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Combined study of the isospin-violating decay $D_s^* \to D_s\pi^0$ and radiative decay $D_s^* \to D_s\gamma$ with intermediate meson loops

Wang and Zhao Phys. Rev. D 111, 096007

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Contents

1 Introduction

In 2023, BESIII reported the first determination of the quantum numbers of the D_s^* meson.
Ablikim et al. Phys. Lett. B 846, 138245

$D_s^{*\pm}$ $I(J^P) = 0(1^-)$

$J^P = 1^-$ established by ABLIKIM 23AZ.

$D_s^{*\pm}$ MASS

The fit includes $D^\pm$, D^0, $D_s^\pm$, $D^{*\pm}$, D^{*0}, $D_s^{*\pm}$, $D_1(2420)^0$, $D_2^*(2460)^0$, and $D_{s1}(2536)^\pm$ mass and mass difference measurements.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
2112.2±0.4 OUR FIT				
2106.6±2.1±2.7	[1] BLAYLOCK	87	MRK3	$e^+e^- \to D_s^\pm \gamma X$

[1] Assuming $D_s^\pm$ mass $= 1968.7 \pm 0.9$ MeV.

The width of D_s^* is still not determined!

$D_s^{*\pm}$ WIDTH

VALUE (MeV)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
< 1.9	90	GRONBERG	95	CLE2	$e^+ e^-$
< 4.5	90	ALBRECHT	88	ARG	$E_{cm}^{ee} = 10.2$ GeV
• • • We do not use the following data for averages, fits, limits, etc. • • •					
< 4.9	90	BROWN	94	CLE2	$e^+ e^-$
<22	90	BLAYLOCK	87	MRK3	$e^+ e^- \to D_s^{\pm} \gamma X$

The dominant decay modes of the D_s^* are the radiative decay $D_s^* \to D_s\gamma$ ($M1$ transition) and the isospin-violating decay $D_s^* \to D_s\pi^0$.

D_s^{*+} DECAY MODES

D_s^{*-} modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D_s^+\gamma$	(93.6 ± 0.4) %
Γ_2	$D_s^+\pi^0$	(5.77 ± 0.35) %
Γ_3	$D_s^+ e^+ e^-$	$(6.7 \pm 1.6) \times 10^{-3}$
Γ_4	$e^+\nu_e$	$(2.1 {}^{+1.2}_{-0.9}) \times 10^{-5}$

BESIII has provided the most precise measurement of branching fraction of $\mathcal{B}(D_s^{*+} \to D_s^+\pi^0)$ relative to $\mathcal{B}(D_s^{*+} \to D_s^+\gamma)$

$$\mathcal{B}(D_s^{*+} \to D_s^+\pi^0)/\mathcal{B}(D_s^{*+} \to D_s^+\gamma) = (6.16 \pm 0.43 \pm 0.18)\%. \quad (1)$$

Ablikim et al. Phys. Rev. D 107, 032011

The radiative decay $D_s^* \to D_s\gamma$ has been well defined in the constituent quark model and there have been a lot of studies of the radiative transition.

Aliev, Iltan, and Pak Phys. Lett. B 334, 169–174
Yu, Li, and Wang Eur. Phys. J. C 75, 243
Deng, Chen, and Deng Chin. Phys. C 38, 013103
Donald et al. Phys. Rev. Lett. 112, 212002
Fayyazuddin and Mobarek Phys. Rev. D 48, 1220–1224
Goity and Roberts Phys. Rev. D 64, 094007
Kamal and Xu Phys. Lett. B 284, 421–426
Wang et al. Phys. Rev. D 100, 016019
Cheng et al. Phys. Rev. D 49, 5857–5881
Choi J. Korean Phys. Soc. 53, 1205
Tran et al. Chin. Phys. C 48, 023103
Cheung and Hwang Eur. Phys. J. C 76, 19
Meng et al. Phys. Rev. D 109, 074511

The isospin-violating decay $D_s^* \to D_s\pi^0$ has not yet been broadly investigated. Several works have been dedicated to this issue based on the $\eta - \pi^0$ mixing.

Cho and Wise Phys. Rev. D 49, 6228–6231
Ivanov arXiv hep-ph/9805347,
Terasaki arXiv 1511.05249,
Cheung and Hwang Eur. Phys. J. C 76, 19

Recently, a heavy meson chiral perturbation theory calculation was presented with the $\mathcal{O}(p^3)$ loop corrections, and it was found that the $\mathcal{O}(p^3)$ corrections may actually be significant. Yang et al. Phys. Rev. D 101, 054019.

Our Work

The isospin-violating decay $D_s^{*+} \to D_s^+ \pi^0$ and the radiative decay $D_s^{*+} \to D_s^+ \gamma$ are studied simultaneously with the intermediate meson loops.

The isospin-violating decay $D_s^{*+} \to D_s^+ \pi^0$

$\eta - \pi^0$ mixing as the leading order and $\mathcal{D}^{(*)}\mathcal{K}^{(*)}$ rescatterings with or without the $\eta - \pi^0$ mixing as the next-to-leading order are considered.

The radiative decay $D_s^{*+} \to D_s^+ \gamma$

In the same frame work of the isospin-violating decay $D_s^{*+} \to D_s^+ \pi^0$ with the vector-meson-dominance(VMD) model.

2 Formalism

Isospin-violating through $\eta - \pi^0$ mixing

For the process $D_s^{*+} \to D_s^+ \eta$, the corresponding effective Lagrangian is

$$\mathcal{L}_{D_s^* D_s \eta} = i g_{\mathcal{D}^* \mathcal{D} \mathcal{P}} \sin \alpha_P D_s (D_s^*)_\mu \partial^\mu \eta \,. \tag{2}$$

where $\alpha_P = 40.6^\circ$ is the $\eta - \eta'$ mixing angle in the SU(3) flavor basis.
For the $\eta \to \pi^0$ process, we use the $\eta - \pi^0$ mixing angle $\theta_{\eta\pi^0}$ which is given by the leading order chiral expansion

$$\tan(2\theta_{\eta\pi^0}) = \frac{\sqrt{3}}{2} \frac{m_d - m_u}{m_s - \hat{m}}. \tag{3}$$

where $\hat{m} = (m_u + m_d)/2$. Gasser and Leutwyler Nucl. Phys. B 250, 465–516
Since $\theta_{\eta\pi^0}$ is very small, we take

$$\theta_{\eta\pi^0} \simeq \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}} \,. \tag{4}$$

Effective Lagrangians

$$\mathcal{L} = -ig_{\mathcal{D}^*\mathcal{D}\mathcal{P}}(\mathcal{D}^i\partial^\mu\mathcal{P}_{ij}\mathcal{D}_\mu^{*j\dagger} - \mathcal{D}_\mu^{*i}\partial^\mu\mathcal{P}_{ij}\mathcal{D}^{j\dagger}) + \frac{1}{2}g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}}\varepsilon_{\mu\nu\alpha\beta}\mathcal{D}_i^{*\mu}\partial^\nu\mathcal{P}^{ij}\overleftrightarrow{\partial}^\alpha\mathcal{D}_j^{*\beta\dagger}$$
$$-ig_{\mathcal{D}\mathcal{D}\mathcal{V}}\mathcal{D}_i^\dagger\overleftrightarrow{\partial}_\mu\mathcal{D}^j(\mathcal{V}^\mu)^i_j - 2f_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu V^\nu)^i_j(\mathcal{D}_i^\dagger\overleftrightarrow{\partial}^\alpha\mathcal{D}^{*\beta j} - \mathcal{D}_i^{*\beta\dagger}\overleftrightarrow{\partial}^\alpha\mathcal{D}^j) \quad (5)$$
$$+ig_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_i^{*\nu\dagger}\overleftrightarrow{\partial}_\mu\mathcal{D}_\nu^{*j}(\mathcal{V}^\mu)^i_j + 4if_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_{i\mu}^{*\dagger}(\partial^\mu\mathcal{V}^\nu - \partial^\nu\mathcal{V}^\mu)^i_j\mathcal{D}_\nu^{*j},$$

where $\mathcal{D}$ and $\mathcal{D}^*$ represent the pseudoscalar and vector charm meson fields, respectively, i.e.

$$\mathcal{D} = (D^0, D^+, D_s^+), \quad \mathcal{D}^* = (D^{*0}, D^{*+}, D_s^{*+}), \quad (6)$$

$\mathcal{P}$ and $\mathcal{V}$ are 3×3 matrices representing the pseudoscalar nonet and vector nonet meson fields

$$\mathcal{P} = \begin{pmatrix} \frac{\sin\alpha_P\eta'+\cos\alpha_P\eta+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\sin\alpha_P\eta'+\cos\alpha_P\eta-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \cos\alpha_P\eta' - \sin\alpha_P\eta \end{pmatrix} \mathcal{V} = \begin{pmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \quad (7)$$

Casalbuoni et al. Phys. Rept. 281, 145–238Cheng, Chua, and Soni Phys. Rev. D 71, 014030

For the light hadron vertices, we adopt the following effective Lagrangian:

$$\begin{aligned}\mathcal{L}_{VPP} =& ig_{VPP}\,\mathrm{Tr}[(\mathcal{P}\partial_\mu\mathcal{P}-\partial_\mu\mathcal{P}\mathcal{P})\mathcal{V}^\mu],\\ \mathcal{L}_{VVP} =& g_{VVP}\varepsilon_{\alpha\beta\mu\nu}\,\mathrm{Tr}[\partial^\alpha\mathcal{V}^\mu\partial^\beta\mathcal{V}^\nu\mathcal{P}],\\ \mathcal{L}_{VVV} =& ig_{VVV}\,\mathrm{Tr}[(\partial_\mu V_\nu-\partial_\nu V_\mu)V^\mu V^\nu].\end{aligned} \tag{8}$$

The following coupling constants are adopted:

$$\begin{aligned}&g_{D^*D^*\pi}=\frac{g_{D^*D\pi}}{\sqrt{m_D m_{D^*}}}=\frac{2g}{f_\pi}, g_{DDV}=g_{D^*D^*V}=\frac{\beta g_V}{\sqrt{2}}, f_{D^*DV}=\frac{f_{D^*D^*V}}{m_{D^*}}=\frac{\lambda g_V}{\sqrt{2}},\\ &g_{D^*D_sK}=\sqrt{\frac{m_{D_s}}{m_D}}g_{D^*D\pi},\quad g_{D_sDV}=\sqrt{\frac{m_{Ds}}{m_D}}g_{DDV},\quad \frac{g_{D_s^*D_s\eta}}{\sqrt{m_{D_s^*}m_{D_s}}}=\frac{g_{D^*D\pi}\sin\alpha_P}{\sqrt{m_{D^*}m_D}}\\ &g_{D_s^*DK}=\sqrt{\frac{m_{D_s^*}}{m_{D^*}}}g_{D^*D\pi},\quad g_V=\frac{m_\rho}{f_\pi},\quad g_{D^*D^*K}=\frac{g_{D^*DK}}{\sqrt{m_D m_{D^*}}}=\frac{2g}{f_K},\\ &g_{D^*D_s^*K^*}=\sqrt{\frac{m_{D_s^*}}{m_{D^*}}}g_{D^*D^*V},\quad f_{D^*D_s^*K^*}=\sqrt{\frac{m_{D_s^*}}{m_{D^*}}}f_{D^*D^*V},.\end{aligned} \tag{9}$$

where $g=0.59, \beta=0.9, f_\pi=132\,\mathrm{MeV}, f_K=155\,\mathrm{MeV}$and $\lambda=0.56\,\mathrm{GeV}^{-1}$.

Cheng, Chua, and Soni Phys. Rev. D 71, 014030Zhang, Wang, Li, and Zhao Phys. Rev. D 85,
Li, and Zhao Phys. Rev. Lett. 102, 172001 074015Isola et al. Phys. Rev. D 68, 114001

The relative strengths and phases of the coupling constants for vector and scalar mesons can be determined by $SU(3)$ flavor symmetry relations, and expressed by overall coupling constants g_{VVP} and g_{VPP} Cao and Zhao Phys. Rev. D 109, 093005,

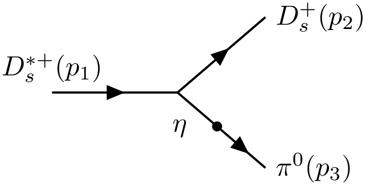
$$\begin{aligned} g_{K^{*+}K^{*-}\pi^0} &= -g_{K^{*0}\bar{K}^{*0}\pi^0} = \frac{1}{\sqrt{2}} g_{VVP}, \\ g_{K^{*0}\bar{K}^0\pi^0} &= -\, g_{K^{*0}\pi^0\bar{K}^0} = g_{\bar{K}^{*0}\pi^0 K^0} = -g_{\bar{K}^{*0}K^0\pi^0} = \frac{1}{\sqrt{2}} g_{VPP}, \\ g_{K^{*+}\pi^0 K^-} &= -\, g_{K^{*+}K^-\pi^0} = g_{K^{*-}K^+\pi^0} = -g_{K^{*-}\pi^0 K^+} = \frac{1}{\sqrt{2}} g_{VPP}. \end{aligned} \tag{10}$$

Tree and loop transition amplitudes of $D_s^{*+} \to D_s^+ \pi^0$

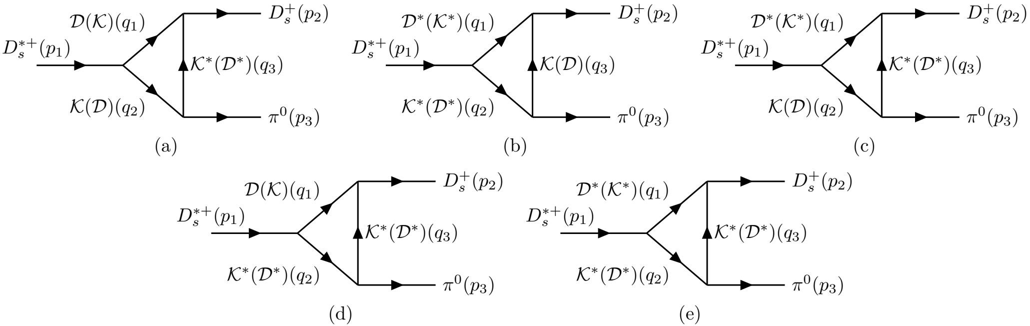
The tree-level amplitude is given by

$$i\mathcal{M}_{\text{tree}} = i g_{D_s^* D_s \eta} \varepsilon_{D_s^*} \cdot p_3 \theta_{\eta\pi^0} = i g_{\text{tree}} \varepsilon_{D_s^*} \cdot (p_2 - p_3),$$

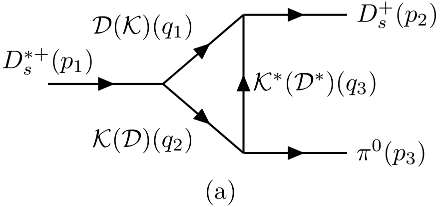
where $g_{\text{tree}} \equiv g_{D_s^* D_s \eta} \theta_{\eta\pi^0}/2$.



The loop diagrams at one-loop level without $\eta - \pi^0$ mixing:



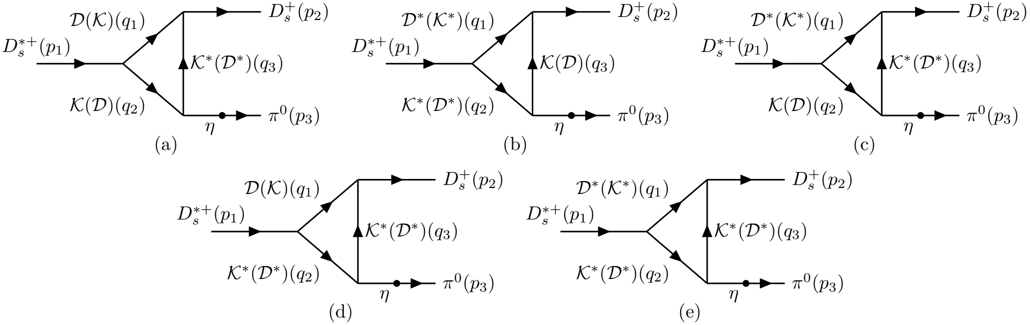
For example , for the first diagram, the transition amplitudes are given below, with a UV cutoff $\mathcal{F}(q_i^2) = \prod_i \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q_i^2} \right)$, and $\Lambda_i \equiv m_i + \alpha \Lambda_{\mathsf{QCD}}$, $\Lambda_{\mathsf{QCD}} = 220\mathrm{MeV}$.



$$i\mathcal{M}_a(\mathcal{D}, \mathcal{K}, \mathcal{K}^*) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{D}_s^* \mathcal{D} \mathcal{K}} q_2 \cdot \varepsilon^s g_{\mathcal{D}_s \mathcal{D} \mathcal{K}^*} (q_1 + p_2)_\alpha \left(g^{\alpha\beta} - \frac{q_3^\alpha q_3^\beta}{m_3^2} \right) g_{\mathcal{K}^* \mathcal{K} \pi} (p_3 + q_2)_\beta}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \mathcal{F}(q_i^2), \tag{11}$$

$$i\mathcal{M}_a(\mathcal{K}, \mathcal{D}, \mathcal{D}^*) = \int \frac{d^4 q_3}{(2\pi)^4} \frac{g_{\mathcal{D}_s^* \mathcal{D} \mathcal{K}} g_{\mathcal{D}^* \mathcal{D}_s \mathcal{K}} g_{\mathcal{D}^* \mathcal{D} \pi} q_1 \cdot \varepsilon^s q_1^\mu \left(g_{\mu\nu} - \frac{q_{3\mu} q_{3\nu}}{m_3^2} \right) p_3^\nu}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)} \mathcal{F}(q_i^2). \tag{12}$$

The loop diagrams at one-loop level with $\eta - \pi^0$ mixing:



$$
\begin{aligned}
i\mathcal{M}_a(\mathcal{D},\mathcal{K},\mathcal{K}^*,\eta) =& i\mathcal{M}_a(\mathcal{D},\mathcal{K},\mathcal{K}^*) \cdot \frac{g_{\mathcal{K}\mathcal{K}^*\eta}\theta_{\eta\pi^0}}{g_{\mathcal{K}\mathcal{K}^*\pi}}, \\
i\mathcal{M}_a(\mathcal{K},\mathcal{D},\mathcal{D}^*,\eta) =& i\mathcal{M}_a(\mathcal{K},\mathcal{D},\mathcal{D}^*) \cdot \frac{g_{\mathcal{D}^*\mathcal{D}\eta}\theta_{\eta\pi^0}}{g_{\mathcal{D}^*\mathcal{D}\pi}}.
\end{aligned}
\tag{13}
$$

Power counting of the loop amplitudes:

Assuming the intermediate mesons are close to being on-shell, the propagator reads

$$1/(t - m_3^2) \simeq -1/m_3^2 \tag{14}$$

The integrand counts:

$$(v^5/v^4) \times p_\pi \cdot p_{D_s}/m_3^2 \simeq v E_{D_s} E_\pi / m_3^2 \tag{15}$$

For the two loop amplitudes which cancel each other, we have

$$v E_{D_s} E_\pi (1/m_{K^{*\pm}}^2 - 1/m_{K^{*0}}^2) \simeq v E_\pi \delta_{K^*}/m_{K^*}^2 \simeq v \underbrace{(m_\pi/m_{K^*})}_{\text{loop power suppression}} \underbrace{(\delta_{K^*}/m_{K^*})}_{\text{Isospin breaking term}}, \tag{16}$$

with $\delta_{K^*} \equiv m_{K^{*0}} - m_{K^{*\pm}}$. The loop amplitude can be expressed as

$$i\mathcal{M}_{\text{loop}} = i g_{\text{loop}} \varepsilon_{D_s^{*+}} \cdot (p_2 - p_3). \tag{17}$$

The total amplitude can be written as

$$i\mathcal{M}_{D_s^{*+} \to D_s^+ \pi^0} = i(g_{\text{tree}} + g_{\text{loop}}) \varepsilon_{D_s^{*+}} \cdot (p_{D_s^+} - p_{\pi^0}) \equiv i g_{\text{total}} \varepsilon_{D_s^{*+}} \cdot (p_{D_s^+} - p_{\pi^0}) \ . \tag{18}$$

Tree and loop amplitudes of $D_s^{*+} \to D_s^+\gamma$ in the VMD model

The radiative decay of D_s^{*+} and its loop correction can be evaluated in the same framework

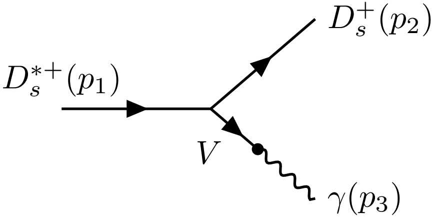
$$i\mathcal{M}^{\gamma}_{\text{tree}} = i g^{\gamma}_{\text{tree}}(\gamma)\varepsilon_{\mu\nu\alpha\beta}p_1^{\mu}p_3^{\nu}\varepsilon_1^{\alpha}\varepsilon_3^{\beta} \ , \qquad (19)$$

where g^{γ}_{tree} can be calculated using the VMD model,

$$g^{\gamma}_{\text{tree}} = i g_{D_s^* D_s V}\frac{e m_V^2}{f_V}G_V \qquad (20)$$

with

$$G_V \equiv \frac{-i}{p_\gamma^2 - m_V^2 + i m_V\Gamma_V} = \frac{-i}{-m_V^2 + i m_V \Gamma_V}. \qquad (21)$$



where $V=\rho,\omega,\phi$, and e/f_V can be determined by experimental data of $\Gamma_{V\to e^+e^-}$:

$$\frac{e}{f_V} = \left[\frac{3\Gamma_{V\to e^+e^-}}{2\alpha_e|\mathbf{p}_e|}\right]^{\frac{1}{2}} \qquad (22)$$

To match the coupling constants in HQEFT, we have

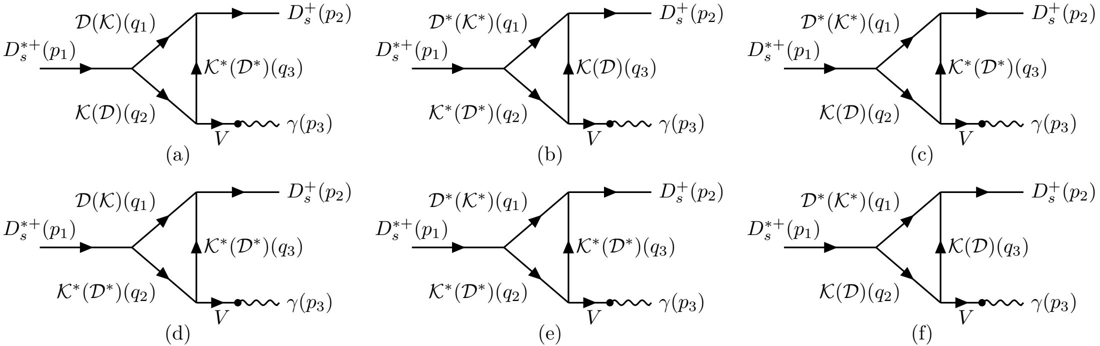
$$g_{D_s^* D_s V} = 4 f_{D_s^* D_s V}. \tag{23}$$

For the tree level $V = \phi,\ J/\psi$, the coupling constant can be extracted:

$$g_{D_s^* D_s \gamma} = i\left(g_{D_s^* D_s \phi} \frac{e m_\phi^2}{f_\phi} G_\phi + g_{D_s^* D_s \psi} \frac{e m_\psi^2}{f_\psi} G_\psi \right), \tag{24}$$

with $g_{D_s^* D_s \psi} = 2 g_{J\psi D\bar{D}} / \tilde{M}, \tilde{M} = \sqrt{M_{D_s^*} M_{D_s}}$
Zhang, Li, and Zhao Phys. Rev. Lett. 102, 172001.

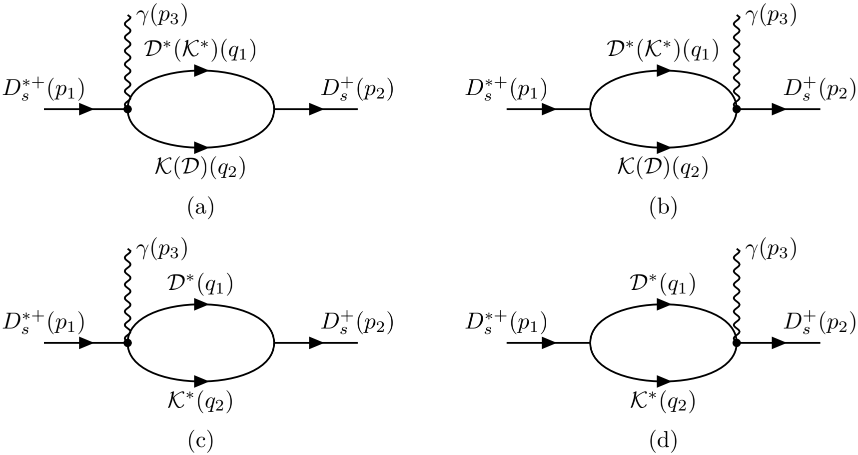
Loop amplitudes based on the VMD:



$$i\mathcal{M}_a(\mathcal{D},\mathcal{K},\mathcal{K}^*,\gamma)=\int\frac{d^4q_3}{(2\pi)^4}\frac{g_{\mathcal{K}\mathcal{K}^*\gamma}\,g_{\mathcal{D}_s^*\mathcal{D}\mathcal{K}}\,g_{\mathcal{D}_s\mathcal{D}\mathcal{K}^*}\,q_2\cdot\varepsilon^s(q_1+p_2)_\alpha\left(g^{\alpha\beta}-\frac{q_3^\alpha q_3^\beta}{m_3^2}\right)\varepsilon_{\beta\sigma\mu\nu}\varepsilon_3^\sigma q_3^\mu p_3^\nu}{(q_1^2-m_1^2)(q_2^2-m_2^2)(q_3^2-m_3^2)}\mathcal{F}(q_i^2),$$

$$i\mathcal{M}_a(\mathcal{K},\mathcal{D},\mathcal{D}^*,\gamma)=\int\frac{d^4q_3}{(2\pi)^4}\frac{g_{\mathcal{D}_s^*\mathcal{D}\mathcal{K}}\,g_{\mathcal{D}^*\mathcal{D}_s\mathcal{K}}\,g_{\mathcal{D}^*\mathcal{D}\gamma}\,q_1\cdot\varepsilon^s q_1^\mu\left(g_{\mu\nu}-\frac{q_{3\mu}q_{3\nu}}{m_3^2}\right)\varepsilon_{\nu\sigma\alpha\beta}\varepsilon_3^\sigma q_3^\alpha p_3^\beta}{(q_1^2-m_1^2)(q_2^2-m_2^2)(q_3^2-m_3^2)}\mathcal{F}(q_i^2).$$

Contact diagrams induced by the requirement of gauge invariance:



(a) ✓, (b) ×, (c) ×, (d) ×

Meson-photon coupling constants in the VMD model:

$$\begin{aligned} g_{K^+K^+\gamma} =& i(g_{\rho^0 K^+K^-}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega K^+K^-}\frac{em_{\omega}^2}{f_{\omega}}G_{\omega} + g_{\phi K^+K^-}\frac{em_{\phi}^2}{f_{\phi}}RG_{\phi})\ , \\ g_{K^0K^0\gamma} =& i(g_{\rho^0 K^0\bar{K}^0}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega K^0\bar{K}^0}\frac{em_{\omega}^2}{f_{\omega}}G_{\omega} + g_{\phi K^0\bar{K}^0}\frac{em_{\phi}^2}{f_{\phi}}RG_{\phi})\ , \\ g_{D^+D^+\gamma} =& i(g_{\rho^0 D^+D^-}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega D^+D^-}\frac{em_{\omega}^2}{f_{\omega}}G_{\omega} + g_{\phi D^+D^-}\frac{em_{\phi}^2}{f_{\phi}}RG_{\phi})\ , \\ g_{D^0D^0\gamma} =& i(g_{\rho^0 D^0\bar{D}^0}\frac{em_{\rho^0}^2}{f_{\rho^0}}G_{\rho^0} + g_{\omega D^0\bar{D}^0}\frac{em_{\omega}^2}{f_{\omega}}G_{\omega} + g_{\phi D^0\bar{D}^0}\frac{em_{\phi}^2}{f_{\phi}}RG_{\phi})\,. \end{aligned} \tag{25}$$

where $R = 0.8$ is the $SU(3)$ flavor-symmetry-breaking parameter for process involving the strange quark pair.
The total radiative decay amplitude can be expressed as

$$i\mathcal{M}_{D_s^{*+}\to D_s^+\gamma} = i(g^{\gamma}_{\text{tree}} + g^{\gamma}_{\text{loop}})\varepsilon_{\mu\nu\alpha\beta}p_1^{\mu}p_3^{\nu}\varepsilon_1^{\alpha}\varepsilon_3^{\beta} \equiv ig^{\gamma}_{\text{total}}\varepsilon_{D_s^{*+}}\cdot(p_{D_s^+} - p_{\pi^0})\ . \tag{26}$$

3 Results and Discussions

The numerical values of the coupling constants

Table: The values of the VPP coupling constants.

Coupling Constant	$g_{D_s^{*+}D^0K}$	$g_{D_s^+D^0K^*}$	$g_{D^{*0}D^0\pi}$	$g_{D^{*-}D^+\pi}$	$g_{D_s^{*+}D^+K}$	$g_{D^{*0}D_s^+K}$	$g_{D_s^+D^+K^*}$	$g_{D^{*+}D_s^+K}$	g_{VPP}
Numerical Value	18.40	3.84	17.29	-17.33	18.42	17.77	3.84	17.78	4.18

Table: The values of the VVP coupling constants.

Coupling Constant	$g_{D_s^{*+}D^{*0}K}$	$f_{D^{*0}D_s^+K^*}$	$f_{D_s^*D^0K^*}$	$g_{D^{*0}\bar{D}^{*0}\pi^0}$	$g_{D^{*+}D^{*-}\pi^0}$	g_{VVP}
Numerical Value(GeV^{-1})	7.81	2.38	2.47	8.94	-8.94	7.93

Table: The values of the VVV coupling constants.

Coupling Constant	g_{VVV}	$g_{D_s^{*+}D^{*0}K^*}$	$f_{D_s^*D^{*0}K^*}$
Numerical Value	4.47	3.83	4.79

Table: The values of the electromagnetic coupling constants in VMD model.

Coupling Constant	$g_{K^{*+}K^+\gamma}$	$g_{K^{*0}K^0\gamma}$	$g_{D^{*0}D^0\gamma}$	$g_{D^{*+}D^+\gamma}$
Numerical Value(GeV^{-1})	$-0.288+0.063i$	$0.369+0.062i$	$-0.383-0.082i$	$0.492+0.082i$
Coupling Constant	$g_{K^{*+}K^{*+}\gamma}$	$g_{K^{*0}K^{*0}\gamma}$	$g_{D^{*0}D^{*0}\gamma}(f_{D^{*0}D^{*0}\gamma})$	$g_{D^{*+}D^{*+}\gamma}(f_{D^{*+}D^{*+}\gamma})$
Numerical Value	$-0.162-0.035i$	$0.208+0.034i$	$-0.139-0.031i(-0.695-0.152i)$	$0.178+0.030i(0.892+0.150i)$
Coupling Constant	$g_{K^+K^+\gamma}$	$g_{K^0K^0\gamma}$	$g_{D^0D^0\gamma}$	$g_{D^+D^+\gamma}$
Numerical Value	$-0.288+0.063i$	$0.369+0.062i$	$-0.383-0.082i$	$0.492+0.082i$

Numerical results

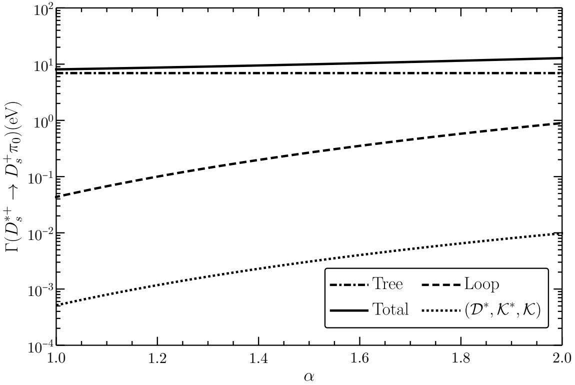


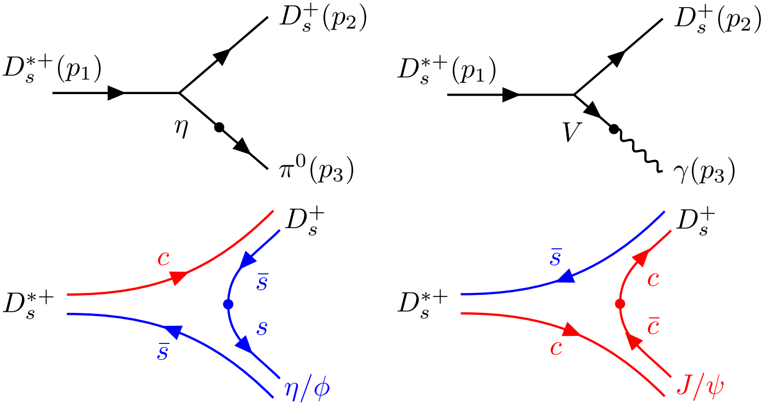
Table: Comparisons of the partial decay width of $D_s^{*+} \to D_s^+ \pi^0$. The decay widths are in the uints of eV. The uncertainties of our result are given by $\alpha = 1.5 \pm 0.15$.

	CM[a]	χPT[b]	Our work
$\Gamma(D_s^{*+} \to D_s^+ \pi^0)$	277^{+28}_{-26}	$8.1^{+3.0}_{-2.6}$	$9.92^{+0.76}_{-0.66}$

[a] Cheung and Hwang Eur. Phys. J. C 76, 19

[b] Yang et al. Phys. Rev. D 101, 054019

Connection and difference between $D_s^{*+} \to D_s^+ \pi^0$ and $D_s^{*+} \to D_s^+ \gamma$



Cancellation occurs between the ϕ and J/ψ term.

The main uncertainties come from the $g_{J/\psi D\bar{D}}$ coupling with $g_{J/\psi D\bar{D}} = 7.0 \sim 7.5$.
Precise measurement of D_s^{*+} width can provide a strong constraint on this coupling.

Table: Experimentally measured branching ratios and relative branching ratios of $D_s^{*+} \to D_s^+ \pi^0$ and $D_s^{*+} \to D_s^+ \gamma$.

	$\mathrm{BR}(D_s^{*+} \to D_s^+ \gamma)$	$\mathrm{BR}(D_s^{*+} \to D_s^+ \pi^0)$	$\mathrm{BR}(D_s^{*+} \to D_s^+ \pi^0)/\mathrm{BR}(D_s^{*+} \to D_s^+ \gamma)$
PDG [2]	$(93.6 \pm 0.4)\%$	$(5.77 \pm 0.35)\%$	$(6.2 \pm 0.4)\%$
BESIII [3]	$(93.54 \pm 0.38 \pm 0.22)\%$	$(5.76 \pm 0.38 \pm 0.16)\%$	$(6.16 \pm 0.43 \pm 0.18)\%$

Table: Contributions of the tree diagram, loop diagrams, and the combination of tree and loop diagrams to the partial decay width of $D_s^{*+} \to D_s^+ \gamma$ with $\alpha = 1.0$, 1.35, 1.5, 1.65 and 2.0 in unit of eV and $g_{J/\psi D\bar{D}} = 7.23$.

α	1.0	1.35	1.5	1.65	2.0
Γ_{tree}	153.89	153.89	153.89	153.89	153.89
Γ_{loop}	0.54	2.22	3.59	5.55	13.14
Γ_{total}	155.30	157.84	159.69	162.16	171.15

[2] Navas et al. Phys. Rev. D 110, 030001
[3] Ablikim et al. Phys. Rev. D 107, 032011

Table: The partial decay widths of $D_s^{*+} \to D_s^+\pi^0$ and $D_s^{*+} \to D_s^+\gamma$, as well as the total width of D_s^{*+} as the sum of both, for $\alpha = 1.5 \pm 0.15$ with $g_{J/\psi D\bar{D}}$ fixed at 7.23, and for $g_{J/\psi D\bar{D}} = 7.23 \pm 0.06$.

	$\Gamma(D_s^{*+} \to D_s^+\pi^0)$	$\Gamma(D_s^{*+} \to D_s^+\gamma)$	$\Gamma_{\text{total}}(D_s^{*+})$
$\alpha = 1.5 \pm 0.15,\ g_{J/\psi D\bar{D}} = 7.23$	$9.92^{+0.76}_{-0.66}$	$159.7^{+2.5}_{-1.8}$	$169.6^{+2.6}_{-2.0}$
$\alpha = 1.5 \pm 0.15,\ g_{J/\psi D\bar{D}} = 7.23 \pm 0.06$	$9.92^{+0.76}_{-0.66}$	160^{+13}_{-12}	170^{+13}_{-12}

Table: The partial decay width of $D_s^{*+} \to D_s^+\gamma$ and the total width of D_s^{*+} obtained using the experimentally measured relative branching ratio and the calculated width of $D_s^{*+} \to D_s^+\pi^0$ with $\alpha = 1.5$.

	$\Gamma(D_s^{*+} \to D_s^+\gamma)$	$\Gamma_{\text{total}}(D_s^{*+})$
PDG[4]	$160^{+10}_{-10}\,\text{eV}$	$172^{+10}_{-10}\,\text{eV}$
BESIII[5]	$161^{+16}_{-16}\,\text{eV}$	$172^{+16}_{-16}\,\text{eV}$

[4] Navas et al. Phys. Rev. D 110, 030001

[5] Ablikim et al. Phys. Rev. D 107, 032011

4 Summary and Outlook

Summary and Outlook

- Isospin-violating decay $D_s^* \to D_s\pi^0$ and radiative decay $D_s^* \to D_s\gamma$ are studied in the same framework.
- Loop corrections are crucial for the $D_s^* \to D_s\pi^0$ decay, while they are negligible in the radiative decay.
- With the experimental data for the branching ratio fraction, we obtain a better constraint on the coupling $g_{J/\psi D\bar{D}}$.
- These mechanisms can also help us understand other isospin-violating decay processes[6].
- The decay $D_s^* \to D_s\pi_0$ is near the threshold of D_s^*, understanding this decay channel will aid in comprehending other near-threshold dynamics.
- Future precise measurement of the total width of D_s^* at BESIII is strongly recommended.

[6] Study of $B_c(1P)^+ \to B_c^{(*)+}\pi^0$ Wang and Zhao arXiv hep-ph, 2507.19952

Thanks