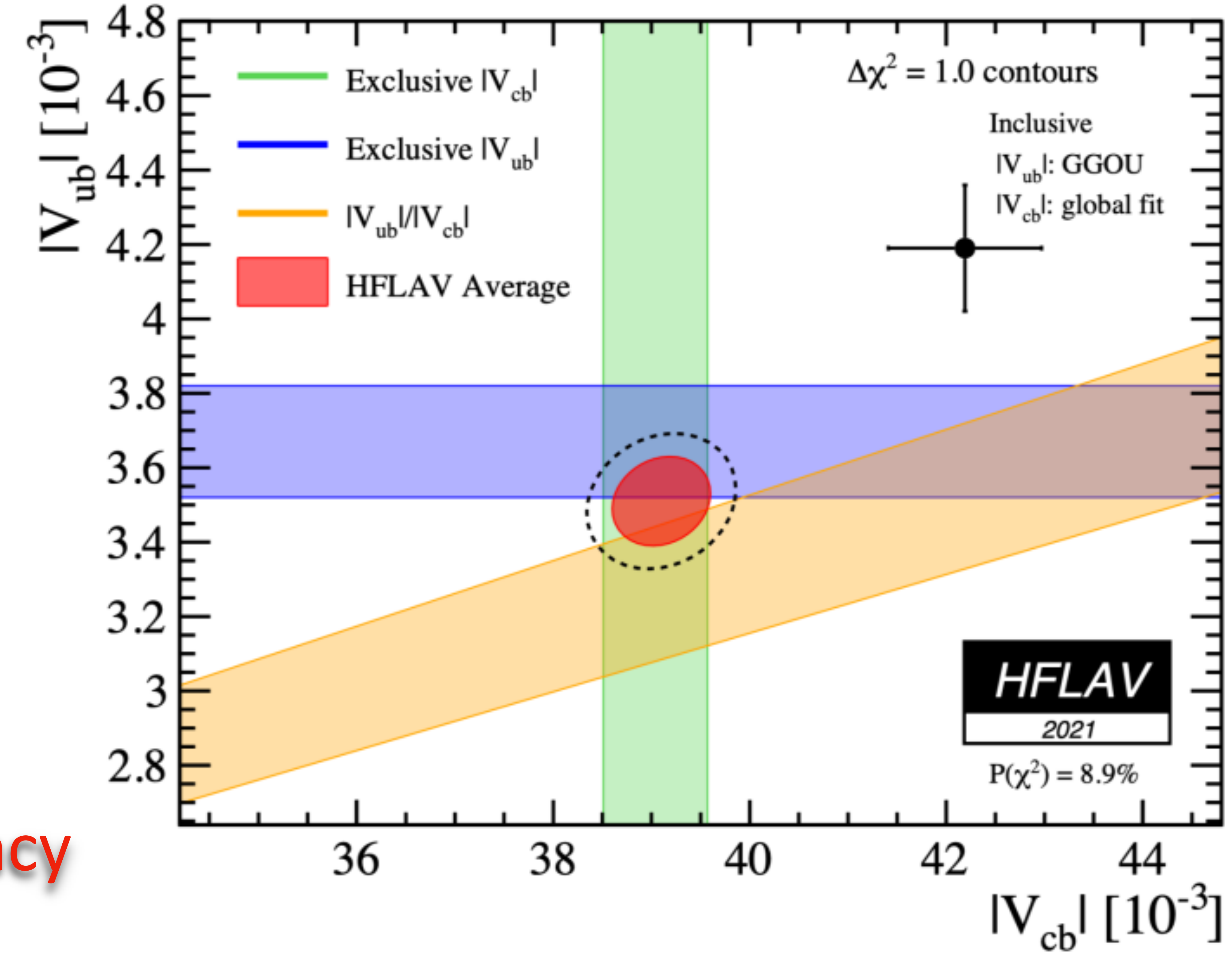
  - From exclusive B, B_s, Λ_b decays

$$|V_{ub}^{\text{Excl.}}| = (3.51 \pm 0.12) \times 10^{-3}$$

$$|V_{cb}^{\text{Excl.}}| = (39.10 \pm 0.50) \times 10^{-3}$$

3.3σ discrepancy

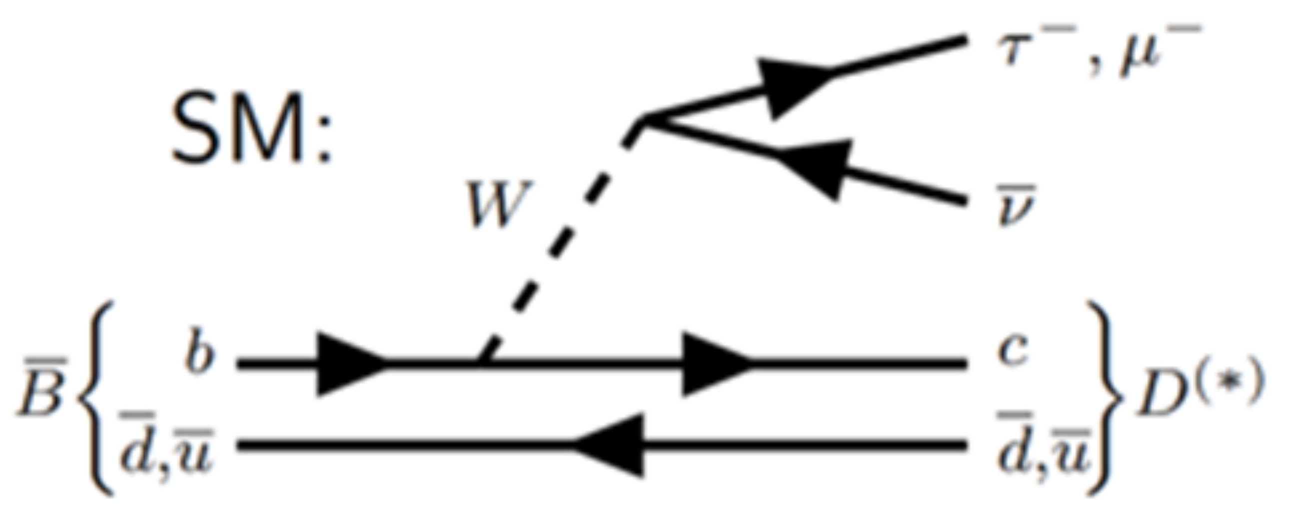
HFLAV, PRD 107, 052008 (2023)



🤔 Why heavy meson structure matters?

$\mathscr{R}(D)$ and $\mathscr{R}(D^*)$

$$\mathscr{R}(D^{(*)}) = \frac{\mathscr{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathscr{B}(B \to D^{(*)}\ell\nu)}$$



- In experimental side

$$\mathscr{R}(D)_{\text{exp}} = 0.342 \pm 0.026$$

$$\mathscr{R}(D^*)_{\text{exp}} = 0.287 \pm 0.012$$

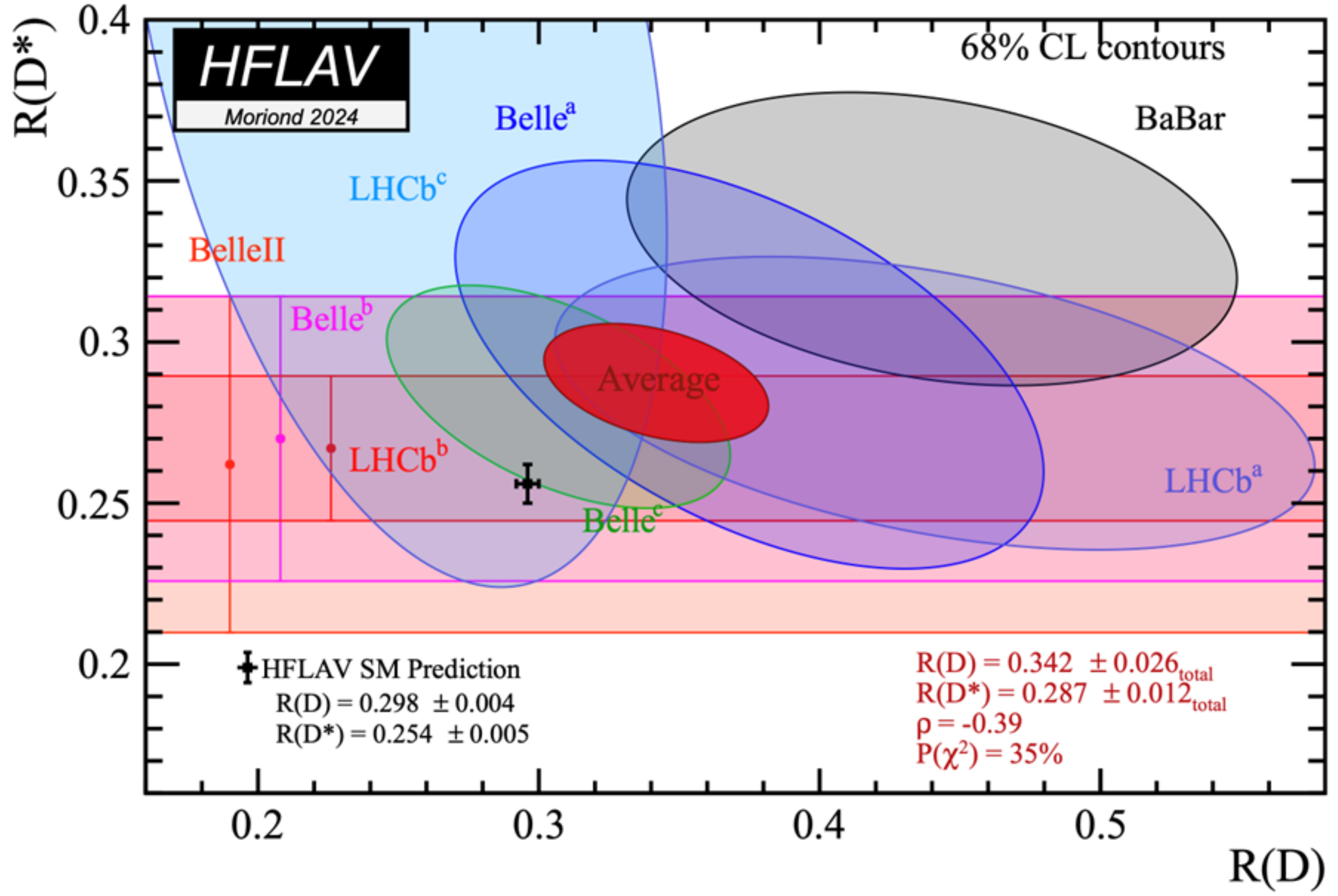
3.31σ discrepancy

- In theoretical side

$$\mathscr{R}(D)_{\text{SM}} = 0.298 \pm 0.004$$

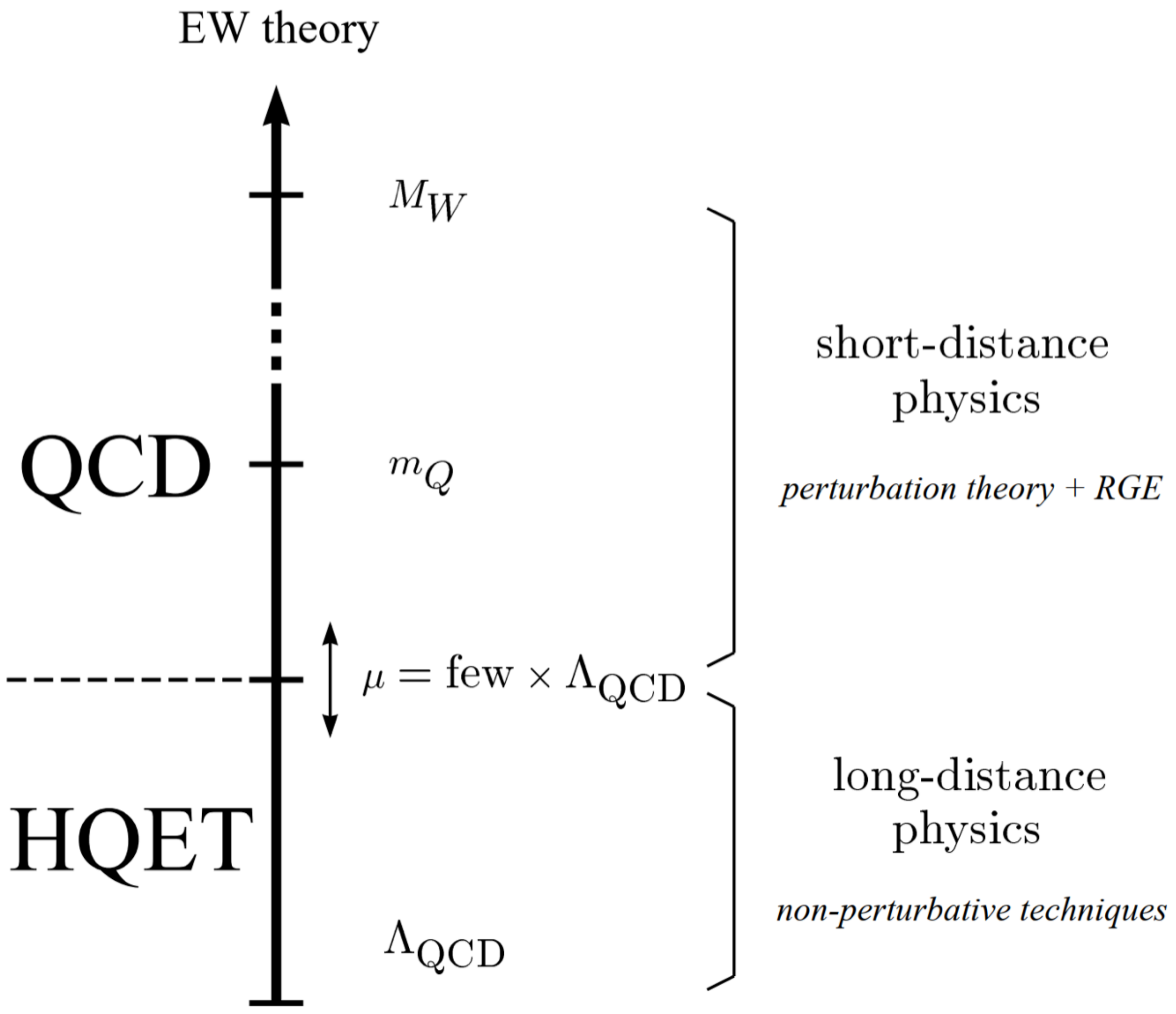
$$\mathscr{R}(D^*)_{\text{SM}} = 0.254 \pm 0.005$$

HFLAV, preliminary average for Moriond 2024



🤔 Why heavy meson LCDAs?

- HQET LCDAs: appear in factorization method



$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx\,T_i^{\rm I}(x)\phi_\pi(x) + \int_0^1 d\xi\,dx\,dy\,T_i^{\rm II}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

Form factor

Heavy meson LCDA

Hard kernel ⊗ LCDA

Hard kernel: Perturbative

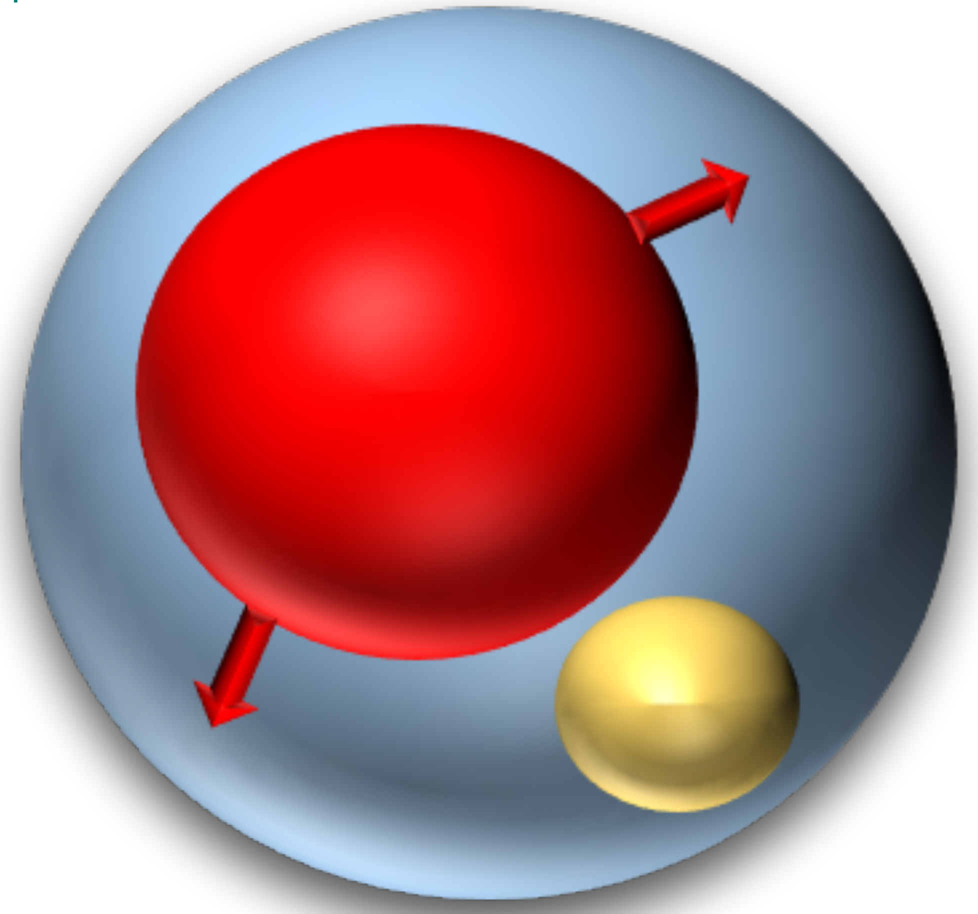
Meson LCDAs: Nonperturbative

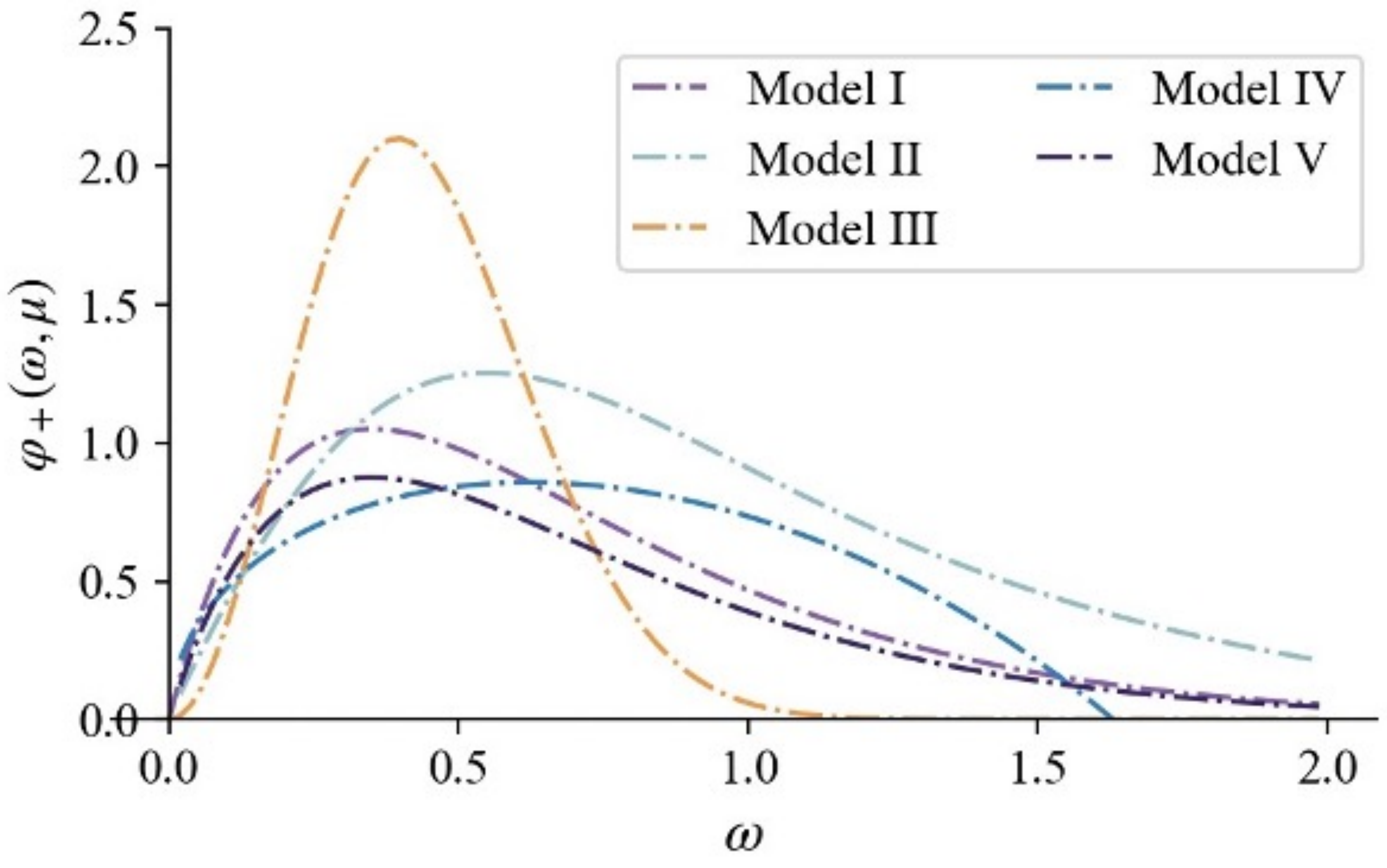
Matrix elements = Hard kernel ⓧ Form factor ⓧ LCDAs

😨 Challenges in profiling the heavy mesons

- Limited understanding of the nonperturbative heavy meson LCDAs

$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_{H_Q} m_{H_Q} n_+\cdot v}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v}\langle 0|\bar{q}(tn_+)\not{n}_+\gamma_5 W_c(tn_+,0)h_v(0)|H_Q(v)\rangle$$





$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Model dependence

$$f^0_{B\to\pi}(0) = 0.122\times\Big[1\pm 0.07\Big|_{S_0^\pi}\pm 0.11\Big|_{\Lambda_q}\pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2}\pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}{}^{+0.06}_{-0.10}\Big|_{\mu_h}\pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big]$$

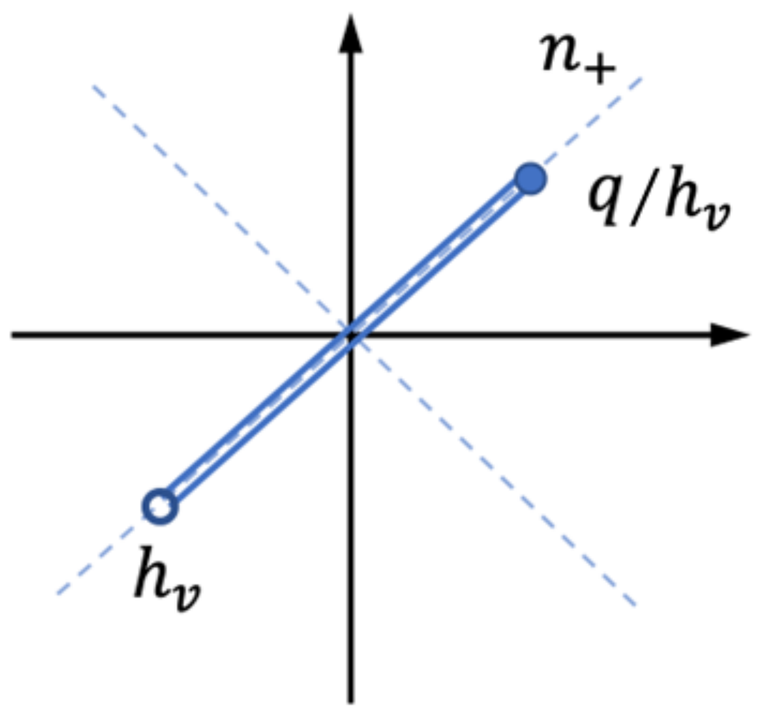
Inverse moment and log moment

Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023

😨 Challenges in First-Principle determination

- Light-cone correlators containing HQET field

$$\langle 0 | \bar{q}\, W_c\, h_v | H_Q \rangle$$



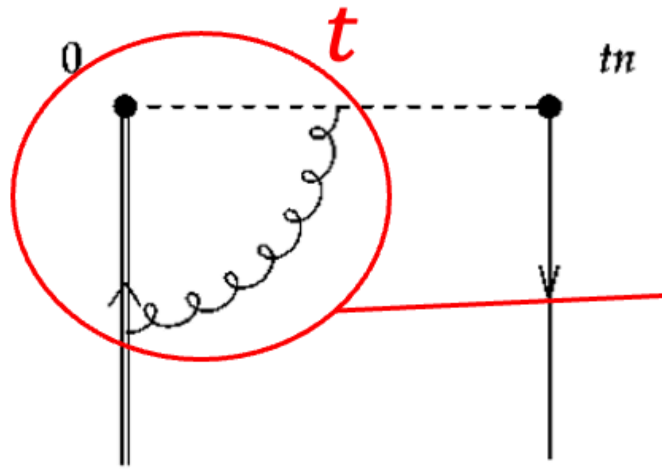
Challenge 1: light-cone correlators

- OPE: expansion into local operators matrix elements — QCD sum rules, lattice QCD
- LaMET: from equal-time correlation functions to light-cone variables — Lattice QCD

Challenge 2: cusp divergence

$$\sim \cosh\theta = \frac{n_+ \cdot v}{\sqrt{n_+^2}\sqrt{v^2}}$$

$$\langle 0|O_v^P(t)|H(p_H)\rangle = i\tilde{f}_H m_H n_+ \cdot v \int_0^\infty d\omega\, e^{i\omega t n_+ \cdot v} \varphi^+(\omega;\mu),$$



Similar to Wilson line

$$O_v^{P,\mathrm{ren}}(t,\mu) = O_v^{P,\mathrm{bare}}(t) + \frac{\alpha_s C_F}{4\pi}\left\{ \left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}}\ln(it\mu)\right) O_v^{P,\mathrm{bare}}(t) - \frac{4}{\hat{\epsilon}}\int_0^1 du \frac{u}{1-u}\left[O_v^{P,\mathrm{bare}}(ut) - O_v^{P,\mathrm{bare}}(t)\right]\right\}$$

No local limit

OPE breaks down

🤔 Take $z^2 \neq 0$, keep h_v; **or** 🤔 No h_v → QCD heavy quark.

HQET on lattice

- Static approximation: $M \to \infty, \vec{v} = \vec{0}$

  Non-relativistic effects in $\mathcal{O}(1/M)$

$$\mathscr{L}_{\mathrm{HQET}} = \mathscr{L}_{\mathrm{stat}} + \frac{1}{2m_Q}\mathscr{L}_{\mathrm{kin}} + \frac{1}{2m_Q}\mathscr{L}_{\mathrm{mag}} + \cdots$$

ALPHA Collaboration

Still not work for heavy meson LCDAs or SFs

HEAVY QUARKS ON THE LATTICE*

E. Eichten
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, IL 60510

Abstract

A method of treating heavy quarks is applied to lattice Q.C.D. for heavy quark masses (m_H) and lattice spacing (a) satisfying the condition $m_H a > 1$. Explicit applications to the measurement of heavy–light meson masses, decay constants, and mixing parameters are presented. Numerical results for B mesons are obtained on a $8^3 \times 16 \times 24$ lattice with $\beta = 5.7$.

*Talk delivered at the "International Symposium of Field Theory on the Lattice," Seillac, France, September 28–October 2, 1987

- Boosted HQET

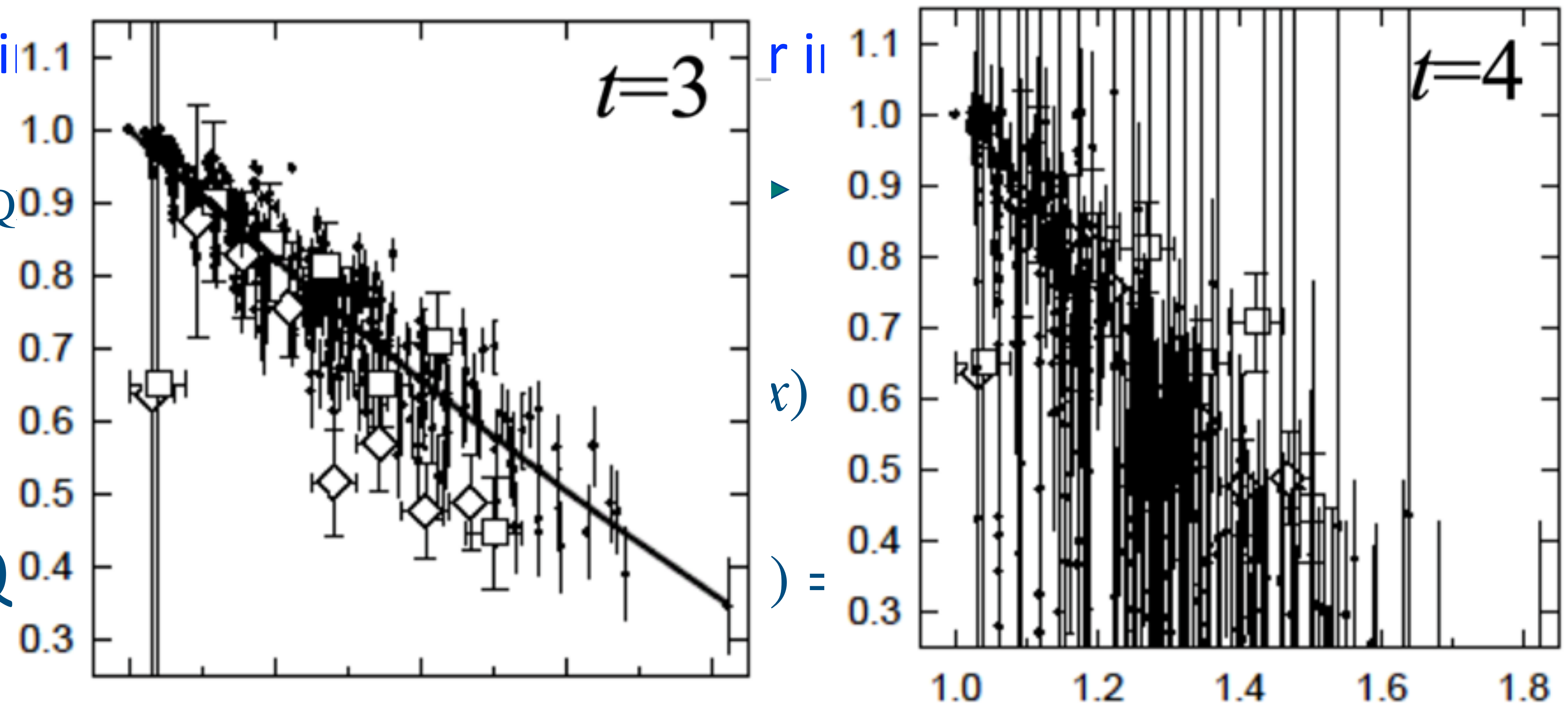
  Significant signal-to-noise problem!

A LATTICE IMPLEMENTATION OF THE ISGUR-WISE LIMIT

Jeffrey E. Mandula
Department of Energy, Division of High Energy Physics, Washington, DC 20585

Michael C. Ogilvie
Department of Physics, Washington University, St. Louis, MO 63130

We construct the Isgur-Wise limit of QCD in a form appropriate for lattice gauge theory. The formulation permits a calculation of heavy quark processes even when the momentum transfers are much larger than the inverse lattice spacing. Applications include semi-leptonic heavy quark decay and scattering processes, including the computation of the nonperturbative part of the Isgur-Wise universal function.



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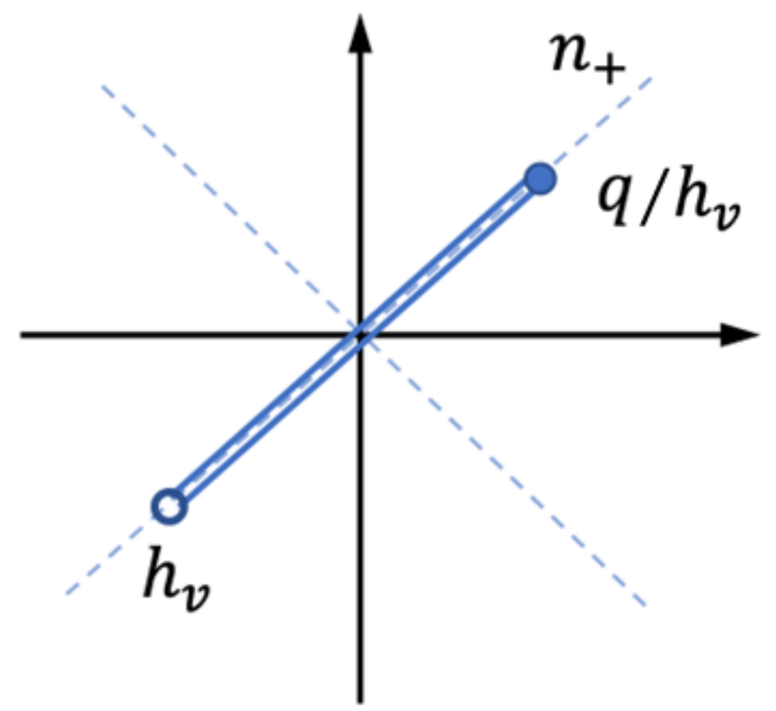

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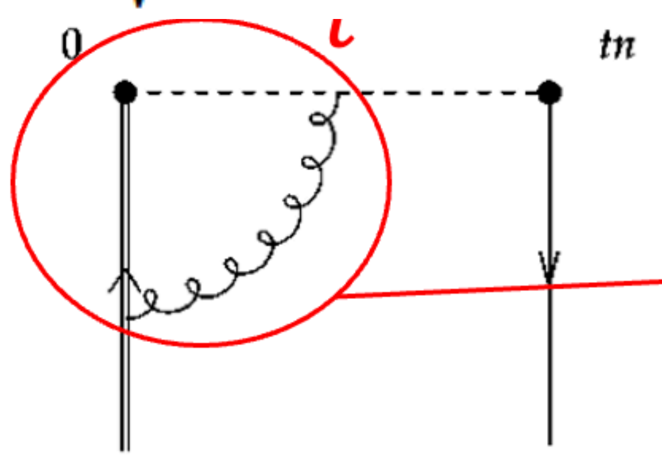

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No local limit

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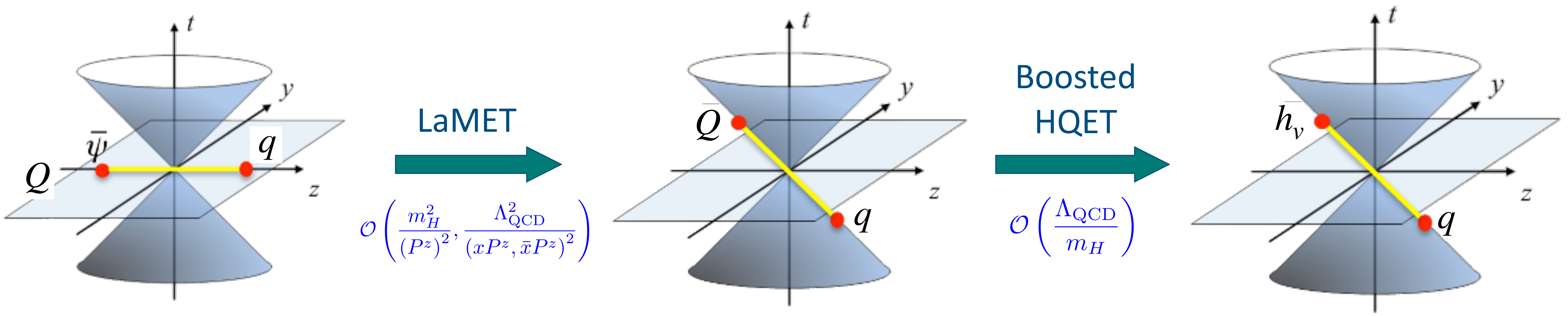
🤔 Take $z^2 \neq 0$, keep h_v; or 🤔 No $h_v \to$ QCD heavy quark. ✅

Theoretical Framework

Sequential effective theory

Sequential effective theory



Equal-time correlator

Light-cone correlator
QCD field

Light-cone correlator
HQET field

XYH, Hua, Ji, Lü, et al., 2024; **XYH**, Hua, Ji, Lü, et al., 2025

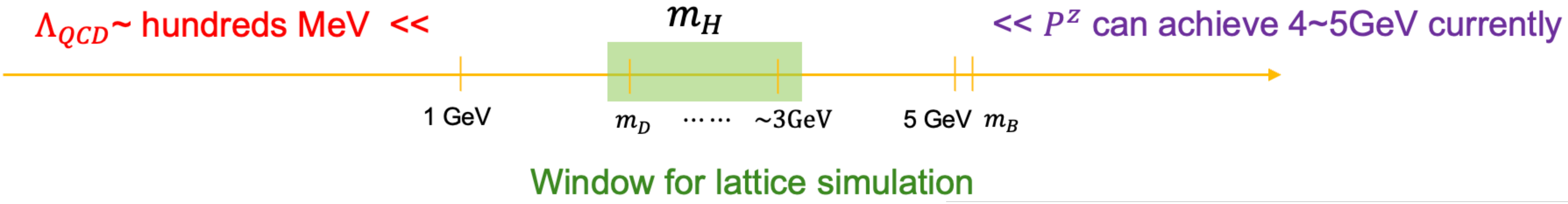
- 3 scales in the equal-time correlator
  - LaMET: $\Lambda_{\rm QCD}, m_Q \ll P^z$, integrate out P^z
  - Boosted HQET: $\Lambda_{\rm QCD} \ll m_Q$, integrate out m_Q

Total hierarchy

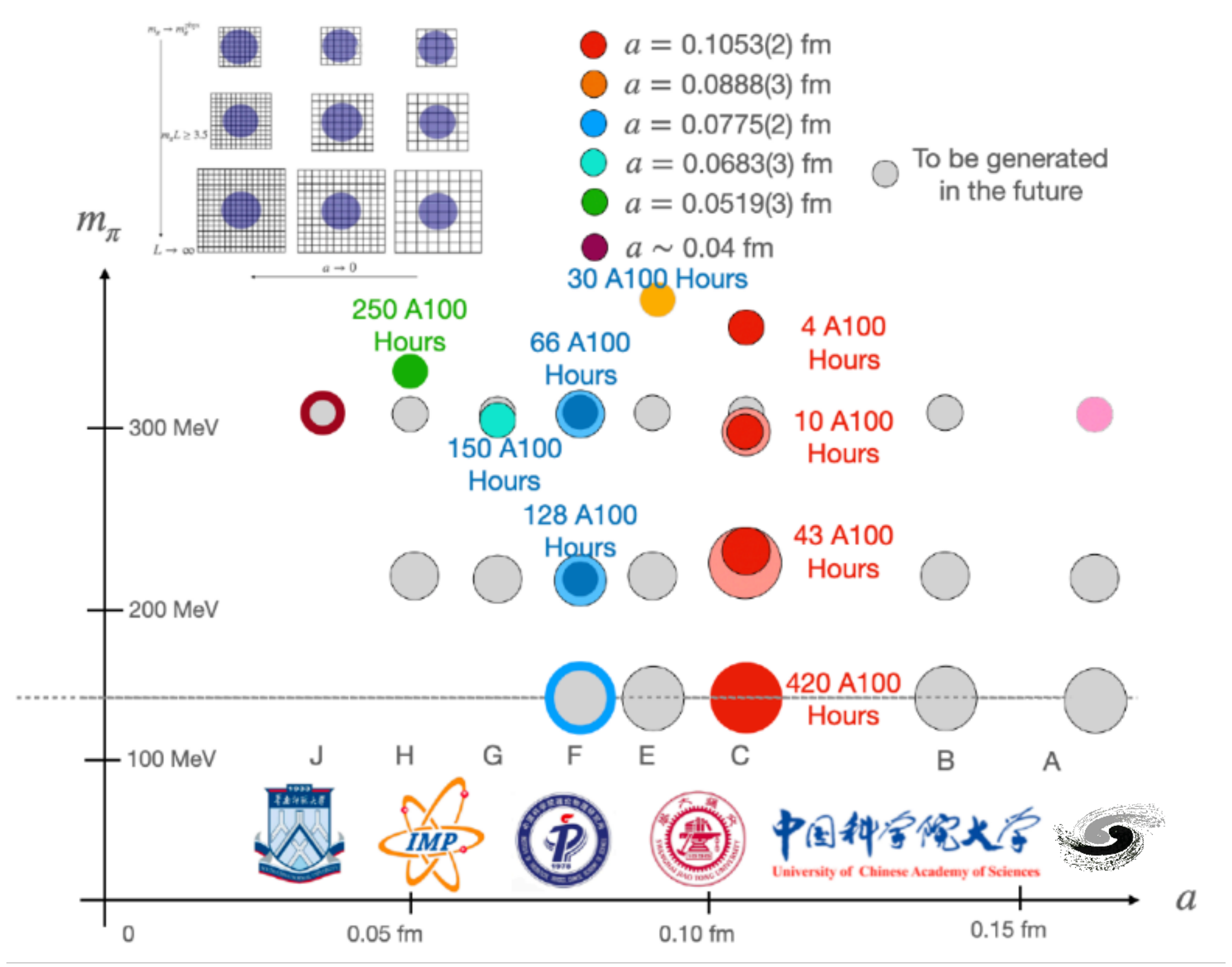
$\Lambda_{\rm QCD} \ll m_Q \ll P^z$

Sequential effective theory

$P^z \ll \pi/a$



- Heavy quark flavor symmetry ⇒ The HQET measurement is independent of heavy quark mass at leading power of $1/m_Q$.
- <u>Determination by charm, application in beauty.</u>



SET for heavy meson LCDAs

Quasi DA —LaMET→ QCD LCDA —bHQET→ HQET LCDA

- Step I: matching in LaMET

$$\tilde{\phi}(x,P^z)=\int_0^1 dy C\left(x,y,\frac{\mu}{P^z}\right)\phi(y,\mu)+\mathcal{O}\left(\frac{m_H^2}{(P^z)^2},\frac{\Lambda_{\rm QCD}^2}{(xP^z,\bar{x}P^z)^2}\right)$$

matching kernel @ NLO:

$$C\left(x,y,\frac{\mu}{P^z}\right)=\delta(x-y)+C_B^{(1)}\left(x,y,\frac{\mu}{P^z}\right)-C_{CT}^{(1)}(x,y)+\mathcal{O}(\alpha_s^2),$$

$$C_B^{(1)}\left(x,y,\frac{\mu}{P^z}\right)=\frac{\alpha_s C_F}{2\pi}\begin{cases}[H_1(x,y)]_+ & x<0<y\\ [H_2(x,y,P^z/\mu)]_+ & 0<x<y\\ \left[H_2\left(1-x,1-y,\frac{P^z}{\mu}\right)\right]_+ & y<x<1\\ [H_1(1-x,1-y)]_+ & y<1<x\end{cases},$$

$$C_{CT}^{(1)}=-\frac{3\alpha_s C_F}{4\pi}\left[\frac{2{\rm Si}[(x-y)z_s P^z]}{\pi(x-y)}\right]_+,$$

XYH, Zhang, et al., PRD111, 034503, (2025)

- Step II: matching in bHQET

$$\varphi^+(\omega,\mu)=\begin{cases}\varphi^+_{\rm peak}(\omega,\mu), & \omega\sim\Lambda_{\rm QCD}\\ \varphi^+_{\rm tail}(\omega,\mu). & \omega\sim m_H\end{cases}$$

with:

$$\varphi^+_{\rm peak}(\omega,\mu)=\frac{1}{m_H}\frac{f_H}{\tilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y,\mu;m_H)$$

$$\varphi^+_{\rm tail}(\omega,\mu)=\frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right)+\frac{4\bar{\Lambda}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right],$$

Beneke, Finauri, Vos, Wei, 2023
Ishaq, Jia, Xiong, Yang, et al., 2020
Lee, Neubert, et al., 2005

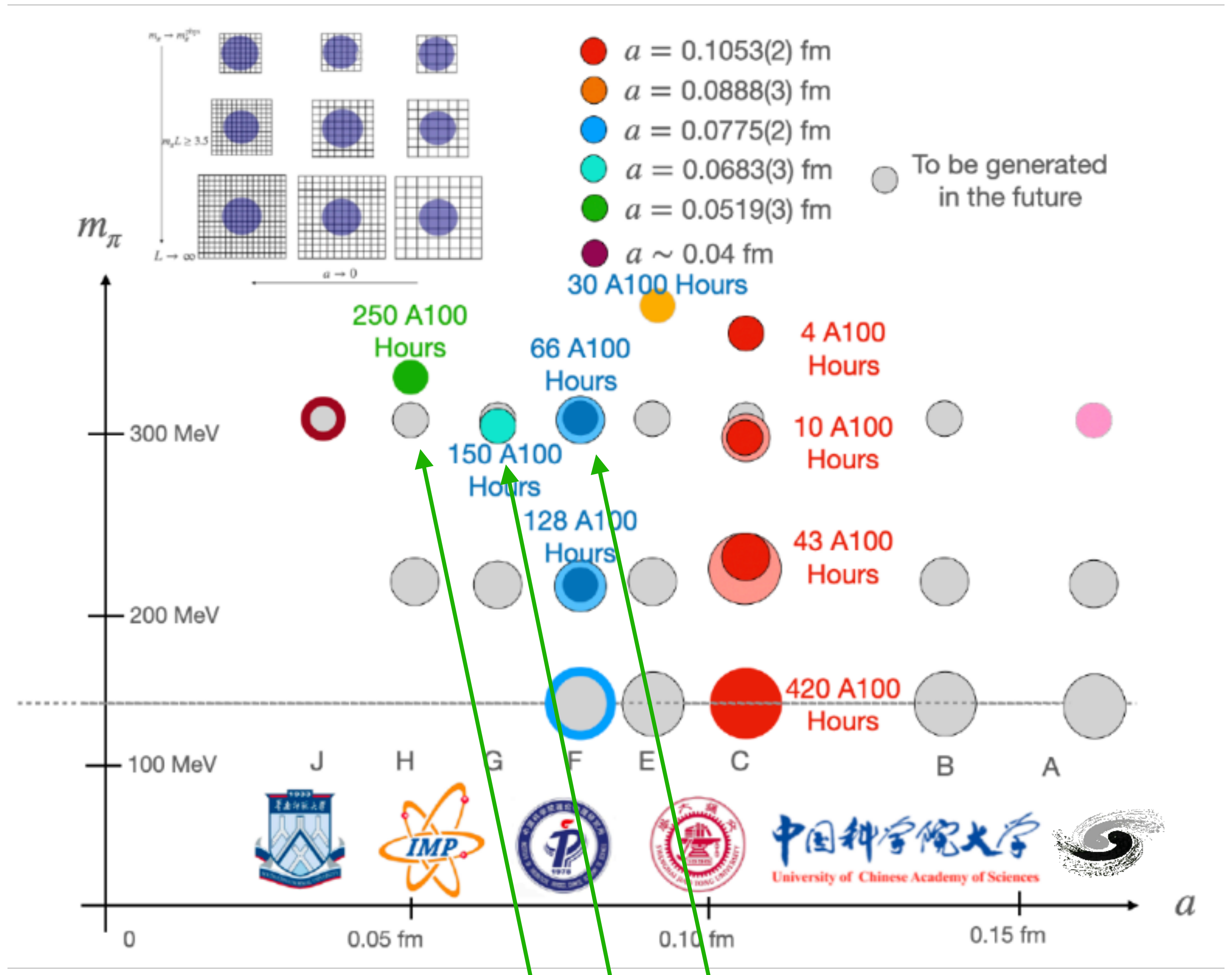
Lattice Implementation

The most advanced lattice simulation

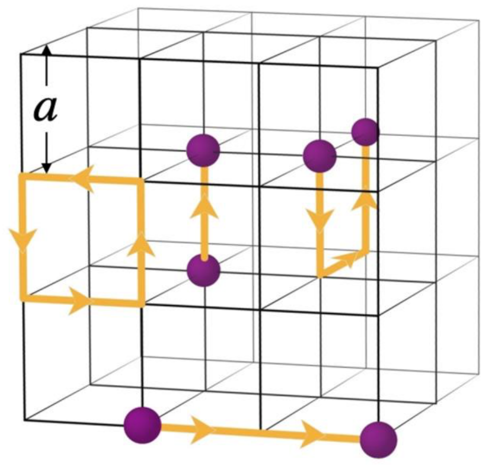
👩‍💻 Lattice Setup

	⬅️ Previous work	➡️ Towards precision calculation
Action	Tadpole improved Wilson clover fermion action	
Improvement	—	HYP smear for Wilson line; Kinematically-enhanced interpolating operators
Lattice spacing	a=0.05187fm	a=0.0775fm, 0.06826fm, 0.05187fm
NPR	Simplified hybrid scheme	Hybrid scheme based on self-renormalization
Pz extrapolation	—	Infinite momentum extrapolation
Statistics	~5k	60k~100k

[**XYH**, Hua, Ji, Lü, et al., 2024;
XYH, Hua, Ji, Lü, et al., 2025]



Ensembles we use



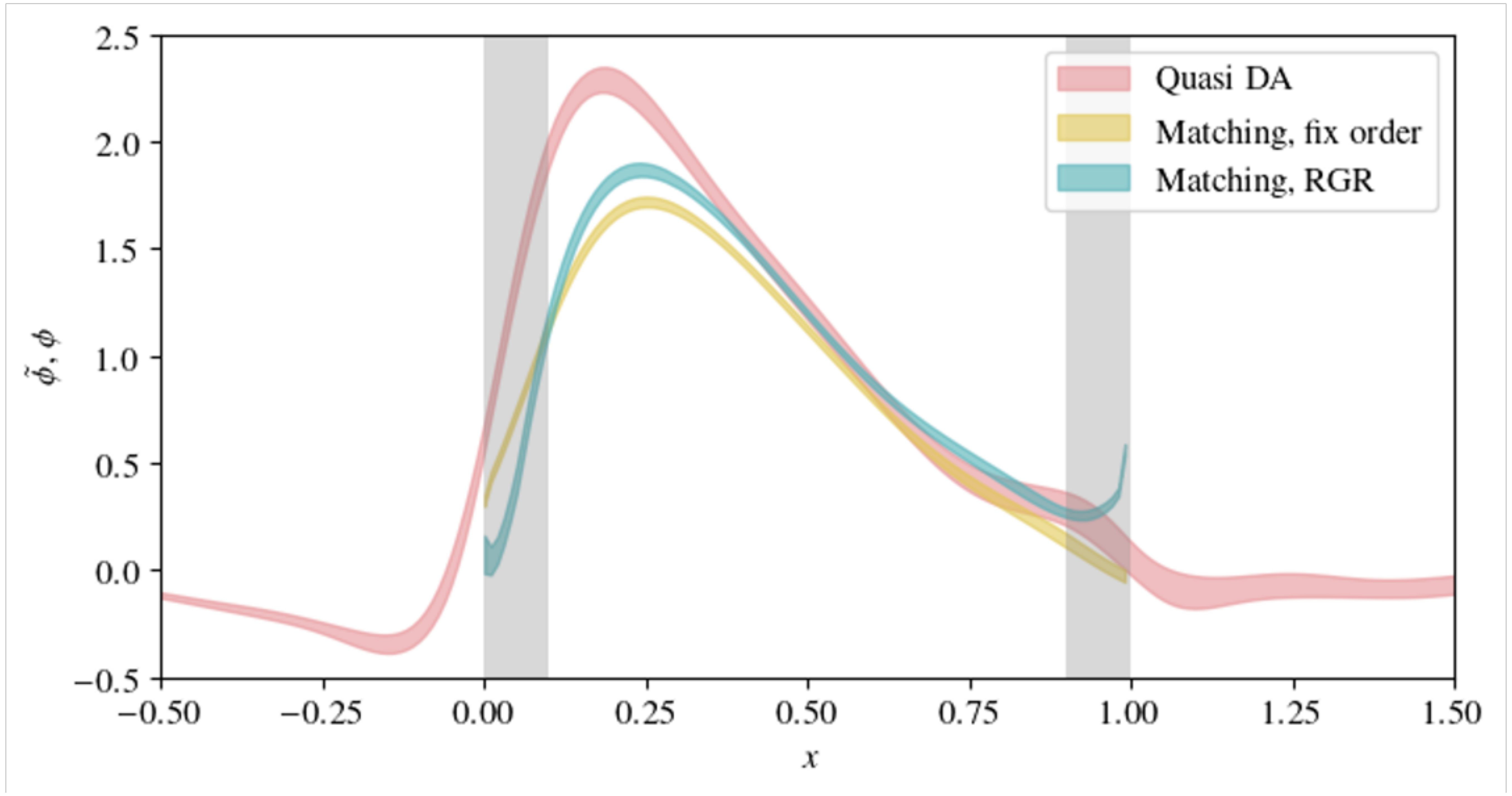
LaMET matching

- LaMET inverse matching

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

$$\phi(y) = \tilde{\phi}(y) - \int dx\, C^{(1)}(y, x)\, \tilde{\phi}(x) + \mathcal{O}(\alpha_s^2)$$

[Liu, Wang, Xu, Zhang, Zhao, 2019]



Fix order: $\mu = 2yP^z$

RGR: $\phi(y,\ 2yP^z) \xrightarrow{\text{ERBL}} \phi(y,\ 2\text{GeV})$

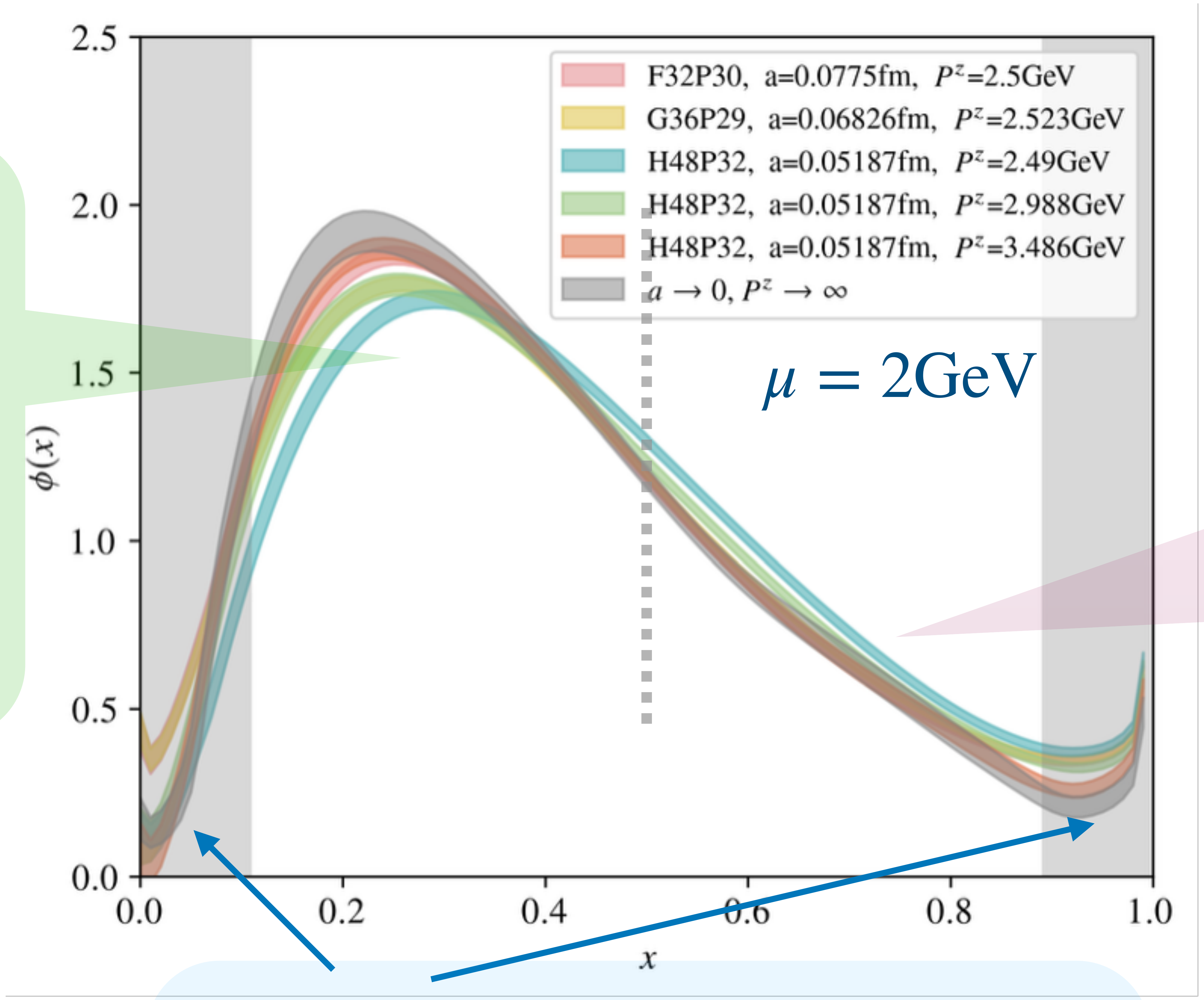
Quasi DA & QCD LCDA

QCD LCDA of D meson

Peak region: $y \sim \frac{\Lambda_{QCD}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA

Beneke, Finauri, Vos, Wei, 2023
Ishaq, Jia, Xiong, Yang, et al., 2020



Tail region: $y \sim 1$

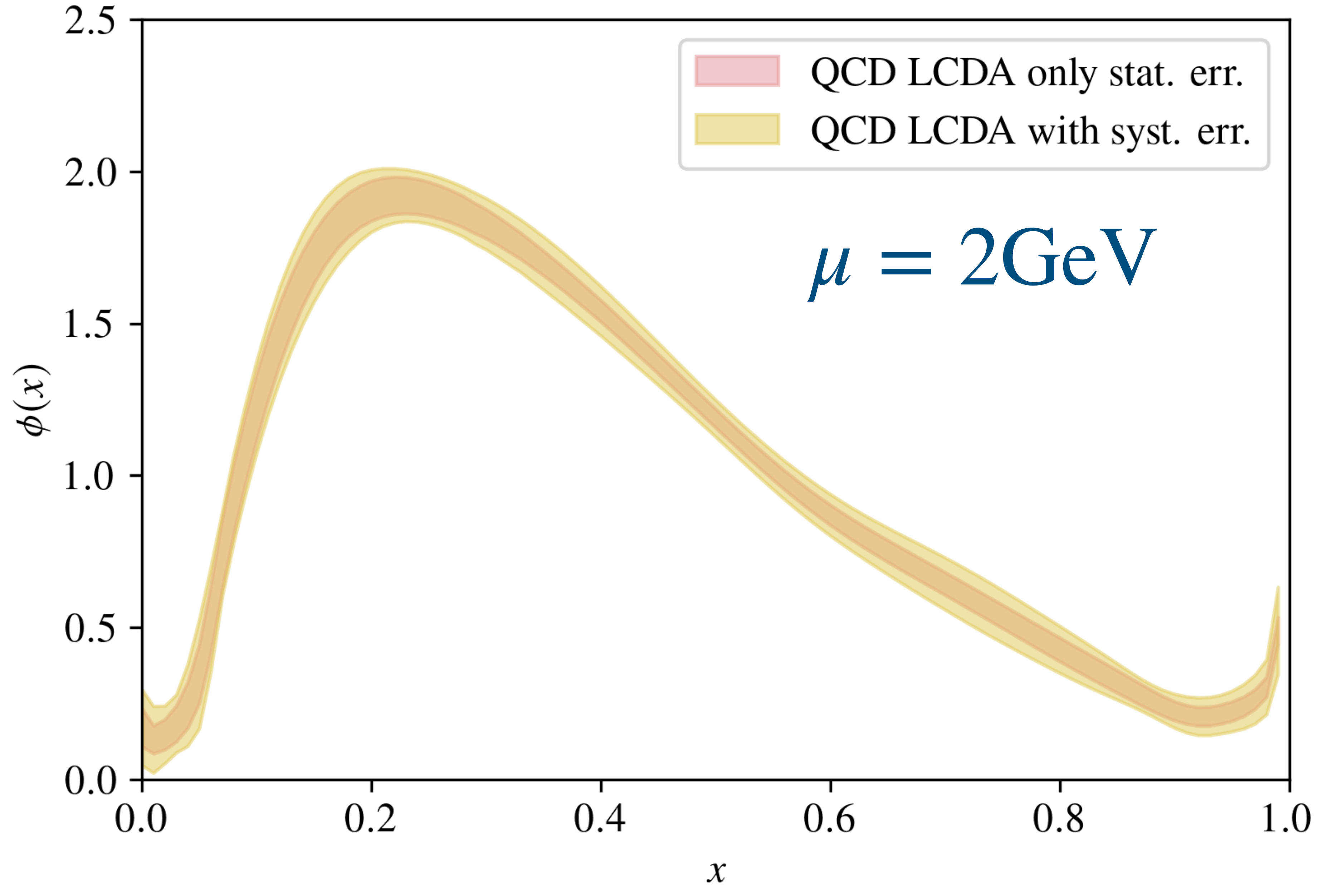
- Contain only hard-collinear physics;
- Suppressed in LCDA.

End-point region

- LaMET matching kernel suffer large power corrections
- Lattice predictions fail

QCD LCDA of D meson

- Analysis of systematic errors



- Systematic errors' origins
  - Hybrid-renormalization
  - λ-extrapolation
  - LaMET matching scale factor $\mu_i = 2cyP^z$, $c = \{1/\sqrt{2}, \sqrt{2}\}$
  - Continuum extrapolation and infinite momentum limit

XYH, Zhang, et al., 2508.xxxxx

- Fit Gegenbauer moments directly

$$\phi(x) = 6x(1-x)\left[1 + \sum_n^{\infty} a_n C_n^{3/2}(2x-1)\right]$$

n	a_1	a_2	a_3	a_4	a_5	a_6
2	-0.397(18)	0.118(10)				
4	-0.412(20)	0.134(16)	-0.016(11)	0.005(8)		
6	-0.391(22)	0.111(18)	0.026(20)	-0.019(15)	0.024(9)	-0.004(7)
8	-0.376(24)	0.109(19)	0.041(22)	-0.034(18)	0.041(11)	-0.012(9)

HQET LCDA

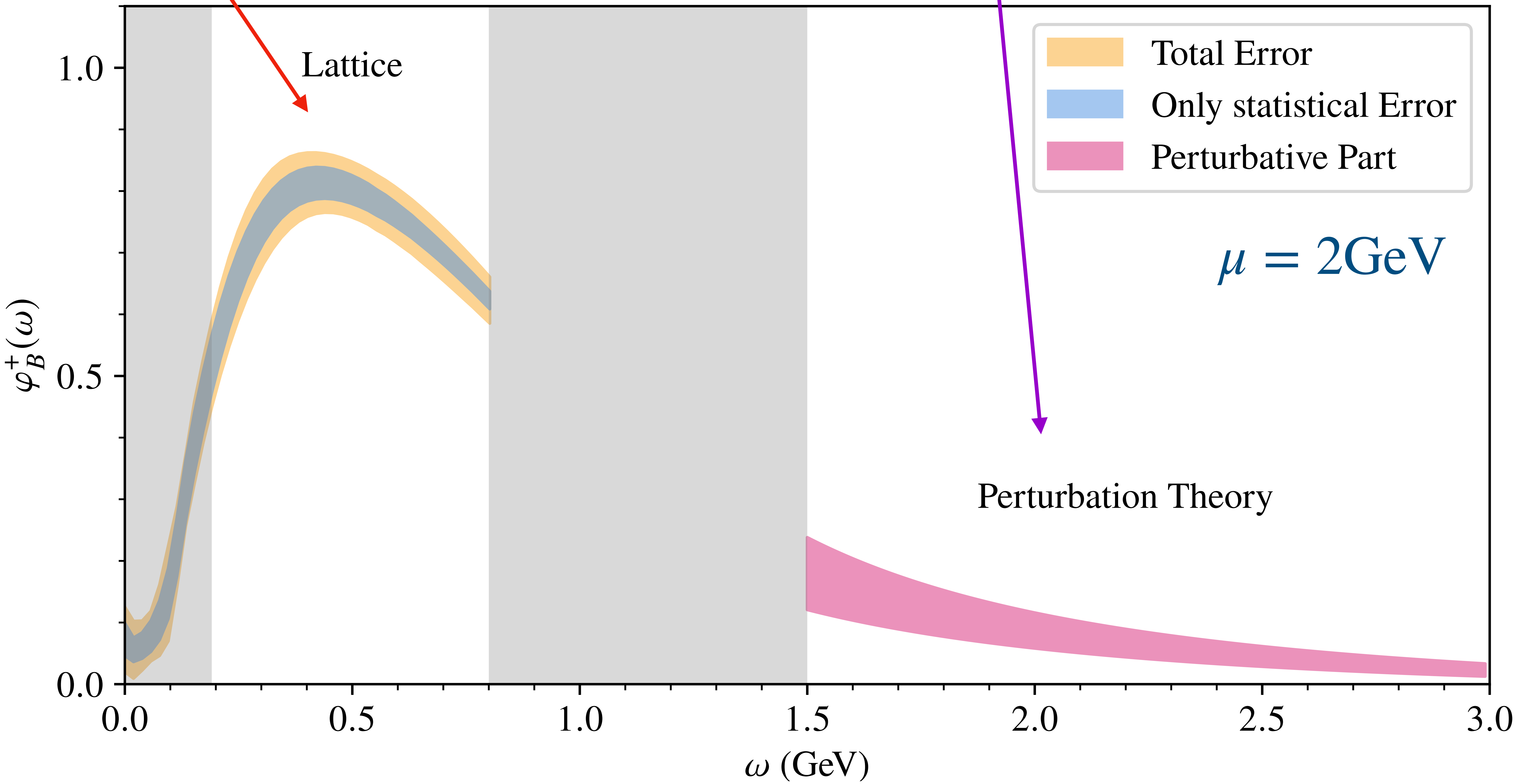
- $\omega \sim \Lambda_{\rm QCD}$

$$\varphi^+_{\rm peak}(\omega,\mu) = \frac{1}{m_H}\frac{f_H}{\tilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y,\mu;m_H)$$

- $\omega \gg \Lambda_{\rm QCD}$

$$\varphi^+_{\rm tail} = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

[Lee, Neubert, 2005]

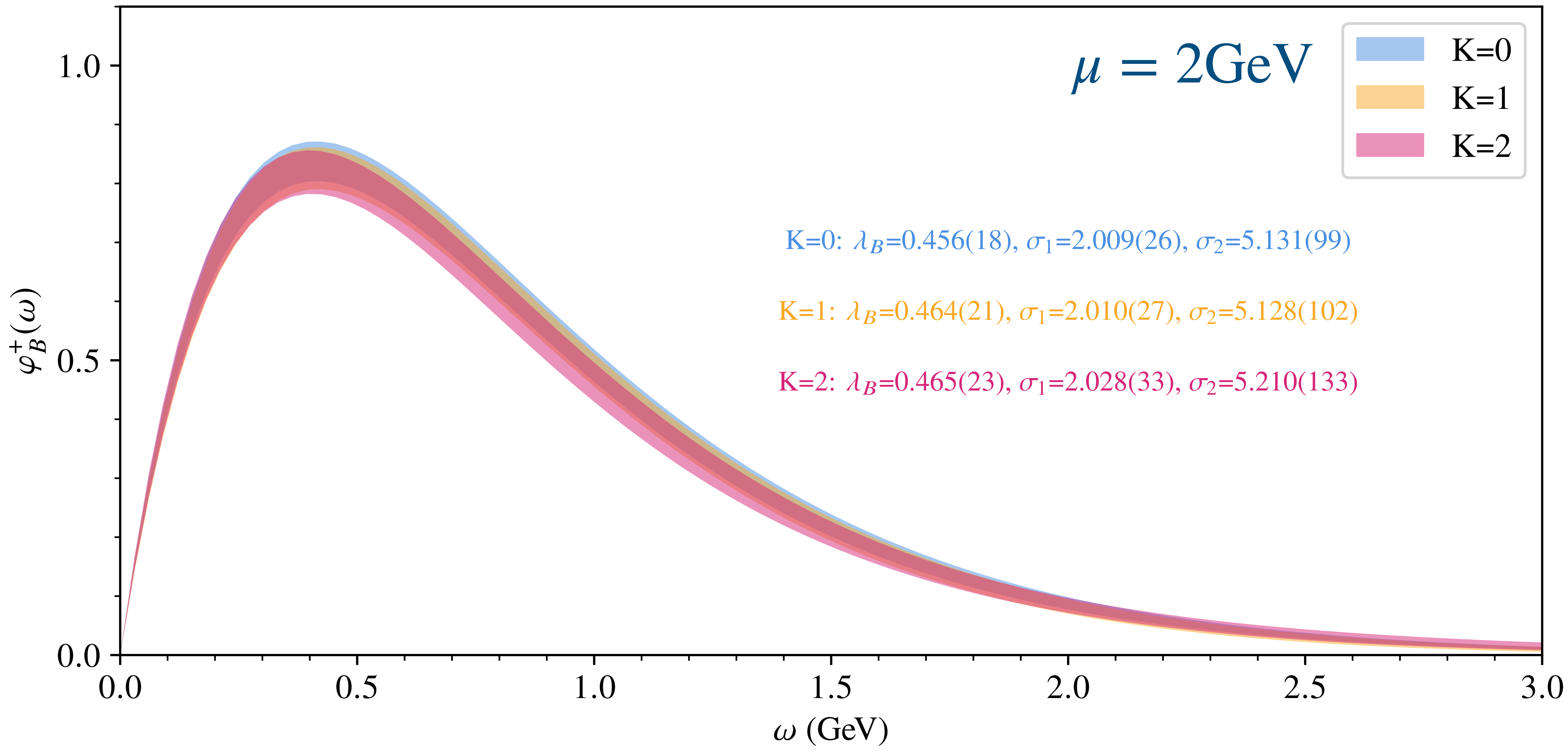


Latest model-based fitting

- Expansion in generalized Laguerre polynomials

$$\varphi^+(\omega,\mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^{K} \frac{a_k(\mu)}{1+k} L_k^{(1)}(2\omega/\omega_0)$$

[Feldmann, Lüghausen, Dyk et al. 2022]



Inverse moment

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega} \varphi^+(\omega,\mu)$$

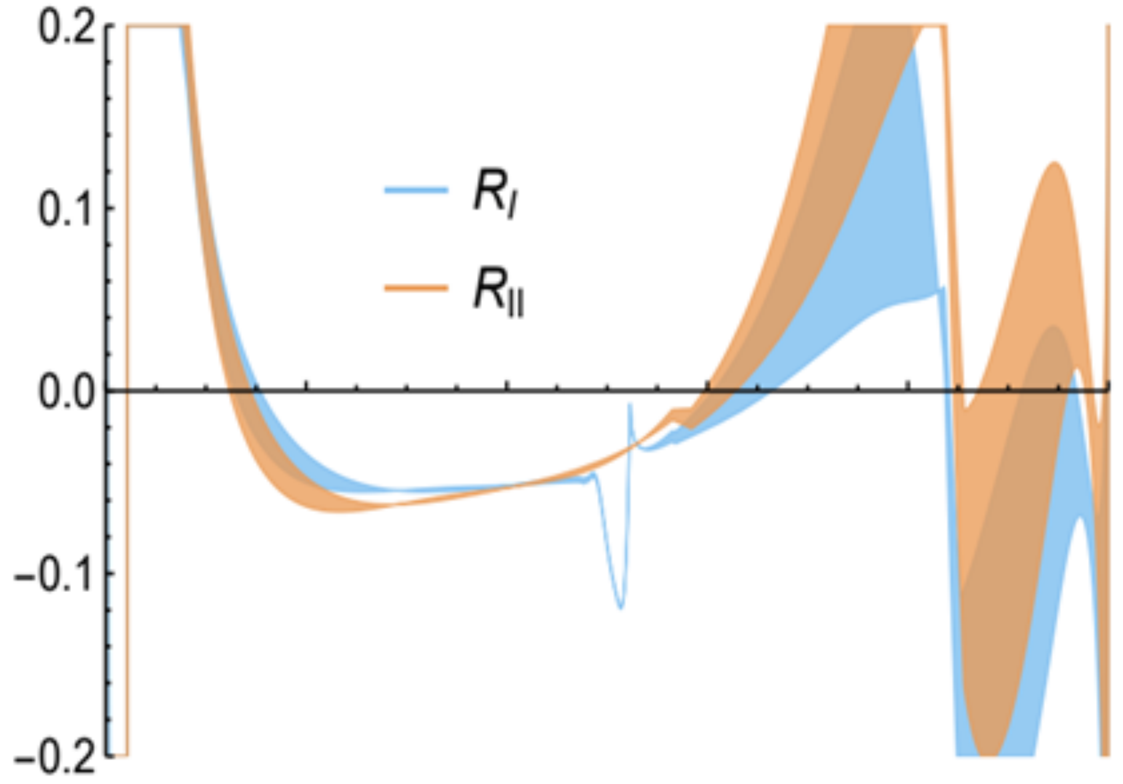
Log moment

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu) \int_0^\infty \frac{d\omega}{\omega} \ln\left(\frac{\mu}{\omega}\right)^n \varphi^+(\omega,\mu)$$

$$\lambda_B = 0.464(21), \sigma_1 = 2.010(27), \sigma_2 = 5.128(102)$$

Power Corrections

- 3 scales in SET intruducing the following powers
  - Heavy hadron mass correction in LaMET: $M^2/(P^z)^2$



Han, Wang, Zhang, Zhang, 2024

    - Smaller than 20% in most regions;
    - Smaller than 10% in the region of boosted matching.
  - Power correction in LaMET: $\Lambda_{\rm QCD}^2/(xP^z)^2$
    - Significant at end-point region $x \to 0, 1$ of the QCD LCDA
    - Can be improved by considering the renormalon resummation, …

Su, Holligan, Ji, Yao, Zhang, Zhang, 2023

  - Heavy quark mass correction in boosted HQET: $\Lambda_{\rm QCD}/m_Q$
    - A possible solution is proposed in [Deng, Wang, Wei, Zeng, PRD110, 114006, (2024)];
    - HQET LCDA shows degeneracy in the Dirac structures due to heavy quark spin symmetry;
    - This power correction can be estimated from HQET LCDAs of pseudoscalar and vector mesons.

🚀 Summary & Outlook

🌱 We propose a sequential effective theory (SET) that bridges lattice calculable Euclidean correlators to the parton distribution profiles of heavy mesons.

🌱 The factorization formula for heavy meson LCDAs within SET has been established. Systematic lattice QCD investigations are now available.

🌱 The theoretical framework for heavy meson shape functions has also been proposed, expecting further validation in lattice QCD calculations.

Thank you for your attention!

Backup

Heavy Quark Mass RGE

As a key scale in HQET, m_Q reflects the intrinsic dynamics in heavy hadrons.

- Theoretically, m_Q is either m_c or m_b. While lattice QCD enables simulations at arbitrary m_Q, this motivates systematic studies of m_Q evolutions in heavy hadronic systems.
- The m_Q-RGE of heavy meson LCDAs:

$$m_Q \frac{\partial}{\partial m_Q}\phi(u, m_Q; \mu) - u\frac{\partial}{\partial u}\phi(u, m_Q; \mu) - (1 + \gamma(m_Q, \mu))\phi(u, m_Q; \mu) = 0,$$

and its solution

$$\phi(u, m_Q; \mu) \approx \exp\left[\frac{2C_F}{\beta_0}\ln\frac{\alpha_s(m_Q)}{\alpha_s(m_{Q_0})} - \frac{4\pi C_F}{\beta_0^2}\left(\frac{1}{\alpha_s(m_{Q_0})}\ln\frac{\alpha_s(\mu)}{\alpha_s(m_{Q_0})e} - \frac{1}{\alpha_s(m_Q)}\ln\frac{\alpha_s(\mu)}{\alpha_s(m_Q)e}\right)\right]\frac{m_Q}{m_{Q_0}}\phi_0\left(u\frac{m_Q}{m_{Q_0}}\right).$$

Wang, Xu, Zhang, Zhao, 2024