School of Nuclear Science and Technology

Phenomenological study on semi-leptonic inclusive decay of charm meson

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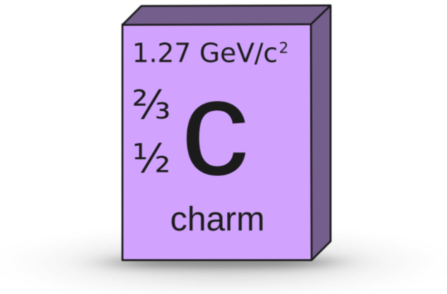
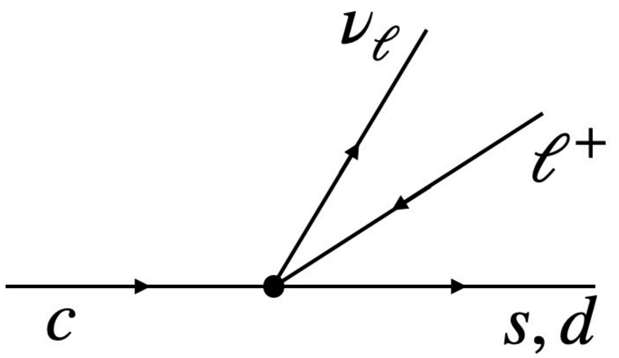


WORKSHOP ON CHARM HADRON PHYSICS AT BESIII 2025 - LANZHOU

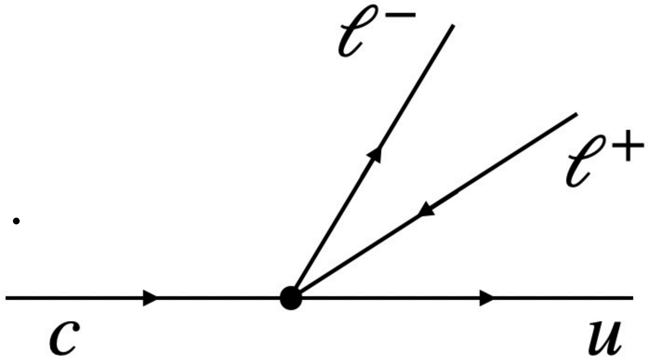
Content

1. What and Why?

What and Why?



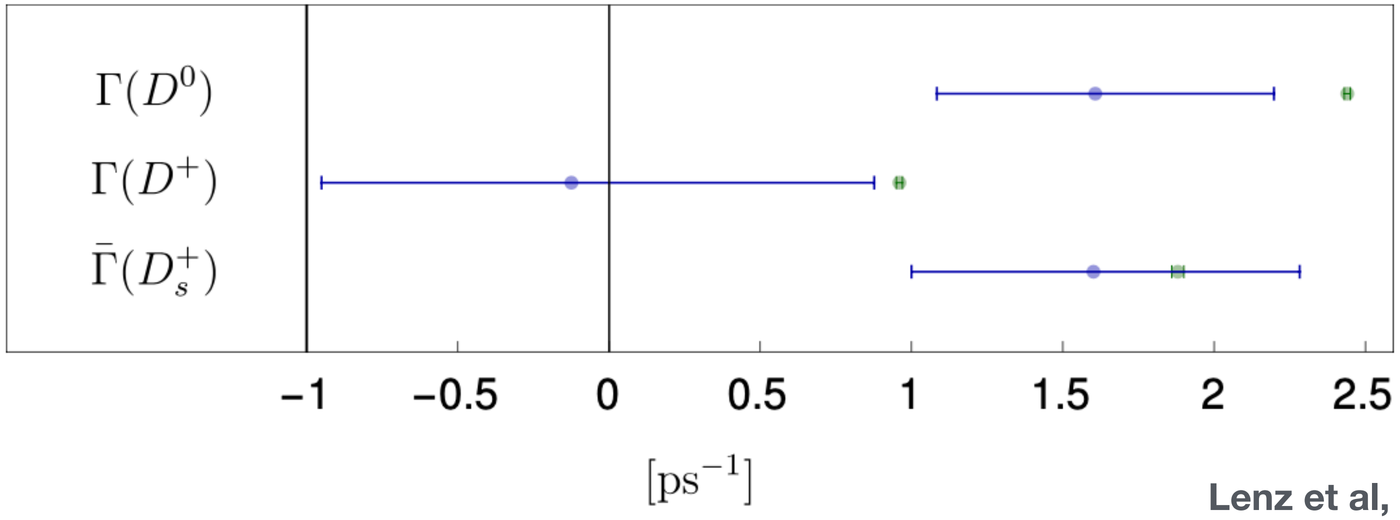
- **Experimental detection of partial final state particles**
  - ➡ $D \to e^+X$ ($D \to e^+\nu_e X$, only e^+ is detected)
- **Sum of a group of exclusive channels**
  - ➡ $D^0 \to e^+X_s = D \to e^+\nu_e K^-, e^+\nu_e K^-\pi^0, e^+\nu_e \bar{K}^0\pi^-, \ldots$
  - ➡ $D^0 \to e^+X_d = D \to e^+\nu_e \pi^-, e^+\nu_e \pi^-\pi^0, e^+\nu_e \pi^-\pi^+\pi^-, \ldots$



- **Compared to exclusive decays: Better theoretical control**
- **Compared to beauty decays: More sensitive to power corrections**

What and Why?

Charmed hadron lifetimes:
theory vs experiment



Lenz et al, ’22]

Fig 1

Meson

Experiment： High precision

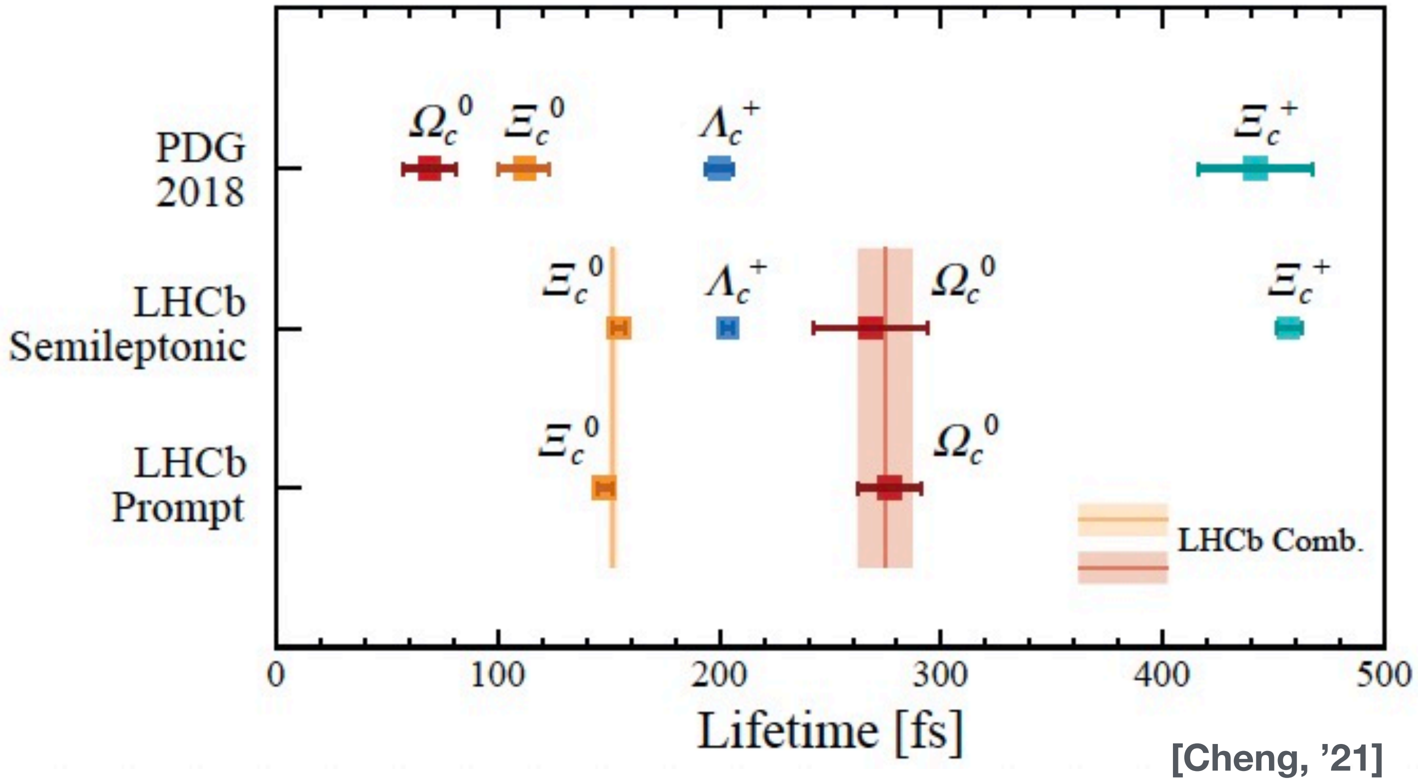
Theory： significant uncertainty

Baryon

Experiment： change in hierarchy

$$\mathcal{O}(1/m_c^3) \;\Rightarrow\; \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\boldsymbol{\Omega_c^0}),$$
$$\mathcal{O}(1/m_c^4) \;\Rightarrow\; \tau(\boldsymbol{\Omega_c^0}) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \;\Rightarrow\; \tau(\Xi_c^+) > \tau(\boldsymbol{\Omega_c^0}) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

[Cheng, ’21]



[Cheng, ’21]

Fig 2

What and Why?

$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \cdots$$

$\langle \mathcal{O}_5 \rangle$

μ_π^2

B meson $\longrightarrow$ D meson

$$\mu_\pi^2(\mathrm{D}) = (0.48 \pm 0.20)\mathrm{GeV}^2$$

$$\mu_\pi^2\left(\mathrm{D}_s^+\right) = (0.57 \pm 0.23)\mathrm{GeV}^2$$

Relative Error: ～40%

μ_G^2

DD* 介子质量差异

$$\mu_G^2(\mathrm{D}) = (0.34 \pm 0.10)\mathrm{GeV}^2$$

$$\mu_G^2\left(\mathrm{D}_s^+\right) = (0.36 \pm 0.10)\mathrm{GeV}^2$$

Relative Error: ～30%

$\langle \mathcal{O}_6 \rangle$ ρ_D^3

B meson $\longrightarrow$ D meson

$$\rho_D^3(\mathrm{D}) = (0.082 \pm 0.035)\mathrm{GeV}^3$$

$$\rho_D^3\left(\mathrm{D}_s\right) = (0.119 \pm 0.052)\mathrm{GeV}^3$$

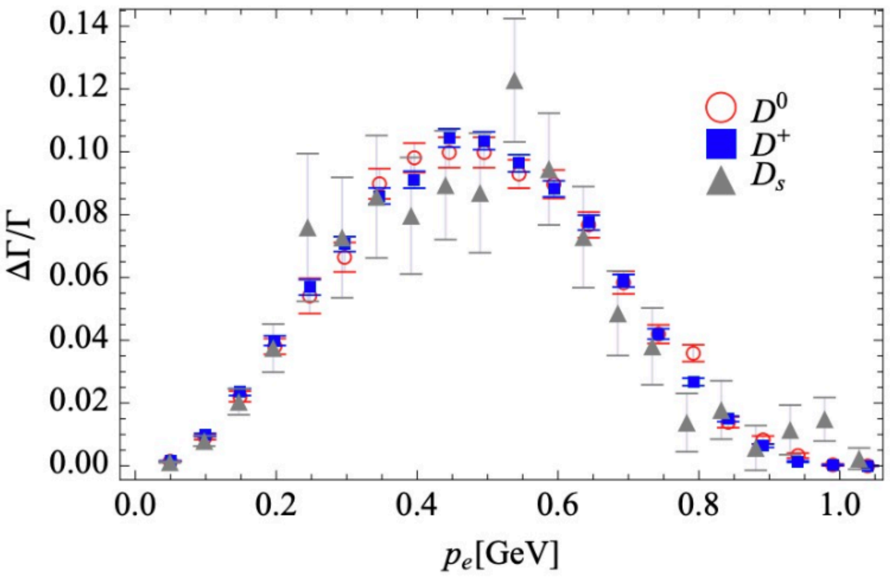
Relative Error: ～40%

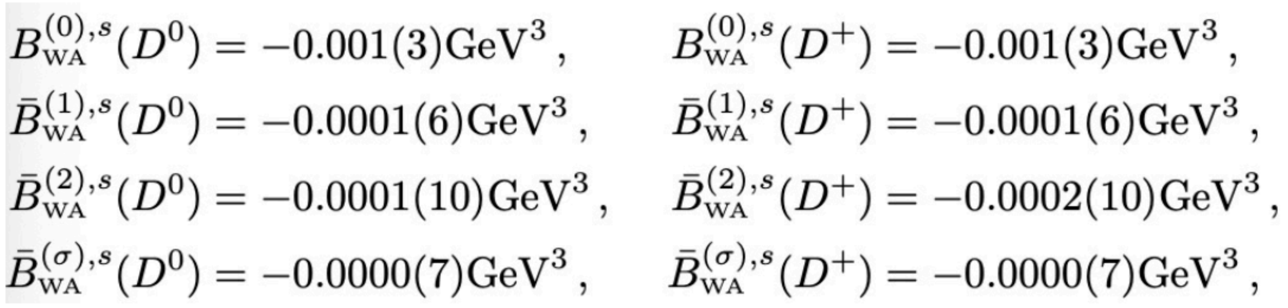
[Lenz et al, '22]

“ Again a more precise experimental determination of μ_π^2 from fits to semi-leptonic D^+, D^0 and D_s^+ meson decays — as it was done for the B^+ and B^0 decays — would be very desirable. ”

What and Why?

- Four quark operators were estimated





$$B^{(0),s}_{\rm WA}(D^0) = -0.001(3)\text{GeV}^3\,, \qquad B^{(0),s}_{\rm WA}(D^+) = -0.001(3)\text{GeV}^3\,,$$
$$\bar{B}^{(1),s}_{\rm WA}(D^0) = -0.0001(6)\text{GeV}^3\,, \qquad \bar{B}^{(1),s}_{\rm WA}(D^+) = -0.0001(6)\text{GeV}^3\,,$$
$$\bar{B}^{(2),s}_{\rm WA}(D^0) = -0.0001(10)\text{GeV}^3\,, \qquad \bar{B}^{(2),s}_{\rm WA}(D^+) = -0.0002(10)\text{GeV}^3\,,$$
$$\bar{B}^{(\sigma),s}_{\rm WA}(D^0) = -0.0000(7)\text{GeV}^3\,, \qquad \bar{B}^{(\sigma),s}_{\rm WA}(D^+) = -0.0000(7)\text{GeV}^3\,,$$

[Gambino, et al ’10]

- Strong interaction running coupling constant have been estimated

$$\alpha_s = 0.377 \pm 0.008 \pm 0.114$$

[Wu, et al ’24]

- Assuming non-perturbative parameters are **identical to those of B meson**
- Global Fitting under **kinetic mass scheme**

?

Starting from dimension-5!

2. Experimental Status and Theoretical Results

Experimental status

CLEO measurements

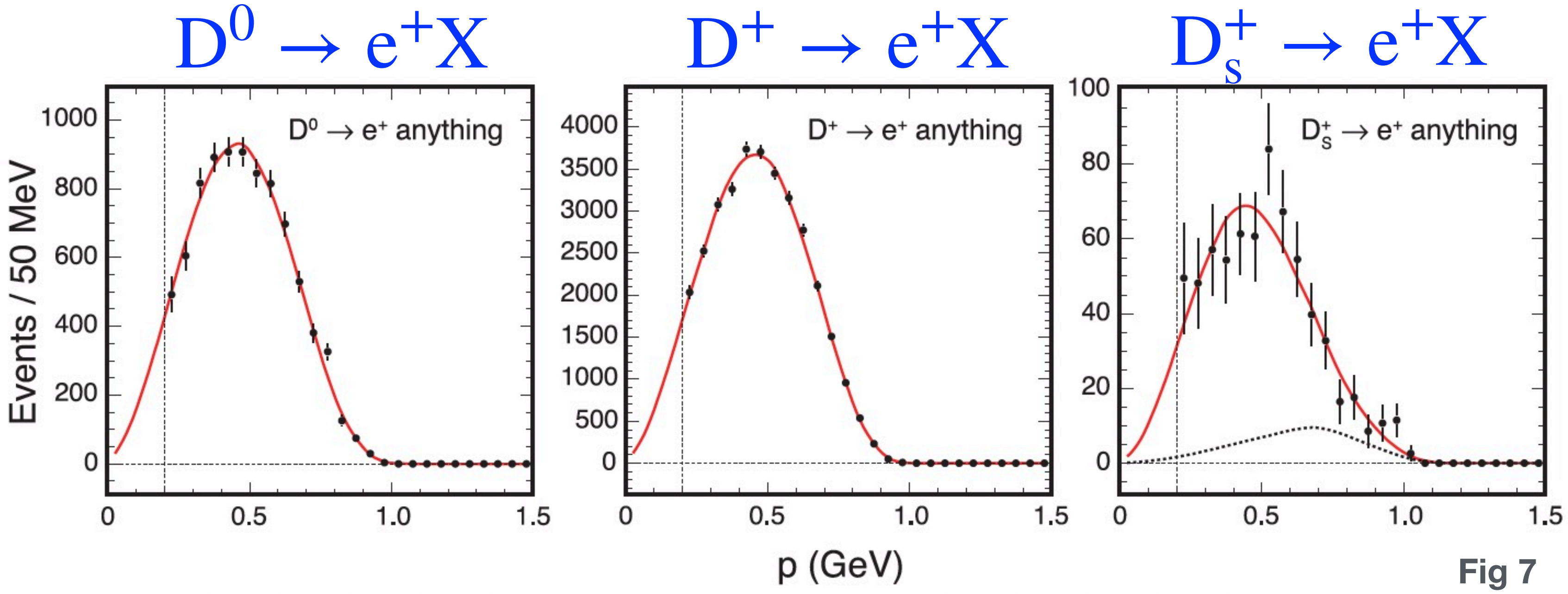
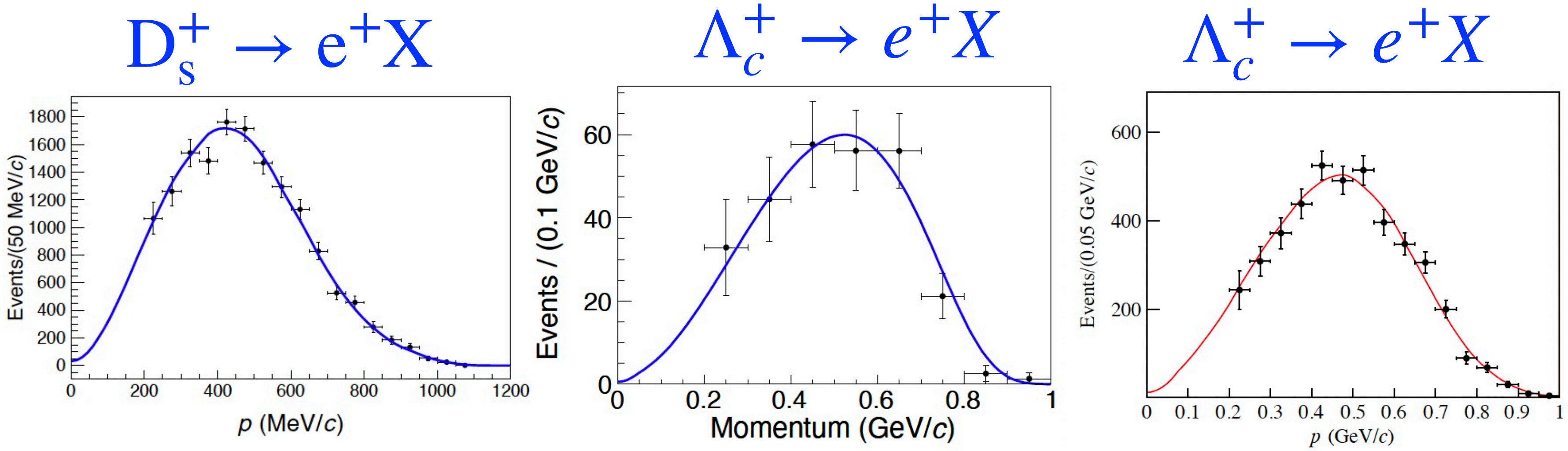

$D^0 \to e^+X$ $D^+ \to e^+X$ $D_s^+ \to e^+X$

Fig 7

$$B\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$
$$B\left(D^+ \to Xe^+\nu_e\right) = (16.13 \pm 0.10 \pm 0.29)\,\%$$
$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.52 \pm 0.39 \pm 0.15)\,\%$$

[CLEO, '09]

BESIII measurements


$D_s^+ \to e^+X$ $\Lambda_c^+ \to e^+X$ $\Lambda_c^+ \to e^+X$

[BESIII, '21]

Fig 8

$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, '21]

$$B\left(\Lambda_c^+ \to Xe^+\nu_e\right) = (3.95 \pm 0.34 \pm 0.09)\,\%$$

[BESIII (567 pb^{-1}), '18]

$$B\left(\Lambda_c^+ \to Xe^+\nu_e\right) = \left(4.06 \pm 0.10_{\text{stat.}} \pm 0.09_{\text{syst}}\right)\%$$

[BESIII (4.5 fb^{-1}), '23]

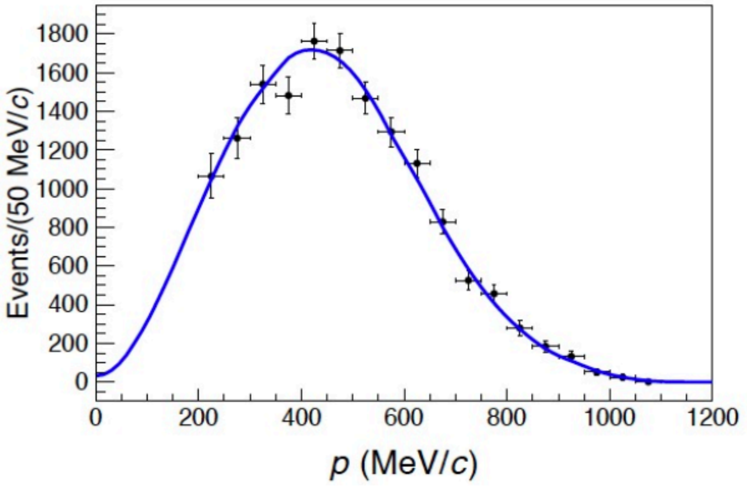
Experimental status

Electronic Energy spectrum $\left(y \equiv 2E_e/m_c\right)$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = 12(1-y)y^2\theta(1-y)$$

$$+\frac{2\mu_\pi^2}{m_c^2}\left[-10y^3\theta(1-y) + 2\delta(1-y)\right.$$

$$-\frac{2\mu_G^2}{3m_c^2}\left[6y^2(6-5y)\theta(1-y)\right] + O\left(\alpha_s, \frac{\Lambda^3}{m_c^3}\right)$$



[BESIII, ’21]

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum

[Neubert, 1995] **[Mannel et al,’94]**

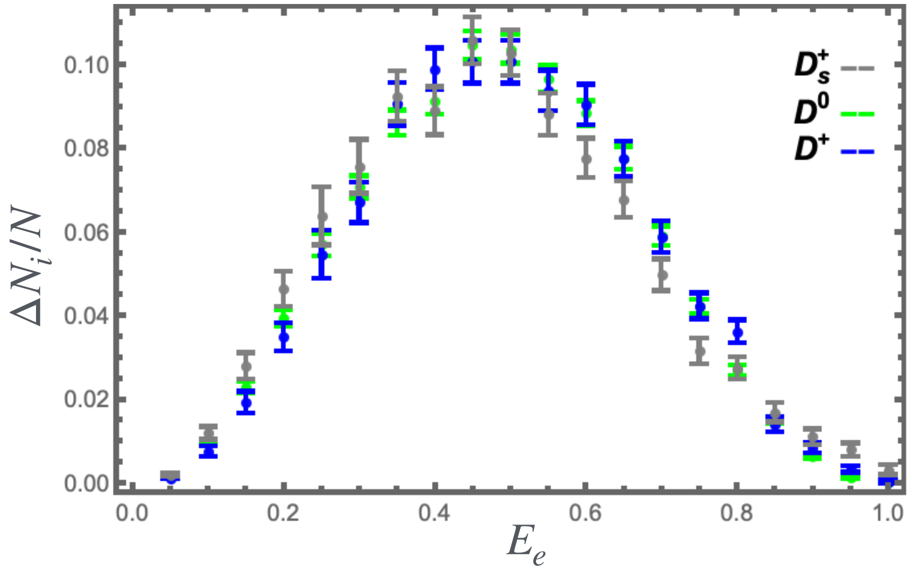
- Observables require integration over final states

$$\Gamma = \int \frac{d\Gamma}{dy}dy, \left\langle E_\ell^n \right\rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^n dy, n = 1,2,3,4$$

[Gambino,Kamenik, ’10]

Experimental status

Our main efforts on data



$$B(D_s \to X e^+ \nu_e) = 0.0631(14) \qquad B(D^0 \to X e^+ \nu_e) = 0.0636(15) \qquad B(D^+ \to X e^+ \nu_e) = 0.1602(32)$$

$$\langle E_e \rangle_{\rm exp}^{D_s} = 0.439(5)\mathrm{GeV}, \quad \langle E_e^2 \rangle_{\rm exp}^{D_s} = 0.223(5)\mathrm{GeV}^2, \quad \langle E_e^3 \rangle_{\rm exp}^{D_s} = 0.124(4)\mathrm{GeV}^3, \quad \langle E_e^4 \rangle_{\rm exp}^{D_s} = 0.074(3)\mathrm{GeV}^4,$$
$$\langle E_e \rangle_{\rm exp}^{D^0} = 0.462(5)\mathrm{GeV}, \quad \langle E_e^2 \rangle_{\rm exp}^{D^0} = 0.242(5)\mathrm{GeV}^2, \quad \langle E_e^3 \rangle_{\rm exp}^{D^0} = 0.138(4)\mathrm{GeV}^3, \quad \langle E_e^4 \rangle_{\rm exp}^{D^0} = 0.084(3)\mathrm{GeV}^4,$$
$$\langle E_e \rangle_{\rm exp}^{D^+} = 0.455(4)\mathrm{GeV}, \quad \langle E_e^2 \rangle_{\rm exp}^{D^+} = 0.236(4)\mathrm{GeV}^2, \quad \langle E_e^3 \rangle_{\rm exp}^{D^+} = 0.134(3)\mathrm{GeV}^3, \quad \langle E_e^4 \rangle_{\rm exp}^{D^+} = 0.081(3)\mathrm{GeV}^4.$$

$$\langle E_e^2 \rangle_{\rm exp,center}^{D_s} = 0.0297(13)\,\mathrm{GeV}^2, \quad \langle E_e^3 \rangle_{\rm exp,center}^{D_s} = 0.0004(4)\,\mathrm{GeV}^3, \quad \langle E_e^4 \rangle_{\rm exp,center}^{D_s} = 0.0021(2)\,\mathrm{GeV}^4,$$
$$\langle E_e^2 \rangle_{\rm exp,center}^{D^0} = 0.0287(12)\,\mathrm{GeV}^2, \quad \langle E_e^3 \rangle_{\rm exp,center}^{D^0} = -0.0001(3)\,\mathrm{GeV}^3, \quad \langle E_e^4 \rangle_{\rm exp,center}^{D^0} = 0.0019(1)\,\mathrm{GeV}^4,$$
$$\langle E_e^2 \rangle_{\rm exp,center}^{D^+} = 0.0291(11)\,\mathrm{GeV}^2, \quad \langle E_e^3 \rangle_{\rm exp,center}^{D^+} = -0.0002(3)\,\mathrm{GeV}^3, \quad \langle E_e^4 \rangle_{\rm exp,center}^{D^+} = 0.0019(1)\,\mathrm{GeV}^4,$$

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Experimental status

Correlations among observables

$$\mathrm{Cor}(D_s^+) = \begin{array}{cccccc}
\Gamma_{\mathrm{sl}} & \langle E_e\rangle & \langle E_e^2\rangle & \langle E_e^3\rangle & \langle E_e^4\rangle & \\
1. & -0.0948363 & -0.0854423 & -0.0692735 & -0.0528099 & \Gamma_{\mathrm{sl}} \\
-0.0948363 & 1. & 0.961103 & 0.877715 & 0.779303 & \langle E_e\rangle \\
-0.0854423 & 0.961103 & 1. & 0.973193 & 0.910118 & \langle E_e^2\rangle \\
-0.0692735 & 0.877715 & 0.973193 & 1. & 0.979699 & \langle E_e^3\rangle \\
-0.0528099 & 0.779303 & 0.910118 & 0.979699 & 1. & \langle E_e^4\rangle
\end{array}$$

Center moment $\langle \mathrm{E}_{\mathrm{e}}^{\mathrm{n}}\rangle_{\mathrm{center}} \equiv \langle (E_e - \langle E_e\rangle)^n\rangle$,

$$\mathrm{Cor}(D_s^+) = \begin{array}{cccccc}
\Gamma_{\mathrm{sl}} & \langle E_e\rangle & \langle E_e^2\rangle_{\mathrm{center}} & \langle E_e^3\rangle_{\mathrm{center}} & \langle E_e^4\rangle_{\mathrm{center}} & \\
1. & -0.0948363 & 0.0254998 & 0.0800786 & 0.00809594 & \Gamma_{\mathrm{sl}} \\
-0.0948363 & 1. & -0.047314 & -0.40486 & -0.0191632 & \langle E_e\rangle \\
0.0254998 & -0.047314 & 1. & 0.0763575 & 0.800281 & \langle E_e^2\rangle_{\mathrm{center}} \\
0.0800786 & -0.40486 & 0.0763575 & 1. & 0.172337 & \langle E_e^3\rangle_{\mathrm{center}} \\
0.00809594 & -0.0191632 & 0.800281 & 0.172337 & 1. & \langle E_e^4\rangle_{\mathrm{center}}
\end{array}$$

$$\boxed{\Gamma_{\mathrm{sl}} \quad \langle E_e\rangle \quad \langle E_e^2\rangle_{\mathrm{center}} \quad \langle E_e^3\rangle_{\mathrm{center}} \quad \langle E_e^4\rangle_{\mathrm{center}}}$$

Theoretical Results

Our main efforts on theory

NLO analytical integration

NNLO numerical results provided by Long Chen

$$\Gamma_{D_i} = \sum_{q=d,s} \hat{\Gamma}_0 |V_{cq}|^2 m_c^5 \Big\{ 1 + \frac{\alpha_s}{\pi}\frac{2}{3}\left(\frac{25}{4}-\pi^2\right) + \frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\frac{2}{3}\left(\frac{25}{4}-\pi^2\right)\log\left(\frac{\mu^2}{m_c^2}\right) + 2.14690 n_l - 29.88311\right]$$

[Chen, Chen, Guan, Ma, ’23]

$$- 8\rho\delta_{sq} - \frac{1}{2}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{3}{2}\frac{\mu_G^2(D_i)}{m_c^2} + \left(6+8\log\left(\frac{\mu^2}{m_c^2}\right)\right)\frac{\rho_D^3(D_i)}{m_c^3} + \frac{\tau_0(D_i)}{m_c^3} + \ldots \Big\},$$

Dim-5, $\Lambda_{\mathrm{QCD}}^2/\mathrm{m}_\mathrm{c}^2$

Dim-6, $\Lambda_{\mathrm{QCD}}^3/\mathrm{m}_\mathrm{c}^3$

$D \to e^+X$ ($D \to e^+\nu_e X$, only e^+ is detected)

Power correlation： up to **dim-6** operators contributions

Perturbative correlation： up to **NNLO** contributions

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Theoretical Results

Our main efforts on theory

$$\langle E_e \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 \, m_c^6 \left[\frac{3}{10} + \frac{\alpha_s}{\pi} a_1^{(1)} + \frac{\alpha_s^2}{\pi^2} a_1^{(2)} - 3\rho\delta_{sq} - \frac{1}{2}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{139}{30} + 4\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3\,(D_i)}{m_c^3} + \frac{3}{10}\frac{\rho_{LS}^3\,(D_i)}{m_c^3} \right.$$
$$\left. + \, \frac{\tau_0}{2m_c^3} + ... \right],$$

$$\langle E_e^2 \rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 \, m_c^7 \left[\frac{1}{10} + \frac{\alpha_s}{\pi} a_2^{(1)} + \frac{\alpha_s^2}{\pi^2} a_2^{(2)} - \frac{6}{5}\rho\delta_{sq} + \frac{1}{12}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{11}{60}\frac{\mu_G^2(D_i)}{m_c^2} + (\frac{17}{6} + 2\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3\,(D_i)}{m_c^3} \right.$$
$$\left. + \frac{7}{30}\frac{\rho_{LS}^3\,(D_i)}{m_c^3} + \frac{\tau_0}{4m_c^3} + ... \right],$$

$$\left\langle E_e^3 \right\rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 \, m_c^8 \left[\frac{1}{28} + \frac{\alpha_s}{\pi} a_3^{(1)} + \frac{\alpha_s^2}{\pi^2} a_3^{(2)} - \frac{1}{2}\rho\delta_{sq} + \frac{1}{14}\frac{\mu_\pi^2\,(D_i)}{m_c^2} - \frac{1}{14}\frac{\mu_G^2\,(D_i)}{m_c^2} + (\frac{223}{140} + \log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3\,(D_i)}{m_c^3} \right.$$
$$\left. + \frac{1}{7}\frac{\rho_{LS}^3\,(D_i)}{m_c^3} + \frac{\tau_0}{8m_c^3} + ... \right],$$

$$\left\langle E_e^4 \right\rangle_{D_i} = \frac{\hat{\Gamma}_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2 \, m_c^9 \left[\frac{3}{224} + \frac{\alpha_s}{\pi} a_4^{(1)} + \frac{\alpha_s^2}{\pi^2} a_4^{(2)} - \frac{3}{14}\rho\delta_{sq} + \frac{3}{64}\frac{\mu_\pi^2\,(D_i)}{m_c^2} - \frac{13}{448}\frac{\mu_G^2\,(D_i)}{m_c^2} + (\frac{481}{560} + \frac{1}{2}\log(\frac{\mu^2}{m_c^2}))\frac{\rho_D^3\,(D_i)}{m_c^3} \right.$$
$$\left. + \frac{9}{112}\frac{\rho_{LS}^3\,(D_i)}{m_c^3} + \frac{\tau_0}{16m_c^3} + ... \right],$$

$$\left\langle E_e^2 \right\rangle_{\text{center}} \equiv \left\langle (E_e - \langle E_e \rangle)^2 \right\rangle$$

3. Phenomenological Analysis

Phenomenological Analysis

- 15 data points

$$\aleph^{D^0}_{theory} := \{\Gamma^{D^{0,+}}, \langle E_e\rangle^{D^{0,+}}, \langle E_e^2\rangle^{D^{0,+}}_{\text{center}}, \langle E_e^3\rangle^{D^{0,+}}_{\text{center}}, \langle E_e^4\rangle^{D^{0,+}}_{\text{center}}\},$$
$$\aleph^{D^+}_{theory} := \{\Gamma^{D^{0,+}}, \langle E_e\rangle^{D^{0,+}}, \langle E_e^2\rangle^{D^{0,+}}_{\text{center}}, \langle E_e^3\rangle^{D^{0,+}}_{\text{center}}, \langle E_e^4\rangle^{D^{0,+}}_{\text{center}}\},$$
$$\aleph^{D_s}_{theory} := \{\Gamma^{D_s}, \langle E_e\rangle^{D_s}, \langle E_e^2\rangle^{D_s}_{\text{center}}, \langle E_e^3\rangle^{D_s}_{\text{center}}, \langle E_e^4\rangle^{D_s}_{\text{center}}\}.$$

- Scenario 1: 4 parameters to be estimated (up to **dim-5** operators)

$$\mu_\pi^2\left(\mathrm{D}^{0,+}\right), \quad \mu_\mathrm{G}^2\left(\mathrm{D}^{0,+}\right), \quad \mu_\pi^2\left(\mathrm{D}_\mathrm{s}^+\right), \quad \mu_\mathrm{G}^2\left(\mathrm{D}_\mathrm{s}^+\right)$$

- Scenario 2: 8 parameters to be estimated (up to **dim-6** operators) VIA

$$\mu_\pi^2\left(\mathrm{D}^{0,+}\right), \quad \mu_\mathrm{G}^2\left(\mathrm{D}^{0,+}\right), \quad \rho_\mathrm{D}^3\left(\mathrm{D}^{0,+}\right), \quad \rho_\mathrm{LS}^3\left(\mathrm{D}^{0,+}\right) \quad \mu_\pi^2\left(\mathrm{D}_\mathrm{s}^+\right), \quad \mu_\mathrm{G}^2\left(\mathrm{D}_\mathrm{s}^+\right), \quad \rho_\mathrm{D}^3\left(\mathrm{D}_\mathrm{s}^+\right), \quad \rho_\mathrm{LS}^3\left(\mathrm{D}_\mathrm{s}^+\right)$$

- Scenario 3: 10 parameters to be estimated (up to **dim-6** operators,WA) HQETSR

$$\mu_\pi^2\left(\mathrm{D}^{0,+}\right), \quad \mu_\mathrm{G}^2\left(\mathrm{D}^{0,+}\right), \quad \rho_\mathrm{D}^3\left(\mathrm{D}^{0,+}\right), \quad \rho_\mathrm{LS}^3\left(\mathrm{D}^{0,+}\right) \quad \mu_\pi^2\left(\mathrm{D}_\mathrm{s}^+\right), \quad \mu_\mathrm{G}^2\left(\mathrm{D}_\mathrm{s}^+\right), \quad \rho_\mathrm{D}^3\left(\mathrm{D}_\mathrm{s}^+\right), \quad \rho_\mathrm{LS}^3\left(\mathrm{D}_\mathrm{s}^+\right)$$
$$\tau_{\text{val}}, \tau_{\text{nonval}}$$

Error from unknown power correlations

- Error from input parameters: $\mu \in [1,2.54]\mathrm{GeV}$

Phenomenological Analysis

The width of semi-leptonic inclusive decays in D mesons is **highly sensitive** to the charm quark mass.

- 1S mass scheme: **well perturbative behaviors** $1/2\,J/\psi$质量

$$\Gamma/\Gamma_{\mathrm{LO}} \approx 1 - 13.1\% - 4.8\% + 1.8\,\%$$

[Hoang,Ligeti,Manohar, ’98; Hoang,Teubner, ’99]

- $\overline{\mathrm{MS}}$ mass scheme: **slow** convergence

$$\Gamma/\Gamma_{\mathrm{LO}} = 1 + 1.35\alpha_s + 3.02\alpha_s^2 + 7.69\alpha_s^3 \approx 1 + 52\% + 46\% + 44\,\%$$

[Melnikov,van Ritbergen, ’99]

- Pole mass scheme**: no** convergence

Naive parameters of HQET

$$\Gamma/\Gamma_{\mathrm{LO}} = 1 - 0.77\alpha_s - 2.38\alpha_s^2 - 10.73\alpha_s^3 \approx 1 - 30\% - 36\% - 62\,\%$$

- Kinetic mass scheme: $\left(\alpha_s/\pi\right)\mu^n/m_c^n$ cut-off scale **somewhat subtle**

[Fael,Schönwald, Steinhauser, ’20]

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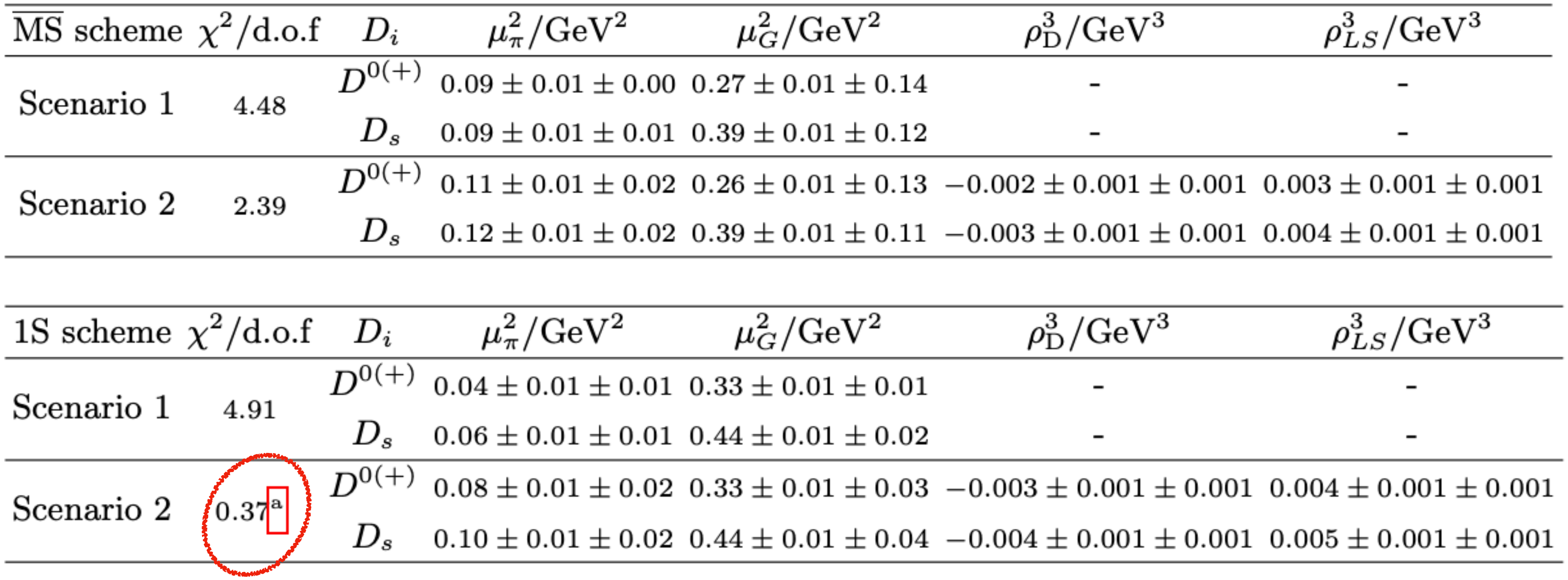
Phenomenological Analysis



$\overline{\text{MS}}$ scheme	χ^2/d.o.f	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	ρ_D^3/GeV^3	ρ_{LS}^3/GeV^3
Scenario 1	4.48	$D^{0(+)}$	$0.09 \pm 0.01 \pm 0.00$	$0.27 \pm 0.01 \pm 0.14$	-	-
		D_s	$0.09 \pm 0.01 \pm 0.01$	$0.39 \pm 0.01 \pm 0.12$	-	-
Scenario 2	2.39	$D^{0(+)}$	$0.11 \pm 0.01 \pm 0.02$	$0.26 \pm 0.01 \pm 0.13$	$-0.002 \pm 0.001 \pm 0.001$	$0.003 \pm 0.001 \pm 0.001$
		D_s	$0.12 \pm 0.01 \pm 0.02$	$0.39 \pm 0.01 \pm 0.11$	$-0.003 \pm 0.001 \pm 0.001$	$0.004 \pm 0.001 \pm 0.001$

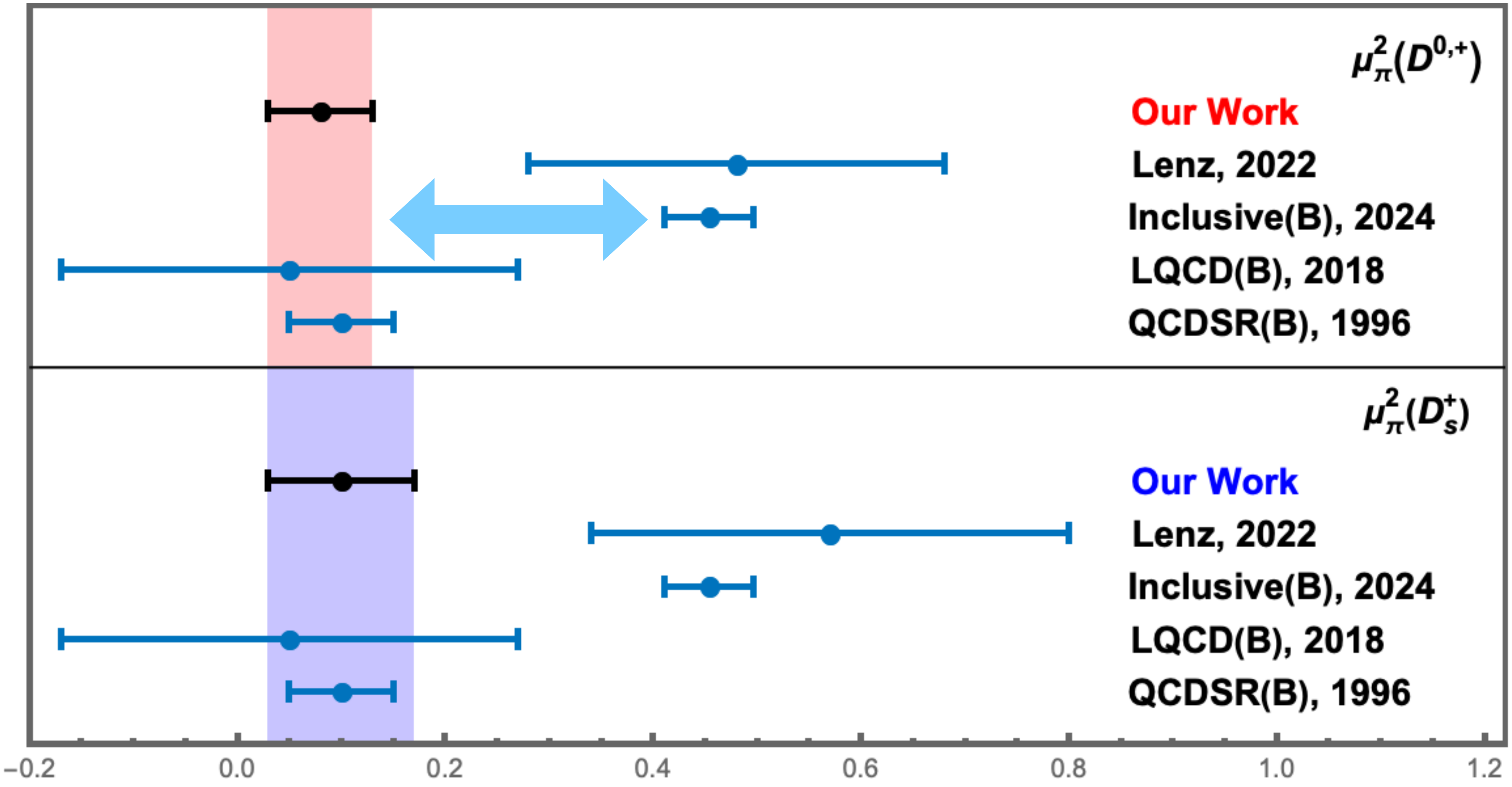
1S scheme	χ^2/d.o.f	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	ρ_D^3/GeV^3	ρ_{LS}^3/GeV^3
Scenario 1	4.91	$D^{0(+)}$	$0.04 \pm 0.01 \pm 0.01$	$0.33 \pm 0.01 \pm 0.01$	-	-
		D_s	$0.06 \pm 0.01 \pm 0.01$	$0.44 \pm 0.01 \pm 0.02$	-	-
Scenario 2	0.37[a]	$D^{0(+)}$	$0.08 \pm 0.01 \pm 0.02$	$0.33 \pm 0.01 \pm 0.03$	$-0.003 \pm 0.001 \pm 0.001$	$0.004 \pm 0.001 \pm 0.001$
		D_s	$0.10 \pm 0.01 \pm 0.02$	$0.44 \pm 0.01 \pm 0.04$	$-0.004 \pm 0.001 \pm 0.001$	$0.005 \pm 0.001 \pm 0.001$

- Discrepancies among central values in the two scenarios
- Stable fitting results under 1S mass scheme

D meson matrix elements of operators in the HQET are hence determined by data **for the first time**

Phenomenological Analysis

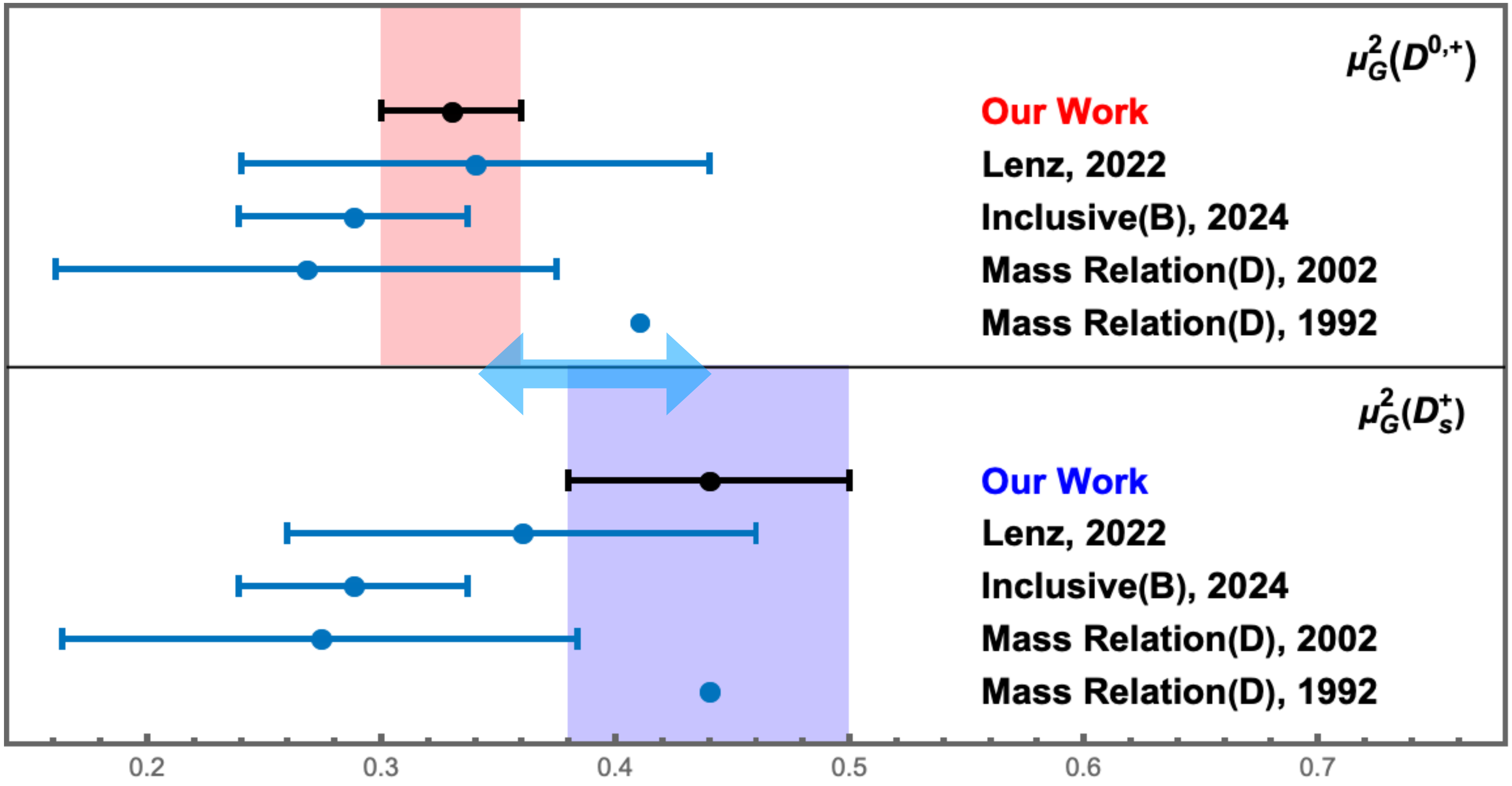
- Data-driven estimation for μ_π^2



- The non-relativistic kinetic energy term in D meson is dominated by **heavy quark contribution**
- Discrepancy between $\mu_\pi^2(\mathrm{D})$ and $\mu_\pi^2(\mathrm{B})$ is **significant**

Phenomenological Analysis

- Data-driven estimation for μ_G^2

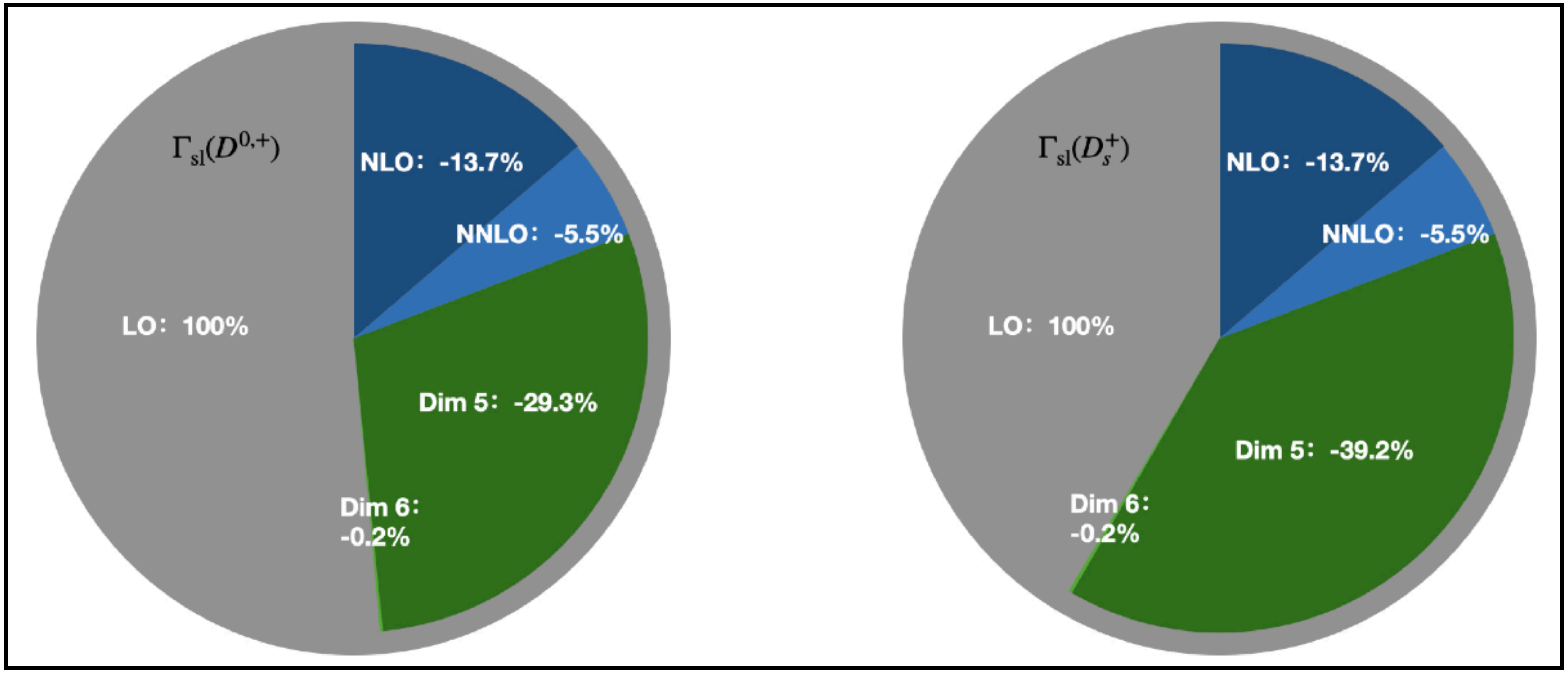


Discrepancy between $\mu_{\mathrm{G}}^2(\mathrm{D})$ and $\mu_{\mathrm{G}}^2(\mathrm{B})$ is **significant**

$SU(3)_f$ Violation effects are also observed

Phenomenological Analysis

The non-perturbative series exhibits **good** convergence behavior in D-meson semi-leptonic inclusive decays under the **1S mass** scheme



We propose the 1S mass scheme as the optimal choice for calculating charmed meson lifetimes

Look Back

Theory:

Optical Theorem

$D \rightarrow X_{d,s} + e^{+}$

OPE

HQET

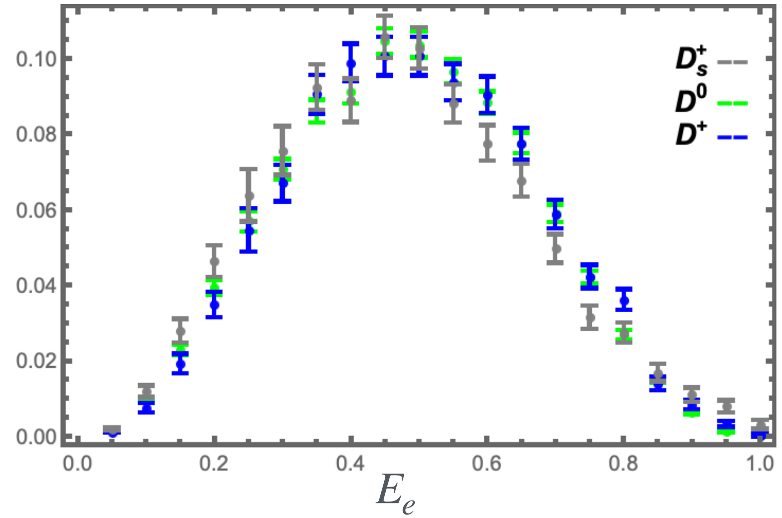
Wilson Coef

Loacl Operators

Analytical Integral

Observables

Data：



Observables

Phenomenological Analysis：

Observables

Global Fitting

Observables

1.Extract Non-perturbative parameters

2.1S mass scheme is a optimal choice

OutLook

- Precision measurements of leptonic energy spectrum **in the rest frame** of charmed hadrons

Collaborate with Prof. Dong Xiao

- q^2 spectrum, good for **higher-dimensional operators**

- Separate X_d, X_s, to give **first measurements** of V_{cd}, V_{cs}

Collaborate with Prof. Liang Sun

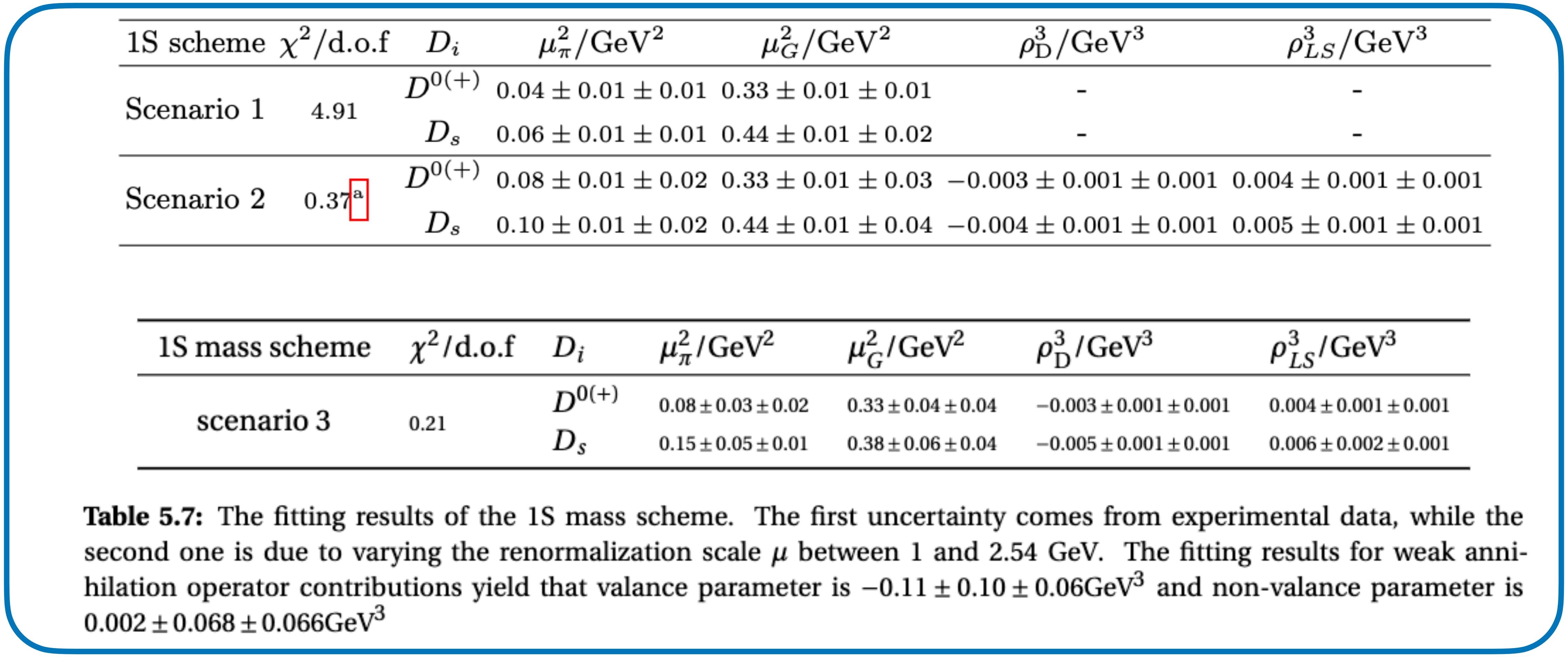
- Rare decays: $D \to X_u \ell \ell$

Prof. Xiao's talk

Thank you!

Appendix

Scenario 3： 1S mass scheme



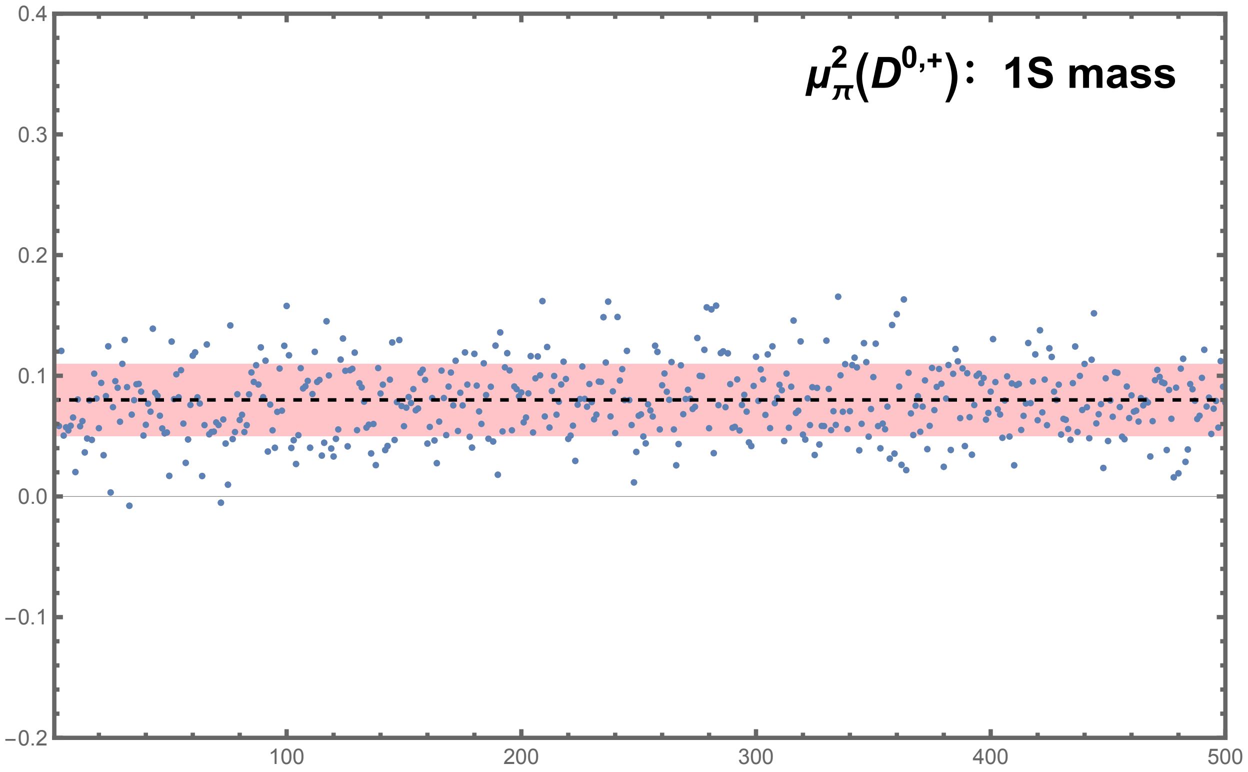
1S scheme	χ^2/d.o.f	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	ρ_D^3/GeV^3	$\rho_{LS}^3/\mathrm{GeV}^3$
Scenario 1	4.91	$D^{0(+)}$	$0.04 \pm 0.01 \pm 0.01$	$0.33 \pm 0.01 \pm 0.01$	-	-
		D_s	$0.06 \pm 0.01 \pm 0.01$	$0.44 \pm 0.01 \pm 0.02$	-	-
Scenario 2	0.37[a]	$D^{0(+)}$	$0.08 \pm 0.01 \pm 0.02$	$0.33 \pm 0.01 \pm 0.03$	$-0.003 \pm 0.001 \pm 0.001$	$0.004 \pm 0.001 \pm 0.001$
		D_s	$0.10 \pm 0.01 \pm 0.02$	$0.44 \pm 0.01 \pm 0.04$	$-0.004 \pm 0.001 \pm 0.001$	$0.005 \pm 0.001 \pm 0.001$

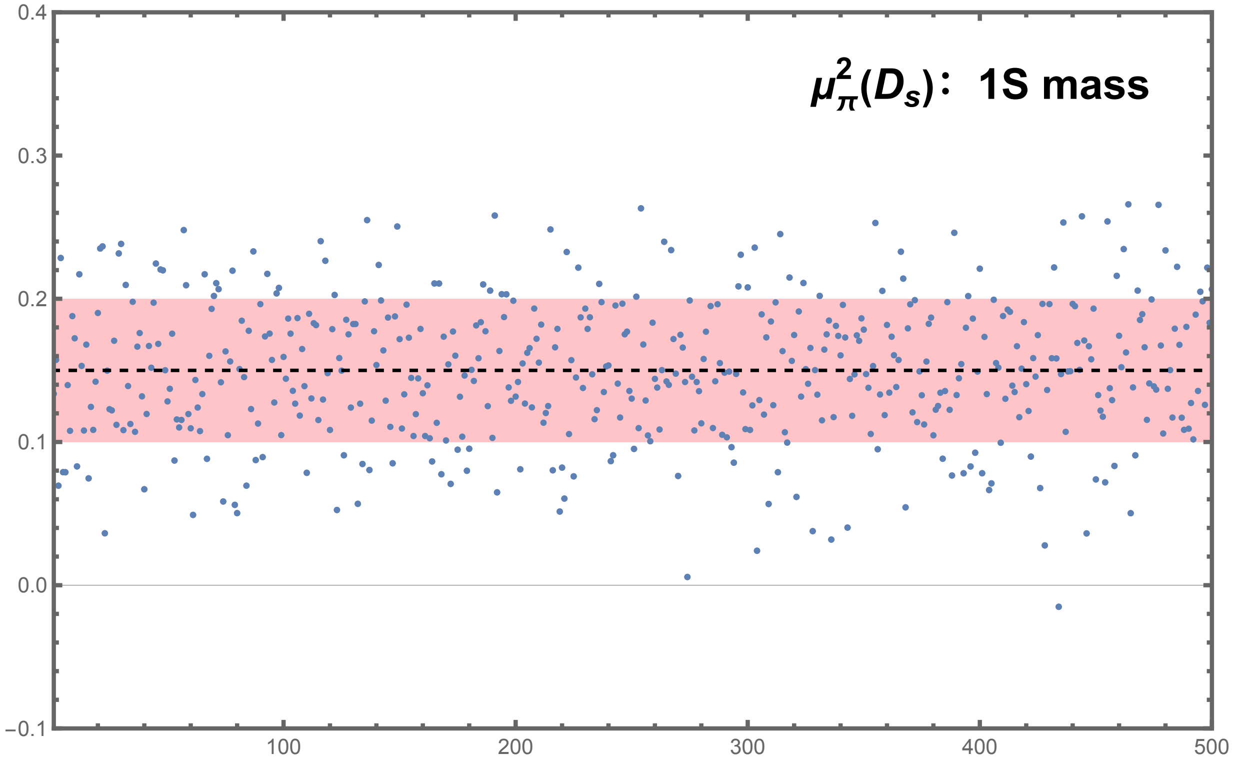
1S mass scheme	χ^2/d.o.f	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	ρ_D^3/GeV^3	$\rho_{LS}^3/\mathrm{GeV}^3$
scenario 3	0.21	$D^{0(+)}$	$0.08 \pm 0.03 \pm 0.02$	$0.33 \pm 0.04 \pm 0.04$	$-0.003 \pm 0.001 \pm 0.001$	$0.004 \pm 0.001 \pm 0.001$
		D_s	$0.15 \pm 0.05 \pm 0.01$	$0.38 \pm 0.06 \pm 0.04$	$-0.005 \pm 0.001 \pm 0.001$	$0.006 \pm 0.002 \pm 0.001$

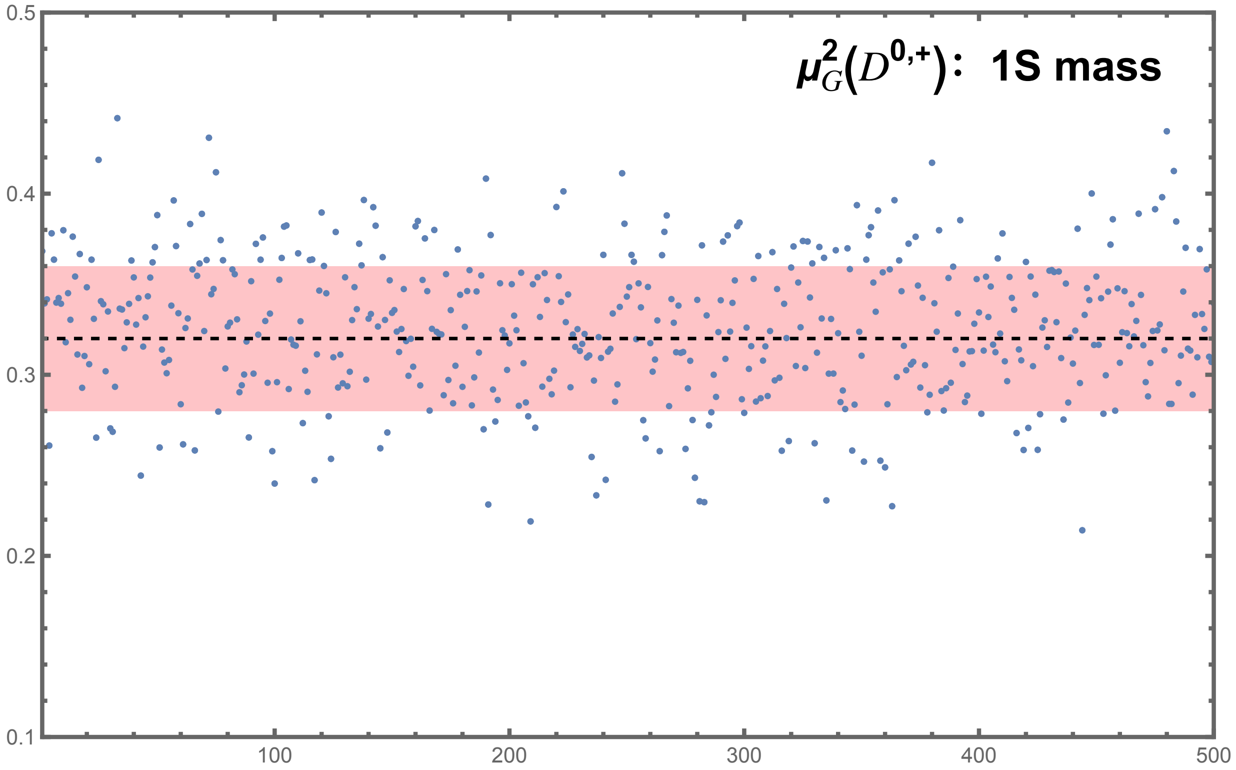
Table 5.7: The fitting results of the 1S mass scheme. The first uncertainty comes from experimental data, while the second one is due to varying the renormalization scale μ between 1 and 2.54 GeV. The fitting results for weak annihilation operator contributions yield that valance parameter is $-0.11 \pm 0.10 \pm 0.06\mathrm{GeV}^3$ and non-valance parameter is $0.002 \pm 0.068 \pm 0.066\mathrm{GeV}^3$

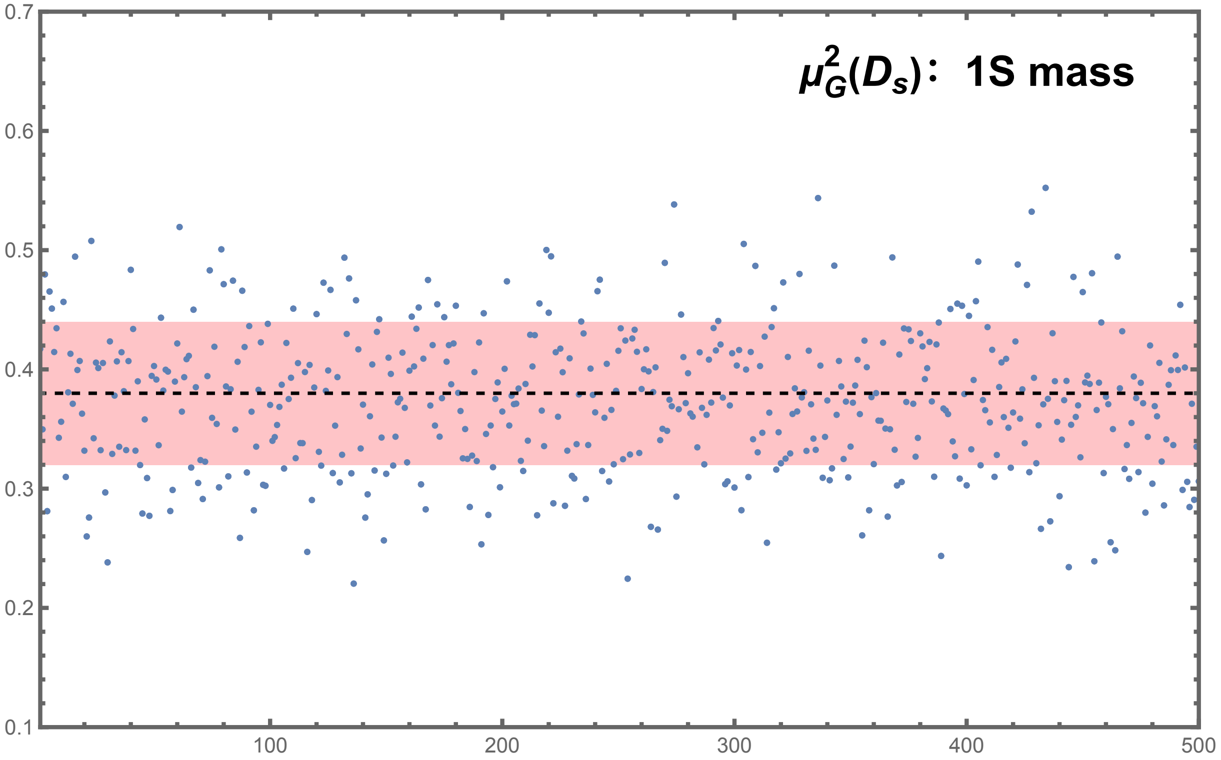
$$\chi^2(\boldsymbol{\theta}) = \sum_{l=u,d,s} \sum_{i=1}^{5} \sum_{j=1}^{5} \left(y_{i,l} - \eta_{i,l}\right) V_{ij,l}^{-1} \left(y_{j,l} - \eta_{j,l}\right) + \left(\frac{\tau_{\text{nonval}} - (-0.18)}{0.65}\right)^2 + \left(\frac{\tau_{\text{val}} - 0.45}{2.10}\right)^2 .$$

1S mass scheme robust Testing： scenario 3

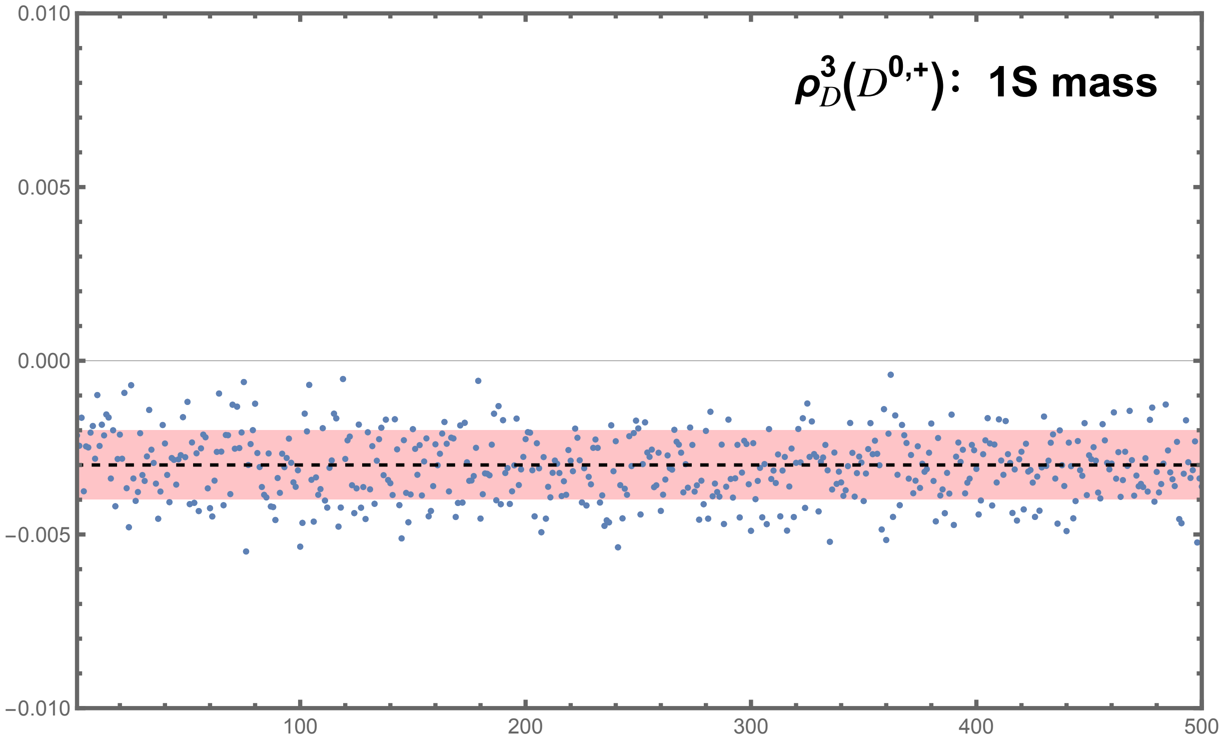


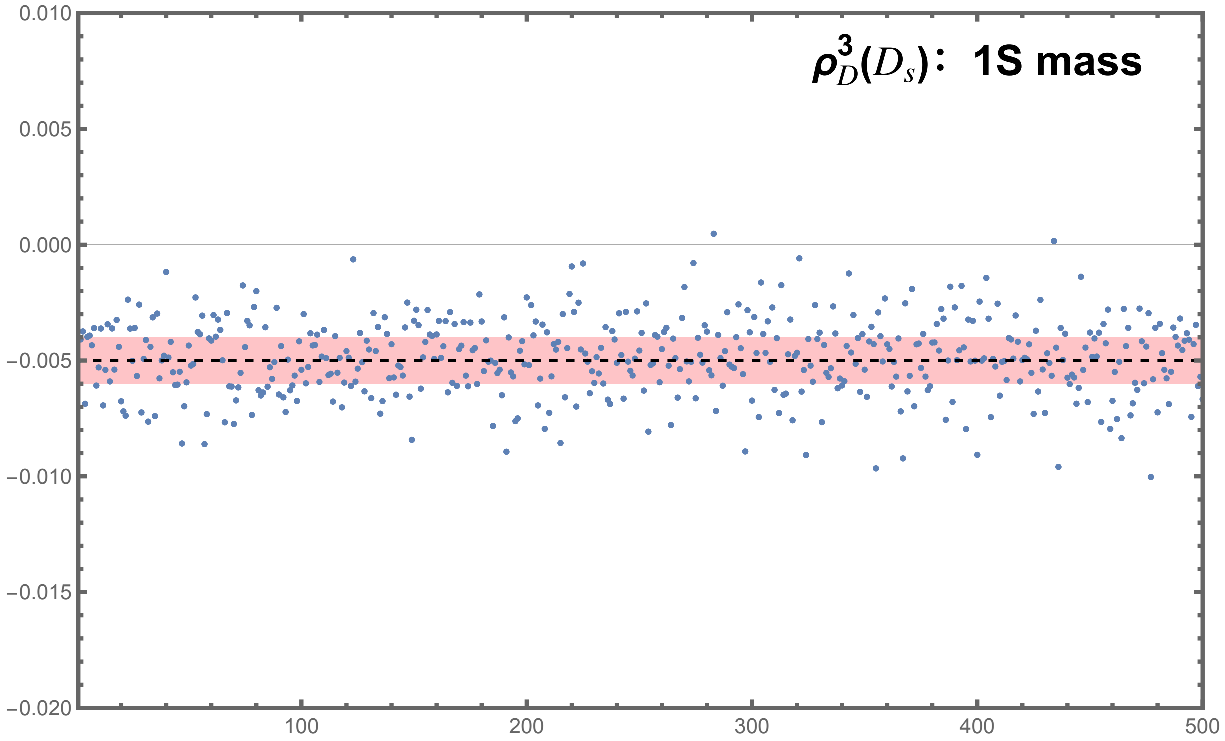


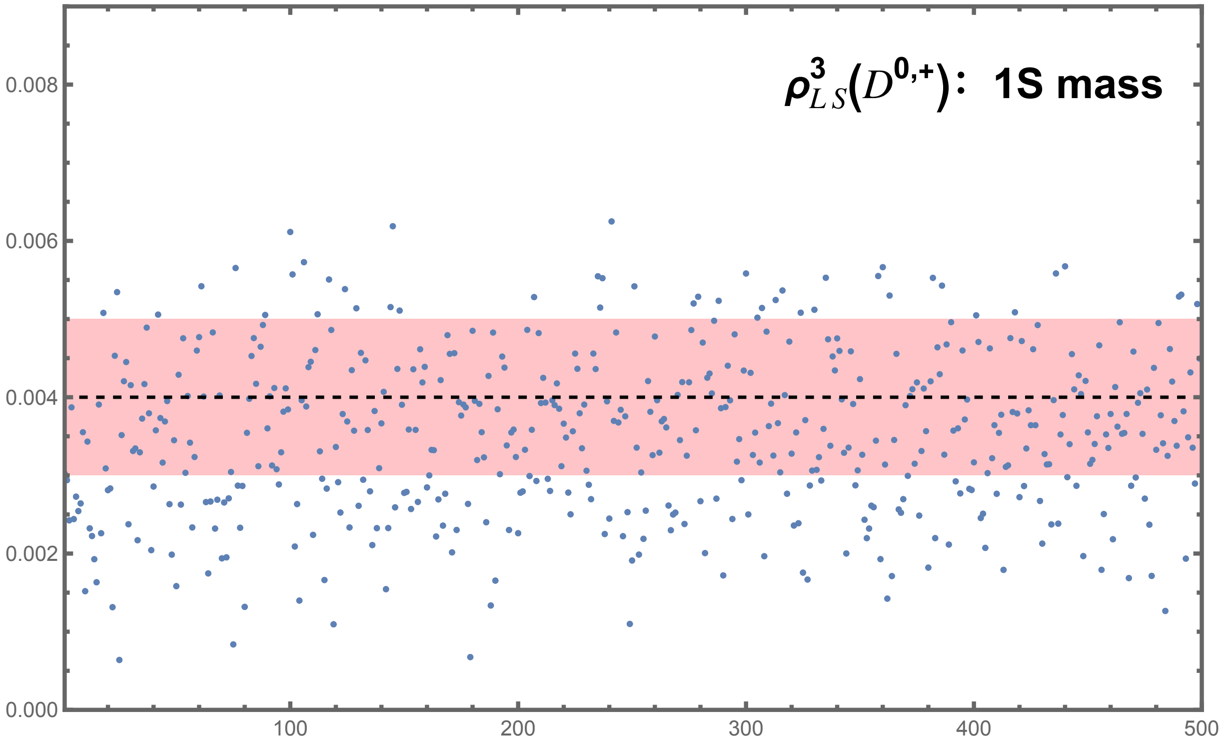


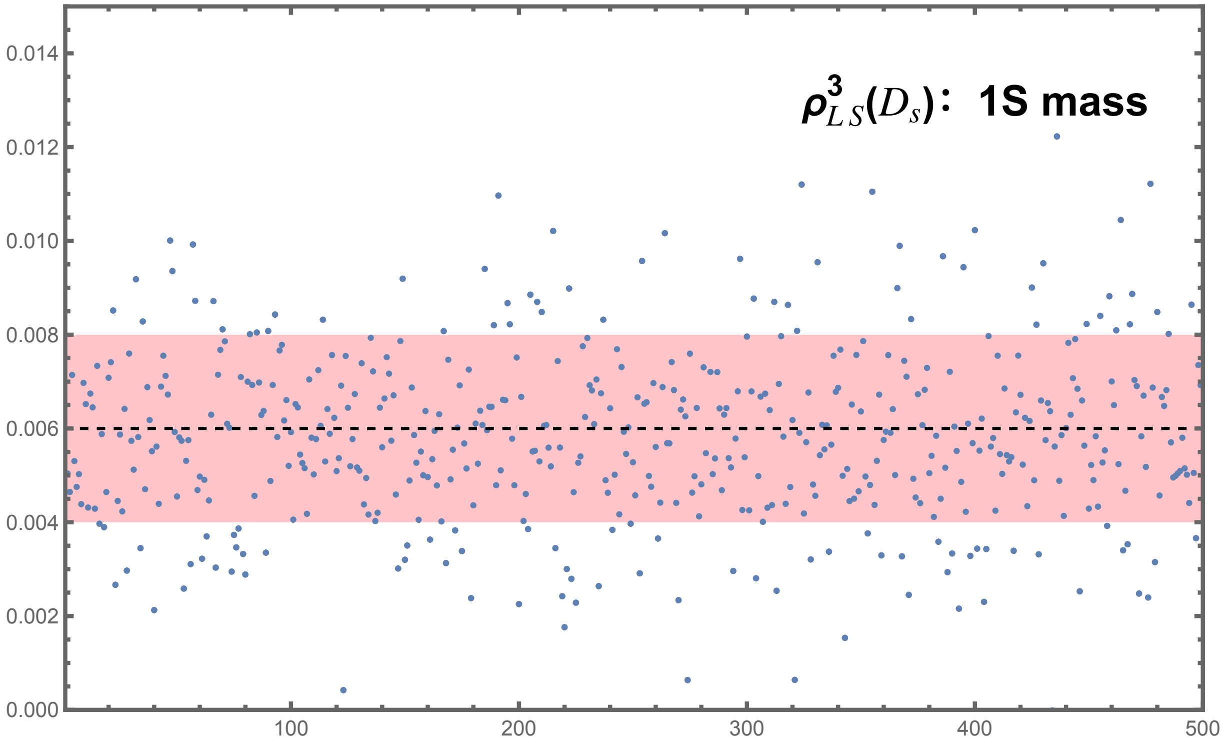


1S mass scheme robust Testing： scenario 3

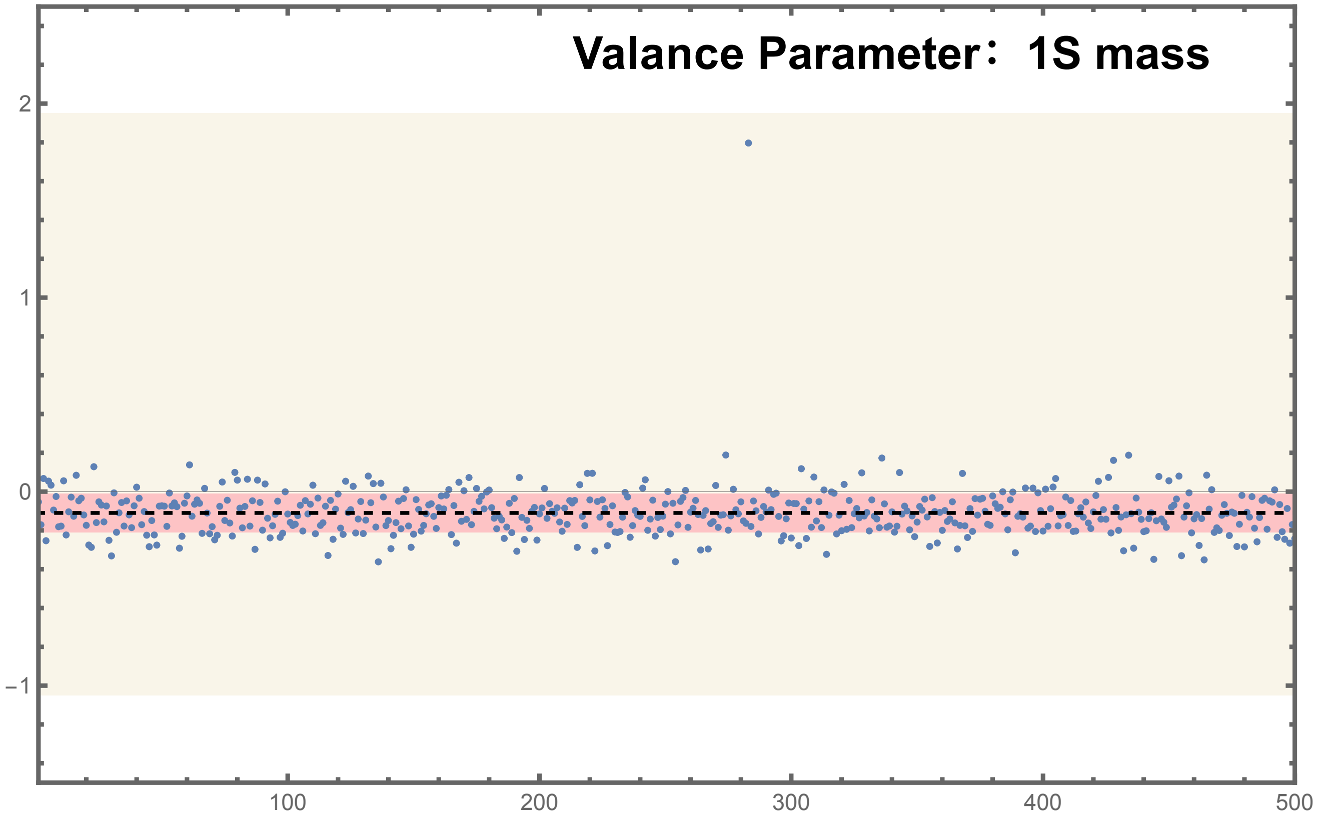


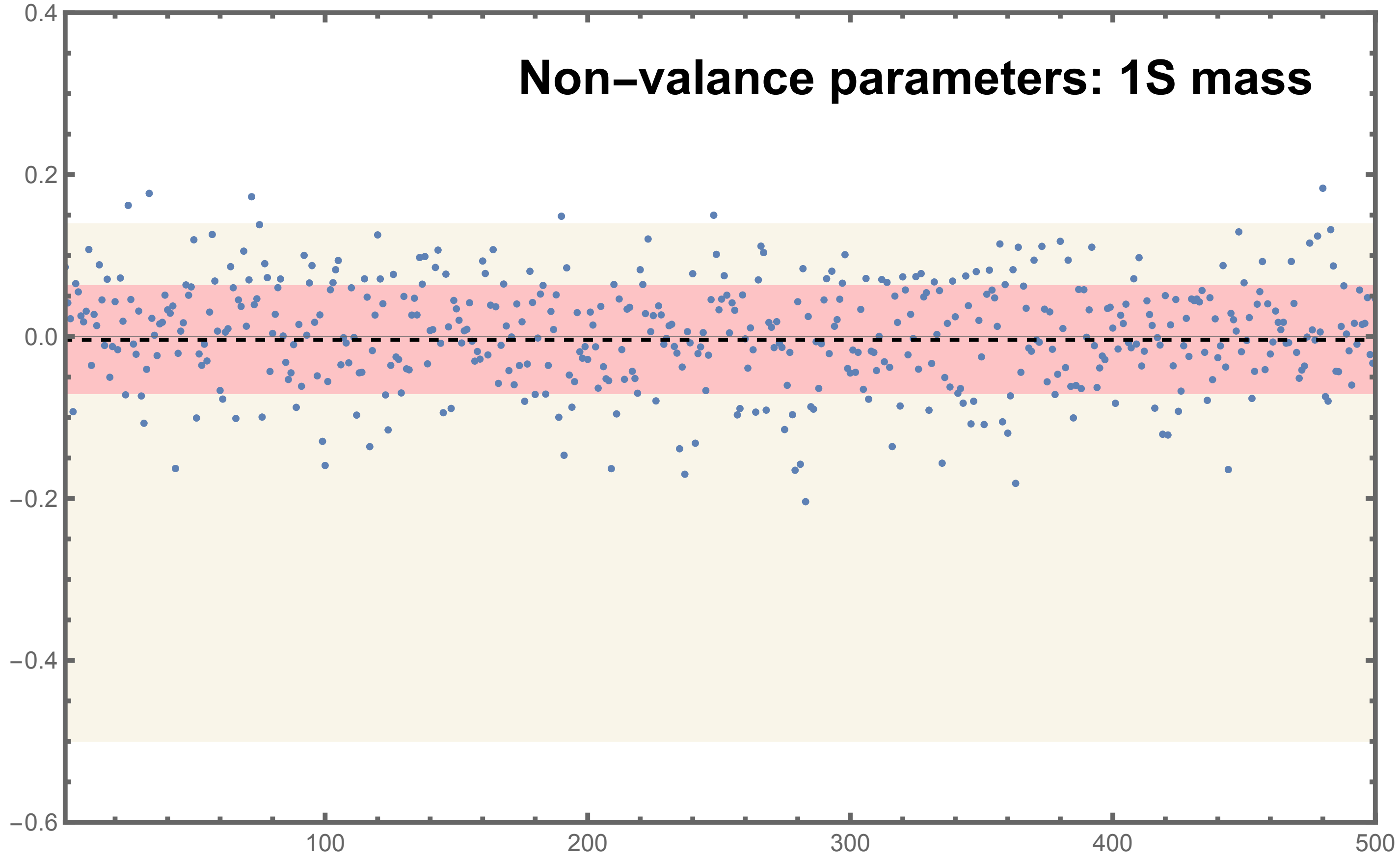




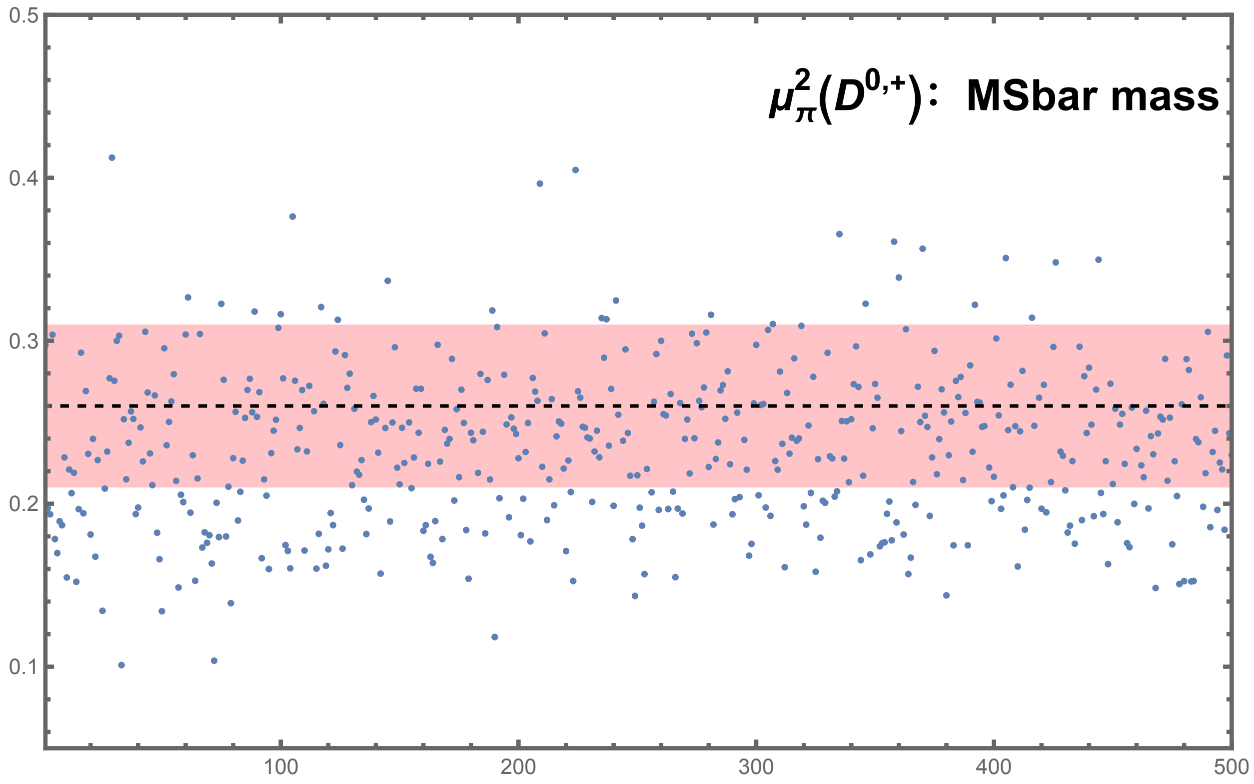


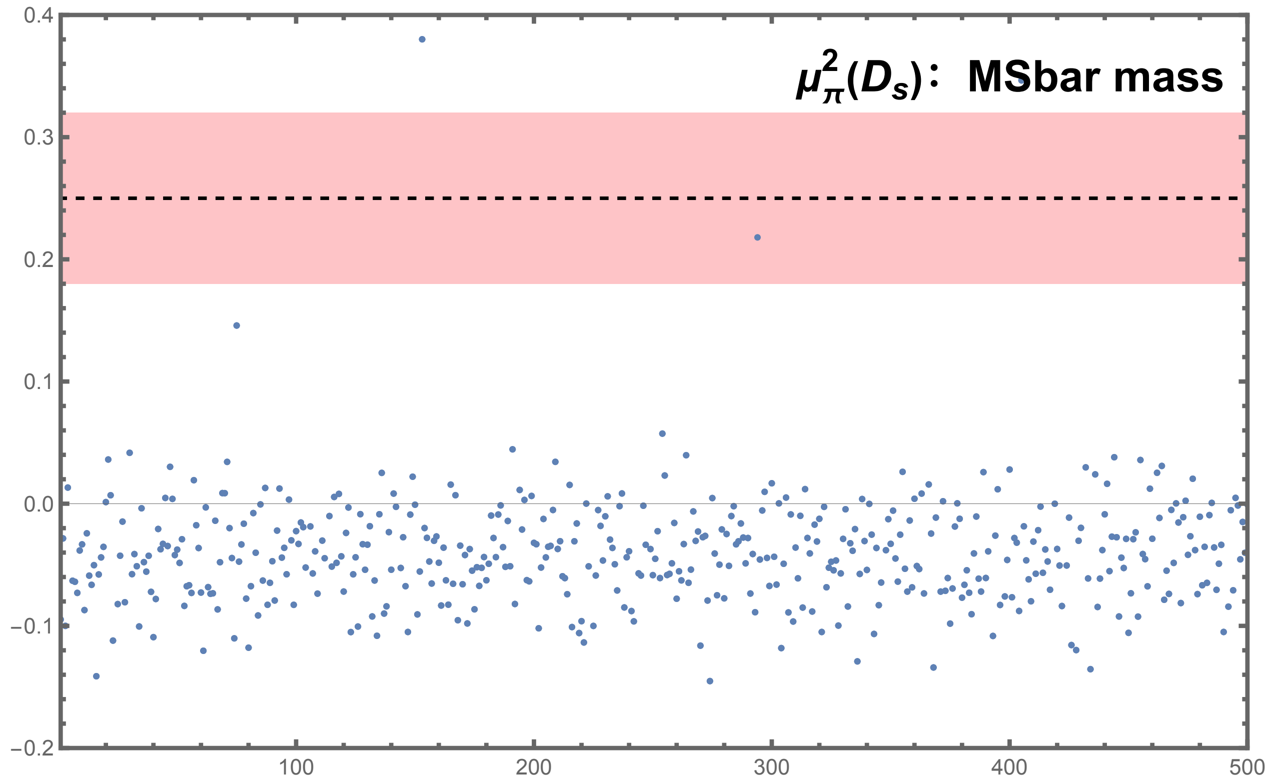
1S mass scheme robust Testing： scenario 3

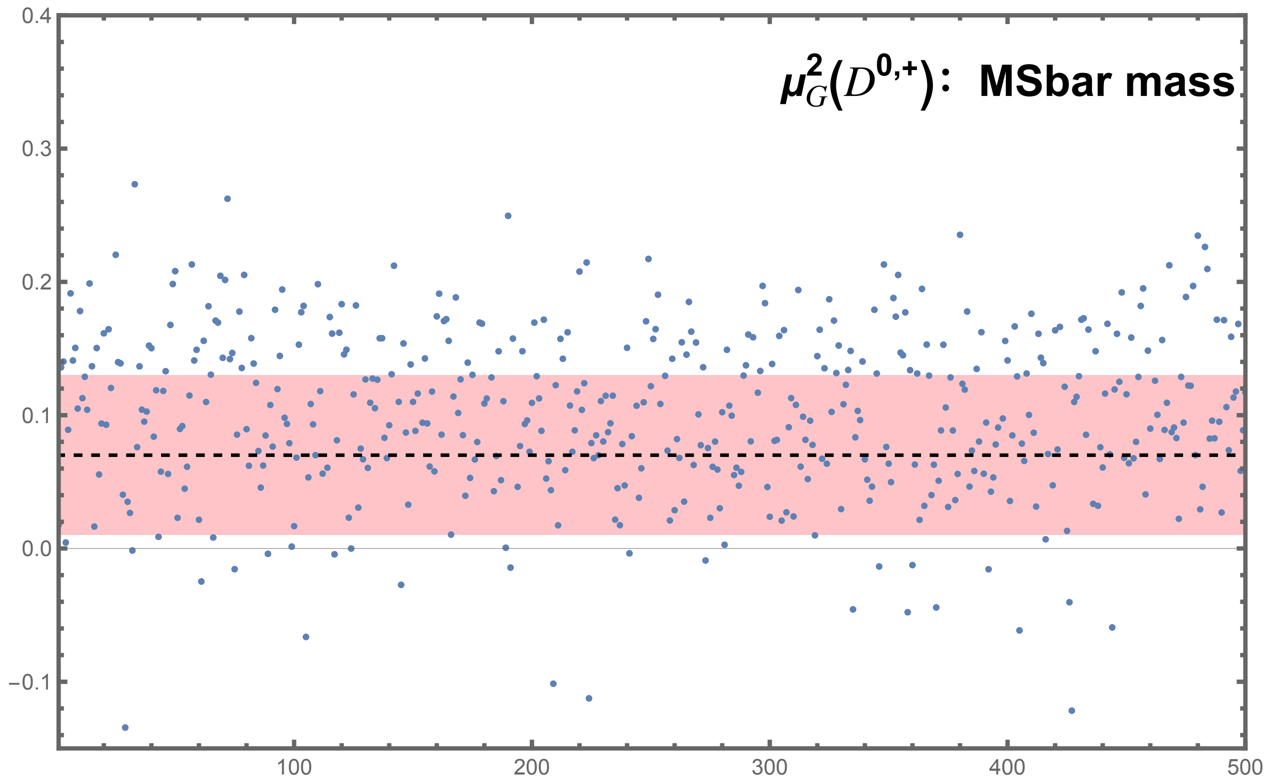


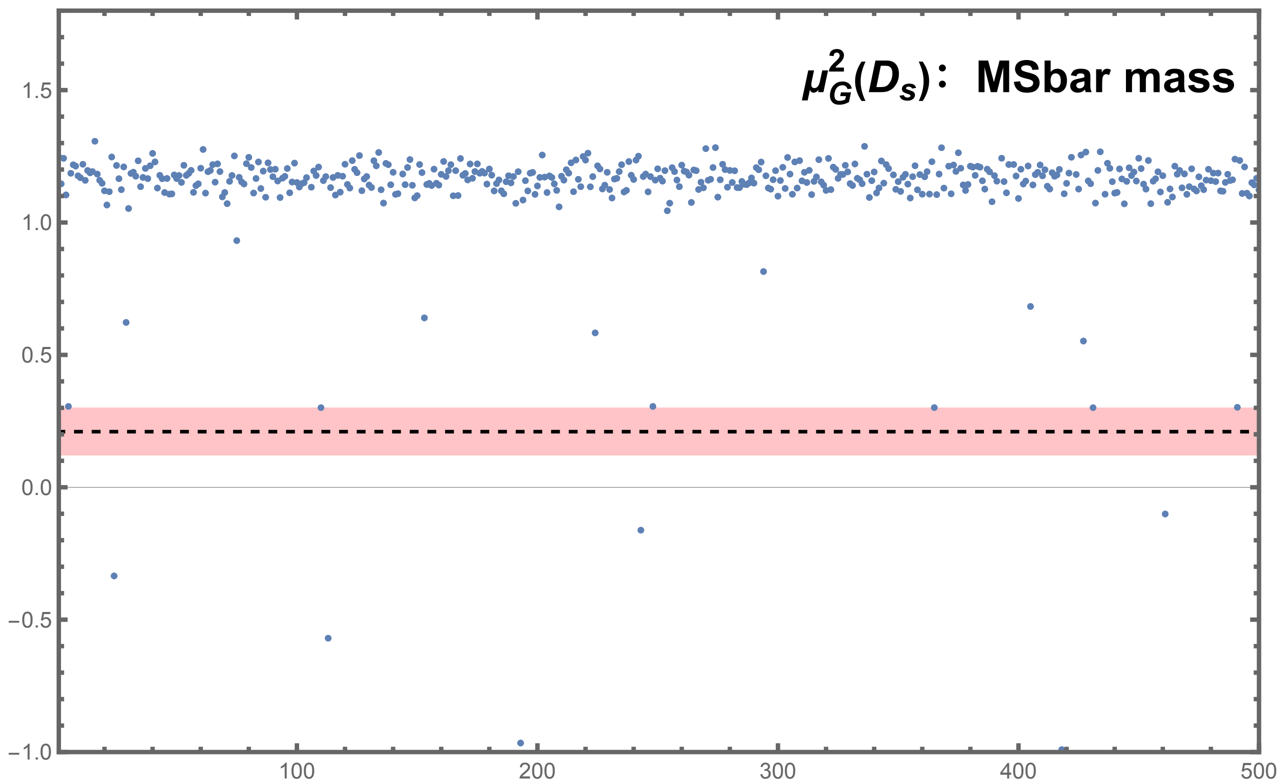


$\overline{\mathrm{MS}}$ mass scheme robust Testing：scenario 3

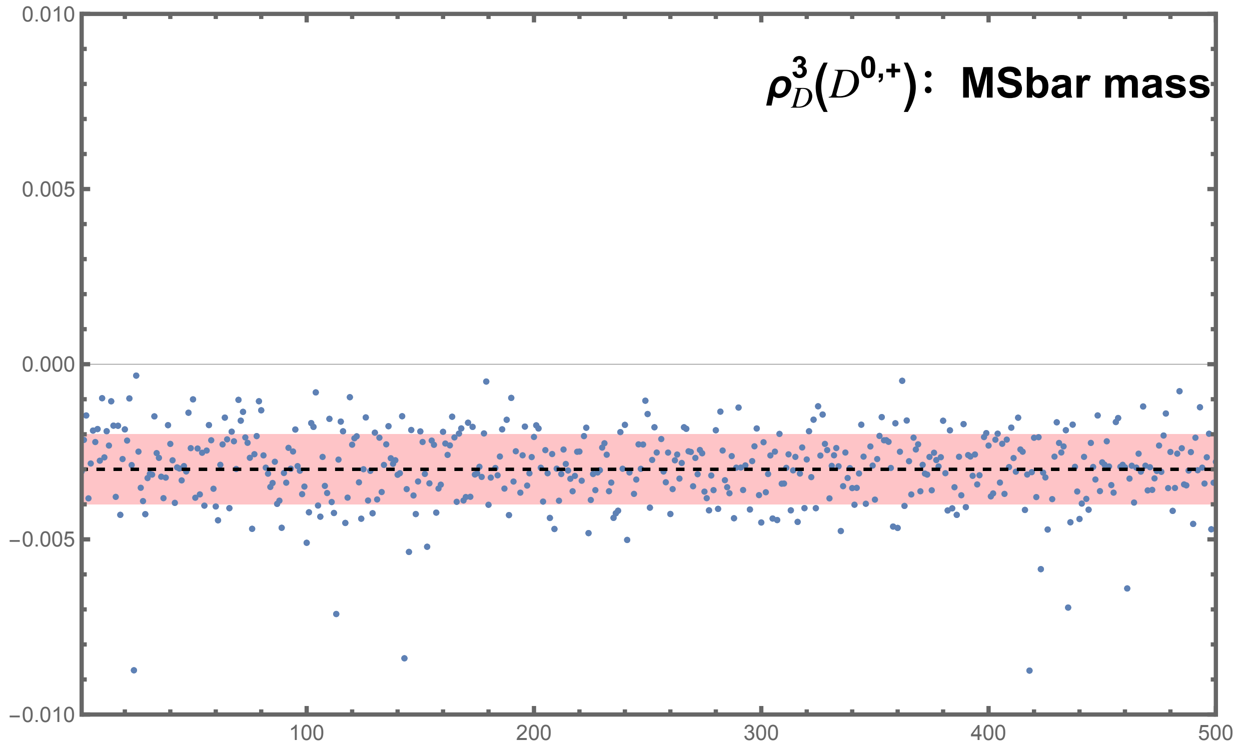


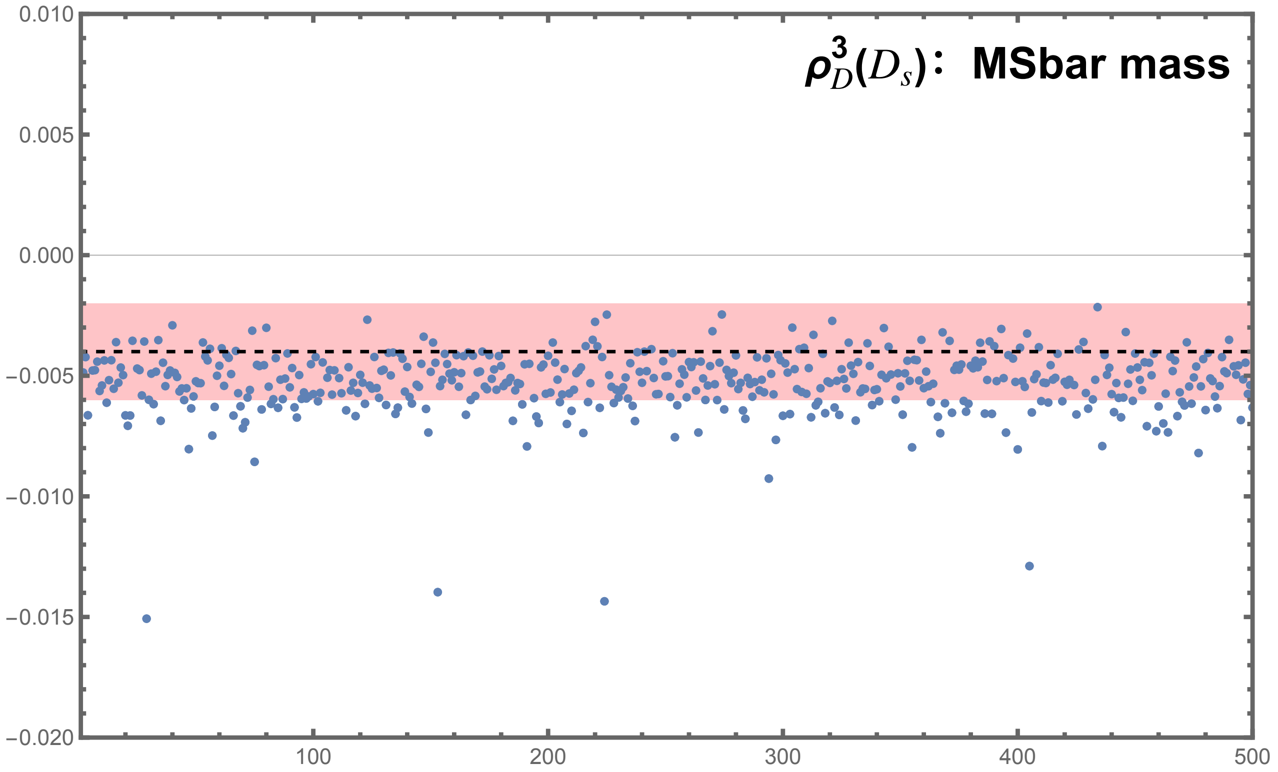


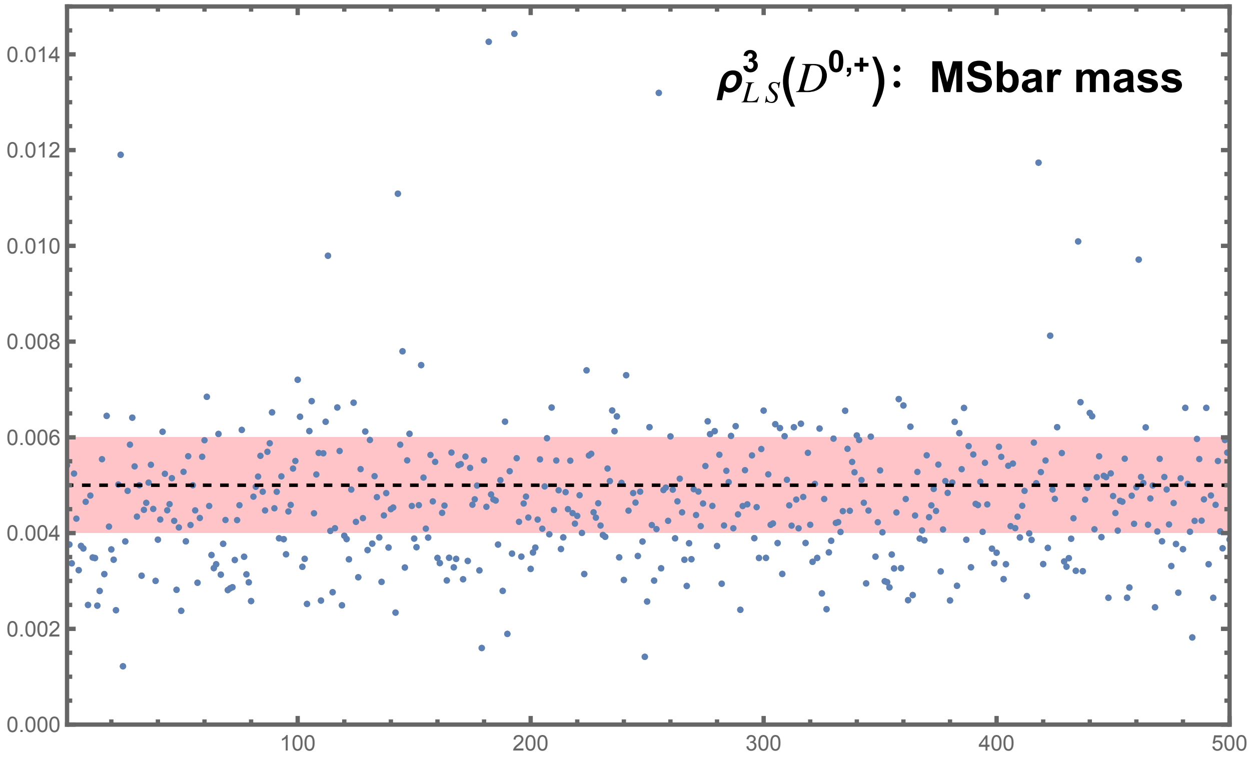


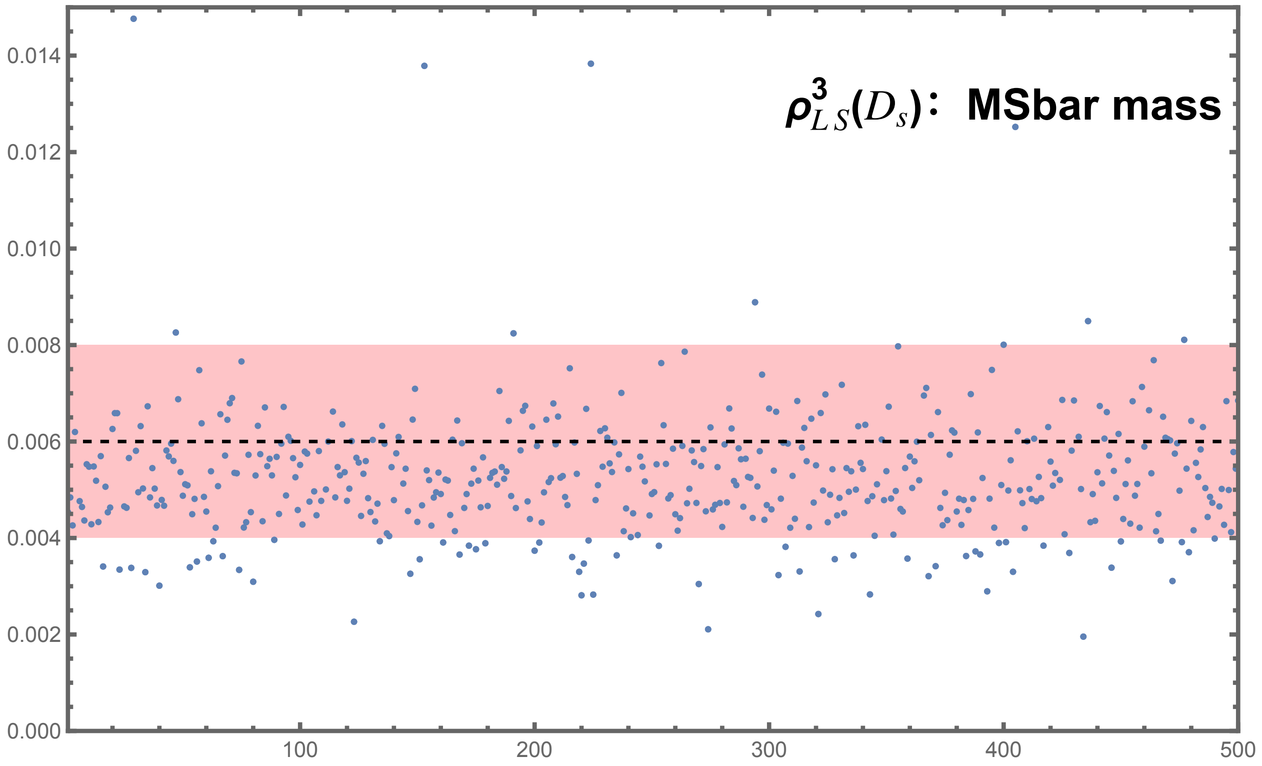


$\overline{\text{MS}}$ mass scheme robust Testing： scenario 3

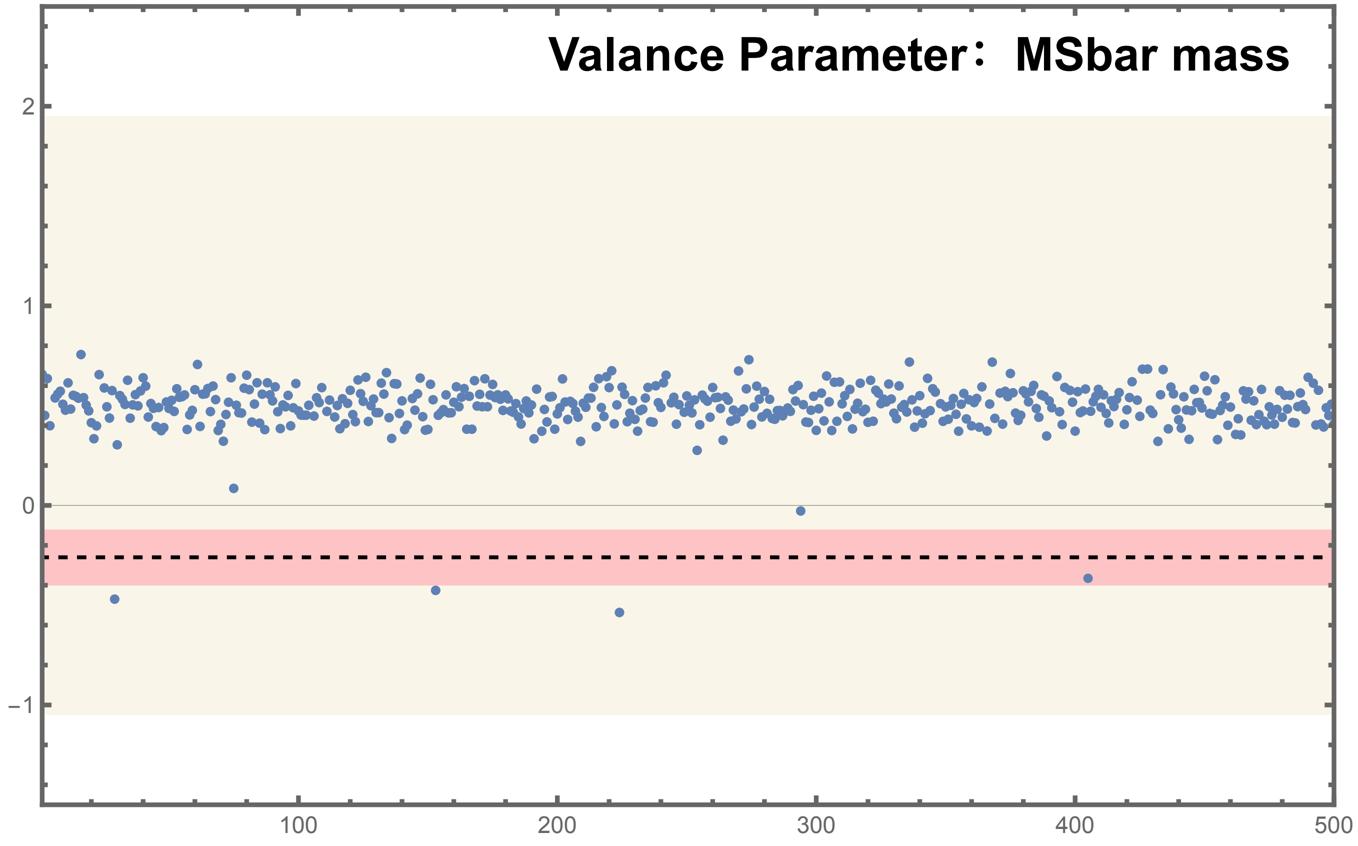


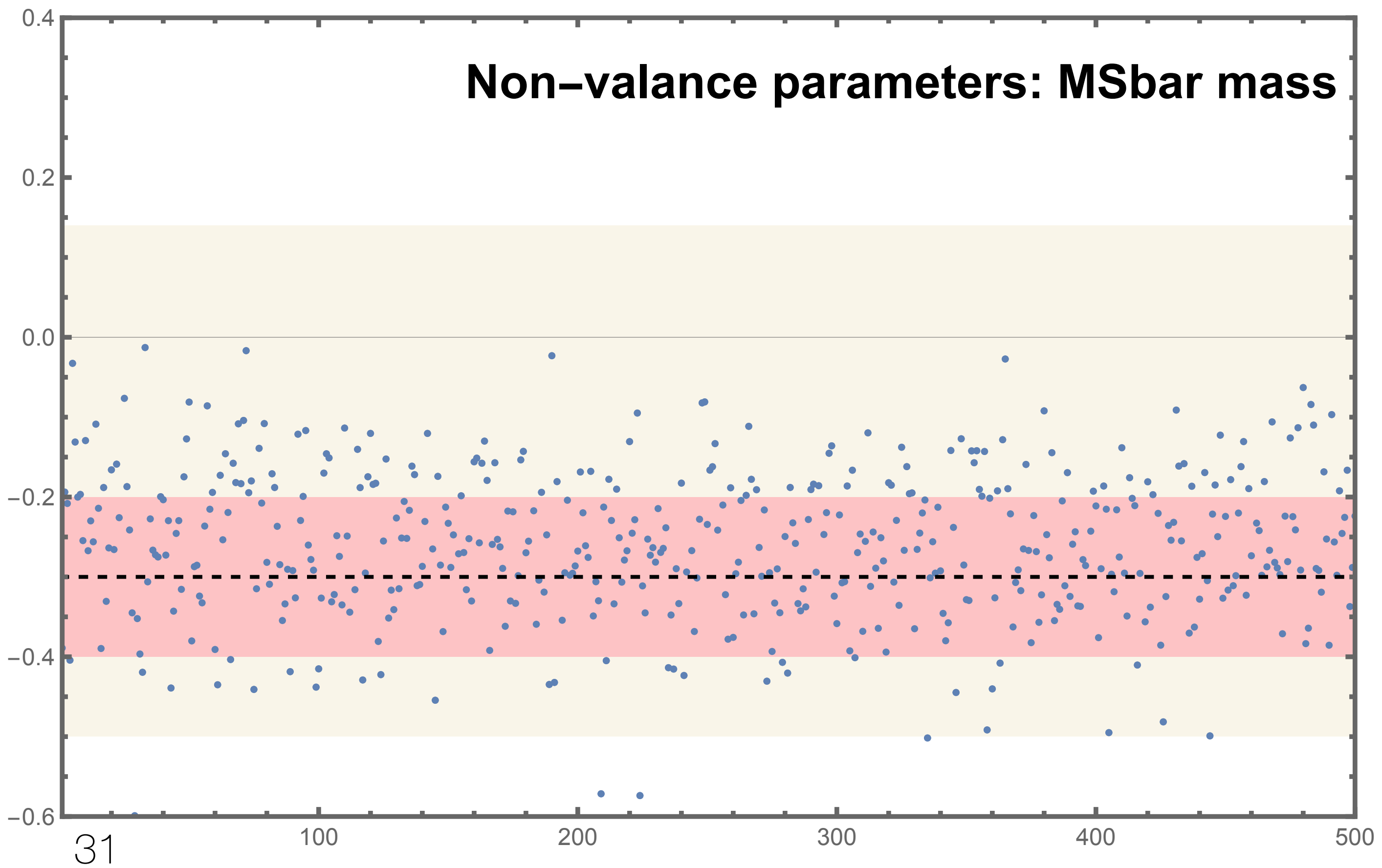




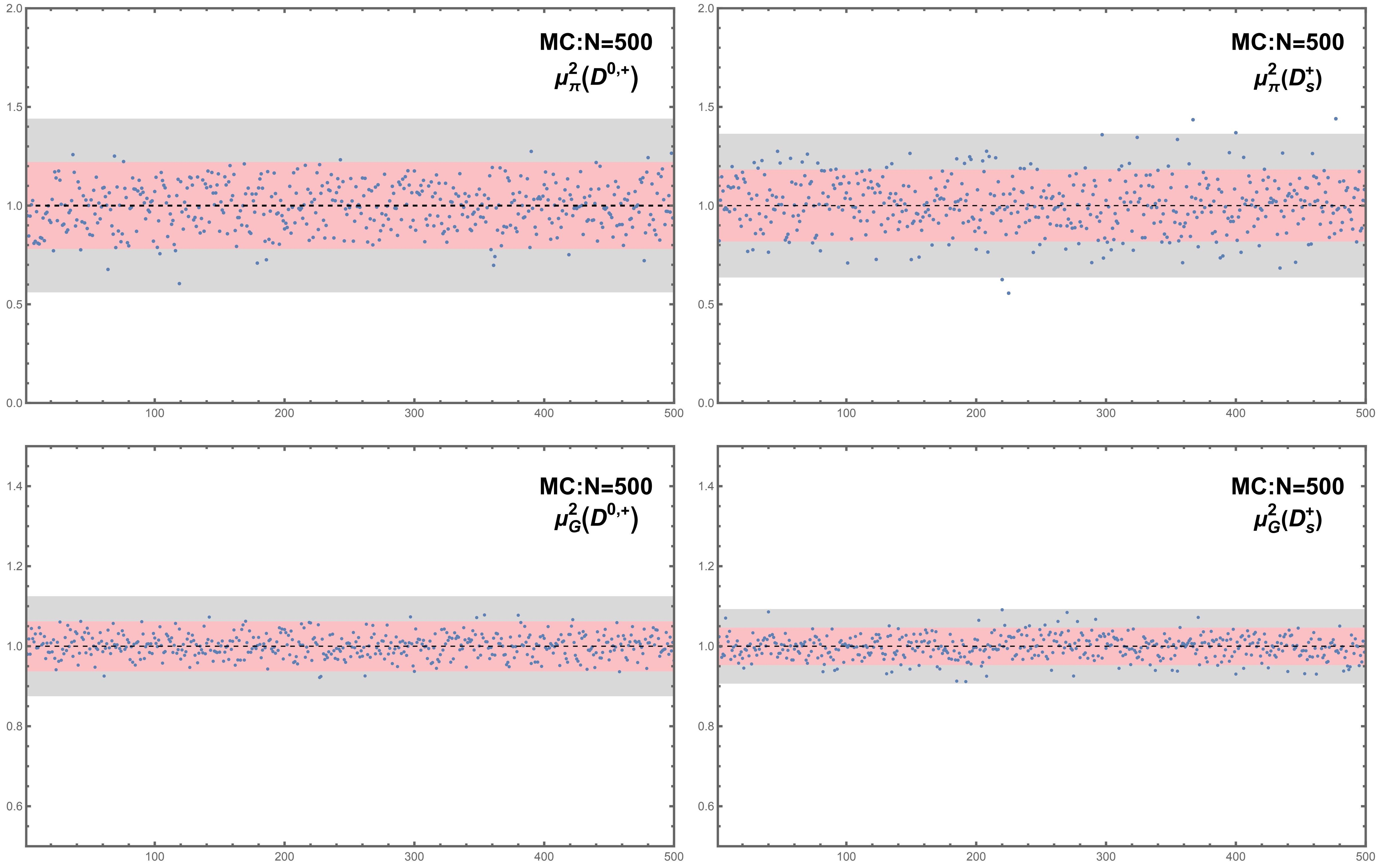


$\overline{\mathrm{MS}}$ mass scheme robust Testing： scenario 3

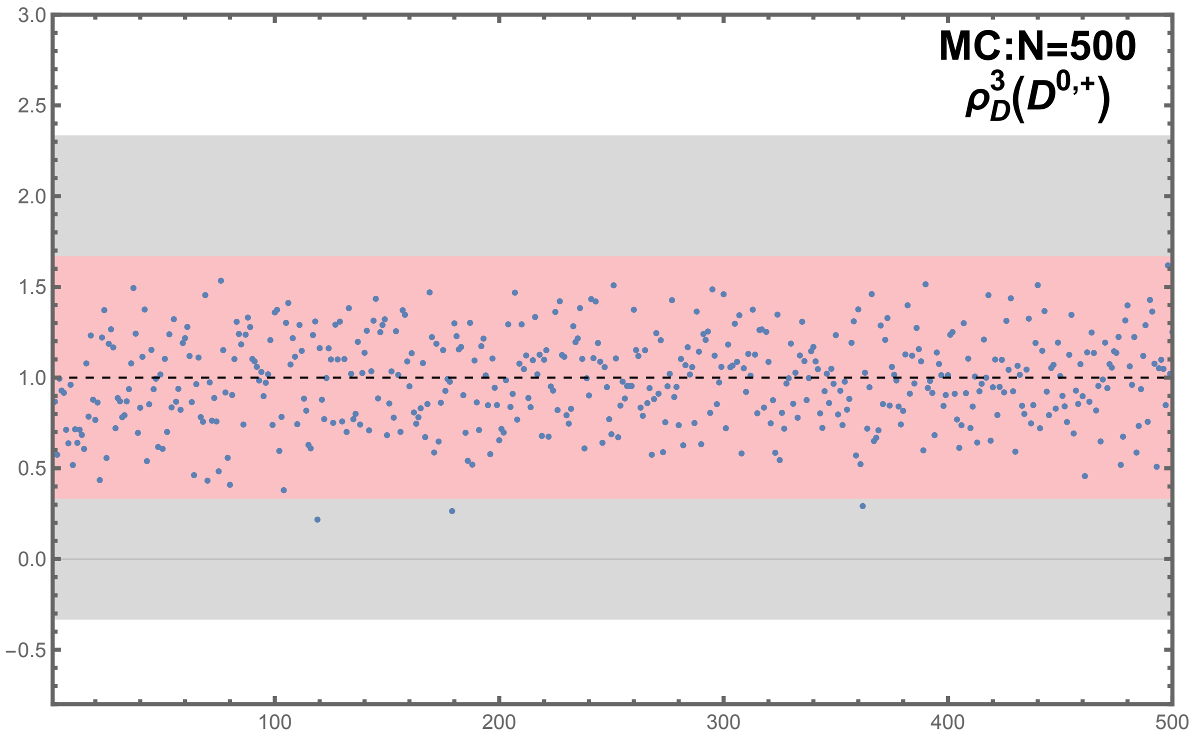


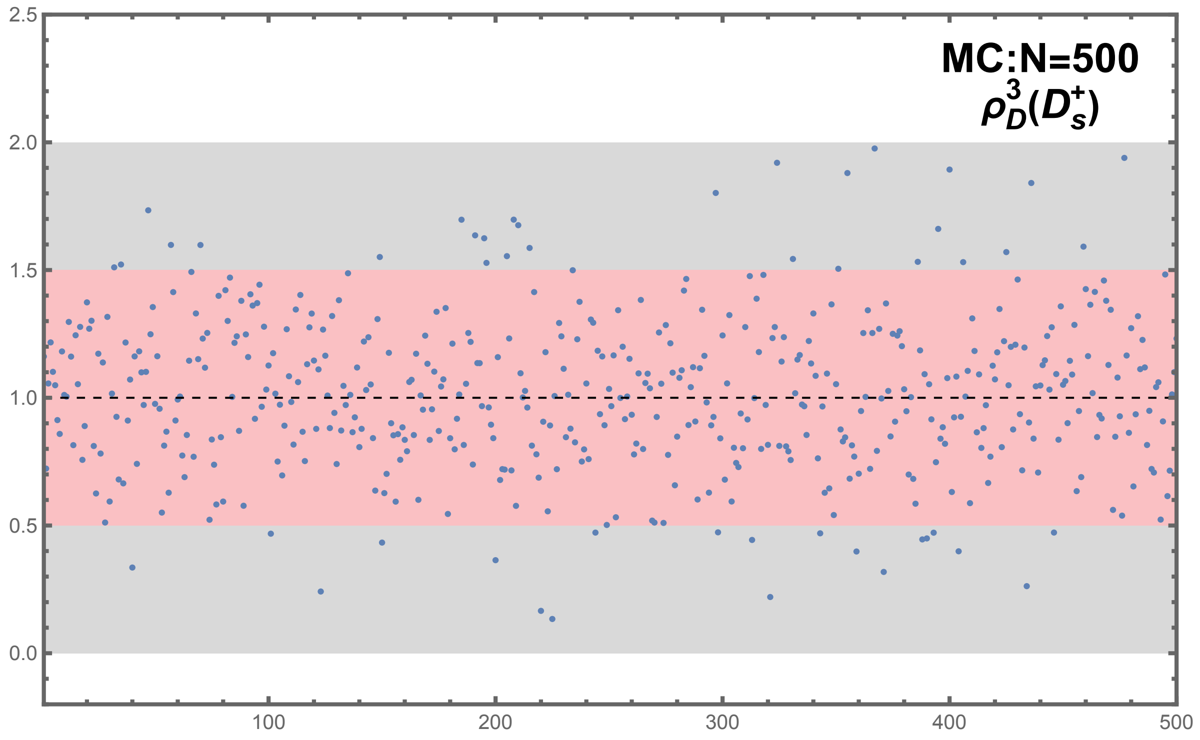


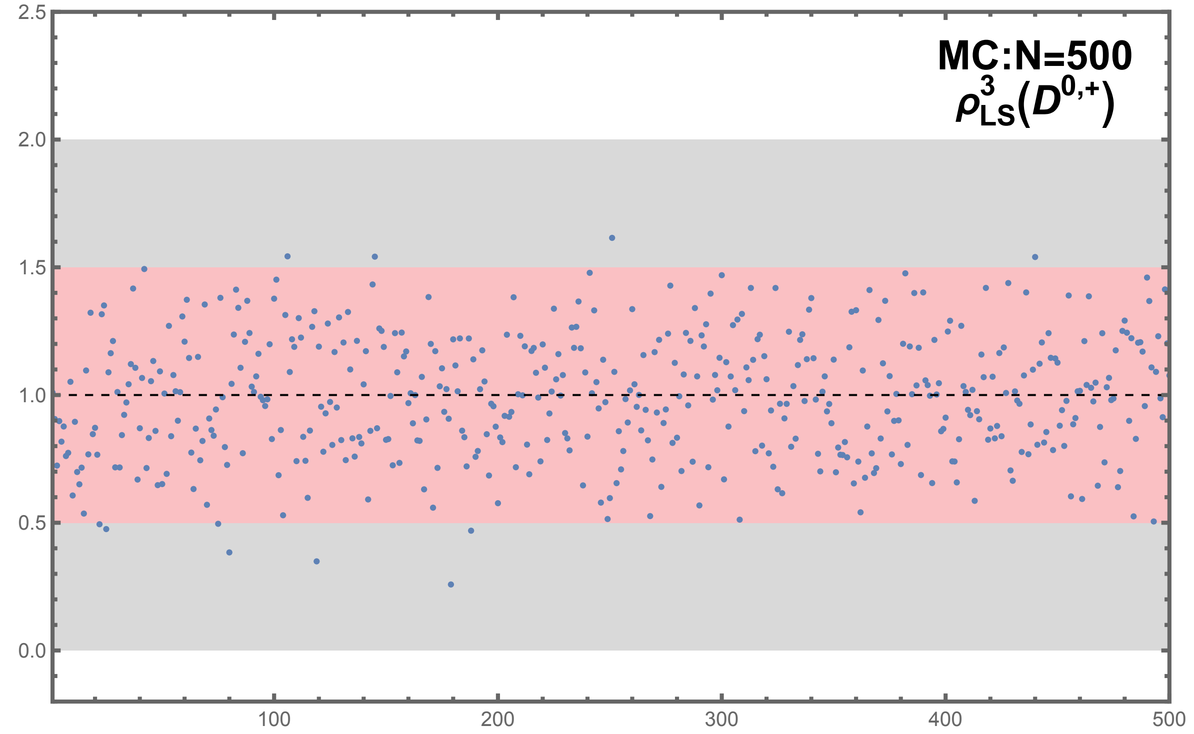
1S mass scheme robust Testing： scenario 2

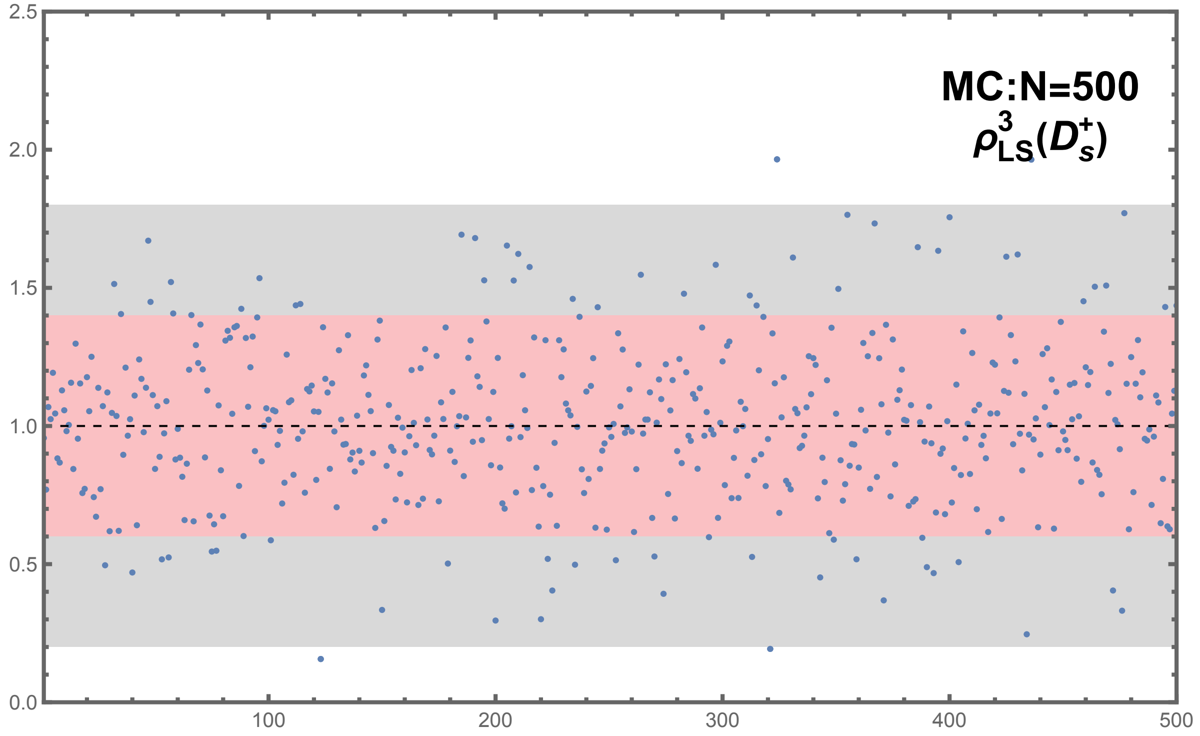


1S mass scheme robust Testing： scenario 2





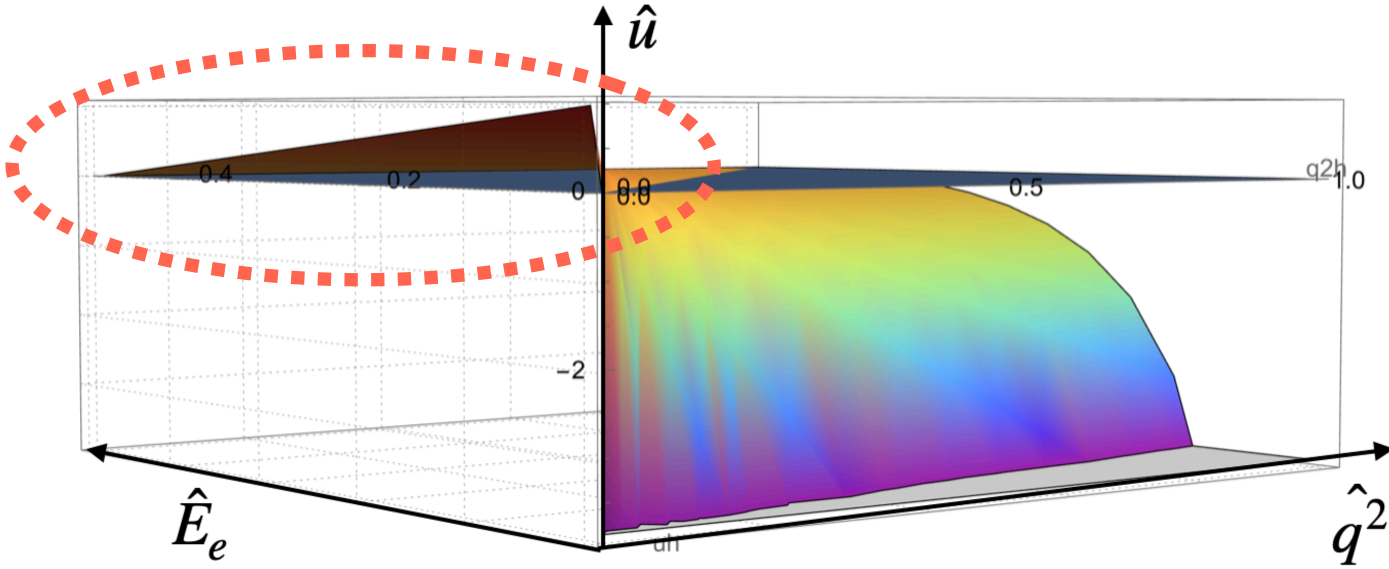




高阶修正和高幂次修正

- 相空间解析积分与电子能谱

$$
\begin{aligned}
\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{E}_\mathrm{e}} =& 96\hat{E}_\mathrm{e}^2(1-2\hat{E}_\mathrm{e})\theta\left(\frac{1}{2}-\hat{E}_\mathrm{e}\right)\\
&+\frac{8\mu_\pi^2}{m_\mathrm{c}^2}\hat{E}_\mathrm{e}^3\left[20\hat{E}_\mathrm{e}+(5-6\hat{E}_\mathrm{e})\delta\underline{\left(\frac{1^-}{2}-\hat{E}_\mathrm{e}\right)}\right]\theta\left(\frac{1}{2}-\hat{E}_\mathrm{e}\right)\\
&-\frac{32\mu_G^2}{m_\mathrm{c}^2}\hat{E}_\mathrm{e}^2(3-5\hat{E}_\mathrm{e})\theta\left(\frac{1}{2}-\hat{E}_\mathrm{e}\right)\\
&+\cdots,
\end{aligned}
$$



- 对于有限幂次修正，电子能谱的末端奇异行为导致其无法直接与实验数据进行对比
- 对相空间进行加权解析积分，抹除其奇异性。电子能量原点矩

$$\Gamma = \int \frac{d\Gamma}{d\hat{E}_e} d\hat{E}_e \qquad \langle E_\mathrm{e}^n \rangle = \frac{1}{\Gamma}\int \hat{E}_e^n \frac{d\Gamma}{d\hat{E}_e}\, d\hat{E}_e$$

- 衰变宽度的三圈图修正数值结果
- 矩的两圈图修正的数值结果

陈龙、马滟青老师

形状函数

b to u 跃迁电子能谱

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\Theta(1-y) - \frac{\lambda_1 + 33\lambda_2}{6m_b^2}\delta(1-y) - \frac{\lambda_1}{6m_b^2}\delta'(1-y)$$

$$\frac{1}{2\Gamma_b}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = yF(y)\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right)$$

$$\delta y = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i = -\frac{\lambda_1+33\lambda_2}{6m_b^2},\quad \sigma_y^2 = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^2 = -\frac{\lambda_1}{3m_b^2}$$

$$\frac{1}{N}\sum_{i=1}^{N}\Theta\left(1-y+\varepsilon_i\right) \xrightarrow{N\to\infty} \vartheta(y) = \Theta(1-y) + S(y)F(1)/F(y)$$

$$S(y) = \left\langle \Theta\left[1-y+\frac{2}{m_b}(v-\hat{p})\cdot iD\right] - \Theta(1-y)\right\rangle + \textbf{ less singular terms}$$

P: 轻子动量

D： b夸克动量

部分子模型

束缚态效应 $\phi\left(\left|\vec{p}_b\right|\right)$

忽略胶子场　A.C.M. 模型

$$\phi\left(\left|\vec{p}_b\right|\right) = \frac{4}{\sqrt{\pi}p_\mathrm{F}^3}\exp\left(-\frac{\left|\vec{p}_b\right|^2}{p_\mathrm{F}^2}\right)$$

$$S_\mathrm{ACM}(y) = \int\cdots\phi\left(\left|\vec{p}_b\right|\right)\cdots\mathrm{d}\vec{p}_b = \left[\frac{1}{2}-\Theta(1-y)\right]\Phi\left(\frac{m_b}{p_\mathrm{F}}\left|y-1\right|\right)$$

Fig.3(b)

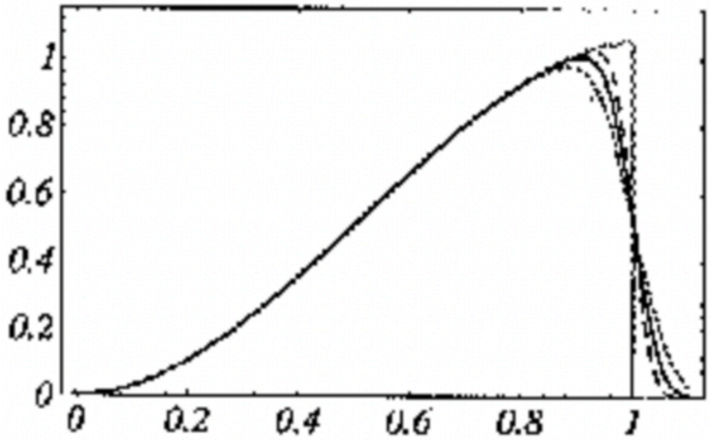
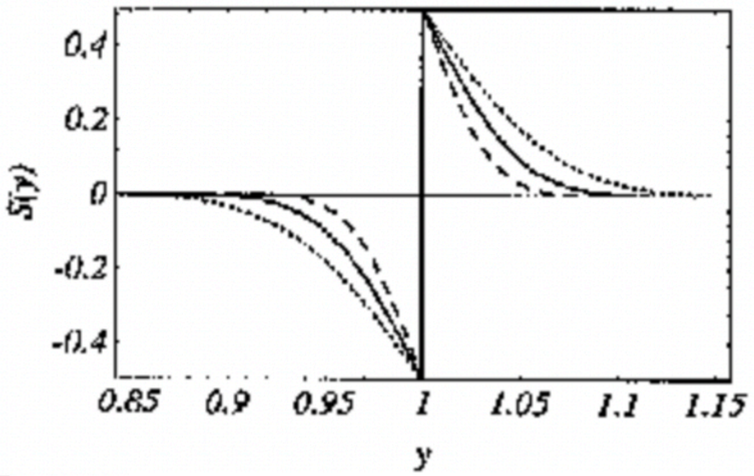


Fig.3(a)



真空插入近似

真空插入近似假设 (vacuum insertion approximation, VIA) 为简化强子内部相互作用，近似假设强子内为真空态，其色场被近似为色单态。具体地，在四夸克算符矩阵元中插入一组完备集，仅保留真空态贡献，将四夸克算符非微扰矩阵元因子化为衰变常数和 Bag 参数的乘积，Bag 参数依赖于算符中的色结构。色单态算符对应的 Bag 参数为假设为 1，色八重态算符对应的 Bag 参数被假设为 0。

粲介子寿命与半轻单举衰变宽度

- 同位旋对称性: $\mathrm{D}^0, \mathrm{D}^+$

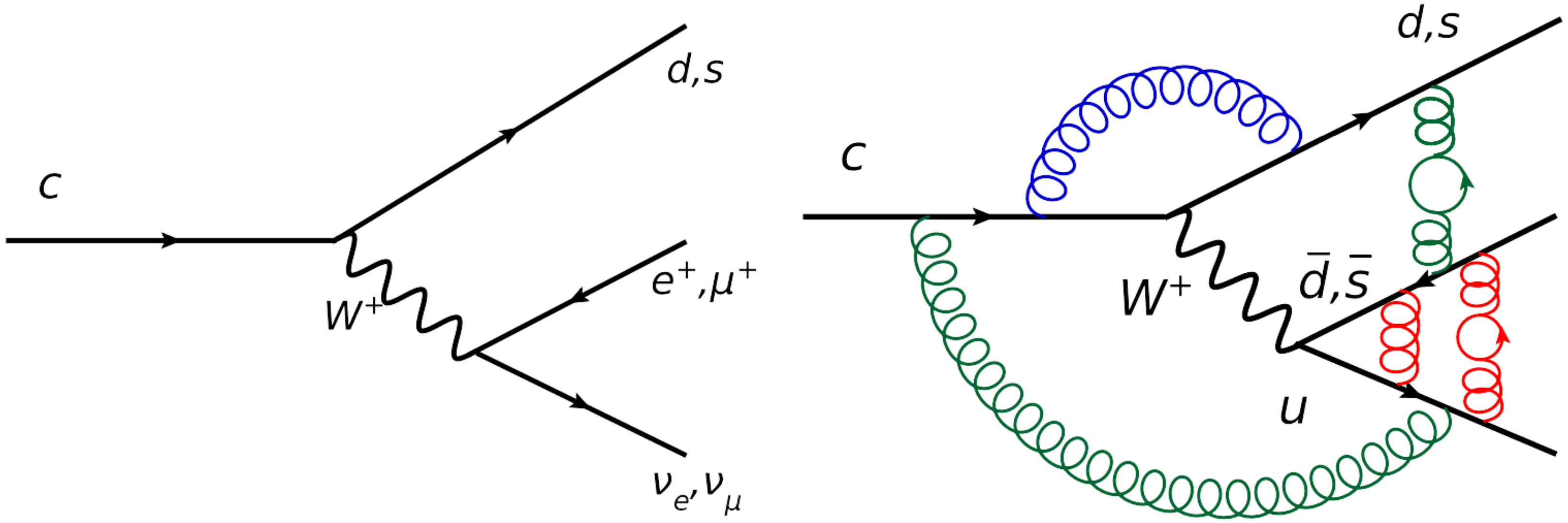
强子化过程中

Quantity	D^0	D^+	D_s^+
$\tau\,[ps]$	0.4101 ± 0.0015	1.040 ± 0.007	0.504 ± 0.004
$\Gamma\,[ps^{-1}]$	2.438 ± 0.009	0.962 ± 0.006	1.984 ± 0.0016
$BR(D_i \to Xe\nu)\,[\%]$	6.49 ± 0.16	16.07 ± 0.30	6.30 ± 0.16
$\Gamma(D_i \to Xe\nu)\,[ps^{-1}]$	0.158 ± 0.004	0.155 ± 0.003	0.125 ± 0.003

旁观者效应贡献很小

- 粲介子寿命差异: 旁观者夸克效应：弱湮灭、弱交换和泡利干涉项

HQETSR, VIA

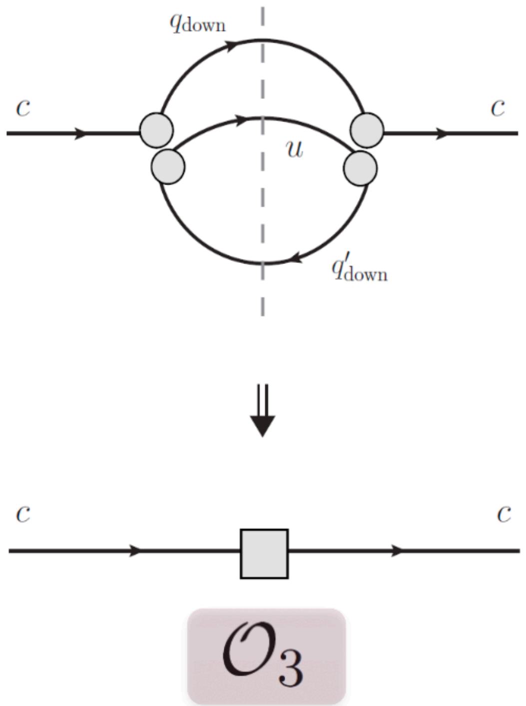


粲介子寿命与半轻单举衰变宽度

HEAVY QUARK EXPANSION (HQE) – systematic expansion in Λ_{QCD}/m_Q and α_s

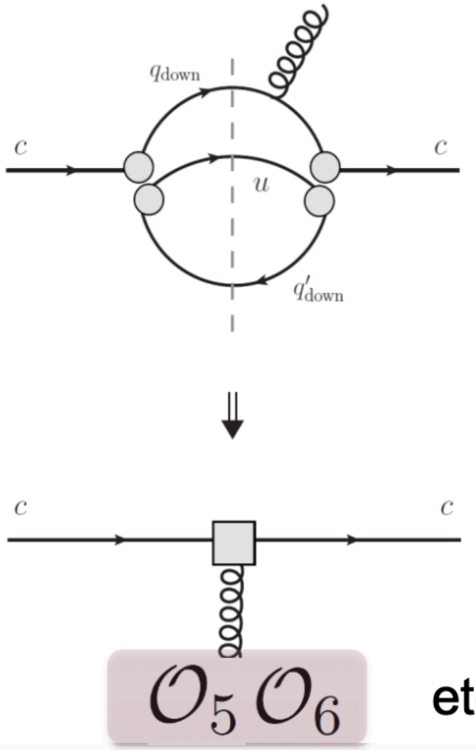
$$\mathcal{T} = \Big(\mathcal{C}_3 \mathcal{O}_3 + \frac{\mathcal{C}_5}{m_Q^2}\mathcal{O}_5 + \frac{\mathcal{C}_6}{m_Q^3}\mathcal{O}_6 + \dots \Big) + 16\pi^2 \Big(\frac{\tilde{\mathcal{C}}_6}{m_Q^3}\tilde{\mathcal{O}}_6 + \frac{\tilde{\mathcal{C}}_7}{m_Q^4}\tilde{\mathcal{O}}_7 + \dots \Big)$$

TWO-QUARK LEADING NON-SPECTATOR CONTRIBUTION



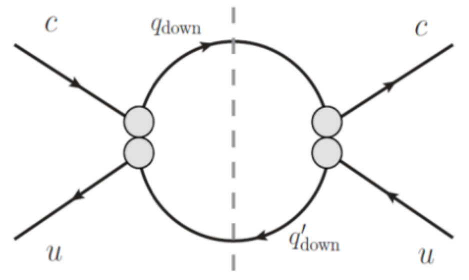
$\mathcal{O}_3$

TWO-QUARK NON-LEADING NON-SPECTATOR CONTRIBUTION **- from 90-ies; HQE**

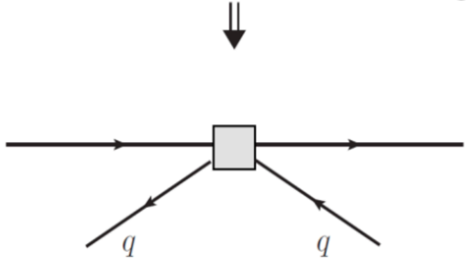


$\mathcal{O}_5\, \mathcal{O}_6$ etc

FOUR-QUARK SPECTATOR CONTRIBUTIONS - **ENHANCED**



light-quarks come into game



$\tilde{\mathcal{O}}_6\, \tilde{\mathcal{O}}_7$ etc

粲介子与粲重子的差异

CALCULATION OF **NON-SPECTATOR (two-quark) MATRIX ELEMENTS**

NON-SPECTATOR PART: - mainly universal – up to SU(3)$_f$ breaking and differences in spins of hadrons

μ_G^2 application of hadron mass formula:

$$m_H = m_c + \bar{\Lambda} + \frac{\mu_\pi^2(H)}{2m_c} - \frac{\mu_G^2(H)}{2m_c} + \mathcal{O}\left(\frac{1}{m_c^2}\right)$$

spin factor: $d_H = -2\left(S_H(S_H+1) - S_h(S_h+1) - S_l(S_l+1)\right)$

$$\mu_G^2(H) \equiv d_H \lambda_2 = d_H \frac{m_{H^*}^2 - m_H^2}{d_H - d_{H^*}}$$

H	D	D^*	$\Lambda_c^+,\, \Xi_c^+,\, \Xi_c^0$	Ω_c^0	Ω_c^{0*}
d_H	3	-1	0	4	-2

μ_π^2 HQET SR: $\mu_\pi^2 \geq \mu_G^2$

ρ_D^3 applying EOM of G$_{\mu\nu}$ and relating it to the dim6 operators:

$$2m_H \rho_D^3 = g_s^2 \langle H | \Big(-\frac{1}{8} O_1^q + \frac{1}{24} \tilde{O}_1^q + \frac{1}{4} O_2^q - \frac{1}{12} \tilde{O}_2^q \Big) | H \rangle + \mathcal{O}(1/m_c)$$

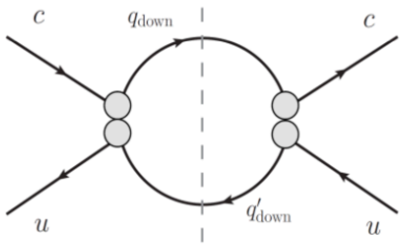
$$\rho_D^3(D_q) = \frac{g_s^2}{18} f_{D_q}^2 m_{D_q} + \mathcal{O}(1/m_c)$$

粲介子寿命与半轻单举衰变宽度

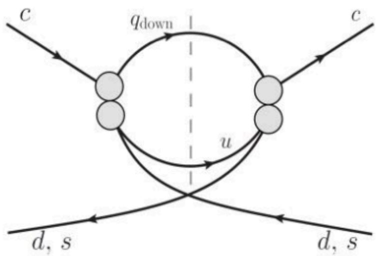
SPECTATOR (u,d,s) FOUR-QUARK CONTRIBUTIONS ARE IMPORTANT :

one-loop i.e 16 π^2 enhanced, although $1/m^3$ (dim6), $1/m^4$ (dim7) suppressed

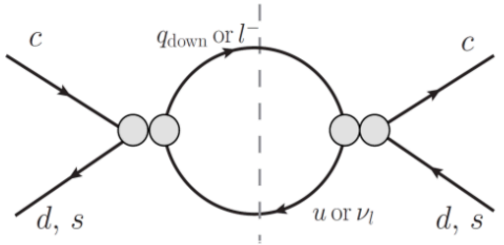
MESONS



WE

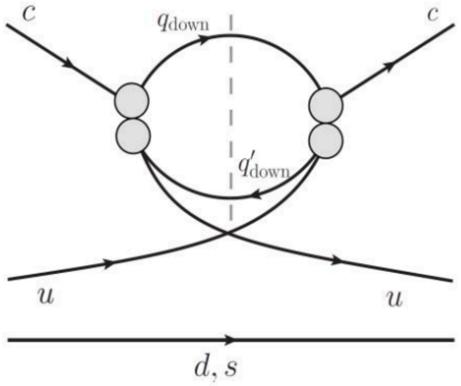


PI

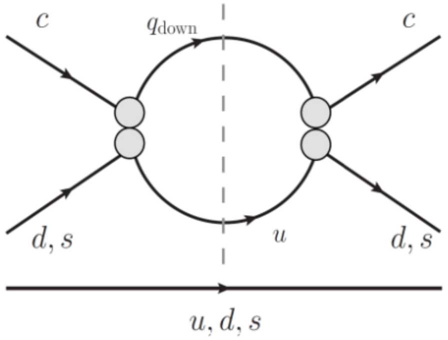


WA

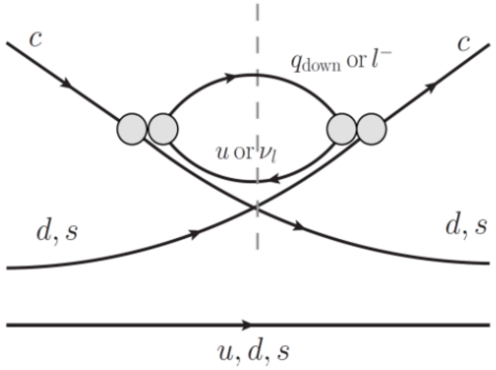
BARYONS



int-



exc



int+

decay H_c	CE NL $c \to s\bar{d}u$	CE SL $c \to s\bar{l}\nu_l$
$\bar{D}^0(u\bar{c})$	$\tilde{\Gamma}_{\rm WE}$	-
$D^-(d\bar{c})$	$\tilde{\Gamma}_{\rm PI}$	-
$D_s^-(s\bar{c})$	$\tilde{\Gamma}_{\rm WA}$	$\tilde{\Gamma}^{\rm SL}_{\rm WA}$
$\Lambda_c^+(udc)$	$\tilde{\Gamma}_{\rm exc}+\tilde{\Gamma}_{\rm int-}$	-
$\Xi_c^+(usc)$	$\tilde{\Gamma}_{\rm int-}+\tilde{\Gamma}_{\rm int+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int+}$
$\Xi_c^0(dsc)$	$\tilde{\Gamma}_{\rm exc}+\tilde{\Gamma}_{\rm int+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int+}$
$\Omega_c^0(ssc)$	$\tilde{\Gamma}_{\rm int+}$	$\tilde{\Gamma}^{\rm SL}_{\rm int+}$

CE = leading; Cabibbo enhanced

* effects are different in different mesons
* effects are different in different baryons
* no helicity suppression for baryons
* effects in SL decays – **different SL BRs !**

粲介子寿命与半轻单举衰变宽度

- 同位旋对称性: D^0, D^+

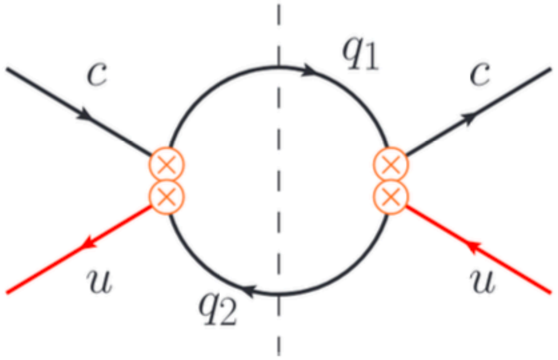
强子化过程中

Quantity	D^0	D^+	D_s^+
$\tau\,[ps]$	0.4101 ± 0.0015	1.040 ± 0.007	0.504 ± 0.004
$\Gamma\,[ps^{-1}]$	2.438 ± 0.009	0.962 ± 0.006	1.984 ± 0.0016
$BR(D_i \to Xe\nu)\,[\%]$	6.49 ± 0.16	16.07 ± 0.30	6.30 ± 0.16
$\Gamma(D_i \to Xe\nu)\,[ps^{-1}]$	0.158 ± 0.004	0.155 ± 0.003	0.125 ± 0.003

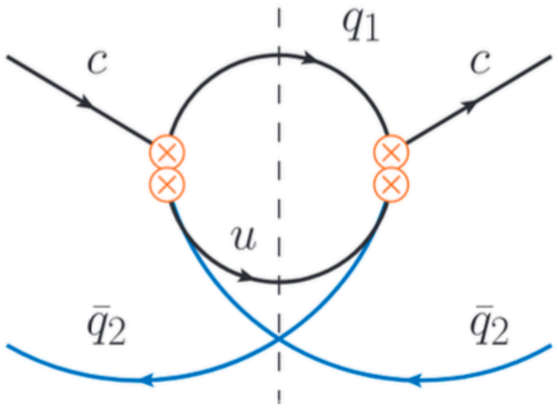
旁观者效应贡献很小

HQETSR, VIA

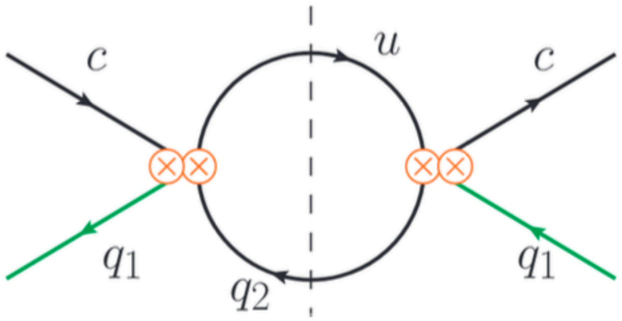
- 粲介子寿命差异: 旁观者夸克效应：弱湮灭、弱交换和泡利干涉项



弱交换



泡利干涉



弱湮灭

半轻衰变过程仅包含弱湮灭过程

粲介子寿命与半轻单举衰变宽度

- **动力学质量方案下粲介子衰变宽度对非微扰矩阵元依赖形式为（$\mu = 0.5\mathrm{GeV}$）：**

$$
\begin{aligned}
\Gamma\left(\mathrm{D}^0\right) = \Gamma_0[& \underbrace{6.15}_{c_3^{\mathrm{Lo}}} + \underbrace{2.95}_{\Delta c_3^{\mathrm{NLO}}} - 1.66\frac{\mu_\pi^2(D)}{\mathrm{GeV}^2} + 0.13\frac{\mu_G^2(D)}{\mathrm{GeV}^2} + 23.6\frac{\rho_D^3(D)}{\mathrm{GeV}^3} \\
& - 1.60\tilde{B}_1^q + 1.53\tilde{B}_2^q - 21.0\tilde{\epsilon}_1^q + 19.2\tilde{\epsilon}_2^q + \underbrace{0.00}_{\mathrm{dim-7,VIA}} \\
& -10.7\tilde{\delta}_1^{qq} + 1.53\tilde{\delta}_2^{qq} + 54.6\tilde{\delta}_3^{qq} + 0.13\tilde{\delta}_4^{qq} - 29.2\tilde{\delta}_1^{sq} + 28.8\tilde{\delta}_2^{sq} + 0.56\tilde{\delta}_3^{sq} + 2.36\tilde{\delta}_4^{sq}] \\
= 6.15\Gamma_0 \Bigg[& 1 + 0.48 - 0.13\frac{\mu_\pi^2(D)}{0.48\mathrm{GeV}^2} + 0.01\frac{\mu_G^2(D)}{0.34\mathrm{GeV}^2} + 0.31\frac{\rho_D^3(D)}{0.082\mathrm{GeV}^3} \\
& - \underbrace{0.01}_{\mathrm{dim\text{-}6,VIA}} - 0.005\frac{\delta\tilde{B}_1^q}{0.02} + 0.005\frac{\delta\tilde{B}_2^q}{0.02} + 0.137\frac{\tilde{\epsilon}_1^q}{-0.04} - 0.125\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{0.00}_{\mathrm{dim-7,VIA}} \\
& - 0.0045r_1^{qq} - 0.0004r_2^{qq} - 0.0035r_3^{qq} + 0.0000r_4^{qq} \\
& - 0.0109r_1^{sq} - 0.0079r_2^{sq} - 0.0000r_3^{sq} + 0.0001r_4^{sq}] \,.
\end{aligned}
$$

粲介子寿命与半轻单举衰变宽度

- 动力学质量方案下粲介子衰变宽度对非微扰矩阵元依赖形式为（$\mu = 0.5\mathrm{GeV}$）：

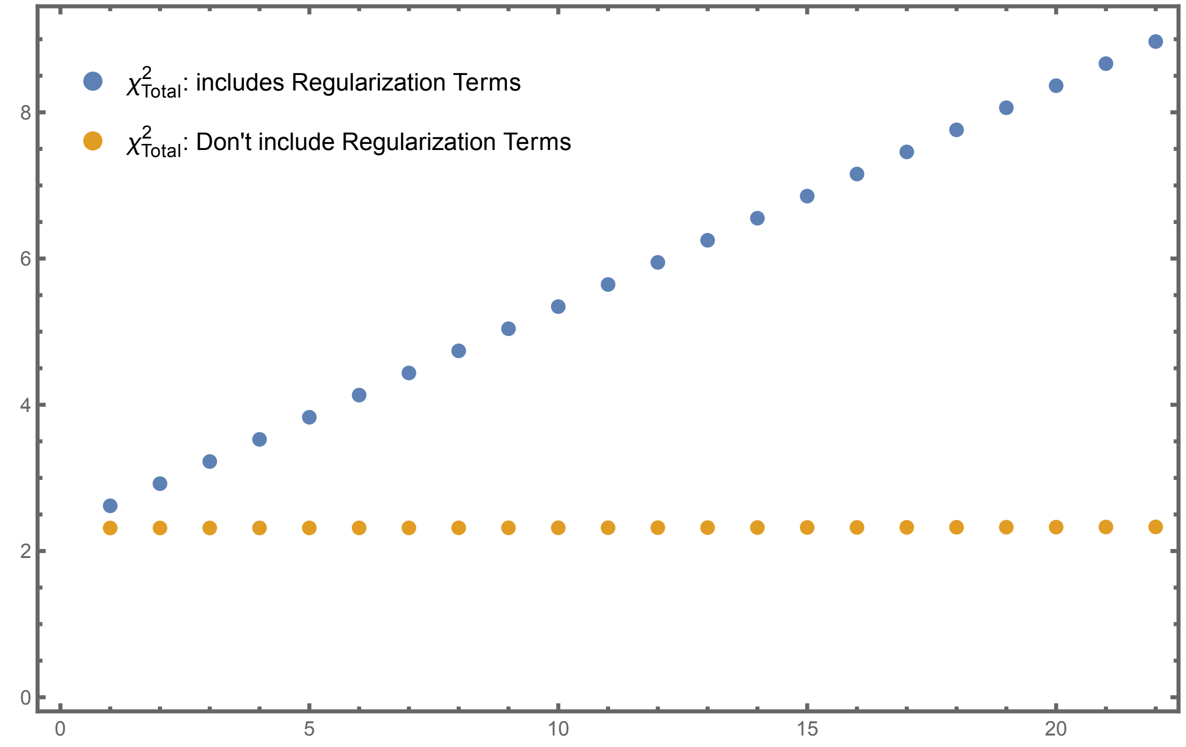
$$
\begin{aligned}
\Gamma\left(\mathrm{D}^{+}\right) = \Gamma_0 [& \underbrace{6.15}_{c_3^{\mathrm{LO}}} + \underbrace{2.95}_{\Delta c_3^{\mathrm{NLO}}} - 1.66 \frac{\mu_\pi^2(D)}{\mathrm{GeV}^2} + 0.13 \frac{\mu_G^2(D)}{\mathrm{GeV}^2} + 23.6 \frac{\rho_D^3(D)}{\mathrm{GeV}^3} \\
& - 16.9 \tilde{B}_1^q + 0.56 \tilde{B}_2^q + 84.0 \tilde{\epsilon}_1^q - 1.34 \tilde{\epsilon}_2^q + \underbrace{6.76}_{\mathrm{dim}-7} \\
& - 0.06 \tilde{\delta}_1^{qq} + 0.06 \tilde{\delta}_2^{qq} - 16.8 \tilde{\delta}_3^{qq} + 16.9 \tilde{\delta}_4^{qq} - 29.3 \tilde{\delta}_1^{sq} + 28.8 \tilde{\delta}_2^{sq} + 0.56 \tilde{\delta}_3^{sq} + 2.36 \tilde{\delta}_4^{sq}] \\
= 6.15 \Gamma_0 \Bigg[& 1 + 0.48 - 0.13 \frac{\mu_\pi^2(D)}{0.48 \mathrm{GeV}^2} + 0.01 \frac{\mu_G^2(D)}{0.34 \mathrm{GeV}^2} + 0.31 \frac{\rho_D^3(D)}{0.082 \mathrm{GeV}^3} \\
& - \underbrace{2.66}_{textdim-6, VIA} - 0.055 \frac{\delta \tilde{B}_1^q}{0.02} + 0.002 \frac{\delta \tilde{B}_2^q}{0.02} - 0.546 \frac{\tilde{\epsilon}_1^q}{-0.04} + 0.009 \frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{1.10}_{\mathrm{dim}-7, \mathrm{VIA}} \\
& - 0.0000 r_1^{qq} - 0.0000 r_2^{qq} + 0.0011 r_3^{qq} + 0.0008 r_4^{qq} \\
& - 0.0109 r_1^{sq} - 0.0080 r_2^{sq} - 0.0000 r_3^{sq} + 0.0001 r_4^{sq}] .
\end{aligned}
$$

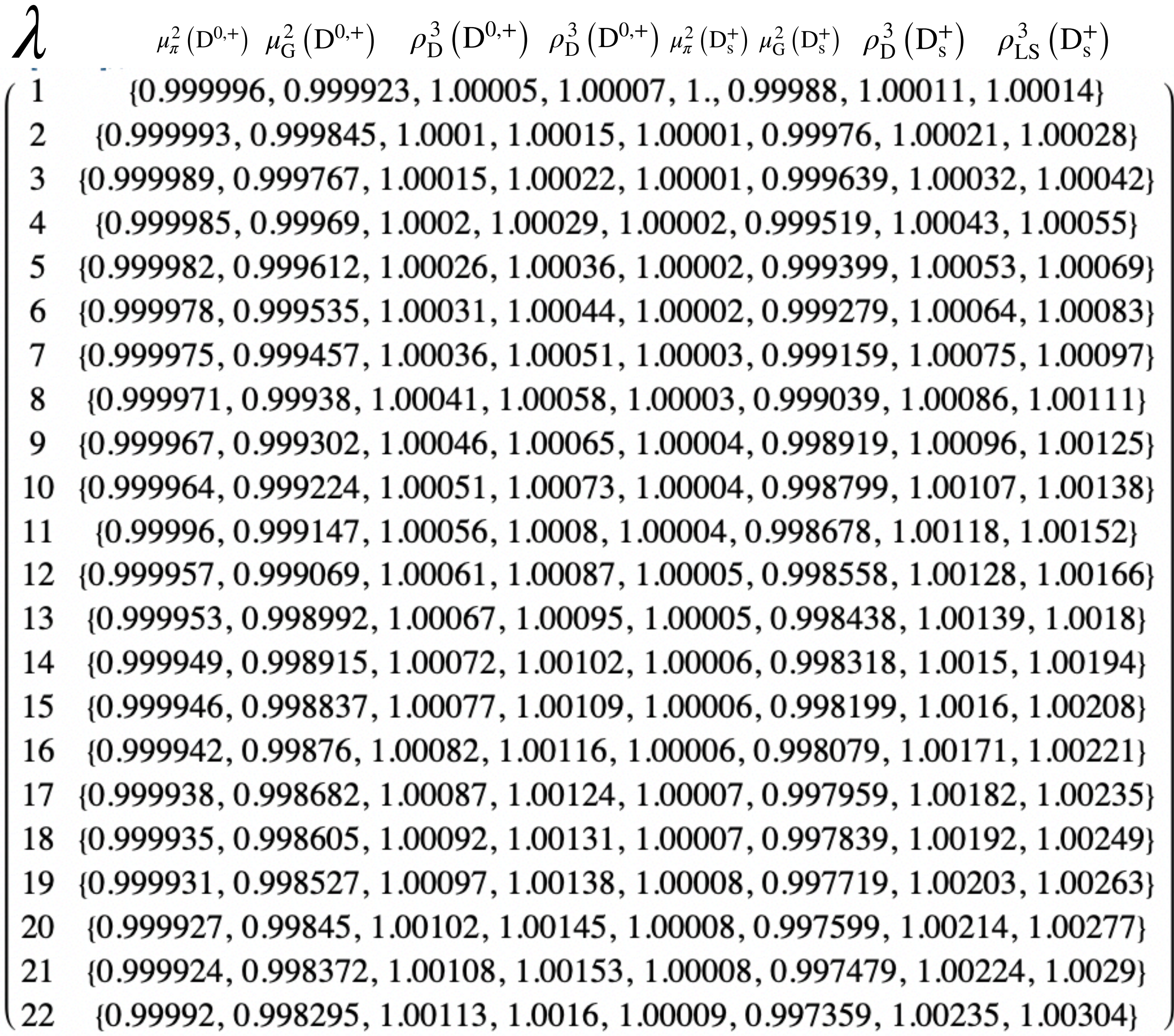
粲介子寿命与半轻单举衰变宽度

- 动力学质量方案下粲介子衰变宽度对非微扰矩阵元依赖形式为（$\mu = 0.5\mathrm{GeV}$）：

$$
\begin{aligned}
\Gamma\left(\mathrm{D}_s^+\right) =\Gamma_0\Big[& \underbrace{6.15}_{c_3^{\mathrm{LO}}} + \underbrace{2.95}_{\Delta c_3^{\mathrm{NLO}}} - 1.66\frac{\mu_\pi^2(D_s)}{\mathrm{GeV}^2} + 0.13\frac{\mu_G^2(D_s)}{\mathrm{GeV}^2} + 23.6\frac{\rho_D^3(D_s)}{\mathrm{GeV}^3} \\
& - 49.6\tilde{B}_1^s + 48.4\tilde{B}_2^s - 13.7\tilde{\epsilon}_1^s + 18.8\tilde{\epsilon}_2^s + \underbrace{0.63}_{\text{dim-7}} \\
& - 15.8\tilde{\delta}_1^{qs} + 2.34\tilde{\delta}_2^{qs} + 55.4\tilde{\delta}_3^{qs} + 25.0\tilde{\delta}_4^{qs}\Big] \\
= 6.15\Gamma_0\Bigg[& 1 + 0.48 - 0.15\frac{\mu_\pi^2(D_s)}{0.57\mathrm{GeV}^2} + 0.01\frac{\mu_G^2(D_s)}{0.36\mathrm{GeV}^2} + 0.46\frac{\rho_D^3(D_s)}{0.119\mathrm{GeV}^3} \\
& - \underbrace{0.20}_{\text{dim-6,VIA}} - 0.161\frac{\delta\tilde{B}_1^s}{0.02} + 0.157\frac{\tilde{B}_2^s}{0.02} + 0.089\frac{\tilde{\epsilon}_1^s}{-0.04} + 0.122\frac{\tilde{\epsilon}_2^s}{0.04} + \underbrace{0.10}_{\text{dim-7,VIA}} \\
& - 0.0064r_1^{qs} - 0.0007r_2^{qs} - 0.0036r_3^{qs} + 0.0012r_4^{qs}\Big].
\end{aligned}
$$

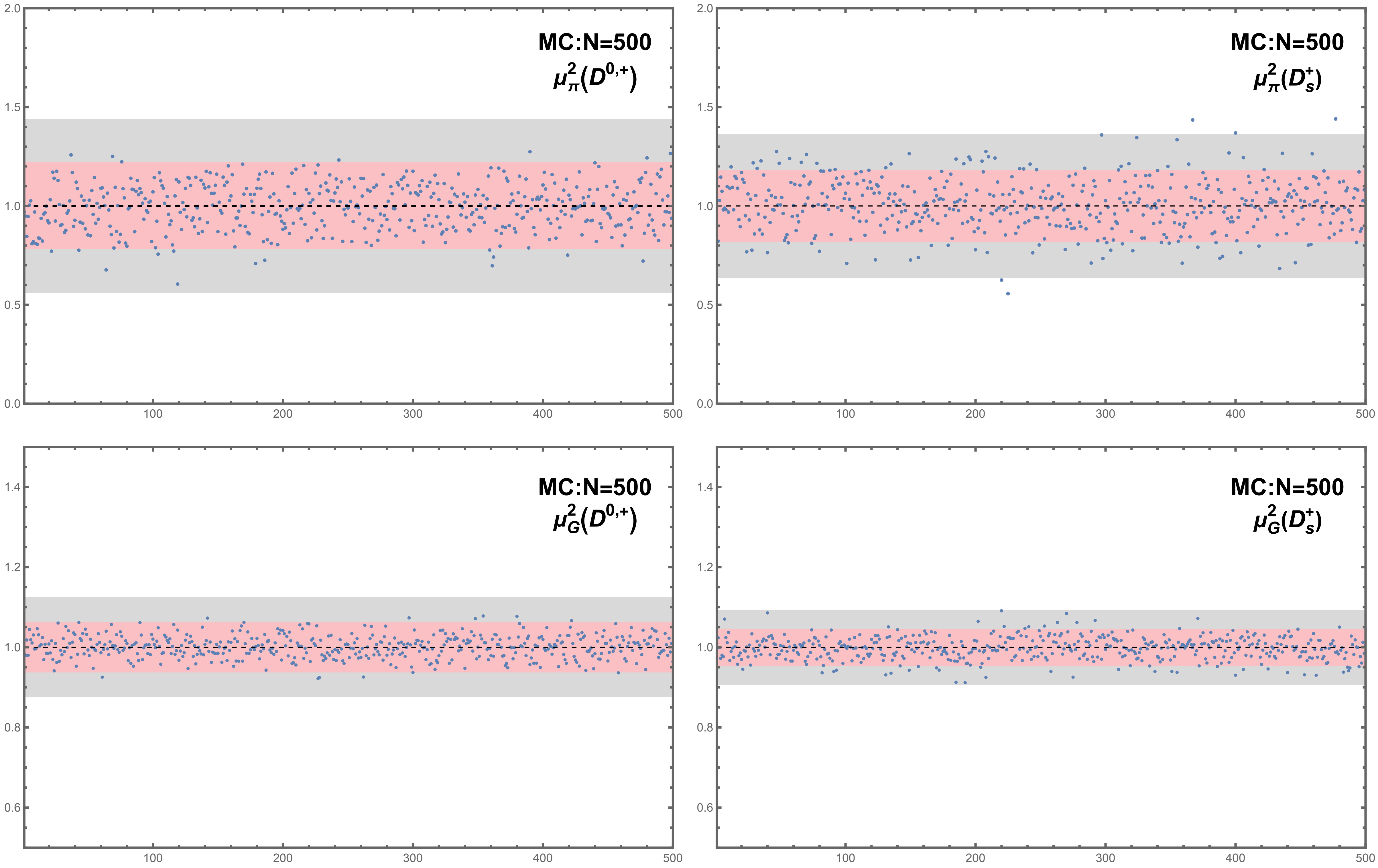
L2正规化



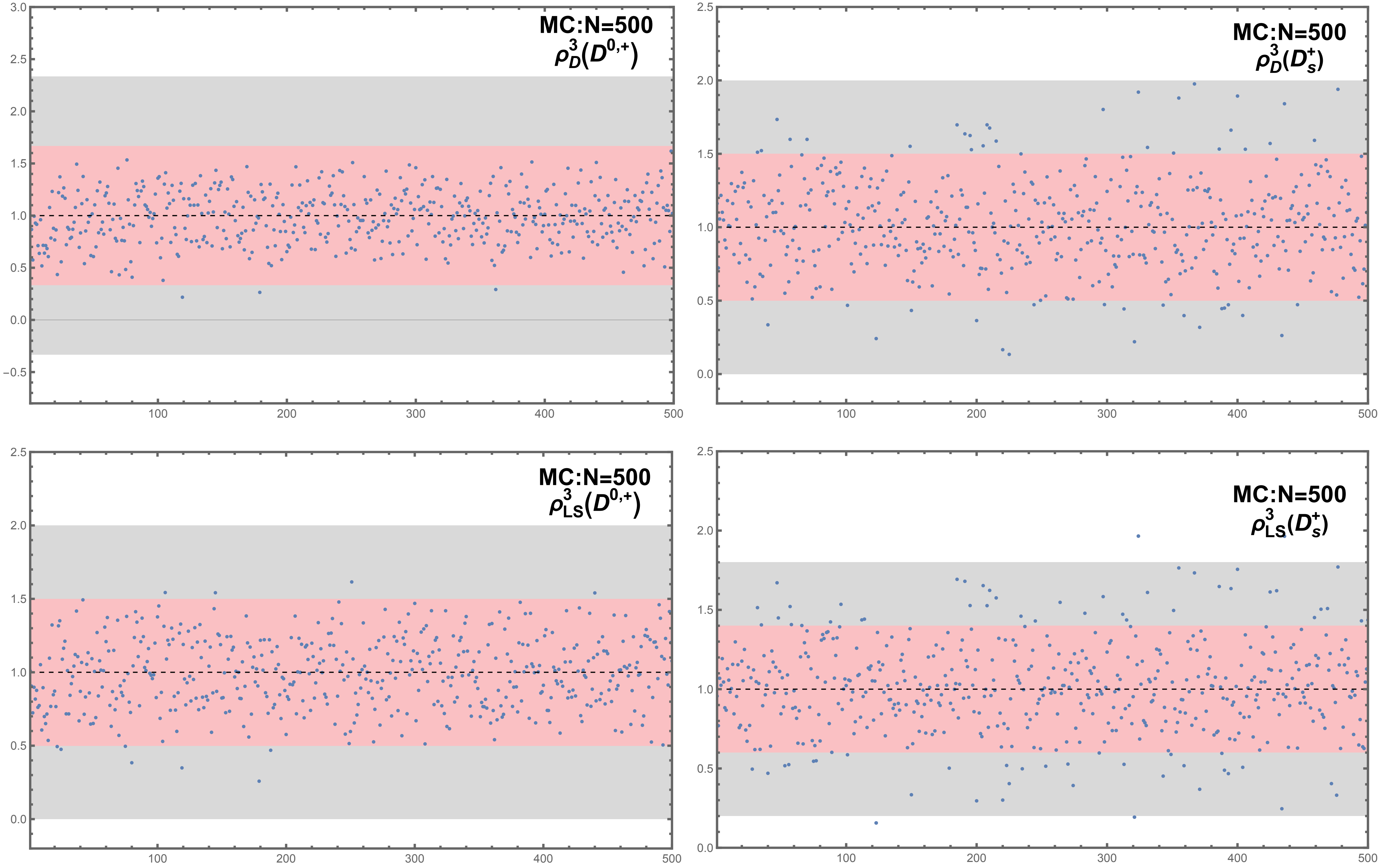


λ	$\mu_\pi^2(D^{0,+})$	$\mu_G^2(D^{0,+})$	$\rho_D^3(D^{0,+})$	$\rho_D^3(D^{0,+})$	$\mu_\pi^2(D_s^+)$	$\mu_G^2(D_s^+)$	$\rho_D^3(D_s^+)$	$\rho_{LS}^3(D_s^+)$
1	{0.999996	0.999923	1.00005	1.00007	1.	0.99988	1.00011	1.00014}
2	{0.999993	0.999845	1.0001	1.00015	1.00001	0.99976	1.00021	1.00028}
3	{0.999989	0.999767	1.00015	1.00022	1.00001	0.999639	1.00032	1.00042}
4	{0.999985	0.99969	1.0002	1.00029	1.00002	0.999519	1.00043	1.00055}
5	{0.999982	0.999612	1.00026	1.00036	1.00002	0.999399	1.00053	1.00069}
6	{0.999978	0.999535	1.00031	1.00044	1.00002	0.999279	1.00064	1.00083}
7	{0.999975	0.999457	1.00036	1.00051	1.00003	0.999159	1.00075	1.00097}
8	{0.999971	0.99938	1.00041	1.00058	1.00003	0.999039	1.00086	1.00111}
9	{0.999967	0.999302	1.00046	1.00065	1.00004	0.998919	1.00096	1.00125}
10	{0.999964	0.999224	1.00051	1.00073	1.00004	0.998799	1.00107	1.00138}
11	{0.99996	0.999147	1.00056	1.0008	1.00004	0.998678	1.00118	1.00152}
12	{0.999957	0.999069	1.00061	1.00087	1.00005	0.998558	1.00128	1.00166}
13	{0.999953	0.998992	1.00067	1.00095	1.00005	0.998438	1.00139	1.0018}
14	{0.999949	0.998915	1.00072	1.00102	1.00006	0.998318	1.0015	1.00194}
15	{0.999946	0.998837	1.00077	1.00109	1.00006	0.998199	1.0016	1.00208}
16	{0.999942	0.99876	1.00082	1.00116	1.00006	0.998079	1.00171	1.00221}
17	{0.999938	0.998682	1.00087	1.00124	1.00007	0.997959	1.00182	1.00235}
18	{0.999935	0.998605	1.00092	1.00131	1.00007	0.997839	1.00192	1.00249}
19	{0.999931	0.998527	1.00097	1.00138	1.00008	0.997719	1.00203	1.00263}
20	{0.999927	0.99845	1.00102	1.00145	1.00008	0.997599	1.00214	1.00277}
21	{0.999924	0.998372	1.00108	1.00153	1.00008	0.997479	1.00224	1.0029}
22	{0.99992	0.998295	1.00113	1.0016	1.00009	0.997359	1.00235	1.00304}

唯象学分析结果的稳定性检验



唯象学分析结果的稳定性检验



唯象学分析理论输入参数的跑动

- 重整化群方程:

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -\left(\beta_0\alpha_s^2 + \beta_1\alpha_s^3 + \beta_2\alpha_s^4 + \beta_3\alpha_s^5 \cdots\right).$$

↓

$$t \equiv \ln \frac{\mu^2}{\Lambda^2}$$

$$\ell = \ln t$$

$$\alpha_s\left(\mu_R^2\right) \simeq \frac{1}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ell}{t} + \frac{\beta_1^2\left(\ell^2 - \ell - 1\right) + \beta_0\beta_2}{\beta_0^4 t^2} + \frac{\beta_1^3\left(-2\ell^3 + 5\ell^2 + 4\ell - 1\right) - 6\beta_0\beta_2\beta_1\ell + \beta_0^2\beta_3}{2\beta_0^6 t^3} + \cdots\right)$$

近似解析解

$$\alpha_s(1.27) = 0.378$$

↓

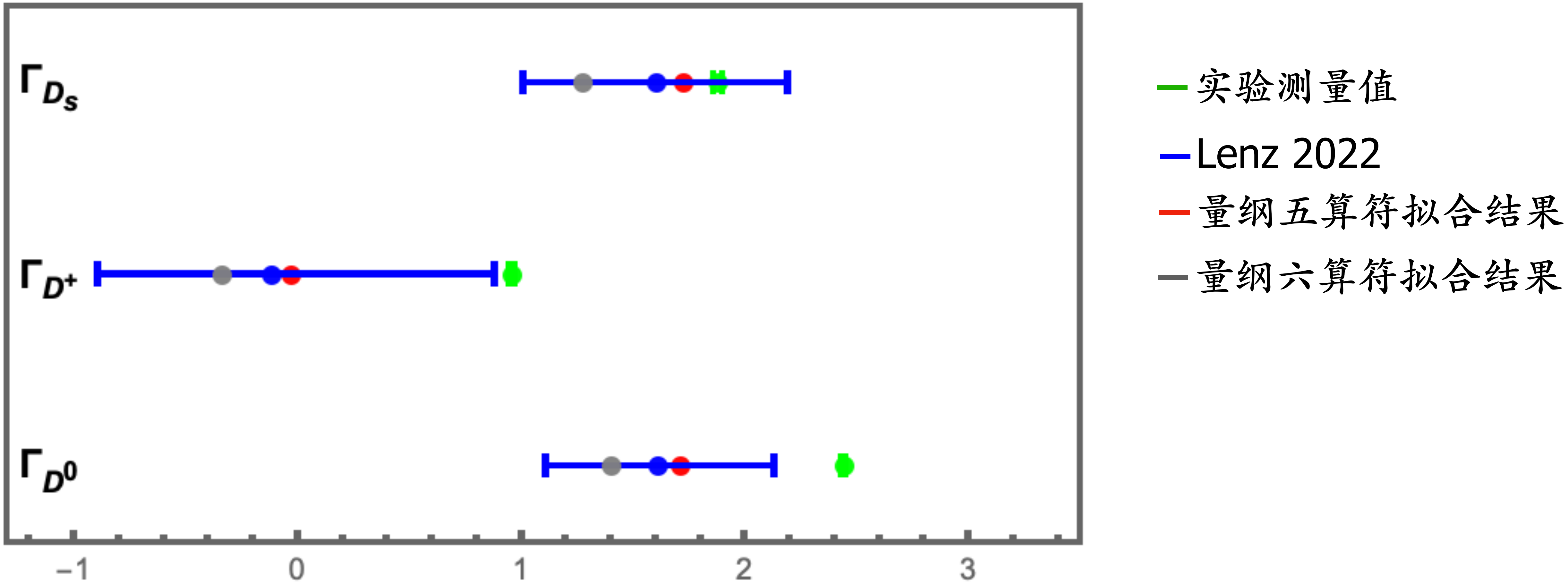
Λ_4 $n_f = 4, n_l = 3$

↓

$\alpha_s(\mu)$ μ被假设服从均匀分布

粲介子寿命的初步定性讨论

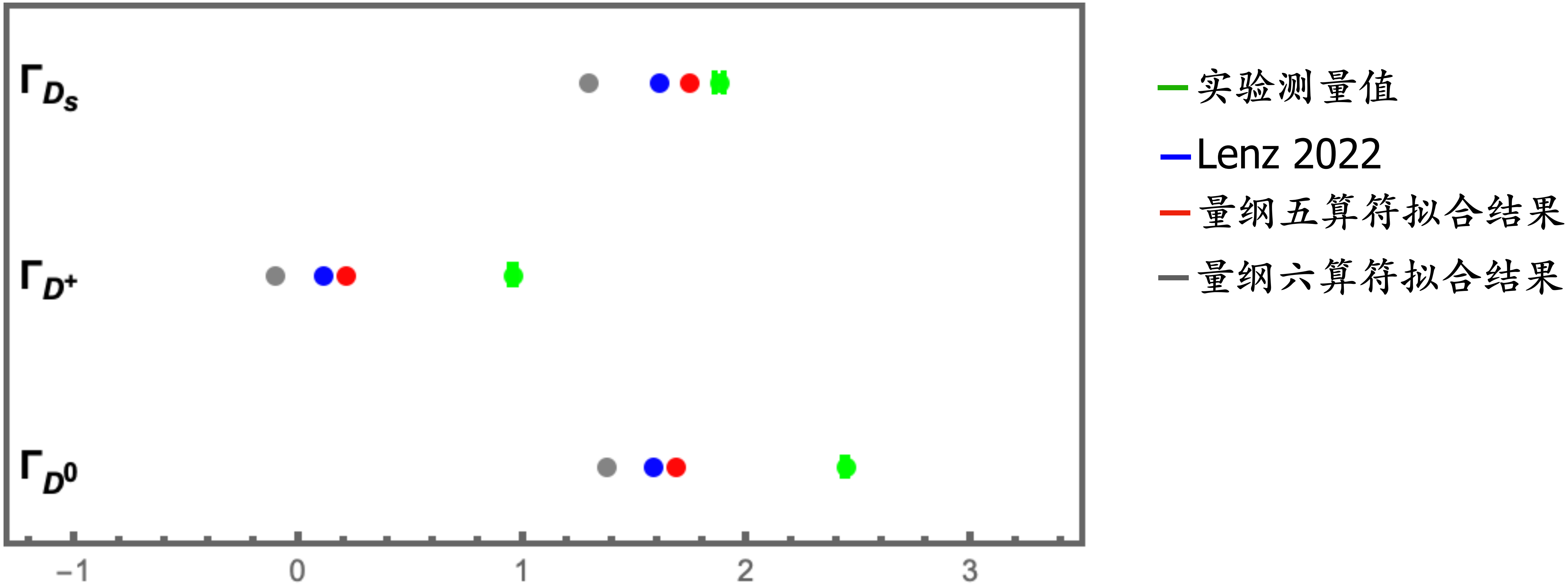
- 动力学质量方案下基于HQETSR对粲介子衰变宽度的理论预言



量纲五算符拟合结果更靠近实验测量值

粲介子寿命的初步定性讨论

- 动力学质量方案下基于VIA对粲介子衰变宽度的理论预言



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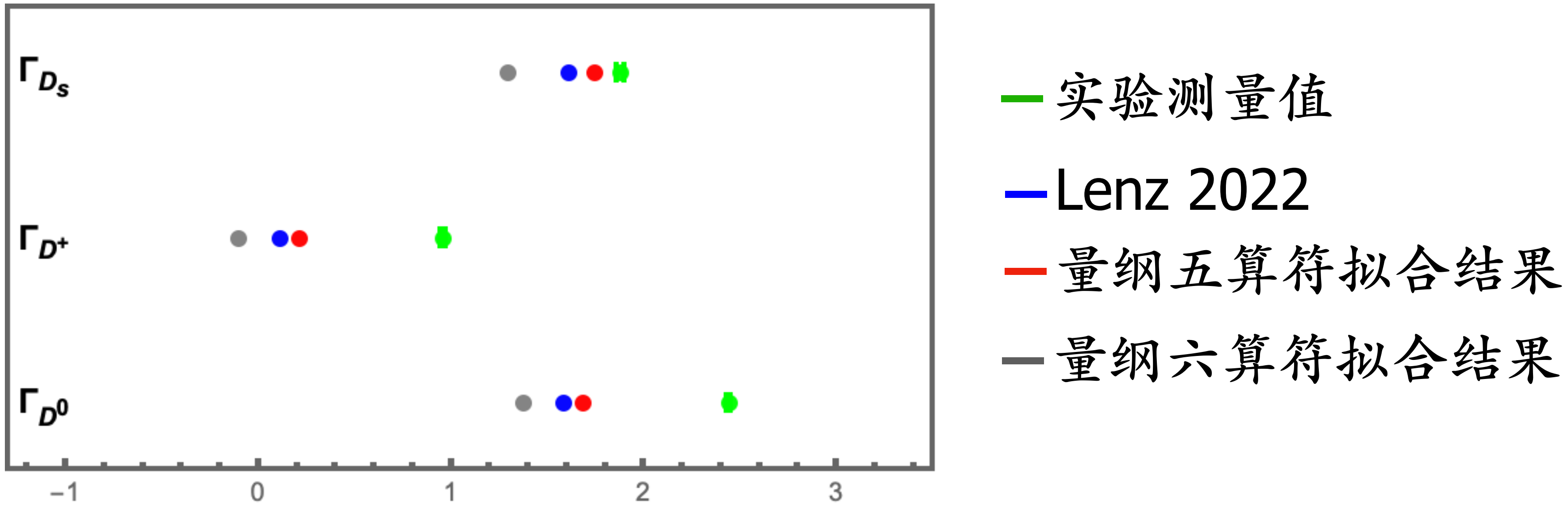
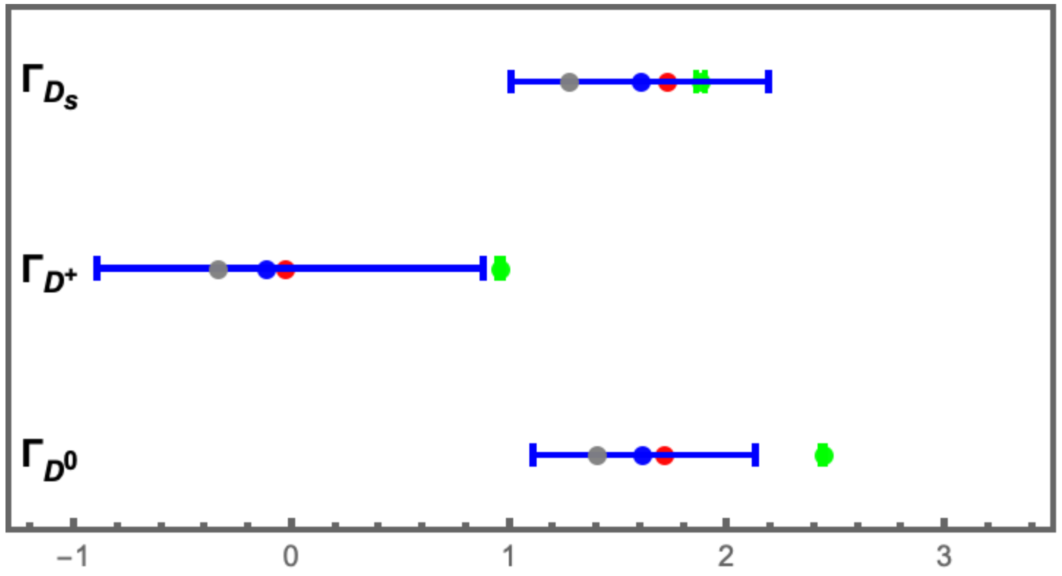


动力学质量方案下基于HQETSR对粲介子衰变宽度的理论预言

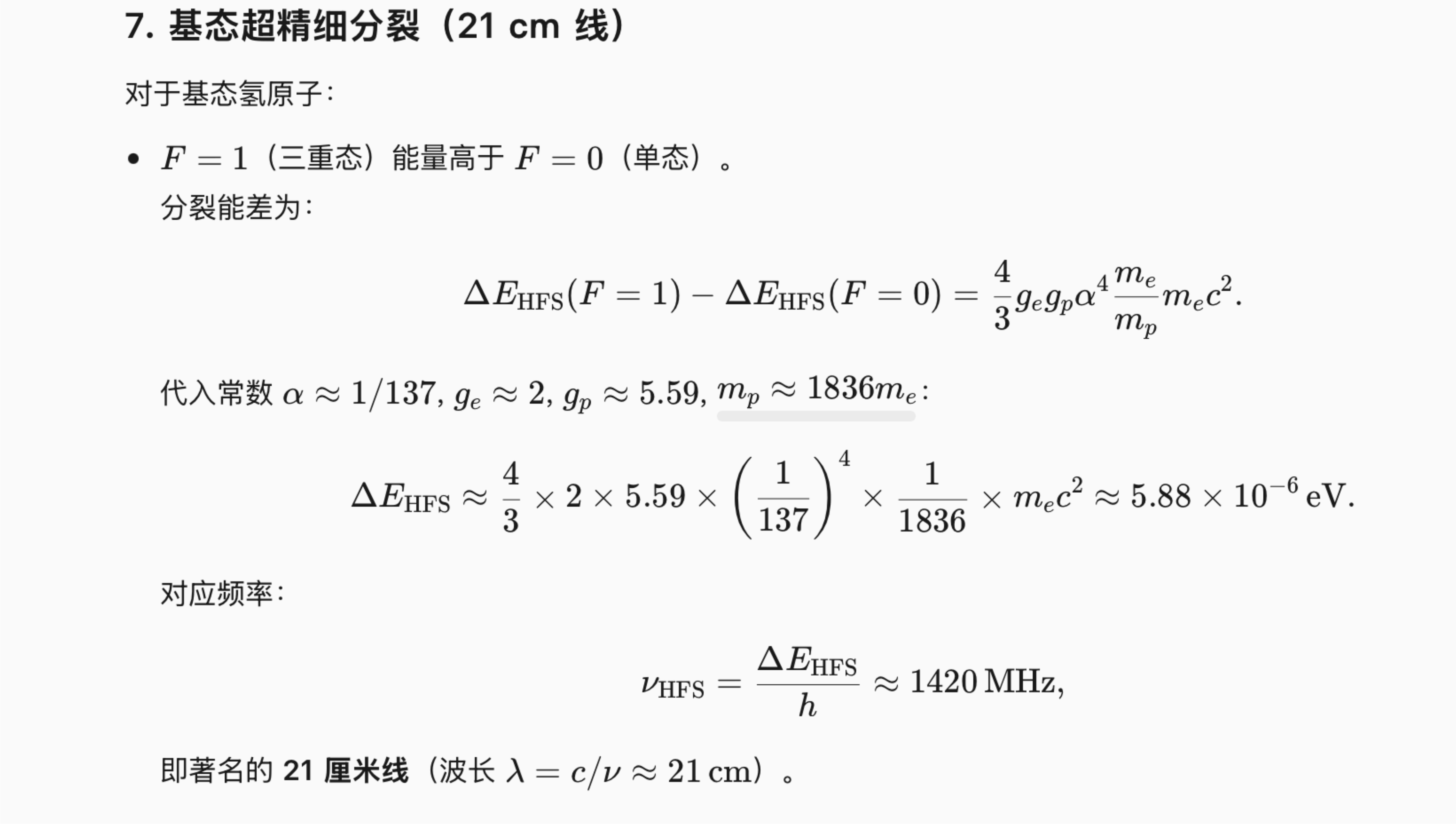


- 实现了量纲五算符非微扰矩阵元模型无关的高精度全局拟合估计
- 该估计值使得衰变宽度更靠近实验测量值
- 该现象没有模型依赖

氢原子超精细结构

超精细分裂的典型能量尺度为

$$\Delta E_{\mathrm{HFS}} \sim \alpha^4 m_e c^2 \left(\frac{m_e}{m_p} \right)$$



7. 基态超精细分裂（21 cm 线）

对于基态氢原子：

- $F = 1$（三重态）能量高于 $F = 0$（单态）。
  分裂能差为：

$$\Delta E_{\mathrm{HFS}}(F=1) - \Delta E_{\mathrm{HFS}}(F=0) = \frac{4}{3} g_e g_p \alpha^4 \frac{m_e}{m_p} m_e c^2.$$

代入常数 $\alpha \approx 1/137$, $g_e \approx 2$, $g_p \approx 5.59$, $m_p \approx 1836 m_e$：

$$\Delta E_{\mathrm{HFS}} \approx \frac{4}{3} \times 2 \times 5.59 \times \left(\frac{1}{137} \right)^4 \times \frac{1}{1836} \times m_e c^2 \approx 5.88 \times 10^{-6}\,\mathrm{eV}.$$

对应频率：

$$\nu_{\mathrm{HFS}} = \frac{\Delta E_{\mathrm{HFS}}}{h} \approx 1420\,\mathrm{MHz},$$

即著名的 **21 厘米线**（波长 $\lambda = c/\nu \approx 21\,\mathrm{cm}$）。

色磁相互作用（自旋自旋）

$$\mu_{\mathrm{G}}^2 = \frac{1}{2\mathrm{M_D}}\langle \mathrm{D}\,|\,\bar{\mathrm{c}}_{\mathrm{v}}\sigma\cdot\mathrm{G}\mathrm{c}_{\mathrm{v}}\,|\,D\rangle$$

$$\sigma^{\mu\nu}G_{\mu\nu} = 2\left(\boldsymbol{\Sigma}\cdot\boldsymbol{B} + i\gamma^0\boldsymbol{\Sigma}\cdot\boldsymbol{E}\right)$$

$\boldsymbol{\Sigma}\cdot\boldsymbol{B}$	直接耦合重夸克自旋与色磁场，色源来自于轻自由度（胶子和轻夸克）
$\boldsymbol{\Sigma}\cdot\boldsymbol{E}$	被v/c速度压低，重夸克自旋与色电场的耦合在非相对论极限下可以忽略

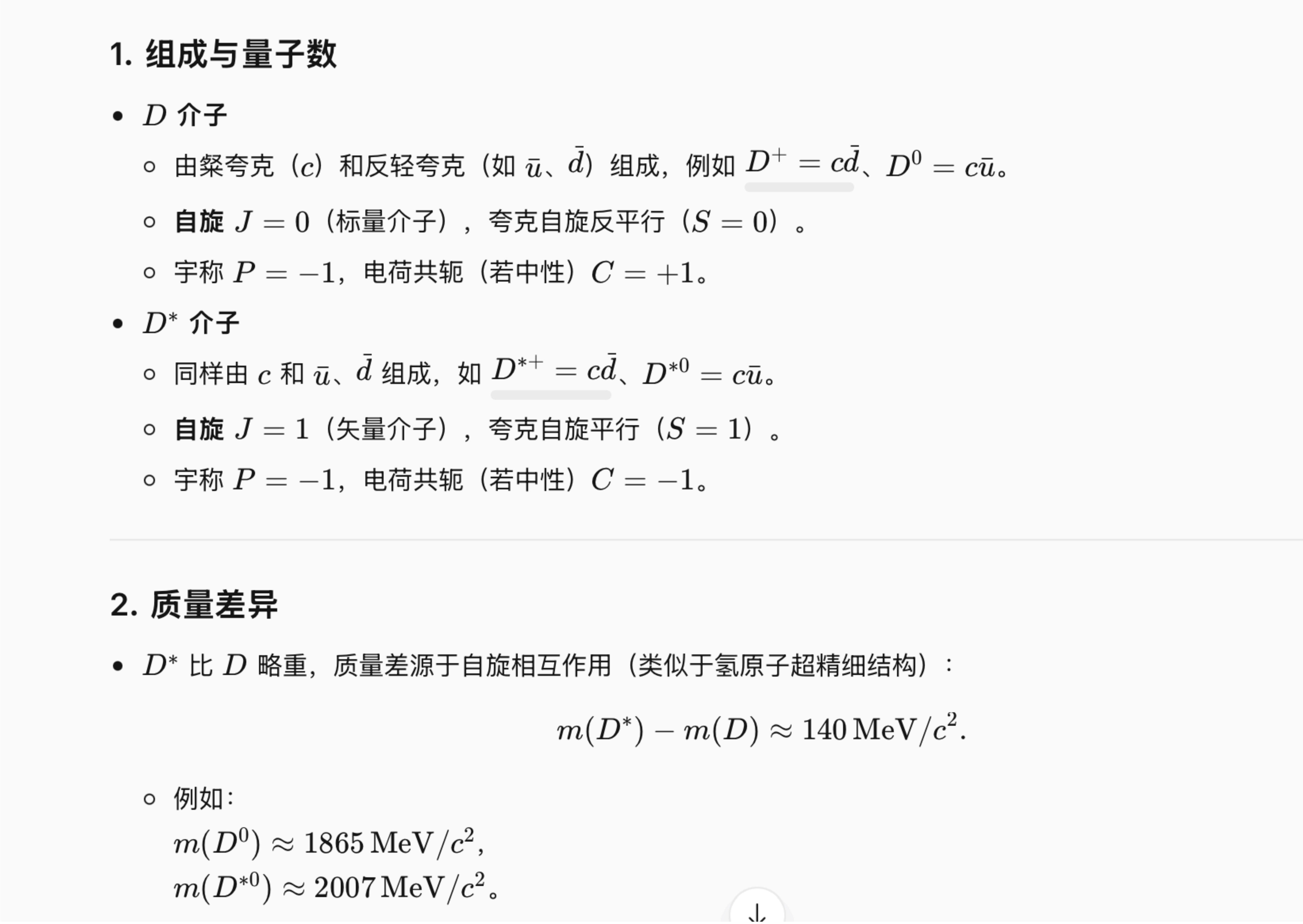
在重夸克极限 $m_Q\to\infty$下，重夸克近似为静态色源，其周围的胶子场主要由 瞬时相互作用（如库伦势）主导。色磁场 B正比于胶子场的空间分量 G_{ij}，且静态胶子场的 B是零能模（非相对论极限下不依赖速度 v/c）。

质量谱劈裂：

$$\mu_G^2\left(D_{(s)}\right) = \frac{3}{4}\left(M_{D_{(s)}^*}^2 - M_{D_{(s)}}^2\right)$$

$$\mu_G^2\left(D_{(s)}\right) = \frac{3}{2}m_c\left(M_{D_{(s)}^*} - M_{D_{(s)}}\right)$$

氢原子超精细结构



1. 组成与量子数

- D 介子
  - 由粲夸克（c）和反轻夸克（如 $\bar{u}$、$\bar{d}$）组成，例如 $D^+ = c\bar{d}$、$D^0 = c\bar{u}$。
  - **自旋** $J = 0$（标量介子），夸克自旋反平行（$S = 0$）。
  - 宇称 $P = -1$，电荷共轭（若中性）$C = +1$。
- D^* 介子
  - 同样由 c 和 $\bar{u}$、$\bar{d}$ 组成，如 $D^{*+} = c\bar{d}$、$D^{*0} = c\bar{u}$。
  - **自旋** $J = 1$（矢量介子），夸克自旋平行（$S = 1$）。
  - 宇称 $P = -1$，电荷共轭（若中性）$C = -1$。

---

2. 质量差异

- D^* 比 D 略重，质量差源于自旋相互作用（类似于氢原子超精细结构）：

$$m(D^*) - m(D) \approx 140\,\mathrm{MeV}/c^2.$$

  - 例如：
    $m(D^0) \approx 1865\,\mathrm{MeV}/c^2$，
    $m(D^{*0}) \approx 2007\,\mathrm{MeV}/c^2$。

自旋轨道耦合项

$$O_{\rho_{LS}} = \frac{1}{2}\bar{c}_v \left\{ iD_\alpha, \left[(ivD), iD_\beta\right] \left(-i\sigma^{\alpha\beta}\right) \right\} c_v$$

轨道角动量算符：$\mathscr{L}^i = \psi^\dagger[\boldsymbol{r} \times (i\boldsymbol{D})]^i \psi,$

$$\mathcal{O}_{\rho_{LS}} = -\frac{g_s}{2}\bar{c}_v \left\{ iD_\alpha, v^\mu G_{\mu\beta}\sigma^{\alpha\beta} \right\} c_v .$$

r一般被积分掉

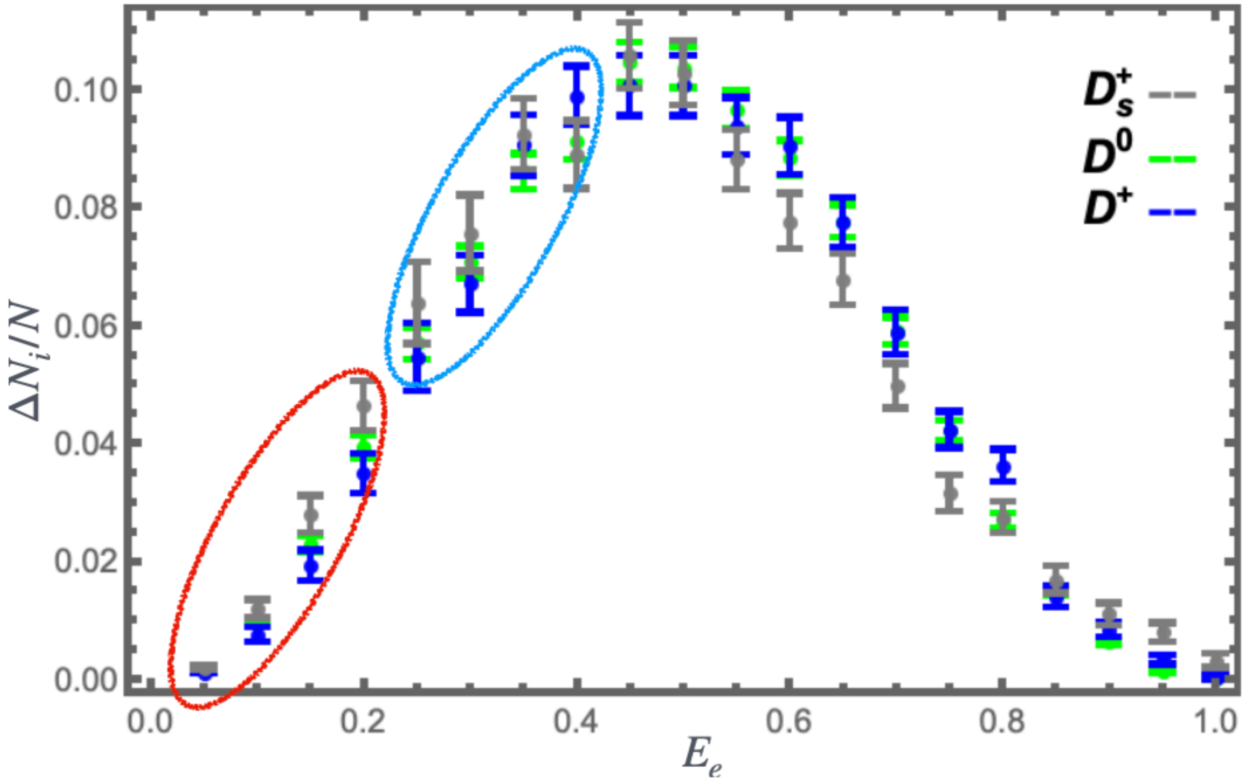
自旋轨道相互作用，自旋-自旋相互作用

数据处理

$$\frac{d\Gamma}{dy} = ay^2(1+by)(1-y)$$

实验原始数据 → 模拟外推 → 完整电子能谱 → 蒙卡模拟 抽取能量矩 → 电子能量矩

[Gambino,Kamenik, '10]



- 假设每个bin的统计误差无关联
- 假设每个bin的系统误差全关联

[Gambino,Kamenik, '10]

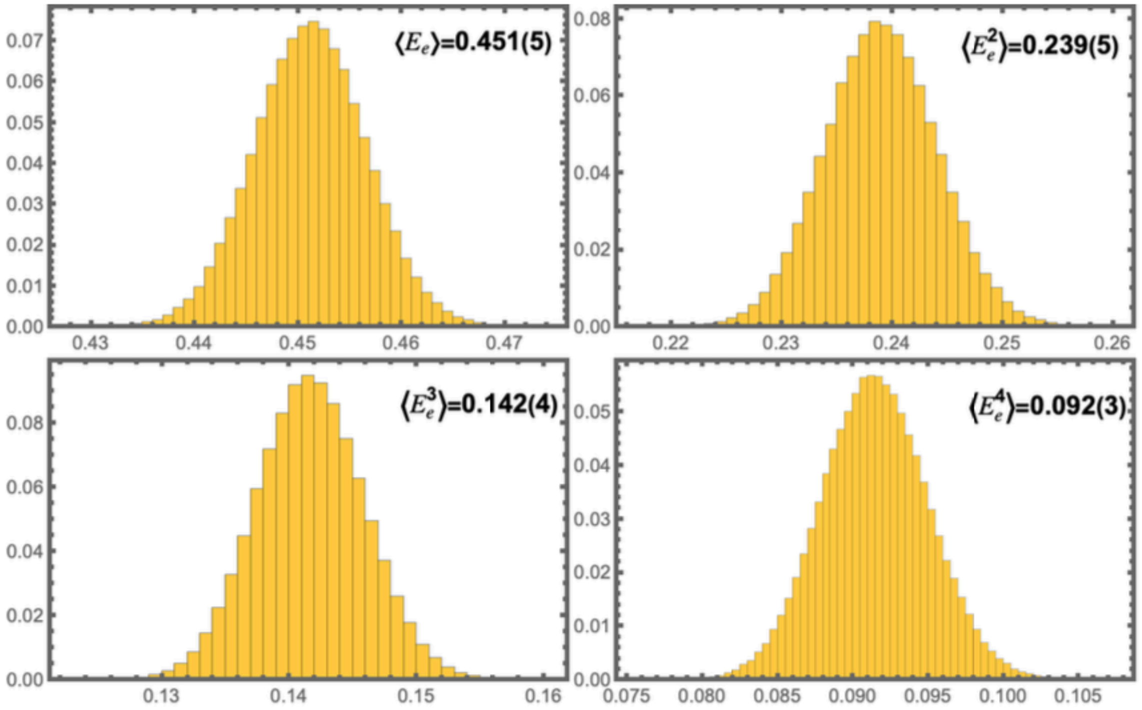
- 假设每个bin内的电子能量服从均匀分布

- 电子能量原点矩

$$\langle \mathrm{E_e^n} \rangle = \frac{1}{\Gamma}\int E_e^n \frac{d\Gamma}{dE_e} dE_e$$

- 实验上：积分离散化为求和

$$\langle \mathrm{E_e^n} \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dE_\mathrm{e}} dE_\mathrm{e} = (E_\mathrm{e}^n)_i \sum_i \frac{\Delta\Gamma_i}{\Gamma} = (E_\mathrm{e}^n)_i \sum_i \frac{\Delta \mathrm{N}_i}{\mathrm{N}}$$



质量方案

- 极质量方案: 微扰展开级数不收敛（重整子任意性问题）

$$m_c^{\text{Pole}} = \bar{m}_c(\bar{m}_c)\left[1+\frac{4}{3}\frac{\alpha_s(\bar{m}_c)}{\pi}+10.43\left(\frac{\alpha_s(\bar{m}_c)}{\pi}\right)^2+116.5\left(\frac{\alpha_s(\bar{m}_c)}{\pi}\right)^3+\cdots\right]$$
$$= \bar{m}_c(\bar{m}_c)\,[1+0.1642+0.1582+0.2176+\cdots].$$

- $\overline{\text{MS}}$质量方案: 微扰展开级数收敛较慢

$$m_c^{\text{pole}} = m_c + m_c\frac{\left(\frac{4}{3}+\log\left[\frac{\mu^2}{m_c^2}\right]\right)\alpha_s}{\pi} + m_c\frac{\alpha_s^2\left(\frac{779}{96}+\frac{\pi^2}{6}+\frac{1}{9}\pi^2\log[2]+\frac{415}{72}\log\left[\frac{\mu^2}{m_c^2}\right]+\frac{37}{24}\log\left[\frac{\mu^2}{m_c^2}\right]^2-\frac{\zeta(3)}{6}\right)}{\pi^2},$$

- 1S 质量方案: 微扰展开级数收敛行为良好：引入$\alpha_s(\mu)^3\Lambda_{\text{QCD}}$量级的修正

$$m_c^{\text{Pole}} = m_c + \frac{2}{9}m_c\epsilon\alpha_s^2 - \frac{\epsilon^2 m_c\left(-\frac{12992\alpha_s^3}{9}-\frac{3200}{3}\log\left[\frac{3\mu}{4m_c\alpha_s}\right]\alpha_s^3-\frac{256\pi\alpha_s^4}{9}\right)}{576\pi},$$

- D介子质量: 引入Λ_{QCD}量级的幂次修正