SOPHIA
HADRON
PHYSICS
GROUP

Inaugural Symposium of the Central China Center for Nuclear Theory (C3NT) on Frontiers in Nuclear Theory
C3NT, CCNU, Wuhan, China
May 17-18, 2025

Dynamics of high-energy nuclear collisions

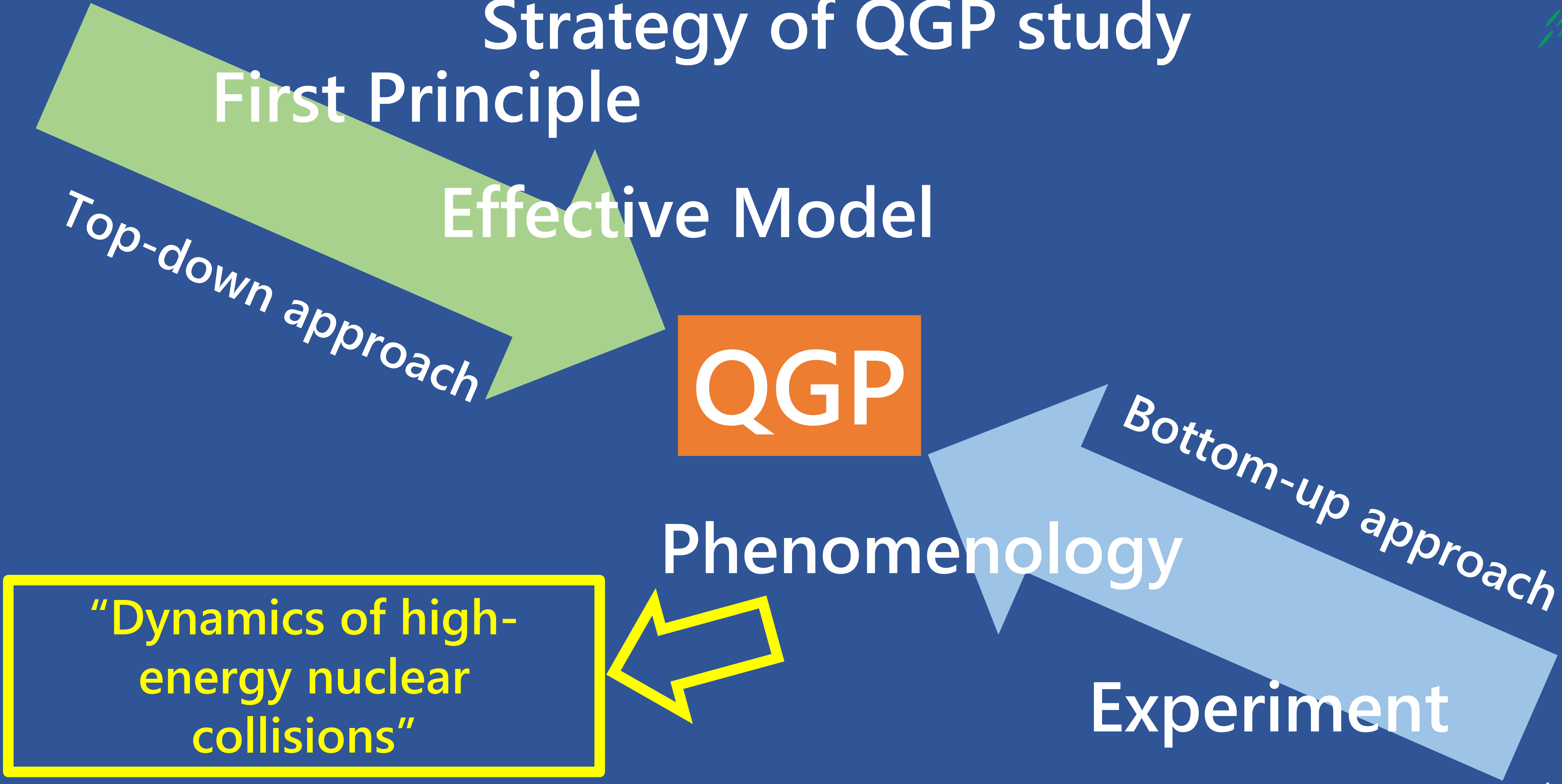
Tetsufumi Hirano (Sophia Univ.)

Plan of this talk

1. Introduction
2. Current Understanding of QGP Properties from Heavy-Ion Data
3. Non-Equilibrium Aspects of Nuclear Collisions
    - Non-linear causality
    - Core-corona picture
4. Summary

1. Introduction

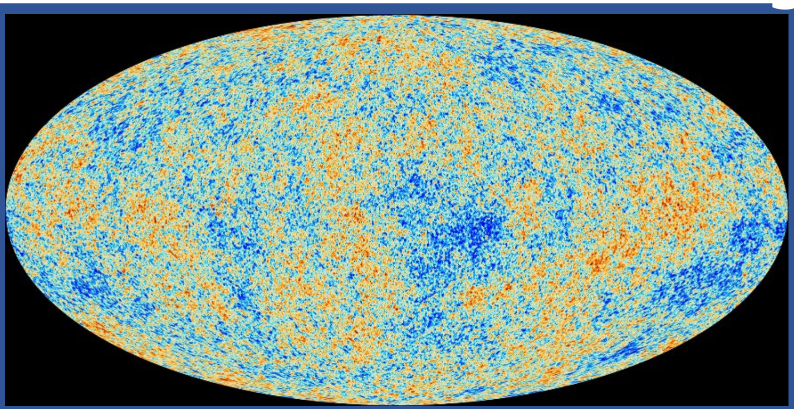
Strategy of QGP study



Bottom-up approach

http://www.esa.int/spaceinimages/Images/2013/04/Planck_CMB_black_background

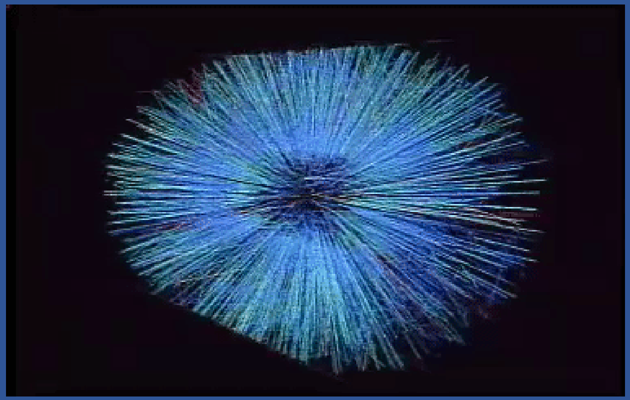
Observational cosmology



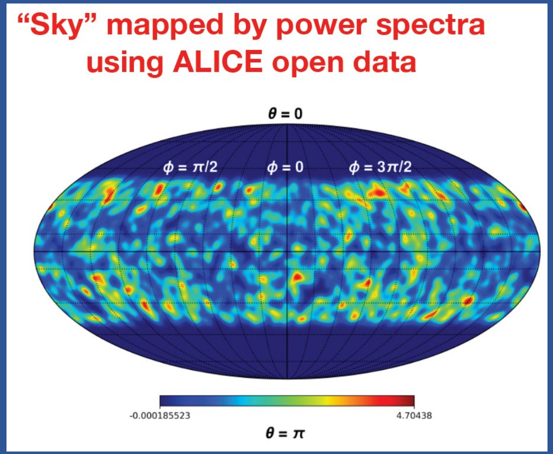
Cosmic Microwave Background
Fluctuations of temperature (Planck)

Cosmological parameters
- Energy budget
- Hubble constant
- Lifetime of Universe
- …

High-energy nuclear collisions



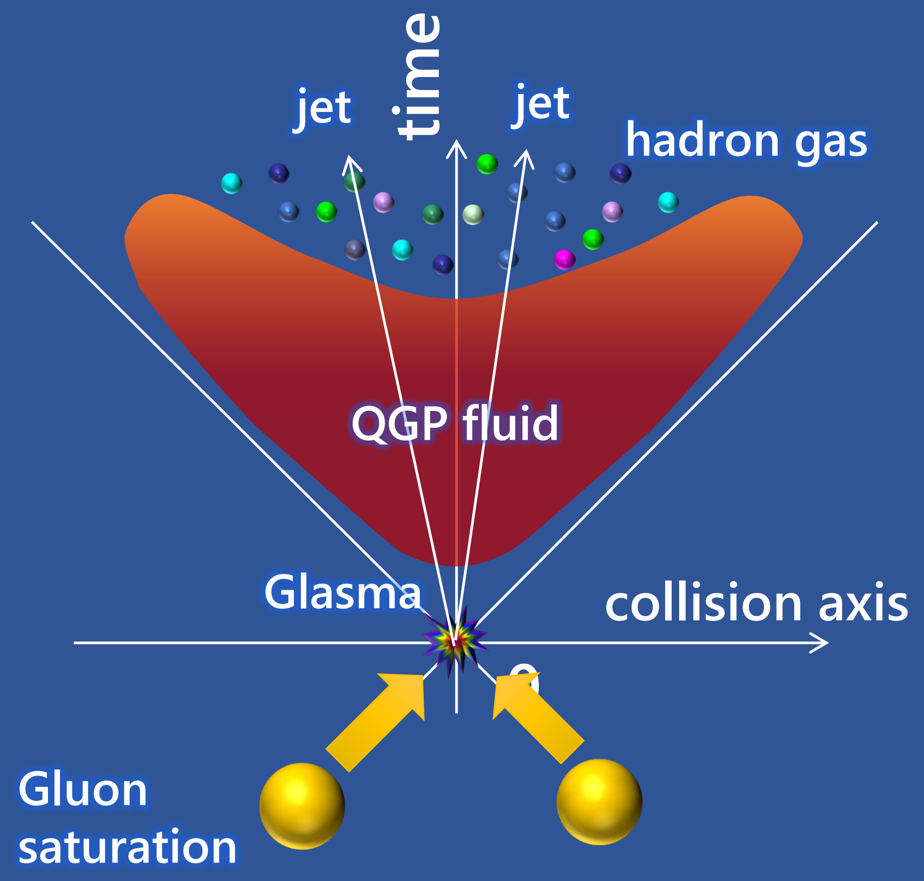
3-D event display (STAR)

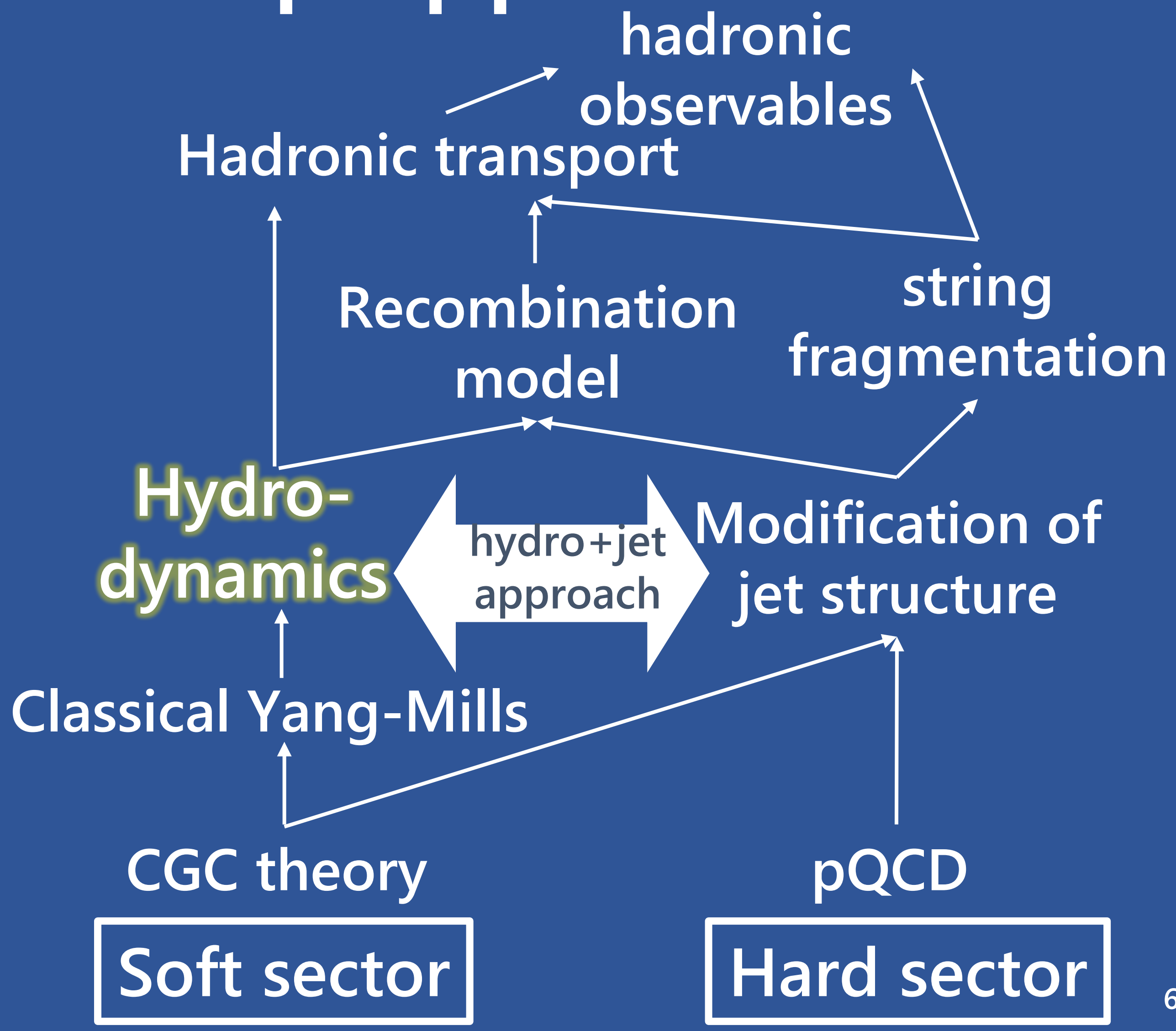


Y.Zhou, talk at QM2018

Properties of the QGP
- Equation of state
- Transport coefficients
- Stopping power
- …

Dynamical modeling as a bottom-up approach





2. Current Understanding of QGP Properties from Heavy-Ion Data

Hydrodynamics

Framework to describe dynamics of macroscopic variables
in the infrared limit ($\omega \to 0$ and $\boldsymbol{k} \to 0$)
← Matter properties directly related to hydrodynamic description

Equation of State

Pressure

Energy density

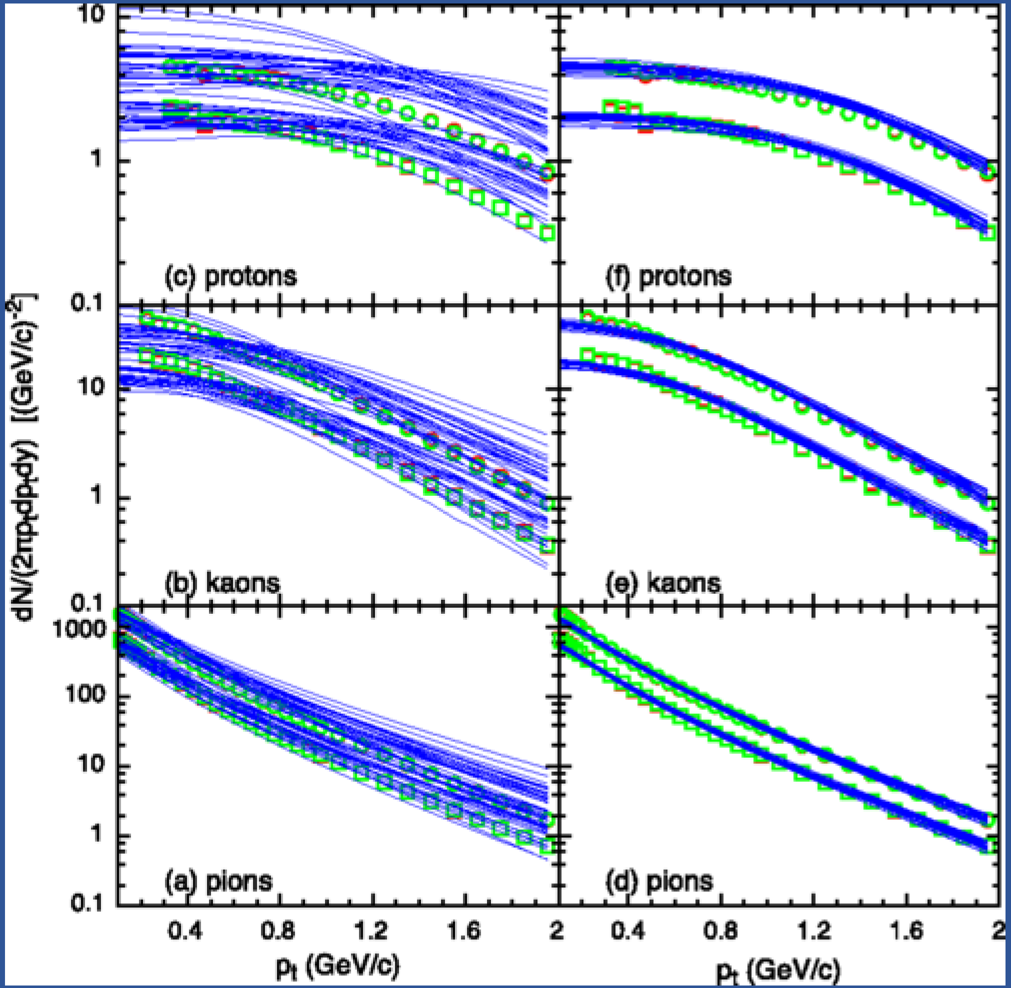
$$p = p(e, n_B, \cdots)$$

Baryon number density

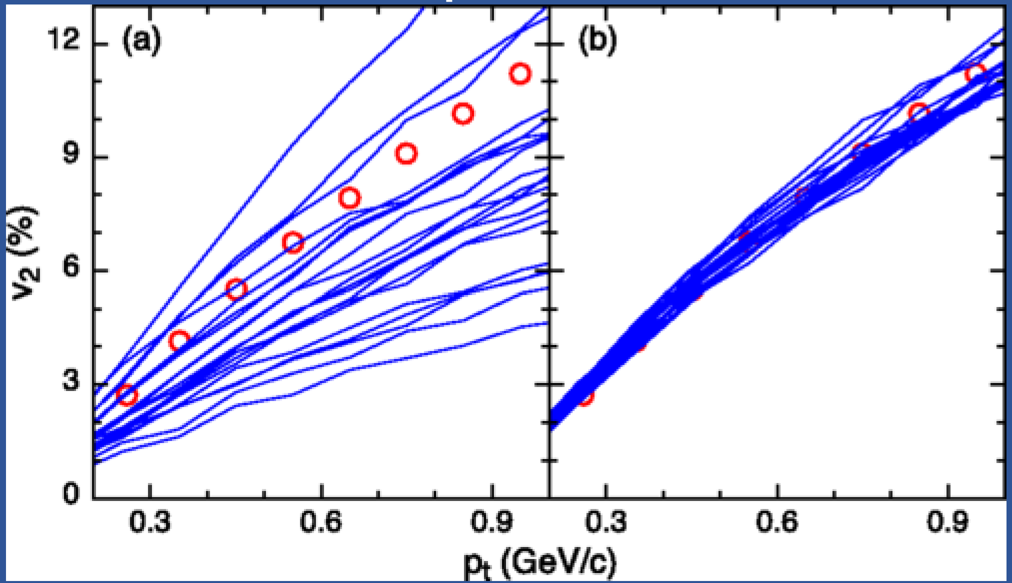
Collective flow generated by pressure gradient

Analysis of heavy-ion data by using dynamical model

p_T spectra



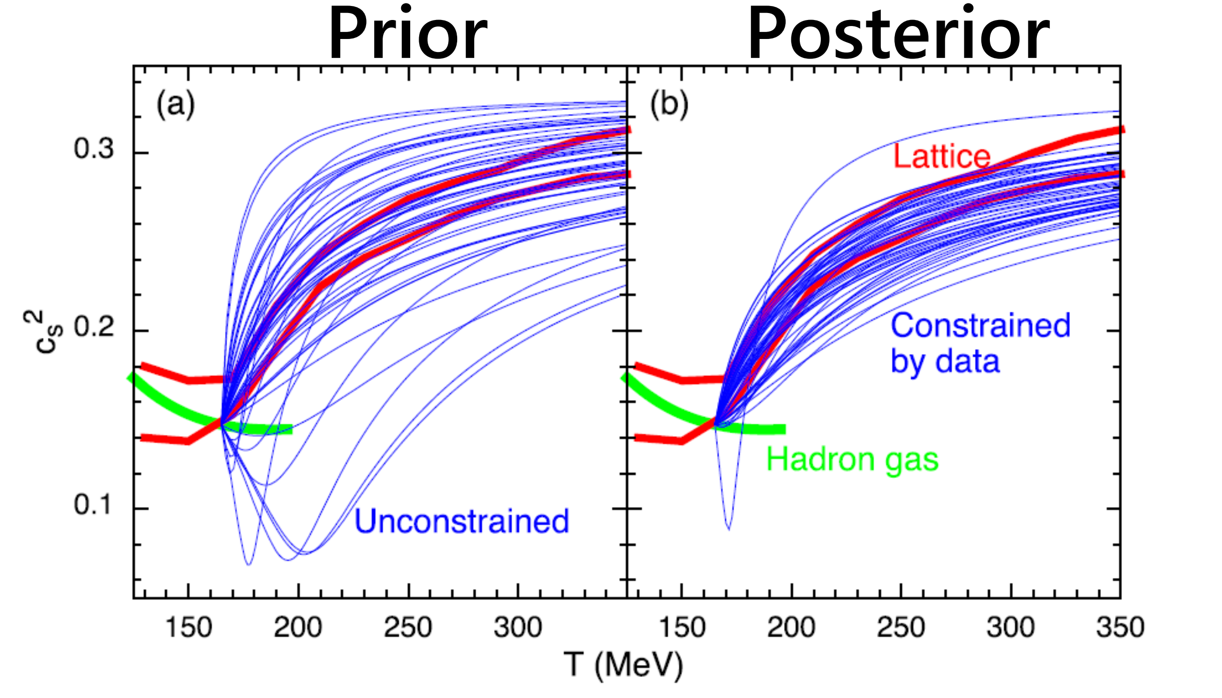
Elliptic flow



S. Pratt *et al.* (2015).

Dynamical model works well in production of yields, spectra and anisotropic flow

Heavy-ion data meet first principle



S. Pratt *et al.* (2015).

Sound velocity $c_s^2 = \partial p / \partial e$ from Bayesian analysis of heavy-ion data

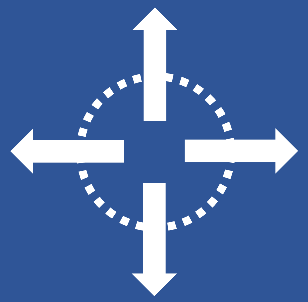
Toward precision stage of QCD matter physics
***Current understanding limited to zero/small baryon density**

Viscous coefficients

(Dissipative current) = (transport coefficient) x (thermodynamic force)

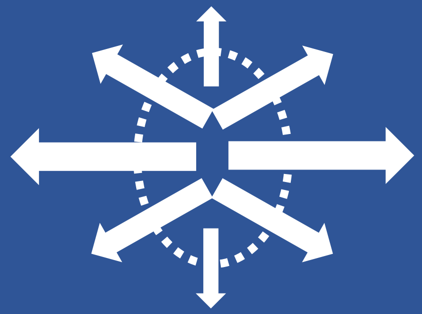
Bulk viscosity

$$\Pi = -\zeta \nabla \cdot u$$



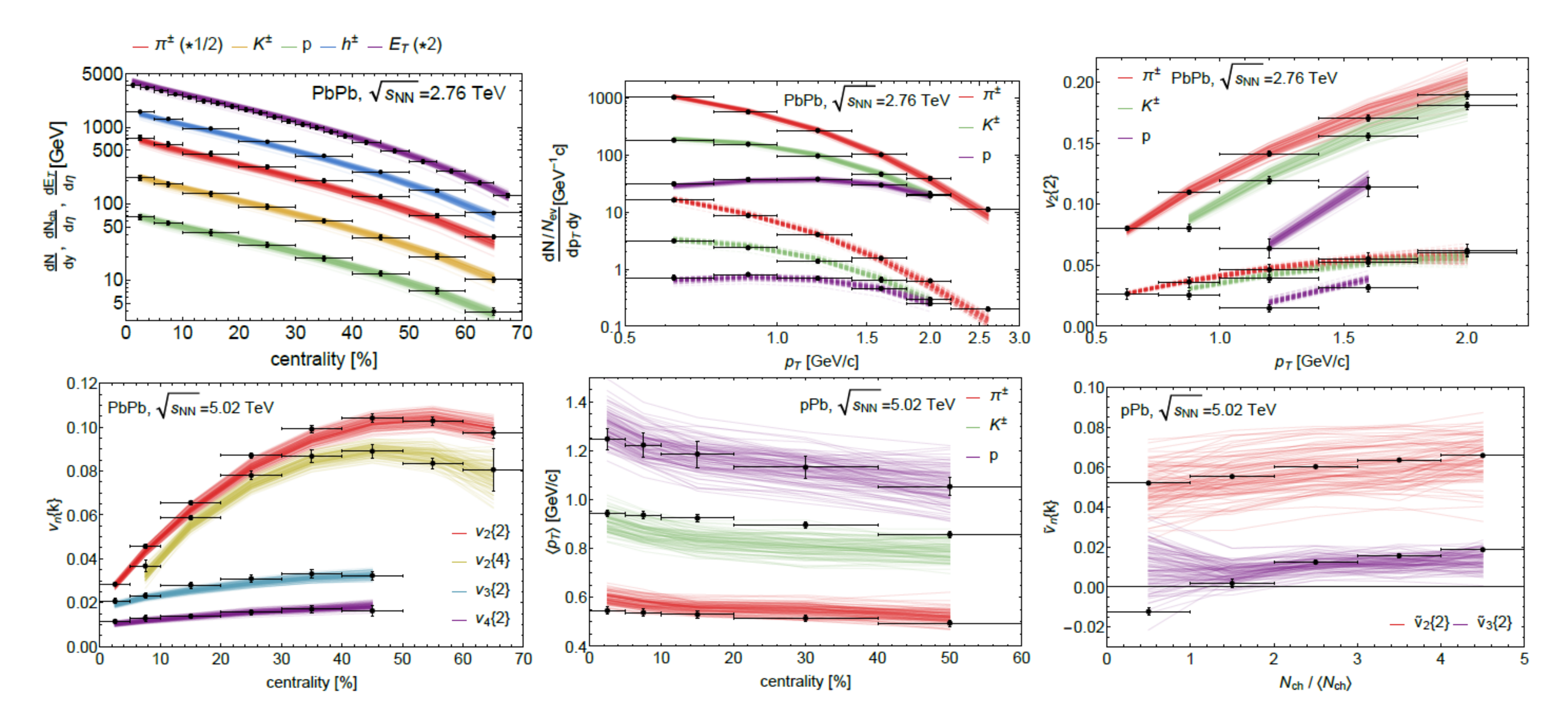
Shear viscosity

$$\pi^{\mu\nu} = 2\eta \nabla^{\langle\mu} u^{\nu\rangle}$$



+ higher order terms for relativistic causality

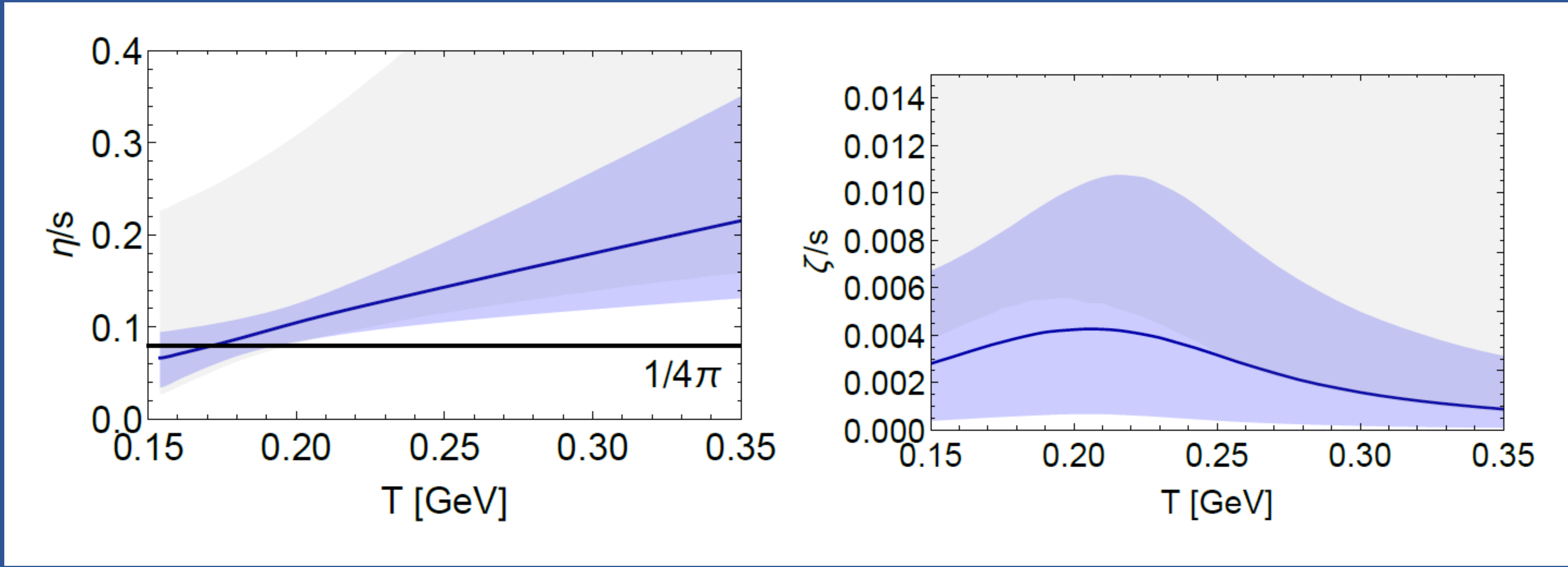
Bayesian analysis of heavy-ion data



G. Nijs *et al.* (2021).

Results using parameters in posterior distribution from Bayesian analysis

Bayesian analysis of viscous coefficients



G. Nijs *et al.*, (2021).

Shear viscosity near T_{pc} well constrained from exp. data
Still need sophistication to constrain bulk viscosity

*See also, G. Nijs and W. van der Schee (2022);
D. Everett *et al.* (JETSCAPE Collaboration) (2021);
J.E. Parkkila *et al.*, (2021); J. Auvinen *et al.*, (2020).

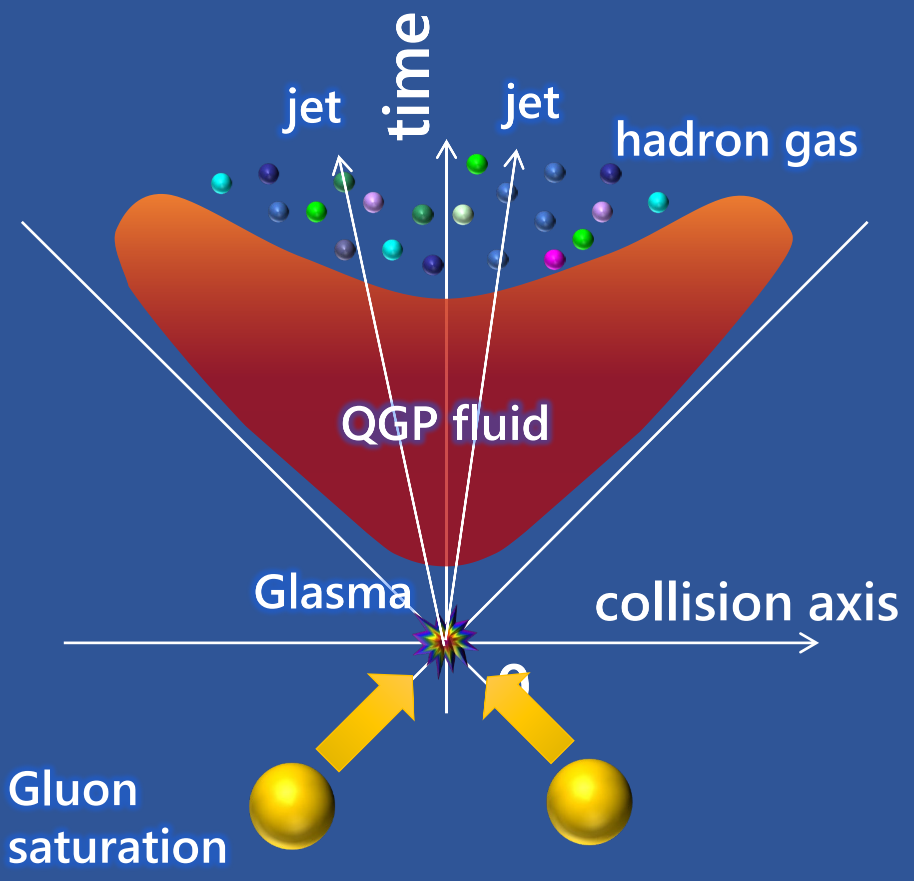
Short summary of current understanding

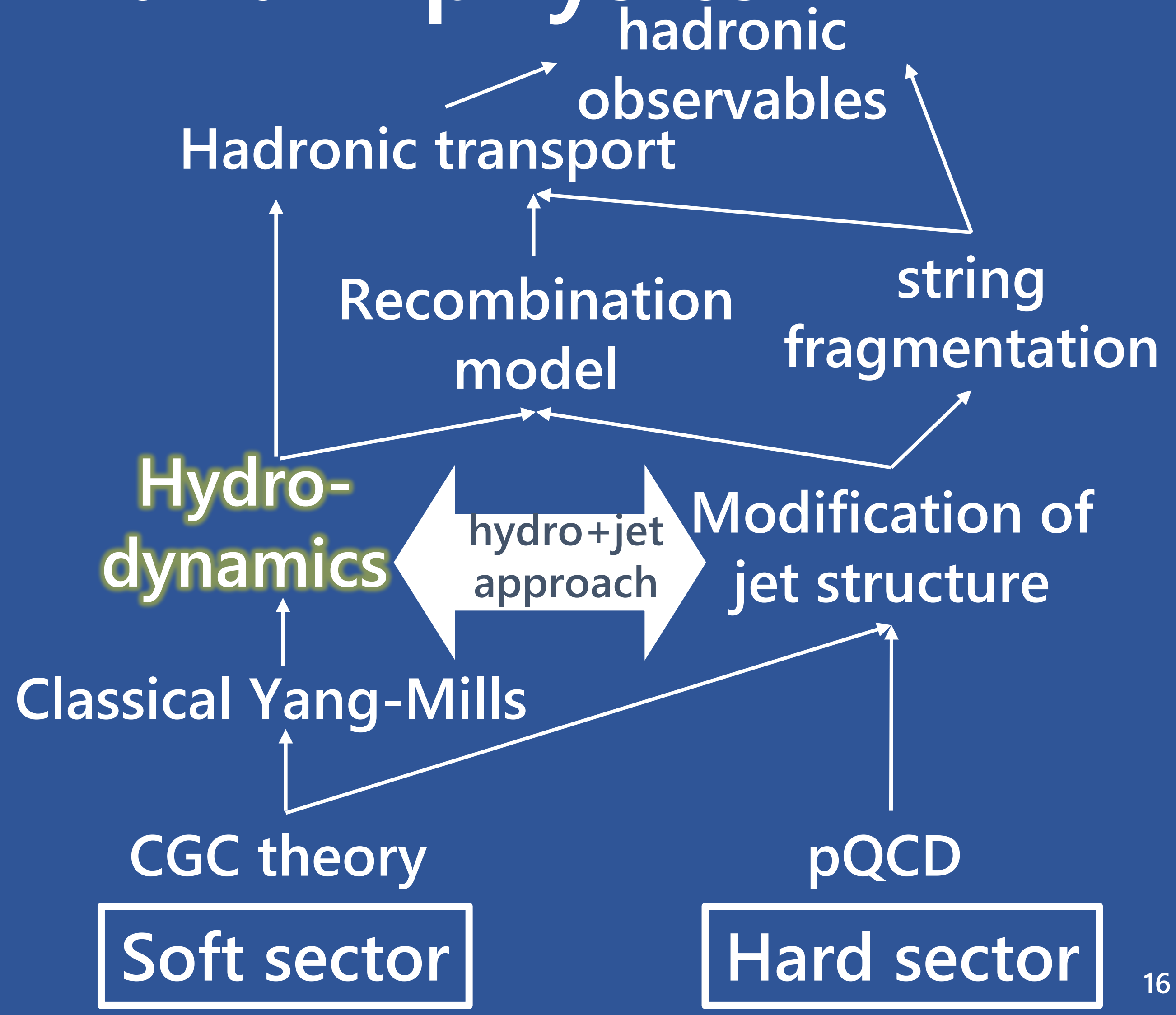
- Heavy-ion data + dynamical modeling → Rich physics of the quark gluon plasma
- Current: Equation of state $(p = p(e))$, shear (η) and bulk (ζ) viscosity
- Future: Baryon diffusion coefficient (σ), EM conductivity (σ_{EM}), …
- Framework for future precision study: spin hydro, resistive magneto hydro, anomalous hydro, …

3. Non-Equilibrium Aspects of Nuclear Collisions

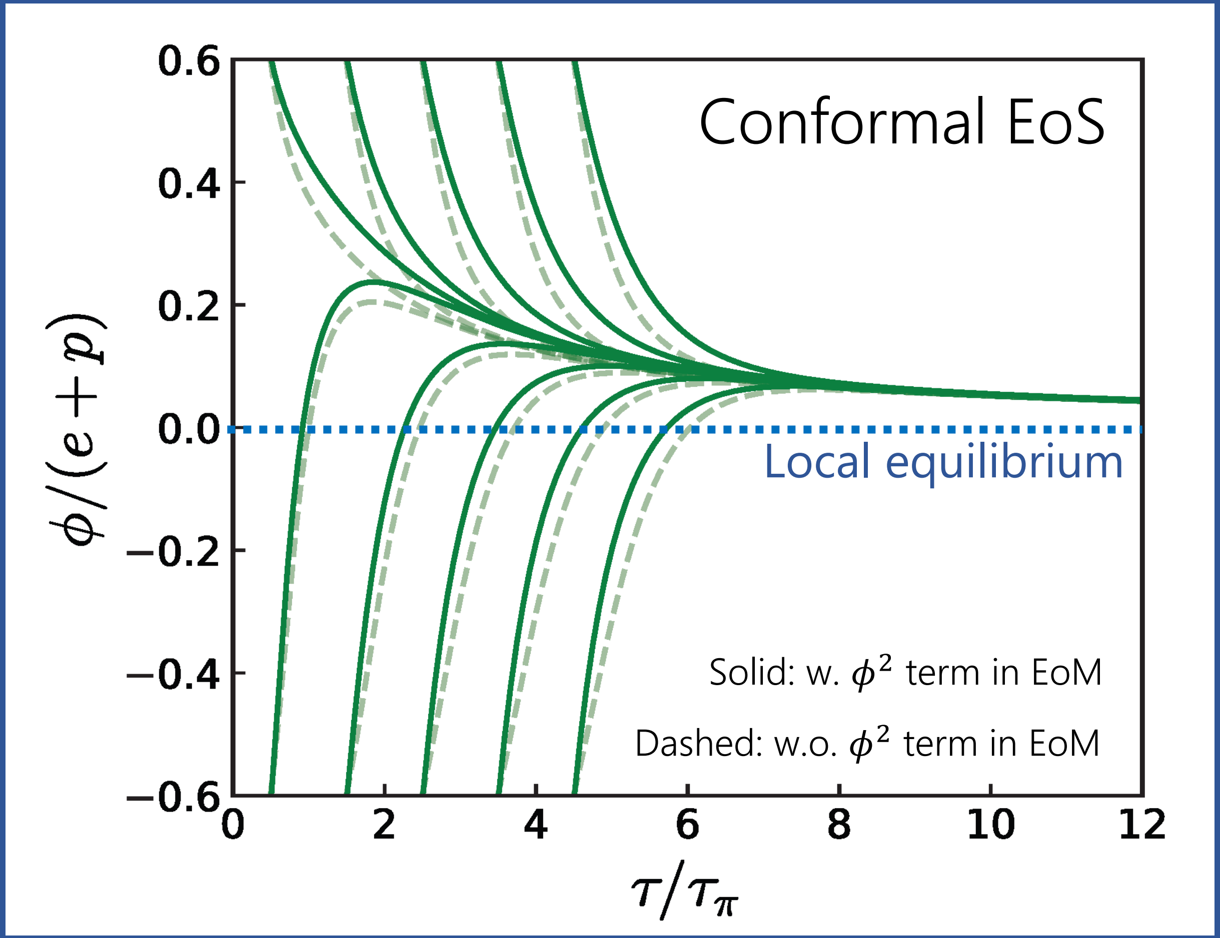
Work done with S. Fujii, T. Hoshino, Y. Kanakubo, and Y. Tachibana

Nuclear collisions as playgrounds of non-equilibrium physics





Far-from-equilibrium description (?)



T. Hoshino, TH (2025).

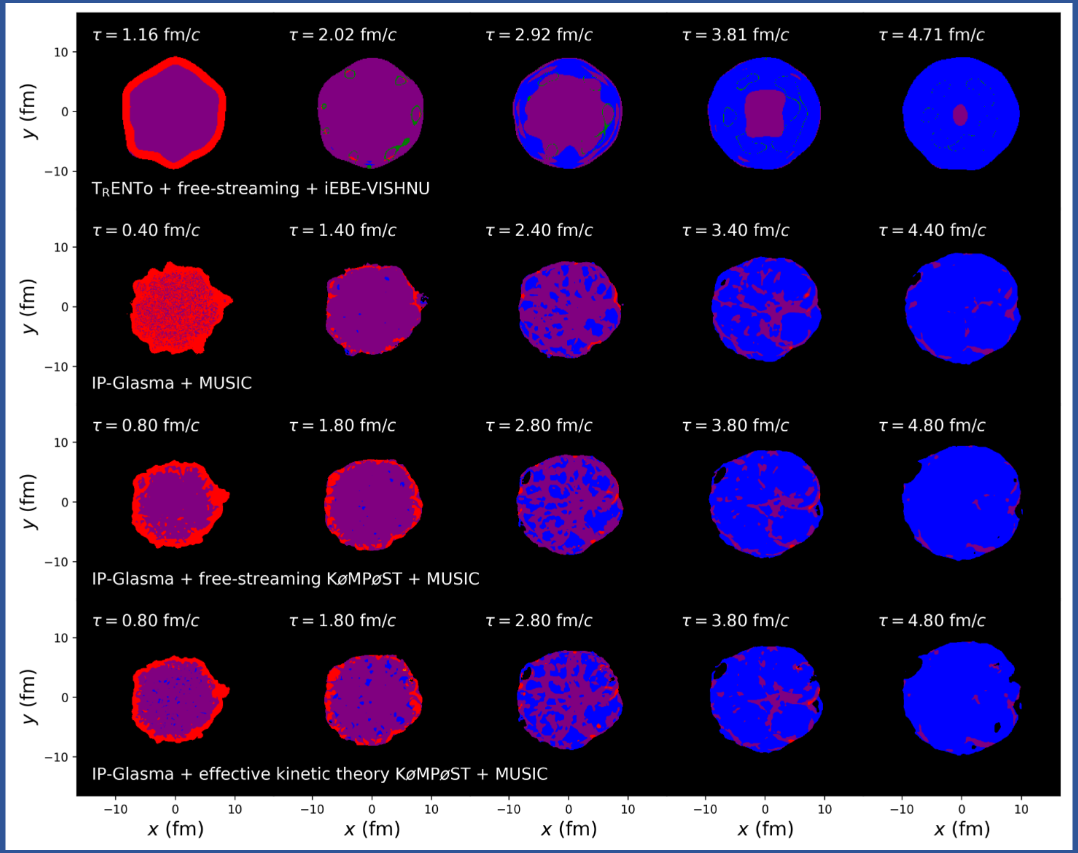
See, *e.g.*, M.P. Heller and M. Spaliński, (2015).

How the system goes toward to local equilibrium?

"Hydrodynamization"

Hydrodynamic **attractor** solution
→ Are (almost) any initial conditions acceptable?

Characteristic velocity and causality



C. Plumberg *et al.* (2022).

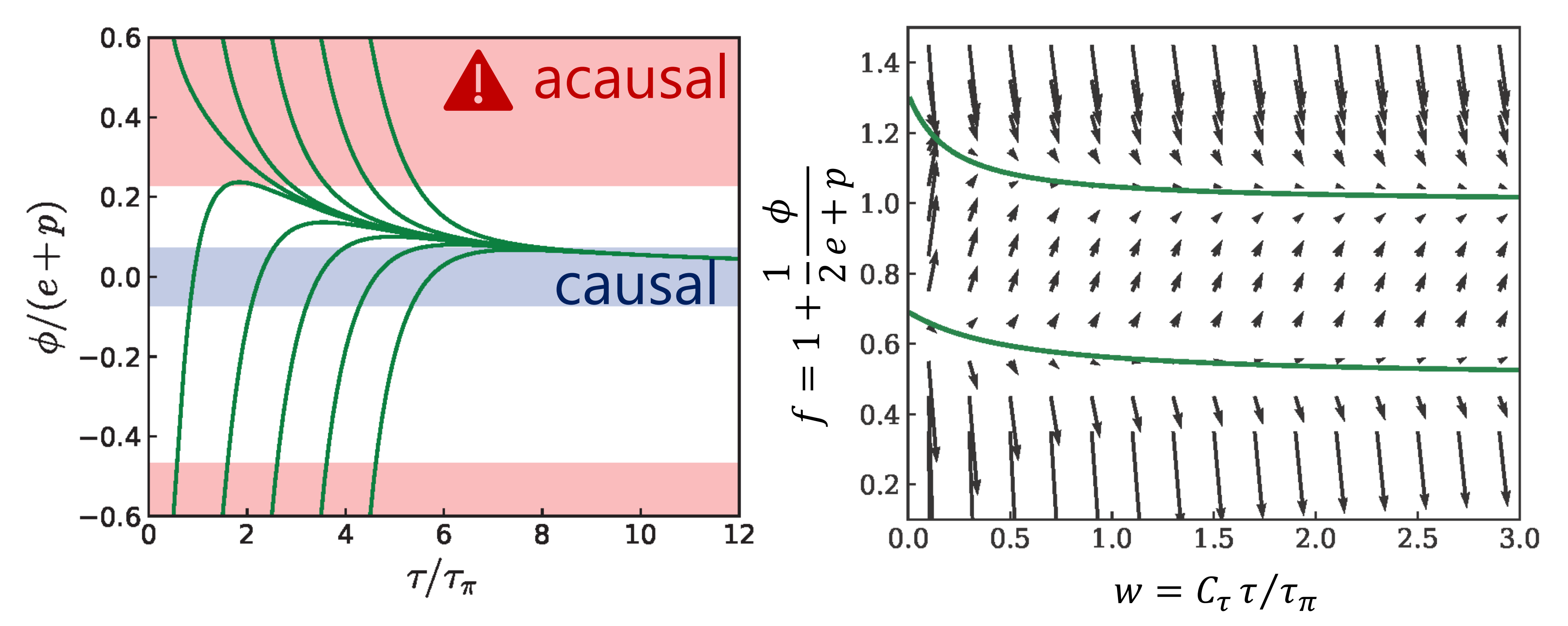
Transverse dynamics of hydro simulations

Characteristic velocities of hydrodynamic equations must obey relativistic causality

F.S. Bemfica *et al.* (2021).

Violations of causality
→ far-from-equilibrium
in the early stage and/or
near the edge

Violation of causality in far-from-equilibrium



T. Hoshino, TH (2025).

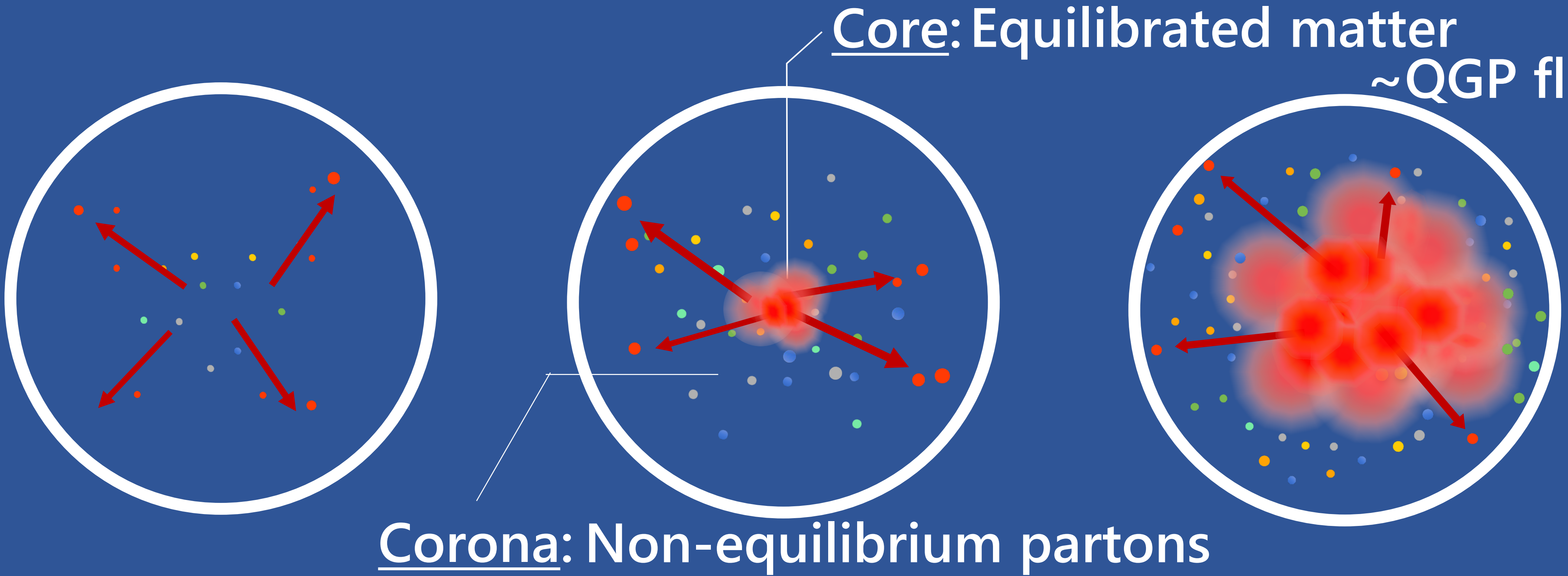
Violation of causality far from equilibrium ($Re^{-1} < 0.23$)
Running away from a hydrodynamic attractor for some initial conditions

Remarks on violation of causality

- Hydrodynamics should be applied closed to local equilibrium (as always!)
- Avoiding far-from-equilibrium fluids would affect extraction of the QGP properties in Bayesian analysis (ExTrEMe Collaboration)
- Necessity of non-equilibrium components/description
  - Boltzmann equation? Caveat: Applicability of Boltzmann eq and BBGKY hierarchy
  - Non-equilibrium field theory?
  - Long-standing issue in high-energy nuclear collisions

Core-corona picture as phenomenological approach

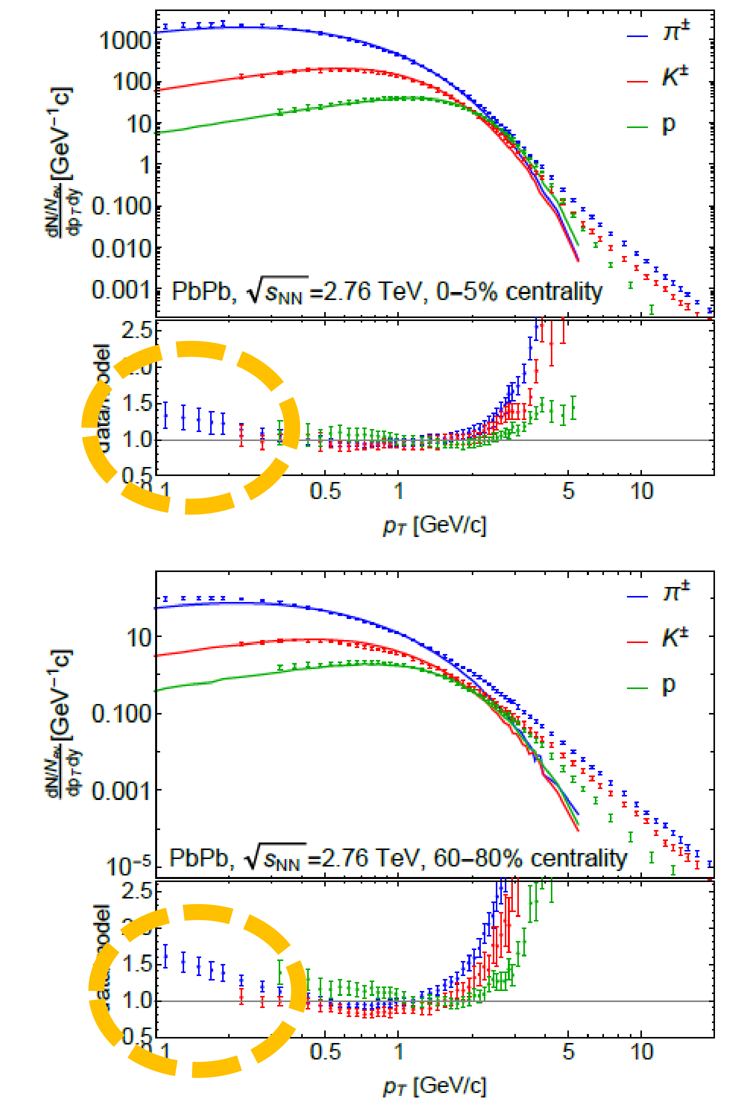
Bozek (2005, 2009), Aichelin, Werner (2009, + many), Becattini, Manninen (2009), Pierog *et al.* (2015), Akamatsu *et al.* (2018), Kanakubo *et al.* (2018, 2020, 2022a, 2022b)



Low ← Multiplicity (The number of particles) → High

Figures: Courtesy of Y.Kanakubo

Spectra at very low p_T



G. Nijs *et al.*, (2021).

Common understanding in low p_T spectra ← Boosted thermal dist.

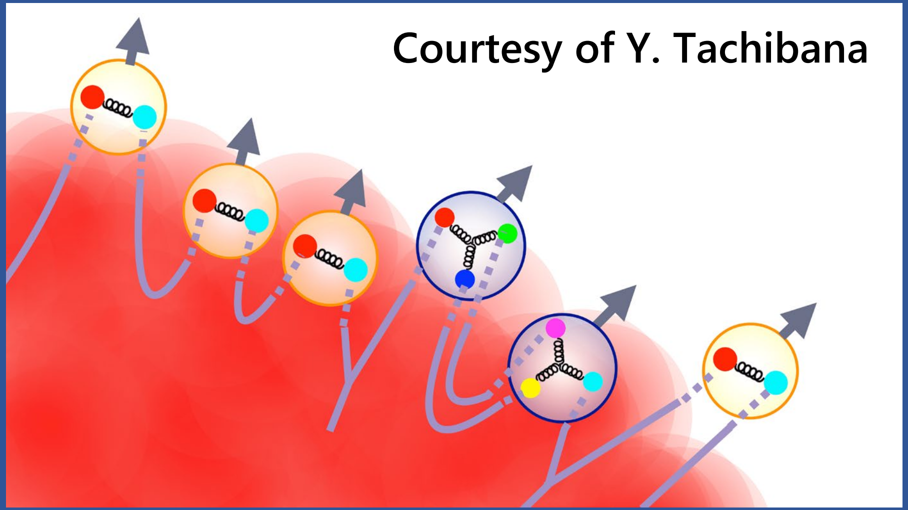
Failure of state-of-the-art hydro model at very low p_T (IR limit!?)

Need to pay attention for mean p_T analysis → Bulk viscosity?

Zimanyi *et al.* (1979), Kataja, Ruuskanen (1990), Gavin, Ruuskanen (1991), Sollfrank *et al.* (1991), Begun *et al.* (2014, 2015), Huovinen *et al.* (2017), Guillen, Ollitrault (2021), Grossi *et al.* (2021).

"Soft-from-corona" components

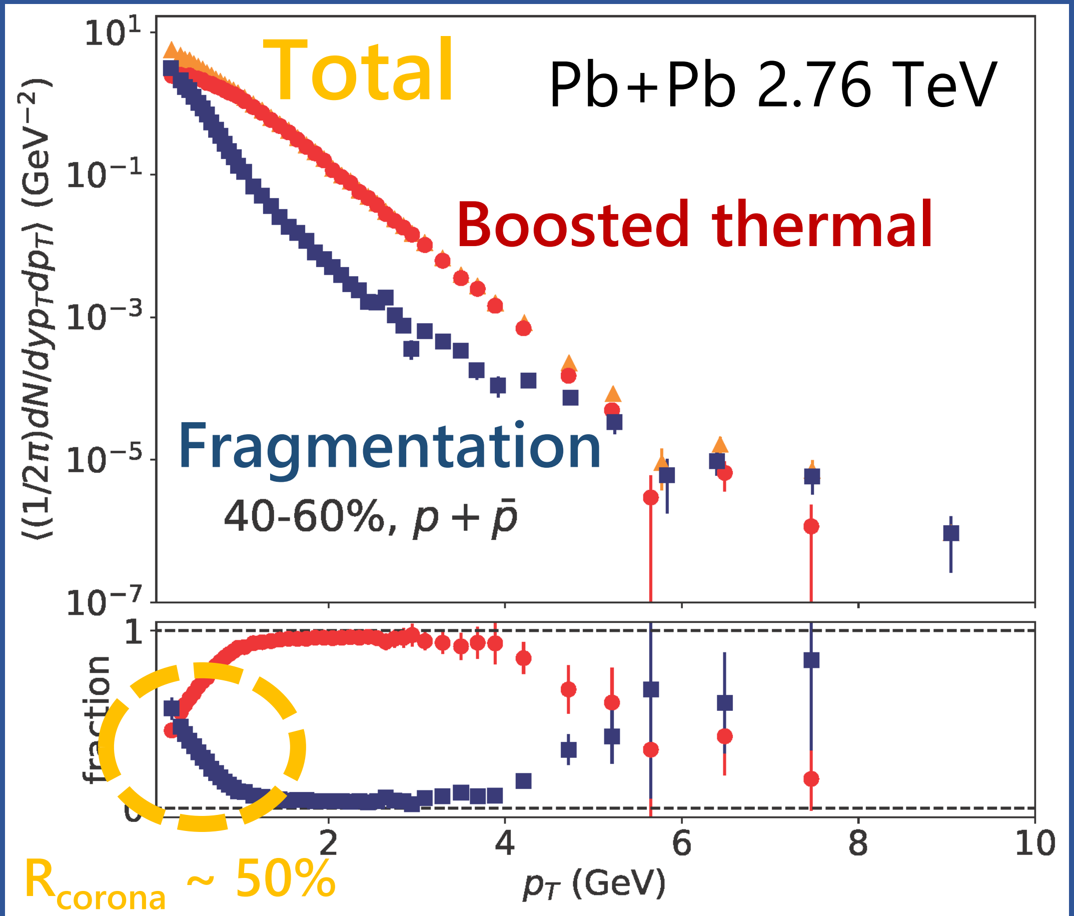
"hard strings" in DCCI2



Traversing hard partons

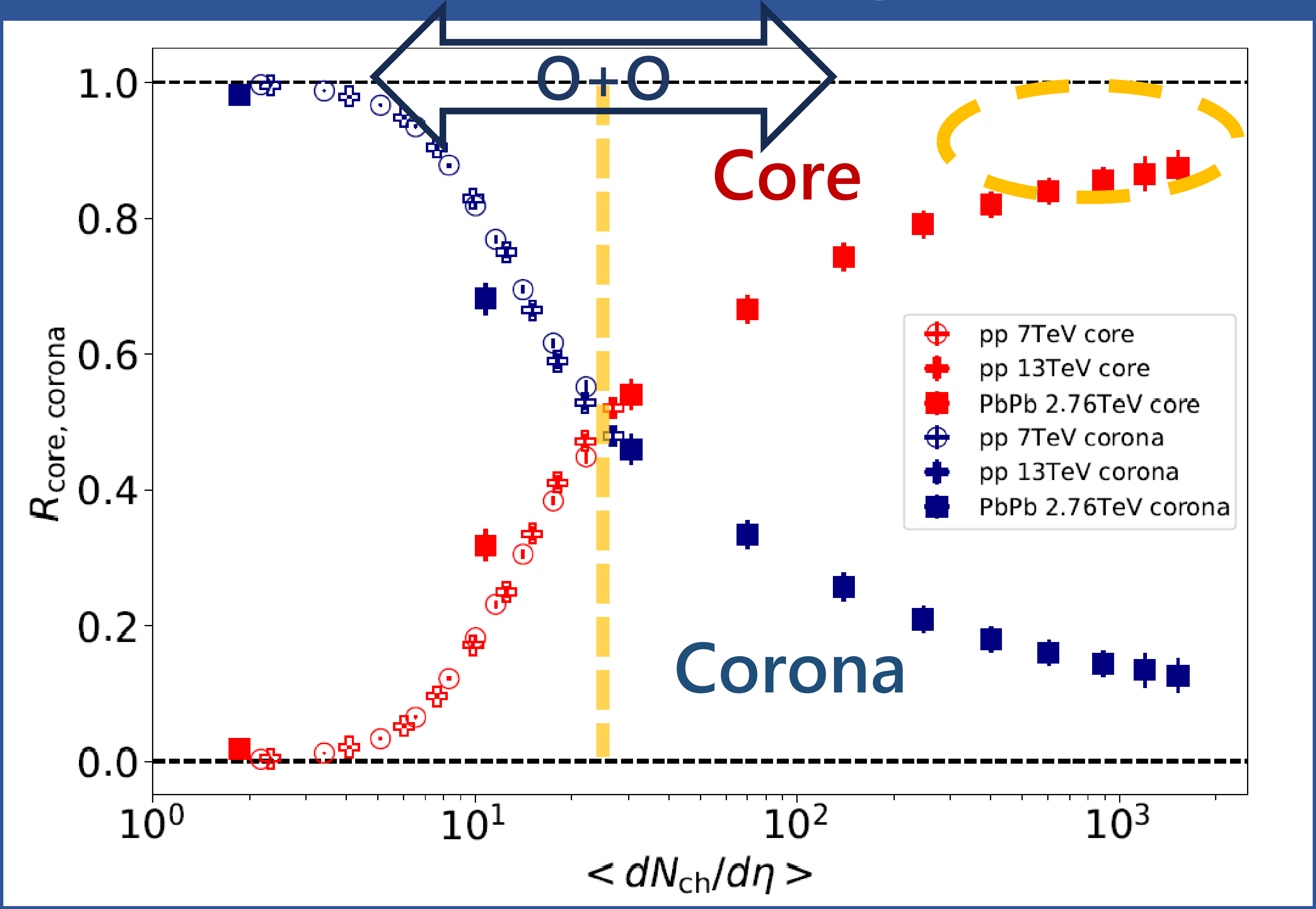
String formation

String fragmentation into soft hadrons
← Power-law shape



Y. Kanakubo *et al.* (2022).

Clear scaling with multiplicity



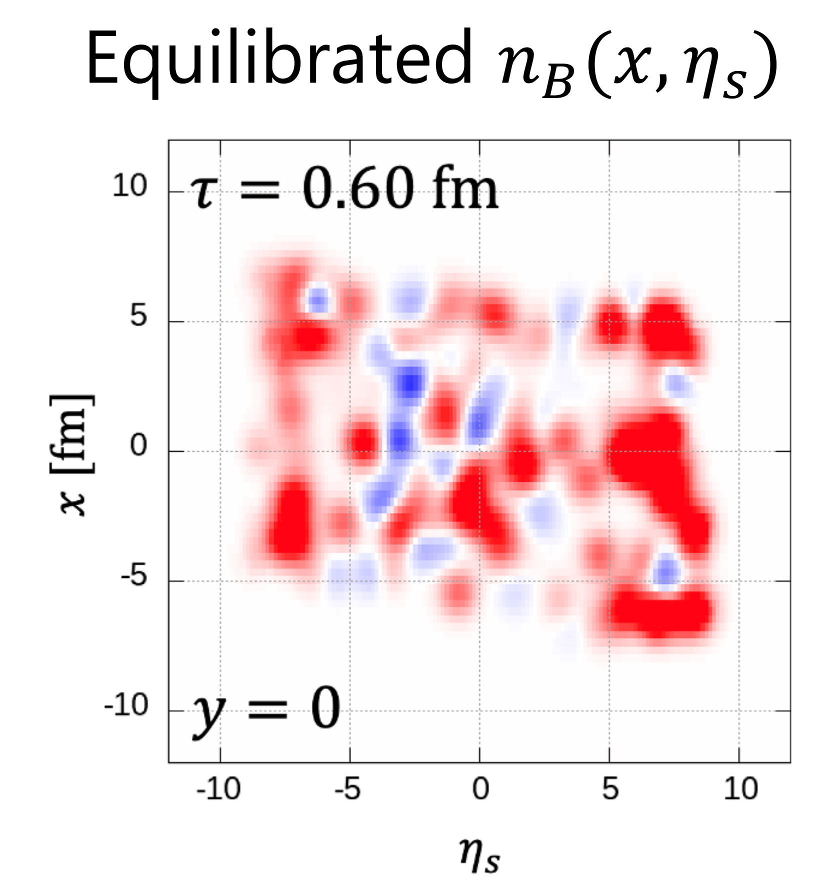
Y. Kanakubo *et al.* (2022).

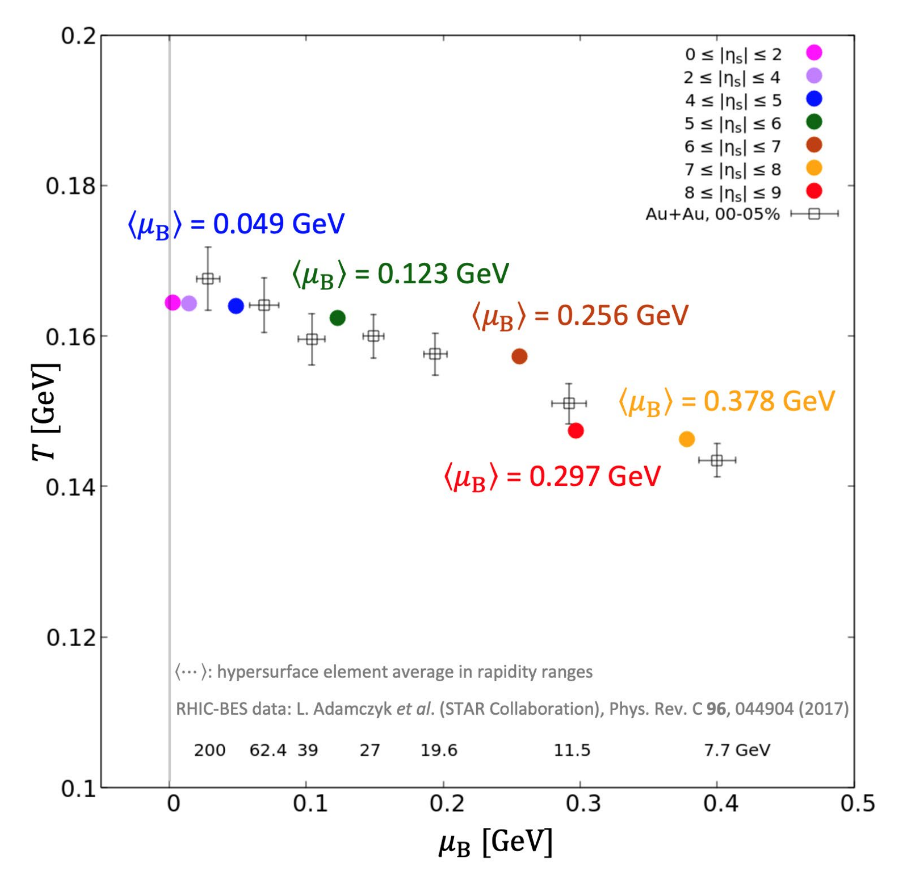
*O+O at sqrt_snn = 7 TeV from PYTHIA Angantyr

Onset of core dominance at $\langle dN_{ch}/d\eta \rangle \sim 20$

$R_{core} \sim 0.85$ at central PbPb?

Rapidity scan with core-corona





S. Fujii *et al.* (2025).

Open a new opportunity to investigate finite baryon density matter in forward regions at LHC (!?)

Go Forward!

Summary

Success of dynamical modeling

✓ Bayesian analysis provides properties of QGP

✓ Dawn of precision era in the physics of high-energy nuclear collisions

Issues of current dynamical modeling

✓ Acausal initial conditions

✓ Lack of very low p_T components

Summary

Success of dynamical modeling

✓ B

QGP

✓ D

high

Issue

✓ A

✓ Lack of very low p_T components

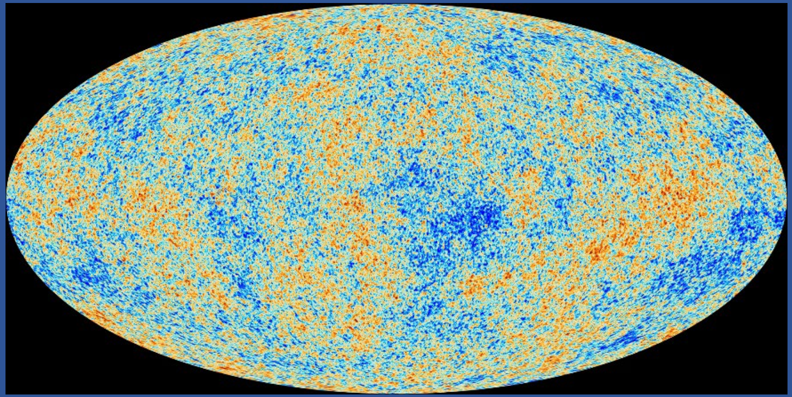
Importance of the non-equilibrium stage to understand equilibrated matter

Extra slides

Remarks on core-corona picture

- ✓ Description of both local equilibrium and non-equilibrated matter at once
- ✓ Necessity of non-equilibrium components even in understanding properties of QGP in equilibrium

Analysis tool in observational cosmology



Cosmic Microwave Background
Fluctuations of temperature (Planck)

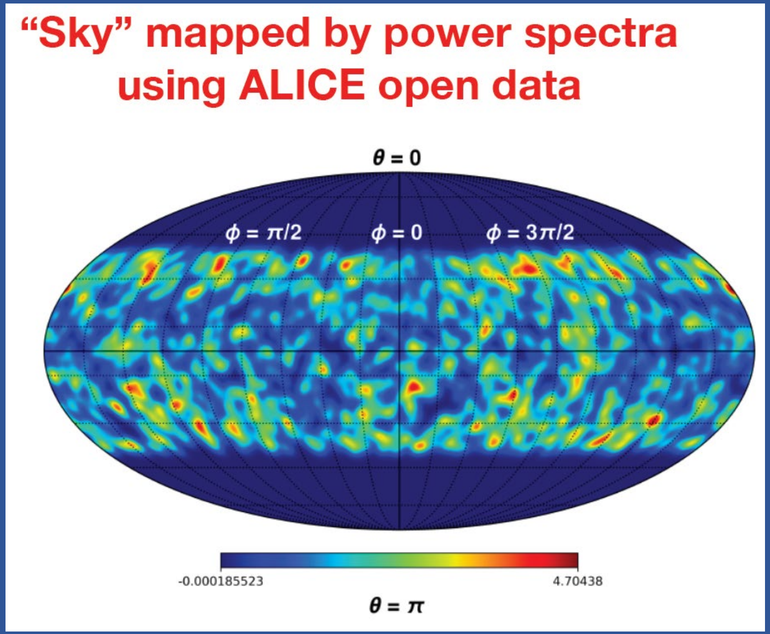
CAMB, CMBFAST, CosmoMC,...

Cosmological parameters

- Energy budget
- Hubble constant (lifetime)
- Curvature (flatness)
- Information about inflation
- ...

http://www.esa.int/spaceinimages/Images/2013/04/Planck_CMB_black_background

Dynamical model of QGP physics in high-energy nuclear collisions



Y.Zhou, talk at QM2018

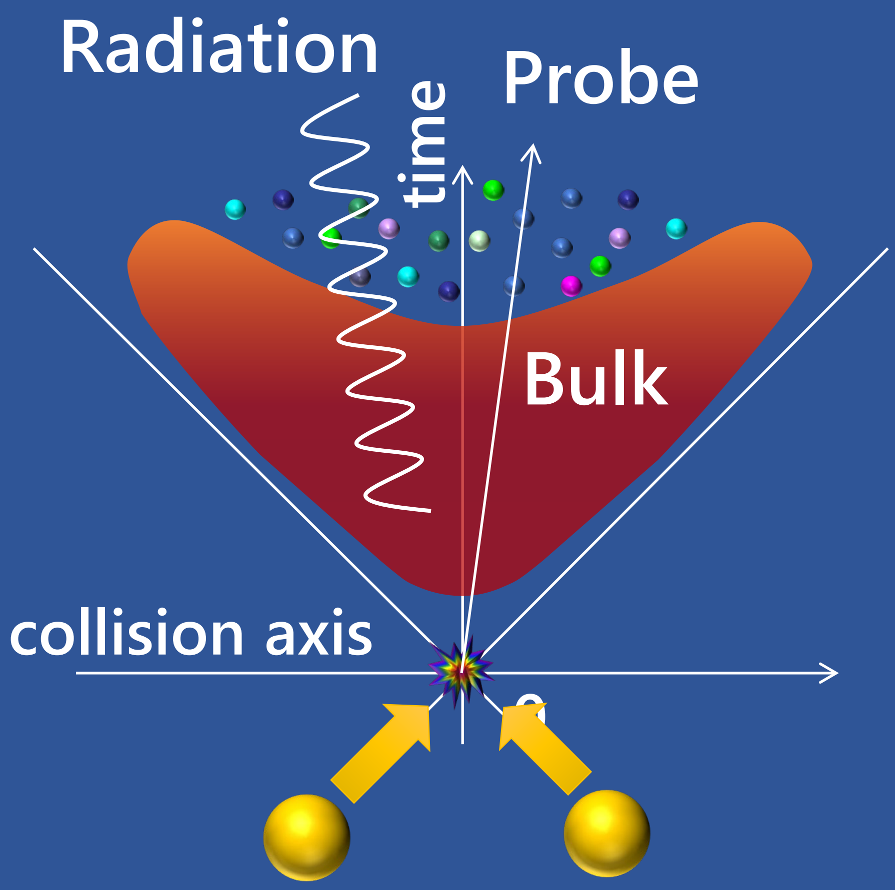
Dynamical model

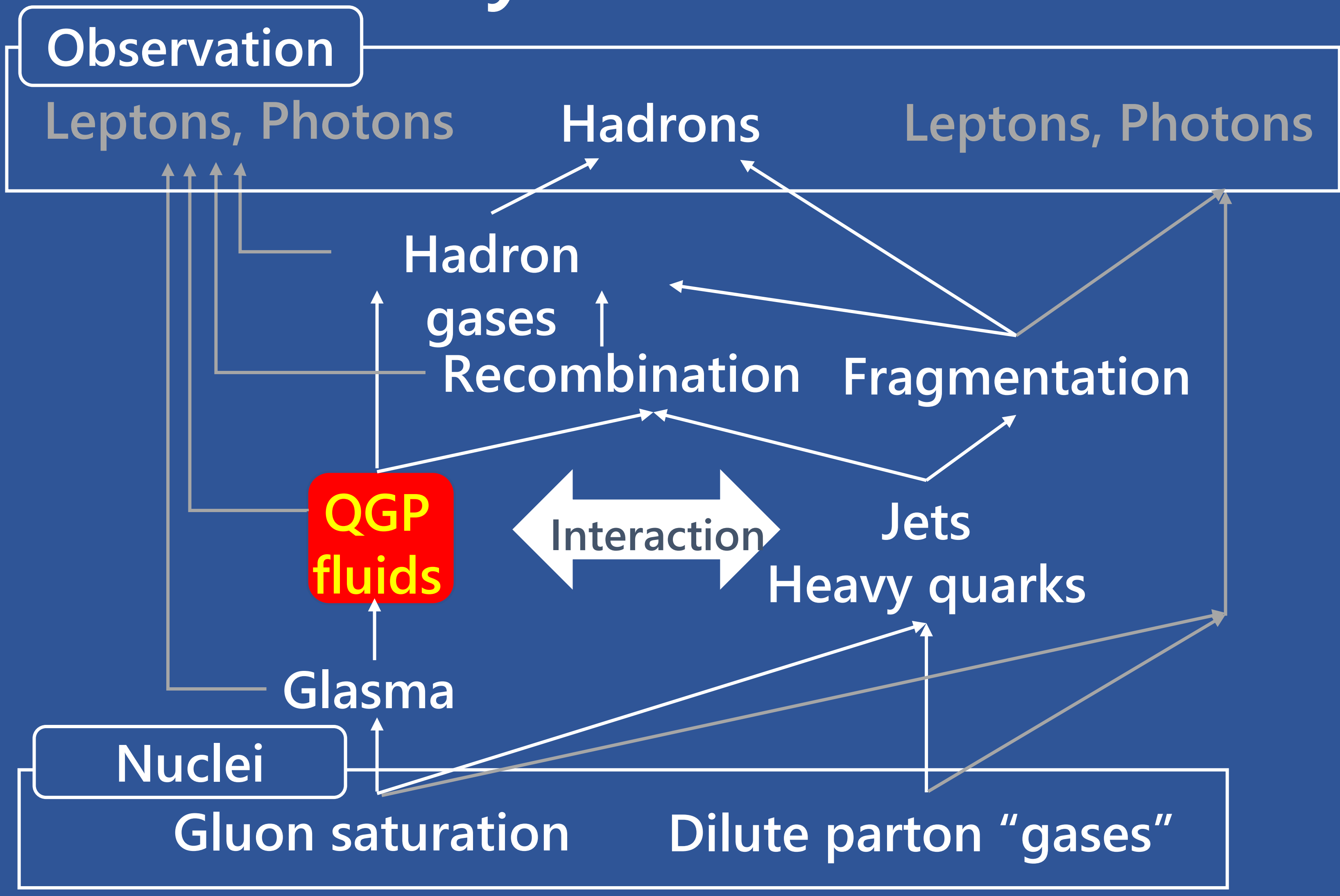
Physics properties of the QGP

- Equation of state
- Shear viscosity
- Bulk viscosity
- Stopping power
- …

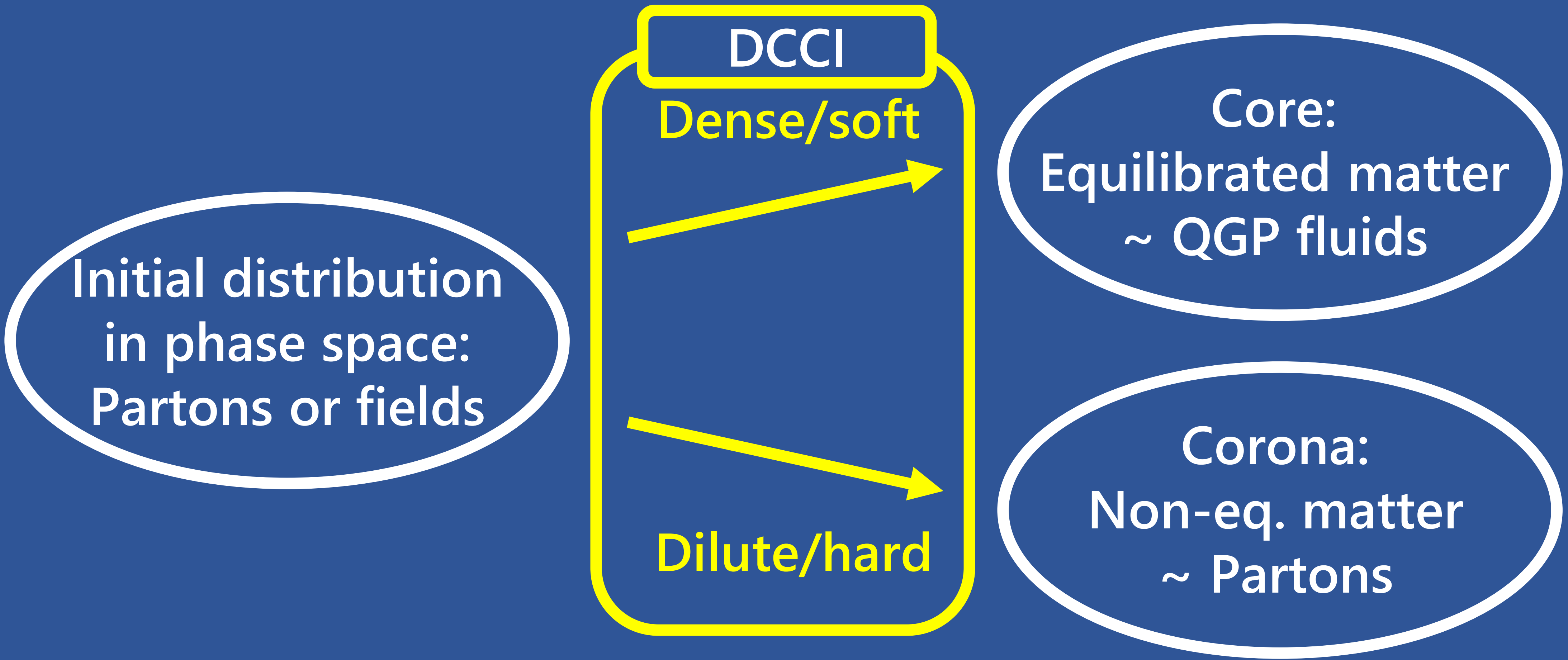
Bottom-up approach in high-energy nuclear collisions

Standard picture of heavy-ion reactions





Dynamical core-corona initialization

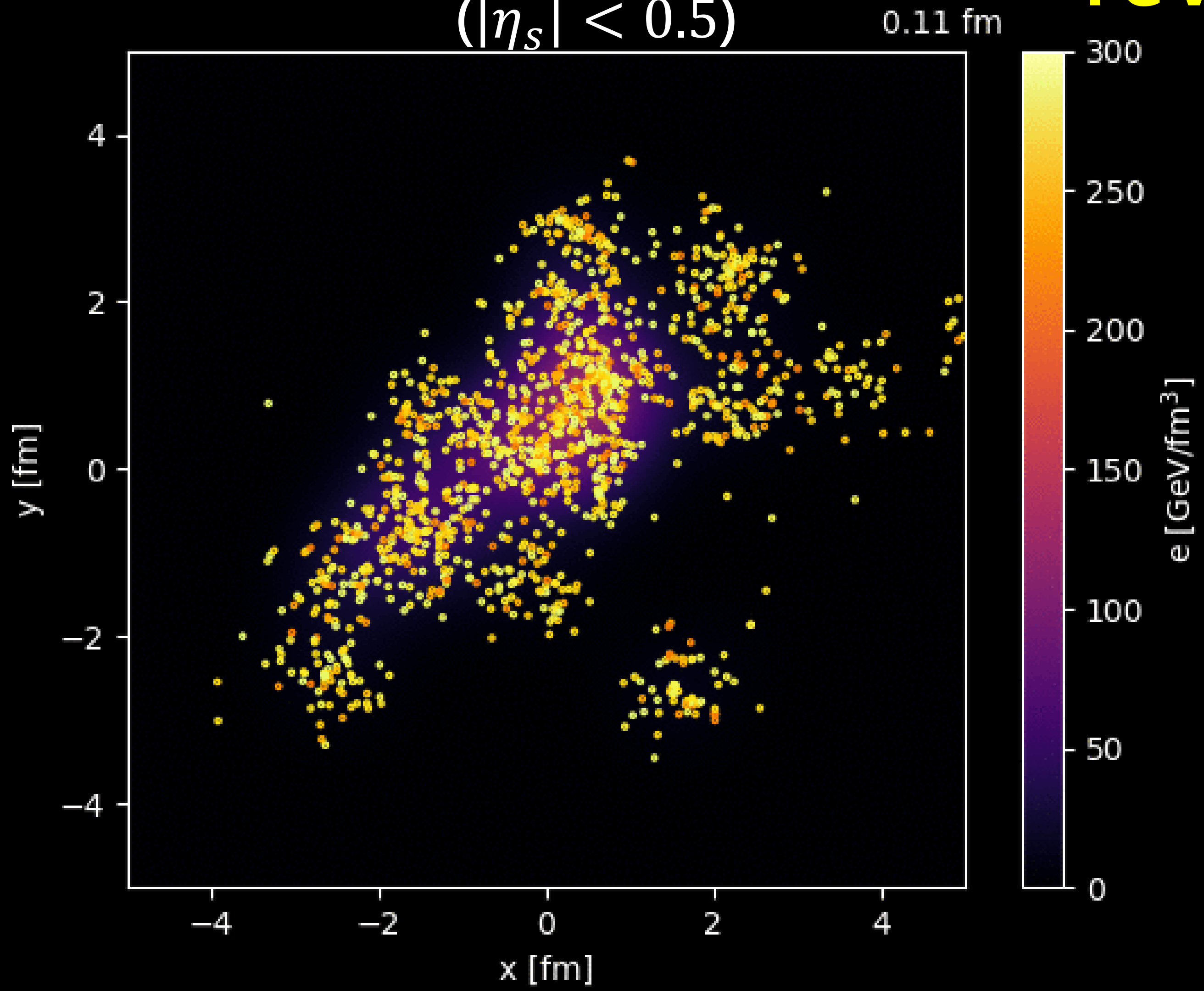


Separation of initial distribution just after collisions dynamically

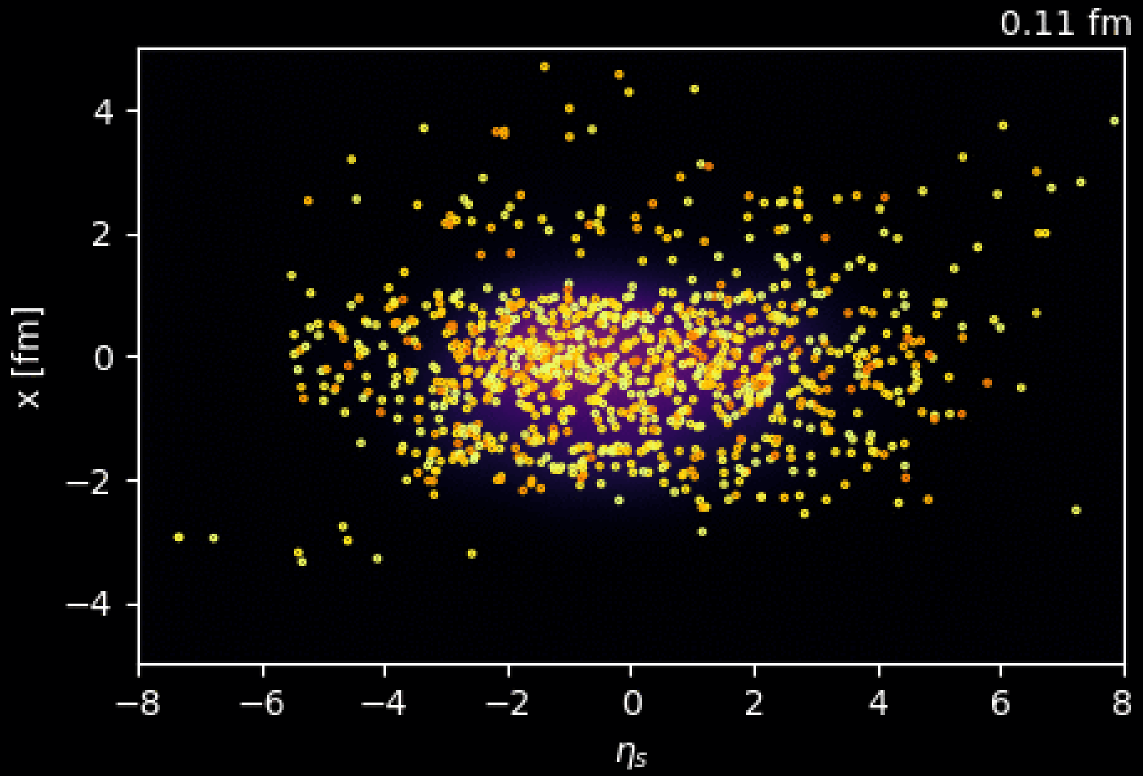
Y. Kanakubo *et al.*, Phys. Rev. C 101, 024912 (2020); Phys. Rev. C 105, 024905 (2022); Phys. Rev. C 106, 054908 (2022).

How DCCI works in AA
Pb+Pb 2.76 TeV

Transverse plane ($|\eta_s| < 0.5$)

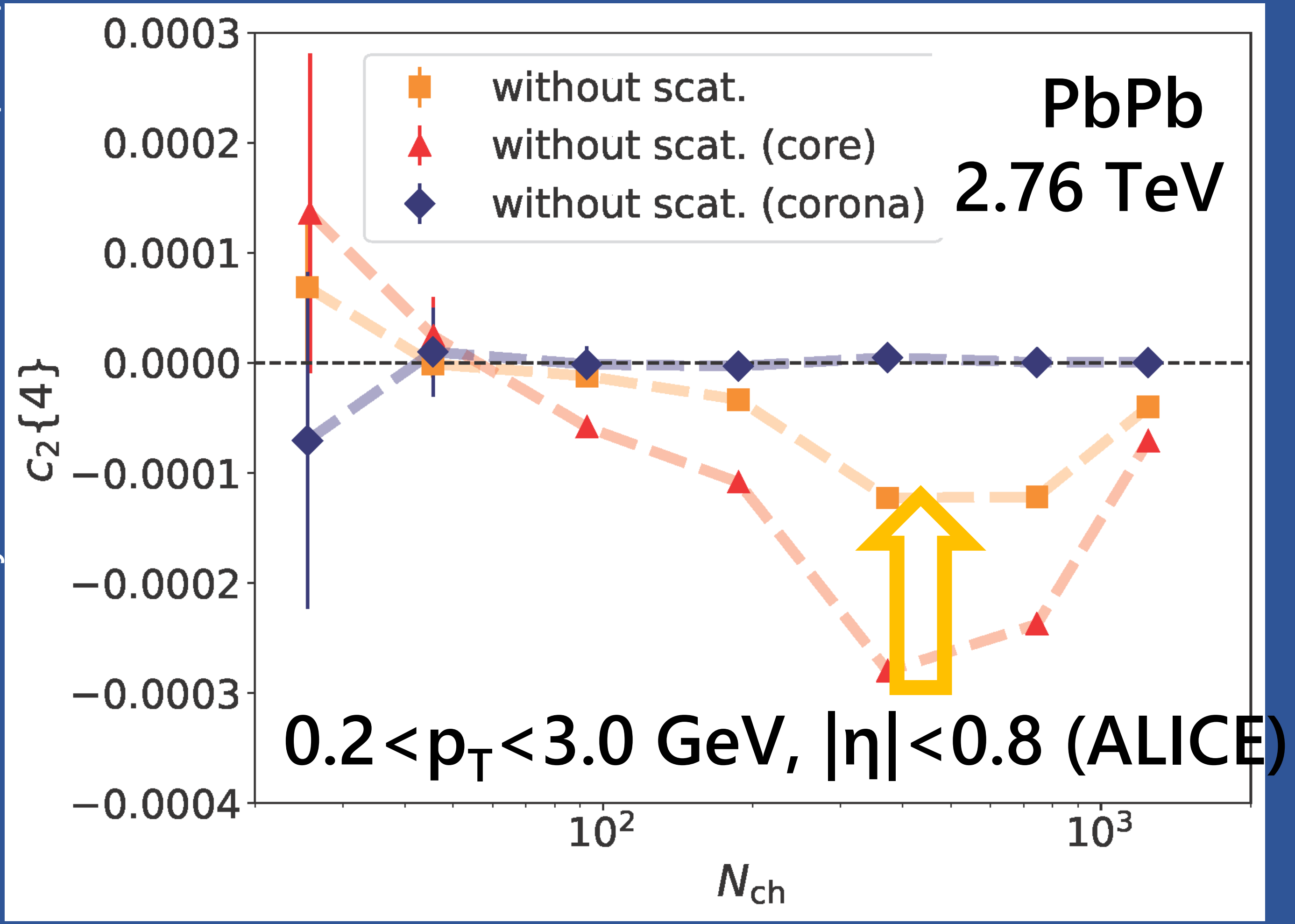


$x - \eta_s$ plane ($|y| < 0.5$)



Movies: Courtesy of Y. Kanakubo

Corona dilutes $c_2\{4\}$



Y. Kanakubo *et al.*, Phys. Rev. C 106, 054908 (2022).

Dilution effect even after non-flow subtraction

Importance in mean p_T, v_n, dN/deta, etc.

Lesson for experimental people
→ Avoid very low p_T

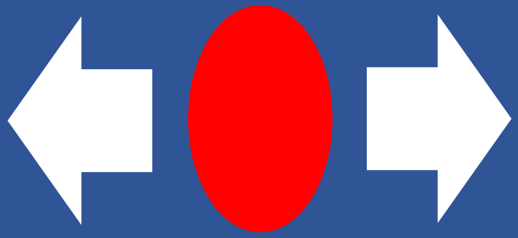
Lesson for theory people
→ Do not ignore corona

Short summary on core-corona picture

- ✓ “Mind the gap”
- ✓ Soft-from-corona components can fill the gap
- ✓ Use p_T *differential* data in Bayesian analysis, otherwise, develop a “full event generator” based on hydro
- ✓ Importance of non-equilibrium process in understanding QGP properties

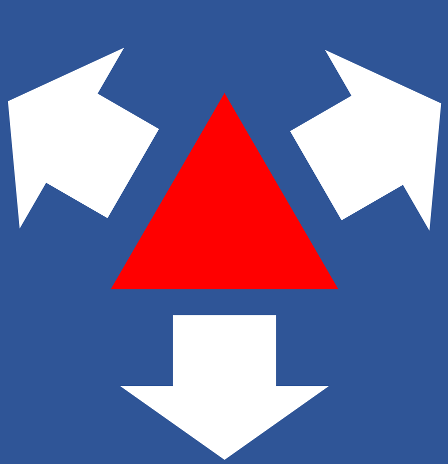
Response to shape of thermalized matter

Flow generated by anisotropic pressure gradient



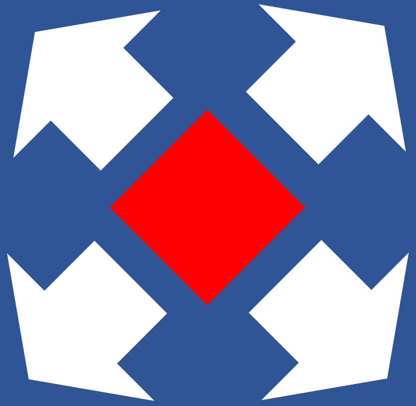
$n = 2$ (quadrupole)
Elliptic flow

Ollitrault (1992)



$n = 3$ (hexapole)
Triangular flow

Alver, Roland (2010)



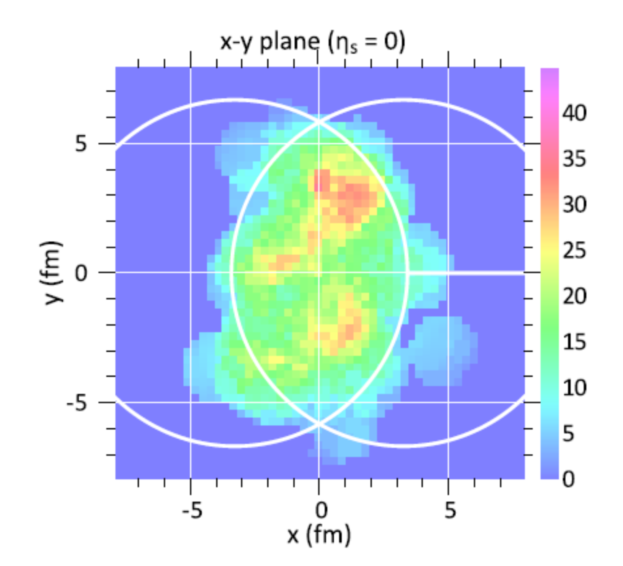
$n = 4$ (octapole)
Quadrangular flow

Kolb (2003)

…

Fourier decomposition of azimuthal distribution
→ Rich information about properties of system

Toward precision study of nuclear shape



TH *et al.* (2013)

Entropy density Distribution From MC-Glauber

$$v_n \approx \kappa_n \varepsilon_n$$

v_n: The nth anisotropic flow

ε_n: The nth deformation parameter

Conventional studies

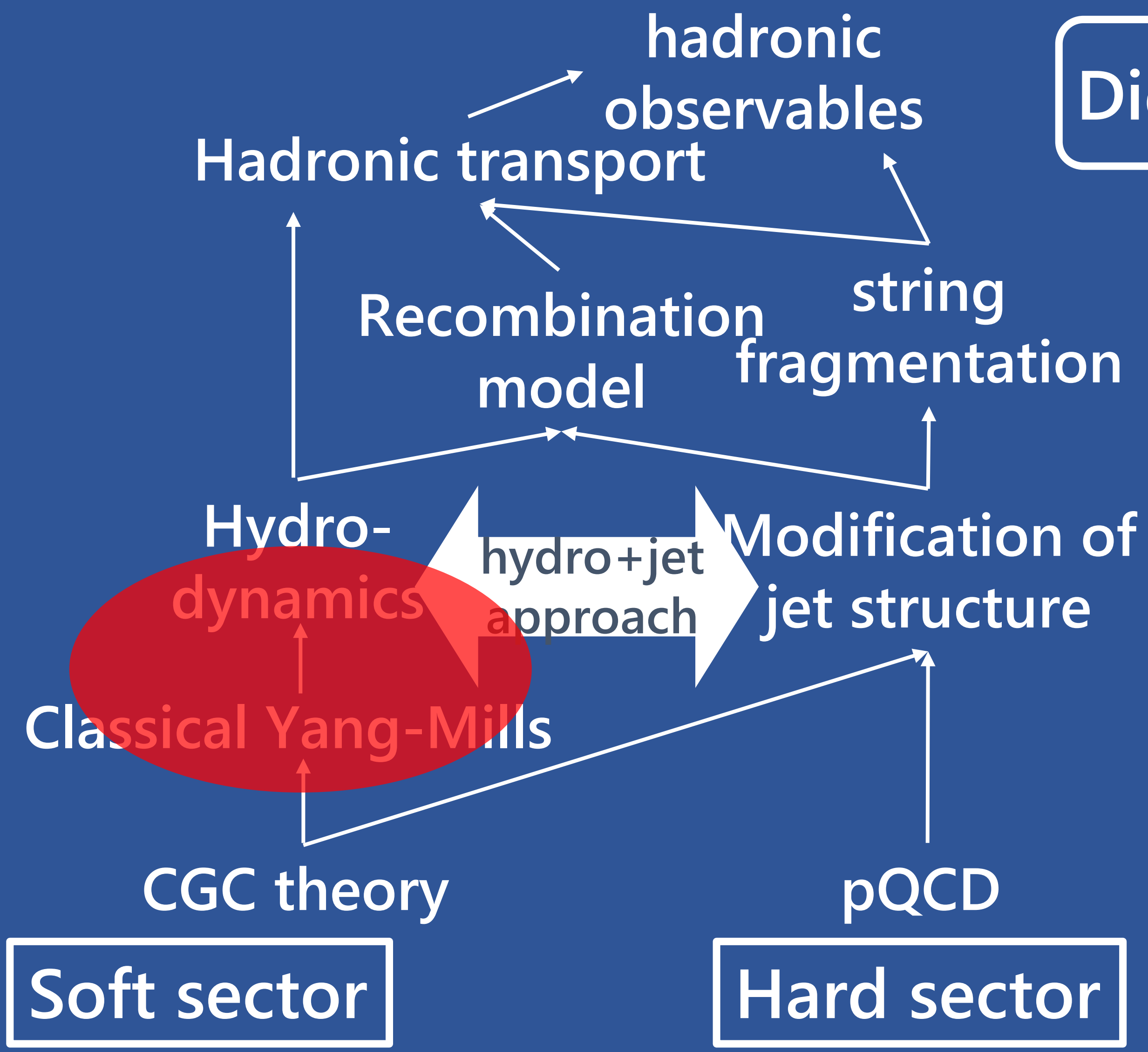
Initial deformation → Hydrodynamic (or transport model) responses to extract bulk and transport properties

Toward precision study

Anisotropic flow → Initial deformation → Nuclear structure? (Supplement to nuclear structure study???)

- Typical size of bumpiness on an event-by-even basis?
- Corona (non-equilibrated) region "deforms" the shape of initial profiles of created matter, in particular, in small colliding system
- How much is the system equilibrated after all?

Outlook



Did we understand dynamics after all?

How much energy deposited?

⇕

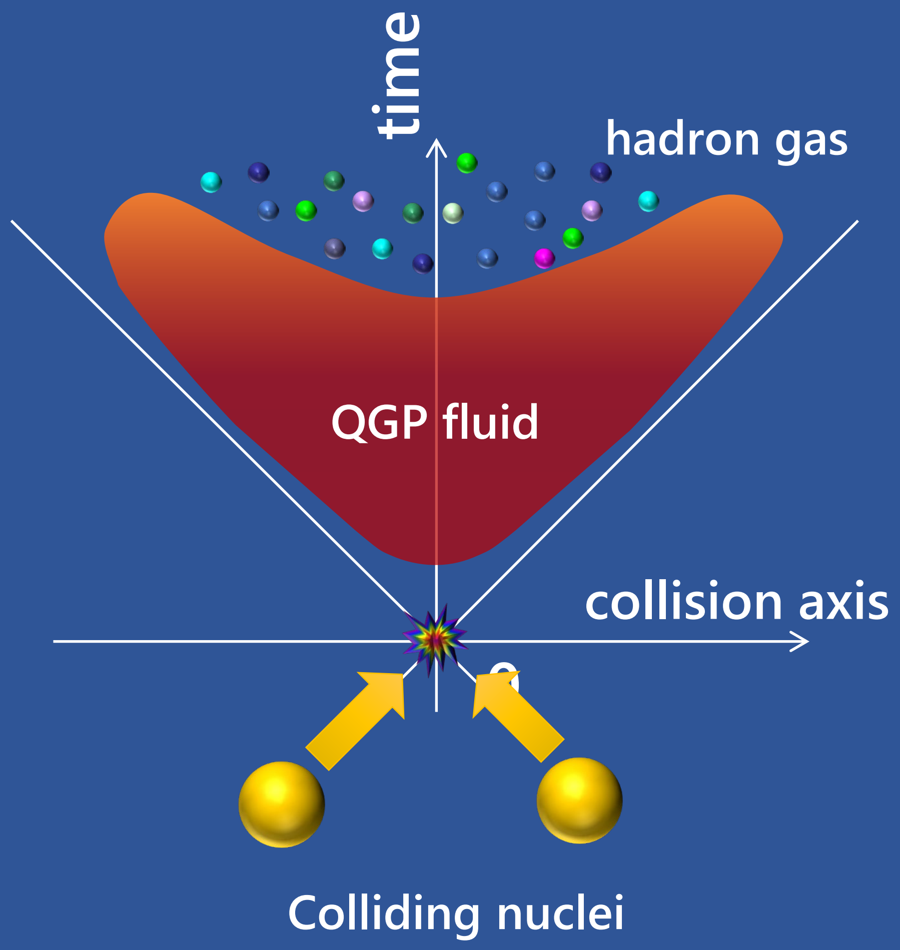
How much initial nuclei lose their energy?

How much deposited energy thermalized?

EIC and LHC forward
(and Forward Physics Facility?)

*The old Roman name for the city of Utrecht

TRAJECTUM* as an example



Hadronic transport
(SMASH)

↑

Relativistic hydrodynamics
(Viscous fluids)

↑

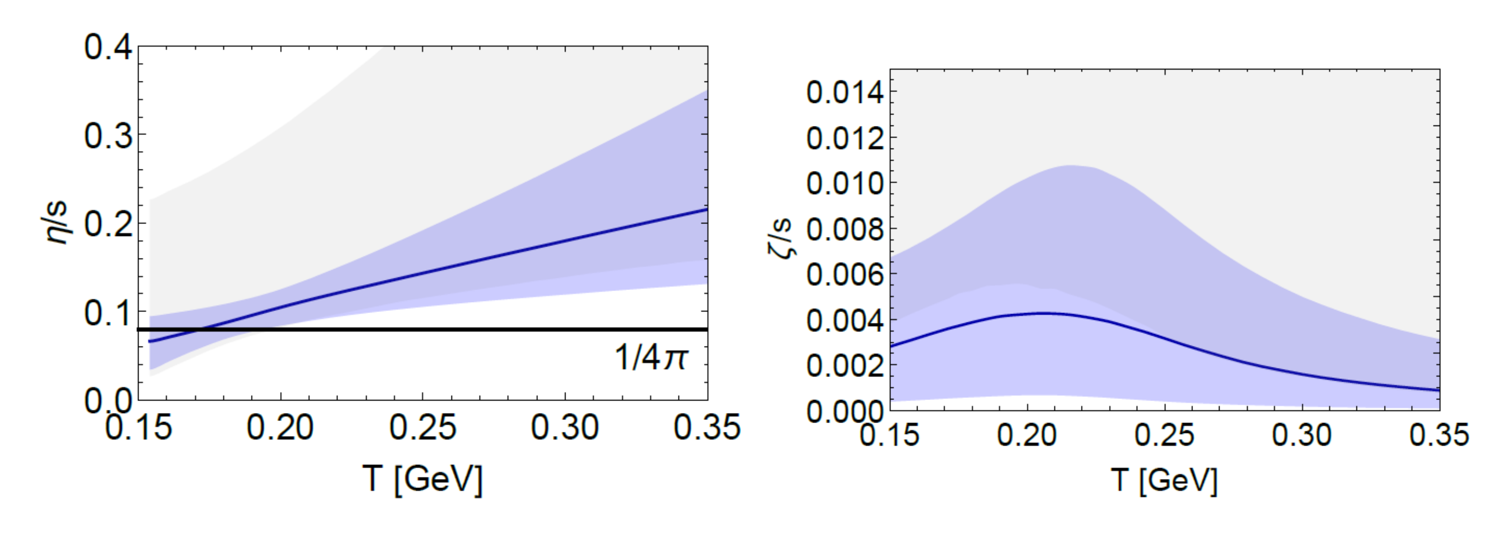
Free streaming

↑

T_RENTo:
CGC⇔Glauber

G. Nijs *et al.*, Phys. Rev. Lett. 126, 202301 (2021); Phys. Rev. C 103, 054909 (2021).

Current understanding of transport coefficients



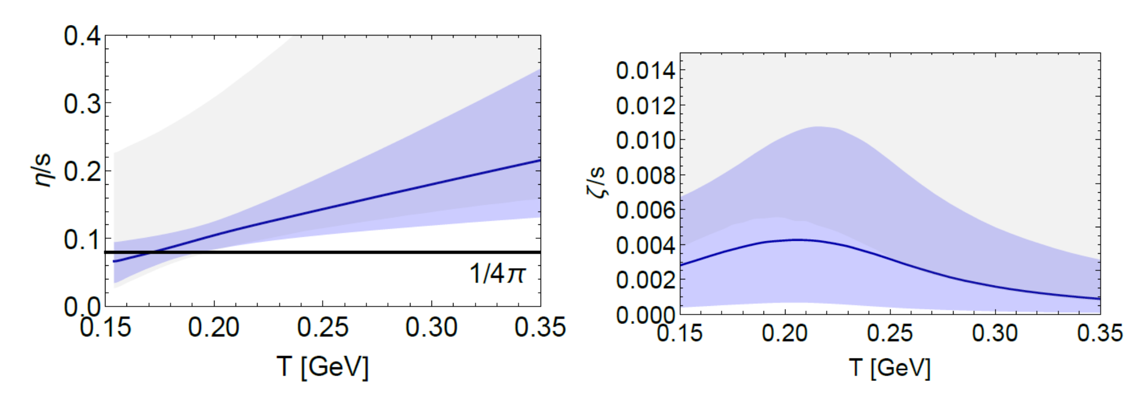
Well constrained by a large volume of data set

- ✓ Dawn of new precision era
- ✓ "Condensed matter physics" of the QGP

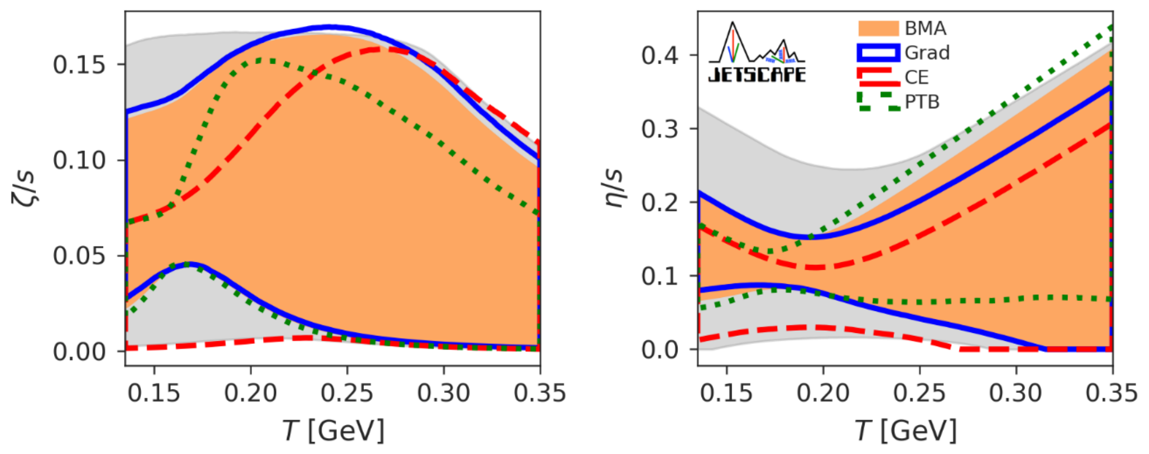
G. Nijs *et al.*, Phys. Rev. Lett. 126, 202301 (2021).

*See also some updates: G. Nijs and W. van der Schee, Phys. Rev. C **106**, 044903 (2022).

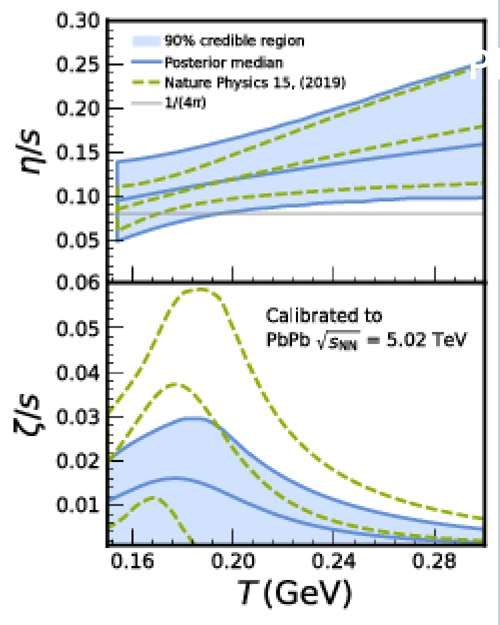
Results from other groups



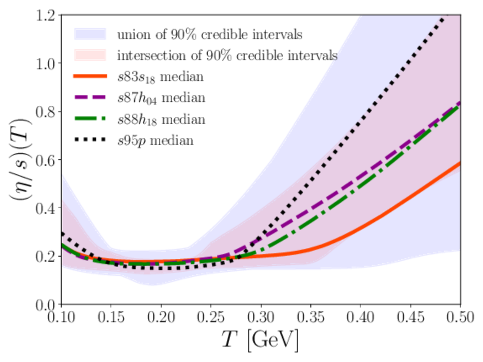
G. Nijs *et al.*,
Phys. Rev. Lett. 126, 202301 (2021).



D. Everett *et al.* (JETSCAPE Collaboration),
Phys. Rev. Lett. 126, 242301 (2021).



J.E. Parkkila *et al.*,
Phys. Rev. C 104, 054904



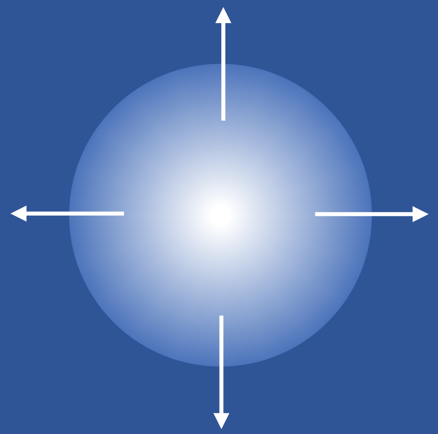
J. Auvinen *et al.*,
Phys. Rev. C 102, 044911 (2020).

- ✓ $\mathcal{O}(10)$ difference in ζ/s
- ✓ Hint for missing piece(s)?

Diffusion coefficient and EM conductivity

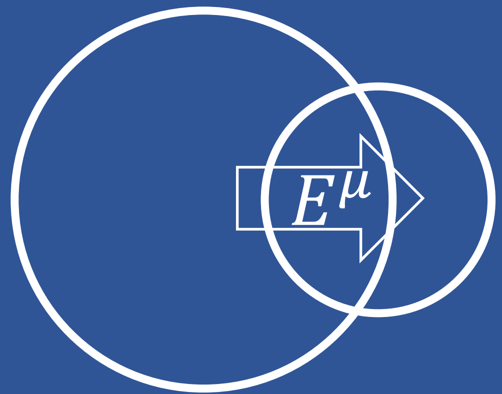
Diffusion coefficient from Fick's law

$$V^\mu = \sigma T \nabla^\mu \frac{\mu}{T}$$



EM conductivity from Ohm's law

$$J^\mu = \sigma_{\mathrm{EM}} F^{\mu\nu} u_\nu$$



*Need (resistive) magneto-hydrodynamics

Hiroshima group, (ECHO-QGP, BHAC-QGP)

Equilibrium measure and inverse Reynolds number

Equilibrium measure

$$f = \frac{3}{2}\tau\frac{1}{w}\frac{dw}{d\tau} = 1 + \frac{3\phi}{8e} = 1 \pm \frac{1}{8}Re_\pi^{-1}$$

Local equilibrium → 1

Out-of-equilibrium → $\frac{1}{8}Re_\pi^{-1}$

$w = \tau T$, e: Energy density, $\phi = \pi^{00} - \pi^{33}$

$$\Rightarrow\ Re_\pi^{-1} = \pm 8(f-1) \qquad +: \phi > 0, \quad -: \phi < 0$$

Note: Various definition of the inverse Reynolds number can be found in the literature.

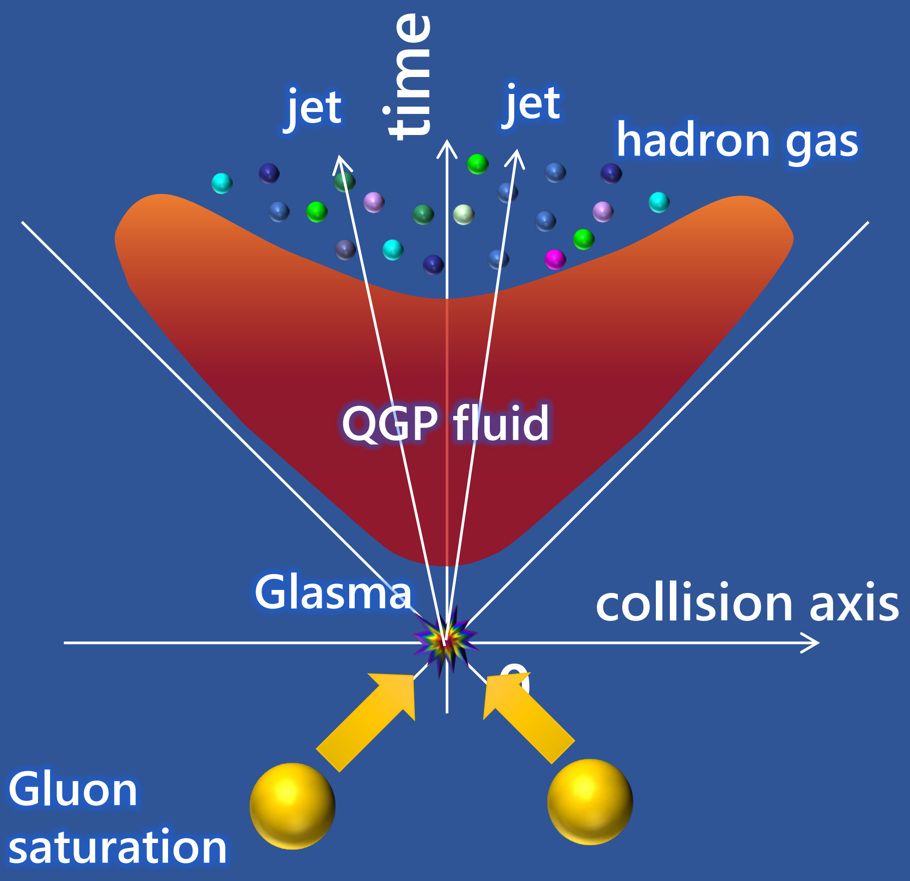
Introduction

Discovery of perfect fluidity announced in 2005*
←Essential role played by "Dynamical Modeling"

- Precision QGP physics
- QGP fluids as thermal media
- Validation of QGP fluidity
- …

*https://www.bnl.gov/newsroom/news.php?a=110303

Dynamical modeling of high-energy nuclear collisions



Final Observables

Dynamical Modeling

Initial Colliding Nuclei

Bayesian parameter estimation

Prior distribution $P(X)$: Flat distribution in parameter space

21 model parameters including initial conditions and transport coefficients

Set of data Y

System: PbPb 2.76 TeV, 5.02 TeV
Observables: multiplicity, transverse energy, PID yields, transverse momentum spectra, mean p_T, v_n,

Likelihood $P(Y|X)$

Bayes' theorem

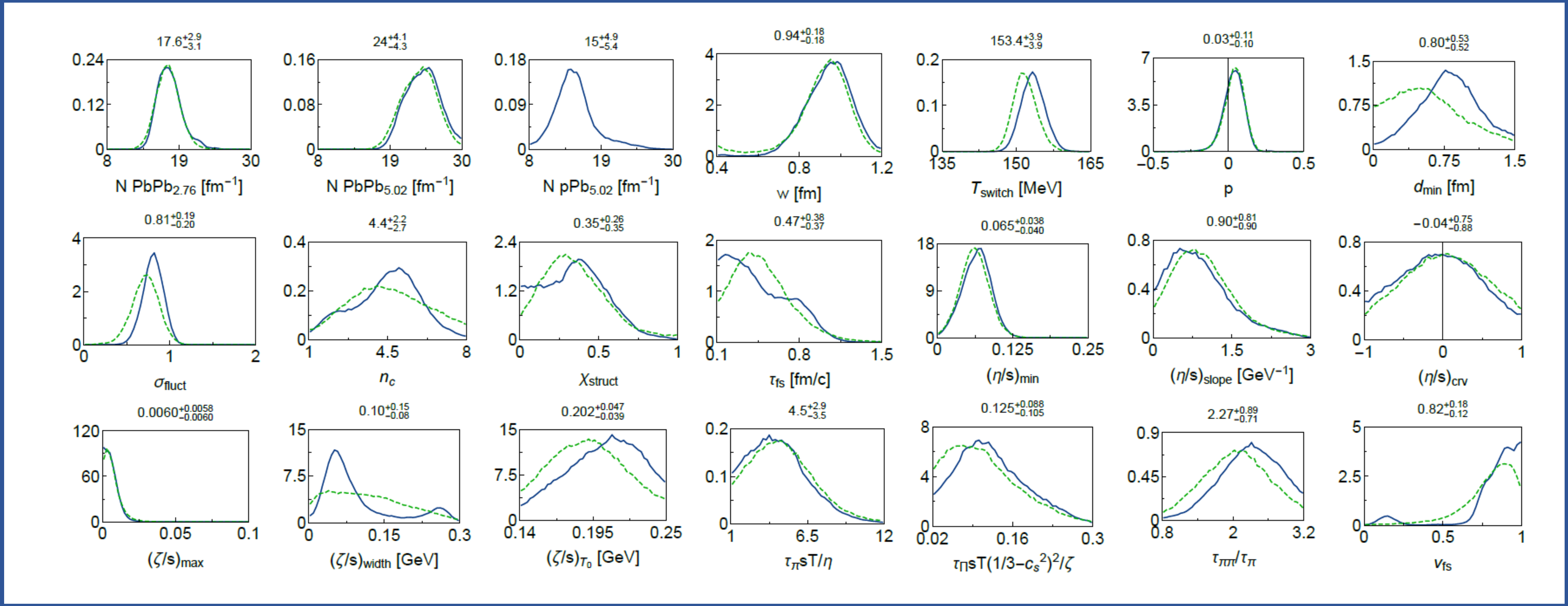
$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Posterior $P(X|Y)$: Probability distribution with 90% confident region of model parameter

J.E. Bernhard, Ph.D thesis, Duke Univ. (2018); arXiv:1804.06469.

G. Nijs *et al.*, arXiv:2010.15130.

Posterior distributions

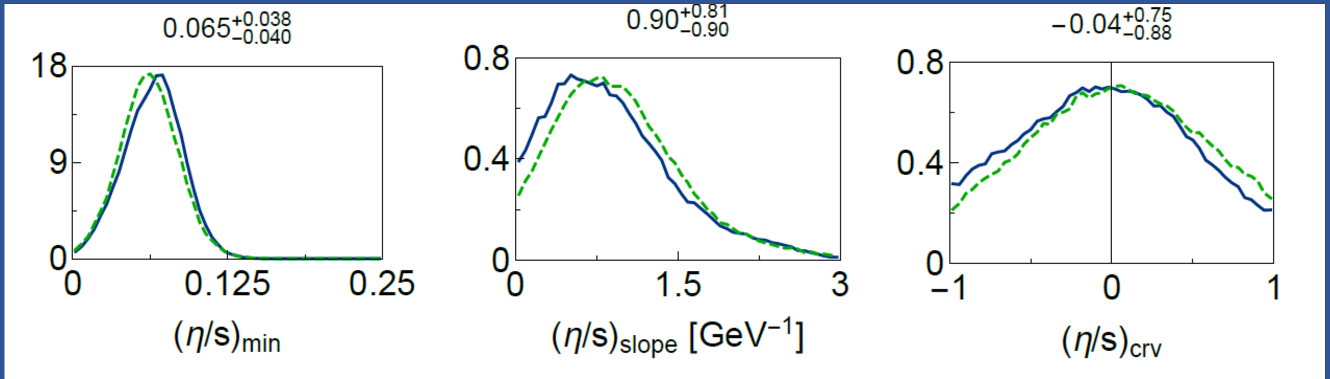


Solid: Pb+Pb, p+Pb
Dashed: Pb+Pb only

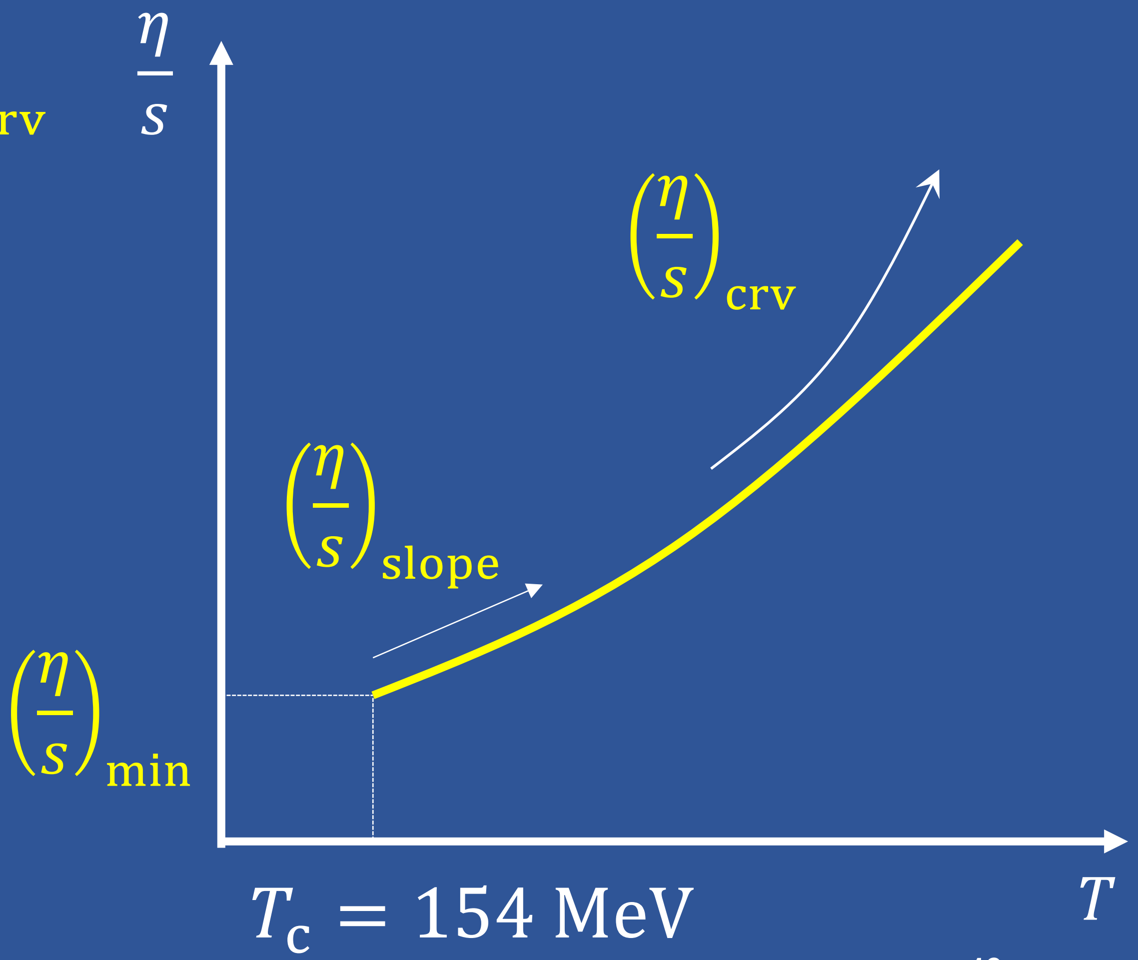
G. Nijs *et al.*, Phys. Rev. Lett. 126, 202301 (2021).

Parametrization of shear viscosity

$$\frac{\eta}{s} = \left(\frac{\eta}{s}\right)_{\mathrm{min}} + \left(\frac{\eta}{s}\right)_{\mathrm{slope}} (T - T_c) \left(\frac{T}{T_c}\right)^{\left(\frac{\eta}{s}\right)_{\mathrm{crv}}}$$

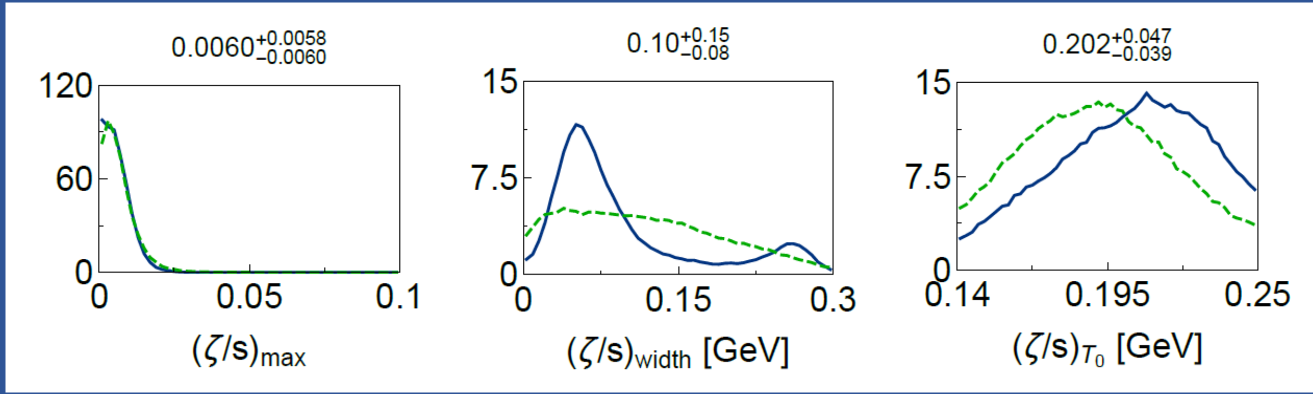


G. Nijs *et al.*, Phys. Rev. Lett. 126, 202301 (2021).

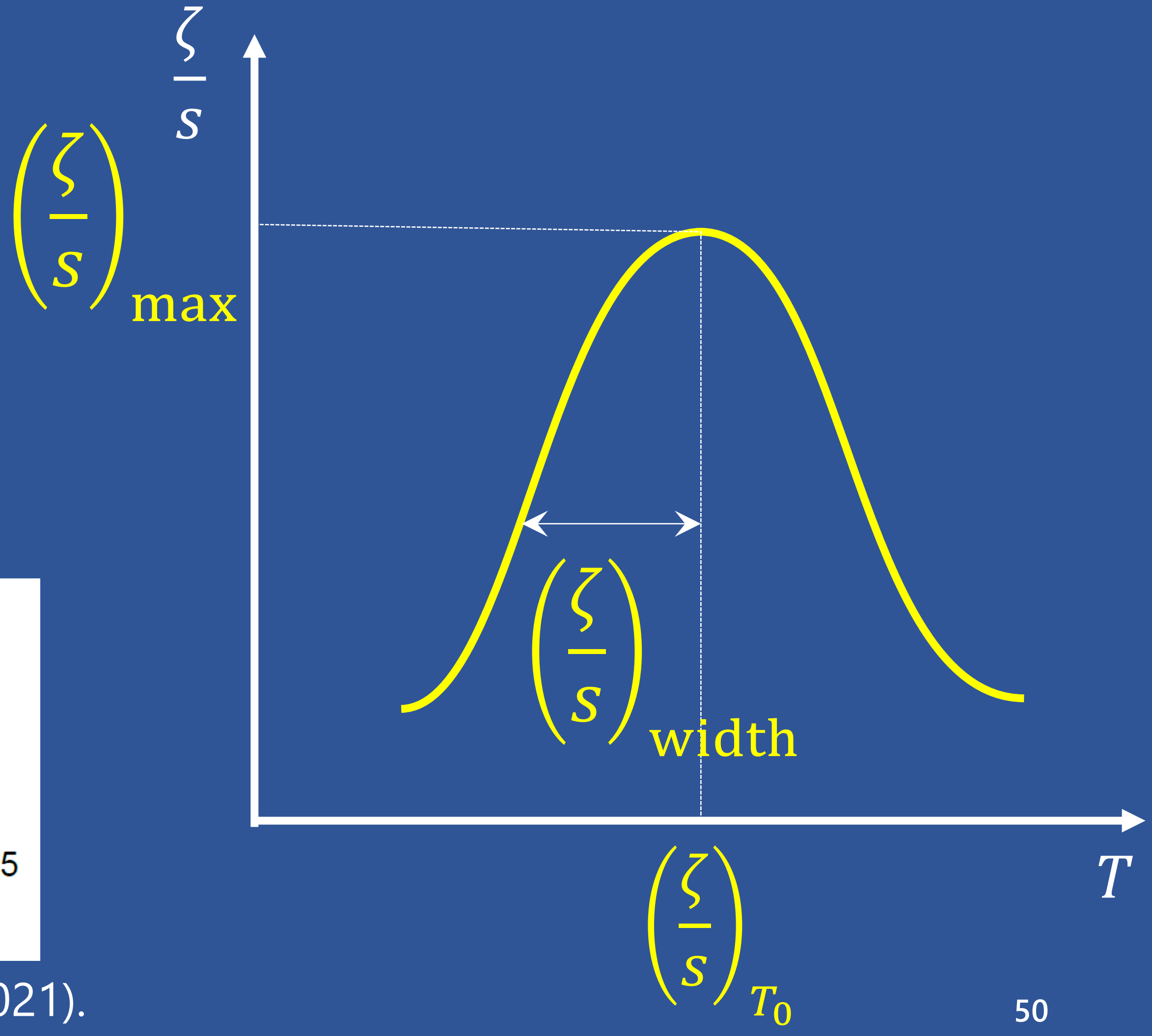


Parametrization of bulk viscosity

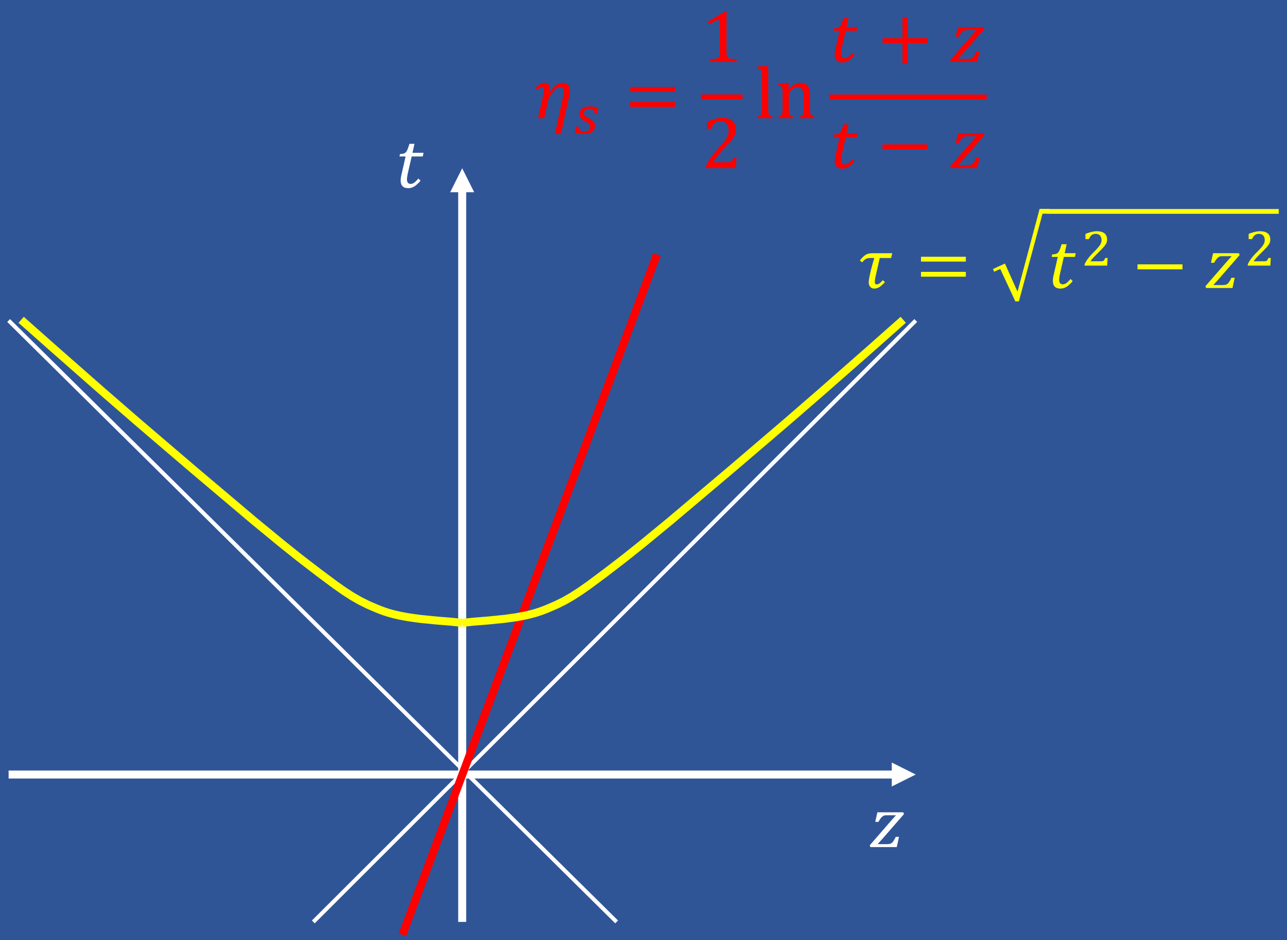
$$\frac{\zeta}{s} = \frac{\left(\frac{\zeta}{s}\right)_{\max}}{1 + \left(\frac{T - \left(\frac{\zeta}{s}\right)_{T_0}}{\left(\frac{\zeta}{s}\right)_{\mathrm{width}}}\right)^2}$$



G. Nijs *et al.*, Phys. Rev. Lett. 126, 202301 (2021).



Boost invariance



Boost invariance along z axis
→ No η_s dependence of thermodynamic variables

Bjorken flow (1D Hubble flow)

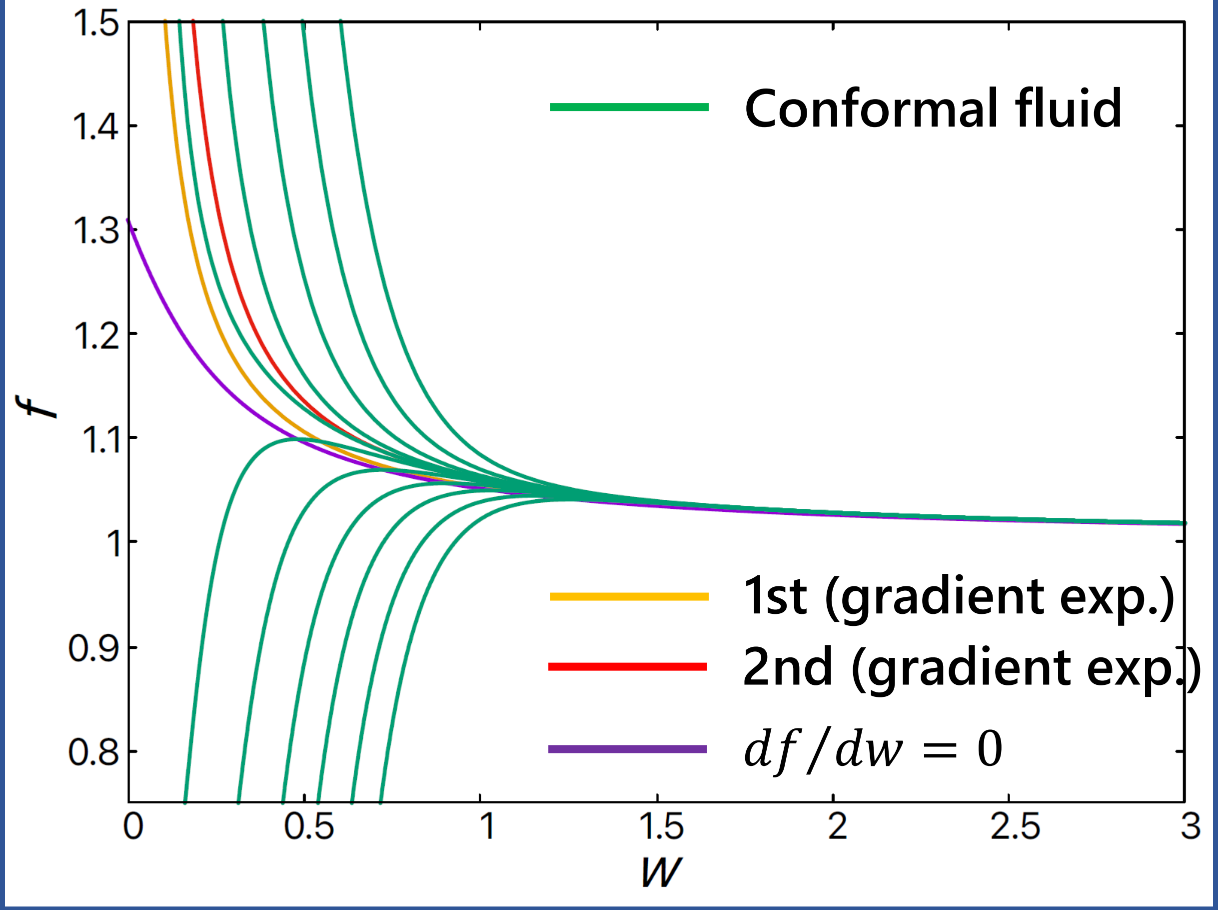
$$u^\mu = \frac{1}{\tau}(t, 0, 0, z)$$

$$e(x) \to e(\tau),\ P(x) \to P(\tau), \text{etc.}$$

→ (0+1)-dimensional problem

J.D. Bjorken, Phys. Rev. D **27**, 140 (1983).

“Time” evolution of equilibrium measure



Relaxation “time”

$$w_\pi = \tau_\pi T = C_{\tau\pi} = \frac{2 - \ln 2}{2\pi}$$

$$\sim 0.208$$

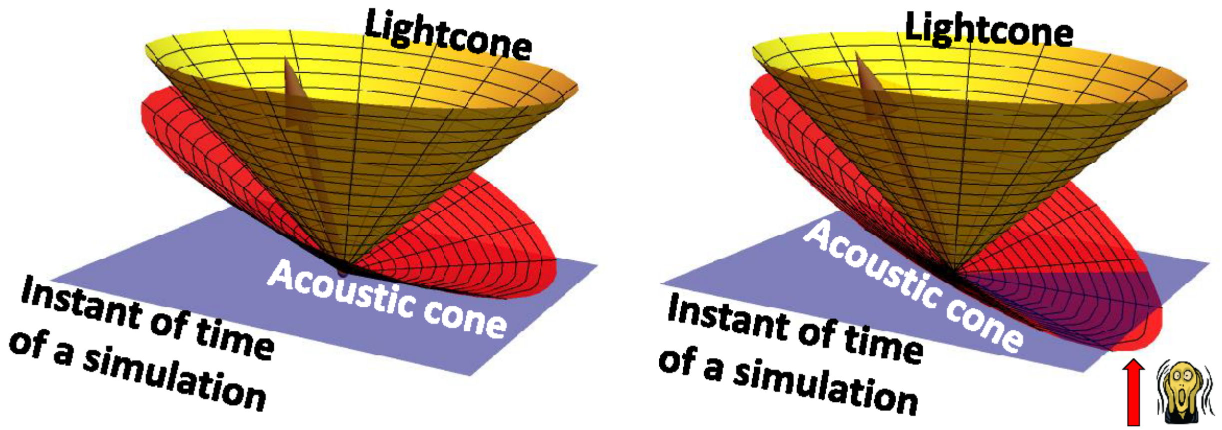
→ Solutions relax to a single curve with relaxation time w_π

“Hydrodynamic attractor”

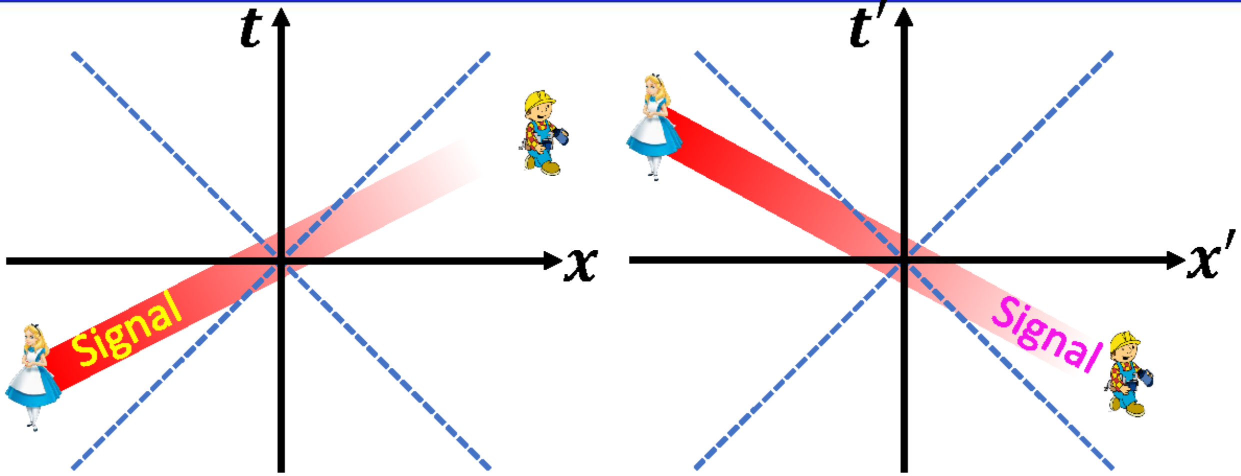
Let's have a closer look at the cone…

Acausality ($w > 1$) but "small" v (specifically: $vw < 1$)

Acausality ($w > 1$) and large v (specifically: $vw \geq 1$)



A thought experiment



In a reference frame where waves propagate backward in time, the fluid description is unstable (LG PRX, 2022).

In some computational frame, the signal exceeds speed of light, but it still propagates forward in time.
← No instability

Appearance of instability in numerical simulations is not a necessary condition for the system to violate causality

Causality in non-linear regime?

Causality of second order hydrodynamics under static equilibrium background in linear perturbation

$$\Pi = 0, \qquad \pi^{\mu\nu} = 0, \qquad u^{\mu} = 0$$

bulk pressure　　shear stress　　four velocity

See, e.g., W.A. Hiscock, L. Lindblom, Annals of Physics 151, 466 (1983).

→ Need to go beyond linear regime to capture full non-linearity of relativistic dissipative hydrodynamic equations

F.S. Bemfica *et al.*, Phys. Rev. Lett. **126**, 222301 (2021).

Characteristic velocity

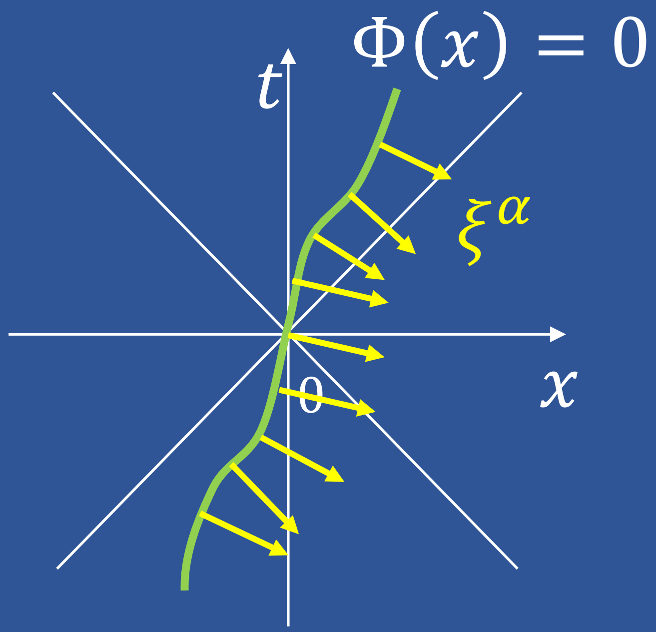
Quasi-linear PDE

$$A^{\alpha}(\Psi)\nabla_{\alpha}\Psi = F$$

$$\Psi = (e, u^{\mu}, \Pi, \pi^{0\mu}, \pi^{1\mu}, \pi^{2\mu}, \pi^{3\mu})$$

Characteristic eqs.

$$\det(A^{\alpha}\xi_{\alpha}) = 0,\ \ \xi^{\alpha} = \nabla^{\alpha}\Phi(x)$$



Normal vector of characteristic surface
→ Space-like vector

$$\xi^{\alpha} = bu^{\alpha} + v^{\alpha},\quad \xi\cdot\xi = b^2 + v\cdot v \le 0$$

Characteristic velocity

$$0 \le k(= -b^2/v\cdot v) \le 1,\ \ 0 \le k = v_c^2 \le 1$$

W.A. Hiscock and T.S. Olson, Phys. Lett. A 141, 125 (1989); F.S. Bemfica *et al.*, Phys. Rev. Lett. **126**, 222301 (2021).

Condition from non-linear causality

$$F_i(e, P, \Pi, \pi^{\mu\nu}, \eta, \zeta, \cdots) \geq 0$$

Necessary and sufficient conditions
in Israel-Stewart type model (DNMR eqs.) from
characteristic velocity
← Inequality among thermodynamic variables,
dissipative currents and transport coefficients.

⬇

Constraint initial conditions from non-linear causality?

F.S. Bemfica *et al.*, Phys. Rev. Lett. **126**, 222301 (2021).

Conformal fluids in Bjorken expansion

Balance eq. (Landau frame) and EoS

$$\partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = e u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}, \quad P = e/3$$

Constitutive eq. (BRSSS eq.)

R. Baier *et al.*, JHEP 0804, 100 (2008).

*Gradient expansion up to 2nd order and "resumed"

**Keep relevant terms in Bjorken expansion and neglect non-linear term

$$\tau_\pi \Delta^{\mu\nu}{}_{\alpha\beta} D\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\nabla^\mu u^\nu - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta$$

Note on derivation of "resumed" BRSSS

R. Baier *et al.*, JHEP 0804, 100 (2008).

Original BRSSS eq. from gradient expansion up to 2nd order

*Keep relevant terms in Bjorken expansion and neglect non-linear term

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} - 2\eta\tau_\pi\left(\Delta^{\mu\nu}{}_{\alpha\beta}D\nabla^{\langle\alpha}u^{\beta\rangle} + \frac{\theta}{3}\nabla^{\langle\mu}u^{\nu\rangle}\right)$$

$$\nabla^{\langle\mu}u^{\nu\rangle} \approx \frac{\pi^{\mu\nu}}{2\eta}$$

$$D\eta \approx -\eta\theta \quad \text{(From } \eta/s = \text{const.)}$$

$$\tau_\pi D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} - \frac{4}{3}\tau_\pi\theta\pi^{\mu\nu}$$

$\pi^{\mu\nu}$ is promoted to a dynamical variable

Hydrodynamic equations under Bjorken expansion

Boost invariant flow $u^\mu = \frac{1}{\tau}(t, 0, 0, z)$

$$\tau = \sqrt{t^2 - z^2}$$

$$\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}$$

J.D. Bjorken, Phys. Rev. D **27**, 140 (1983).

$$\begin{cases} \frac{d}{d\tau}e(\tau) = -\frac{4}{3\tau}e(\tau) + \frac{1}{\tau}\phi(\tau) \\ \left(1 + \tau_\pi \frac{d}{d\tau}\right)\phi(\tau) = -\frac{4\tau_\pi}{3\tau}\phi(\tau) + \frac{4\eta}{3\tau} \end{cases}$$

$$\phi = \pi^{00} - \pi^{33}$$

Note: Transverse pressure

$$P_T = \frac{e}{3} + \frac{\phi}{2}$$

Longitudinal pressure

$$P_L = \frac{e}{3} - \phi$$

Variable transformation

"Conformal time": $w = \tau T$

"Equilibrium measure": $f = \frac{3}{2}\tau\frac{1}{w}\frac{dw}{d\tau}$

$$C_{\tau\pi} w f \frac{df}{dw} + 4C_{\tau\pi} f^2 + \left(\frac{3}{2}w - 8C_{\tau\pi}\right) f - C_\eta + 4C_{\tau\pi} - \frac{3}{2}w = 0$$

Transport coefficients: $\eta = C_\eta s, \quad \tau_\pi = \frac{C_{\tau\pi}}{T}$

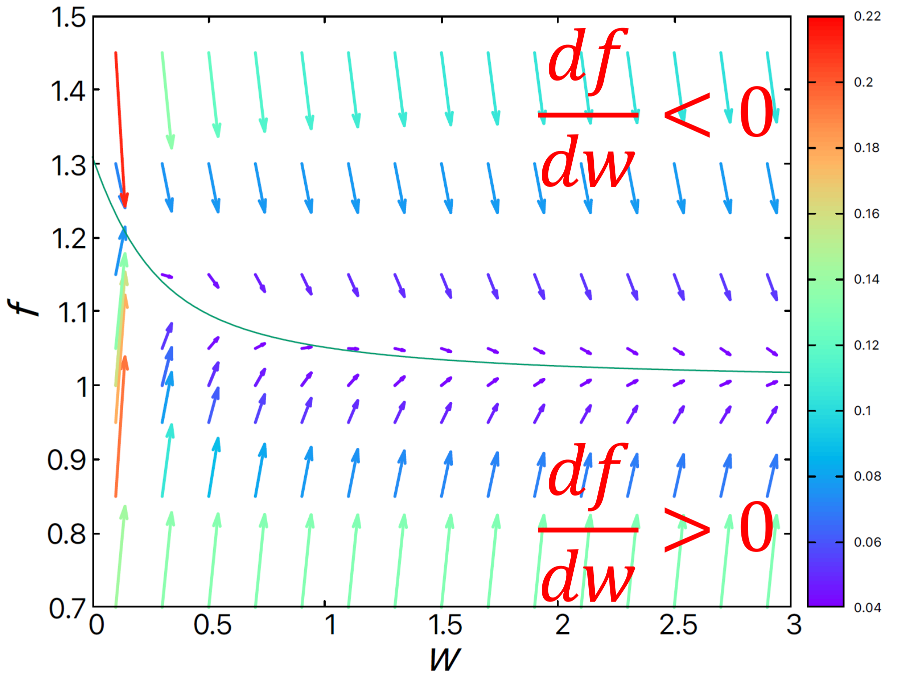
M.P. Heller and M. Spaliński, Phys. Rev. Lett. **115**, 072501 (2015).

Note 1: In ideal hydrodynamics, $w \propto \tau^{2/3}$ from $T \propto \tau^{-1/3}$

Note 2: Different normalization employed for f

Behavior of solutions

$$\Delta f = \left(-4\frac{f}{w} - \frac{3}{2C_{\tau\pi}} + \frac{8}{w} + \frac{C_\eta}{C_{\tau\pi} w f} - \frac{4}{fw} + \frac{3}{2C_{\tau\pi} f} \right) \Delta w$$



"Flow vector" $(\Delta w, \Delta f)$

Transport coefficients from AdS/CFT

$$C_\eta = \frac{1}{4\pi}$$

P.Kovtun *et al.*, Phys. Rev. Lett. 94, 111601 (2005).

$$C_{\tau\pi} = \frac{2 - \ln 2}{2\pi}$$

R. Baier *et al.*, JHEP 0804, 100 (2008).

Solutions from gradient expansion

Constitutive eq. (BRSSS eq. with relevant terms in Bjorken expansion

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} - 2\eta\tau_\pi\Delta^{\mu\nu}{}_{\alpha\beta}D\nabla^{\langle\alpha}u^{\beta\rangle} - \frac{2}{3}\eta\tau_\pi\theta\nabla^{\langle\mu}u^{\nu\rangle}$$

$$f(w) = 1 + \frac{2}{3}C_\eta w^{-1} + \frac{4}{9}C_{\tau\pi}C_\eta w^{-2} + \mathcal{O}(w^{-3})$$

1st order in gradient

2nd order in gradient

Solution directly from EoM

$$C_{\tau\pi} w f \frac{df}{dw} + 4C_{\tau\pi} f^2 + \left(\frac{3}{2}w - 8C_{\tau\pi}\right) f - C_\eta + 4C_{\tau\pi} - \frac{3}{2}w = 0$$

$$f(w) = \sum_n r_n w^{-n}$$

$$f(w) = 1 + \frac{2}{3} C_\eta w^{-1} + \frac{4}{9} C_{\tau\pi} C_\eta w^{-2} + \mathcal{O}(w^{-3})$$

Equilibrium measure and inverse Reynolds number

$$Re_\pi^{-1} = \sqrt{\frac{6|\pi_{\mu\nu}\pi^{\mu\nu}|}{e^2}} \Rightarrow Re_\pi^{-1} = \frac{3|\phi|}{e}, \quad \phi = \pi^{00} - \pi^{33}$$

*Normalized such that $Re_\pi^{-1} = 1$ at $P_L = 0$

E.g.) V.E. Ambrus *et al.*, Phys. Rev. Lett. 130, 152301 (2023).

Equilibrium measure

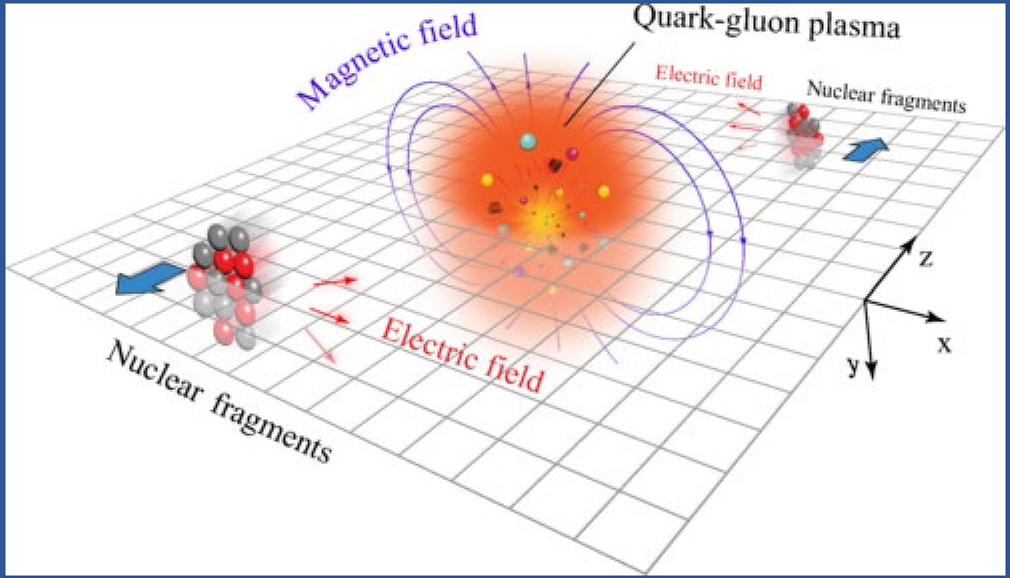
$$f = \frac{3}{2}\tau\frac{1}{w}\frac{dw}{d\tau} = 1 + \frac{3\phi}{8e} = 1 \pm \frac{1}{8}Re_\pi^{-1}$$

Local equilibrium (1) — Out-of-equilibrium ($\frac{1}{8}Re_\pi^{-1}$)

$$\Rightarrow Re_\pi^{-1} = \pm 8(f-1) \qquad +: \phi > 0, \quad -: \phi < 0$$

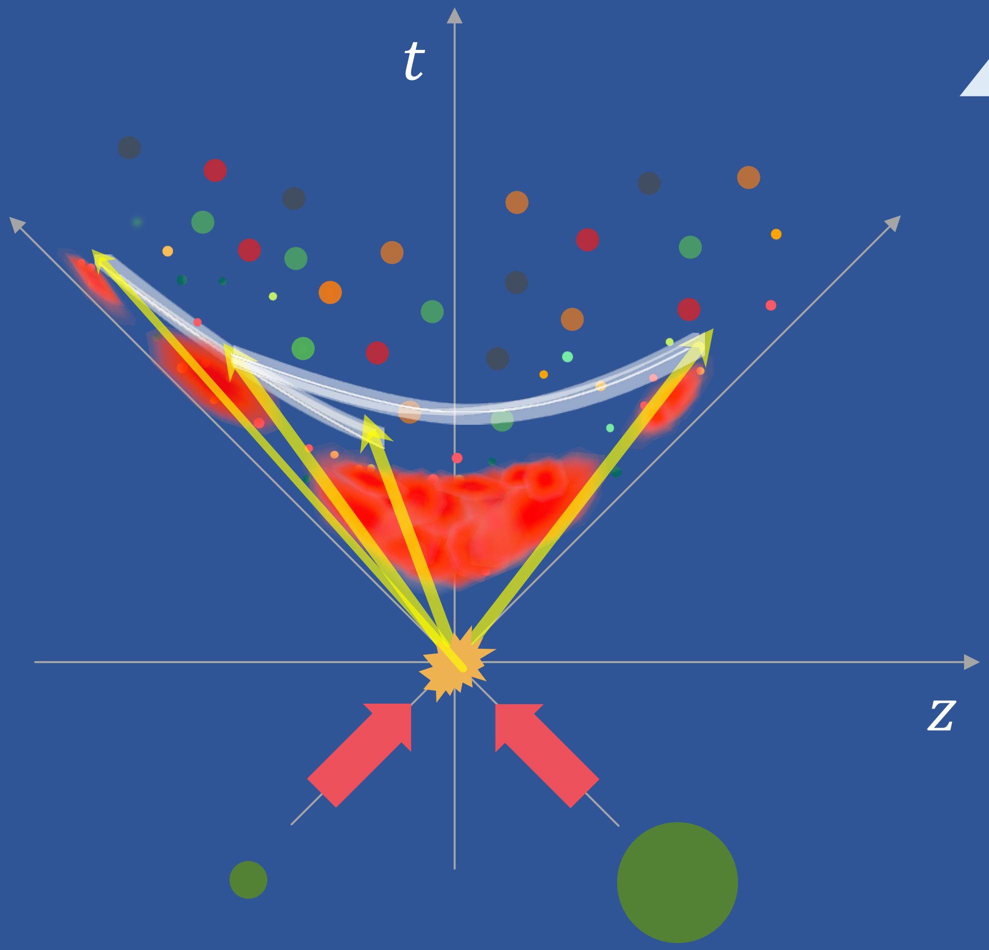
Note: Various definition of the inverse Reynolds number can be found in the literature.

Observation of EM field effect



STAR Collaboration (2024)

Dynamical Core Corona Initialization (DCCI2) model



Figures: Courtesy of Y. Kanakubo

Hadronic afterburner

JAM

Y. Nara *et al.*,Phys. Rev. C61, 024901 (2000)

Hadronization

PYTHIA8 (string fragmentation)

iS3D (thermal hadron sampling)

M. McNelis *et al.*, Comput. Phys.Commun. 258, 107604 (2021).

DCCI + QGP dynamics

(3+1)-D hydro with source terms

Y. Tachibana and TH, Phys. Rev. C 90, 021902 (2014).

Initial parton production

PYTHIA8/PYTHIA8 Angantyr

T. Sjöstrand *et al.*, Comput. Phys. Commun. 191, 159 (2015);
C. Bierlich *et al.*, JHEP 1610 139 (2016)

M.Okai *et al.* (2018)

Dynamical initialization

Conservation of the whole system

$$\partial_\mu \left(T^{\mu\nu}_{\mathrm{fluid}} + T^{\mu\nu}_{\mathrm{parton}} \right) = 0 \quad \Rightarrow \quad \begin{aligned} \partial_\mu T^{\mu\nu}_{\mathrm{fluid}} &= J^{\nu}_{\mathrm{p}\to\mathrm{f}} \\ \partial_\mu T^{\mu\nu}_{\mathrm{parton}} &= -J^{\nu}_{\mathrm{p}\to\mathrm{f}} \end{aligned}$$

Hydrodynamic equations with source terms

Step 1
Generate initial partons
$T^{\mu\nu}_{\mathrm{parton}}(0^+)$

⇒

Step 2
Model source term
$J^{\nu}_{\mathrm{p}\to f} = -\partial_\mu T^{\mu\nu}_{\mathrm{parton}}(t)$

⇒

Step 3
Solve hydro eqs. with vacuum initial condition

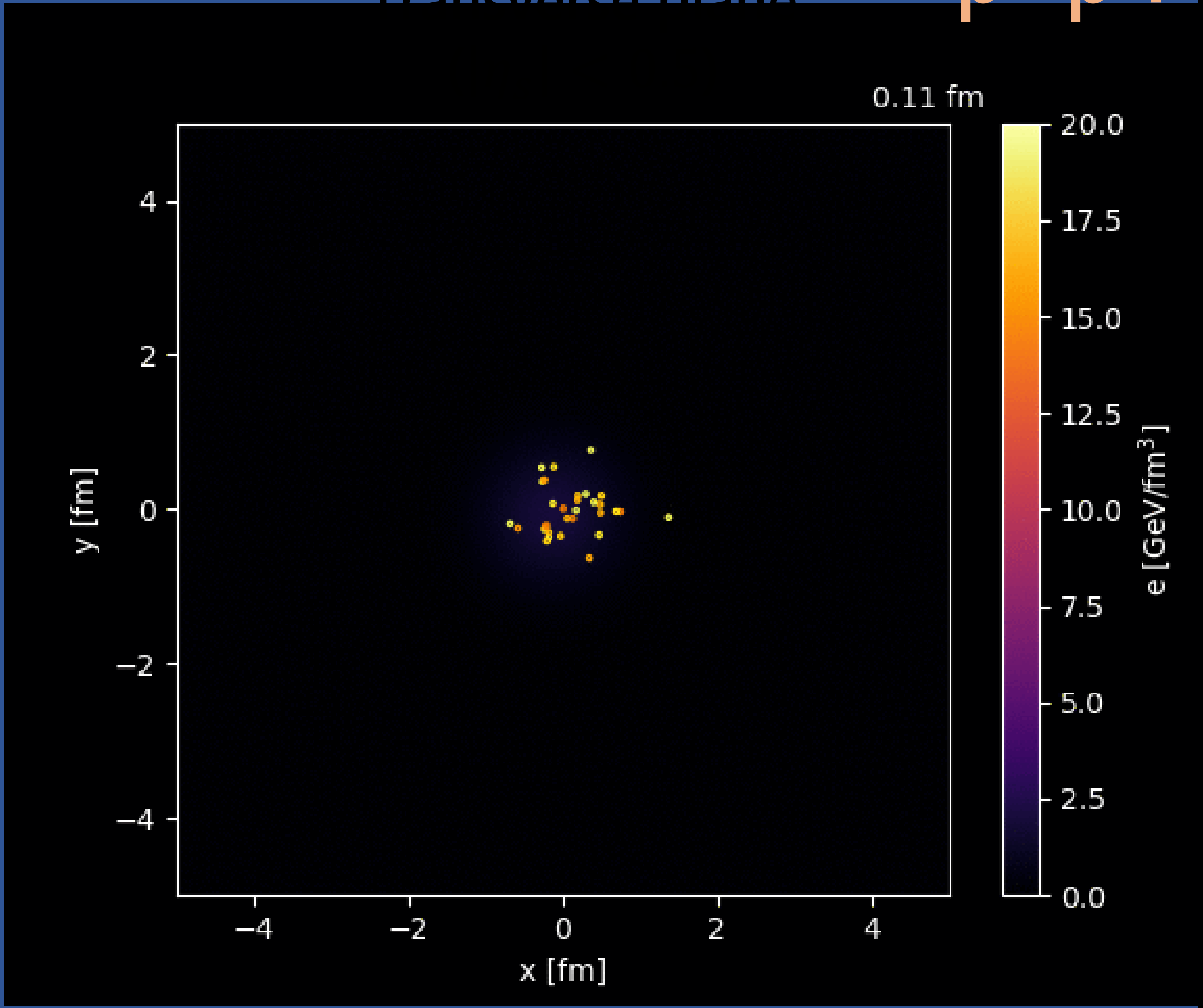
$$\partial_\mu T^{\mu\nu}_{\mathrm{fluid}} = J^{\nu}_{\mathrm{p}\to\mathrm{f}}$$

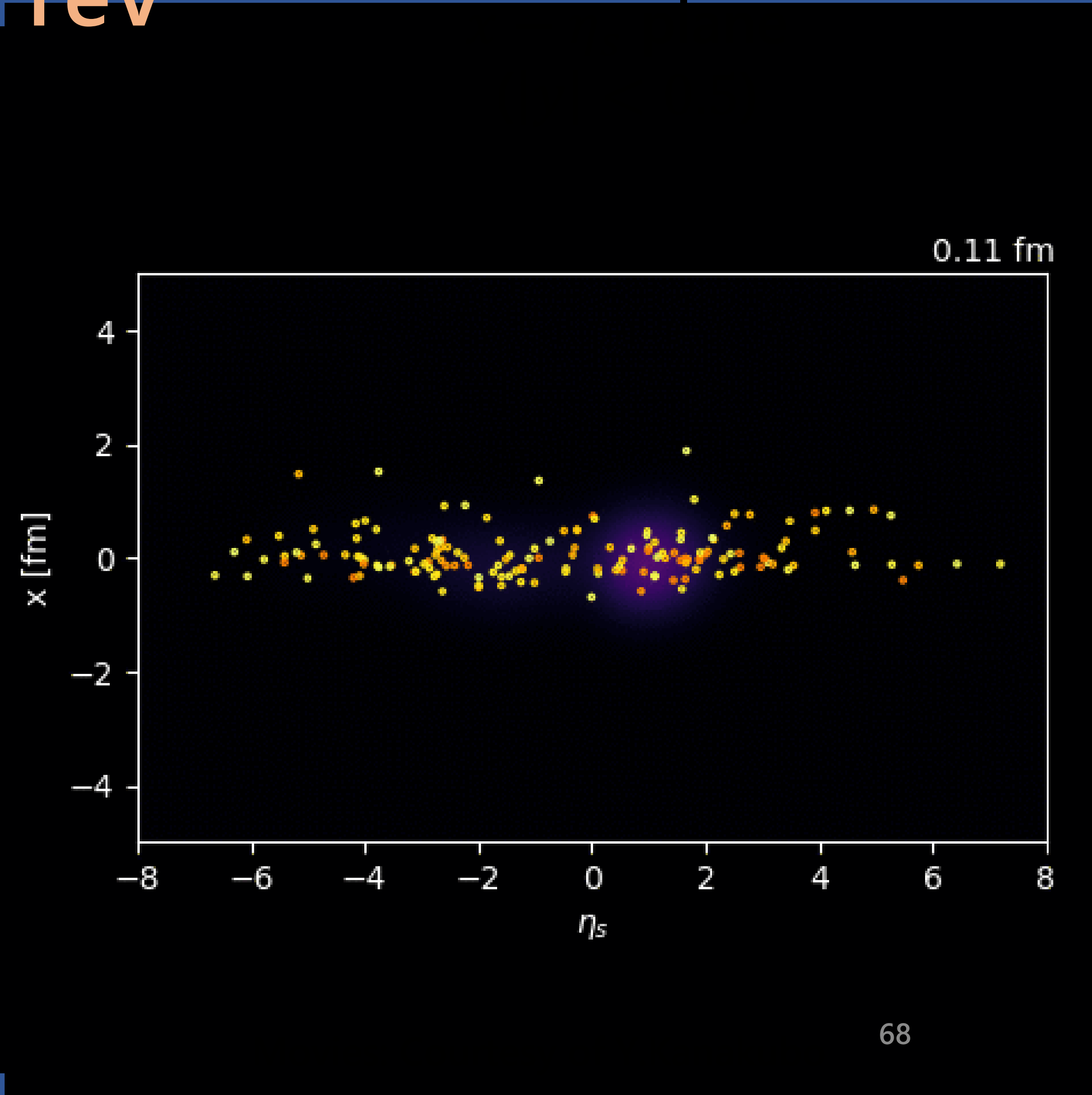
with $T^{\mu\nu}_{\mathrm{fluid}}(t_0) = 0$

How DCCI works in pp

p+p 7 TeV

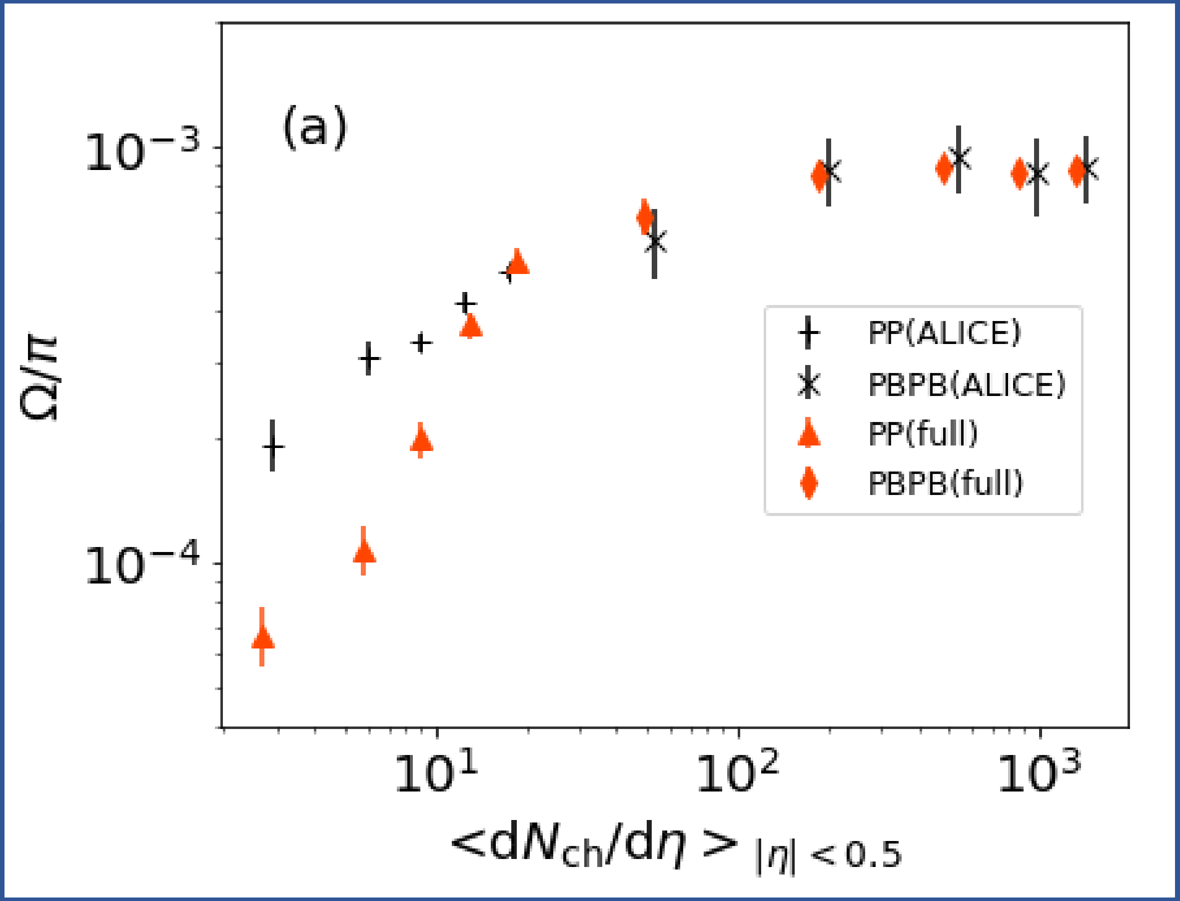
Transverse plane





Ω/π ratio from p+p to Pb+Pb

J. Adam *et al.*, (ALICE Collaboration), Nature Phys. 13 535-539 (2017)
B. Abelev *et al.* (ALICE Collaboration), Phys. Lett. B 728, 216-227 (2014); erratum 734, 409-410 (2014)

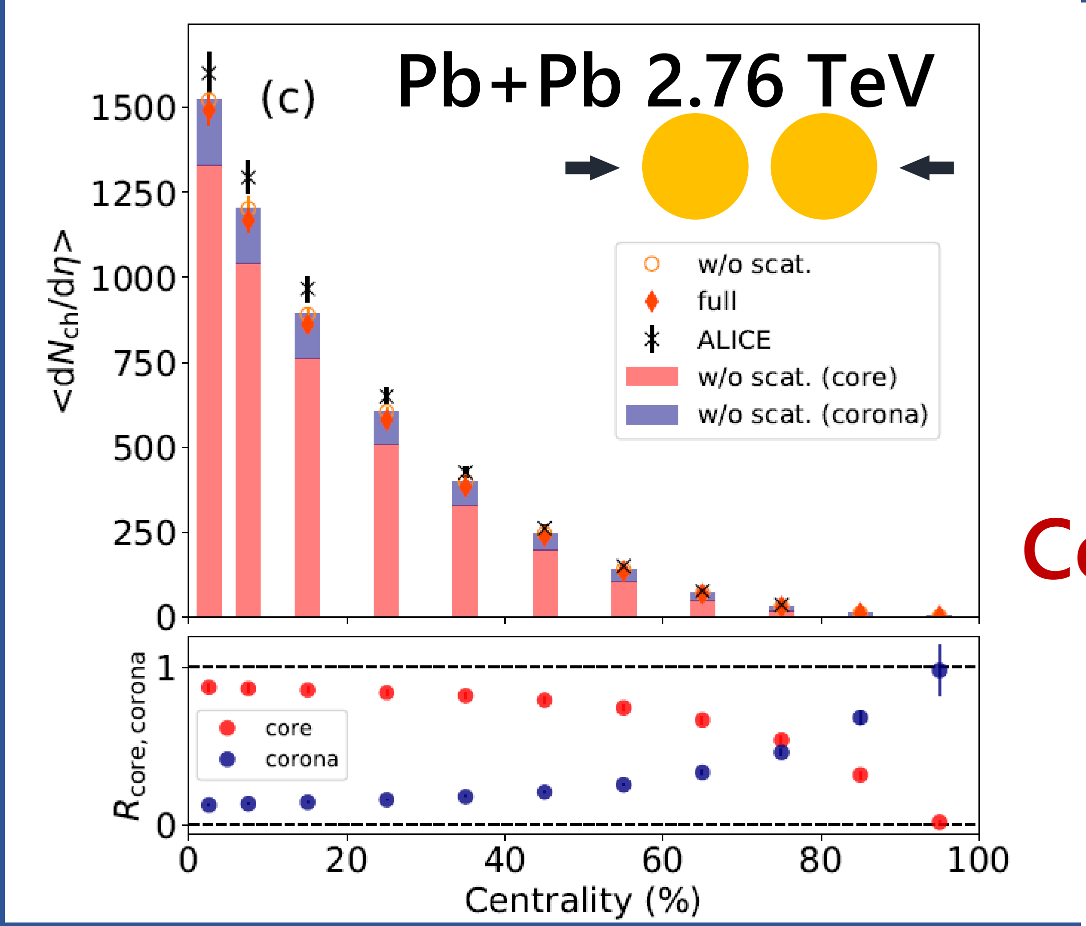


*Deviation from data at low multiplicity can be attributed to PYTHIA default tuning

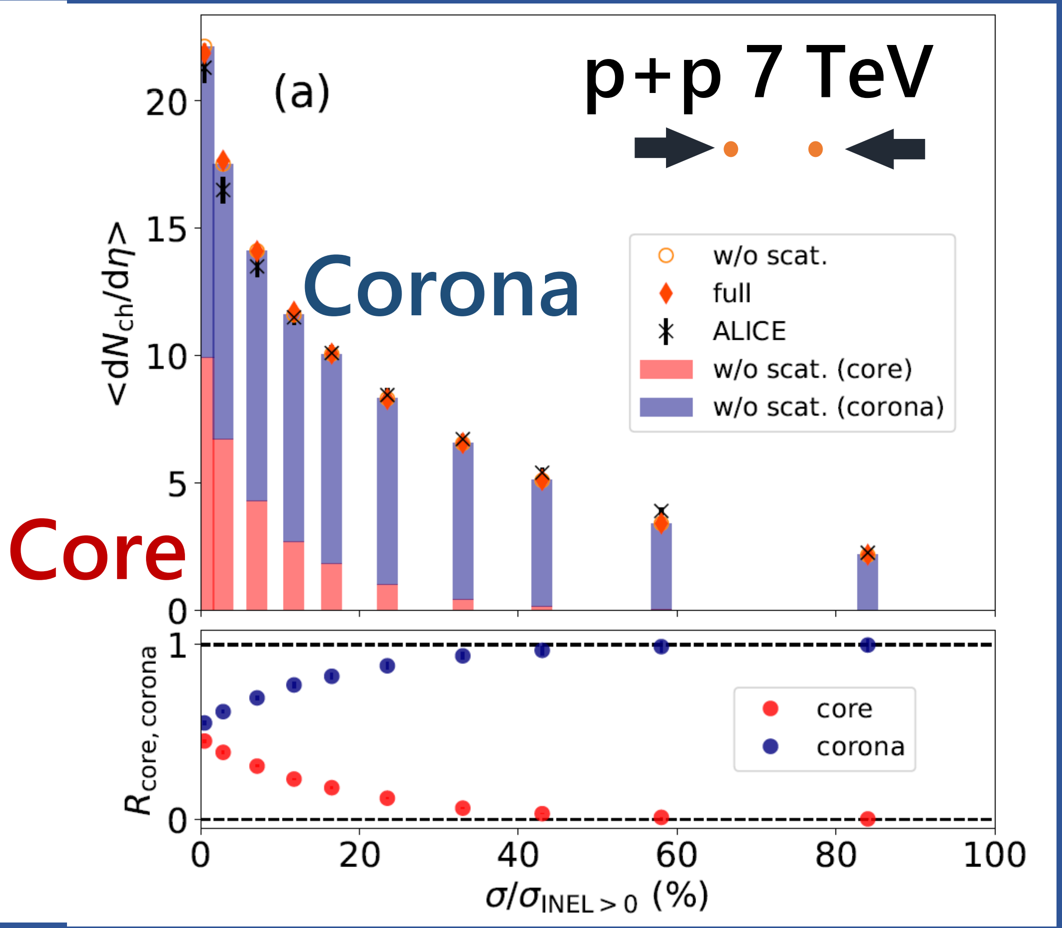
Fixing parameters to control fraction of core/corona (Backup slides for details)

Smooth increase of core contribution
→ Smooth enhancement of Ω/π

Fraction of core and corona components



K. Aamodt *et al.*, (ALICE Collaboration), Phys. Rev. Lett. 106, 032301 (2011)

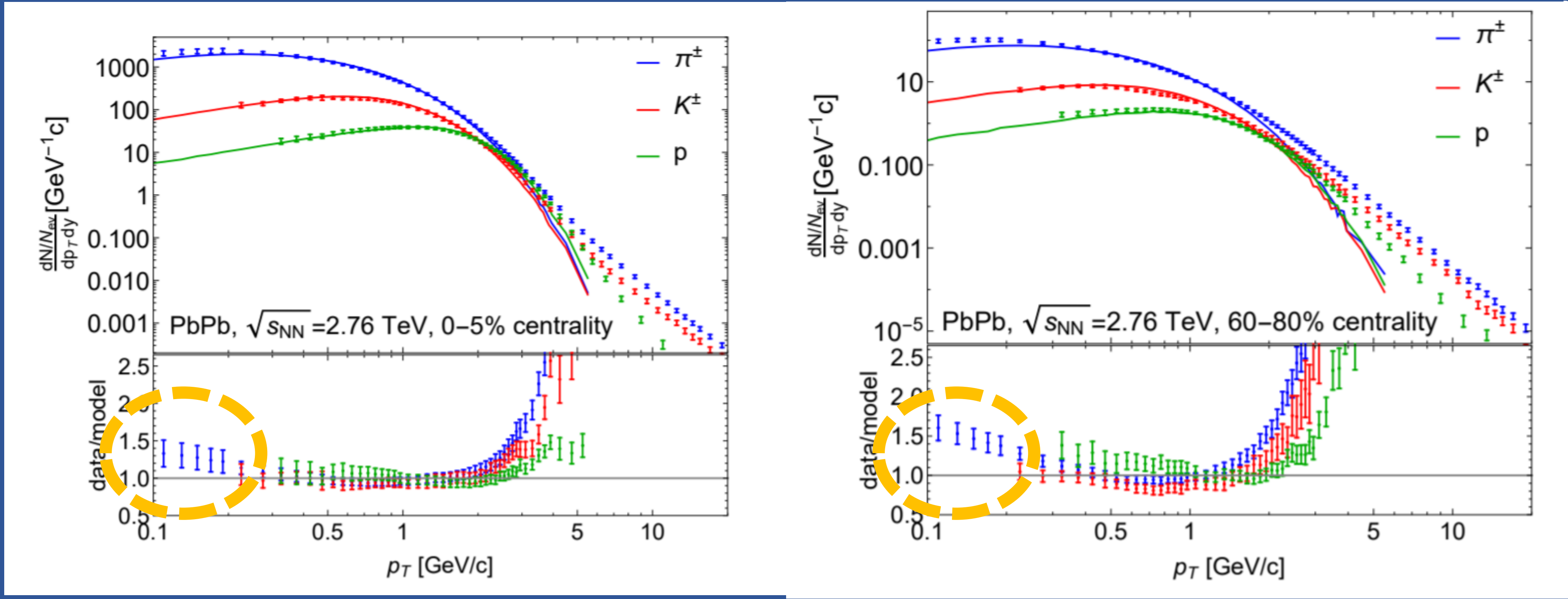


J. Adam *et al.*, (ALICE Collaboration), Nature Phys. 13, 535-539 (2017)

PbPb: corona ~ 20% at 40-60% centrality

pp: core/corona ~ 50% at σ/σ_{INEL} = 0-0.95%

Lessons from a cutting edge model: TRAJECTUM



G. Nijs *et al.*, Phys. Rev. C 103, 054909 (2021)

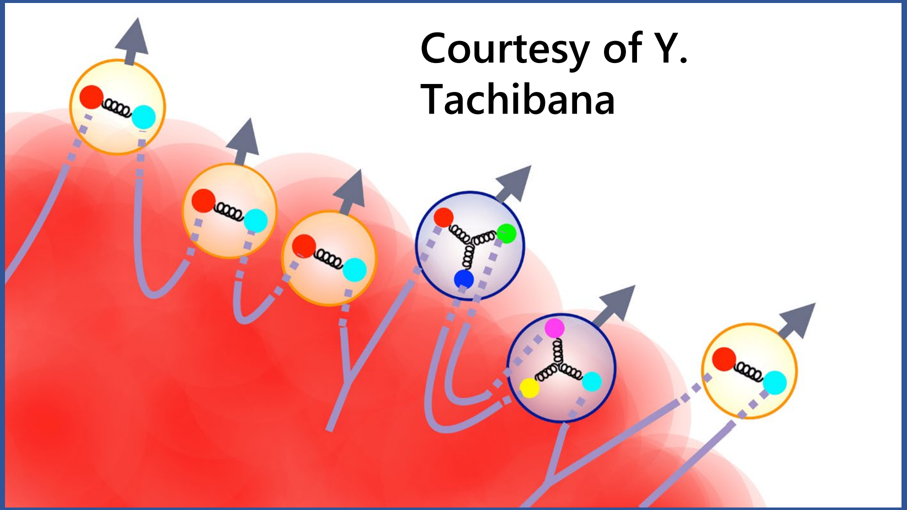
Prevailing paradigm: 99% soft (~core) components

Current understanding: Lack of core at very low p_T ($\lesssim$0.5 GeV)

Question: Soft components always from core?

"Soft-from-corona" components

"hard strings" in DCCI2

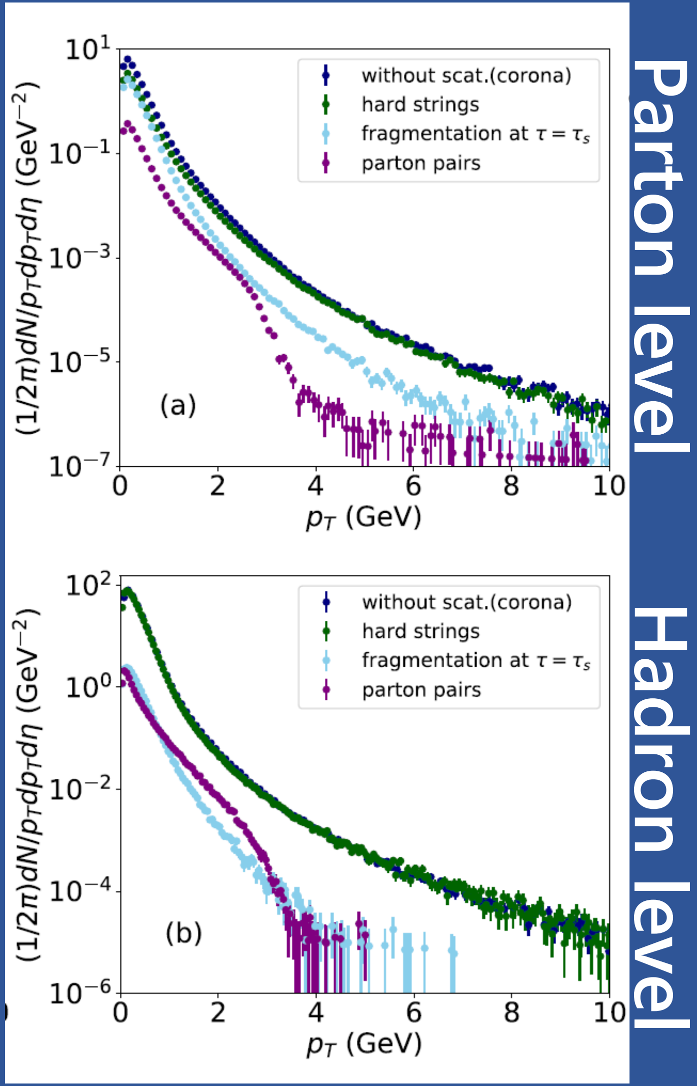


Traversing hard partons ($p_T \gtrsim$ 3 GeV)

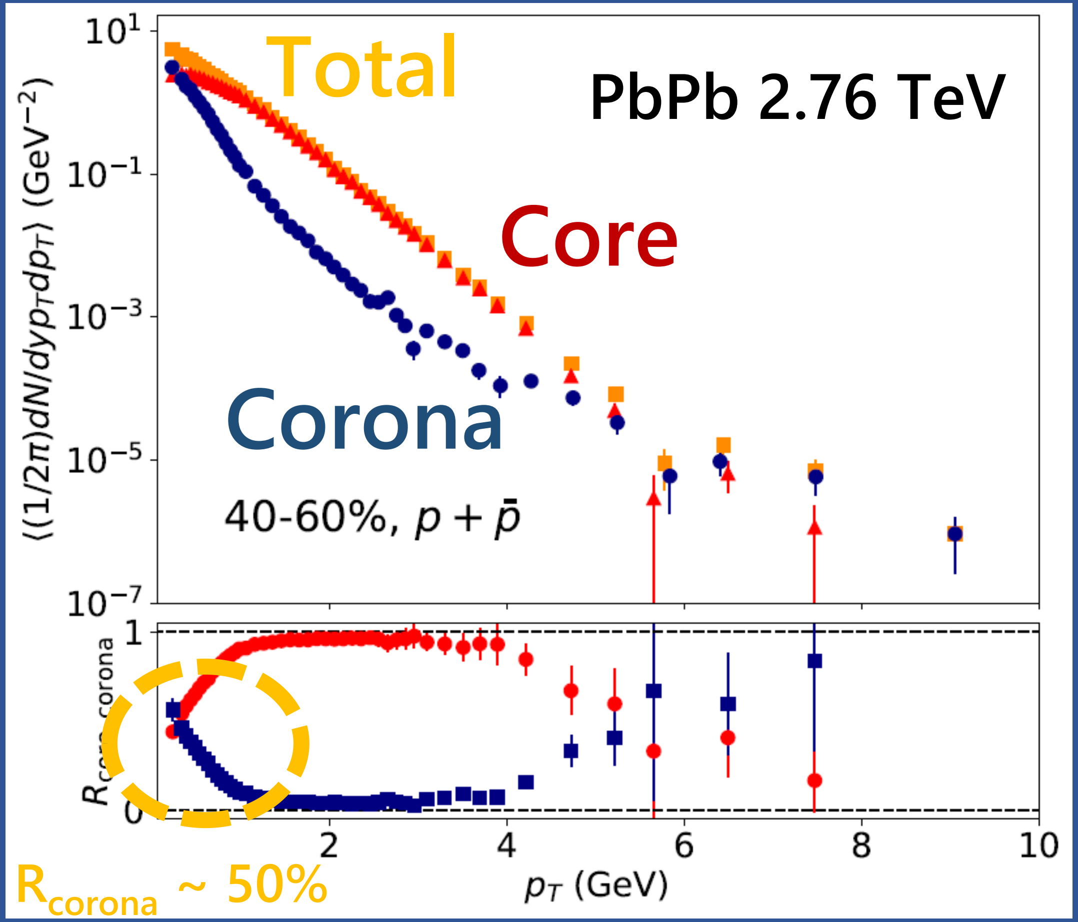
String formation

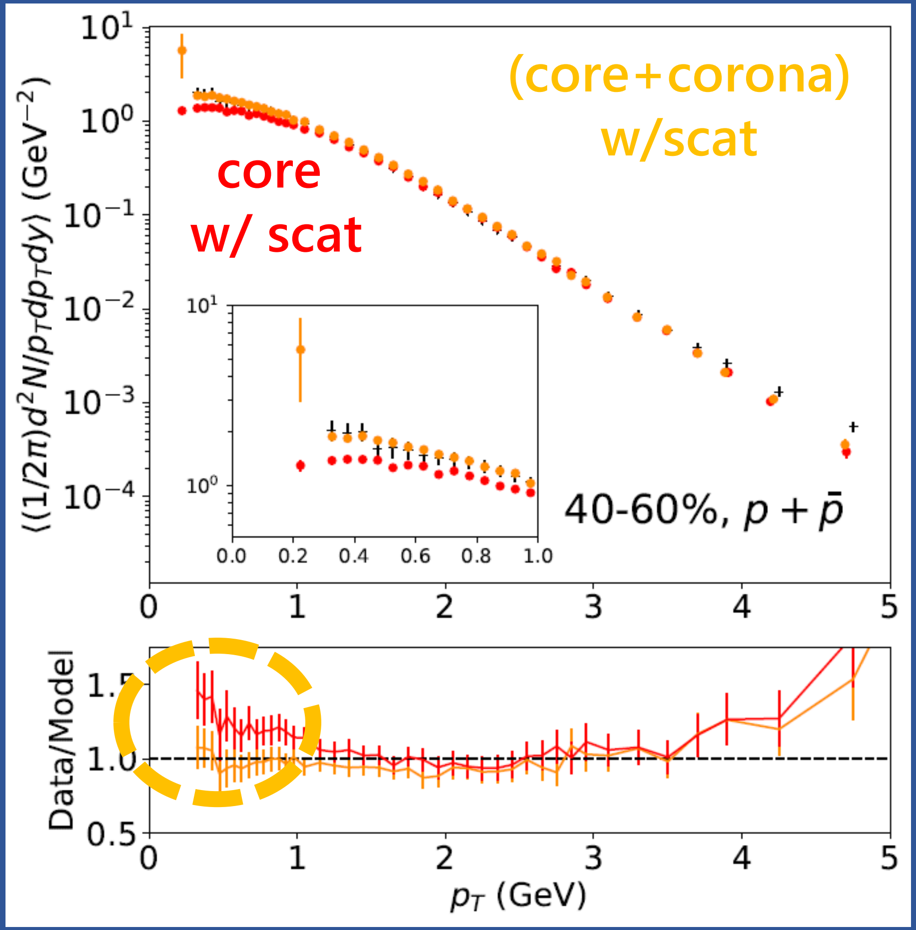
Fragment into soft hadrons

← Power-law shape



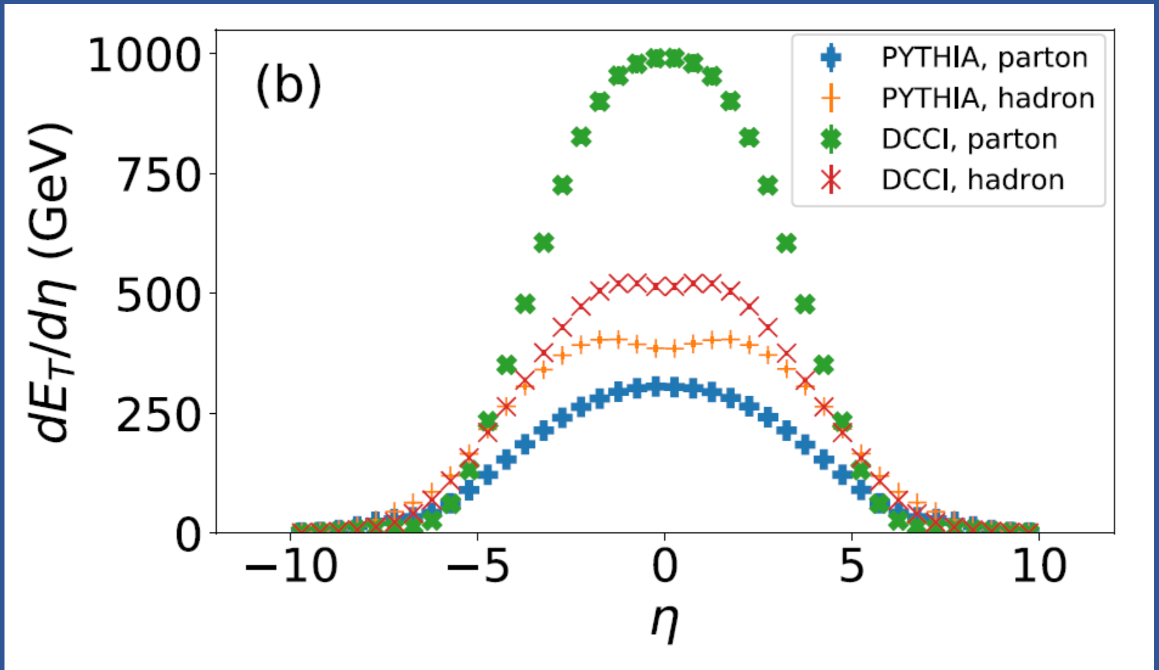
Low p_T enhancement from corona



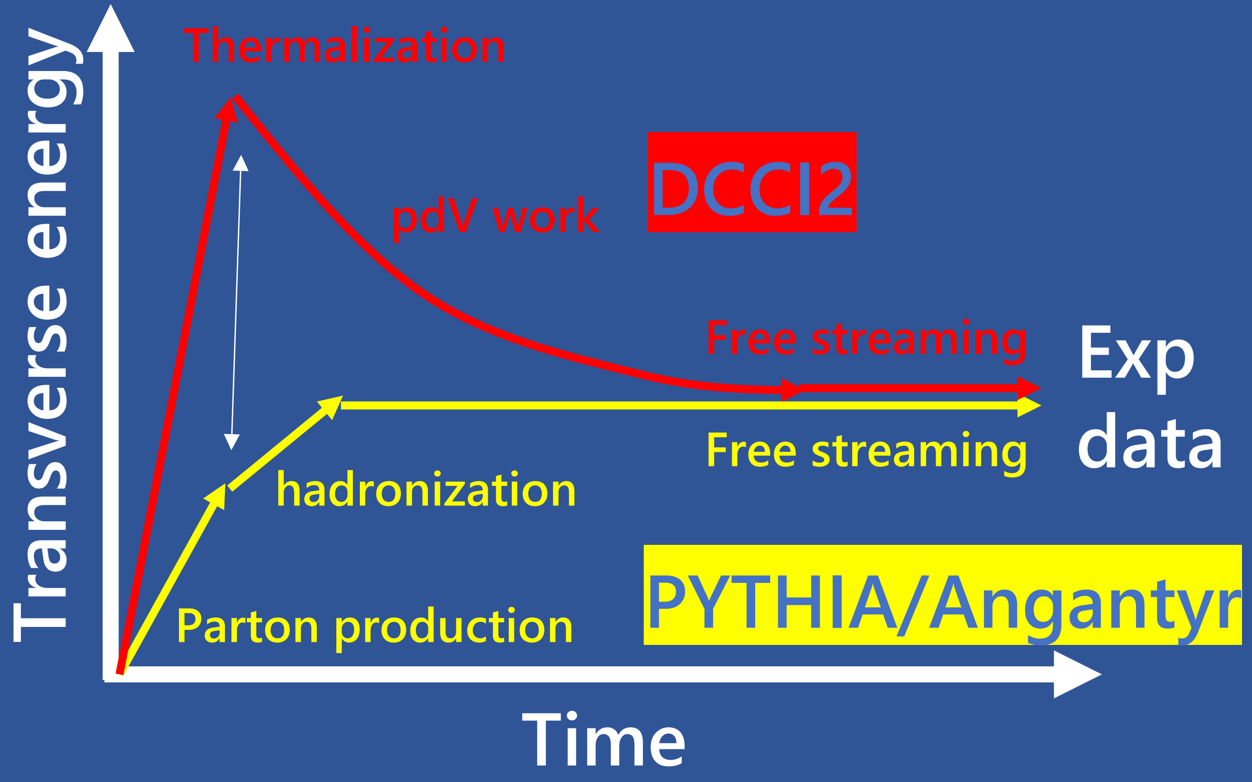


Note: Not so better agreement in pion/kaon cases
← Partly due to lack of hard parton in PYTHIA Angantyr

Transverse energy evolution

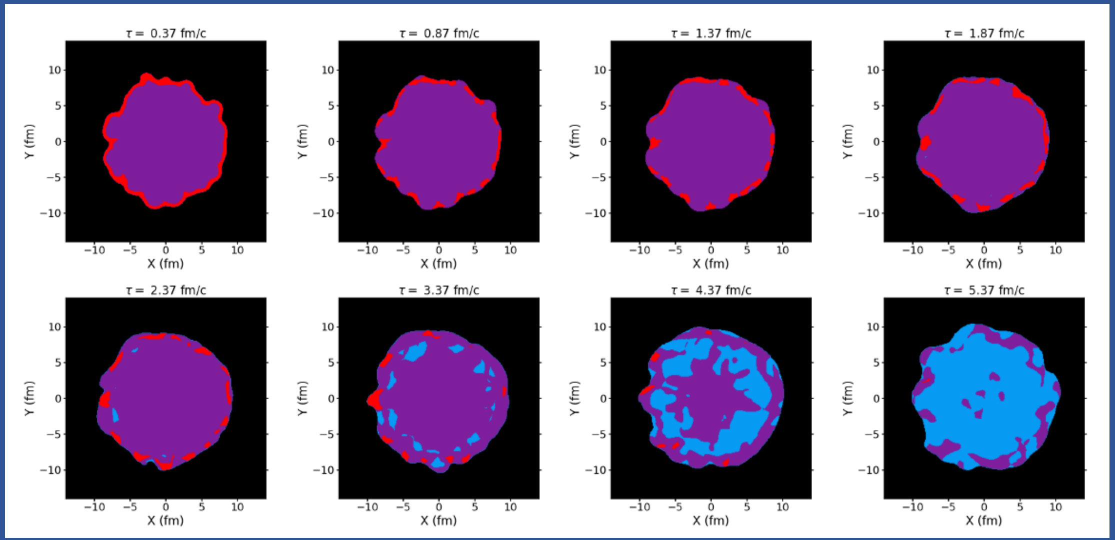


Y. Kanakubo *et al.*, Phys. Rev. C 105, 024905 (2022).



How much matter thermalized is highly correlated with initial production under constraint from exp data.

Issue 1:
Violation of causality in hydro simulations



Transverse profile in Pb+Pb collisions at 5.02 TeV by using T_RENTo + Free-streaming + MUSIC

Criteria from non-linear causality

Red: Acausal
Purple: Intermediate
Blue: Causal

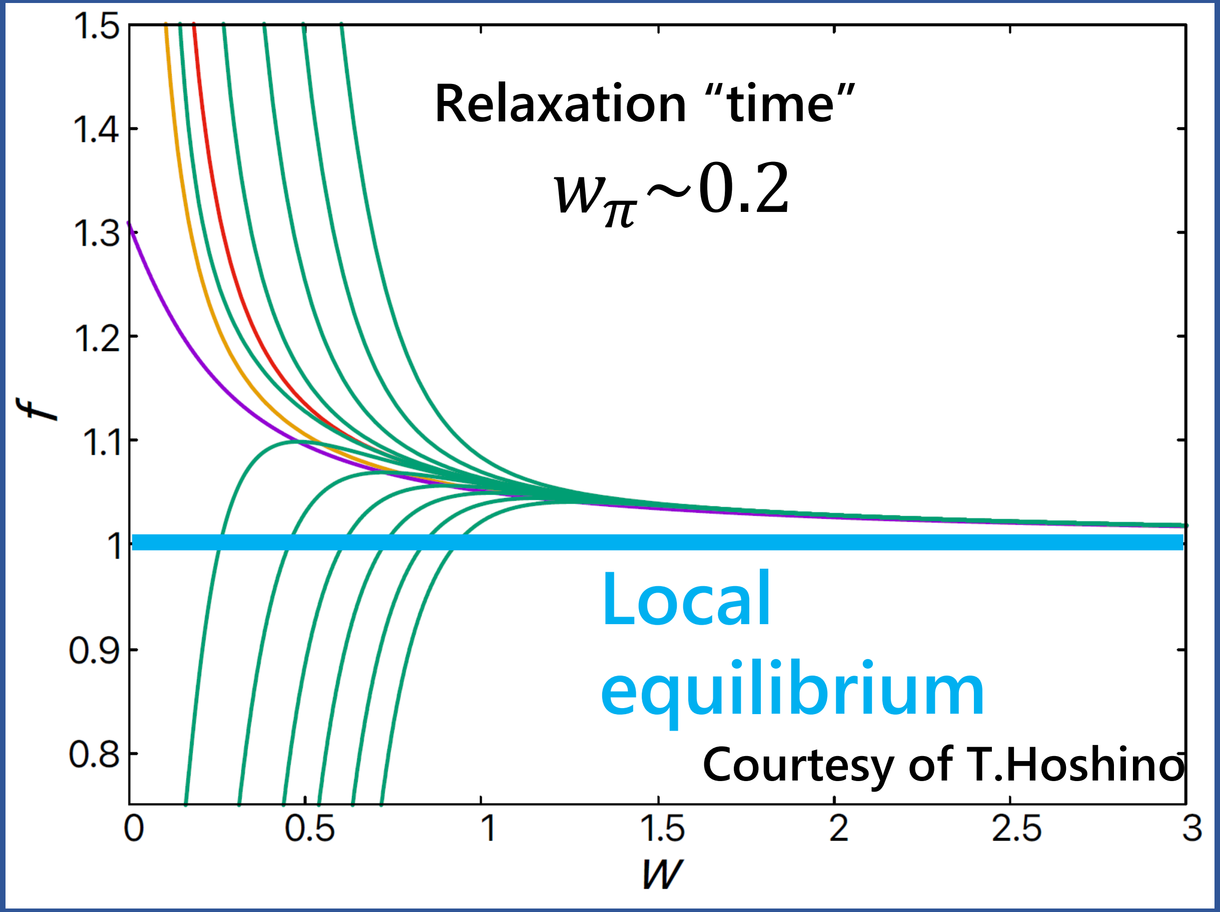
Tendency for violation of causality near the edge regions
→ Importance of non-equilibrium physics?

R. Krupczak *et al.* (The ExTrEMe Collaboration), arXiv: 2311.02210 [nucl-th]

F.S. Bemfica *et al*, Phys. Rev. Lett. 126, 222301 (2021).

Hydrodynamic attractor

"equilibrium measure"



"time" ~ (tau)$^{2/3}$

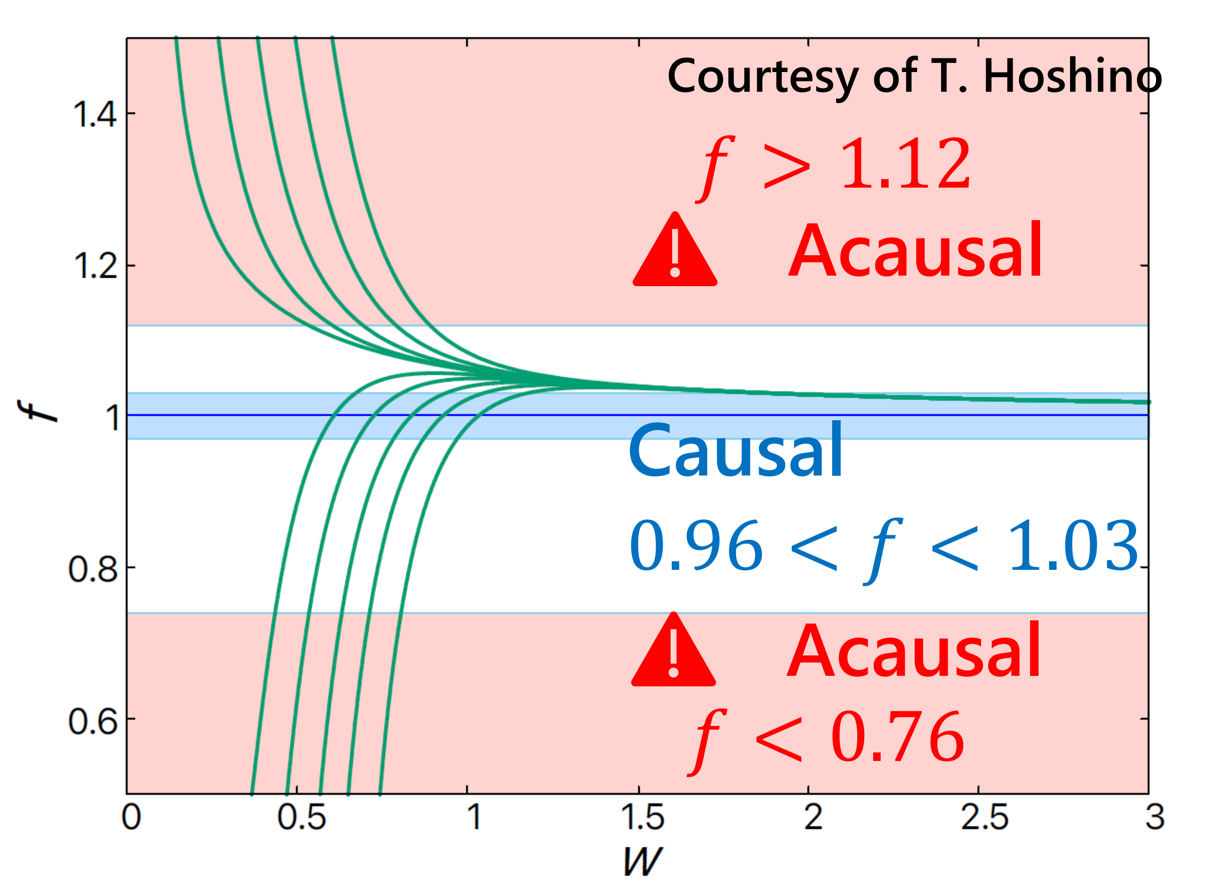
Hydrodynamic attractor
→ Fluid dynamics far from equilibrium is justified(?)
→ Almost any initial conditions are acceptable (?)

⇩

Scrutinize from non-linear causali

M.P. Heller and M. Spaliński, Phys. Rev. Lett. **115**, 072501 (2015).

Relation between causality and Reynolds number



$$Re_\pi^{-1} = \pm 8(f-1)$$

Acausal from necessary conditions

$\Rightarrow$ $Re_\pi^{-1} > 0.96$ $(\phi > 0)$

Causal from sufficient conditions

$\Rightarrow$ $Re_\pi^{-1} < 0.24$ $(\phi > 0)$

✓ Constraint inverse Reynolds number from causality

Short summary 1

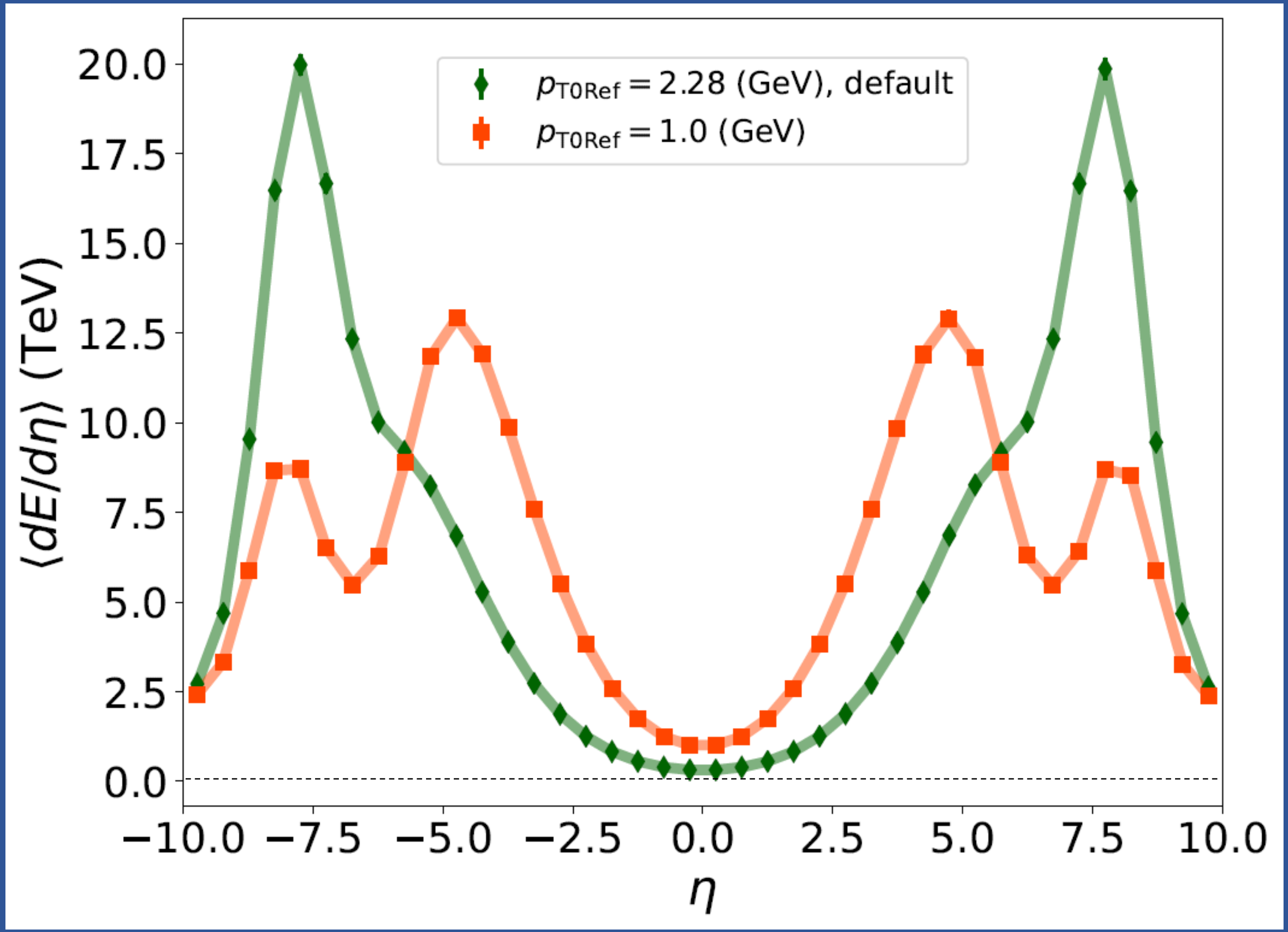
- ✓ Constraint of the inverse Reynolds number from non-linear causality
- ✓ Importance of non-equilibrium description prior to hydrodynamic evolution
- ✓ Effects of non-linear causality on Bayesian analysis*

*R. Krupczak *et al.* (The ExTrEMe Collaboration), arXiv: 2311.02210 [nucl-th]

Go “Forward”

Toward understanding dynamics as a whole

Energy distribution



PYTHIA Angantyr Pb+Pb 2.76 TeV
Minimum bias, parton level
Courtesy of Y.Kanakubo

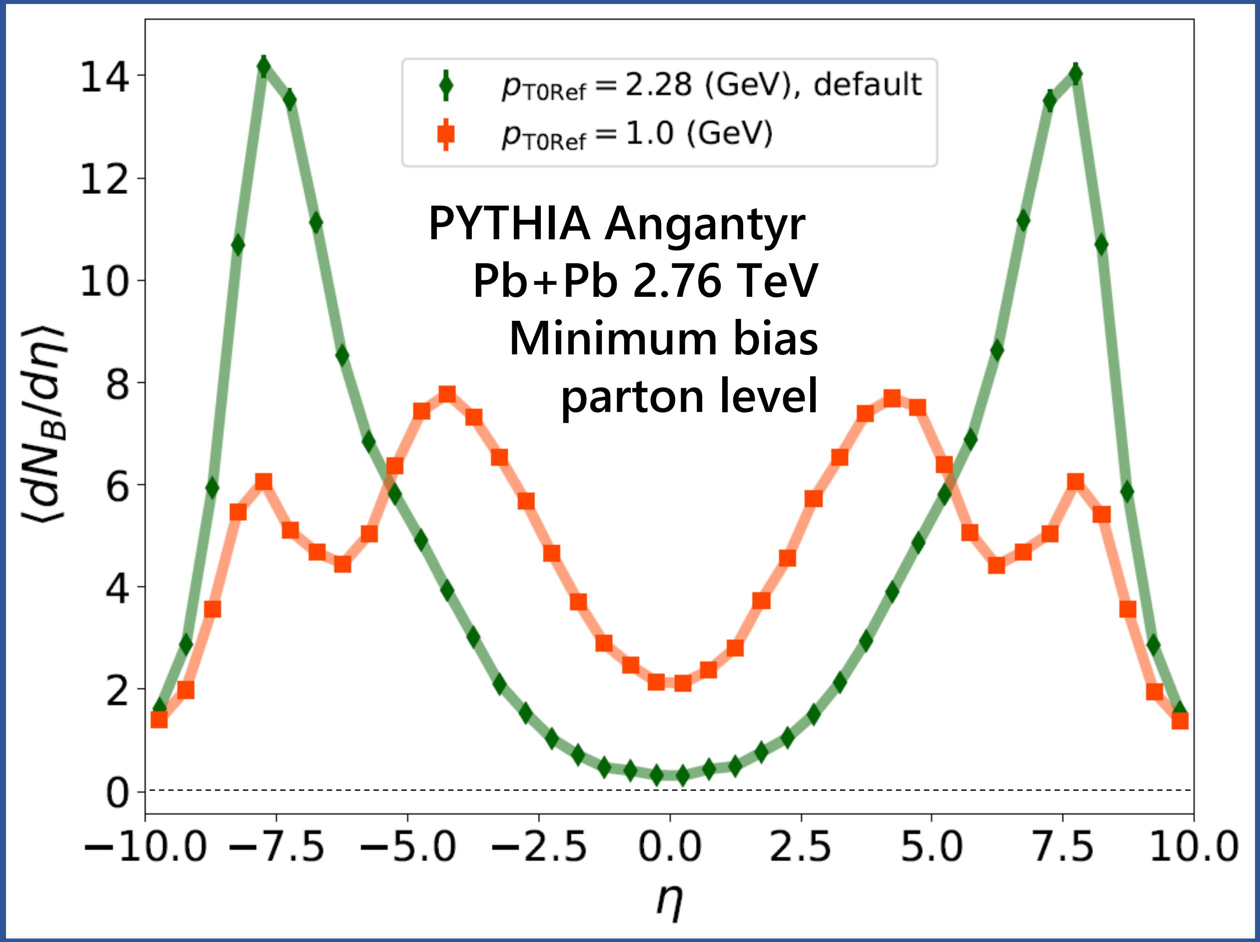
Energy deposited at $-1 < \eta < 1$

→ 0.2-0.3% of total energy
(minimum bias case)

Small change in forward
→ Large change in central

Importance of modeling at primary collisions
ex.) Highly depends on MPI parameters in PYTHIA

Understanding central region from forward baryon production



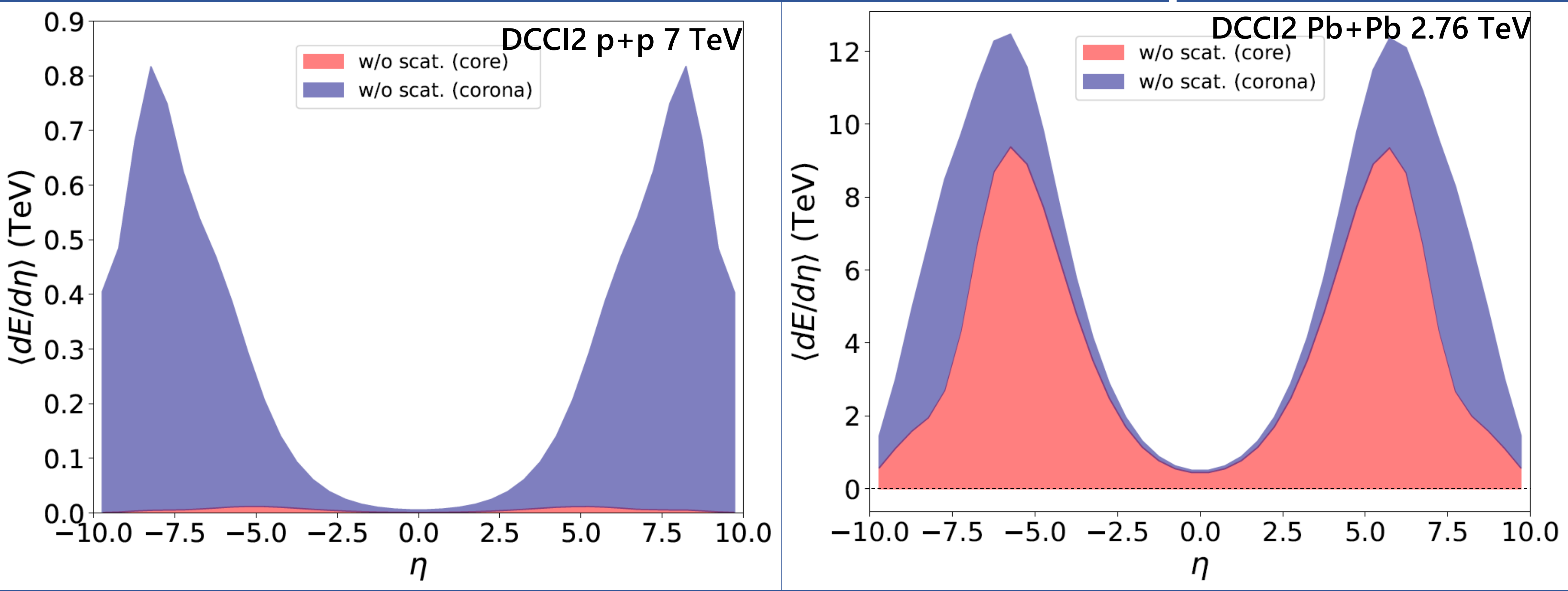
Courtesy of Y.Kanakubo

Change of net baryon distribution due to MPI parameters

Initial energy deposited in the central region is highly correlated with baryon stopping

The more minijet are produced, the more baryons stop

Fraction of core and corona components



Important in quantifying how much matter thermalized in forward

Courtesy of Y. Kanakubo

Correlation between central and forward

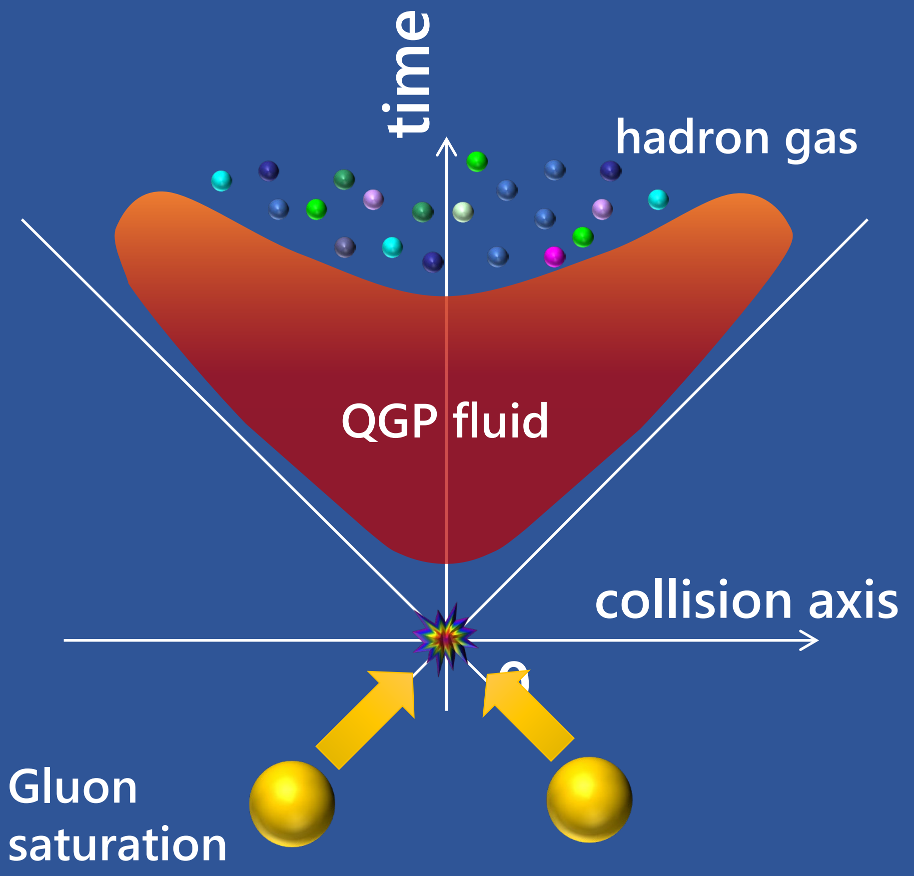
Central
More mini-jet production

Forward
More baryon stopping

Importance of **particle identification at forward**

$$\frac{dN_{p-\bar{p}}}{dy} \qquad R_{Y/\pi}(y)$$

Did we understand basic dynamics after all?



Evolution of thermalized energy?
Baryon transport, pressure gradient or diffusion?

How much deposited energy thermalized?

How much energy deposited?
←→ How much initial nuclei lose their energy?

These basic questions must be also important in EIC and LHC forward region.