粲介子半轻衰变的 QCD 计算与现象学研究

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Overview

I Why semileptotic $D_{(s)}$ decays

II Opportunities in Semileptotic D decays

i $D_s \to [f_0, \cdots \to] \pi\pi e^+\nu$

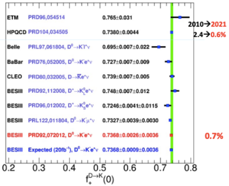
ii $D \to \rho l^+\nu$ and $D_{(s)} \to K^* l^+\nu$

III Conclusions and Prospects

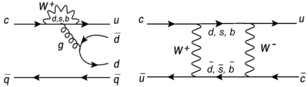
Semileptotic $D_{(s)}$ decays

play a crucial role in the precision era of particle physics

- fundamental parameters, like the CKM matrix element $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022]
  - the result measured via $D \to Kl\nu$ and $D_s \to \mu\nu_\mu$ consist with each other ($\sim 1.5\sigma$ derivation)
  - $\sim 3\sigma$ tension three years ago [PDG 2020, 2021]
  - the improvement mainly due to the high precision of $D \to K$ form factor from lattice evaluation and the f_{D_s} from the BESIII



- new physical mechanism via the FCNC
  - anomalous measured in $B \to K^* \mu^+ \mu^-$, 3.6σ derivation of $d\mathcal{B}/dq^2$ in $q^2 \in [1, 6]$ GeV^2, 1.9σ derivation of $p_5' = S_5/\sqrt{F_L(1 - F_L)}$ in $[4, 8]$ GeV^2
  - a plausible effect in up-type FCNC process $c \to ull$ [Bharucha 2011.12856] SM $\mathcal{B}(D \to \pi l^+ l^-) \sim \mathcal{O}(10^{-9})$, current best-world limit $\mathcal{O}(10^{-8})$
  - LCSRs prediction $\mathcal{B}(D \to \pi\mu\mu) = 1.33^{+0.17}_{-0.24} \times 10^{-8}$, at the same order of LHCb limit 6.7×10^{-8} [A. Bansal, A. Khodjamirian and T. Mannel 2505.21369[hep-ph]],
  - first measurement of $D^0 \to \pi^+\pi^- e^+ e^-$ [LHCb 2412.09414] $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ



New physics hunter $D \to \pi\mu^+\mu^-$

- my talk in the " 超级陶粲装置研讨会" at LZU, July 8th, 2024

  - Experimentail potentials

Experiment	Measurement	Sensitivity
LHCb	Angular observables	$\sim 0.2\%$ with $50\,\text{fb}^{-1}$, $\sim 0.08\%$ with $300\,\text{fb}^{-1}$
LHCb	Branching ratio	$\sim 10^{-8}$ with $50\,\text{fb}^{-1}$, $\sim 3\times10^{-9}$ with $300\,\text{fb}^{-1}$
Belle-II	Branching ratio	$\sim 10^{-8}$ (rescaling BaBar)

[talk at Towards the Ultimate Precision in Flavour Physics, Durham U.K. (2019)]

[BABAR Collaboration 1107.4465]

Run 4 $\sim$ 2030
Run 5 $\sim$ 2038

$N(D\bar{D}) \sim 10^9/\text{ab}^{-1}$ angular observables $\sim 0.2\%$

  - BESIII Collaboration in the electron channel [BESIII Collaboration 1802.09752]
    $\mathcal{B}(D \to \pi^+\pi^-e^+e^-) < 0.7\times10^{-5}$ with $N(c\bar{c}) = 2\times10^7$ at 3.7 GeV

3.770	1	$D^0\bar{D}^0$	3.6	3.6×10^9	
		$D^+\bar{D}^-$	2.8	2.8×10^9	
		$D^0\bar{D}^0$		7.9×10^8	Single Tag
		$D^+\bar{D}^-$		5.5×10^8	Single Tag

STCF $N(D\bar{D}) \sim 8\times10^9$ Branching ratio $\sim 10^{-8}$

- STCF is still competitive in hunting the NP via $D \to \pi\mu^+\mu^-, \pi\pi\mu^+\mu^-$

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Semileptotic $D_{(s)}$ decays

- a clean environment to study scalar mesons *see En Wang and Chu-wen's talks*
  - $f_0(1370), f_0(1500), a_0(1450), K_0^*(1430)$ form a $SU(3)$ flavor nonet
  - $f_0(500)/\sigma, f_0(980), a_0(980), K_0^*(700)/\kappa$ form another flavor nonet

    compact tetraquark and $K\bar{K}$ bound state are favored from spectral analysises, $s\bar{s}$ is dominated in B_s decay
  - how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?

- depends on the precise pertubative QCD calculation, more important is **the accurate nonperturbative description of the structure**
  - the signal channel is $D_{(s)} \to [f_0 \to] \pi\pi, K\bar{K}l\nu$ invariant mass
  - dynamics of H_{l4} is governed by $H \to \pi\pi$ **form factors**, a big task of the QCD methods
    [S. Faller, et.al., 1310.6660, X.W. Kang, et.al., 1312.1193]

- **Dipion LCDAs** are introduced in the LCSRs predictions of $B, D \to \pi\pi$ transitions
  - [**SC**, A. Khodjamirian and J. Virto, 1701.01633[hep-ph]] *B*-meson LCSRs
  - [C. Hambrock and A. Khodjamirian, 1511.02509[hep-ph]] 2πDAs LCSRS of $F_{\parallel,\perp}$
  - [**SC**, A. Khodjamirian and J. Virto, 1709.00173[hep-ph]] timelike-helicity FF F_t and F_0
  - [**SC**, 1901.06071[hep-ph]] 2πDAs updates and $B \to [\pi\pi]_{\mathrm{S,P}}$ FFs
  - [**SC** and J.M Shen, 1907.08401[hep-ph]],
    [**SC** and S.L Zhang, 2307.02309[hep-ph]] $D_s \to [f_0 \to] \pi\pi l\nu$ leading twist
  - [**SC**, 2502.07333[hep-ph]] first study of twist-three 2πDAs and $|V_{ub}|$ extraction
  - [**SC**, L.Y Dai and S.L Zhang, to be appearing] $D_s \to [f_0 \to] \pi\pi l\nu$ sub-leading twist

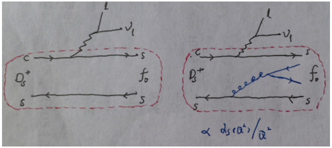
Opportunities in SL $D_{(s)}$ decays
(private opinions)

i $D_s \to [f_0, \cdots \to] \, \pi\pi e^+\nu$

ii $D \to \rho l^+\nu$ and $D_{(s)} \to K^* l^+\nu$

$$D_s \to [f_0, \cdots \to] \pi\pi e^+ \nu$$

$D_s \to [f_0, \cdots \to] \pi\pi e\nu$



- ⋆ in SL B_s decays
- ⋆ tetraquark contribution is suppressed doubly by strong coupling and power
- ⋆ FSI is weak too

$$|f_0(980)\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$

$$\psi^n_{f_0}(x_i, k_{\perp i}, \lambda_i) = \langle n, x_i, k_{\perp i}, \lambda_i | f_0\rangle$$

⋆ physical observables are usually written in a QCD convolution

$$\frac{d\sigma}{d\Omega} = \sum_t \int_0^1 dx_i \, \mathcal{H}^t(x_i, Q) \psi^t(x_i, \mu)$$

⋆ ψ^t is universal, however H^t is process dependent, hence different observables might highlike the contributions from different components

- **how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?**

$D_s \to [f_0, \cdots \to] \pi\pi e\nu$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons
  - $D_s \to f_0 e^+\nu$ [CLEO '09], $D_{(s)} \to a_0 e^+\nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+\nu$[BESIII '19]
  - $D_s \to f_0(\to \pi^0\pi^0, K_s K_s)e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- Theoretical considerations: single particle (narrow width limit)

$$\frac{d\Gamma(D_s^+ \to f_0 l^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2\lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2, \; D_s \to f_0 \;\mathrm{FF}$$

- Improvement with the width effect by resonant model

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}}\, l^+\nu)}{ds dq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1\beta_\pi(s)}{|m_{\mathrm{S}}^2 - s + i\,(g_1\beta_\pi(s)) + g_2\beta_K(s))\,|^2}, \;\mathrm{BESIII}$$

- Improvement with the width effect by $\pi\pi$ invariant mass spectral

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_{\mathrm{S}}\, l^+\nu)}{dk^2 dq^2} = \frac{G_F^2|V_{cs}|^2\,\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}}q^2}{3(4\pi)^5 m_{D_s}^3}\sum_{\ell=0}^{\infty}|F_0^{(\ell)}(q^2, k^2)|^2, \; D_s \to \pi\pi \;\mathrm{FF}$$

$D_s \to f_0$ form factor and $D_s^+ \to (f_0, [\pi\pi]_S)e^+\nu_e$ decay

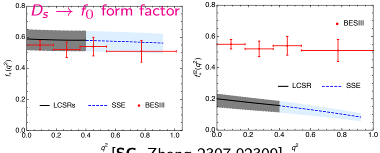
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\}\mathrm{GeV}^2$ *see Hai-bing Fu's talk* for the LCSRs

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

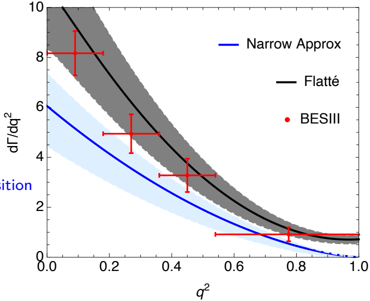
- the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

  different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)
  we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



[**SC**, Zhang 2307.02309]



Differential decay width of $D_s^+ \to (f_0, [\pi\pi]_S)e^+\nu_e$

- Twist-3 LCDAs give dominate contribution in $D_s \to f_0$ transition
  - the uncertainty estimation is conservative
  - without NLO correction
  - we need a model independent calculation
  - not only for the QCD understanding
  - but also for the future partial-wave measurement

$D_s \to [\pi\pi]_{\rm S}$ form factors

- Definition of $D_s \to [\pi\pi]_{\rm S}$ form factor

$$\langle[\pi(k_1)\pi(k_2)]_{\rm S}\,|\bar{s}\gamma_\mu(1-\gamma_5)c|D_s^+(p)\rangle = -iF_t k_\mu^t - iF_0(q^2,s,\zeta)k_\mu^0 - iF_\parallel k_\mu^\parallel$$

$$k_\mu^t = \frac{q_\mu}{\sqrt{q^2}},\, k_\mu^0 = \frac{2\sqrt{q^2}}{\sqrt{\lambda_{D_s}}}\left(k_\mu - \frac{k\cdot q}{q^2}q_\mu\right),\, k_\mu^\parallel = \frac{1}{\sqrt{k^2}}\left(\bar{k}_\mu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_{D_s}}k_\mu + \frac{4k^2(q\cdot\bar{k})}{\lambda_{D_s}}q_\mu\right)$$

- LCSRs calculations start with the correlation functions

$$\Pi_\mu^{ab}(q,k_1,k_2) = i\int d^4x e^{iq\cdot x}\langle\pi^a(k_1)\pi^b(k_2)|T\{j_{1,\mu}(x),j_2(0)\}|0\rangle$$

- The chiral even two quark isoscalar 2πDAs
  leading twist, twist-three LCDAs are available now [**SC**, arXiv: 2502.07333]

$$\langle\left[\pi^a(k_1)\pi^b(k_2)\right]_{\rm S}|\bar{s}(xn)\gamma_\mu s(0)|0\rangle = 2\delta^{ab}k_\mu\int du e^{iux(k\cdot n)}\Phi^{I=0}_{\parallel,[\pi\pi]_{\rm S}}(u,\zeta,k^2)$$

$$\Phi^{I=0}_{\parallel,[\pi\pi]_{\rm S}} = 6u(1-u)\sum_{n=1,\rm odd}^{\infty}\sum_{l=0,\rm odd}^{n+1}B^{I=0}_{\parallel,nl}(k^2,\mu)C_n^{3/2}(2u-1)C_l^{1/2}(2\zeta-1)$$

- Do the LCSRs QCD calculations and consider the partial-wave expansion

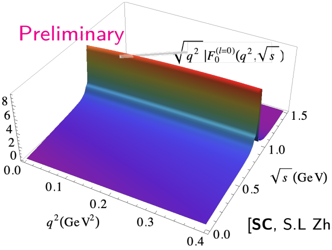
$$F_0(q^2,k^2,\zeta) = \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\,F_{0,t}^{(\ell)}(q^2,k^2)P_\ell^{(0)}(\cos\theta_\pi)$$

$D_s \to [\pi\pi]_{\rm S}$ form factor and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay

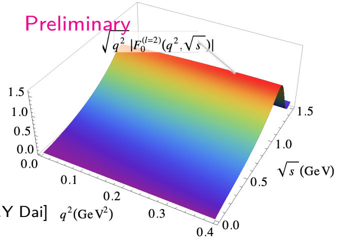
- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_{\rm S}$ form factors ($\ell' = \text{even}$ & $\ell' \leqslant n+1$)

$$\sqrt{q^2} F_0^{(\ell')}(q^2, k^2) = \frac{m_c(m_c + m_s)\sqrt{q^2}\sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1,\text{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2, k^2, M^2, s_0)\, B_{n\ell,\parallel}^{l=0}(k^2)\, I_{\ell\ell'}$$

- Near threshold, B_{nl} can be determined from the low-energy effective theory of pions interacting with massive "constituent" quarks, based on the instanton model of the QCD vacuum
- For the resonant regions, Watson's theorem yields the k^2-dependence via Omnés solutions with the phase shifts mplemented through subtracted dispersion relations



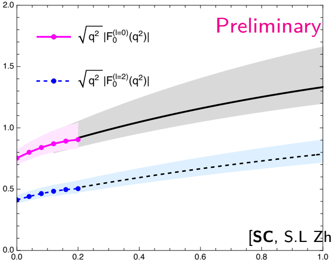
[**SC**, S.L Zhang and L.Y Dai]

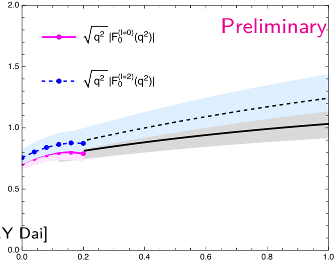


- S- and D-wave FFs: $\sqrt{q^2}|F_0^{(l=0)}(\sqrt{s}, q^2)|$ and $\sqrt{q^2}|F_0^{(l=2)}(\sqrt{s}, q^2)|$

$D_s \to [\pi\pi]_{\rm S}$ form factor and $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ decay

- The q^2-dependence of FFs obtained by adopting the phase shifts from $\pi^- p \to \pi^0\pi^0 n$ reaction (peak) and $\pi\pi$ scattering (dip)





[**SC**, S.L Zhang and L.Y Dai]

- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- the FFs at the two-particle level are one-order lower than the required
- the conventional $q\bar{q}$ is not the dominate component in the charm decays
- we have to go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)
- much different in B decays leading twist dominated [**SC**, arXiv: 2502.07333]

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$

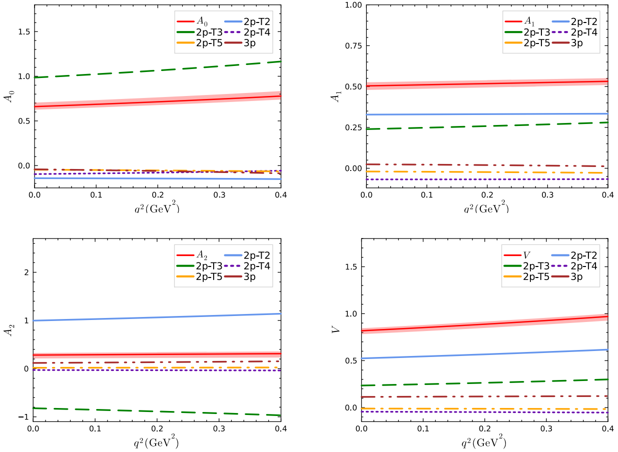
- motived by the recent BESIII measurements

Decay Mode	Collaboration	Year	Reference
$D^0 \to K^{*-}\mu^+\nu_\mu$	BESIII	2025	PRL 134(2025)1,011803
$D^0 \to \rho^- e^+\nu_e$	BESIII	2024	PRD 110(2024)11,112018
$D_s^+ \to K^{*0} e^+\nu_e$	BESIII	2019	PRL 122 (2019) 6, 061801
$D^+ \to \rho^0 e^+\nu_e$	CLEO	2013	PRL 110(2013)13,131802
$D^+ \to K^{*0}\mu^+\nu_\mu$	FOCUS	2006	PLB 637(2006)32-38

- state-of-the-art LCSRs calculation of $D \to V$ form factors with high twist LCDAs
- to examine the width effects and the non-resonant QCD backgrounds in the D_{l4} decays ($D_{(s)} \to V[\to P_1 P_2] l\nu$)

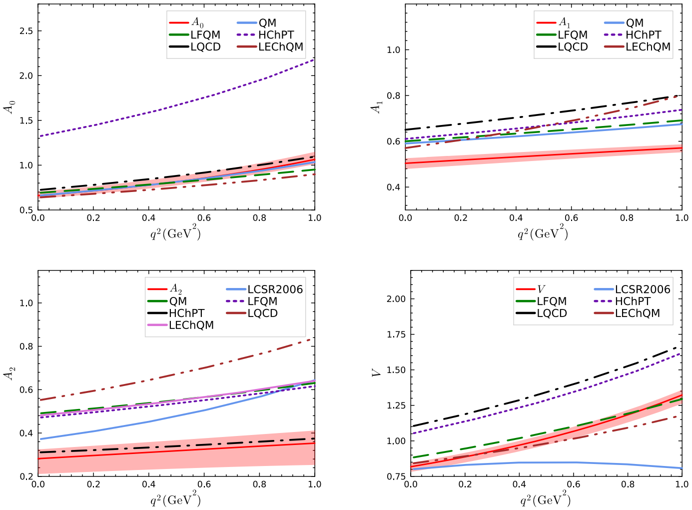
$D^0 \to \rho^-$ form factors from different twist (power) contributions

[W. Lin, X.E Huang, **SC** and D.L Yao, 2505.01329]



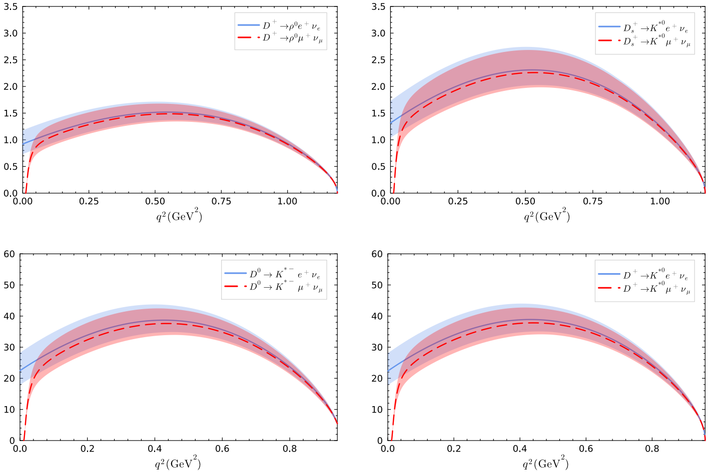
- both the three-particle LCDAs and the power correction from the heavy quark expansion at $\mathcal{O}(1/m_c)$ contribute significantly to the form factors A_1 and A_2.
- The OPE expansion exhibits good convergence: while two-particle twist-three LCDAs contribute sizeably or even dominantly in the axial-vector transition, the higher-twist effects remain well under control

$D^0 \to \rho^-$ form factors



- LCSRs predictions in comparing to the results obtained from other approaches

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$



- Differential decay widths $d\Gamma/dq^2$ of $D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$ decays
- the lepton mass effect is mostly prominent in the large-recoil region of the semileptonic charm to vector meson decays

$D \to \rho l^+ \nu$ and $D_{(s)} \to K^* l^+ \nu$

- branching ratios of semileptonic charm decays (in unit of 10^{-6})

	$D^+ \to \rho^0 \ell^+ \nu_\ell$	$D_s^+ \to K^{*0} \ell^+ \nu_\ell$	$D^0 \to K^{*-} \ell^+ \nu_\ell$	$D^+ \to \bar{K}^{*0} \ell^+ \nu_\ell$
This work	$2.30^{+0.32}_{-0.25}$	$1.55^{+0.30}_{-0.20}$	$17.6^{+2.4}_{-1.9}$	$45.2^{+6.2}_{-5.0}$
	$2.20^{+0.30}_{-0.23}$	$1.48^{+0.29}_{-0.19}$	$16.6^{+2.2}_{-1.8}$	$42.7^{+5.7}_{-4.5}$
LCSR2006 [21]	$2.29^{+0.23}_{-0.16}$	$2.33^{+0.29}_{-0.30}$	21.2 ± 0.9	$53.7^{+2.4}_{-2.3}$
	$2.20^{+0.21}_{-0.16}$	$2.24^{+0.27}_{-0.29}$	20.1 ± 0.9	$51.0^{+2.3}_{-2.1}$
CLFQM [51]	2.32	1.90	—	73.2
	2.22	1.82		69.3
HChPT [32]	2.50	2.20	22.0	56.0
PDG [2]	1.90 ± 0.10	2.15 ± 0.28	21.5 ± 1.60	54.0 ± 1.00
	2.40 ± 0.40	—	18.9 ± 2.40	52.7 ± 1.50

see Xian-wei Kang's talk

- 10%-20% discrepancy when confronting our LCSR predictions with experimental measurements
- a sizable $SU(3)$ flavor-breaking effect is observed in $D_{(s)} \to K^* l\nu$ decays
- the necessity to further implement the effects of vector meson widths and non-resonant QCD background

Conclusions and Prospects

- **The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons in H_{l4} processes**

◦ a new booster on the accurate calculation in flavor physics

◦ improvement study in the CKM determinations and the flavor anomalies

- The studies of 2πDAs and H_{l4} decays are now at two-particle component

◦ universal phase shift in $\pi\pi$ scattering and heavy decay ?

- Go further to high twist LCDAs associated to multi-particle components

◦ $B \to \pi\pi l\nu, B \to [\rho\rho \to] \to 4\pi,\ D_s \to \pi\pi l\nu, D \to K\pi\mu\nu,\ D \to \pi\pi e^+e^-$ et al.

Thank you for your patience.