

# Heavy ion collisions and nuclear structure

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on Frontiers in Nuclear Theory

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# Heavy ion collisions and nuclear structure

**Low-energy structure of nuclei affects outcome of high-energy collisions between nuclei**

**Numerous evidences for the influence of "intrinsic" nuclear shapes, e.g Ru/Zr ratios**



**Observations made at colliders impact our knowledge of nuclear structure**

**The large sensitivity to initial configurations of nucleons allows for a precise determination of deformation parameters, neutron skin, etc**

**That fine details of nuclear structure survive the complexity of a nucleus-nucleus collision at high energy is amazing...**

**[For a representative publication with many references see [arXiv 2402.05995](https://arxiv.org/abs/2402.05995)]**

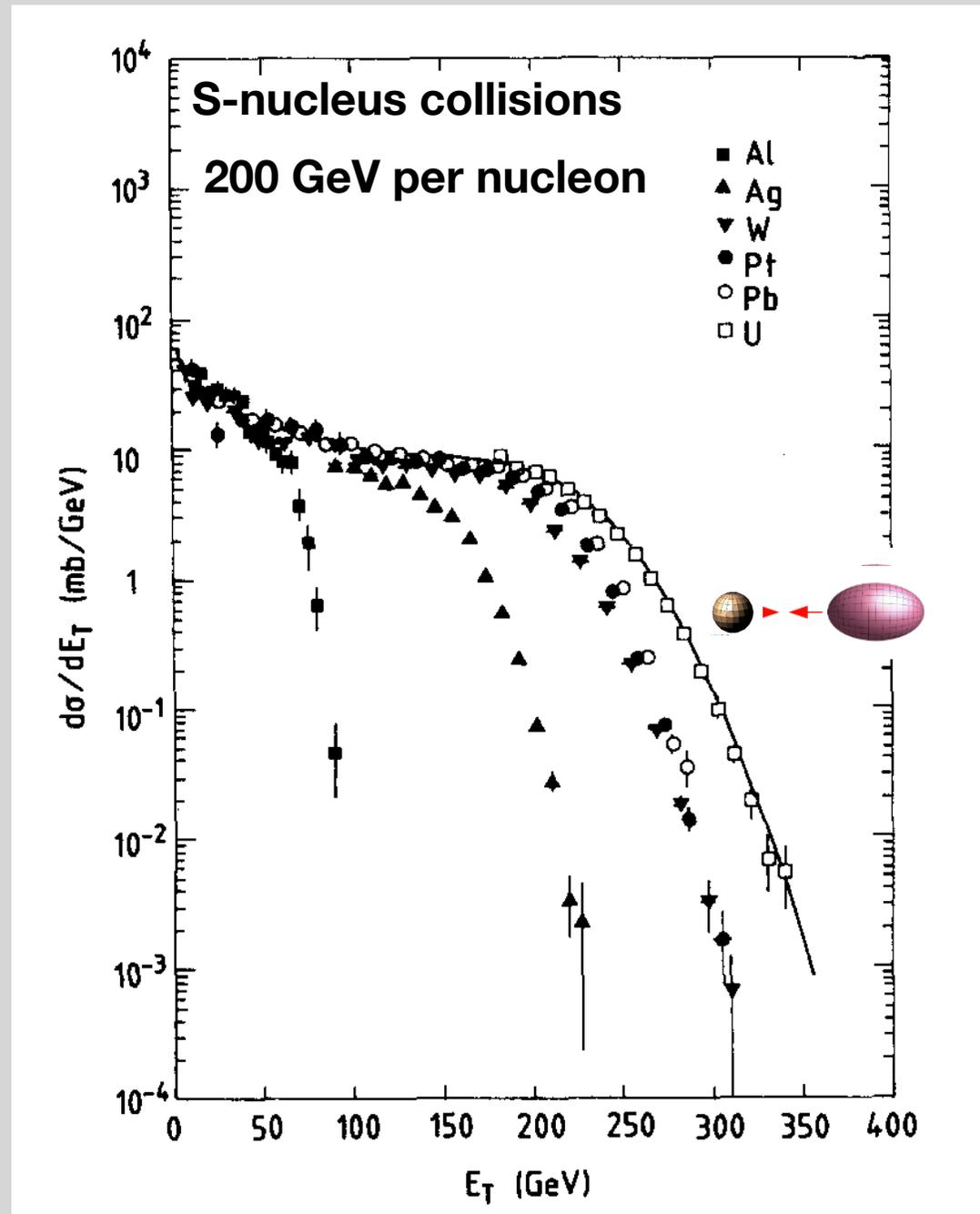
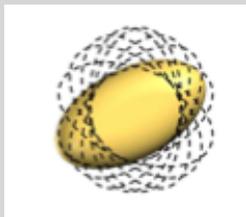
# Shape of nuclei matters

(an old story)

The tail of the transverse energy distribution depends on the orientation of the Uranium nucleus

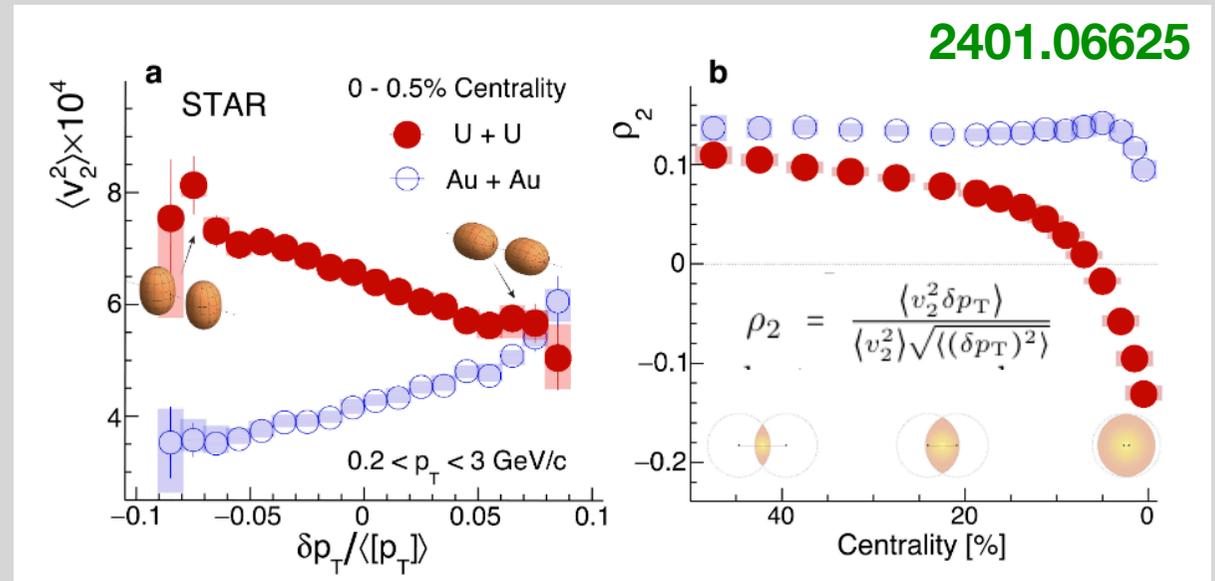
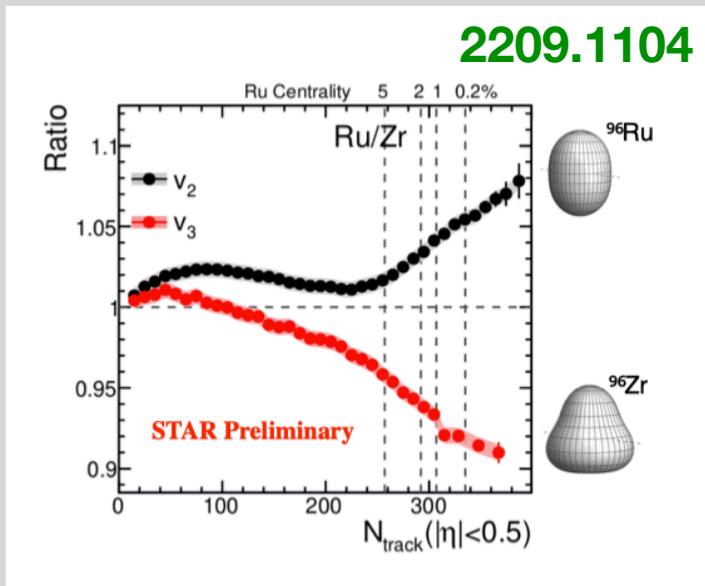
HELIOS collaboration,  
(CERN SPS)  
Phys. Lett. B 214 (1988) 295

Analogous finding in  
electron scattering on  
deformed nuclei  
[Hofstadter, 1956]



# Shape of nuclei matters (WHAT IS NEW ?)

High statistics, allowing measurements of correlations with high precision



High energy, allowing us to resolve dynamics on very short time scales

# The importance of time scales

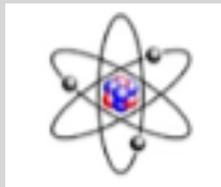
# IMAGING NUCLEI ON YOCTOSECOND time scale

Nobel prize 2023 (P. Agostini, F. Krausz, A l'Huillier)

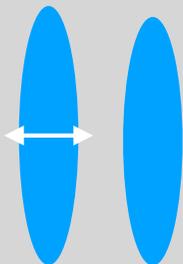
“for experimental methods that generate attosecond pulses of light for the study of electron dynamics in matter.”



fs =  $10^{-15}$ s [Molecule internal dynamics]



as =  $10^{-18}$ s [Electronic motion]



[Heavy Ion collisions at LHC]

ys =  $10^{-24}$ s  $\simeq$  0.3fm/c

Very short time scale as compare to the time scale of internal nucleus dynamics

Typical time for a nucleon to cross a nucleus

$$\Delta t \simeq 10 \text{ ys} \quad (v_F/c \approx 0.3)$$

Time scale associated with excitation energy of 1 MeV

$$\Delta t \simeq 200 \text{ fm/c} \approx 650 \text{ ys}$$

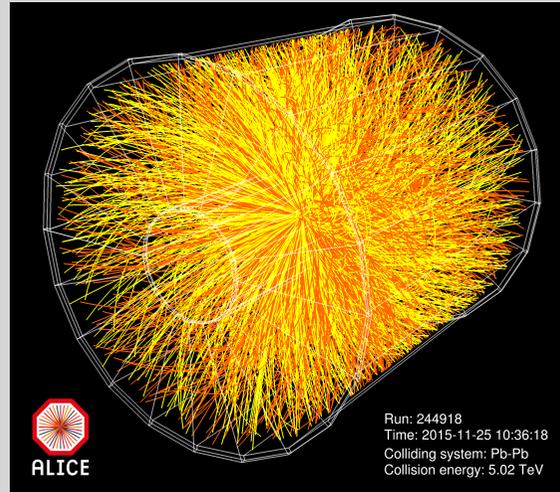
$$\Delta x = (2R)/\gamma \simeq 10^{-2} \text{ fm}$$

# Azimuthal structure of particle production

# What do we measure in HI collisions?

## One-particle distribution

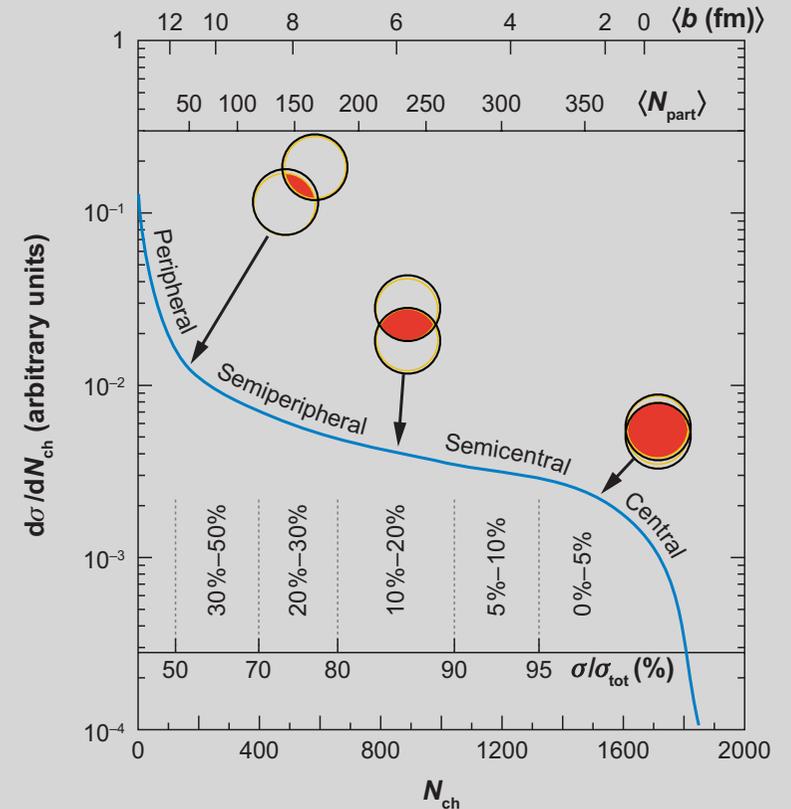
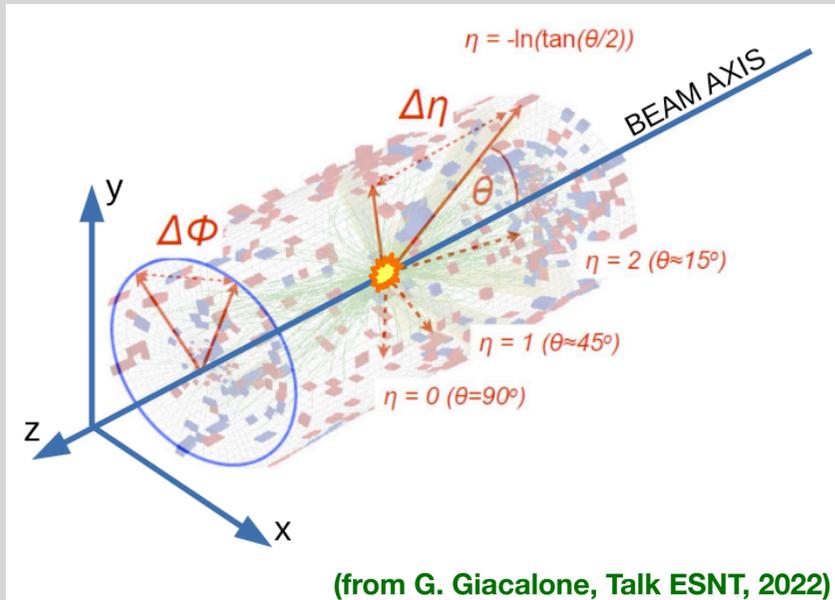
$$\frac{dN}{p_T dp_T d\varphi d\eta}$$



## Transverse energy distribution

$$\frac{d\sigma}{dN_{ch}} \sim \frac{d\sigma}{dE_T}$$

$$|\eta| < 1$$

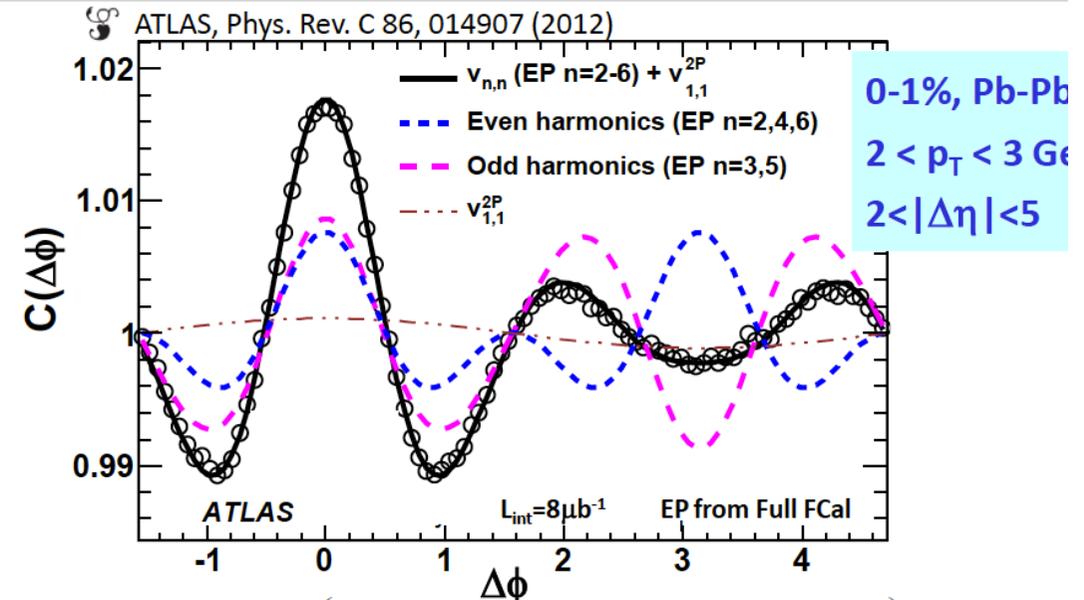
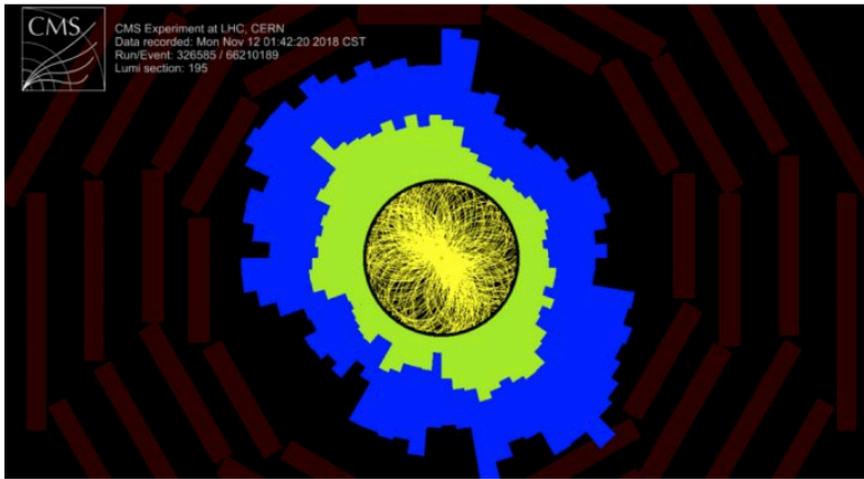


(Miller et al., Annu. Rev. Nucl. Part. Sci. 2007. 57:205-43)

# Non trivial azimuthal distribution (1)

Single event is not symmetric

Fourier analysis  
"Multiple harmonics"



$$P_{\Psi}(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} V_n e^{-in\varphi}$$

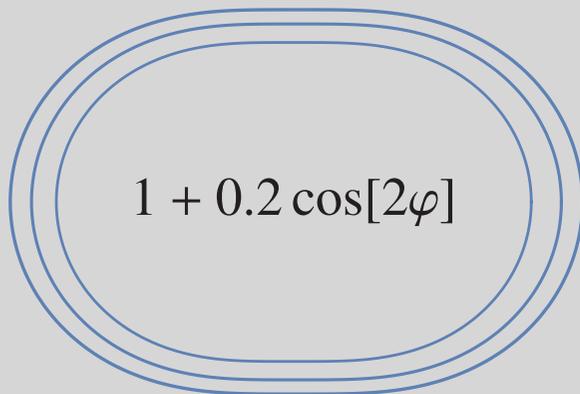
$$V_n = \int_0^{2\pi} d\varphi P_{\Psi}(\varphi) e^{in\varphi}$$

$$V_n = v_n e^{in\Psi_n}$$

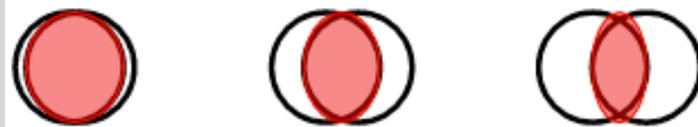
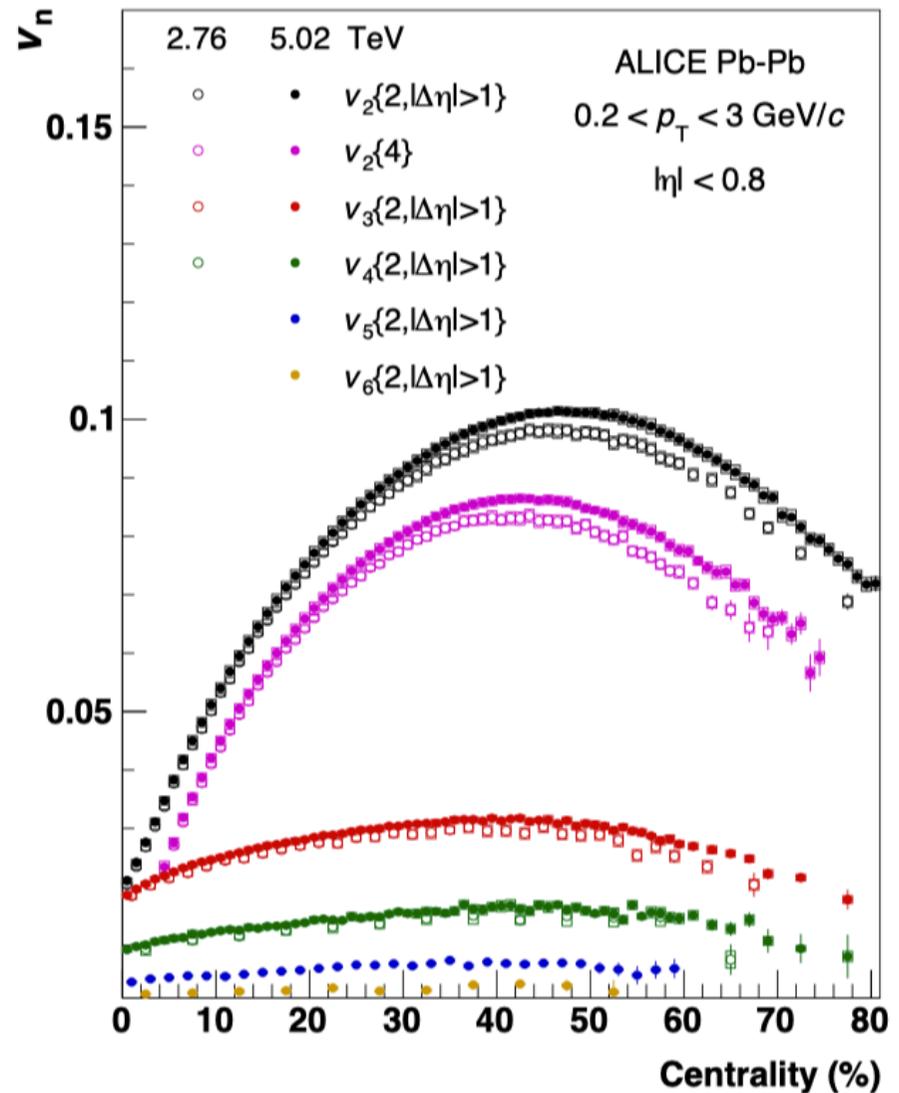
$$\frac{1}{N} \frac{dN}{d\varphi} = \frac{1}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)] \right]$$

# Non trivial azimuthal distribution (2)

$$\frac{1}{N} \frac{dN}{d\varphi} = \frac{1}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)] \right]$$



The magnitudes of the coefficients  $v_n$  are correlated with the impact parameter of the collision

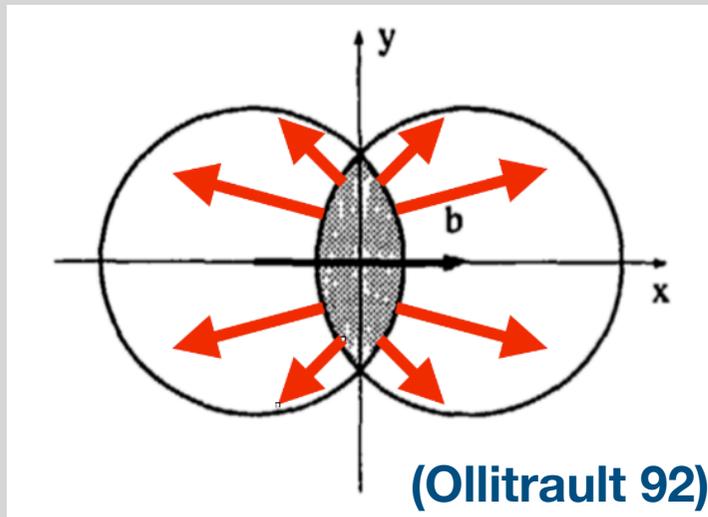


# QUESTIONS

- WHAT IS THE PHYSICAL ORIGIN OF THE EFFET?
- HOW DOES ONE DETERMINE  $V_n$  ?

# HYDRODYNAMIC FLOW (1)

The shape of the collision zone determines the pressure gradients which accelerate particles



$$u_x = u \cos \varphi \quad u_y = u \sin \varphi$$

( $u$  = flow velocity)

$$\nabla_x P \gg \nabla_y P \longrightarrow |u_x| \gg |u_y|$$

$$\langle \cos 2\varphi \rangle = \langle \cos^2 \varphi - \sin^2 \varphi \rangle > 0$$

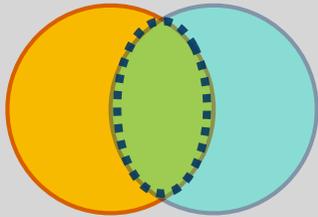
Hence a non vanishing value of the "elliptic flow"  $v_2$

$$\frac{1}{N} \frac{dN}{d\varphi} = \frac{1}{2\pi} [1 + 2v_2 \cos[2(\varphi - \Psi_2)]]$$

$$v_2 = \langle \cos 2\varphi \rangle = \int \frac{d\varphi}{2\pi} \frac{1}{N} \frac{dN}{d\varphi} \cos(2\varphi)$$

# HYDRODYNAMIC FLOW (2)

- The magnitude of  $v_2$  is sensitive to the "shape" of the interaction region (in the transverse plane)



$$\varepsilon_2 = \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

Average done over the energy density in the transverse plane

- Hydrodynamics response is such that  $v_2 \propto \varepsilon_2$
- Hence the **dependence of  $v_2$  on the impact parameter**
- Hydrodynamics then provides a link between the initial distribution of the energy density (its "shape") and the azimuthal structure of the momentum distribution of the observed particles.

**HOW DOES ONE DETERMINE  $V_n$  ?**

**CORRELATIONS**

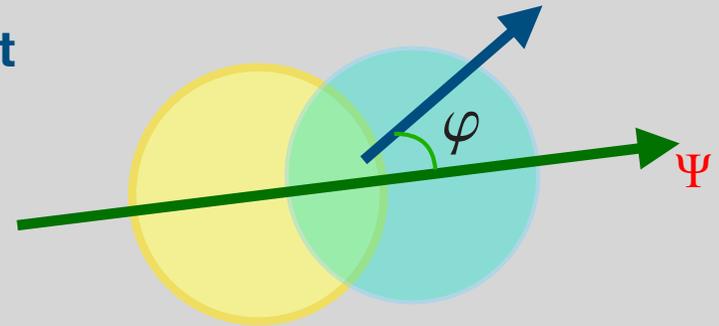
**AND**

**AVERAGE OVER EVENTS**

# TWO KINDS OF AVERAGE

Distribution of particles produced in a single event

$$P_{\Psi}(\varphi_1, \varphi_2, \dots, \varphi_N)$$



In a given event emissions of particles are **uncorrelated**

$$P_{\Psi}(\varphi_1, \varphi_2, \dots, \varphi_N) = p_{\Psi}(\varphi_1) \cdots p_{\Psi}(\varphi_N)$$

One-point function

$$p_{\Psi}^{(1)}(\varphi) = \int d\varphi_2 \cdots d\varphi_N P_{\Psi}(\varphi_1, \varphi_2, \dots, \varphi_N) = p_{\Psi}(\varphi)$$

For elliptic flow alone

$$p_{\Psi}(\varphi) = \frac{1}{2\pi} [1 + 2v_2 \cos 2(\varphi - \Psi)]$$

If there were enough particles in the event, one could reconstruct the one-point distribution, that is  $v_2$  and  $\Psi$ .

# EMERGENCE OF CORRELATIONS

- **Start from uncorrelated 2-point function**

$$p_{\Psi}^{(2)}(\varphi_1, \varphi_2) = p_{\Psi}^{(1)}(\varphi_1)p_{\Psi}^{(1)}(\varphi_2)$$

- **Integration over  $\Psi$  generates correlations**

$$p^{(2)}(\varphi_1, \varphi_2) = \int d\Psi p_{\Psi}^{(2)}(\varphi_1, \varphi_2) = 1 + 2v_2^2 \cos 2\Delta\varphi = p^{(2)}(\Delta\varphi)$$

$$\Delta\varphi \equiv \varphi_1 - \varphi_2$$

- **One can determine  $v_2$  from a correlation function  
(count pairs instead of single particles)**

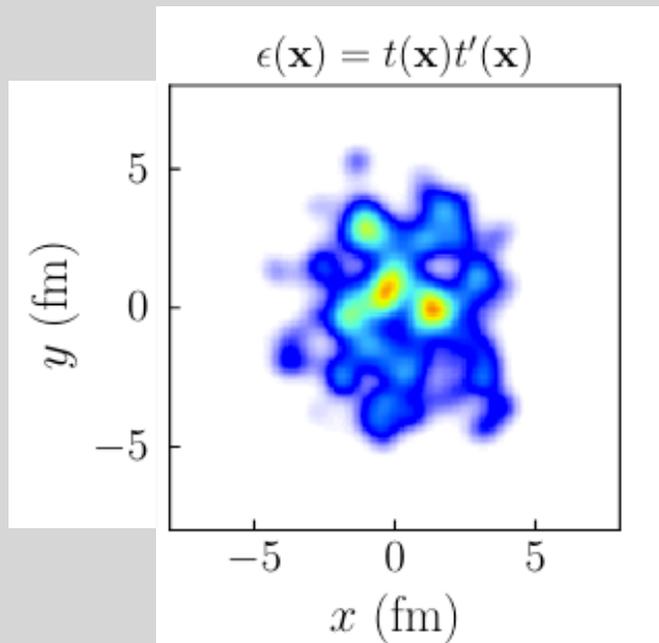
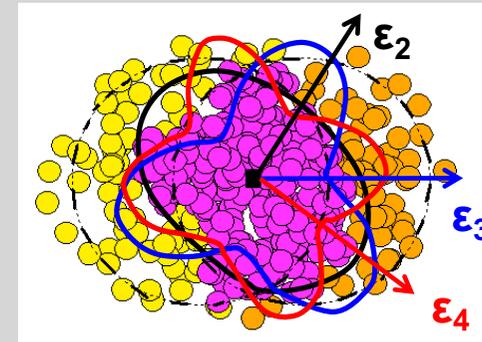
$$\left\langle \sum_{i \neq j} \cos 2(\Delta\varphi_{ij}) \right\rangle = N(N-1) \int \frac{d\Delta\varphi}{2\pi} p^{(2)}(\Delta\varphi) \cos(2\Delta\varphi) = N(N-1)v_2^2$$

# INITIAL FLUCTUATIONS

- **RANDOM POSITIONS OF NUCLEONS**
- **RANDOM ORIENTATION OF NUCLEI**
- **ZERO POINT COLLECTIVE OSCILLATIONS**

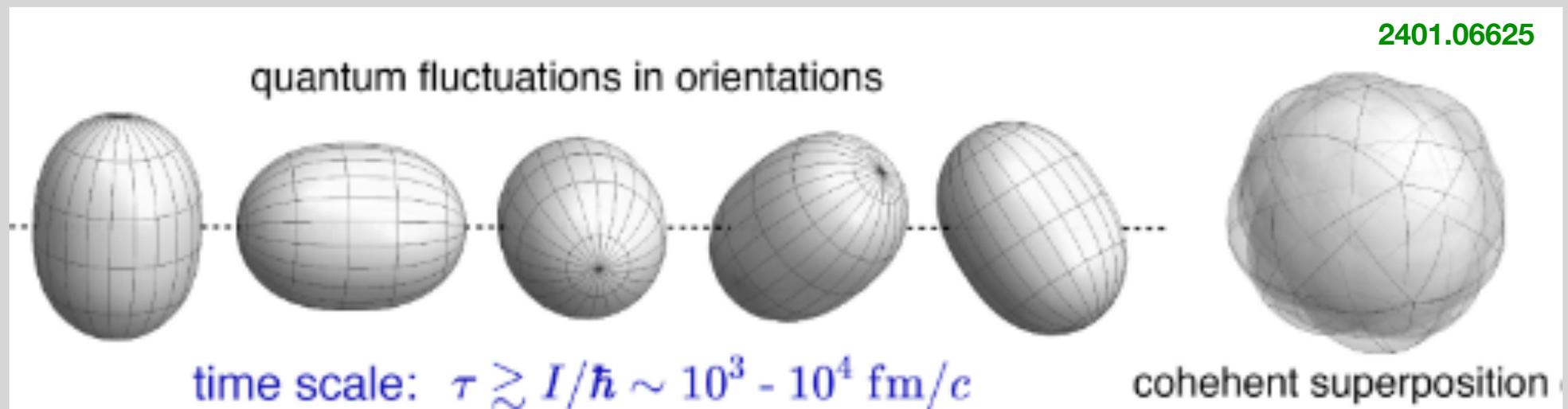
# GLAUBER INITIAL FLUCTUATIONS

- Energy deposition is a **random process**, with local fluctuations in energy density



- The **pattern of fluctuations is strongly correlated** with that of initial positions, as given by the Glauber sampling
- As a result of fluctuations, "shapes" emerge, even at **zero impact parameter**, leading to measurable flow effects
- Short wavelength fluctuations average out. What remains, after some transient evolution, are the **long wavelength fluctuations** (low multipoles, "collective variables") that characterize the "shape" of the collision zone.
- The **deformation of nuclei** leads to another source of fluctuations

# PROBING THE RANDOM ORIENTATIONS OF DEFORMED NUCLEI



... ASSUMING THE PICTURE OF A WELL DEFORMED INTRINSIC STATE IS VALID

# Connection to nuclear structure deformed nuclei

# Why are nuclei "deformed"

- If nuclei were "liquid drops", their equilibrium shapes would be spherical (the qualification "deformed" refers to deviation from spherical shape)
- Deformation is intimately connected with single particle motion in a self-consistent mean field
- A deformed nucleus is characterised by a non vanishing quadrupole moment of the one-body density

$$Q = r^2 P_2(\cos \theta)$$

$$\langle Q \rangle = \int d^3 r \rho(\vec{r}) Q(\vec{r}) \neq 0 \quad \rho(\vec{r}) = \int d^3 r_2 \cdots d^3 r_N |\Phi(\vec{r}, \vec{r}_2, \cdots, \vec{r}_N)|^2$$

where  $\Phi(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_N)$  is the "deformed" independent particle wave function.

- But  $\Phi$  cannot be the ground state of the nuclear Hamiltonian since the ground state carries zero angular momentum

$$\langle \Psi_{J=0} | Q | \Psi_{J=0} \rangle = 0$$

- Way out:  $\Phi$  is considered as an "intrinsic" state, function of **intrinsic coordinates**. The full wave function contains a factor that describe the **collective rotation** of the system.

# Conceptual issues

- The collective model is **physically well motivated**, but it remains a model. No unique description. Collective coordinates are (most often) redundant.
- Descriptions based on self-consistent mean fields (or density functional theories) involve **spontaneous symmetry breaking**.
- Symmetry breaking in finite systems is an **approximate concept**. Symmetry has to be **restored**, one way or another (collective model, projection techniques, etc).
- One could even describe nuclear properties **without any reference to an intrinsic state**. This is the case for instance of shell model wave functions.  
(see e.g. A. Poves et al. "Limits on assigning a shape to a nucleus", arXiv: 1906.07542)
- One touches here a general issue, that of the choice of basis in quantum mechanics. In some basis the "physics" is more "manifest" than in others...
- Deformation can be inferred from invariant moments (Kumar 1972)

$$\langle Q \rangle = 0 \quad \langle Q^2 \rangle \neq 0 \quad (\langle Q^4 \rangle - \langle Q^2 \rangle^2, \langle Q^6 \rangle - \langle Q^3 \rangle^2) \longrightarrow (\Delta\beta, \Delta\gamma)$$

and more broadly by **correlation functions**

$$\langle Q^2 \rangle = \int_{\mathbf{r}_1 \mathbf{r}_2} q(\mathbf{r}_1) q(\mathbf{r}_2) S(\mathbf{r}_1, \mathbf{r}_2)$$

$$S(\mathbf{r}_1, \mathbf{r}_2) = \langle \hat{\rho}(\mathbf{r}_1) \hat{\rho}(\mathbf{r}_2) \rangle - \langle \hat{\rho}(\mathbf{r}_1) \rangle \langle \hat{\rho}(\mathbf{r}_2) \rangle$$

# From wave functions to correlation functions

# Correlation functions

- In the intrinsic state the nucleons are **uncorrelated** (mean field picture), but the average potential has some "orientation"

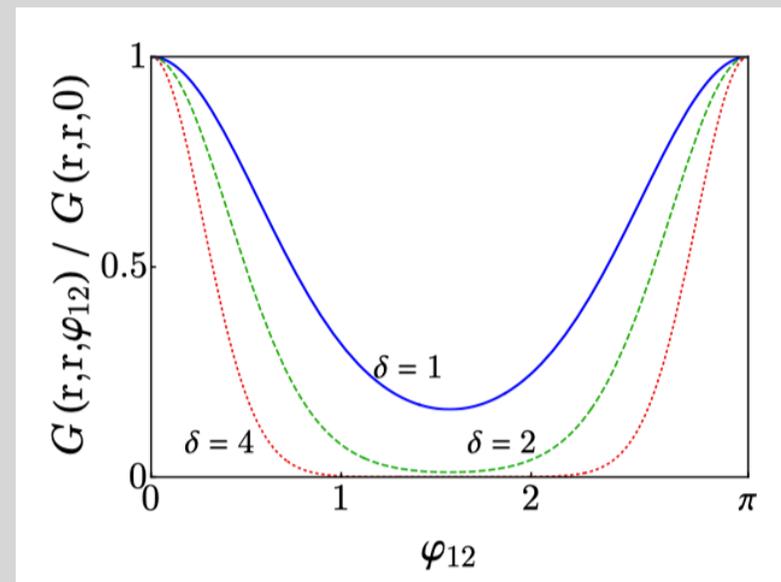
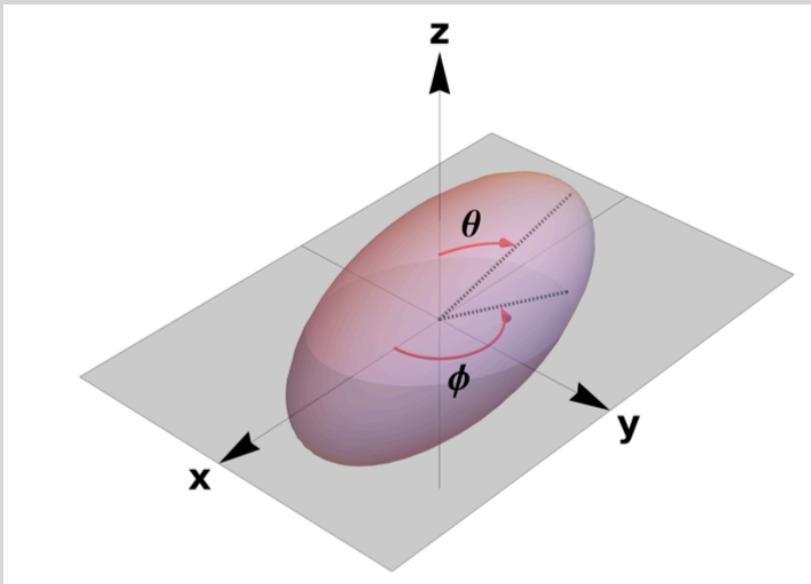
$$P_{\Omega}(r_1, r_2, \dots, r_N) = \left| \Phi_{\Omega}^{\text{int}}(r_1, r_2, \dots, r_N) \right|^2$$

- Averaging over the collective wave function generates correlations (of all orders).

$$P(r_1, r_2, \dots, r_N) = \int \frac{d\Omega}{4\pi} \left| \Phi_{\Omega}^{\text{int}}(r_1, r_2, \dots, r_N) \right|^2$$

**NB.** This average projects onto a spherical state

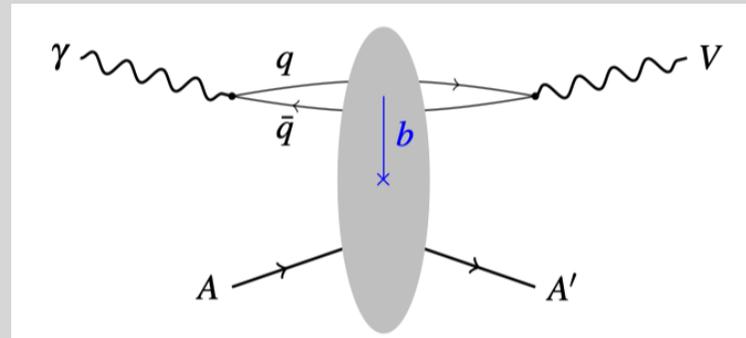
- Calculations based on this (approximate) procedure yield characteristic angular dependence of the density-density correlation function. (JPB, G. Giacalone 2504.15421)



# Direct measurement of correlation functions

[Based on old idea by Caldwell and H. Kowalski, (2010)]

[See also H. Mantysaari, et al. (2023)]



- The incoherent diffractive production of a vector meson gives access to the density-density correlation function (projected onto transverse plane)

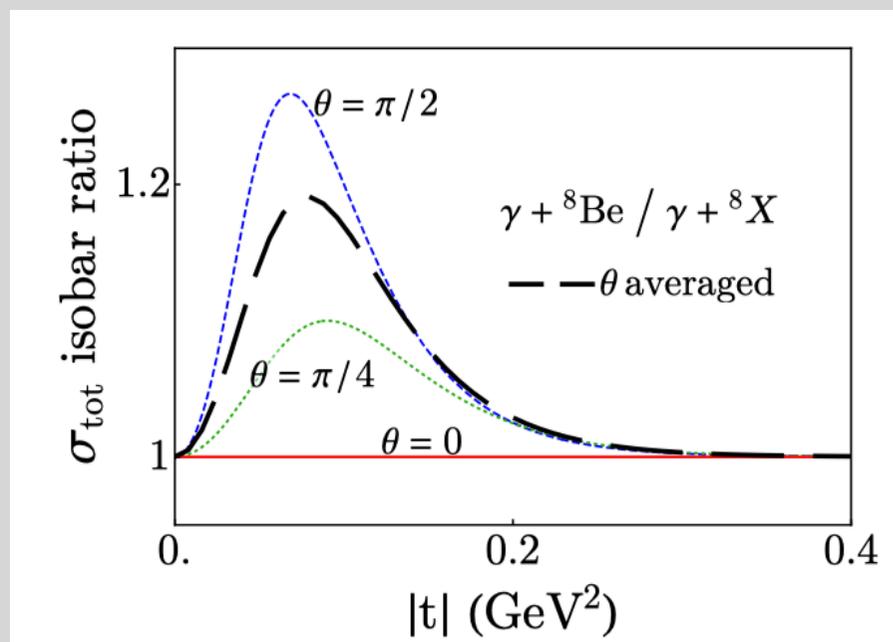
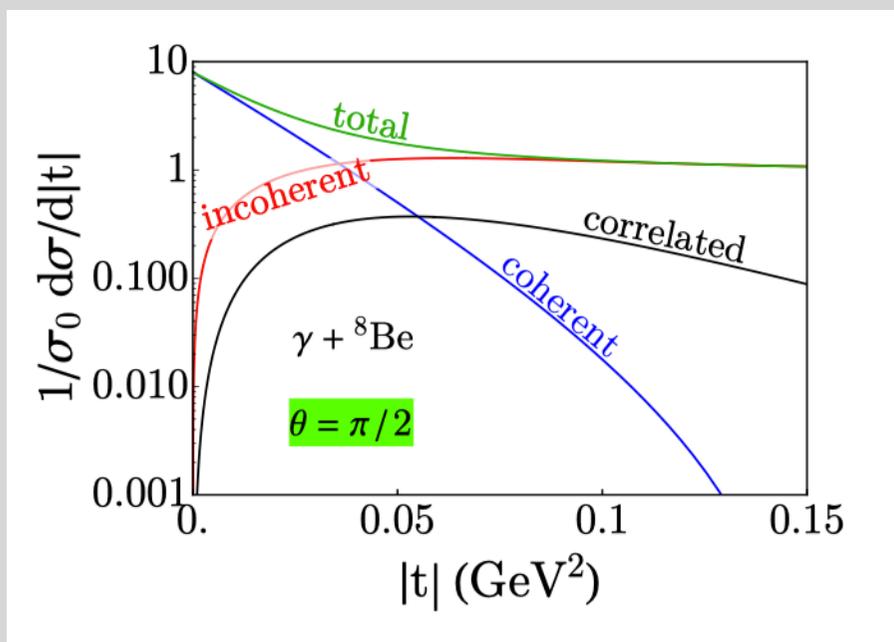
$$\langle T_A(s)T_A(s') \rangle - \langle T_A(s) \rangle \langle T_A(s') \rangle$$

- If the wave function is approximated by that of a deformed state, an angular average is needed which generates long range correlations (JPB, G. Giacalone 2504.15421)

$$\int_{\Omega} \langle T_A(s)T_A(s') \rangle_{\Omega} - \int_{\Omega} \langle T_A(s) \rangle_{\Omega} \int_{\Omega} \langle T_A(s') \rangle_{\Omega}$$

# A TOY MODEL STUDY

## Diffractive production of vector mesons on Beryllium



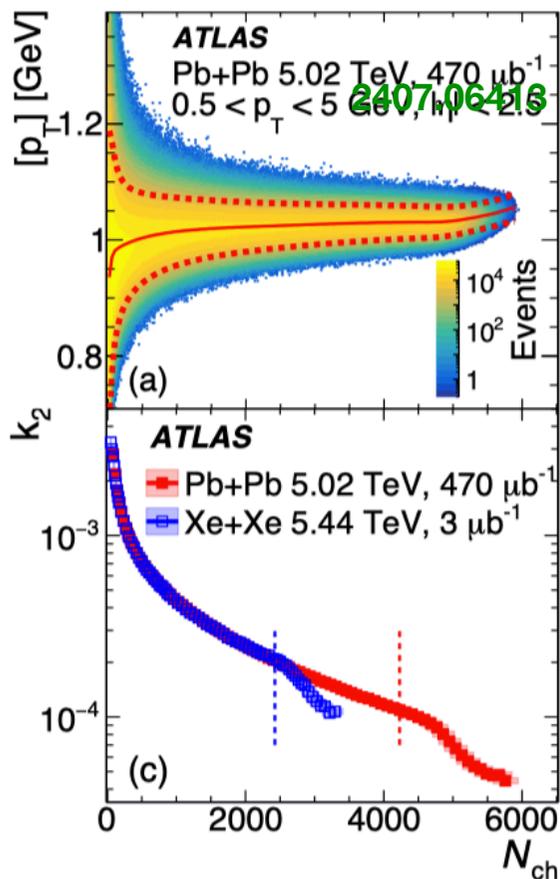
${}^8\text{X}$  is a fictitious isobar of  ${}^8\text{Be}$

# zero point oscillations in nuclear ground states

# ultracentral collisions and $v_0$

- See talk by Huichao Song, and 2504.19644 (A "breathing" octupole...)
- B.G. Zakharov (2008.07304)
- Somewhat more speculative... fluctuations of the initial volume

(inspired by 2407.17313, Parida, Samanta, Ollitrault)



Volume fluctuations (at zero impact parameters) could be related to the giant monopole resonance in Pb

They can be estimated accurately (JPB, Phys.Rept. 64 (1980))

$$q \equiv r^2 - \langle r^2 \rangle_0 \quad \frac{\sqrt{\langle q^2 \rangle}}{\langle r^2 \rangle_0} \simeq 0.03$$

This looks compatible with ATLAS measurements of  $v_0$

$$k_2 = \frac{\langle (\delta p_T)^2 \rangle}{\langle [p_T] \rangle^2} = v_0$$

NB. In Oxygen, 0.03  $\rightarrow$  0.1

# Conclusions

## A FASCINATING DEVELOPMENT

- ★ **Heavy ion collisions may offer us the possibility to capture the shapes of deformed nuclei in a more direct way than any other previous experiment.**
- ★ **Not only does one "see" the deformed shapes, but the values of deformation parameters can be determined with surprisingly high precision.**
- ★ **One might be able to access/measure magnitude of zero point fluctuations in nuclear ground states**
- ★ **The information about the (quantum) state of the nuclei is captured on a very short time scale. How can this information survive the complexity of the matter evolution?**
- ★ **Much remains to be understood/explored....**

