QCD Sum Rule predictions for some charmed meson semileptonic decays

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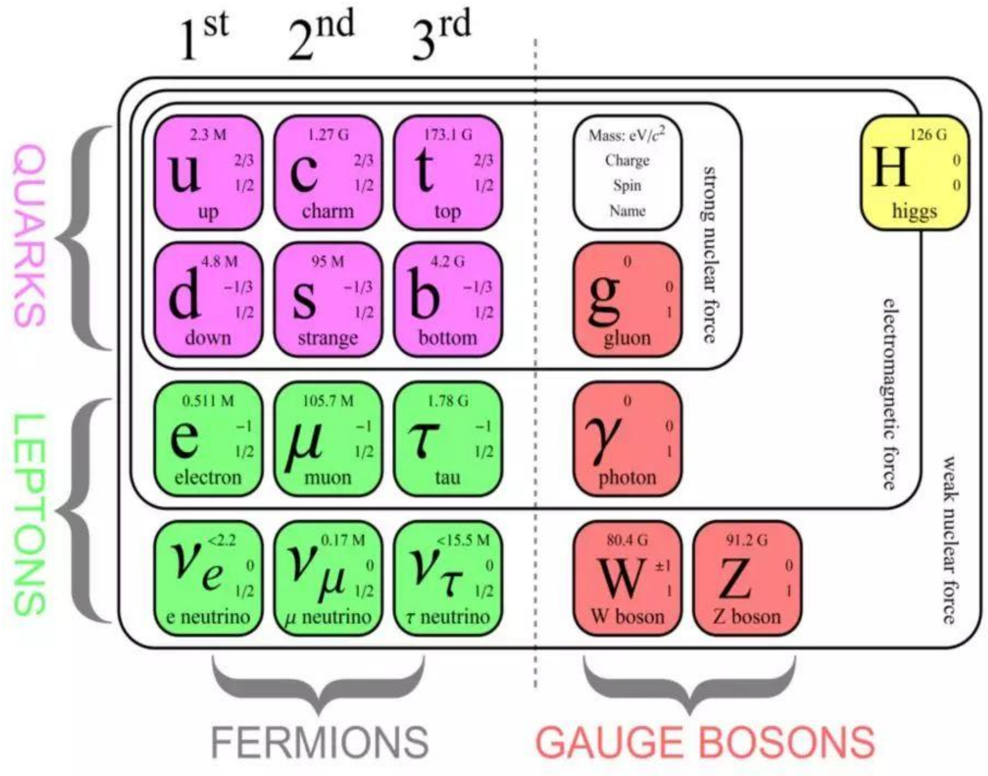
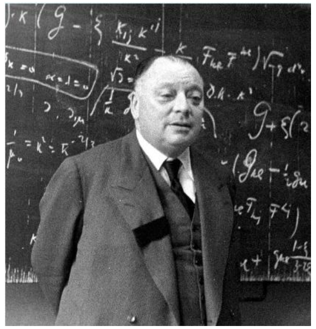
I. Motivation

QCD求和规则(sum rule) 是计算强子非微扰参数的唯象方法，创立于1970年代后期。

Nucl. Phys. B 147, 385 (1979)

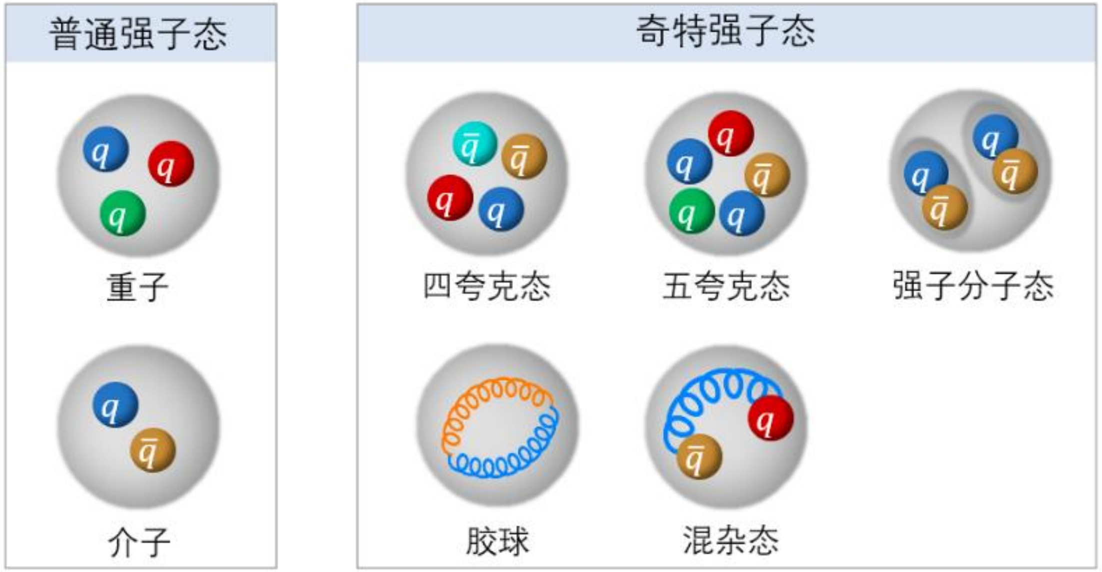




Shifman

Vainshtein

Zakharov



可计算的非微扰参数包括: 强子质量谱，衰变常数，形状因子，强子耦合常数...

I. Motivation

研究方法：QCD求和规则

QCD求和规则 →

- SVZ求和规则
- 光锥求和规则

→

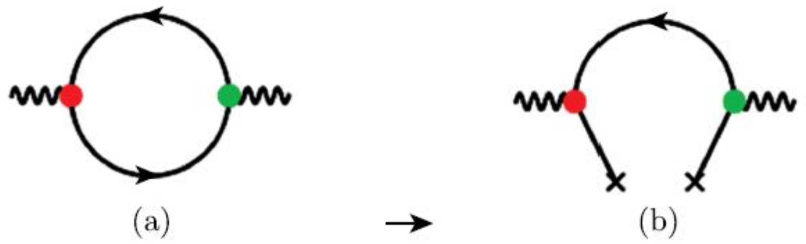
- 构造关联函数
- 算符乘积展开
- 强子表示
- 色散关系

SVZ求和规则(真空-真空) → 介子twist-2,3分布振幅

光锥求和规则(真空-强子态) → 衰变过程中的跃迁形状因子 → 衰变宽度、衰变分支比或CKM矩阵元等

研究方法①：背景场理论框架下的SVZ求和规则

- 按照微扰QCD图形切断其中夸克内线或胶子内线得到切断后的费曼图形
- 被切断的夸克和胶子与物理真空相互作用形成夸克凝聚和胶子凝聚
- 夸克和胶子是在充满夸克-反夸克对和胶子的物理真空中传播
- 夸克及胶子传播子，含有非微扰效应



(a)微扰QCD图形；(b)切断(a)中夸克内线后的图形

研究方法①：背景场理论框架下的SVZ求和规则

背景场方法的基本思想

用满足理论运动方程的经典背景场来描述非微扰效应，以量子场描述在此基础上的量子起伏，即微扰效应。

理论的拉氏量

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu}^{A} G^{A\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$

$$G_{\mu\nu}^{a} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + g f^{abc} A_{\mu}^{b} A_{\nu}^{c}$$

$$D_{\mu} = \partial_{\mu} - i g A_{\mu}^{a} T^{a}$$

$$\Downarrow$$

$$A_{\mu}^{a} \to A_{\mu}^{a} + \phi_{\mu}^{a}$$

$$\psi(x) \to \psi(x) + \eta(x)$$

其中，$A_\mu^a(x)$, $\psi(x)$分别表示胶子，夸克的背景场； $\phi_\mu^a(x)$, $\eta(x)$分别表示相应的量子起伏。

I. Motivation

研究方法①：背景场理论框架下的SVZ求和规则

当背景场存在时，关于它们的量子化已经被讨论 [*Ann. Phys., 1983, 145:340*]。

胶子的量子场满足背景场规范：

$$D_{\mu}^{ab}(A)\phi_{b}^{\mu}=0$$
$$D_{\mu}^{ab}(A)=\delta^{ab}\partial_{\mu}-gf^{abc}A_{\mu}^{c}$$

背景场规范的优点：使理论明显地具有背景场规范变换下的不变性，使得计算所得的物理量与规范无关。

关联函数的中的夸克场替换为背景场和量子场，进行算符乘积展开：

$$\Pi^{(n,0)}(z,x)=i\int d^{4}x e^{iq\cdot x}\langle 0|T\{J_{n}(x),J_{0}^{\dagger}(0)\}|0\rangle$$

$$\Downarrow$$

$$\bar{q}_{1}(x)\rightarrow\bar{q}_{1}(x)+\bar{\eta}_{1}(x),q_{1}(0)\rightarrow q_{1}(0)+\eta_{1}(0)$$
$$q_{2}(x)\rightarrow q_{2}(x)+\eta_{2}(x),\bar{q}_{2}(0)\rightarrow\bar{q}_{2}(0)+\bar{\eta}_{2}(0)$$

其中，背景场不收缩，贡献为真空凝聚；量子场收缩为传播子。接下来的计算步骤与传统的SVZ求和规则一致。

研究方法②：利用光锥求和规则计算跃迁形状因子

光锥求和规则方法(LCSR)

- 依据不同的初末态，构建关联流和关联函数
- 插入强子态完备集，根据跃迁矩阵元的基本定义，给出形状因子的强子表示
- 在光锥附近进行微扰展开，即算符乘积展开，其中非微扰部分吸收进不同扭度的分布振幅当中
- 对初态介子动量做Borel变换，得到有限的结果，
- 将两者进行匹配，最终按照不同的洛伦兹结构，给出跃迁形状因子的解析表达式

PHYSICAL REVIEW LETTERS **132,** 141901 (2024)

Study of the $f_0(980)$ and $f_0(500)$ Scalar Mesons through the Decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$

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Using e^+e^- collision data corresponding to an integrated luminosity of 7.33 fb^{-1} recorded by the BESIII detector at center-of-mass energies between 4.128 and 4.226 GeV, we present an analysis of the decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$, where the D_s^+ is produced via the process $e^+e^- \to D_s^{*\pm}D_s^{\mp}$. We observe the $f_0(980)$ in the $\pi^+\pi^-$ system and the branching fraction of the decay $D_s^+ \to f_0(980)e^+\nu_e$ with $f_0(980) \to \pi^+\pi^-$ measured to be $\boxed{(1.72 \pm 0.13_{\mathrm{stat}} \pm 0.10_{\mathrm{syst}}) \times 10^{-3}}$, where the uncertainties are statistical and systematic, respectively. The dynamics of the $D_s^+ \to f_0(980)e^+\nu_e$ decay are studied with the simple pole parametrization of the hadronic form factor and the Flatté formula describing the $f_0(980)$ in the differential decay rate, and the product of the form factor $f_+^{f_0}(0)$ and the $c \to s$ Cabibbo-Kobayashi-Maskawa matrix element $|V_{cs}|$ is determined for the first time to be $\boxed{f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017_{\mathrm{stat}} \pm 0.035_{\mathrm{syst}}}$. Furthermore, the decay $D_s^+ \to f_0(500)e^+\nu_e$ is searched for the first time but no signal is found. The upper limit on the branching fraction of $D_s^+ \to f_0(500)e^+\nu_e$, $f_0(500) \to \pi^+\pi^-$ decay is set to be 3.3×10^{-4} at 90% confidence level.

I. Motivation

1. 不同的夸克组分

$$f_0(980) = \bar{s}s$$

$$f_0(980) = \frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right)\bar{s}s$$

$$\boxed{f_0(980) = \sin\theta\left[\frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right)\right] + \cos\theta\left(\bar{s}s\right) \quad \sigma(500) - f_0(980)\ \text{mixing}} \Longrightarrow \text{考虑混合效应}$$

2. 实验测量：$f_0(980)$作为中间态

CLEO(2009) : $\mathcal{B}(D_s \to f_0(980)e^+\nu_e) \times \mathcal{B}(f_0(980) \to \pi^+\pi^-) = (0.13 \pm 0.04 \pm 0.01)\%$ *PhysRevD.80.052007*

CLEO(2009) : $\mathcal{B}(D_s \to f_0(980)e^+\nu_e) \times \mathcal{B}(f_0(980) \to \pi^+\pi^-) = (0.20 \pm 0.04 \pm 0.01)\%$ *PhysRevD.80.052009*

BESIII(2019) : $\mathcal{B}(D_s \to f_0(980)e^+\nu_e) \times \mathcal{B}(f_0(980) \to \pi^+\pi^-) < 2.8 \times 10^{(-5)}$ *Phys.Rev.Lett.122,062001*

BESIII(2022) : $\mathcal{B}(D_s \to f_0(980)e^+\nu_e) \times \mathcal{B}(f_0(980) \to \pi^0\pi^0) = (0.79 \pm 0.14 \pm 0.04) \times 10^{-3}$ *PhysRevD.105.L031101*

BESIII(2024) : $\mathcal{B}(D_s \to f_0(980)e^+\nu_e) \times \mathcal{B}(f_0(980) \to \pi^+\pi^-) = (0.172 \pm 0.013 \pm 0.01)\%$ *PhysRevLett.132.141901*

3. 理论预测

Light-front quark model(LFQM) *Phys.Rev.D.80.074030 (2009)*

Covariant light-front dynamics (CLFD) *Phys.Rev.D.79.076004(2009)*

Three point sum rule (3PSR) *Phys.Lett.B* 579 (2004) 59-66 ($\bar{s}s$) *PRD68.036001* (混合态)

Light-cone sum rules(LCSR) *Phys.Rev.D.81.074001(2010)*

II. $D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e$ decay process

1. 得到构建关联函数的流:

$$f_0(980) = \sin\theta[\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)] + \cos\theta(\bar{s}s) \qquad \theta = (19.7 \pm 12.8)^\circ$$

Phys.Rev.Lett.132.141901

Twist-2，3分布振幅矩阵元的定义：

$$\langle f_0(980)|\bar{q}_2(z_2)q_1(z_1)|0\rangle = m_{f_0}\bar{f}_{f_0}\int_0^1 dx e^{i(xp\cdot z_2 + \bar{x}p\cdot z_1)}\phi^p_{3;f_0}(x)$$

$$\langle f_0(980)|\bar{q}_2(z_2)\gamma_\mu q_1(z_1)|0\rangle = p_\mu\bar{f}_{f_0}\int_0^1 dx e^{i(xp\cdot z_2 + \bar{x}p\cdot z_1)}\phi_{2;f_0}(x)$$

$z^2 \to 0$，对两边进行泰勒展开：

$$\langle f_0(980)|\bar{q}_2(0)q_1(0)|0\rangle = m_{f_0}\bar{f}_{f_0}\langle\xi^{p,0}_{3;f_0}\rangle$$

$$\langle f_0(980)|\bar{q}_2(0)\not{z}(iz\cdot\overleftrightarrow{D})^n q_1(0)|0\rangle = \bar{f}_{f_0}(z\cdot p)^{n+1}\langle\xi^n_{2;f_0}\rangle$$

$$\Downarrow$$

$$\boxed{J_n^{(s\bar{s})}(x) = \bar{s}(x)\not{z}(iz\cdot\overleftrightarrow{D})^n s(x) \qquad \hat{J}_0^{(s\bar{s}),\dagger}(0) = \bar{s}(0)s(0)}$$

2. 对关联函数作算符乘积展开(OPE):

$$
\begin{aligned}
\Pi^{(s\bar{s})}_{2;f_0}(z,q) &= i\int d^4x e^{iq\cdot x}\langle 0|T\{\bar{s}(x)\not{z}(iz\cdot \overleftrightarrow{D})^n s(x), \bar{s}(0)s(0)\}|0\rangle \\
&\Downarrow s \to s+\eta_s, \qquad \bar{s}\to\bar{s}+\bar{\eta}_s \\
&= i\int d^4x e^{iq\cdot x}\langle 0|T\{[\bar{s}(x)+\bar{\eta}_s(x)]\not{z}(iz\cdot \overleftrightarrow{D})^n[s(x)+\eta_s(x)], \\
&\qquad [\bar{s}(0)+\bar{\eta}_s(0)][s(0)+\eta_s(0)]\}|0\rangle
\end{aligned}
$$

s是夸克场，η_s是对应的量子起伏。展开总共有16项，但只有三项有贡献

$$
\begin{aligned}
\Pi^{(s\bar{s})}_{2;f_0}(z,q) = i\int d^4x e^{iq\cdot x}\{ &-\mathrm{Tr}\langle 0|S^s_F(0,x)\not{z}(iz\cdot \overleftrightarrow{D})^n S^s_F(x,0)|0\rangle \\
&+\mathrm{Tr}\langle 0|\bar{s}(x)s(0)\not{z}(iz\cdot \overleftrightarrow{D})^n S^s_F(x,0)|0\rangle \\
&+\mathrm{Tr}\langle 0|S^s_F(0,x)\not{z}(iz\cdot \overleftrightarrow{D})^n \bar{s}(0)s(x)|0\rangle \\
&+\cdots
\end{aligned}
$$

II. $D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e$ decay process

3.强子表示

对关联函数插入强子态完备集可得：

$$\mathrm{Im} I_{2;f_0}^{\mathrm{had}}(s) = \pi m_{f_0} \delta(s - m_{f_0}^2) \bar{f}_{f_0}^{(s\bar{s})2} \langle \xi_{2;f_0}^{(s\bar{s}),n} \rangle|_\mu \langle \xi_{3;f_0}^{p(s\bar{s}),0} \rangle|_\mu + \mathrm{Im} I_{2;f_0}^{\mathrm{pert}}(s) \theta(s - s_{f_0})$$

4.色散关系

利用色散关系令两种表示相等，

$$\frac{1}{\pi}\frac{1}{M^2}\int_{4m_s^2}^{s_{f_0}} ds e^{-s/M^2} \mathrm{Im} I_{2;f_0}^{\mathrm{had}}(s) = L_{M^2} I_{2;f_0}^{\mathrm{QCD}}(q^2)$$

再通过Borel变换压低高量纲与凝聚的贡献，可得最终的求和规则表达式，即：

Twist-2分布振幅的矩$\langle \xi_{3;f_0}^{p(s\bar{s}),0} \rangle \langle \xi_{2;f_0}^{(s\bar{s}),n} \rangle$

Twist-3分布振幅的零阶矩$\langle \xi_{3;f_0}^{p(s\bar{s}),0} \rangle$ $\longrightarrow$ 利用其对称性，即归一的物理性质，来确定连续态阈值s_{f_0}

II. $D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e$ decay process

关于形状因子计算，由如下的关联函数出发：

$$\begin{aligned}\Pi_\mu(p,q) &= i\int d^4x e^{iq\cdot x}\langle f_0(980)(p)|T\{\bar{s}(x)\gamma_\mu\gamma_5 c(x), \bar{c}(0)i\gamma_5 s(0)\}|0\rangle \\ &= F[q^2,(p+q)^2]p_\mu + \widetilde{F}[p^2,(p+q)^2]q_\mu.\end{aligned}$$

在经过OPE和强子表示后，将其等价，最终可得到形状因子表达式：

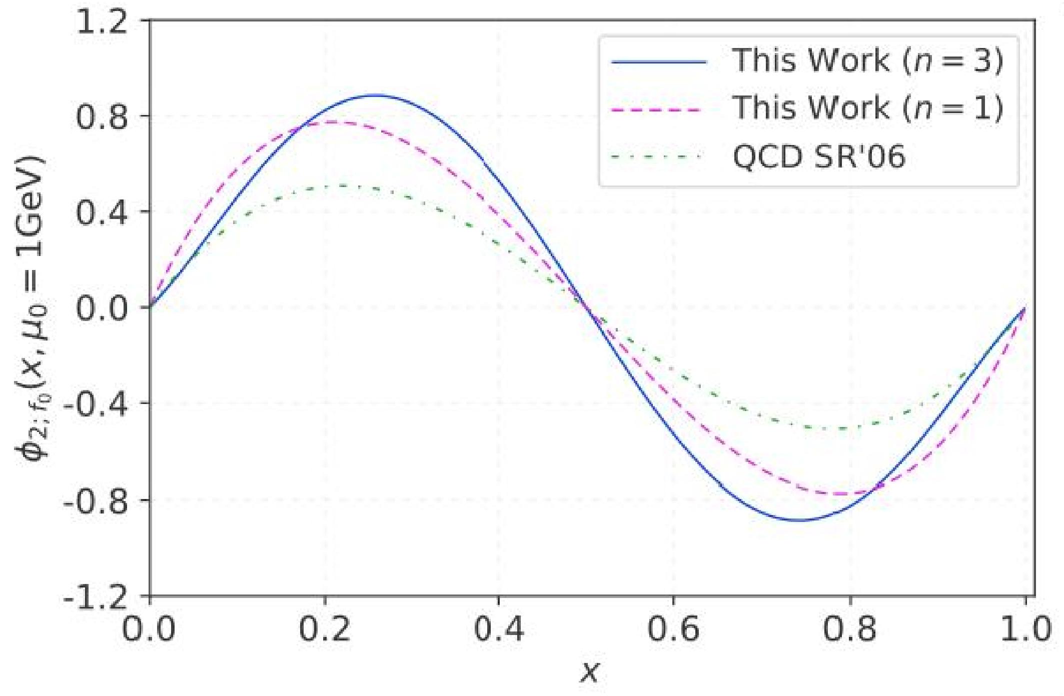
$$\begin{aligned}f_+(q^2) \;=\; & \frac{(m_c+m_s)\bar{f}_{f_0}}{2m_{D_s}^2 f_{D_s}} e^{m_{D_s^+}^2/M^2}\Bigg\{\int_{u_0}^1 du e^{-(m_c^2-\bar{u}q^2+u\bar{u}m_{f_0}^2)/(uM^2)}\Bigg[-m_c\frac{\phi_{2;f_0}(u)}{u} \\ & +m_{f_0}\left(\frac{2}{u}+\frac{4um_c^2m_{f_0}^2}{(m_c^2-q^2+u^2m_{f_0}^2)^2}-\frac{m_c^2+q^2-u^2m_{f_0}^2}{m_c^2-q^2+u^2m_{f_0}^2}\frac{d}{du}\right)\frac{\phi_{3;f_0}^\sigma(u)}{6}+m_{f_0} \\ & \times\phi_{3;f_0}^p(u)\Bigg]+e^{-(m_c^2-\bar{u}q^2+u\bar{u}m_{f_0}^2)/(uM^2)}\frac{m_{f_0}}{6m_c}\frac{m_c^2+q^2-u^2m_{f_0}^2}{m_c^2-q^2+u^2m_{f_0}^2}\phi_{3;f_0}^\sigma(u)\Big|_{u\to 1}\Bigg\}.\end{aligned}$$

II. $D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e$ decay process

1. $f_0(980)$twist-2分布振幅

$$\phi_{2;f_0}(x,\mu) = \bar{f}_{f_0}(\mu) 6x\bar{x}\left[B_0(\mu) + \sum_{n=1}^{\infty} B_n(\mu) C_n^{3/2}(\xi)\right]$$

$$B_1(\mu) = \frac{5}{3}\langle\xi^1_{2;f_0}\rangle|_\mu,$$
$$B_3(\mu) = \frac{3}{4}(7\langle\xi^3_{2;f_0}\rangle|_\mu - 3\langle\xi^1_{2;f_0}\rangle|_\mu)$$



$$\langle\xi^1_{2;f_0}\rangle|_{\mu_0} = -0.693^{+0.095}_{-0.081},$$
$$\langle\xi^1_{2;f_0}\rangle|_{\mu_k} = -0.542^{+0.074}_{-0.063},$$
$$\langle\xi^3_{2;f_0}\rangle|_{\mu_0} = -0.261^{+0.097}_{-0.095},$$
$$\langle\xi^3_{2;f_0}\rangle|_{\mu_k} = -0.217^{+0.079}_{-0.063}.$$

$$B_1(\mu_0) = -1.156^{+0.157}_{-0.133},$$
$$B_1(\mu_k) = -0.903^{+0.123}_{-0.104},$$
$$B_3(\mu_0) = 0.191^{+0.294}_{-0.317},$$
$$B_3(\mu_k) = 0.079^{+0.252}_{-0.269}.$$

预测的twist-2分布振幅是反对称行为，与QCDSR预测行为一致

II. $D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e$ decay process

2. $D_s \to f_0(980)$跃迁形状因子(TFF)

	$f_+(0)$
This work (LCSR)	$0.516^{+0.027}_{-0.024}$
BESIII'24 [28]	$0.518 \pm 0.018 \pm 0.036$
LCSR'10 [29]	0.300 ± 0.030
3PSR'04 [32]	0.50 ± 0.13
3PSR'10 [33]	0.48 ± 0.23
LCSR'24-s1 [35]	0.580 ± 0.070
LCSR'24-s2 [35]	0.400 ± 0.060
LCSR'24-s3 [35]	$0.780^{+0.130}_{-0.100}$

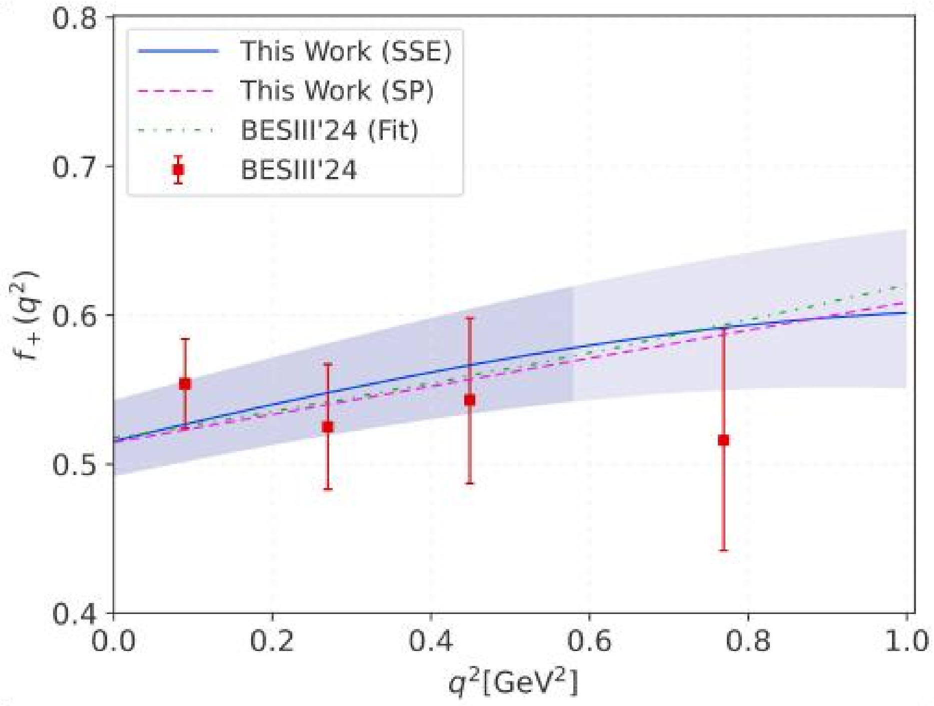
$$s_0 = 6.5 \pm 0.5\ GeV$$

$$M^2 = 7.0 \pm 1.0\ GeV^2$$

端点处的跃迁形状因子接近于最新的实验预测结果

简单级数展开参数化(SSE)

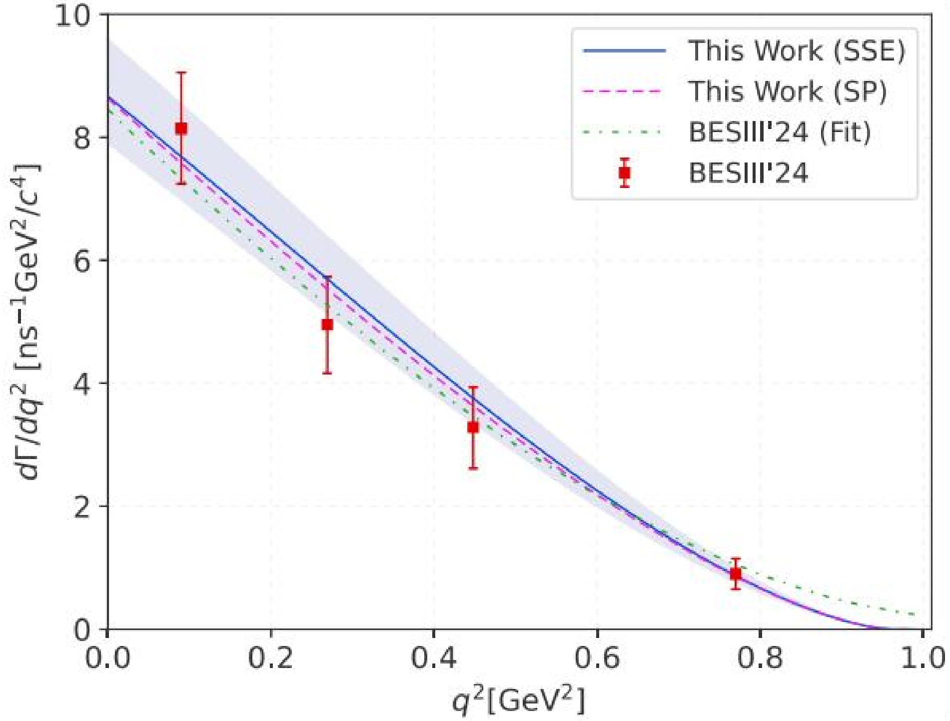
$$f_+(q^2) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{k=1}^{2} \alpha_k z^k(t, t_0)$$



我们的预测结果与BESIII最新的预测很接近

II. $D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e$ decay process

3. $D_s \to f_0(980)e^+\nu_e$的衰变宽度与分支比



	$\mathcal{B}(D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e)$
This work(SSE)	$(1.783^{+0.227}_{-0.189}) \times 10^{-3}$
This work(SP)	$(1.743^{+0.190}_{-0.157}) \times 10^{-3}$
CLEO'09 [24]	$(1.3 \pm 0.4 \pm 0.1) \times 10^{-3}$
CLEO'09 [25]	$(2.0 \pm 0.3 \pm 0.1) \times 10^{-3}$
BESIII'24 [28]	$(1.72 \pm 0.13 \pm 0.10) \times 10^{-3}$

- 我们预测的微分衰变宽度中心值包含误差均在BESIII'24的误差范围之内；
- 宽度和分支比与实验实验数据的主要区别来自于对不变质量s的取值范围；
- 衰变分支比与BESIII'24的预测值很接近，SSE参数化预测中心值只存在4%的误差；
- 相较于SSE参数化，单极点参数化(SP)更接近于实验预测

arXiv: 2506.15967

III. $D_s \to \phi l^+ \nu_l$ decay process

1. 实验与理论组得到的分支比有一定差距

2018 BESIII 2015 CLEO
2008 BaBar 2022 PDG

$\longrightarrow$ $D_s \to \phi \iota v_\iota \in [2.14,\ 2.61] \times 10^{-2}$

CLFQM CCQM 3PSR
CQM HQEFT LCSR

$\longrightarrow$ $D_s \to \phi \iota v_\iota \in [1.80, 3.10] \times 10^{-2}$

2. 目前只有一组理论组用LCSR方法计算$D_s \to \phi$的TFFs, γ_V, γ_2 与实验结果相差较大

理论组：LQCD HQEFT 3PSR CQM HM$_\chi$T LCSR(2004)

3.对于ϕ介子分布振幅的研究比较少

P. Ball predicted

$$a_{2;\phi}^{\parallel} = 0.18(8)$$
$$a_{2;\phi}^{\perp} = 0.14(7)$$

III. $D_s \to \phi l^+ \nu_l$ decay process

模型I

$$a^n_{2;\phi}(\mu) = \frac{\int_0^1 dx \phi^{\|}_{2;\phi}(x,\mu) C_n^{3/2}(2x-1)}{\int_0^1 dx 6x(1-x)[C_n^{3/2}(2x-1)]^2}. \rightarrow \boxed{b^2_{2;\phi} \quad b^4_{2;\phi}}$$

$$\langle \xi^{\|;2}_{2;\phi} \rangle|_\mu = \frac{1}{5} + \frac{12}{35} a^2_{2;\phi}(\mu) \qquad \langle \xi^{\|;4}_{2;\phi} \rangle|_\mu = \frac{3}{35} + \frac{8}{35} a^2_{2;\phi}(\mu) + \frac{8}{77} a^4_{2;\phi}(\mu),$$

$$\langle \xi^{\|;6}_{2;\phi} \rangle|_\mu = \frac{1}{21} + \frac{12}{77} a^2_{2;\phi}(\mu) + \frac{120}{1001} a^4_{2;\phi}(\mu) + \frac{64}{2145} a^6_{2;\phi}(\mu)$$

模型II

$$\chi^2(\theta) = \sum_{i=1}^{5} \frac{(y_i - \mu(x_i,\theta))^2}{\sigma_i^2}$$

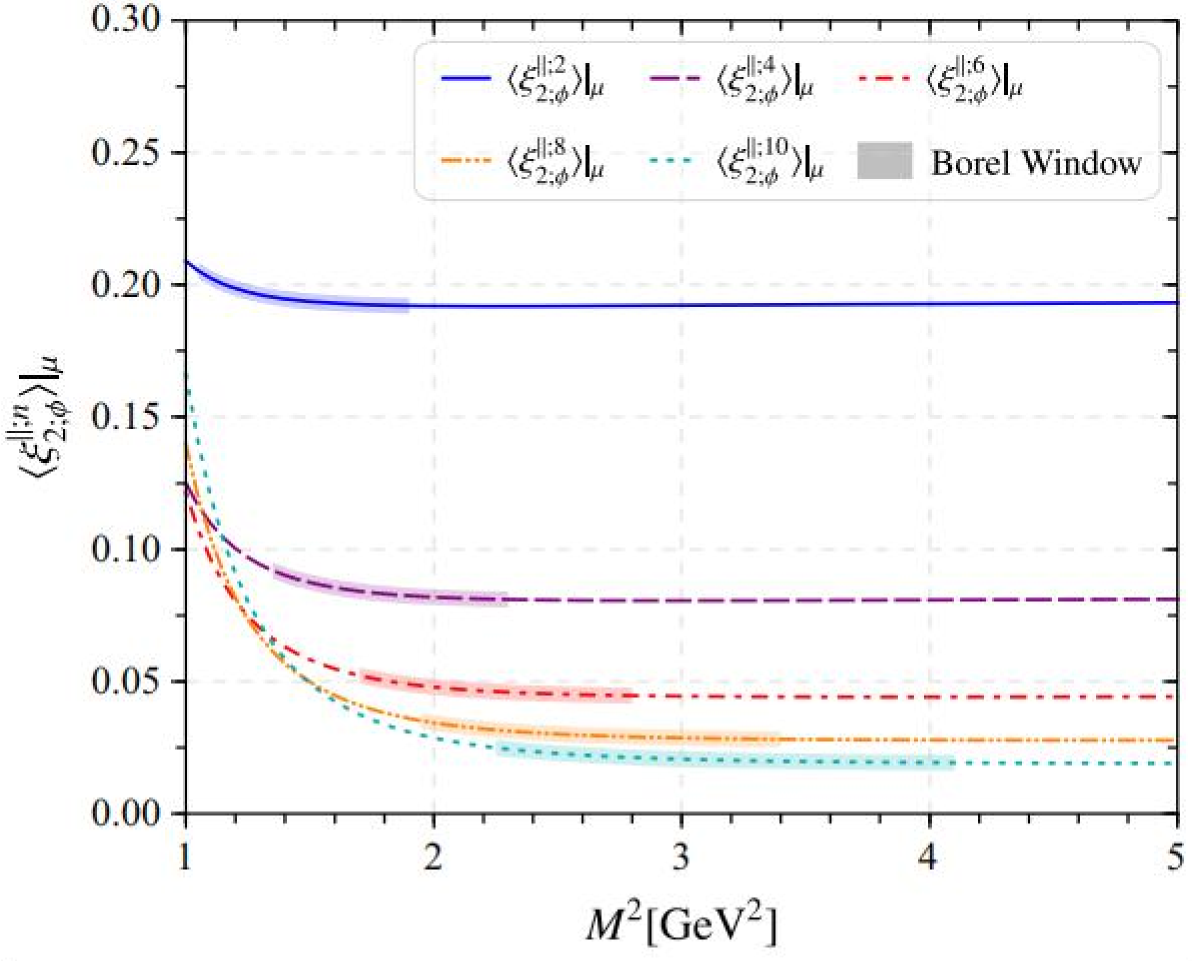
$$P_{\chi^2} = \int_{\chi^2}^{\infty} f(y; n_d) dy$$

$$f(y; n_d) = \frac{1}{\Gamma\left(\frac{n_d}{2}\right) 2^{\frac{n_d}{2}}} y^{\frac{n_d}{2}-1} e^{-\frac{y}{2}}$$

$$\Rightarrow \boxed{\alpha_{2;\phi} \quad B^2_{2;\phi}}$$

III. $D_s \to \phi l^+ \nu_l$ decay process

ϕ-介子前五阶分布振幅矩的整体行为以及具体数值



$$\langle\xi^{\|;2}_{2;\phi}\rangle|_{\mu_0} = 0.199 \pm 0.010,$$

$$\langle\xi^{\|;2}_{4;\phi}\rangle|_{\mu_0} = 0.086 \pm 0.006,$$

$$\langle\xi^{\|;2}_{6;\phi}\rangle|_{\mu_0} = 0.049 \pm 0.004,$$

$$\langle\xi^{\|;2}_{8;\phi}\rangle|_{\mu_0} = 0.032 \pm 0.003,$$

$$\langle\xi^{\|;2}_{10;\phi}\rangle|_{\mu_0} = 0.022 \pm 0.003.$$

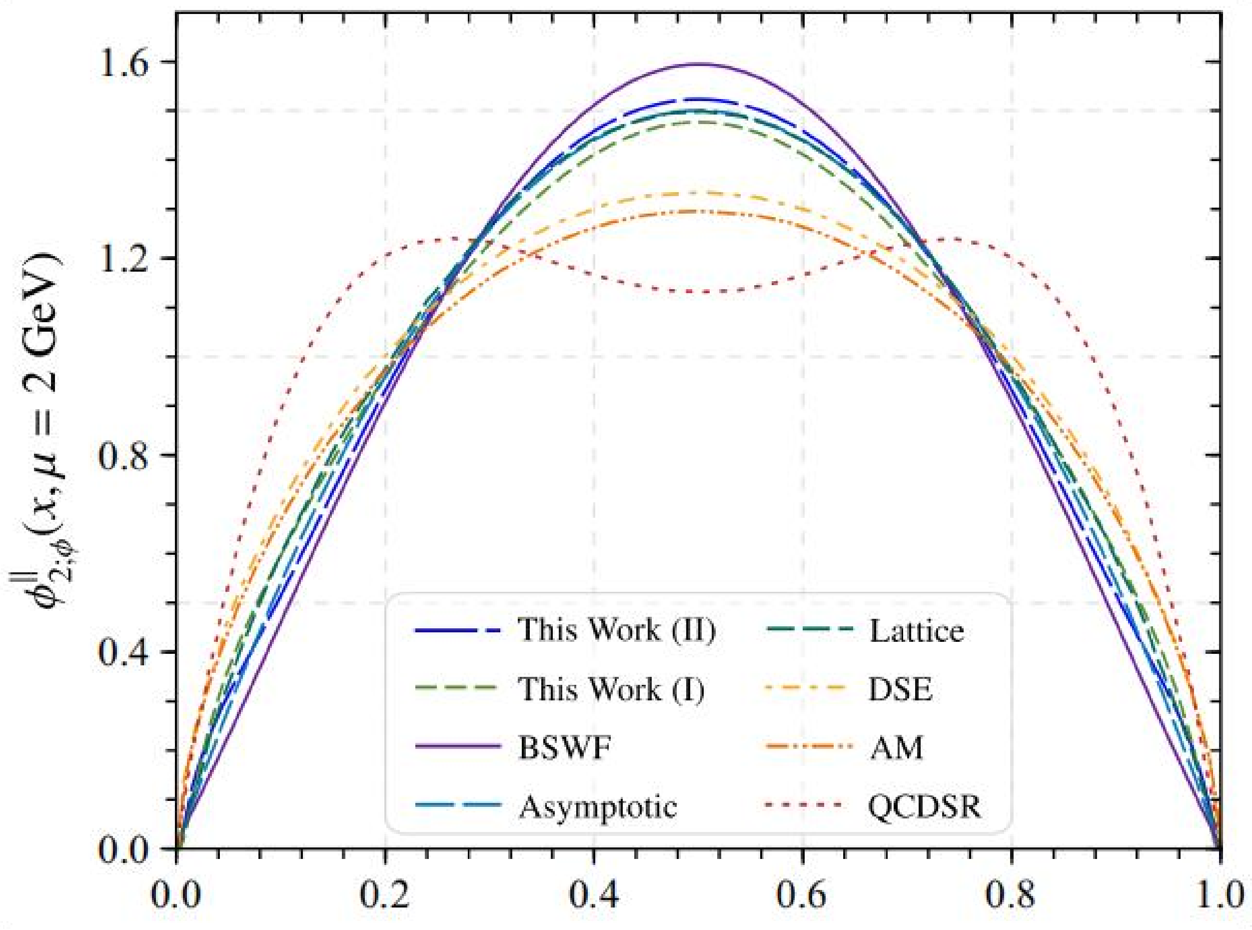
- 连续阈值$s_\phi = 2.1\ GeV^2$
- 连续态贡献分别不超过20%，25%，30%，35%，40%，六维凝聚贡献不超过5%

III. $D_s \to \phi l^+ \nu_l$ decay process

ϕ-介子twist-2的分布振幅

模型I：$A_{2;\phi}^{\parallel(\mathrm{I})} = 11.508\ \mathrm{GeV}^{-1},\quad \beta_{2;\phi}^{(\mathrm{I})} = 1.212\ \mathrm{GeV},\quad b_{2;\phi}^{2} = 0.061,\quad b_{2;\phi}^{4} = 0.020.$

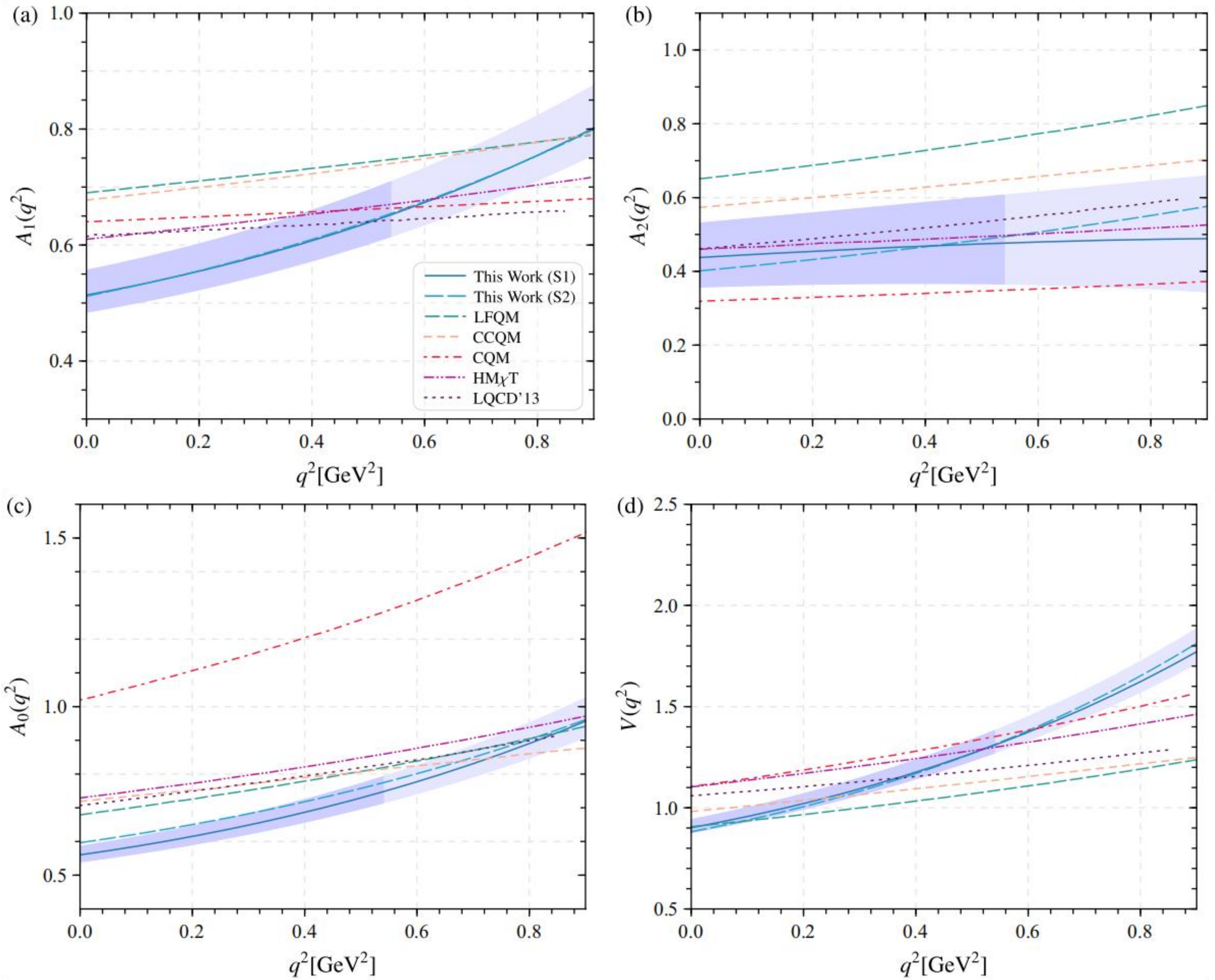
模型II $A_{2;\phi}^{\parallel(\mathrm{II})} = 2.515\ \mathrm{GeV}^{-1},\quad \alpha_{2;\phi} = -0.940,\quad B_{2;\phi}^{2} = -0.149,\ \beta_{2;\phi}^{(\mathrm{II})} = 1.207\ \mathrm{GeV}$



III. $D_s \to \phi l^+ \nu_l$ decay process

$D_s \to \phi$跃迁形状因子

	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
This work (S1)	$0.902^{+0.040}_{-0.024}$	$0.560^{+0.025}_{-0.021}$	$0.514^{+0.024}_{-0.016}$	$0.438^{+0.093}_{-0.080}$
This work (S2)	$0.882^{+0.040}_{-0.036}$	$0.596^{+0.025}_{-0.020}$	$0.512^{+0.030}_{-0.020}$	$0.402^{+0.078}_{-0.067}$
LQCD(2001) [22]	0.85(14)	0.63(2)	0.63(2)	0.62(78)
LQCD(2011) [23]	0.903(67)	0.686(17)	0.594(22)	0.401(80)
LQCD(2013) [24]	1.059(124)	0.706(37)	0.615(24)	0.457(78)
HQEFT [12]	$0.778^{+0.057}_{-0.062}$	$-0.757^{+0.029}_{-0.039}$	$0.569^{+0.046}_{-0.049}$	$0.304^{+0.021}_{-0.017}$
HMχT [14]	1.10	1.02	0.61	0.32
CQM [15]	1.10	0.73	0.64	0.47
3PSR [10]	1.21(33)	0.53(12)	0.55(15)	0.59(11)
CLFQM(2008) [18]	0.91	0.62	0.61	0.58
CLFQM(2011) [19]	0.98	0.72	0.69	0.57
LFQM [21]	1.24	0.71	0.77	0.66
LCSR [13]	0.70(10)	0.53(9)	0.54(9)	0.57(9)
CCQM [17]	0.91	0.68	0.68	0.67
RQM [27]	0.999	0.713	0.643	0.492
SCI [28]	1.00	0.66	0.61	0.44

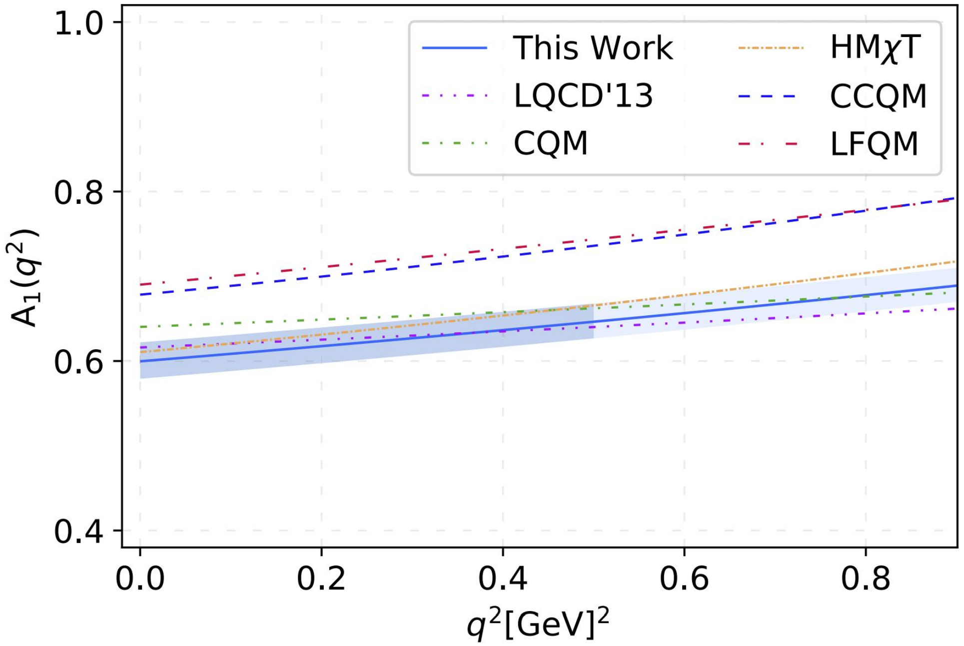


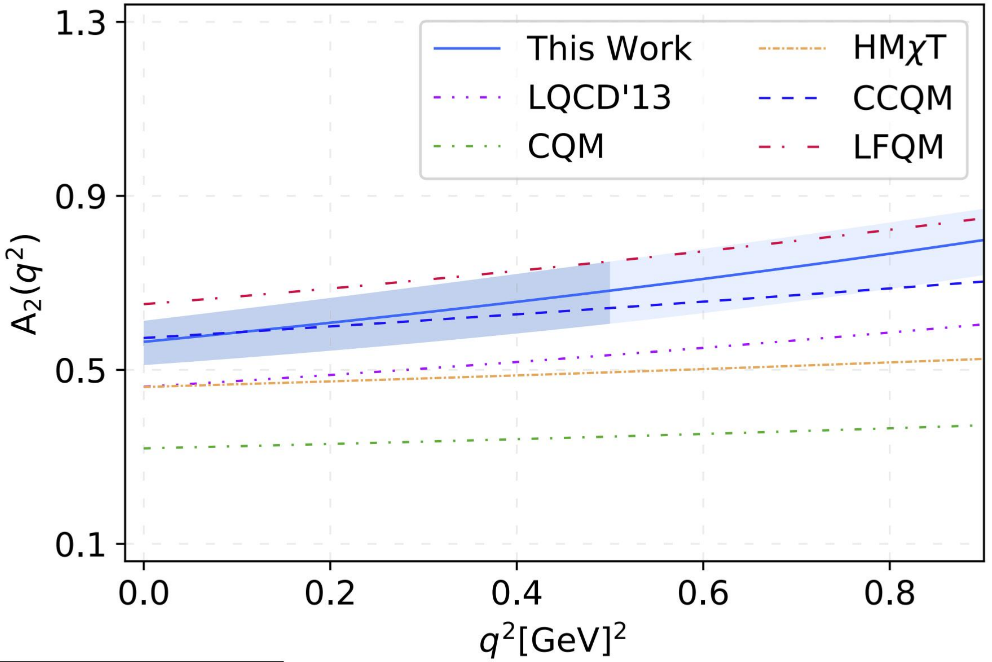
$D_s \to \phi l \nu_l$的分支比

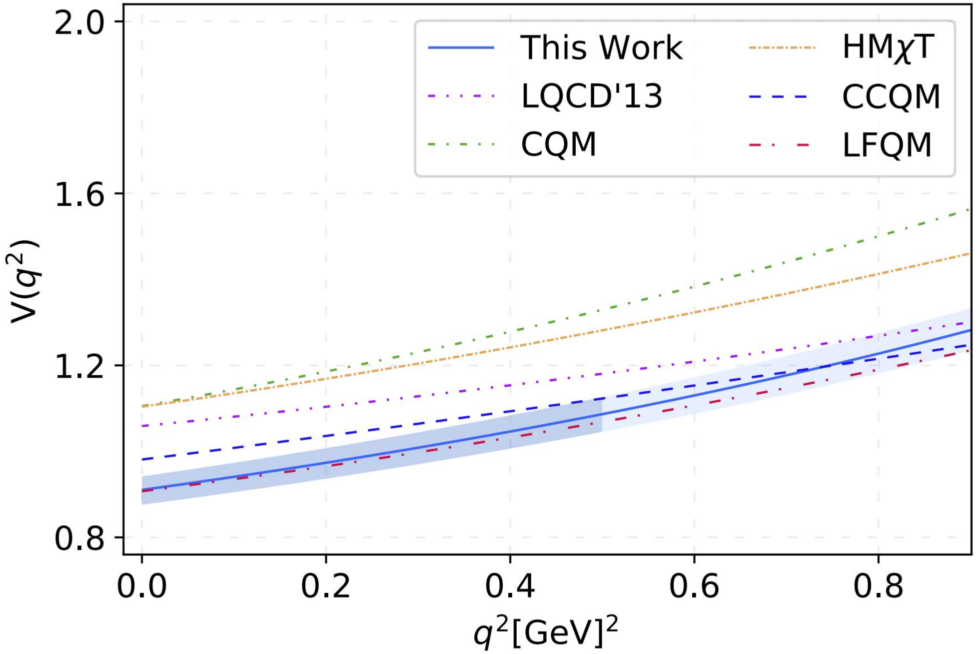
	$\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$	$\mathcal{B}(D_s^+ \to \phi \mu^+ \nu_\mu)$
This work (S1)	$2.347^{+0.342}_{-0.191}$	$2.330^{+0.341}_{-0.190}$
This work (S2)	$2.367^{+0.256}_{-0.132}$	$2.349^{+0.255}_{-0.132}$
BABAR [6]	$2.61 \pm 0.11 \pm 0.15$	$\cdots$
CLEO [7]	$2.14 \pm 0.17 \pm 0.08$	$\cdots$
BESIII(2017) [8]	$2.26 \pm 0.45 \pm 0.09$	1.94 ± 0.54
BESIII(2023) [9]	$\cdots$	$2.25 \pm 0.09 \pm 0.07$
PDG [79]	2.39 ± 0.16	1.90 ± 0.5
CLFQM(2017) [20]	3.1 ± 0.3	2.9 ± 0.3
3PSR(2004) [10]	1.80 ± 0.50	$\cdots$
CLFQM(2008) [18]	2.30	$\cdots$
LCSR [13]	$2.15^{+0.27}_{-0.31}$	$\cdots$
HQEFT [12]	$2.53^{+0.37}_{-0.40}$	$2.40^{+0.35}_{-0.40}$
CCQM [17]	3.01	2.85
CQM [15]	2.57	2.57
χUA [26]	2.12	1.94
RQM [27]	2.69	$\cdots$
SCI [28]	2.45	2.30

arXiv: 2403.10003

III. $D_s \to \phi l^+ \nu_l$ decay process







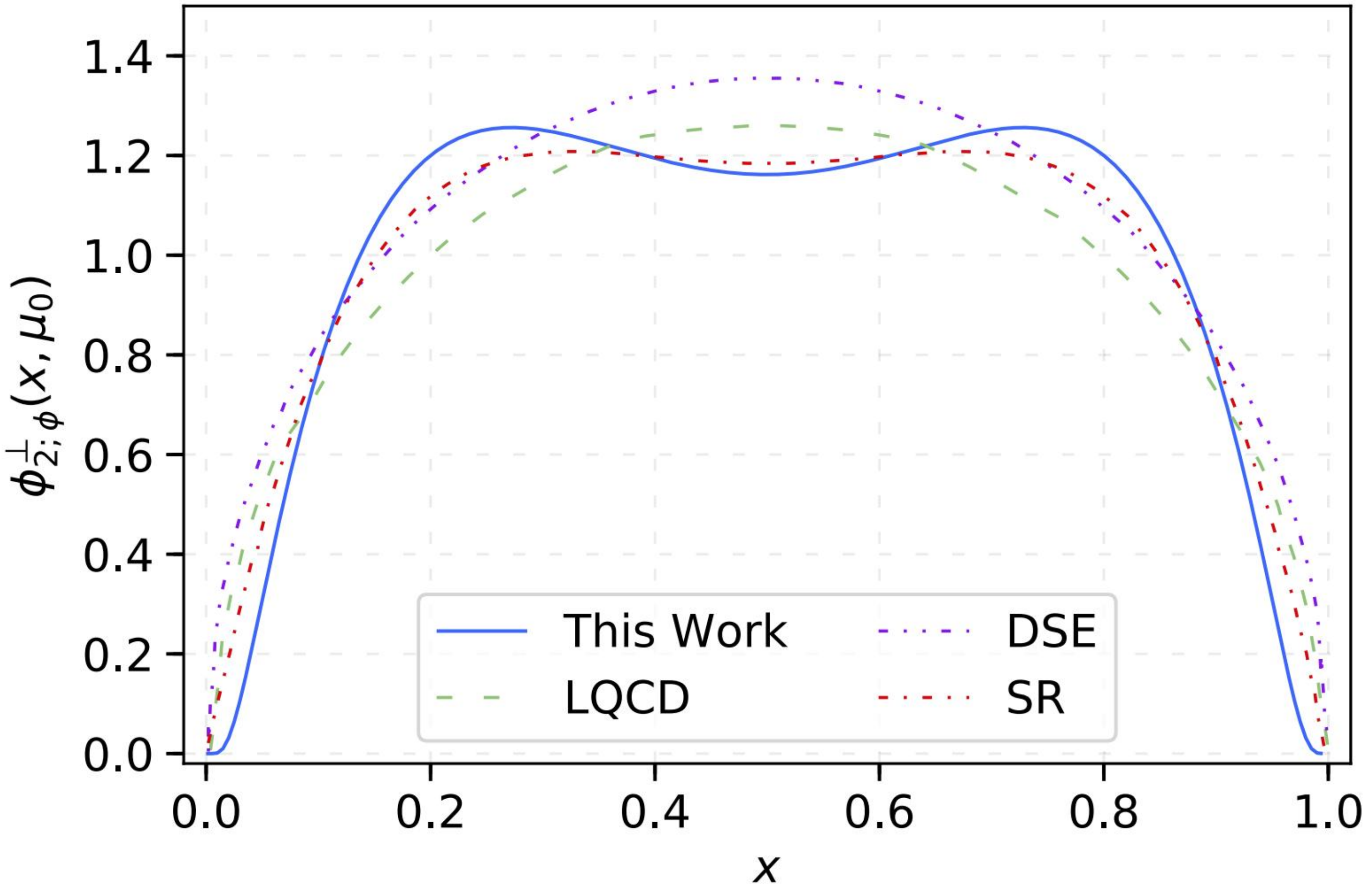
arXiv: 2505.15014

IV. Summary

- 背景场理论框架下采用SVZ求和规则计算$f_0(980)$介子的twist-2分布振幅的矩；
- 光锥求和规则计算$D_s \to f_0(980)$跃迁形状因子及半轻衰变过程的观测量
- ϕ介子的twist-2分布振幅，以及$D_s \to \phi$形状因子、衰变宽度、分支比以及极化和不对称性参数

Thanks for your attention!

IV. Summary



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\phi^{\perp}_{2;\phi}(x,\mu_0)$
0.0
0.2
0.4
0.6
0.8
1.0
x
This Work
LQCD
DSE
SR