含粲少体强子分子态性质的研究

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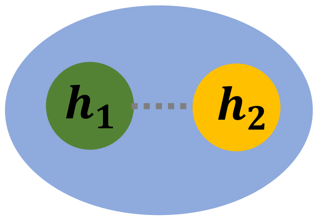
报告提纲

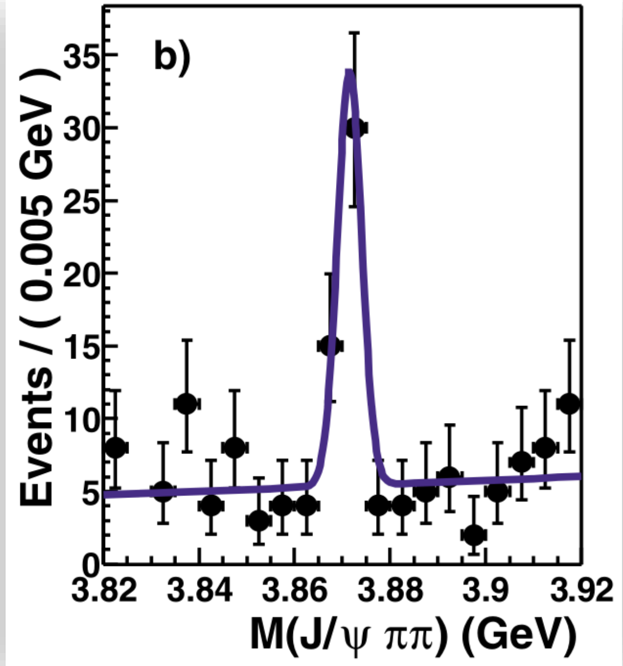
1. 研究背景
2. 模型方法
3. 数值结果
4. 总结

1. 研究背景

- 强子分子态：由若干个色单态的强子组成的松散物质
- 典型特征：质量接近阈值



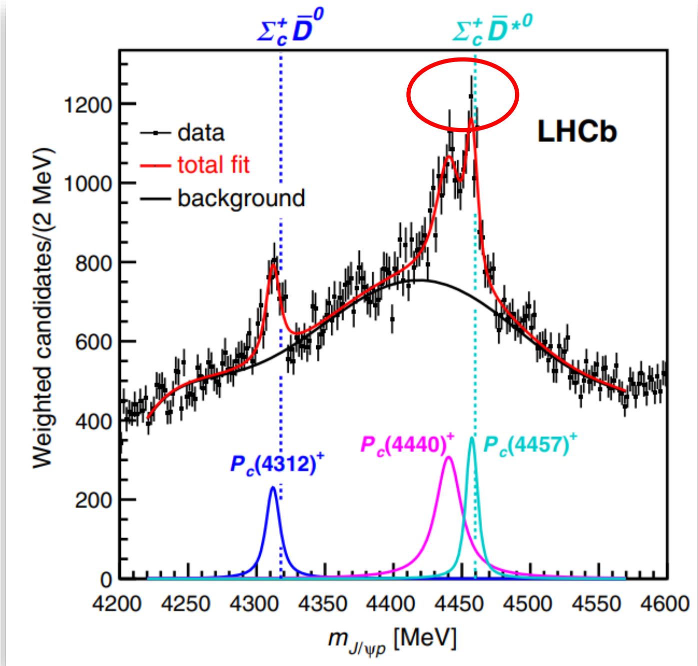
2003年：$X(3872)$



[Belle] Phys. Rev. Lett. 91, 262001 (2003)

- 质量接近$D\bar{D}^*$的阈值
- 首个XYZ粒子
- 为奇特强子态的研究打开了新的大门

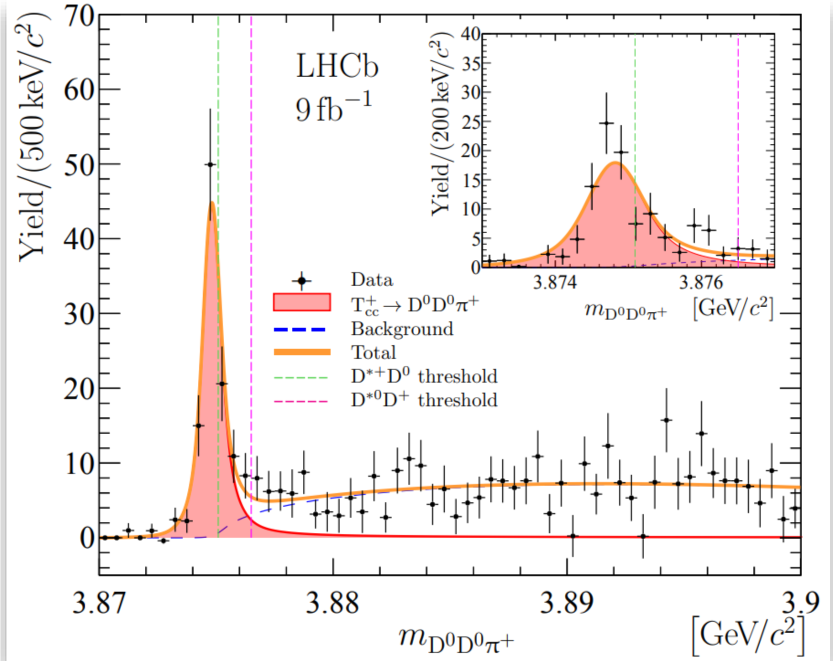
2019年：P_c



[LHCb] Phys. Rev. Lett. 122, 222001 (2019)

- 发现了$P_c(4450)$的子结构
- 为强子分子态提供了直接证据

2021年：T_{cc}^+



[LHCb] Nature Phys. 18, 751-754 (2022)
[LHCb] Nature Commun. 13, 3351 (2022)

- 质量接近DD^*的阈值
- 首次发现双粲四夸克物质

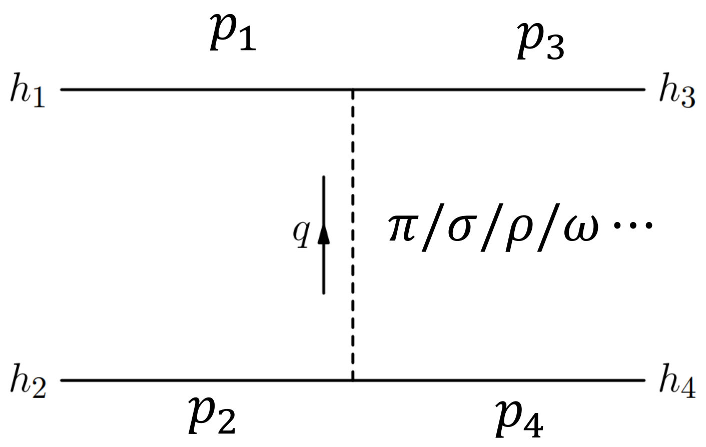
2. 模型方法

2.1 强子-强子间的相互作用势

常用方法：

1. 格点QCD
2. QCD求和规则
3. 共振群(RGM)方法
4. 单玻色子交换模型

⋮



（单玻色子交换模型）

动量表象下强子-强子间的有效势可以通过散射振幅得到：

$$V(\mathbf{q}) = -\frac{\mathcal{M}(h_1 h_2 \to h_3 h_4)}{\sqrt{\Pi 2M_f \Pi 2M_i}},$$

考虑形状因子，并通过Fourier变换，我们得到坐标表象下的有效势：

$$V(\mathbf{r}) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} V(\mathbf{q}) \mathcal{F}^2(q^2, m_E^2),$$

$$\mathcal{F}(q^2, m_E^2) = (\Lambda^2 - m_E^2)/(\Lambda^2 - q^2)$$

（形状因子）

2.3 高斯展开法

$$\phi_{nlm}(\mathbf{r}) = \phi_{nl}(r)Y_{lm}(\hat{\mathbf{r}})$$

$$\phi_{nl}(r) = N_{nl}r^l e^{-\nu_n r^2}$$

$$N_{nl} = \sqrt{\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)!!}}$$

$$\nu_n = \frac{1}{r_n^2} \quad r_n = r_1 a^{n-1} \quad (n = 1 - n_{\max})$$

优点：

1. 无节点，使得积分的速度较快
2. 指数上的二次形式便于Jacobi坐标变换，使其可以推广到包含多个空间自由度的系统
3. 仅使用少量的高斯基就可以使得特征值收敛至预期的精度

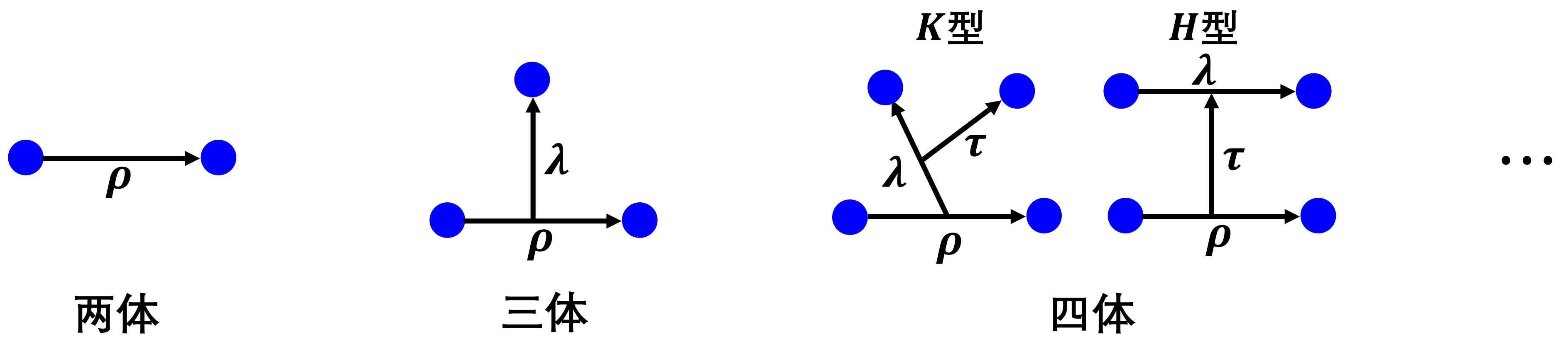
应用：

1. 原子物理
2. 分子物理
3. 原子核物理
4. 强子物理

⋮

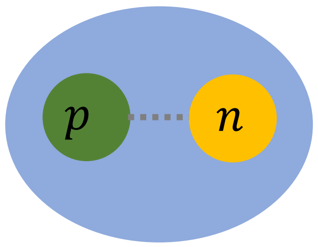
E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223-307 (2003)

Jacobi坐标



若系统包含多个空间自由度，则使用多个高斯基耦合

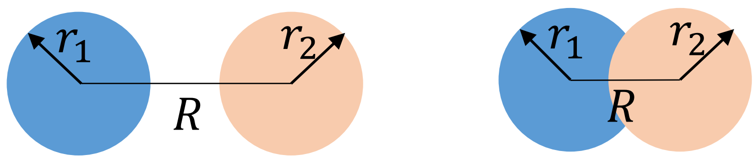
- 典型的强子分子态：氘核



氘核

- Cutoff Λ的范围：≈1 GeV

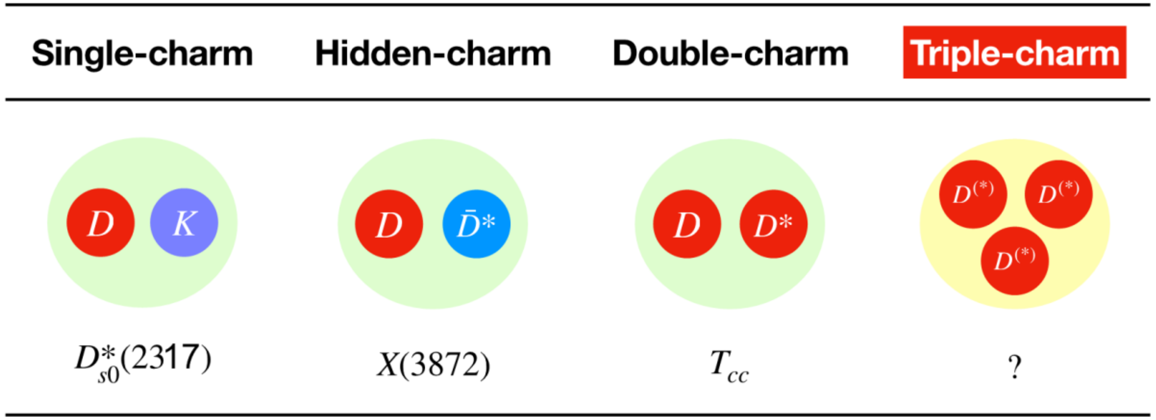
- 方均根半径：R不能远小于$r_1 + r_2$



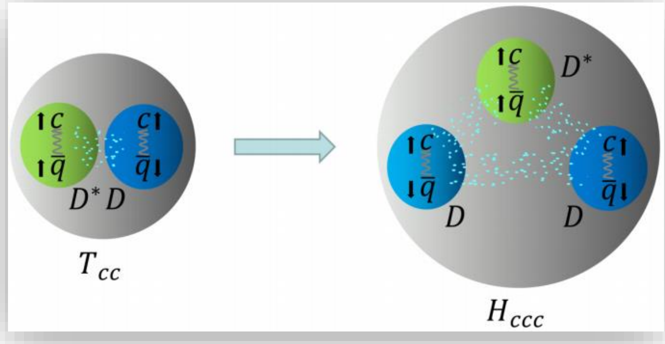
- 基矢选择：单道S波→S-D混合→耦合道效应

3.数值结果（$D^{(*)}D^{(*)}D^{*}$、$DD^{*}N$、类氢强子物质）

3.1 $D^{(*)}D^{(*)}D^{*}$构型的强子分子态



单粲、隐粲、两粲的分子态均存在好的候选态，那么是否存在三粲分子态候选态(H_{ccc})?



利用T_{cc}束缚能的实验值为相互作用势的参数定标

将参数运用到 H_{ccc} 中，计算三个粲介子能否形成束缚态

有效势：

$$V^{DD\to DD} = -g_\sigma^2 Y_\sigma + \frac{1}{2}\beta^2 g_V^2 \left(C_1(I)Y_\rho + C_0(I)Y_\omega\right),$$

$$V^{DD^*\to DD^*} = -g_\sigma^2 \mathcal{O}_1 Y_\sigma + \frac{1}{2}\beta^2 g_V^2 \mathcal{O}_1 \left(C_1(I)Y_\rho + C_0(I)Y_\omega\right),$$

$$\begin{aligned} V^{DD^*\to D^*D} =& \frac{g^2}{3f_\pi^2}(\mathcal{O}_2\hat{\mathcal{P}} + \mathcal{O}_3\hat{\mathcal{Q}})C_1'(I)Y_{\pi1} \\ &+ \frac{2}{3}\lambda^2 g_V^2 (2\mathcal{O}_2\hat{\mathcal{P}} - \mathcal{O}_3\hat{\mathcal{Q}})\left(C_1'(I)Y_{\rho1} + C_0'(I)Y_{\omega1}\right), \end{aligned}$$

$$\begin{aligned} V^{DD^*\to D^*D^*} =& \frac{g^2}{3f_\pi^2}(\mathcal{O}_4\hat{\mathcal{P}} + \mathcal{O}_5\hat{\mathcal{Q}})C_1(I)Y_{\pi2} \\ &+ \frac{2}{3}\lambda^2 g_V^2 (2\mathcal{O}_4\hat{\mathcal{P}} - \mathcal{O}_5\hat{\mathcal{Q}})\left(C_1(I)Y_{\rho2} + C_0(I)Y_{\omega2}\right), \end{aligned}$$

$$\begin{aligned} V^{D^*D^*\to D^*D^*} =& -g_\sigma^2 \mathcal{O}_6 Y_\sigma + \frac{1}{2}\beta^2 g_V^2 \mathcal{O}_6 \left(C_1(I)Y_\rho + C_0(I)Y_\omega\right) \\ &- \frac{g^2}{3f_\pi^2}(\mathcal{O}_7\hat{\mathcal{P}} + \mathcal{O}_8\hat{\mathcal{Q}})C_1(I)Y_\pi \\ &- \frac{2}{3}\lambda^2 g_V^2 (2\mathcal{O}_7\hat{\mathcal{P}} - \mathcal{O}_8\hat{\mathcal{Q}})\left(C_1(I)Y_\rho + C_0(I)Y_\omega\right). \end{aligned}$$

耦合常数和介子质量：

耦合常数	数值 [1–4]	介子	质量(GeV) [5]
g	0.6	π	0.140
f_π	0.132 GeV	σ	0.600
g_σ	3.4	ρ	0.770
βg_V	5.2	ω	0.780
λg_V	3.133 GeV^{-1}	D	1.867
		D^*	2.009

算符和参数：

$$\begin{aligned} \mathcal{O}_1 &= \epsilon_4^\dagger \cdot \epsilon_2, \\ \mathcal{O}_2 &= \epsilon_3^\dagger \cdot \epsilon_2, \\ \mathcal{O}_3 &= S(\mathbf{r}, \epsilon_3^\dagger, \epsilon_2), \\ \mathcal{O}_4 &= \epsilon_3^\dagger \cdot (i\epsilon_4^\dagger \times \epsilon_2), \\ \mathcal{O}_5 &= S(\mathbf{r}, \epsilon_3^\dagger, i\epsilon_4^\dagger \times \epsilon_2), \\ \mathcal{O}_6 &= (\epsilon_3^\dagger \cdot \epsilon_1)(\epsilon_4^\dagger \cdot \epsilon_2), \\ \mathcal{O}_7 &= (\epsilon_3^\dagger \times \epsilon_1)\cdot(\epsilon_4^\dagger \times \epsilon_2), \\ \mathcal{O}_8 &= S(\mathbf{r}, \epsilon_3^\dagger \times \epsilon_1, \epsilon_4^\dagger \times \epsilon_2), \end{aligned}$$

$$S(\mathbf{r}, \mathbf{a}, \mathbf{b}) = \frac{3(\mathbf{a}\cdot\mathbf{r})(\mathbf{b}\cdot\mathbf{r})}{r^2} - \mathbf{a}\cdot\mathbf{b}$$

$$\Lambda_i = \sqrt{\Lambda^2 - q_i^2} \qquad m_{Ei} = \sqrt{m_E^2 - q_i^2}$$

Λ (高动量截断因子)：使用T_{cc}的束缚能定标

$$q_1 = m_{D^*} - m_D$$

$$q_2 = (m_{D^*}^2 - m_D^2)/(4m_{D^*})$$

$$\hat{\mathcal{P}} = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}, \quad \hat{\mathcal{Q}} = r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}$$

$$C_1(0) = -\frac{3}{2},\ C_1'(0) = +\frac{3}{2},\ C_0(0) = +\frac{1}{2},\ C_0'(0) = -\frac{1}{2},$$

$$C_1(1) = +\frac{1}{2},\ C_1'(1) = +\frac{1}{2},\ C_0(1) = +\frac{1}{2},\ C_0'(1) = +\frac{1}{2}.$$

$$Y_i = \frac{e^{-m_{Ei}r}}{4\pi r} - \frac{e^{-\Lambda_i r}}{4\pi r} - \frac{\Lambda_i^2 e^{-\Lambda_i r}}{8\pi\Lambda_i} + \frac{m_{Ei}^2 e^{-\Lambda_i r}}{8\pi\Lambda_i}$$

[1] M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie and L. S. Geng, Phys. Rev. D 99, 094018 (2019)
[2] D. O. Riska and G. E. Brown, Nucl. Phys. A 679, 577-596 (2001)
[3] M. Gell-Mann and M. Levy, Nuovo Cim. 16, 705 (1960)
[4] C. Isola, M. Ladisa, G. Nardulli and P. Santorelli, Phys. Rev. D 68, 114001 (2003)
[5] P. A. Zyla *et al.* [Particle Data Group], PTEP 2020, 083C01 (2020)

➢ 模型在DD^*系统中的验证

DD^*的束缚能

S-wave			S-D mixing		
Λ (MeV)	B (MeV)	r_{DD^*} (fm)	Λ (MeV)	B (MeV)	r_{DD^*} (fm)
976	0.3	5.94	945	0.3	6.09
998	1.0	3.47	970	1.0	3.55
1013	1.7	2.72	986	1.7	2.81

得到$\Lambda \approx$1 GeV

PHYSICAL REVIEW D **88**, 114008 (2013)

Coupled-channel analysis of the possible $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$ and $D^{(*)}\bar{B}^{(*)}$ molecular states

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(Received 22 November 2012; revised manuscript received 17 September 2013; published 3 December 2013)

I	J^P		$D^{(*)}D^{(*)}$ OPE				$D^{(*)}D^{(*)}$ OBE			
	0^+		* * *				* * *			
0	1^+	Λ (GeV)	1.05	1.10	1.15	1.20	0.95	1.00	1.05	1.10
		B.E. (MeV)	1.24	4.63	11.02	20.98	0.47	5.44	18.72	42.82
		M (MeV)	3874.61	3871.22	3864.83	3854.87	3875.38	3870.41	3857.13	3833.03
		$r_{\rm rms}$ (fm)	3.11	1.68	1.12	0.84	4.46	1.58	0.91	0.64
		P_1 (%)	96.39	92.71	88.22	83.34	97.97	92.94	85.64	77.88
		P_2 (%)	0.73	0.72	0.57	0.42	0.58	0.55	0.32	0.15
		P_3 (%)	2.79	6.45	11.07	16.11	1.41	6.42	13.97	21.91
		P_4 (%)	0.08	0.13	0.14	0.13	0.04	0.09	0.08	0.05

CHIN. PHYS. LETT. Vol. 38, No. 9 (2021) 092001 Views & Comments

Perfect DD^* Molecular Prediction Matching the T_{cc} Observation at LHCb

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In 2012, we investigated the possible molecular states composed of two charmed mesons [Phys. Rev. D 88 (2013) 114008; 2012 arXiv:1211.5007 [hep-ph]]. The D^*D system with the quantum numbers of $I(J^P) = 0(1^+)$ was found to be a good candidate of the loosely bound molecular state. This state is very close to the D^*D threshold with a binding energy around 0.47 MeV. This prediction was confirmed by the new LHCb observation of T_{cc}^+ [see Franz Muheim's talk at the European Physical Society conference on high energy physics 2021].

DOI: 10.1088/0256-307X/38/9/092001

➢ DDD^*

$$I(1/2,-1/2): -\sqrt{\frac{2}{3}}D^0D^0D^{*+} + \sqrt{\frac{1}{6}}(D^+D^0 + D^0D^+)D^{*0}$$

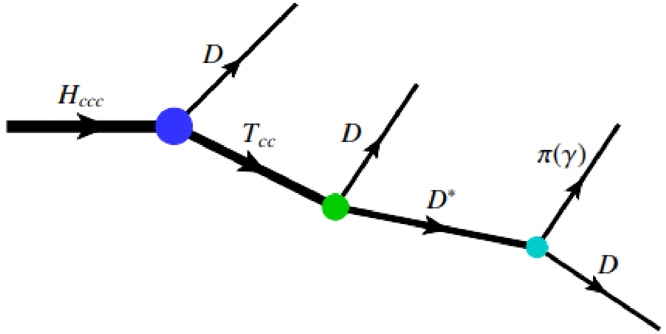
Λ(MeV)	B(MeV)	r_{DD^*}	r_{DD}	$\langle T\rangle$	$\langle V_{DD^*}\rangle$	$\langle V_{DD}\rangle$
976	0.56	5.64	7.17	15.95	−15.90	−0.60
998	1.87	3.28	4.13	29.18	−29.57	−1.49
1013	2.60	2.60	3.27	38.05	−39.10	−2.13

$$I(1/2,1/2): \sqrt{\frac{2}{3}}D^+D^+D^{*0} - \sqrt{\frac{1}{6}}(D^+D^0 + D^0D^+)D^{*+}$$

Λ(MeV)	B(MeV)	r_{DD^*}	r_{DD}	$\langle T\rangle$	$\langle V_{DD^*}\rangle$	$\langle V_{DD}\rangle$
976	0.16	8.83	10.74	7.65	−7.81	−0.00
998	1.09	4.50	5.86	23.65	−24.14	−0.60
1013	2.22	3.15	4.04	33.34	−34.40	−1.16

存在库仑排斥力

衰变：



1. $I(J^P)=\frac{1}{2}(1^-)$的DDD^*可以形成束缚态
2. 我们建议通过$DDD\pi$或$DDD\gamma$寻找DDD^*

➢ D^*D^*D

		S-wave				S-D mixing				coupled channels			
I	J^P	Λ (GeV)	B (MeV)	$r_{D^*D^*}$ (fm)	r_{D^*D} (fm)	Λ (GeV)	B (MeV)	$r_{D^*D^*}$ (fm)	r_{D^*D} (fm)	Λ (GeV)	B (MeV)	$P_{D^*D^*D}$ (%)	$P_{D^*D^*D^*}$ (%)
$\frac{1}{2}$	0^-	0.98	0.45	7.73	6.11	0.95	0.44	7.64	6.08	0.92	0.84	99.34	0.66
		1.03	3.60	3.91	3.03	1.00	3.38	3.81	2.98	0.97	5.44	97.95	2.05
		1.08	9.65	2.55	1.99	1.05	9.10	2.51	1.97	1.02	14.79	95.86	4.14
	1^-	0.98	0.86	3.52	3.88	0.95	0.44	5.11	5.25	0.93	0.81	99.29	0.71
		1.03	6.84	1.55	1.82	1.00	5.16	1.81	2.11	0.98	6.22	97.95	2.05
		1.08	17.43	1.10	1.31	1.05	14.21	1.23	1.46	1.03	18.03	95.50	4.50
	2^-	0.97	0.82	4.36	3.67	0.94	0.57	5.34	4.45	0.90	0.48	99.26	0.74
		1.02	6.30	2.00	1.71	0.99	5.04	2.28	1.94	0.95	7.05	96.32	3.68
		1.07	16.19	1.41	1.22	1.04	13.54	1.56	1.35	1.00	22.16	91.42	8.58
$\frac{3}{2}$	0^-	1.76	0.41	5.26	4.44	1.75	0.37	5.33	4.52	1.76	0.63	99.99	0.01
		1.81	1.94	3.27	2.70	1.80	1.81	3.31	2.74	1.81	2.28	99.98	0.02
		1.86	4.48	2.50	2.03	1.85	4.21	2.52	2.06	1.86	4.91	99.98	0.02
	1^-	1.85	0.46	11.75	8.68	1.85	0.54	11.98	8.80	1.85	0.57	~100	~0
		1.90	2.20	11.23	8.09	1.90	2.41	1.88	14.38	1.90	2.41	~100	~0
		1.95	4.97	10.93	7.81	1.95	5.61	1.35	14.26	1.95	5.61	~100	~0
	2^-	1.49	0.86	2.26	2.29	1.48	0.66	2.48	2.51	1.47	0.31	99.97	0.03
		1.54	5.16	1.32	1.34	1.53	4.48	1.39	1.42	1.52	3.68	99.96	0.04
		1.59	12.46	0.98	1.00	1.58	11.10	1.03	1.04	1.57	9.70	99.96	0.04

1. $I(J^P)=\frac{1}{2}(0^-,1^-,2^-)$的$D^*D^*D$可以形成束缚态；
2. 我们建议通过如下过程寻找D^*D^*D：
   (a) 三个粲介子+π或γ；
   (b) 双粲强子分子态+粲介子；
   (c) 三个粲介子。

- $D^*D^*D^*$

I	J^P	S-wave Λ (GeV)	S-wave B (MeV)	S-wave $r_{D^*D^*}$ (fm)	S-D Mixing Λ (GeV)	S-D Mixing B (MeV)	S-D Mixing $r_{D^*D^*}$ (fm)
$\frac{1}{2}$	0^-	1.02	0.67	9.75	1.00	0.84	9.92
		1.07	4.48	9.06	1.05	4.57	9.17
		1.12	10.72	8.81	1.10	10.63	8.88
	1^-	1.00	0.67	5.52	0.97	0.44	6.02
		1.05	5.02	2.51	1.02	4.30	2.61
		1.10	12.83	1.73	1.07	11.52	1.78
	2^-	0.98	0.37	4.71	0.96	0.51	4.53
		1.03	5.94	1.79	1.01	5.80	1.87
		1.08	16.33	1.25	1.06	15.55	1.31
	3^-	1.84	0.51	9.74	1.02	0.35	12.19
		1.89	2.39	9.18	1.07	4.03	11.72
		1.94	5.38	8.90	1.12	10.23	11.63
$\frac{3}{2}$	1^-	1.80	0.20	7.82	1.81	0.46	7.21
		1.85	1.40	6.11	1.86	1.89	5.59
		1.90	3.59	4.92	1.91	4.37	4.45
	2^-	1.85	0.81	9.60	1.84	0.61	9.89
		1.90	2.89	9.13	1.89	2.51	9.29
		1.95	6.13	8.88	1.94	5.53	8.98
	3^-	1.48	0.23	2.95	1.48	0.78	2.31
		1.53	4.21	1.38	1.53	4.90	1.34
		1.58	11.28	1.00	1.58	11.93	1.00

1. $I(J^P)=\frac{1}{2}(0^-,1^-,2^-,3^-)$的$D^*D^*D^*$容易形成束缚态；
2. $D^*D^*D^*$的质量较大，因此，$D^*D^*D^*$与DDD^*和D^*D^*D拥有相似的衰变模式。

3.2 DD^*N构型的强子分子态

$\Sigma_c(2800)$的质量和宽度
$(m_D + m_N \approx 2808$ MeV)

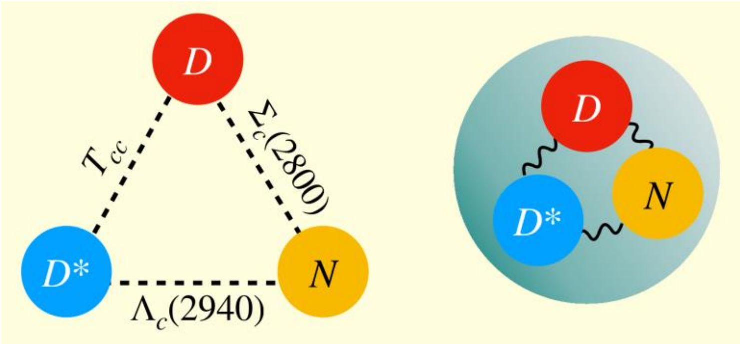
		PDG [1]	Belle [2]	BaBar [3]
Σ_c^{++}	Mass (MeV)	2801^{+4}_{-6}	$2801^{+3.4+2.8}_{-3.1-4.9}$	
	Width (MeV)		75^{+18+12}_{-13-11}	
Σ_c^{+}	Mass (MeV)	2792^{+14}_{-5}	$2792^{+5.8+12.4}_{-4.6-2.0}$	
	Width (MeV)		62^{+37+52}_{-23-38}	
Σ_c^{0}	Mass (MeV)	2806^{+5}_{-7}	$2802^{+3.2+2.1}_{-3.1-6.0}$	$2846 \pm 8 \pm 10$
	Width (MeV)	72^{+22}_{-15}	61^{+18+22}_{-13-13}	$86^{+33}_{-22} \pm 12$

$\Lambda_c(2940)$的质量和宽度
$(m_{D^*} + m_N \approx 2949$ MeV)

Collaboration	Mass (MeV)	width (MeV)
PDG [1]	$2939.6^{+1.3}_{-1.5}$	20^{+6}_{-5}
BaBar [4]	$2939.8 \pm 1.3 \pm 1.0$	$17.5 \pm 5.2 \pm 5.9$
Belle [5]	$2938.0 \pm 1.3^{+2.0}_{-4.0}$	13^{+7+27}_{-5-7}
LHCb [6]	$2944.8^{+3.5}_{-2.5} \pm 0.4^{+0.1}_{-4.6}$	$27.7^{+8.2}_{-6.0} \pm 0.9^{+5.2}_{-10.4}$

T_{cc}^{+} 的质量和宽度 [7, 8]

$$m_{\mathrm{BW}} = (m_{D^{*+}} + m_{D^0}) - 273 \pm 61 \pm 5^{+11}_{-14}\ \mathrm{keV}$$
$$\Gamma_{\mathrm{BW}} = 410 \pm 165 \pm 43^{+18}_{-38}\ \mathrm{keV}$$
$$m_{\mathrm{pole}} = (m_{D^{*+}} + m_{D^0}) - 360 \pm 40^{+4}_{-0}\ \mathrm{keV}$$



D、D^*和N的两两组合均存在强子分子态候选态，是否存在DD^*N的强子分子态候选态？

[1] [Particle Data Group] PTEP 2020, 083C01 (2020)
[2] [Belle] Phys. Rev. Lett. 94, 122002 (2005)
[3] [BaBar] Phys. Rev. D 78, 112003 (2008)
[4] [BaBar] Phys. Rev. Lett. 98, 012001 (2007)
[5] [Belle] Phys. Rev. Lett. 98, 262001 (2007)
[6] [LHCb] JHEP 1705, 030 (2017)
[7] [LHCb] Nature Phys. 18, 751-754 (2022)
[8] [LHCb] Nature Commun. 13, 3351 (2022)

相互作用势

$$V^{DD^*\to DD^*} = -g_\sigma^2 O_1 Y_\sigma + \frac{1}{2}\beta^2 g_V^2 O_1 \mathcal{H}_V$$

$$V^{DD^*\to D^*D} = \frac{g^2}{3f_\pi^2}(O_2\hat{O} + O_3\hat{\mathcal{P}})\mathcal{H}'_{P1} + \frac{2}{3}\lambda^2 g_V^2 (2O_2\hat{O} - O_3\hat{\mathcal{P}})\mathcal{H}'_{V1},$$

$$V^{DD^*\to D^*D^*} = \frac{g^2}{3f_\pi^2}(O_4\hat{O} + O_5\hat{\mathcal{P}})\mathcal{H}_{P2} + \frac{2}{3}\lambda^2 g_V^2 (2O_4\hat{O} - O_5\hat{\mathcal{P}})\mathcal{H}_{V2},$$

$$\mathcal{H}_{Pi}^{(\prime,\prime\prime)} = C_1^{(\prime,\prime\prime)}(I)Y_{\pi i} + \frac{1}{3}C_0^{(\prime,\prime\prime)}(I)Y_{\eta i},$$

$$\mathcal{H}_{Vi}^{(\prime,\prime\prime)} = C_1^{(\prime,\prime\prime)}(I)Y_{\rho i} + C_0^{(\prime,\prime\prime)}(I)Y_{\omega i},$$

$$Y_i = \frac{e^{-m_{Ei}r}}{4\pi r} - \frac{e^{-\Lambda_i r}}{4\pi r} - \frac{\Lambda_i^2 e^{-\Lambda_i r}}{8\pi\Lambda_i} + \frac{m_{Ei}^2 e^{-\Lambda_i r}}{8\pi\Lambda_i}$$

$$V^{DN\to DN} = g_{\sigma NN}g_\sigma\left(1 - \frac{\boldsymbol{\sigma}\cdot\mathbf{L}}{4m_N^2}\hat{Q}\right)Y_\sigma + \left[-h_{vNN}\beta g_V - \frac{(h_{vNN}+2f_{vNN})\beta g_V}{8m_N^2}(\hat{\mathcal{P}} + 2\boldsymbol{\sigma}\cdot\mathbf{L}\hat{Q})\right]\mathcal{H}''_V,$$

$$V^{DN\to D^*N} = \frac{g_{\pi NN}g_\pi}{3\sqrt{2}m_N f_\pi}(O_6\hat{O} + O_7\hat{\mathcal{P}})\mathcal{H}''_{P3} + \left[-\frac{2h_{vNN}\lambda g_V}{m_N}O_8\hat{Q} + \frac{(h_{vNN}+f_{vNN})\lambda g_V}{3m_N}(2O_6\hat{O} - O_7\hat{\mathcal{P}})\right]\mathcal{H}''_{V3},$$

$$V_{D^*N\to D^*N} = g_{\sigma NN}g_\sigma O_9\left(1 - \frac{\boldsymbol{\sigma}\cdot\mathbf{L}}{4m_N^2}\hat{Q}\right)Y_\sigma + \left[-h_{vNN}\beta g_V O_9 - \frac{(h_{vNN}+2f_{vNN})\beta g_V}{8m_N^2}O_9(\hat{\mathcal{P}} + 2\boldsymbol{\sigma}\cdot\mathbf{L}\hat{Q})\right]\mathcal{H}''_V + \frac{g_{\pi NN}g_\pi}{3\sqrt{2}m_N f_\pi}(O_{10}\hat{O} + O_{11}\hat{\mathcal{P}})\mathcal{H}''_P + \left[-\frac{2h_{vNN}\lambda g_V}{3m_N}O_{12}\hat{Q} + \frac{(h_{vNN}+f_{vNN})\lambda g_V}{3m_N}(2O_{10}\hat{O} - O_{11}\hat{\mathcal{P}})\right]\mathcal{H}''_V.$$

算符

$$O_1 = \boldsymbol{\epsilon}_4^\dagger\cdot\boldsymbol{\epsilon}_2,$$

$$O_2 = \boldsymbol{\epsilon}_3^\dagger\cdot\boldsymbol{\epsilon}_2,\ O_3 = S(\mathbf{r},\boldsymbol{\epsilon}_3^\dagger,\boldsymbol{\epsilon}_2),$$

$$O_4 = \boldsymbol{\epsilon}_3^\dagger\cdot(i\boldsymbol{\epsilon}_4^\dagger\times\boldsymbol{\epsilon}_2),\ O_5 = S(\mathbf{r},\boldsymbol{\epsilon}_3^\dagger, i\boldsymbol{\epsilon}_4^\dagger\times\boldsymbol{\epsilon}_2),$$

$$O_6 = \boldsymbol{\epsilon}_3^\dagger\cdot\boldsymbol{\sigma},\ O_7 = S(\mathbf{r},\boldsymbol{\epsilon}_3^\dagger,\boldsymbol{\sigma}),\ O_8 = \boldsymbol{\epsilon}_3^\dagger\cdot\mathbf{L}$$

$$O_9 = \boldsymbol{\epsilon}_3^\dagger\cdot\boldsymbol{\epsilon}_1,\ O_{10} = i\boldsymbol{\epsilon}_3^\dagger\times\boldsymbol{\epsilon}_1\cdot\boldsymbol{\sigma},$$

$$O_{11} = S(\mathbf{r}, i\boldsymbol{\epsilon}_3^\dagger\times\boldsymbol{\epsilon}_1,\boldsymbol{\sigma}),\ O_{12} = i\boldsymbol{\epsilon}_3^\dagger\times\boldsymbol{\epsilon}_1\cdot\mathbf{L},$$

参数 [1-3]

coupling constants		meson masses	
$\frac{g}{f_\pi}$ =4.545 GeV^{-1}	$g_{\pi NN}$ =-13.07	m_π =0.140 GeV	m_η =0.548 GeV
g_σ =0.76	$g_{\sigma NN}$=-8.46	m_σ=0.600 GeV	m_D =1.867 GeV
βg_V =5.2	h_{VNN} =3.25	m_ρ =0.770 GeV	m_{D^*} =2.009 GeV
λg_V =3.133 GeV^{-1}	f_{VNN} =19.83	m_ω=0.780 GeV	m_N =0.939 GeV

[1] M. Z. Liu, T. W. Wu, M. Pavon Valderrama, J. J. Xie and L. S. Geng, Phys. Rev. D 99, no.9, 094018 (2019)
[2] J. He, Y. T. Ye, Z. F. Sun and X. Liu, Phys. Rev. D 82, 114029 (2010)
[3] P. A. Zyla *et al.* [Particle Data Group], PTEP 2020, no.8, 083C01 (2020)

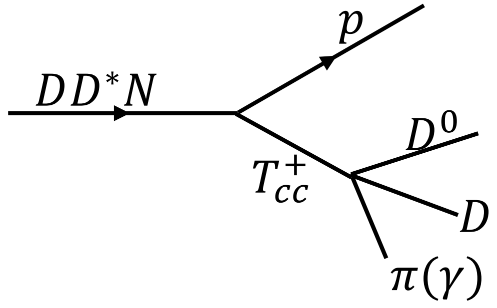
➢ 数值结果

I	J^P	S-wave					S-D Mixing Effect					Coupled Channel Effect			
		Λ	E	r_{DD^*}	r_{DN}	r_{D^*N}	Λ	E	r_{DD^*}	r_{DN}	r_{D^*N}	Λ	E	P_{DD^*N} (%)	$P_{D^*D^*N}$ (%)
$\frac{1}{2}$	$\frac{1}{2}^+$	1.19	-0.11	4.31	10.24	10.20	1.02	-1.13	11.64	11.75	2.40	1.00	-0.41	97.75	2.25
		1.24	-2.35	2.10	8.55	8.54	1.07	-21.19	11.56	11.59	1.26	1.05	-11.40	99.98	0.02
		1.29	-6.62	1.38	6.34	6.33	1.12	-56.46	11.55	11.57	0.98	1.10	-40.43	~100	~0
	$\frac{3}{2}^+$	1.20	-0.37	3.81	10.24	10.18	0.89	-0.19	9.77	10.22	4.82	0.89	-0.22	99.98	0.02
		1.25	-2.83	1.99	9.51	9.50	0.94	-3.25	9.22	9.34	2.47	0.94	-3.32	99.96	0.04
		1.30	-7.12	1.37	9.13	9.12	0.99	-9.25	9.00	9.05	1.69	0.99	-9.44	99.83	0.17
$\frac{3}{2}$	$\frac{1}{2}^+$	1.84	-0.11	10.71	11.12	5.17	1.77	-0.18	9.86	10.25	4.61	1.77	-0.21	99.99	0.01
		1.89	-0.77	10.43	10.67	3.77	1.82	-0.94	9.63	9.85	3.47	1.82	-0.98	99.98	0.02
		1.94	-1.72	10.24	10.38	2.87	1.87	-1.99	9.45	9.59	2.70	1.87	-2.04	99.98	0.02
	$\frac{3}{2}^+$	2.56	-1.32	2.13	9.01	9.00	1.90	-0.18	14.00	14.21	3.66	1.90	-0.18	~100	~0
		2.61	-17.07	0.68	8.59	8.59	1.95	-1.16	13.77	13.90	2.82	1.95	-1.16	~100	~0
		2.66	-41.08	0.47	8.53	8.53	2.00	-2.44	13.65	13.73	2.26	2.00	-2.44	~100	~0

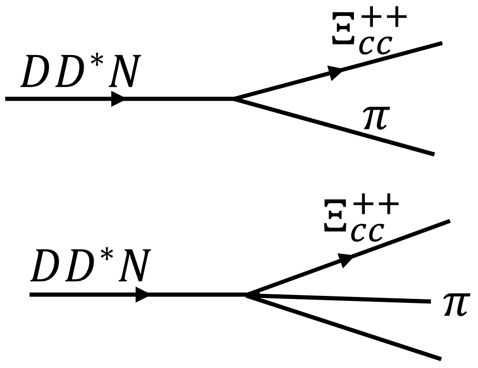
1. $I(J^P) = \frac{1}{2}(\frac{1}{2}^+, \frac{3}{2}^+)$的$DD^*N$是较好的分子态候选态，$I(J^P) = \frac{3}{2}(\frac{1}{2}^+, \frac{3}{2}^+)$的$DD^*N$较难形成束缚态；
2. S-D混合、耦合道效应对结果的影响较小；
3. 我们建议通过如下过程寻找DD^*N：
   (a) DDp+π或γ； (b) 双粲重子+π； (c) 单粲重子+单粲介子。

➢ 衰变

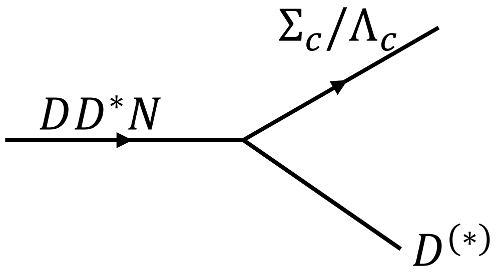
(a)



(b)

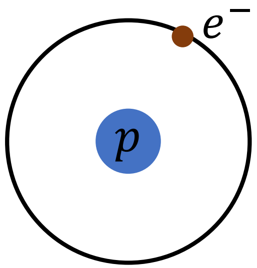


(c)

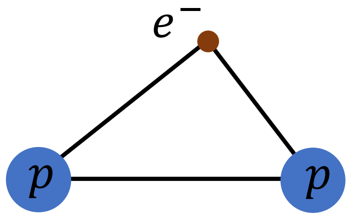


3.3 类氢强子物质

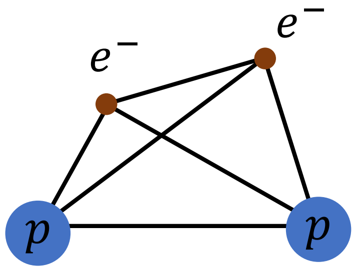
➢ **经典的原子-分子物质**



氢原子(H)



氢分子离子(H_2^+)

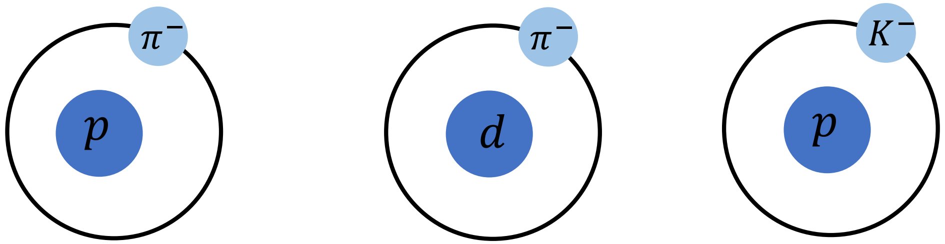


氢分子(H_2)

	氢原子(H)	氢分子离子(H_2^+)	氢分子(H_2)
束缚能(eV)	13.6	16.25	31.65

主要由核子和电子组成，在库仑力下形成束缚态

由强子构成的原子-分子物质



ϵ (eV)	$-7 \pm 0.028 \pm 0.036$	2.43 ± 0.10	$-283 \pm 36 \pm 6$
Γ (eV)	$0.97 \pm 0.010 \pm 0.05$	1.02 ± 0.21	$541 \pm 89 \pm 22$

[1] D. Sigg et al., Nucl. Phys. A 609, 269-309 (1996)
[2] A. W. Thomas and R. H. Landau, Phys. Rept. 58, 121 (1980)
[3] M. Bazzi *et al.* [SIDDHARTA], Phys. Lett. B 704, 113-117 (2011)
[4] M. Bazzi et al., Nucl. Phys. A 881, 88-97 (2012)
[5] Z. W. Liu, J. J. Wu, D. B. Leinweber and A. W. Thomas, Phys. Lett. B 808, 135652 (2020)
[6] M. Bazzi, et al., Nucl. Phys. A 881, 88-97 (2012)
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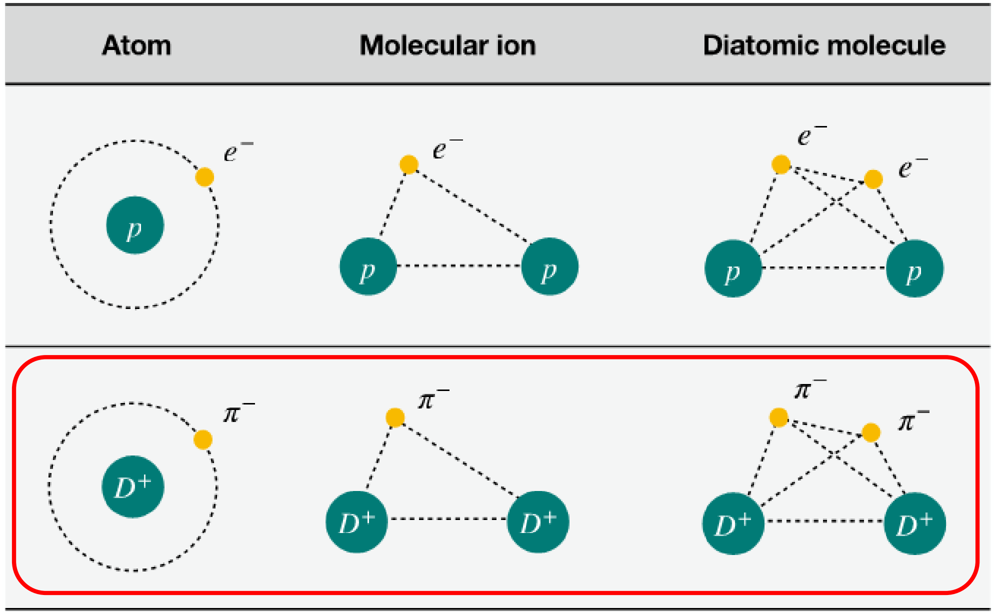
[10] C. Curceanu et al., Rev. Mod. Phys. 91, 025006 (2019)
[11] D. Chatellard et al., Nucl. Phys. A 625, 855-872 (1997)



$$p \to D^+ \quad e^- \to \pi^- \ (e^- \to K^-)$$

特点：

1. 系统以电磁相互作用(库仑势)为主导形成束缚态，强相互作用可以微扰处理；
2. 强相互作用会使得束缚能产生一定的偏移(energy shift ϵ)，实验上可以精准地抽取该参数；
3. 系统方均根半径位于100 fm量级，而传统强子的方均根半径在fm的量级，故通过此类系统为强相互作用的研究提供了新视角。

相互作用势

$$V_{\text{Coul}} = \frac{e_i e_j \alpha}{r}$$

$$V^{D^+\pi^-} = \frac{g_{DD\sigma} g_{\pi\pi\sigma}}{2m_\pi} Y(\Lambda, m_\sigma, r) + \frac{1}{2}\beta g_V g_{\rho\pi\pi} Y(\Lambda, m_\rho, r),$$

$$V^{D^+K^-} = \frac{g_{DD\sigma} g_{KK\sigma}}{2m_K} Y(\Lambda, m_\sigma, r) + \frac{1}{2}\beta g_V g_{KK\rho} Y(\Lambda, m_\rho, r) - \frac{1}{2}\beta g_V g_{KK\omega} Y(\Lambda, m_\omega, r),$$

$$V^{D^+D^+} = -g_{DD\sigma} g_{DD\sigma} Y(\Lambda, m_\sigma, r) + \frac{1}{4}\beta^2 g_V^2 Y(\Lambda, m_\rho, r) + \frac{1}{4}\beta^2 g_V^2 Y(\Lambda, m_\omega, r),$$

$$V^{\pi^-\pi^-} = -\frac{g_{\pi\pi\sigma}^2}{4m_\pi^2} Y(\Lambda, m_\sigma, r) + g_{\pi\pi\rho}^2 Y(\Lambda, m_\rho, r),$$

$$V^{K^-K^-} = -\frac{g_{KK\sigma}^4}{2m_K^2} Y(\Lambda, m_\sigma, r) + g_{KK\rho}^2 Y(\Lambda, m_\rho, r) + g_{KK\omega}^2 Y(\Lambda, m_\omega, r),$$

耦合常数

$$\boldsymbol{\beta g_V = 5.2}$$
$$\boldsymbol{g_{\pi\pi\rho} = -6.01}$$
$$\boldsymbol{g_{\pi\pi\sigma} = -3.52}$$

$$\boldsymbol{g_{KK\rho} \approx g_{KK\omega} \approx \frac{1}{2} g_{\pi\pi\rho}}$$
$$\boldsymbol{g_{KK\sigma} \approx \frac{m_K}{2m_\pi} g_{\pi\pi\rho}}$$

两体衰变

如果强子AB形成一个束缚态，记为$[AB]$，该复合粒子衰变到CD，则该衰变振幅可以同散射过程联系起来：

$$\mathcal{M}_{[AB]\to CD} = \int \frac{1}{(2\pi)^{3/2}} \frac{\sqrt{2m_{[AB]}}}{\sqrt{2m_A}\sqrt{2m_B}} \psi_{AB}(\mathbf{p}') \times \mathcal{M}_{AB\to CD}(\mathbf{p}, \mathbf{p}') \mathrm{d}^3\mathbf{p}'.$$

衰变宽度为：

$$\Gamma_{[AB]\to CD} = \frac{p}{32\pi^2 m_{[AB]}^2} \int |\mathcal{M}_{[AB]\to CD}|^2 \mathrm{d}\Omega.$$

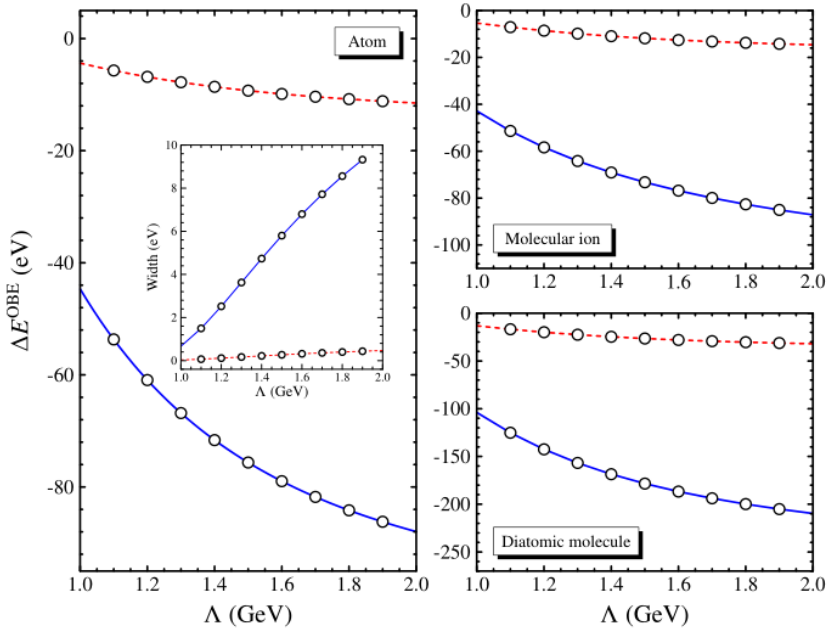
$D^+\pi^- \to D^0\pi^0$和$D^+K^- \to D^0K^0$散射振幅为：

$$i\mathcal{M}_{D^+\pi^-\to D^0\pi^0} = [i^3 g_{\pi\pi\rho}(p_{\pi^-}^0 + p_{\pi^0}^0)] \frac{-ig_{00} + i\frac{q_0 q_0}{m_\rho^2}}{q^2 - m_\rho^2} \times [-i\sqrt{2}\beta g_V m_D] \mathcal{F}^2(q^2, \Lambda^2, m_\rho^2),$$

$$i\mathcal{M}_{D^+K^-\to D^0\bar{K}^0} = [-i^3\sqrt{2} g_{KK\rho}(p_{K^-}^0 + p_{\bar{K}^0}^0)] \frac{-ig_{00} + i\frac{q_0 q_0}{m_\rho^2}}{q^2 - m_\rho^2} \times [-i\sqrt{2}\beta g_V m_D] \mathcal{F}^2(q^2, \Lambda^2, m_\rho^2).$$

➢ 数值结果

		hydrogen		charm-pion		charm-kaon	
		Expt.	Theo.		Theo.		Theo.
Atom	E (eV)	-13.6	-13.6	E^{QED} (keV)	-3.458	E^{QED} (keV)	-10.421
	R (nm)		0.09	R (fm)	360.6	R (fm)	119.6
				ΔE^{OBE} (eV)	-4.4~-11.5	ΔE^{OBE} (eV)	-44.7~-88.0
				Γ (eV)	0.03~0.47	ΔE^{OBE} (eV)	0.7~10.0
Molecular ion	B (eV)	-16.25	-16.20	E^{QED} (keV)	-3.848	E^{QED} (keV)	-11.182
	R^{pp} (nm)		0.11	$R^{D^+D^+}$ (fm)	613.0	$R^{D^+D^+}$ (fm)	259.2
	R^{pe^-} (nm)		0.10	$R^{D^+\pi^-}$ (fm)	496.0	$R^{D^+K^-}$ (fm)	197.2
				ΔE^{OBE} (eV)	-5.3~-14.6	ΔE^{OBE}	-42.8~-87.2
Diatomic molecule	E (eV)	-31.65	-31.60	E^{QED} (keV)	-7.517	E^{QED} (keV)	-21.889
	R^{pp} (nm)		0.127	$R^{D^+D^+}$ (fm)	574.3	$R^{D^+D^+}$ (fm)	187.3
	$R^{e^-e^-}$ (nm)		0.076	$R^{\pi^-\pi^-}$ (fm)	435.4	$R^{K^-K^-}$ (fm)	214.3
	R^{pe^-} (nm)		0.094	$R^{D^+\pi^-}$ (fm)	433.8	$R^{D^+K^-}$ (fm)	164.7
				ΔE^{OBE}	-13.1~-32.0	ΔE^{OBE} (eV)	-103.9~-209.7



虚线-圆圈为D^+-π^-系统
实线-圆圈为D^+-K^-系统

1. 高斯展开法可以很好地重复出氢分子离子和氢分子的束缚能；
2. D^+-π^-和D^+-K^-组成的原子、分子离子、双原子分子在库仑力下形成束缚态；
3. D^+-π^-系统在强相互作用下的质量偏移的计算值大约位于若干到几十eV，
   D^+-K^-系统在强相互作用下的质量偏移的计算值大约位于几十到几百eV；
4. D^+-π^-原子的宽度理论值大约位于0.1 eV的量级，D^+-K^-原子的宽度理论值大约位于1 eV的量级。

4. 总结

1. $I(J^P)=\frac{1}{2}(1^-)$的DDD^*、$I(J^P)=\frac{1}{2}(0^-,1^-,2^-)$的$D^*D^*D$、$I(J^P)=\frac{1}{2}(0^-,1^-,2^-,3^-)$的$D^*D^*D^*$是较好的分子态候选态；
2. $I(J^P)=\frac{1}{2}(\frac{1}{2}^+,\frac{3}{2}^+)$的$DD^*N$是较好的分子态候选态；
3. D^+-π^-和D^+-K^-组成的原子、分子离子、双原子分子可以在电磁相互作用下形成束缚态，在强相互作用下产生质量平移。

谢谢大家，敬请批评指正