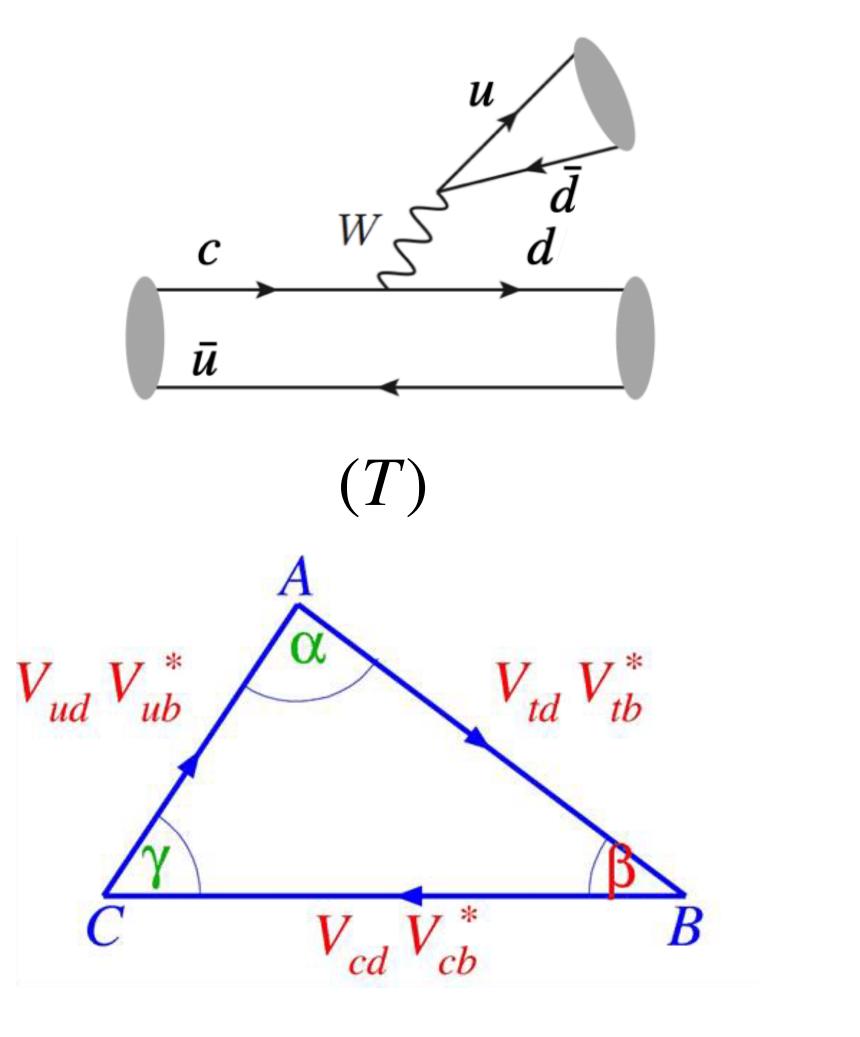
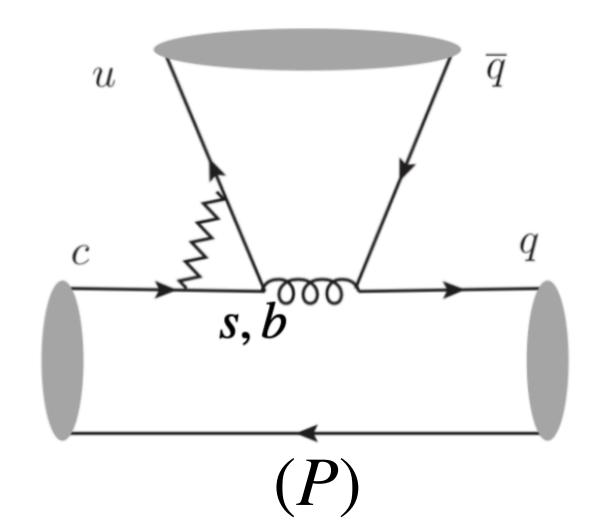


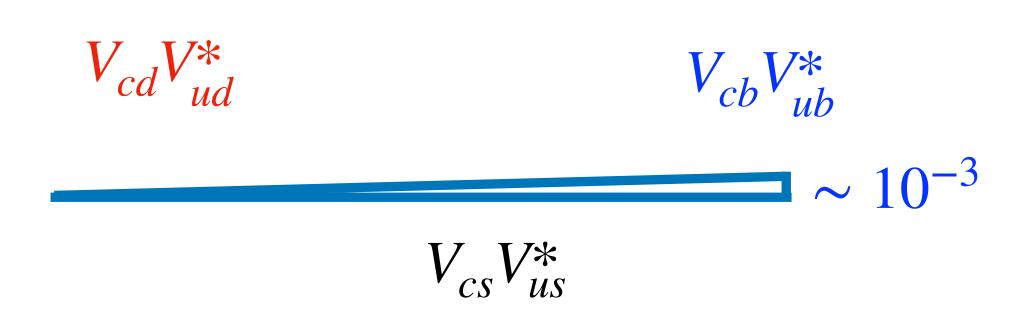
$$a_{CP} \approx 1.5 \times 10^{-3} \times \text{Im}(P/T)$$

$$A(D^0 \to \pi^+ \pi^-) = V_{cd}^* V_{ud} T + V_{cb}^* V_{ub} P$$



CKM triangle for $b \rightarrow d$





CKM triangle for $c \rightarrow u$

See Fu-Sheng's talk for details.

• Charming physics - CP violation $a_{CP} \approx 1.5 \times 10^{-3} \times \text{Im}(P/T)$

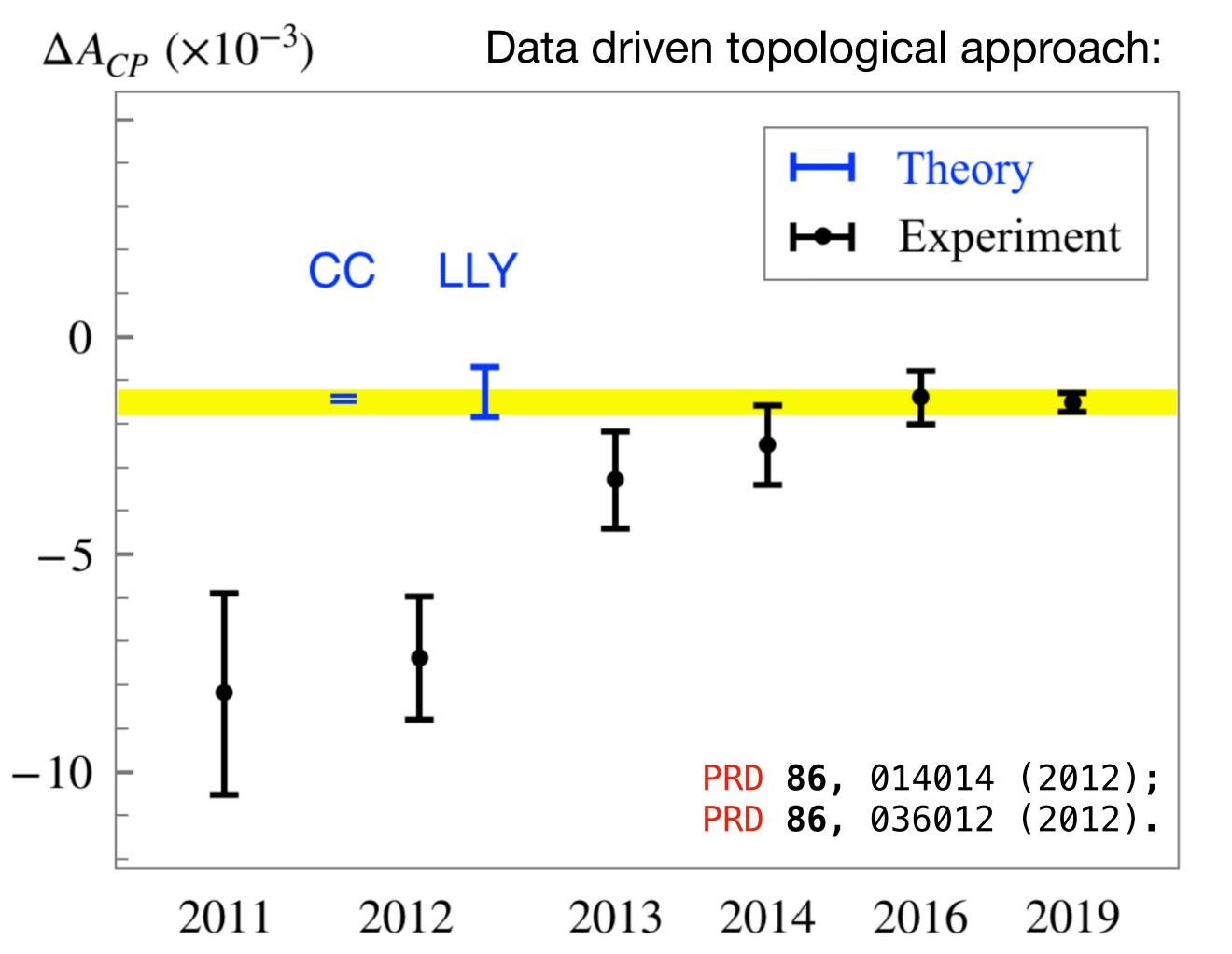
$$a_{CP} \approx 1.5 \times 10^{-3} \times \text{Im}(P/T)$$

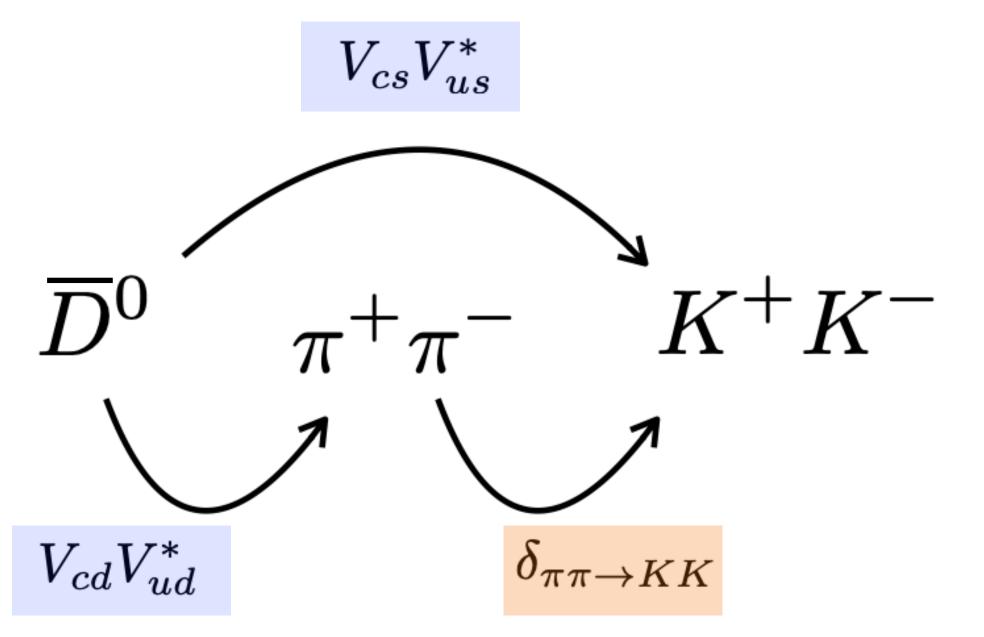
$$a_{CP}(D^0 \to K^+K^-) - a_{CP}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$



PRL **122**, 211803 (2019);

• $|P/T| \approx 1$, an order larger than naive expectation!





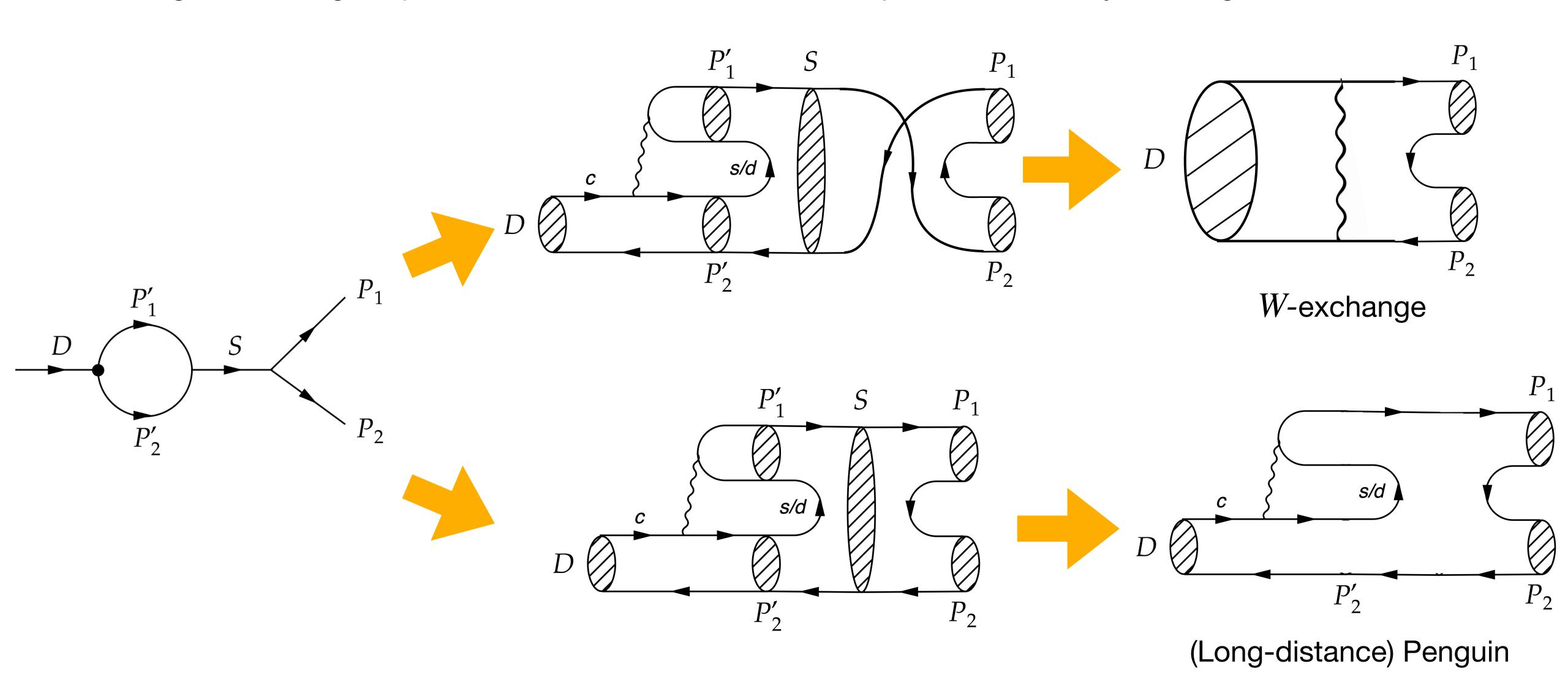
$$a_{CP} \propto \sin \delta_{\text{weak}} \sin \delta_{\text{strong}}$$

Two necessary and sufficient conditions for CPV:

CKM phases and strong phases.

PRL 131, 051802 (2023).

Cheng and Chiang conjectured P=E in 2012, which was proved in 2021 by Di Wang.



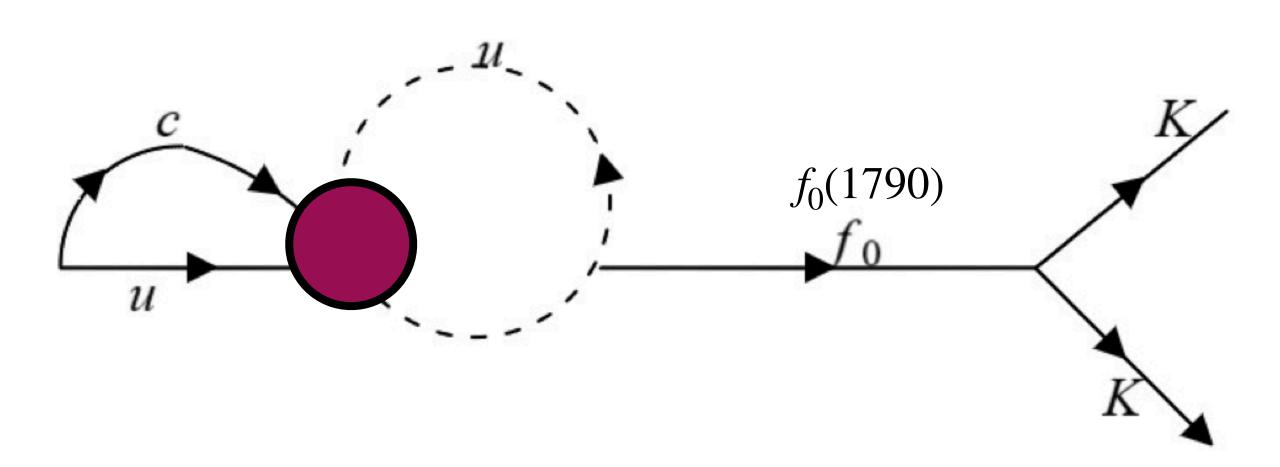
PRD 86, 014014 (2012), 2111.11201, 2505.07150.

Reasons to go beyond charmed mesons:

$$a_{CP}^{KK} = (7.7 \pm 5.7) \times 10^{-4}, \quad a_{CP}^{\pi\pi} = (23.2 \pm 6.1) \times 10^{-4}$$

PRL **131**, 091802 (2023)

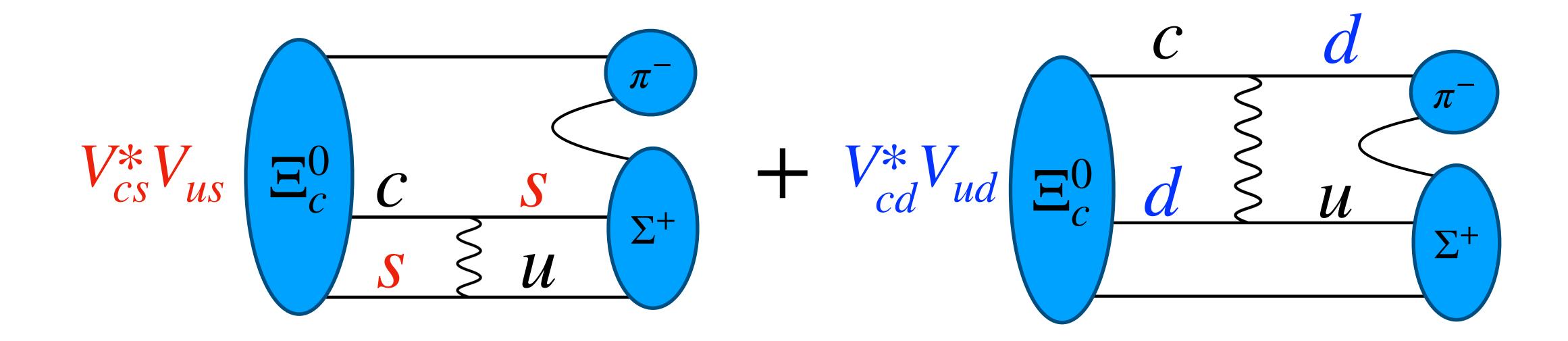




PRD 81, 074021 (2010), PLB 825, 136855 (2022).

- 1. Relative sign of a_{CP}^{KK} and $a_{CP}^{\pi\pi}$ contradicts to the theoretical expectations.
- 2. f_0 might be a glueball which mainly decays to kaons. Leading order amplitude $\propto m_{s}$.
- 3. Its mass is too close to D meson, enhancing SU(3) breaking effects from mass splitting.

Reasons to go beyond charmed mesons:



- 4. Quark structure provides CKM phase at tree level.
- 5. Unlike $D^0 \to h^+h^-$, CP-even phase shifts in baryon decays can be directly measured.

Very important inputs and driven force in the study of charm baryons!



Experimental status of charmed baryon decays

2023: The *first* measurement of CP violation in charmed baryon two-body decays

Sci. Bull. **68**, 583-592 (2023)

$$A_{CP}(\Lambda_c^+ \to \Lambda K^+) = 0.021 \pm 0.026$$



* The most precise CP asymmetries in branching fractions by far in charmed baryons.

• 2024: Measurements of the strong phase in $\Lambda_c^+ o \Xi^0 K^+$

PRL **132**, 031801 (2024)

$$\delta_P - \delta_S = -1.55 \pm 0.27(+\pi), \quad \alpha = 0.01 \pm 0.16$$



* CP even and Cabibbo-favored, but very important to studies of CP violation!

See Pei-Rong's talk.

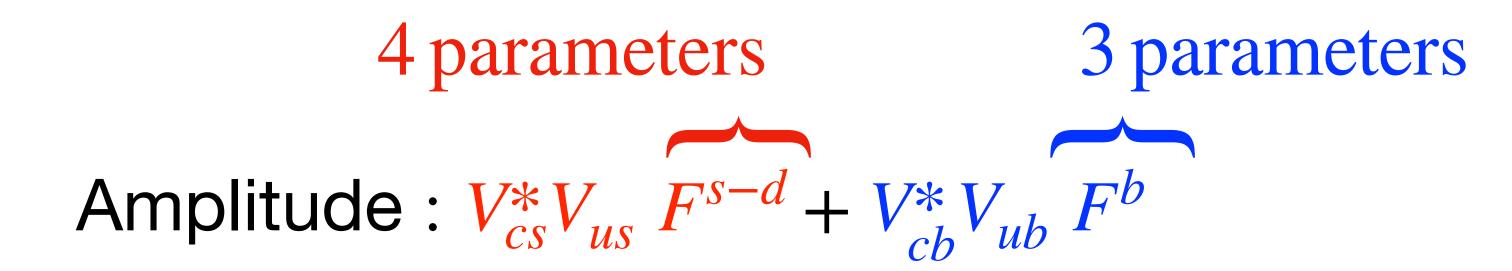
2024: Measurements of *strong phases* in $\Lambda_c^+ \to \Lambda \pi^+, \Lambda K^+$

PRL 133, 261804 (2024)

$$(\beta_{\pi}, \beta_{K}) = (0.368 \pm 0.019 \pm 0.008, 0.35 \pm 0.12 \pm 0.04).$$



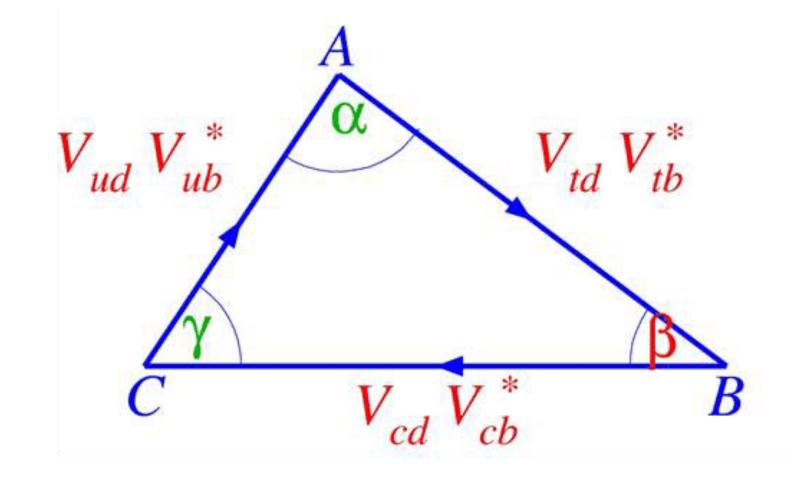
* Confirmed the discovery of large strong phases in charmed baryon decays.



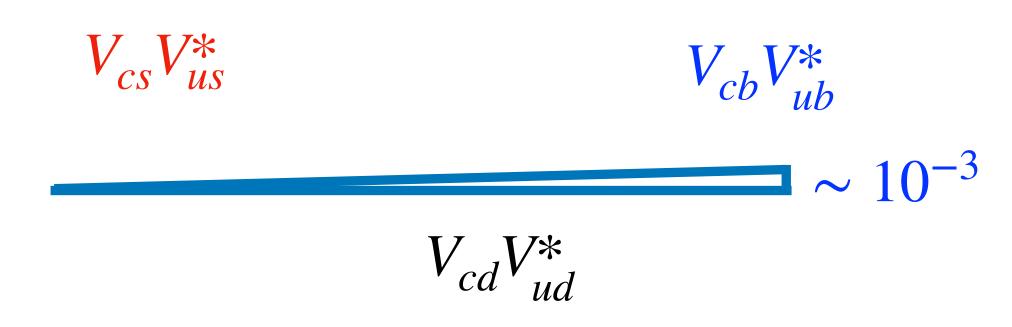
Do not need to consider F^b in studying CP-even quantities.



 F^b cannot be determined with CP-even quantities.



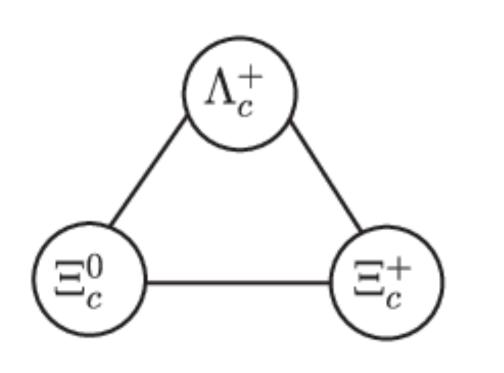
CKM triangle for
$$b \rightarrow d$$



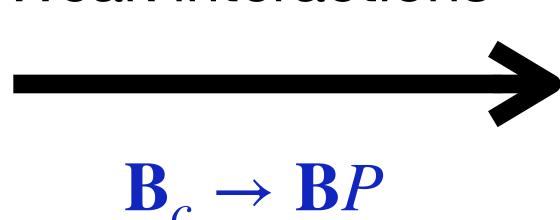
CKM triangle for $c \rightarrow u$

Focus on the leading CKM contributions, i.e. $V_{ch}^* V_{ub} = 0$.

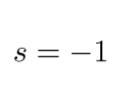
PRD 93, 056008 (2016), NPB 956, 115048 (2020) JHEP 09, 035 (2022), JHEP 03, 143 (2022) ...



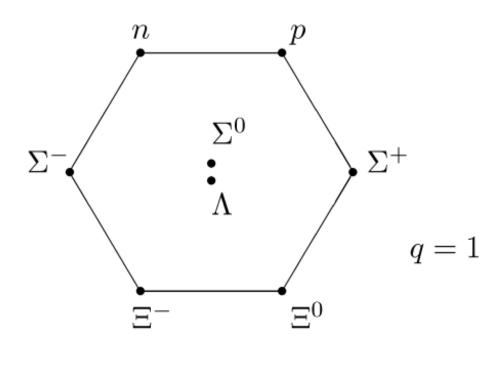
Weak interactions



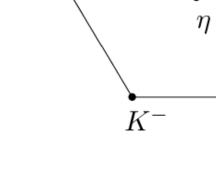
$$s = 0$$



$$s = -2$$



q = -1

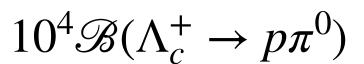


$$q = -1$$
 $q = 0$



$$\alpha(\Lambda_c^+ \to pK_S^0)$$







$$10^4 \mathcal{B}(\Lambda_c^+ \to p\pi^0)$$

$$10^3 \mathcal{B}(\Lambda_c^+ \to \Lambda K_S^0 \pi^+)$$



$$10^3 \mathcal{B}(\Xi_c^0 \to \Xi^0 \eta)$$



$$10^3 \mathcal{B}(\Xi_c^0 \to \Xi^0 \eta')$$

PDG (2023)

$$0.18 \pm 0.45$$

None

None

None

Theory (2023)

$$-0.40 \pm 0.49$$

$$1.6 \pm 0.2$$

$$1.97 \pm 0.38$$

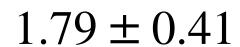
$$2.94 \pm 0.97$$

$$5.66 \pm 0.93$$

Data (2024)

q = 0

$$-0.744 \pm 0.015$$



$$1.73 \pm 0.28$$

$$1.6 \pm 0.5$$

$$1.2 \pm 0.4$$









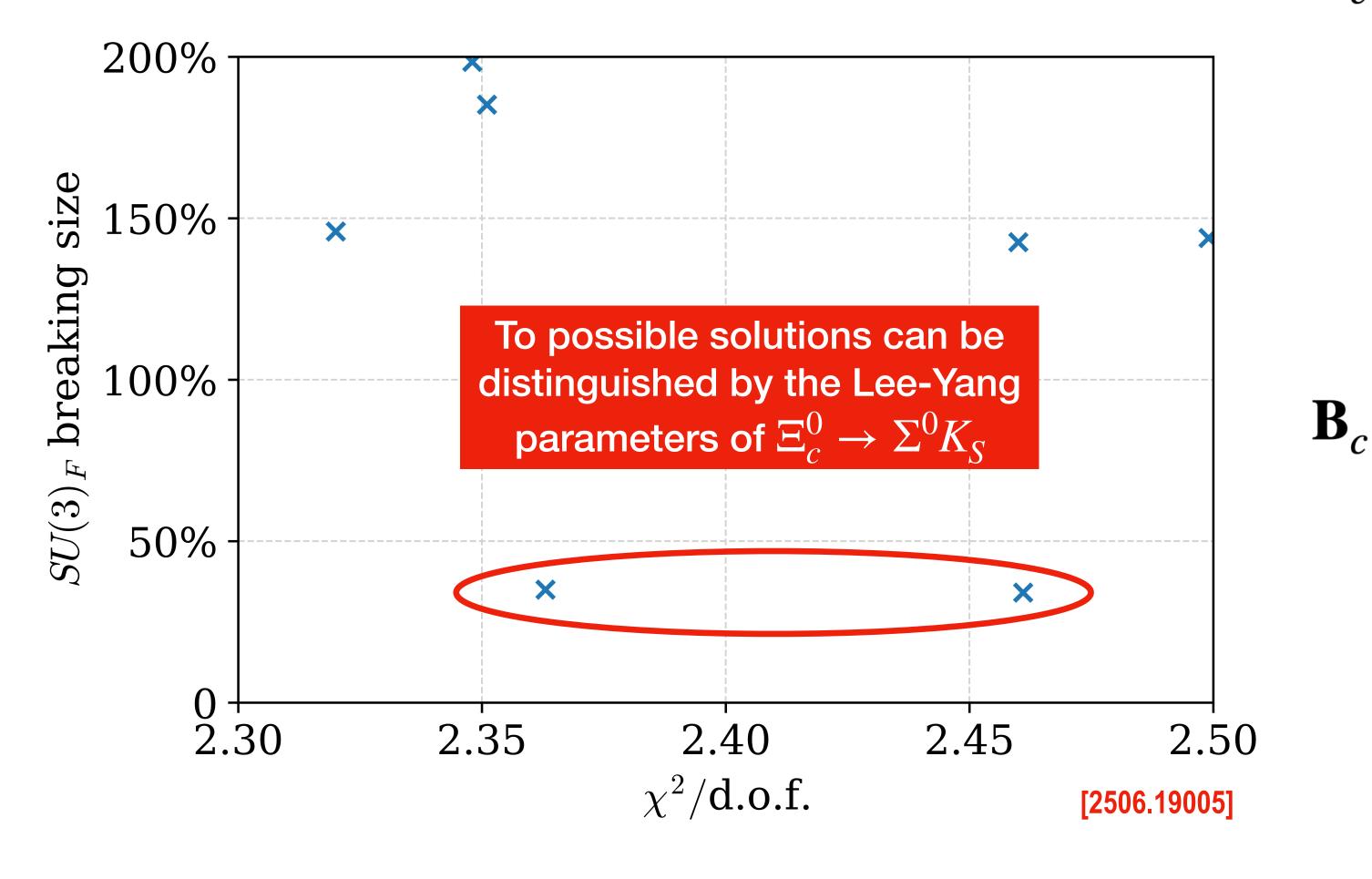


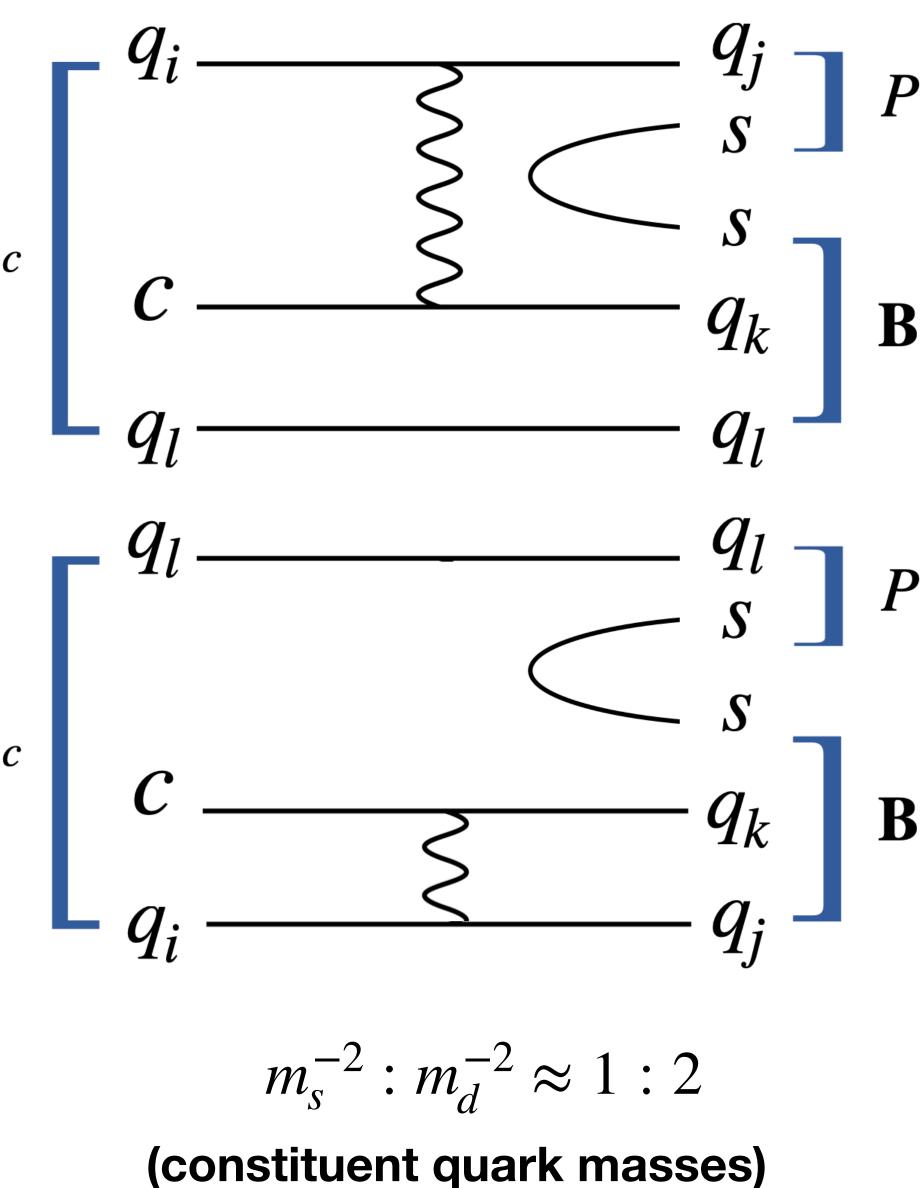
There are some shortcomings in $SU(3)_F$ symmetry approach.



The $SU(3)_F$ is an approximate symmetry with errors in 10^{-1} .

We propose a new scenario that incorporates the $SU(3)_F$ breaking of strange quark pair production from the vacuum.







The large χ^2 is mainly contributed by two channels:

	PDG	$SU(3)_F$ conserved	$SU(3)_F$ broken	
$10^2 \mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)$	1.43 ± 0.32	2.72 ± 0.09	2.9 ± 0.1	
$10^2 \mathcal{B}(\Xi_c^+ \to \Xi^- \pi^+ \pi^+)$	2.9 ± 1.3	6.82 ± 0.36	6.0 ± 0.4	

Both of them are the normalized channels in $\Xi_c^{0,+}$! It is important for a second group to crosscheck. \longleftrightarrow

Same underestimations occurs in $\Xi_c^0 \to \Xi^- \mathcal{E}^+ \nu_{\ell}$.

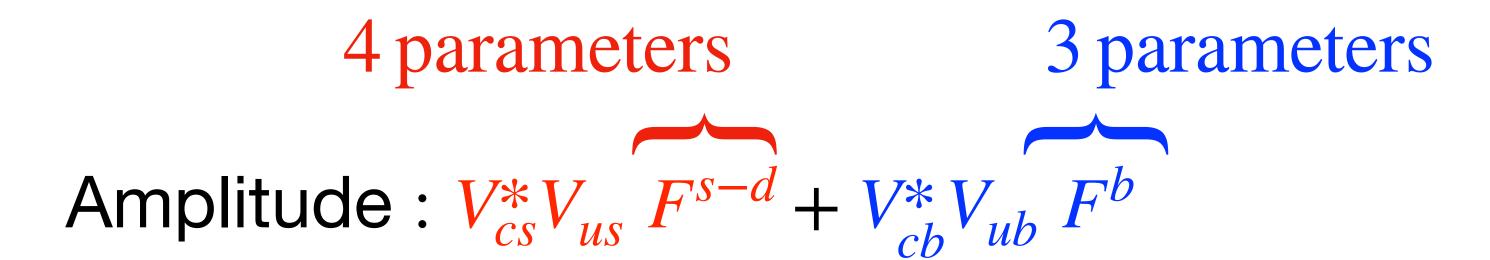
PDG
$$SU(3)_F$$
 Lattice Lattice 1.05 ± 0.20 $2.12 \pm 0.13*$ Lattice 1.05 ± 0.20 2.38 ± 0.44 3.58 ± 0.12

*Using $\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+) = (2.9 \pm 0.1) \%$

[2110.04179]

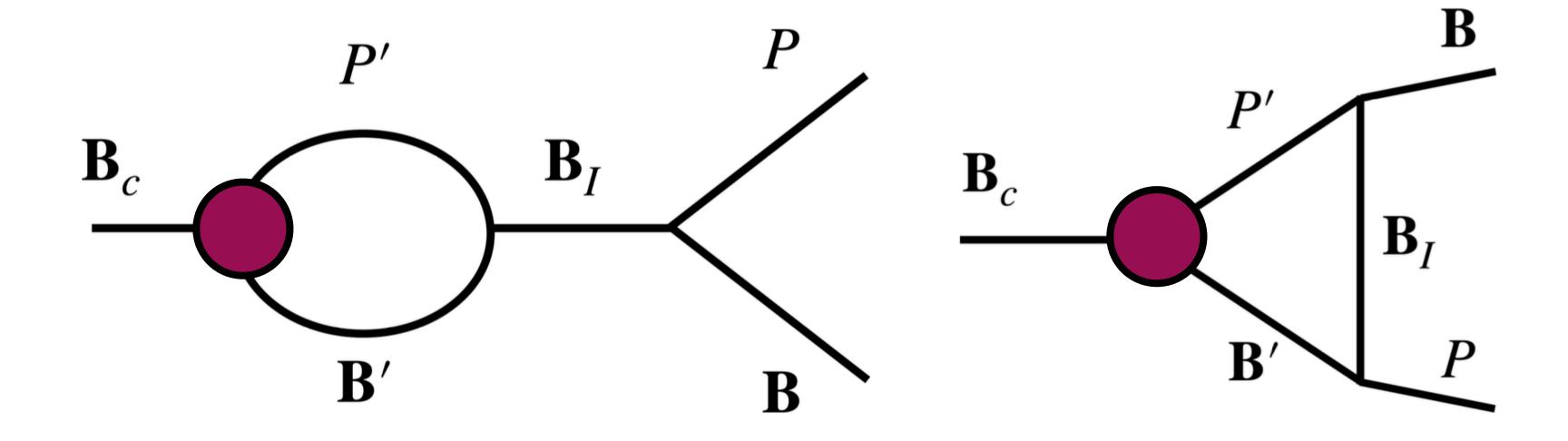
[2103.07064]

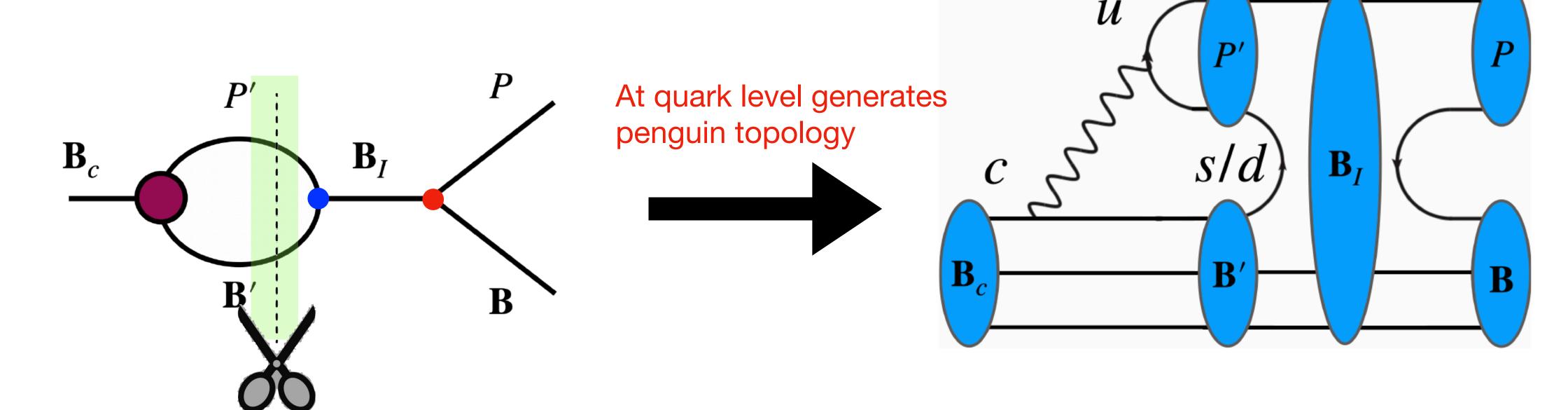
[2504.07302]



Four parameters have been extracted from the CP-even data.

Three parameters need to be determined with models.

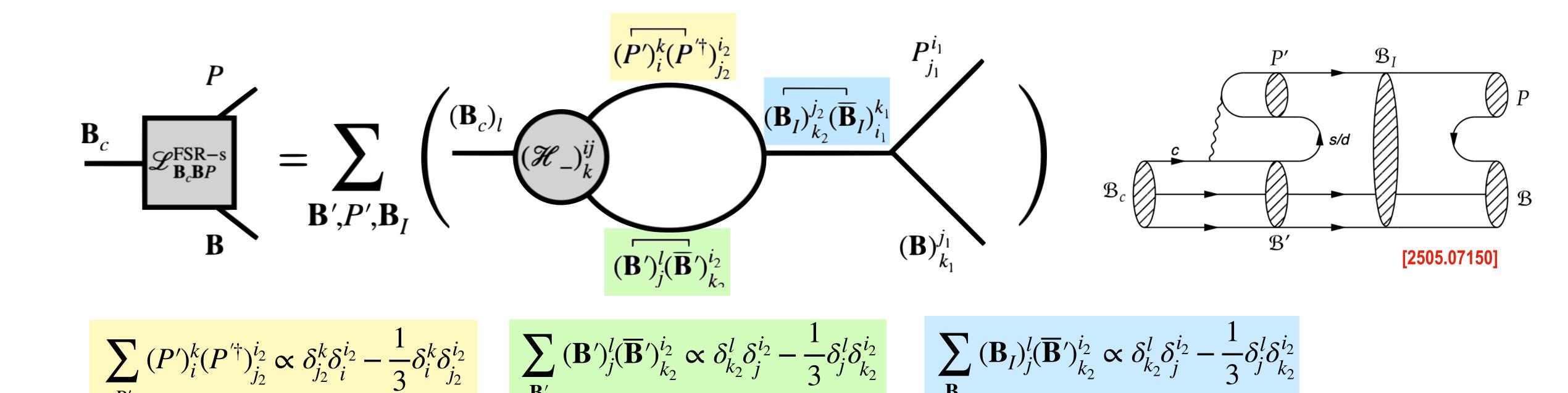




Generate necessary strong phase!

$$\langle \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\mathrm{FSR-s}} \rangle = \sum_{\mathbf{B}_{l},\mathbf{B}',P'} \overline{u}_{\mathbf{B}} \left(\int \frac{d^{4}q}{(2\pi)^{4}} \frac{g_{\mathbf{B}_{l}}^{\mu} p_{\mathbf{B}'}^{\mu} + m_{I}}{p_{\mathbf{B}_{c}}^{2} - m_{I}^{2}} \frac{q^{\mu} \gamma_{\mu} + m_{\mathbf{B}'}}{q^{2} - m_{\mathbf{B}'}^{2}} \frac{1}{(q - p_{\mathbf{B}_{c}})^{2} - m_{P'}^{2}} F_{\mathbf{B}_{c}\mathbf{B}'P'}^{\mathrm{Tree}} \right) u_{\mathbf{B}_{c}}$$

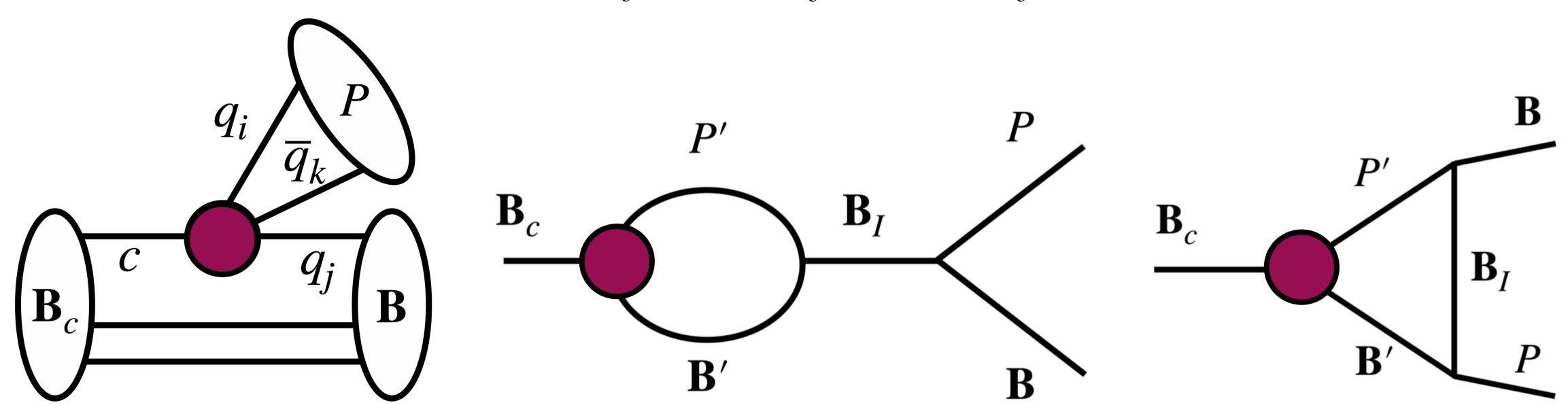
 $F_{\mathbf{B},\mathbf{B}'P'}^{\mathrm{Tree}}$ and $g_{\mathbf{B}_I\mathbf{B}'P'}$ depend on q^2 otherwise a cut-off has to be introduced.



Assumptions:

- 1. $\mathbf{B}_I \in \text{lowest-lying baryons of both parities.}$
- 2. The rescattering is closed, i.e. $\mathbf{B}'P'$ belong to the same $SU(3)_F$ group of $\mathbf{B}P$.

$$\mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P} = \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\text{Tree}} + \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\text{FSR-s}} + \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\text{FSR-t}}$$



Induce two parameters:

 F_V^{\pm} , including effective color number and form factors.

Induce one parameter:

 \tilde{S}^- , containing the q^2 dependencies of couplings.

Induce one parameter:

 \tilde{T}^- , containing the q^2 dependencies of couplings.

Described by f 4 complex parameters, having the same number of parameters with the $SU(3)_F$ analysis!

Amplitudes: $\frac{\lambda_s - \lambda_d}{2} \tilde{f}^{b,c,d,e} + \lambda_b \tilde{f}^{b,c,d}_3$

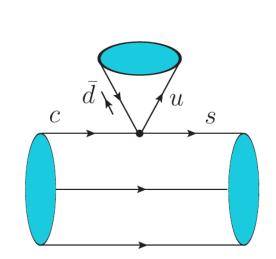
SU(3) leading \rightarrow rescattering parameters \rightarrow SU(3) suppressed

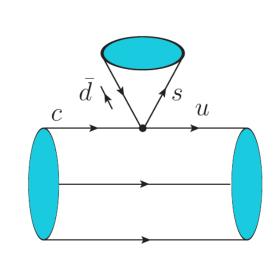
$$(\tilde{f}^b, \tilde{f}^c, \tilde{f}^d, \tilde{f}^e) \longrightarrow (\tilde{F}_V^+, \tilde{F}_V^-, \tilde{S}^-, \tilde{T}^-) \longrightarrow (\tilde{f}_3^b, \tilde{f}_3^c, \tilde{f}_3^d)$$

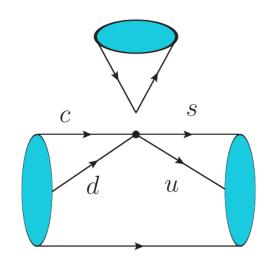
$$\tilde{f}^b = \tilde{F}_V^- - (r_- + 4)\tilde{S}^- + \sum_{\lambda = \pm} (2r_\lambda^2 - r_\lambda)\tilde{T}_\lambda^-,$$

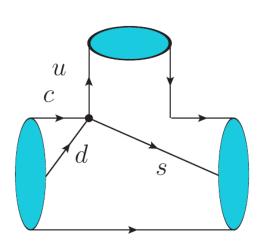
$$|\tilde{f}^c| = -r_-(r_- + 4)\tilde{S}^- + \sum_{\lambda=+} (r_\lambda^2 - 2r_\lambda + 3)\tilde{T}_\lambda^-,$$

$$|\tilde{f}^d| = |\tilde{F}_V^-| + \sum_{\lambda} (2r_{\lambda}^2 - 2r_{\lambda} - 4)\tilde{T}_{\lambda}^-, \quad |\tilde{f}^e| = \tilde{F}_V^+$$

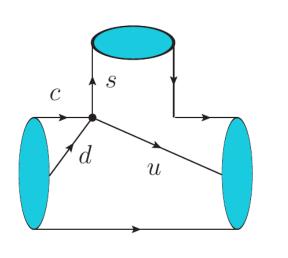


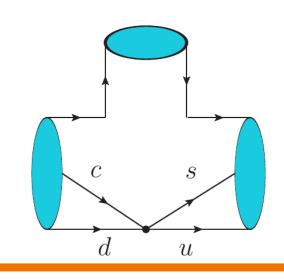


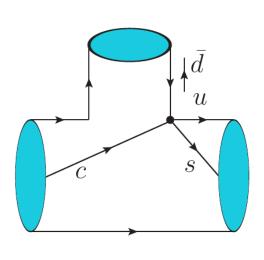




PRD **100**, 093002 (2019)



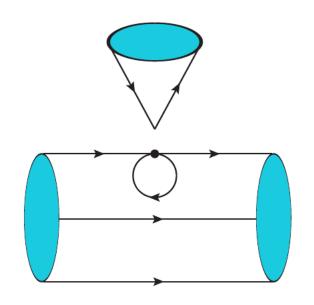


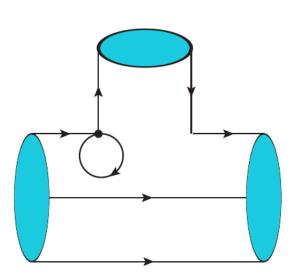


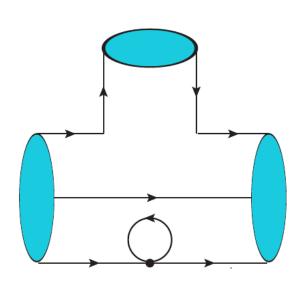
$$|\tilde{f}_{3}^{b}| = (1 - \frac{7r_{-}}{2})\tilde{S}^{-} + \sum_{\lambda = \pm} (r_{\lambda}^{2} - 5r_{\lambda}/2 + 1)\tilde{T}_{\lambda}^{-},$$

$$\frac{\tilde{f}_{3}^{c}}{6} = \frac{(r_{-}+1)(7r_{-}-2)}{6}\tilde{S}^{-} - \sum_{\lambda=\pm} \frac{r_{\lambda}^{2}+11r_{\lambda}+1}{6}\tilde{T}_{\lambda}^{-},$$

$$|\tilde{f}_3^d| = \frac{2r_- - 7r_-^2}{2}\tilde{S}^- + \sum_{i=1}^{\infty} \frac{(r_{\lambda} + 1)^2}{2}\tilde{T}_{\lambda}^- - \frac{\tilde{F}_V^+ + 2\tilde{F}_V^-}{4}.$$







Much more complicated compared to $P^{LD} = E$ in D mesons!

Rescattering, numerical results

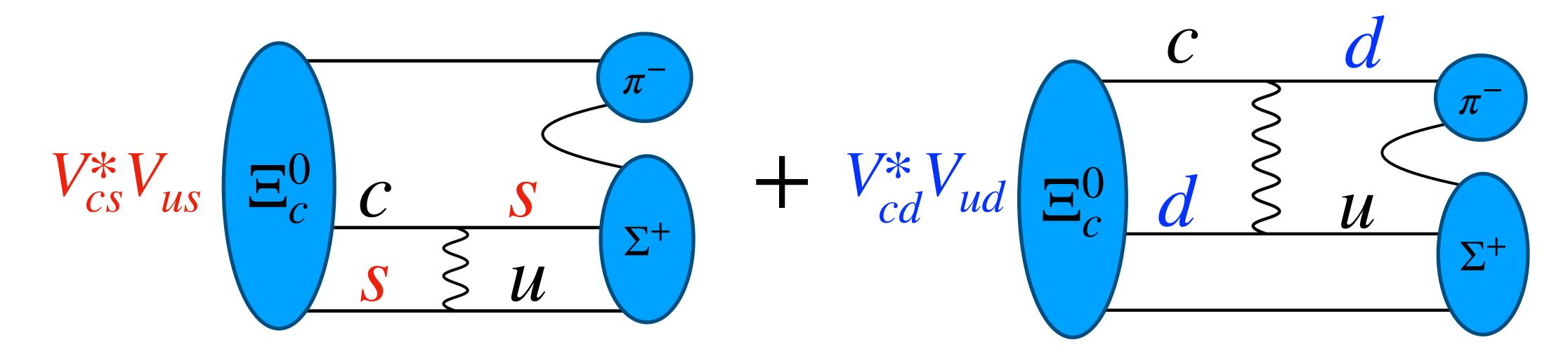
• A_{CP} in the same size with the ones in D meson!

$$A_{CP} \left(\Xi_c^0 \to \Sigma^+ \pi^- \right) = (0.71 \pm 0.16) \times 10^{-3}$$

 $A_{CP} \left(\Xi_c^0 \to pK^- \right) = (-0.73 \pm 0.19) \times 10^{-3}$

In the U-spin limit, we have that

$$A_{CP}\left(\Xi_{c}^{0}\to\Sigma^{+}\pi^{-}\right)=-A_{CP}\left(\Xi_{c}^{0}\to pK^{-}
ight).$$
 EPJC 79, 429 (2019)



Two topological diagrams are in the same size, leads to $A_{CP} \sim \left| 2 {\rm Im} (V_{cs}^* V_{us} / V_{cd}^* V_{ud}) \right| \sim 10^{-3}$.

• Rescattering, numerical results

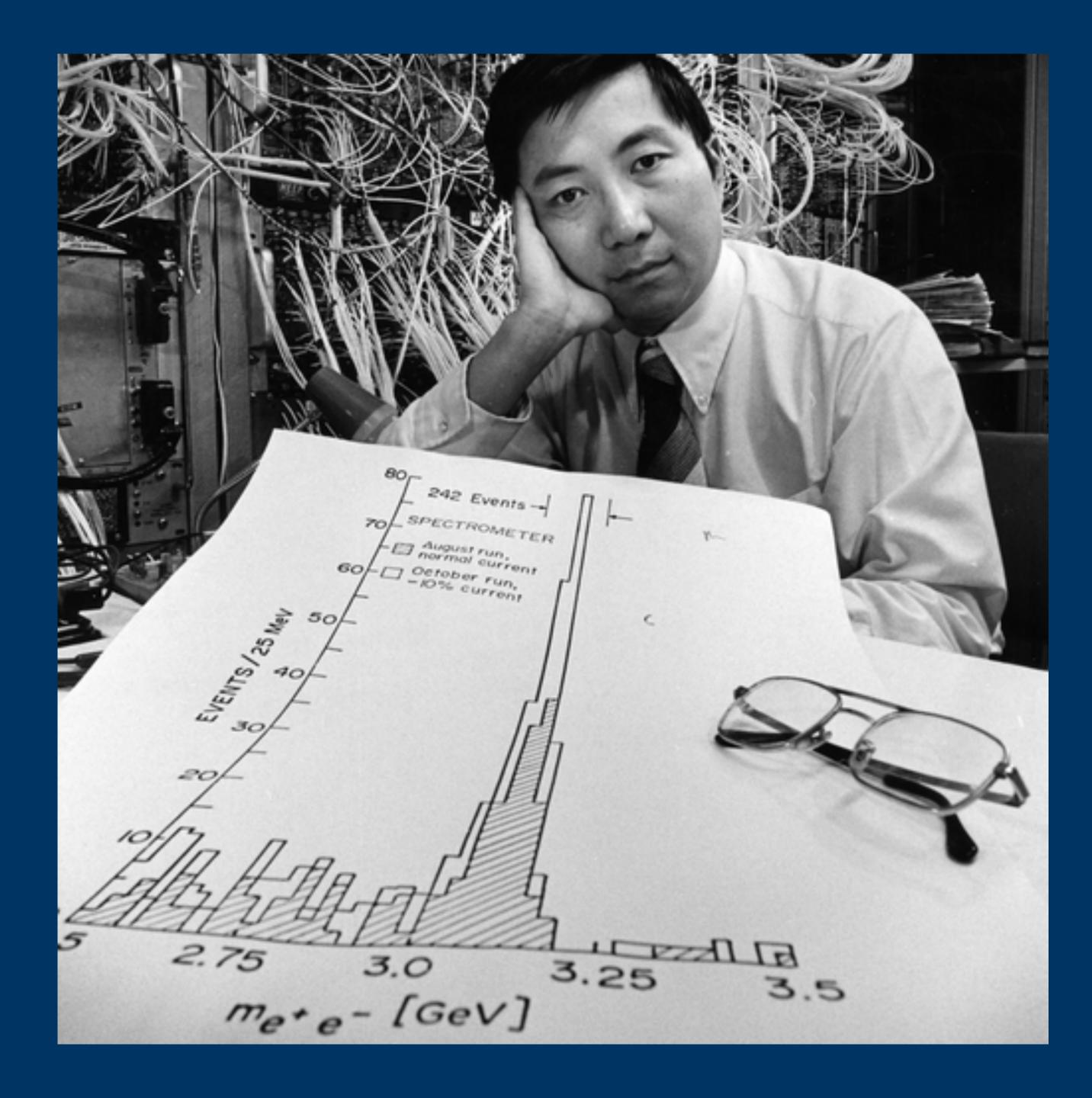
Channels	\mathcal{B}	A_{CP}	A^{lpha}_{CP}	Channels	\mathcal{B}	A_{CP}	A_{CP}^{lpha}
$\Lambda_c^+ o p \pi^0$	0.18(2)	-0.01(7) $0.01(15)(45)$	-0.15(13) $0.55(20)(61)$	$\Xi_c^0 \to \Sigma^+ \pi^-$	0.26(2)	$0 \\ 0.71(15)(6)$	0 $-1.83(10)(15)$
$\Lambda_c^+ \to n \pi^+$	0.68(6)	0.0(1) $-0.02(7)(28)$	0.03(2) $0.30(13)(41)$	$\Xi_c^0 \to \Sigma^0 \pi^0$	0.34(3)	-0.02(4) $0.44(24)(17)$	0.01(1) $-0.43(31)(16)$
$\Lambda_c^+ \to \Lambda K^+$	0.62(3)	0.00(2) $-0.15(13)(9)$	0.03(2) $0.50(9)(21)$	$\Xi_c^0 \to \Sigma^- \pi^+$	1.76(5)	0.01(1) $0.12(6)(2)$	-0.01(1) $-0.22(5)(21)$
$\Xi_c^+ \to \Sigma^+ \pi^0$	2.69(14)	-0.02(6) $0.05(7)(8)$	0.07(4) $-0.23(3)(15)$	$\Xi_c^0 o \Xi^0 K_{S/L}$	0.38(1)	$0 \\ 0.18(3)(5)$	0 -0.38(2)(11)
$\Xi_c^+ \to \Sigma^0 \pi^+$	3.14(10)	0.00(1) $0.05(8)(7)$	-0.02(1) $-0.24(6)(13)$	$\Xi_c^0\to\Xi^-K^+$	1.26(4)	0.00(1) $-0.12(5)(2)$	0.01(1) $0.21(4)(2)$
$\Xi_c^+ \to \Xi^0 K^+$	1.30(10)	0.00(0) $0.01(6)(17)$	-0.02(1) -0.23(9)(52)	$\Xi_c^0 \to pK^-$	0.31(2)	0 $-0.73(18)(6)$	0 $1.74(11)(14)$
$\Xi_c^+ \to \Lambda \pi^+$	0.18(3)	-0.01(2) $-0.31(21)(13)$	0.0(0) $0.96(25)(44)$	$\Xi_c^0 \to n K_{S/L}$	0.86(3)	0 -0.14(3)(4)	$0 \\ 0.27(2)(7)$
$\Xi_c^+ o pK_s$	1.55(7)	0 -0.13(3)(4)	$0 \\ 0.22(3)(7)$	$\Xi_c^0 \to \Lambda \pi^0$	0.06(2)	0.02(3) $-0.12(18)(10)$	0.0(1) $0.69(8)(43)$

Quantitative change leads to qualitative change.









Final-state rescattering

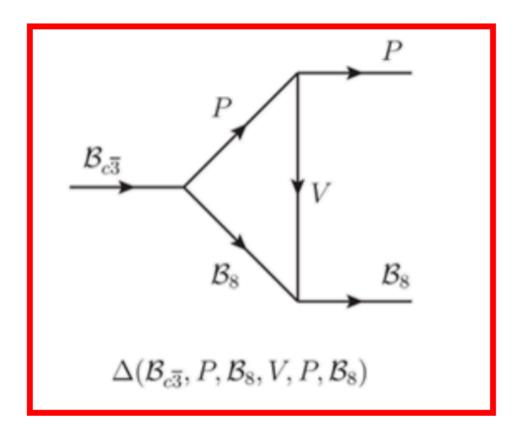
Wang's Slide

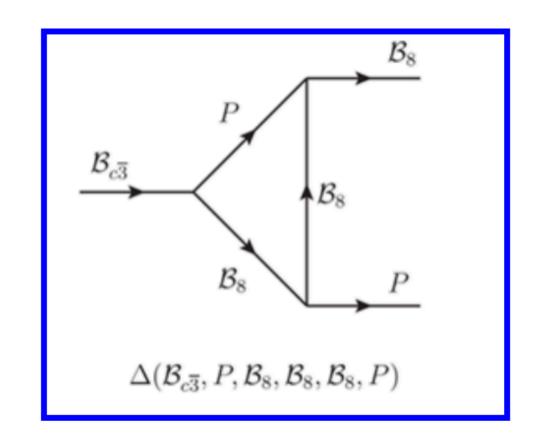
$$\mathcal{A}_{L}(\Lambda_{c}^{+} \to \Sigma^{0} K^{+}) = \frac{1}{\sqrt{2}} \lambda_{d} \Delta_{\alpha^{+}, \gamma^{-}}(\Lambda_{c}^{+}, \pi^{+}, n, \overline{K}^{*0}, K^{+}, \Sigma^{0}) + \frac{1}{\sqrt{2}} \lambda_{d} \Delta_{(\beta^{+} - \beta^{-}), \beta^{-}}(\Lambda_{c}^{+}, \pi^{+}, n, \Sigma^{-}, \Sigma^{0}, K^{+})$$

...

$$\mathcal{A}_{L}(\Lambda_{c}^{+} \to \Sigma^{+} K^{0}) = \frac{1}{2} \lambda_{d} \Delta_{(\beta^{+} + \beta^{-}), (2\beta^{+} - \beta^{-})} (\Lambda_{c}^{+}, \pi^{+}, n, \Lambda^{0}, \Sigma^{+}, K^{0}) + \frac{1}{2} \lambda_{d} \Delta_{(\beta^{+} - \beta^{-}), \beta^{-}} (\Lambda_{c}^{+}, \pi^{+}, n, \Sigma^{0}, \Sigma^{+}, K^{0})$$

...





arXiv:2507.06914.

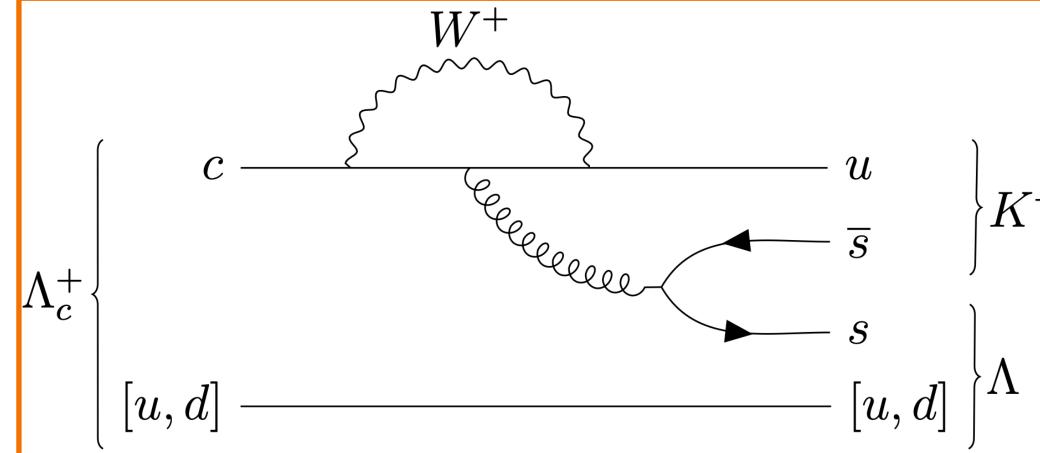
Amplitudes: $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

$$\tilde{f}^b = \tilde{F}_V^- + \tilde{S}^- - \sum_{\lambda = \pm} (2r_\lambda^2 - r_\lambda) \tilde{T}_\lambda^-,$$

$$\tilde{f}^c = r_- \tilde{S}^- - \sum_{\lambda=+} (r_\lambda^2 - 2r_\lambda + 3) \tilde{T}_\lambda^-,$$

$$\tilde{f}^d = \tilde{F}_V^- - \sum_{\lambda = \pm} (2r_{\lambda}^2 - 2r_{\lambda} - 4)\tilde{T}_{\lambda}^-, \quad \tilde{f}^e = \tilde{F}_V^+,$$

Corrections to A_{CP} are around 10%



$$\tilde{f}_{\mathbf{3}}^{b} = \frac{7r_{-} - 2}{8 + 2r_{-}} \tilde{S}^{-} - \sum_{\lambda = \pm} (r_{\lambda}^{2} - 5r_{\lambda}/2 + 1) \tilde{T}_{\lambda}^{-},$$

$$\tilde{f}_{\mathbf{3}}^{c} = \frac{(r_{-}+1)(2-7r_{-})}{24+6r_{-}}\tilde{S}^{-} + \sum_{\lambda=\pm}^{1} \frac{1}{6}(r_{\lambda}^{2}+11r_{\lambda}+1)\tilde{T}_{\lambda}^{-},$$

$$\tilde{f}_{\mathbf{3}}^{d} = \frac{r_{-}(7r_{-}-2)}{8+2r_{-}}\tilde{S}^{-} - \sum_{\lambda=\pm}^{\lambda=\pm} \frac{1}{2}(r_{\lambda}+1)^{2}\tilde{T}_{\lambda}^{-} - \frac{1}{4}\left(\tilde{F}_{V}^{+} + 2\tilde{F}_{V}^{-}\right) \left(1 + \frac{\left(3C_{4} + C_{3}\right)m_{c} - \frac{2m_{K}^{2}}{m_{s} + m_{u}}\left(3C_{6} + C_{5}\right)}{\left(C_{+} + C_{-}\right)m_{c}}\right)$$

$$(\tilde{f}^b, \tilde{f}^c, \tilde{f}^d, \tilde{f}^e) \longleftrightarrow (\tilde{F}_V^+, \tilde{F}_V^-, \tilde{S}^-, \tilde{T}^-) \longrightarrow (\tilde{f}_3^b, \tilde{f}_3^c, \tilde{f}_3^d)$$

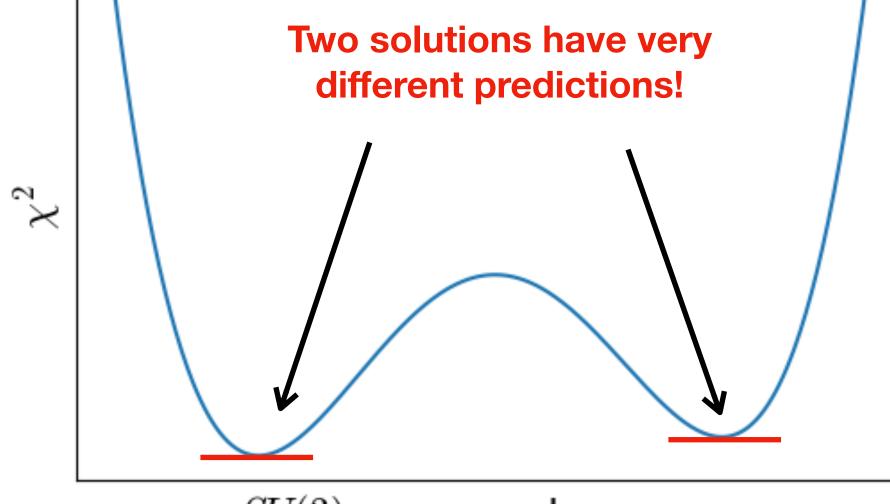
$$\left(1 + \frac{\left(3C_4 + C_3\right)m_c - \frac{2m_K^2}{m_s + m_u}\left(3C_6 + C_5\right)}{(C_+ + C_-)m_c}\right)$$

Much more complicated compared to $P^{LD} = E$ in D mesons!

The $SU(3)_F$ is an approximate symmetry with errors in 10^{-1} .



There exhibits \mathbb{Z}_2 ambiguities:



 $SU(3)_F$ parameter space

$$\Gamma \propto |F^{2}| + \kappa^{2}|G^{2}|, \quad \alpha = \frac{2\kappa \text{Re}(F^{*}G)}{|F^{2}| + \kappa^{2}|G^{2}|}, \quad \beta = \frac{2\kappa \text{Im}(F^{*}G)}{|F^{2}| + \kappa^{2}|G^{2}|}, \quad \gamma = \frac{|F^{2}| - \kappa^{2}|G^{2}|}{|F^{2}| + \kappa^{2}|G^{2}|}.$$

 Γ and α are invariant under $(F,G) \to (F^*,G^*)$ and $F \leftrightarrow \kappa G^*$ but β and γ flip signs. In general, the amplitudes cannot be fully reconstructed without β and γ as input.



Precise β and γ data can break the ambiguities, highlighting the importance of



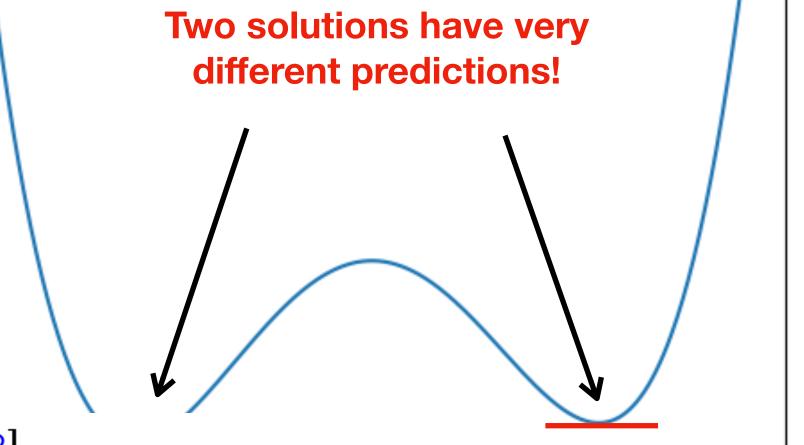


Nevertheless, there are still a few ambiguities.

Measurement of Λ_b^0 , Λ_c^+ , and Λ Decay Parameters Using $\Lambda_b^0 \to \Lambda_c^+ h^-$ Decays PRL **133**, 261804 (2024)

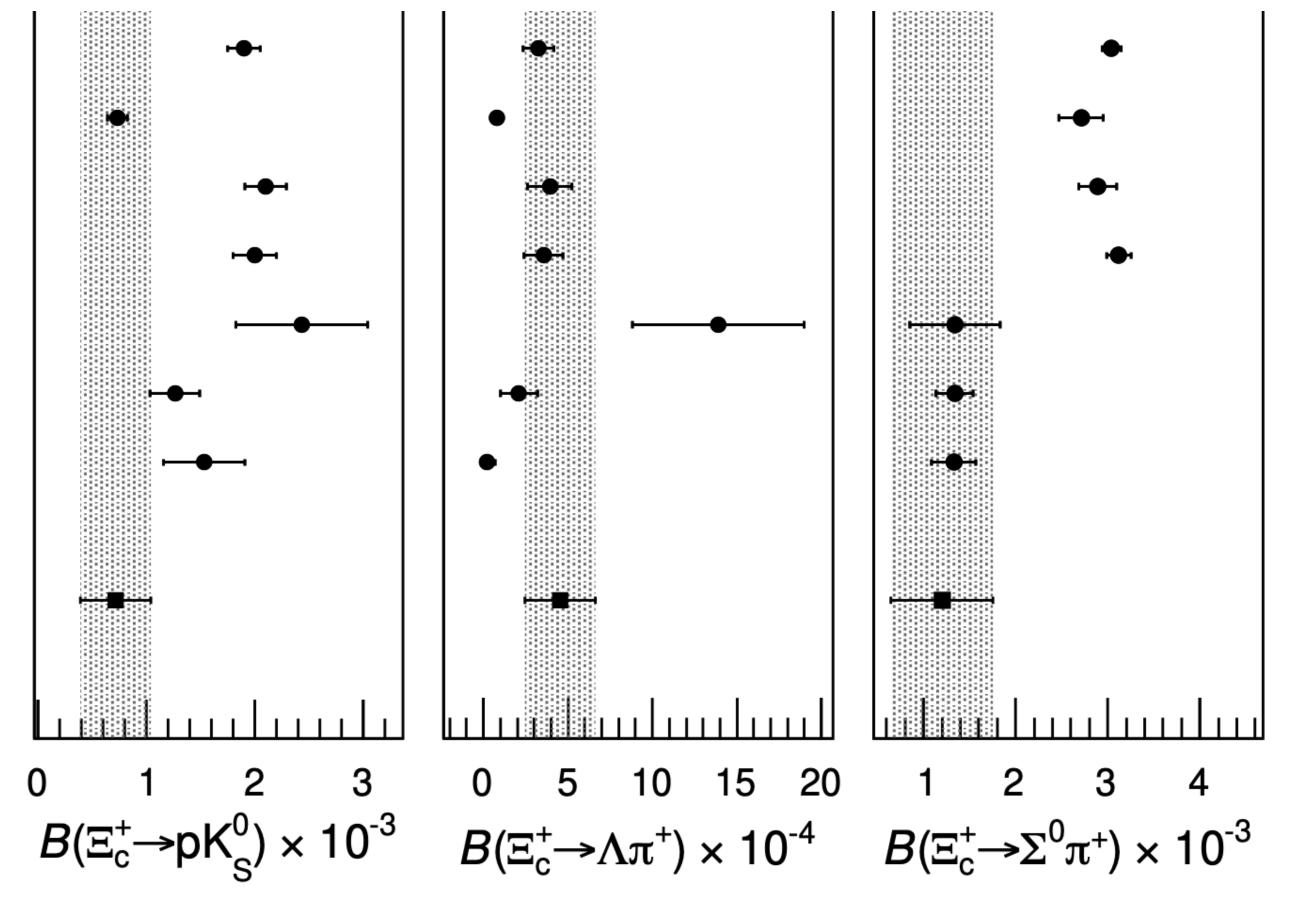


The $SU(3)_F$ is an approximate symmetry with errors in 10^{-1} .





There exhibits \mathbb{Z}_2 ambiguities:



Geng et.al [18]

Liu [19]

Zhong et.al (I) [20]

Zhao et.al [21]

Hsiao et.al (I) [22]

Hsiao et.al (II) [22]

Belle and Belle II combined measurement

 $)_F$ parameter space

Zhong et.al (I) [20]
$$= \frac{|F^{2}| - \kappa^{2} |G^{2}|}{|F^{2}| + \kappa^{2} |G^{2}|}.$$

nd γ flip signs. out.

Tuch

Parameters Using $\Lambda_b^0 \to \Lambda_c^+ h^-$ Decays PRL **133**, 261804 (2024)

$$\mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P} = \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\text{Tree}} + \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\text{FSR-s}} + \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\text{FSR-t}} + \mathcal{L}_{\mathbf{B}_{c}\mathbf{B}P}^{\text{FSR-u}} + \dots (?)$$

