Spin physics in nuclear reaction

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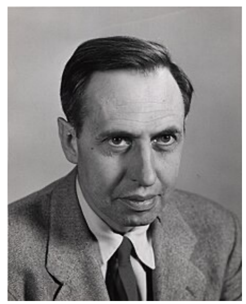
2025.05.17

Discovery of spin in physics

This year marks the 100th anniversary of the discovery of spin and the 20th anniversary of the proposal for spin polarization in relativistic heavy-ion collisions.

Wolfgang Pauli

Ralph Kronig



George Uhlenbeck

Samuel Goudsmit

Llewellyn H. Thomas

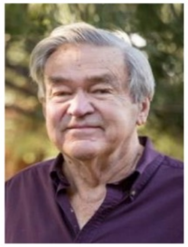
Wolfgang Pauli

- **1924** — two valued -ness not describable classically
- **1925** — self-rotation of the electron
- **1925** — Published the first paper on spin
- **1926** — Thomas procession
- **1927** — spin in quantum mechanics

Spin in high energy physics

Polarization data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection.

----- J.D. Bjorken



Striking spin effects have been observed in high energy reactions since 1970s.

Proton spin crisis

Quark Model:

Sum spin of quark Σ

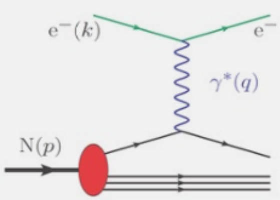
= spin of proton S_p

DIS exp:

$\Sigma \sim 0$ (1989)

$\Sigma \sim 0.2\, S_p$(NOW)

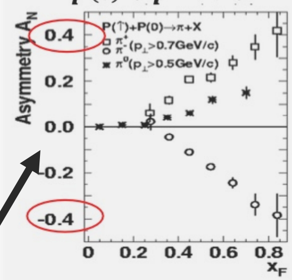
$$e + p \rightarrow e + X$$

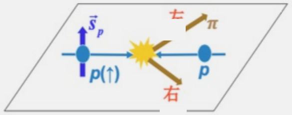


EMC, PLB (1988)

Single spin left-right asymmetry

$$p(\uparrow) + p \rightarrow \pi + X$$



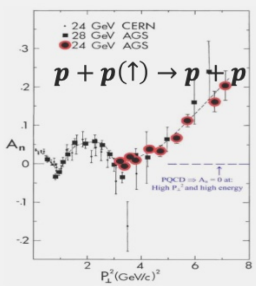


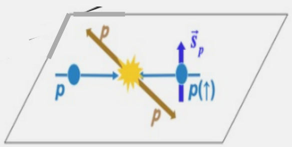
$$A_N \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

FNAL E704; PLB264, 462 (1991)

Prediction of QCD in 1970s ~ 0

Spin analyzing power in $pp \rightarrow pp$



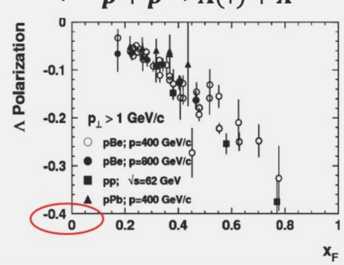


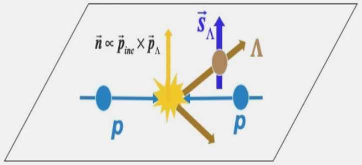
$$A_N \equiv -\frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

Grab, PRL (1987)

Transverse polarization of Λ in $pp \rightarrow \Lambda X$

$$p + p \rightarrow \Lambda(\uparrow) + X$$

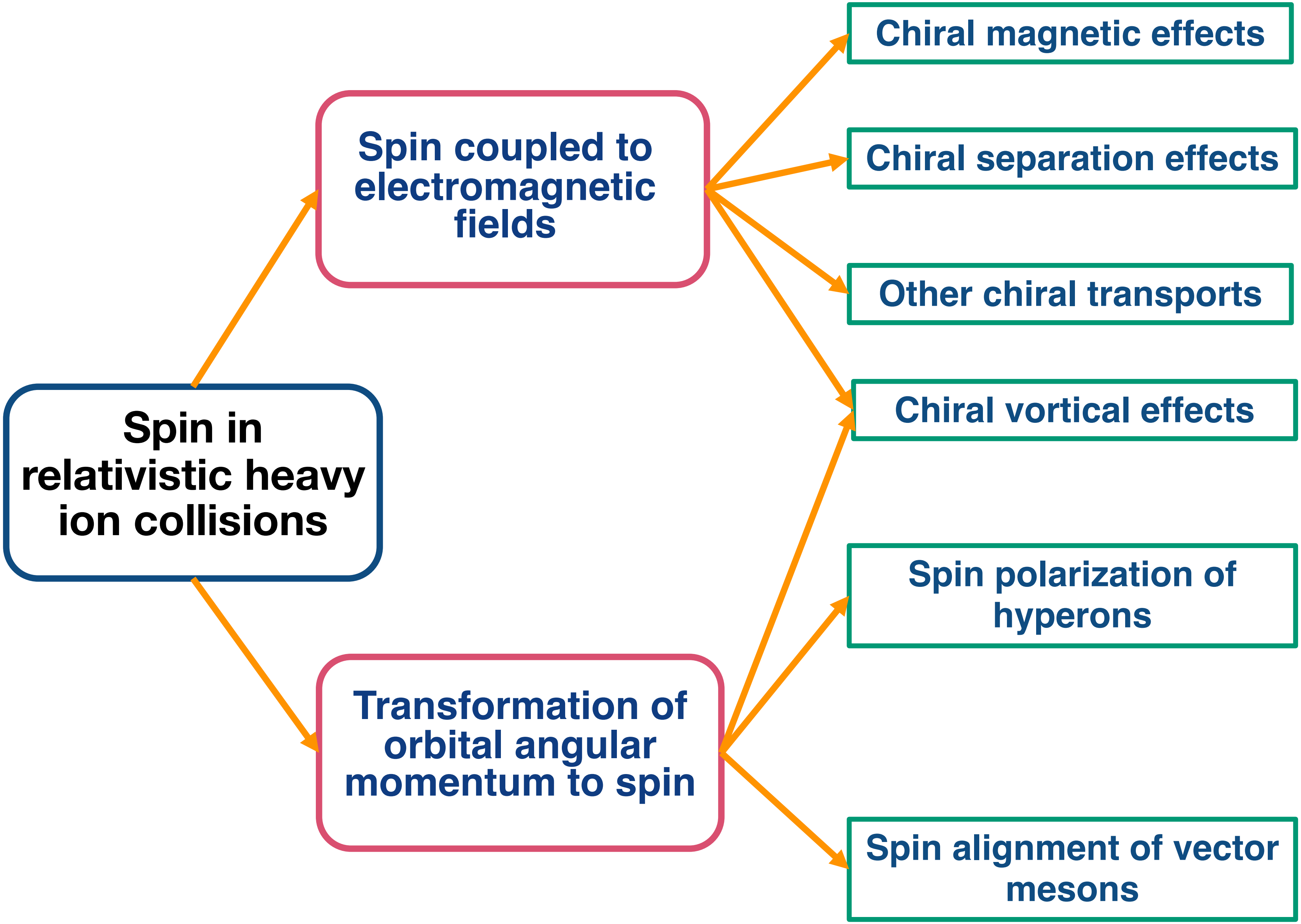




$$P_\Lambda \equiv \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$

Gourlay, PRL (1986)

Slides copy from Zuo-tang Liang's review talk on Spin2023

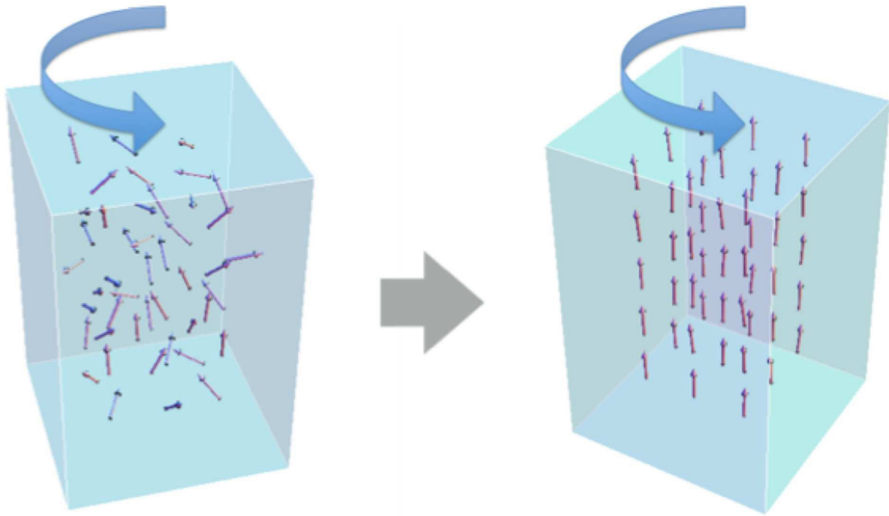

Spin in relativistic heavy ion collisions
Spin coupled to electromagnetic fields
Transformation of orbital angular momentum to spin
Chiral magnetic effects
Chiral separation effects
Other chiral transports
Chiral vortical effects
Spin polarization of hyperons
Spin alignment of vector mesons

Outline

- Global and local spin polarization
- Spin alignment
- Summary

Global spin polarization

Barnet and Einstein-de Hass effects



Barnett effect:

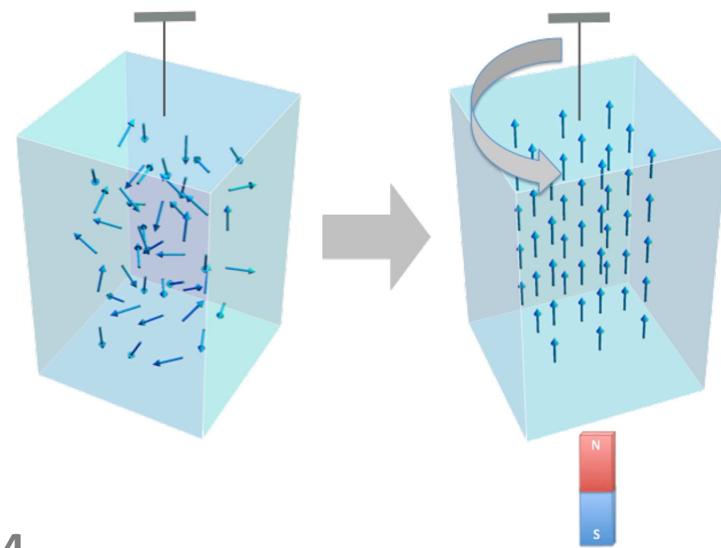
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

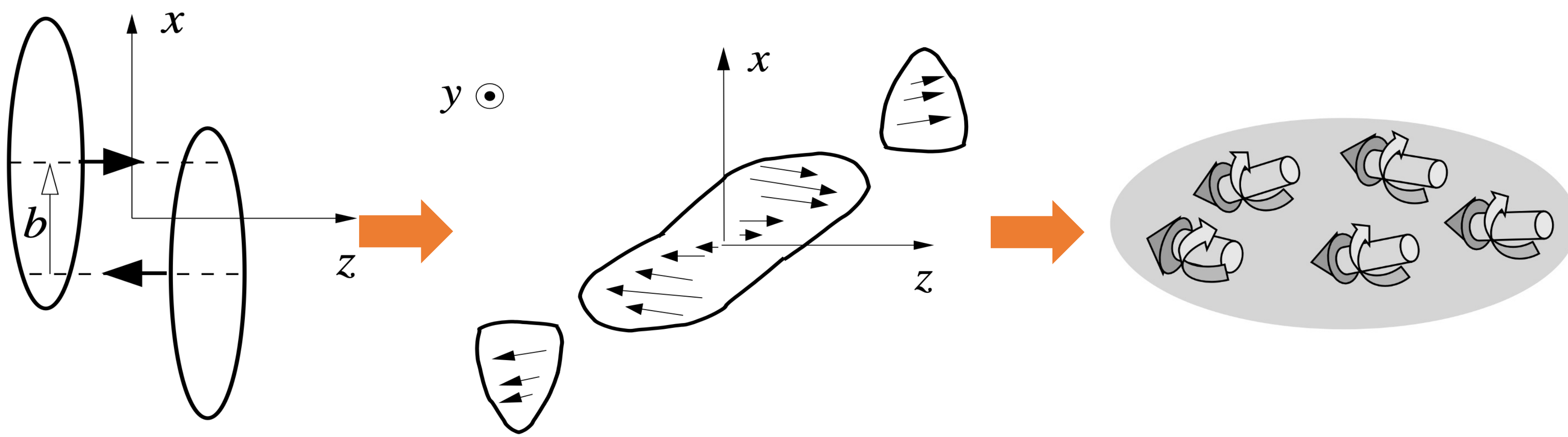
Magnetization $\Longrightarrow$ Rotation

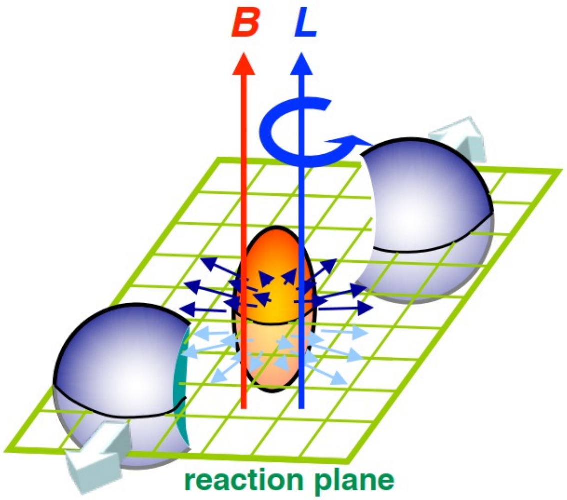
Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: from paper doi: 10.3389/fphy.2015.00054

OAM to spin polarization in HIC

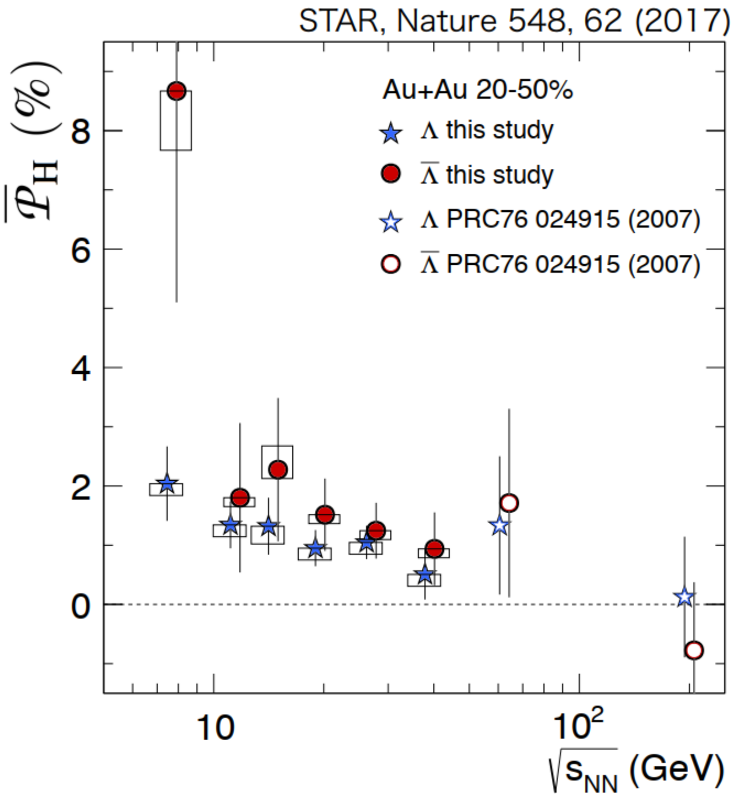




- **Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.**
- **Global orbital angular momentum leads to the polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbital coupling.**

Liang, Wang, PRL (2005); PLB (2005);
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization for Λ and $\bar{\Lambda}$ hyperons



- **Estimation given by Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)**

$$\mathbf{P}_\Lambda \simeq \frac{\omega}{2T} + \frac{\mu_\Lambda \mathbf{B}}{T}$$
$$\mathbf{P}_{\bar{\Lambda}} \simeq \frac{\omega}{2T} - \frac{\mu_\Lambda \mathbf{B}}{T}$$

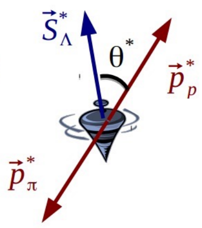
- **ω = (9 ± 1)x10^{21}/s, greater than previously observed in any system.**
- **QGP is most vortical fluid so far.**

parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

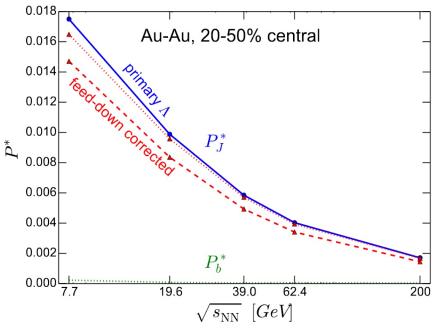
α: Λ decay parameter (=0.642±0.013)
P$_\Lambda$: Λ polarization
p$_p^*$: proton momentum in Λ rest frame



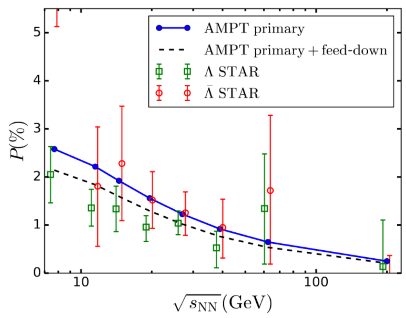
$\Lambda \to p + \pi^+$
(BR: 63.9%, c τ ~7.9 cm)

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

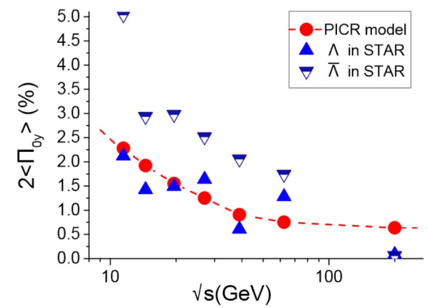
Phenomenological models for global polarization



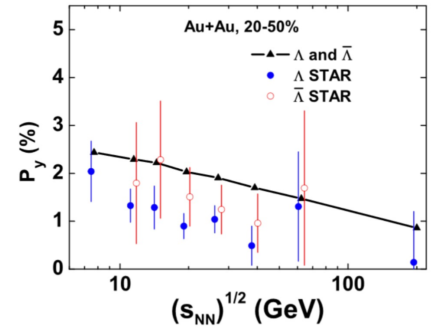
Karpenko, Becattini, EPJC(2017)



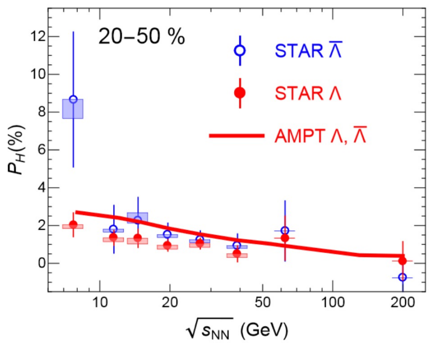
Li, Pang, Wang, Xia PRC(2017)



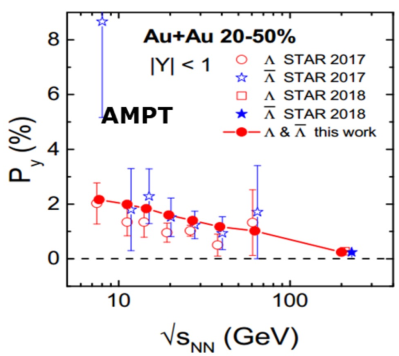
Xie, Wang, Csernai, PRC(2017)



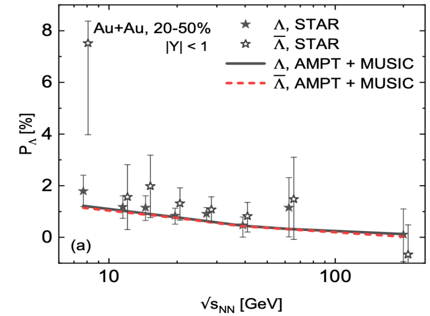
Sun, Ko, PRC(2017)



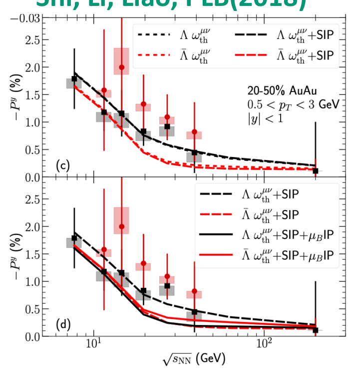
Shi, Li, Liao, PLB(2018)



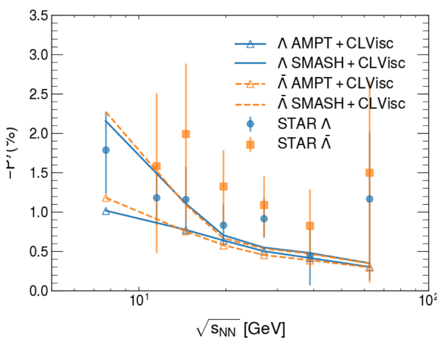
Wei, Deng, Huang, PRC(2019)



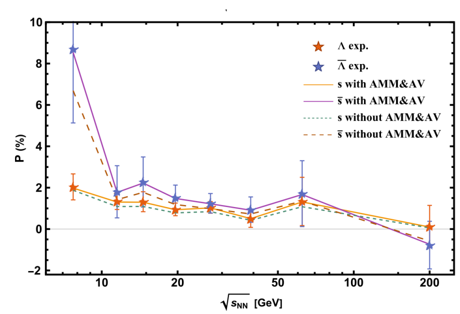
Fu, Xu, Huang, Song, PRC (2021)



S. Ryu, V. Jupic, C. Shen, PRC (2021)



Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

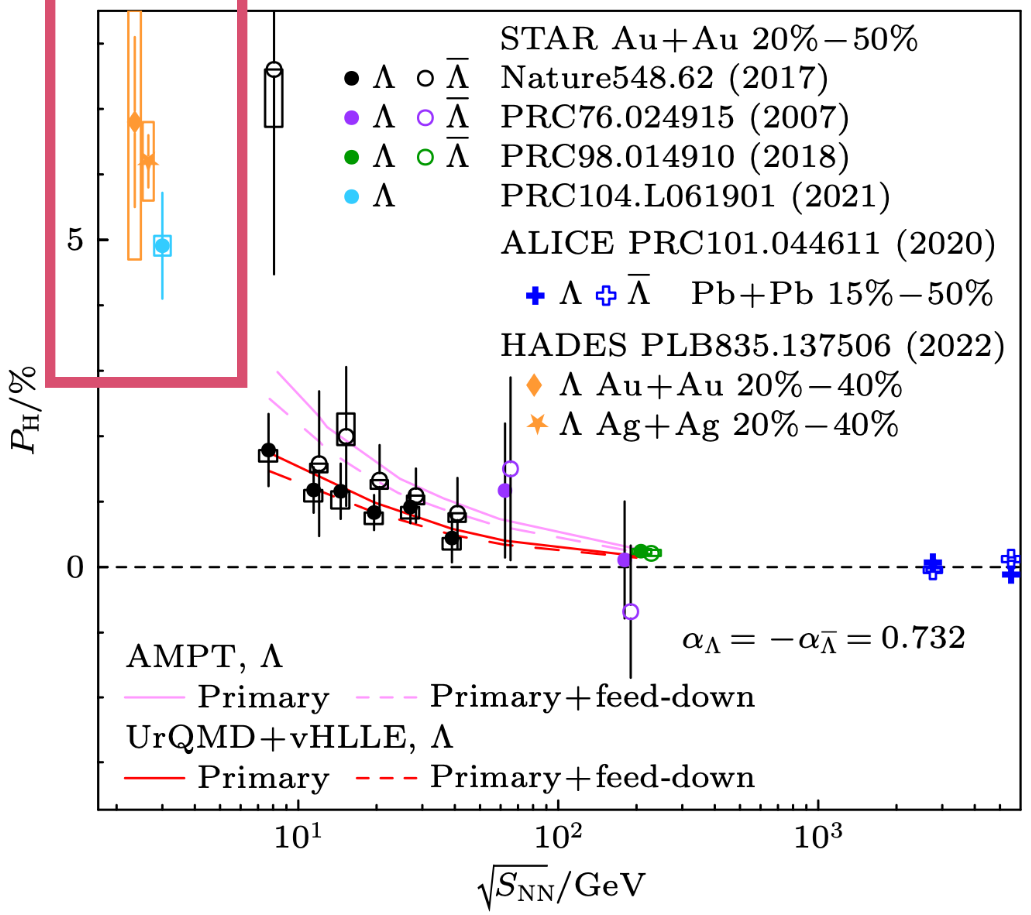


Xu, Lin, Huang, Huang, PRDL (2022)

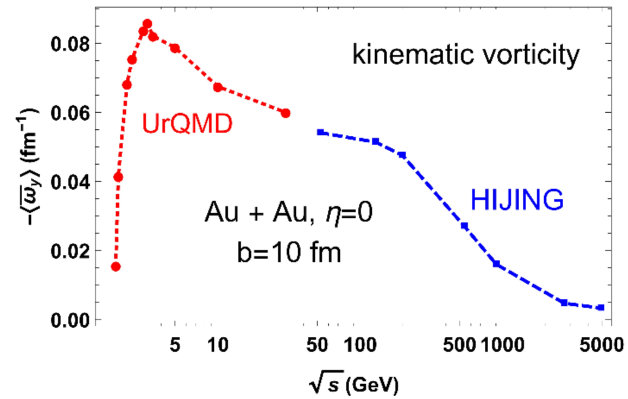
Polarization at low energies

Will the polarization of Λ be nonzero as $\sqrt{s_{NN}} \to 0$?

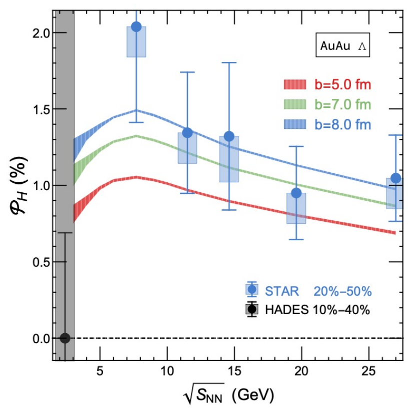
If not, how large the will "critical $\sqrt{s_{NN}}$" be?

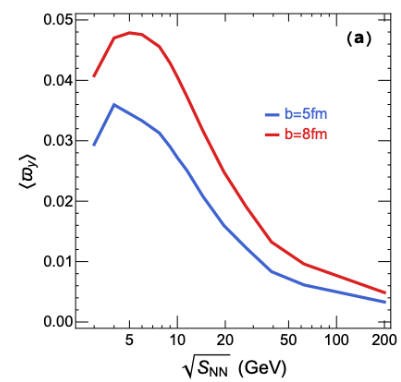


Sun Xu et al., Acta Phys. Sin. 72(7), 072401 (2023)



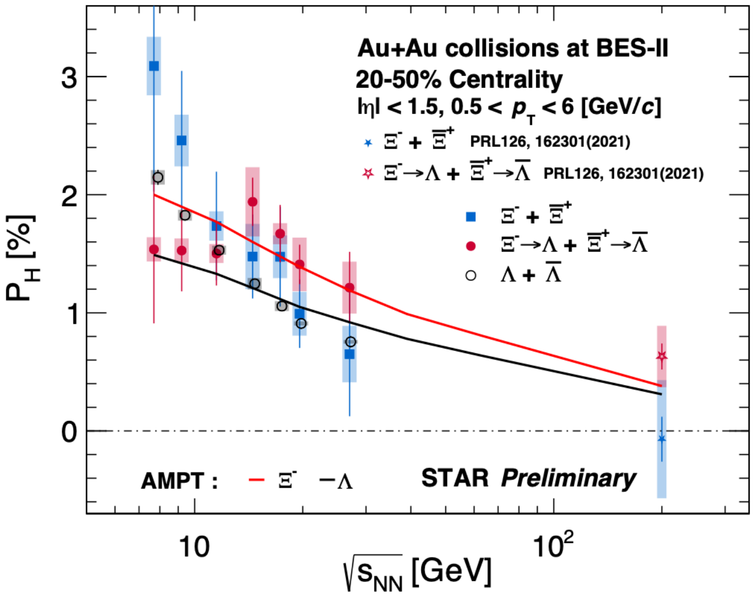
Deng, Huang PRC 2016, PRC 2020

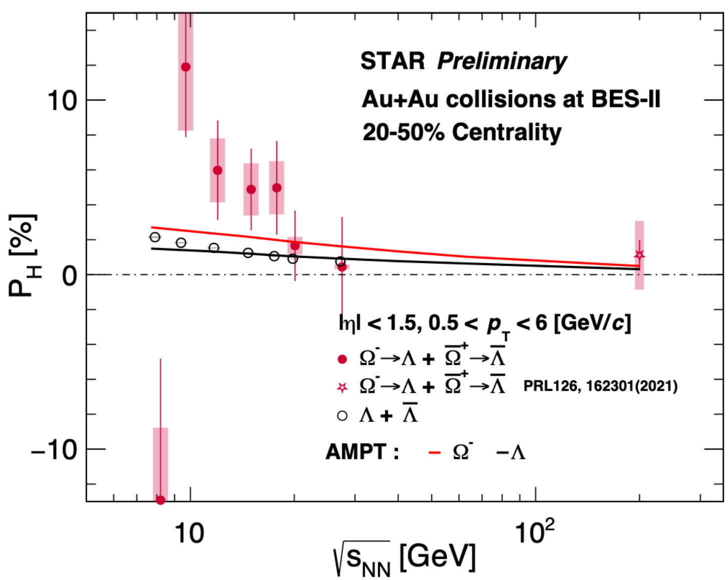




Guo, Liao, et. al, PRC 2021

Global polarization for other hyperons





In the quark model,

$$P_\Lambda \cong P_s$$

assuming that $P_{u,d} \sim P_s$, one can get

$$P_\Sigma \sim P_\Lambda$$

$$P_\Xi \sim P_\Lambda$$

$$P_\Omega \sim \frac{5}{3} P_\Lambda$$

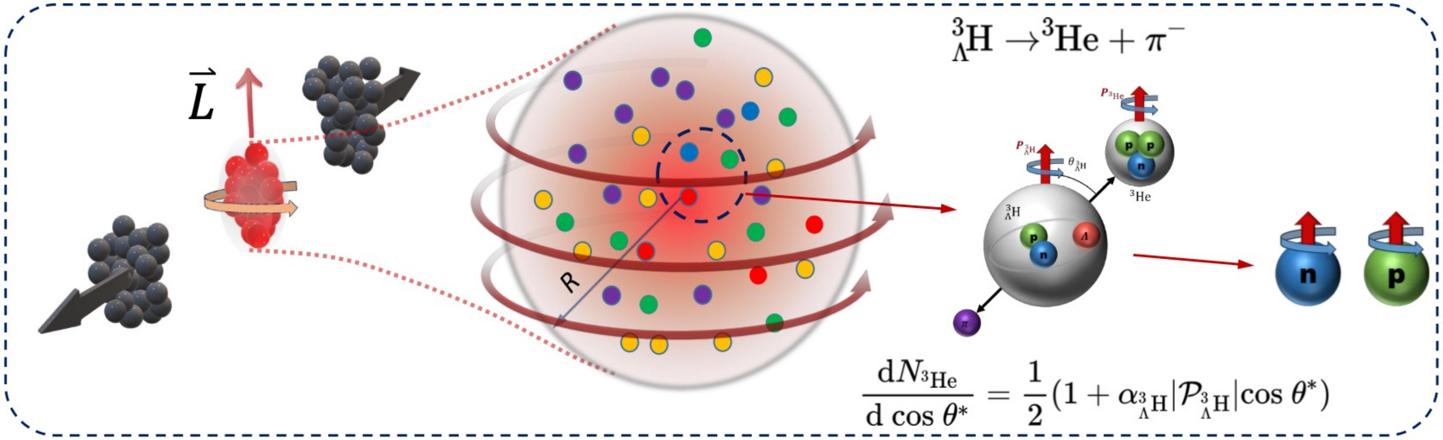
Liang, Wang, PRL (2005)
Also see:
Li, Xia, Huang, Huang, PLB (2022)

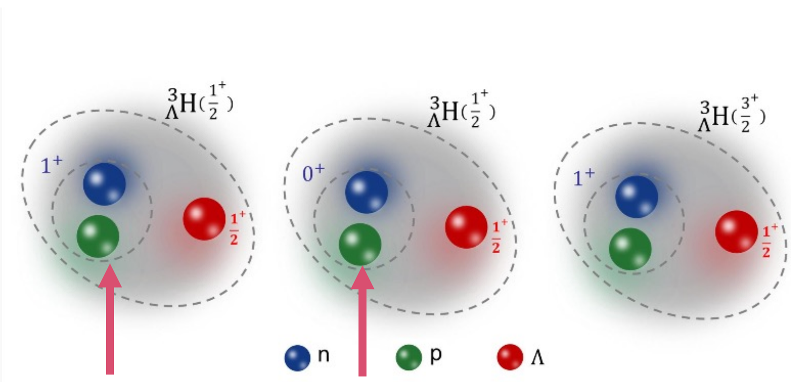
First measurement in BES-II energy:

- **No significant difference between Ξ and Λ global polarization**
- **Hint of larger global polarization of Ω than Λ hyperon**

Global polarization for ${}^{3}_{\Lambda}\mathrm{H}$

The measurement of hypertriton polarization provides a novel method to uniquely determine its internal spin structure.





spin triplet

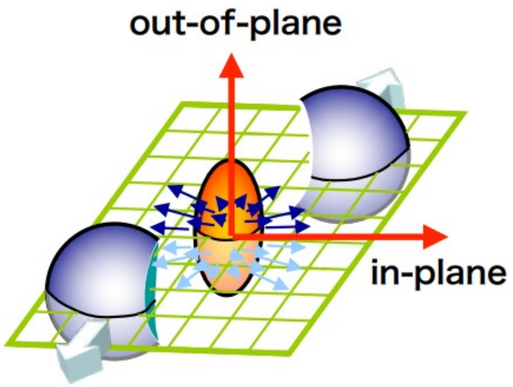
spin singlet

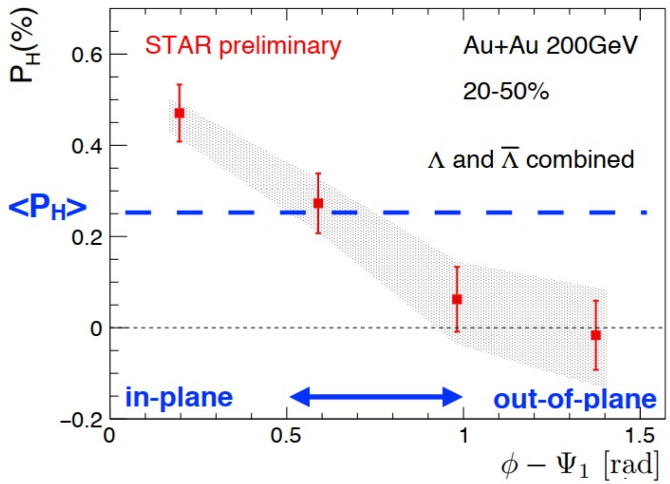
J^P	structure	decay mode	$\frac{dN}{d\cos\theta^*}$
$\frac{1}{2}^+$	$\Lambda(\frac{1}{2}^+) - np(1^+)$	${}^{3}_{\Lambda}\mathrm{H} \to \pi^- + {}^{3}\mathrm{He}$	$\frac{1}{2}(1 - \boxed{\frac{1}{2.58}\alpha_\Lambda \mathcal{P}_\Lambda \cos\theta^*})$
$\frac{1}{2}^+$	$\Lambda(\frac{1}{2}^+) - np(0^+)$	${}^{3}_{\Lambda}\mathrm{H} \to \pi^- + {}^{3}\mathrm{He}$	$\frac{1}{2}(1 + \boxed{\alpha_\Lambda \mathcal{P}_\Lambda \cos\theta^*})$
$\frac{3}{2}^+$	$\Lambda(\frac{1}{2}^+) - np(1^+)$	${}^{3}_{\Lambda}\mathrm{H} \to \pi^- + {}^{3}\mathrm{He}$	$\frac{1}{2}(1 - \boxed{\mathcal{P}_\Lambda^2(3\cos^2\theta^* - 1)})$
$\frac{1}{2}^-$	$\bar{\Lambda}(\frac{1}{2}^-) - \overline{np}(1^-)$	${}^{3}_{\bar{\Lambda}}\overline{\mathrm{H}} \to \pi^+ + {}^{3}\overline{\mathrm{He}}$	$\frac{1}{2}(1 - \frac{1}{2.58}\alpha_{\bar{\Lambda}} \mathcal{P}_{\bar{\Lambda}} \cos\theta^*)$
$\frac{1}{2}^-$	$\bar{\Lambda}(\frac{1}{2}^-) - \overline{np}(0^-)$	${}^{3}_{\bar{\Lambda}}\overline{\mathrm{H}} \to \pi^+ + {}^{3}\overline{\mathrm{He}}$	$\frac{1}{2}(1 + \alpha_{\bar{\Lambda}} \mathcal{P}_{\bar{\Lambda}} \cos\theta^*)$
$\frac{3}{2}^-$	$\bar{\Lambda}(\frac{1}{2}^-) - \overline{np}(1^-)$	${}^{3}_{\bar{\Lambda}}\overline{\mathrm{H}} \to \pi^+ + {}^{3}\overline{\mathrm{He}}$	$\frac{1}{2}(1 - \mathcal{P}_{\bar{\Lambda}}^2(3\cos^2\theta^* - 1))$

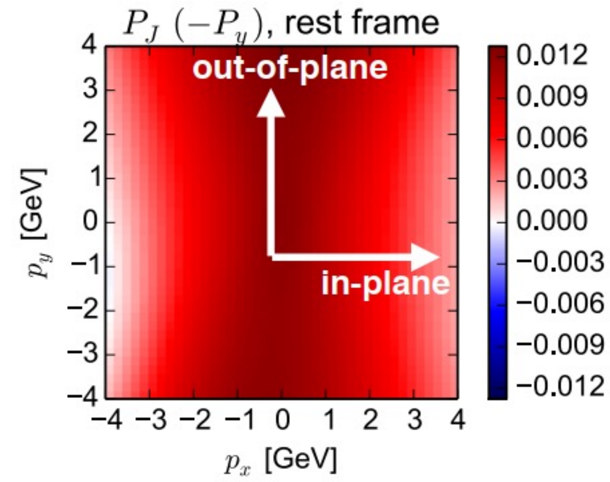
Prediction from Fudan's group: Sun, Liu, Zhen, Chen, Ko, Ma, PRL (2025)

Local polarization

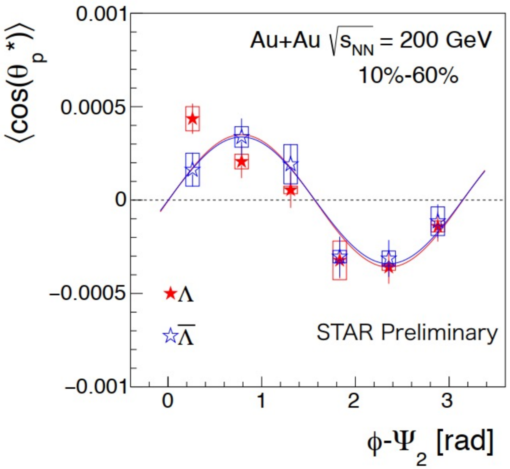
Local polarization and sign problem

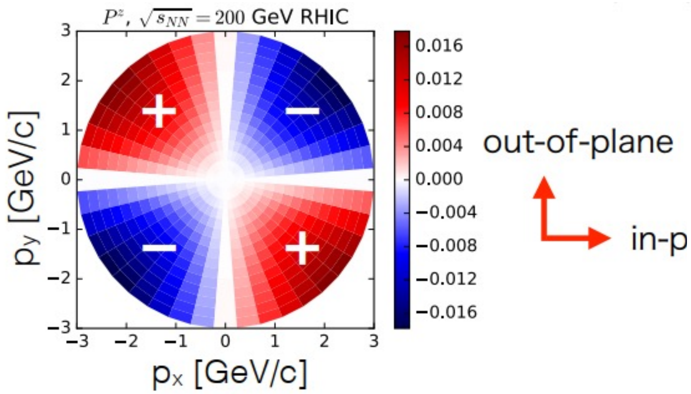


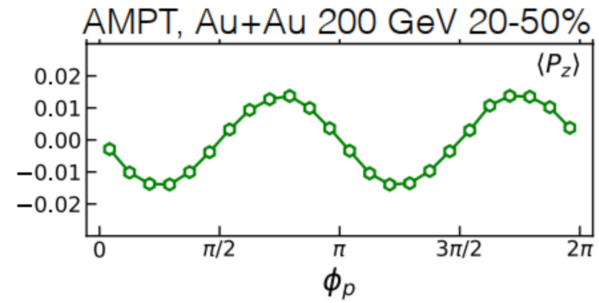




STAR, PRL 123, 132301 (2019)







- **UrQMD :**
  Becattini, Karpenko, PRL (2018)
- **AMPT:**
  Xia, Li, Tang, Wang, PRC (2018)

Polarization and axial current

- **The polarization tensor is connected to the axial current in phase space by modified Cooper-Frye formula**
  Karpenko, Becattini, EPJC. (2017); Fang, Pang, Wang, Wang, PRC (2016)

$$\mathcal{S}^{\mu}(\mathbf{p})=\frac{\int d\Sigma\cdot p\mathcal{J}_{5}^{\mu}(p,X)}{2m_{\Lambda}\int d\Sigma\cdot\mathcal{N}(p,X)},$$

Polarization pseudo-vector ~ Spin tensor in phase space

- **The expression for the spin tensor in phase space can be computed using statistical quantum field theory, quantum kinetic theory, Kubo formula and other effective theories.**

Becattini's group, Rischke's group, Florkowski's group, …
Chinese groups:
Xin-Nian Wang @ CCNU, Zuo-Tang Liang @ SDU, Peng-fei Zhuang @ THU, Qun Wang, SP @ USTC, Mei Huang, Feng Li @ CAS, Xu-Guang Huang, Li Yan @ Fudan, Yin Yi @ CUHK, Huichao Song @ PKU, Defu Hou @CCNU, Jianhua Gao @ SDU, Shu Lin @ SYSU, Shuai Liu @ HNU, …

Polarization induced by different sources

$$\mathcal{S}^{\mu}(\mathbf{p}) = \mathcal{S}^{\mu}_{\rm thermal} + \mathcal{S}^{\mu}_{\rm shear} + \mathcal{S}^{\mu}_{\rm accT} + \mathcal{S}^{\mu}_{\rm chemical} + \mathcal{S}^{\mu}_{\rm EB}$$

Yi, SP, Yang, PRC (2021); Wu, Yi, Qin, SP, PRC (2022); Hidaka, SP, Yang, PRD (2018)

Thermal vorticity： main contribution to global spin polarization of hyperons

$$\mathcal{S}^{\mu}_{\rm thermal}(\mathbf{p}) = \frac{\hbar}{8m_{\Lambda}N}\int d\Sigma^{\sigma}p_{\sigma}f_{V}^{(0)}(1-f_{V}^{(0)})\boxed{\epsilon^{\mu\nu\alpha\beta}p_{\nu}\partial_{\alpha}\frac{u_{\beta}}{T}},$$

Shear viscous tensor

$$\mathcal{S}^{\mu}_{\rm shear}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}\boxed{(\partial_{\sigma}u_{\nu}+\partial_{\nu}u_{\sigma})-Du_{\nu}}\}$$

Fluid acceleration

$$\mathcal{S}^{\mu}_{\rm accT}(\mathbf{p}) = -\frac{\hbar}{8m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{1}{T}\epsilon^{\mu\nu\alpha\beta}p_{\nu}u_{\alpha}\boxed{\left(Du_{\beta}-\frac{1}{T}\partial_{\beta}T\right)},$$

Gradient of chemical potential

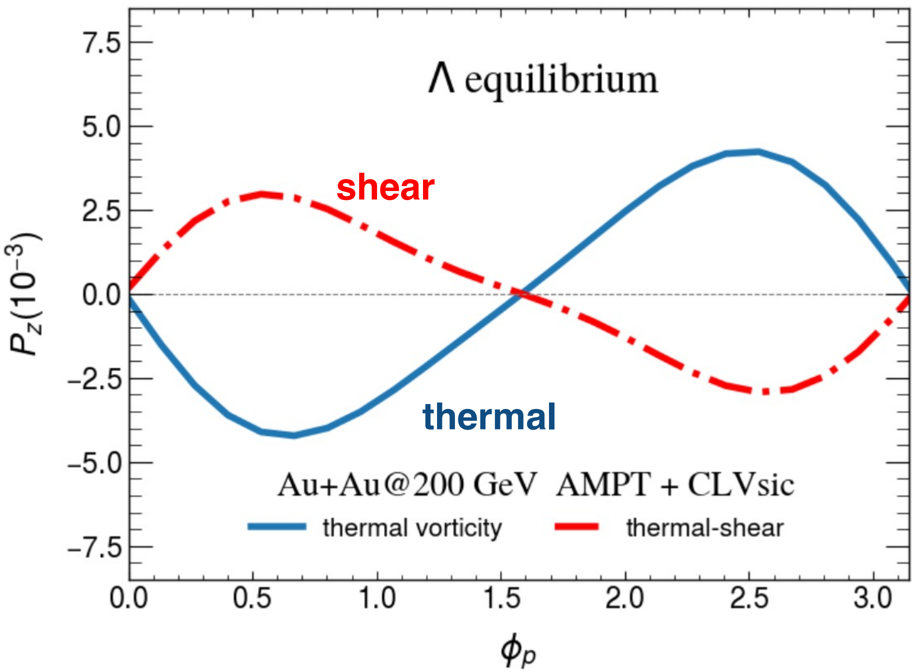
$$\mathcal{S}^{\mu}_{\rm chemical}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{\partial_{\nu}\frac{\mu}{T}},$$

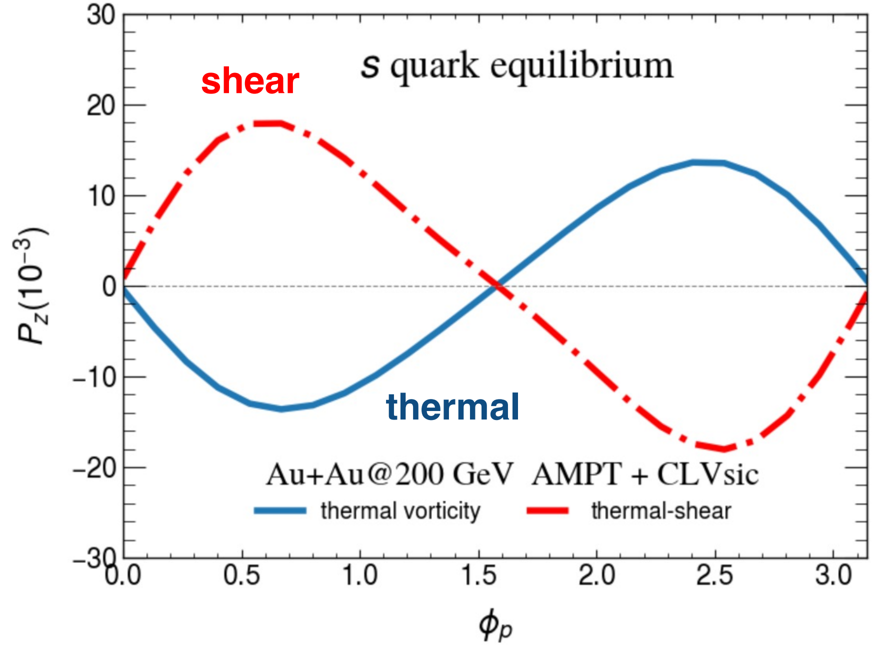
Electromagnetic fields

$$\mathcal{S}^{\mu}_{\rm EB}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\left(\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{E_{\nu}+\frac{B^{\mu}}{T}}\right)$$

Shear induced polarization: s quark scenario

s quark equilibrium: The spin of Λ hyperons is assumed to be primarily carried by the constituent s quark. One needs to take the s quark's mass instead of mass of Λ in the simulation. Fu, Liu, Pang, Song, Yin, PRL 2021





$$\mathcal{S}^{\mu}_{\text{thermal}} \propto \frac{1}{\boxed{m_N}}\int d\Sigma\cdot p ... \epsilon^{\mu\nu\alpha\beta}p_{\nu}\partial_{\alpha}\frac{u_{\beta}}{T},$$

$$\mathcal{S}^{\mu}_{\text{shear}} \propto \frac{1}{\boxed{m_N}}\int d\Sigma\cdot p ... \frac{1}{\boxed{(u\cdot p)}}\epsilon^{\mu\nu\alpha\beta}...$$

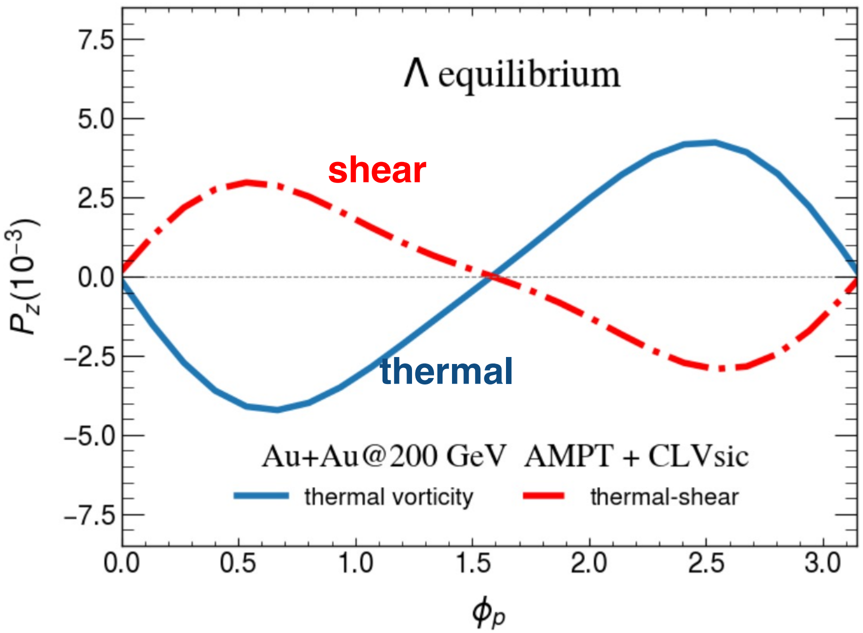
Once we replace the m_{Λ} by m_s, both thermal and shear induced polarization are enhanced. But, there is an extra mass term in the denominator for shear effects, which results in a 'double' enhancement of the shear-induced polarization.

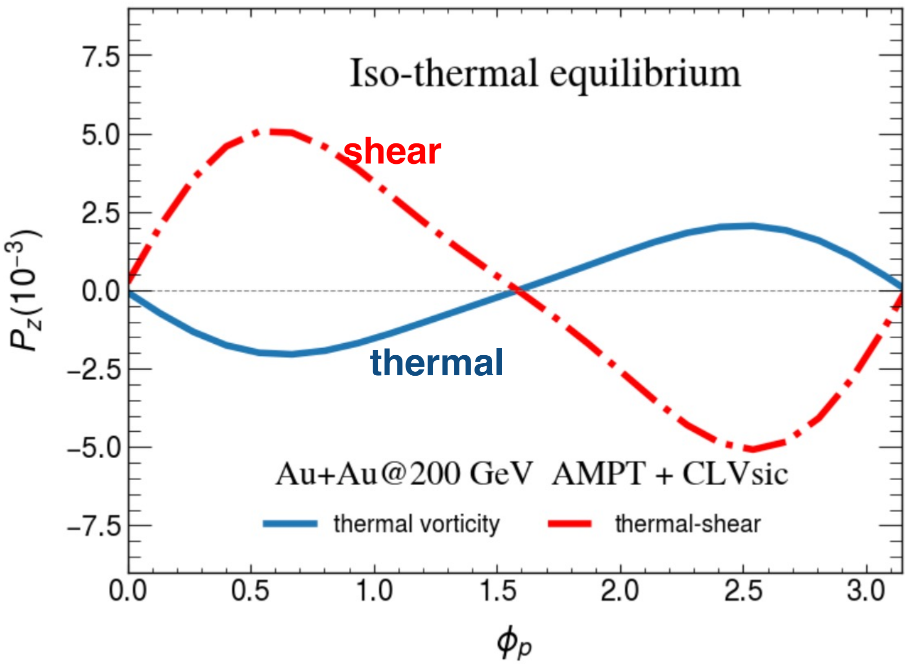
Also see: Yi, Pu, Yang, PRC (2021); Yi, Wu, Qin, Pu, PRC (2022); Ryu, Jupic, Shen, PRC (2021)

Shear induced polarization: isothermal equilibrium

Isothermal equilibrium: The temperature of the system at the freeze-out hyper-surface is assumed to be constant.

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021



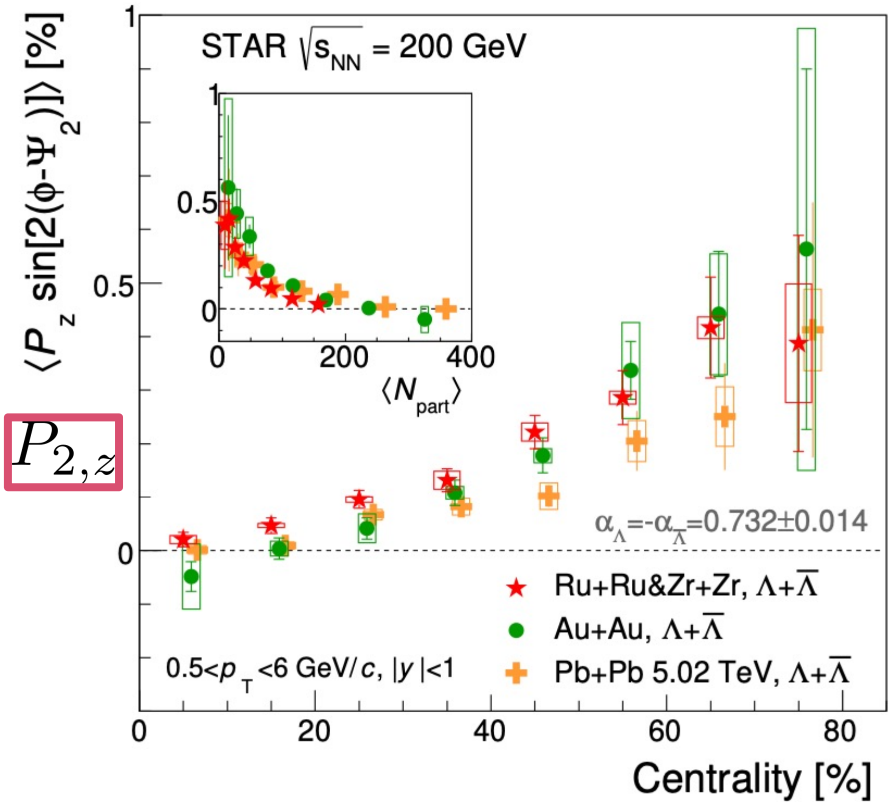


Lessons learned:

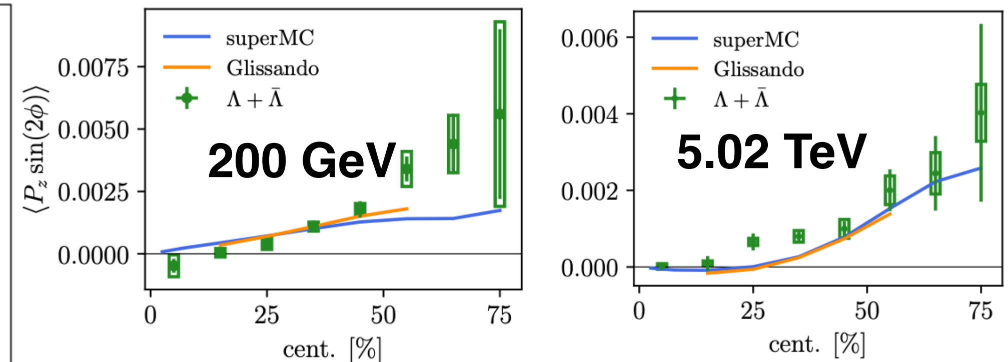
Temperature gradient plays a crucial role to the local spin polarization.

This finding is consistent with earlier studies on polarization induced by kinetic vorticity, Wu, Pang, Huang, Wang, PRR (2019) and also aligns with results from the polarization computed using the blast wave model by Arslan, Dong, Ma, SP, Wang, PRC (2025).

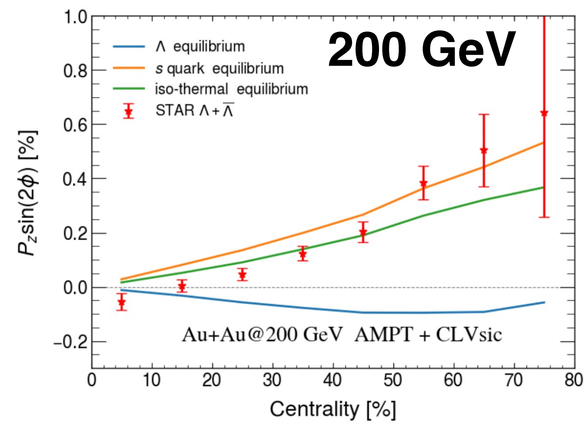
Local polarization VS centrality



STAR, PRL 123, 132301 (2019)
ALICE, PRL 128, 172005 (2022)



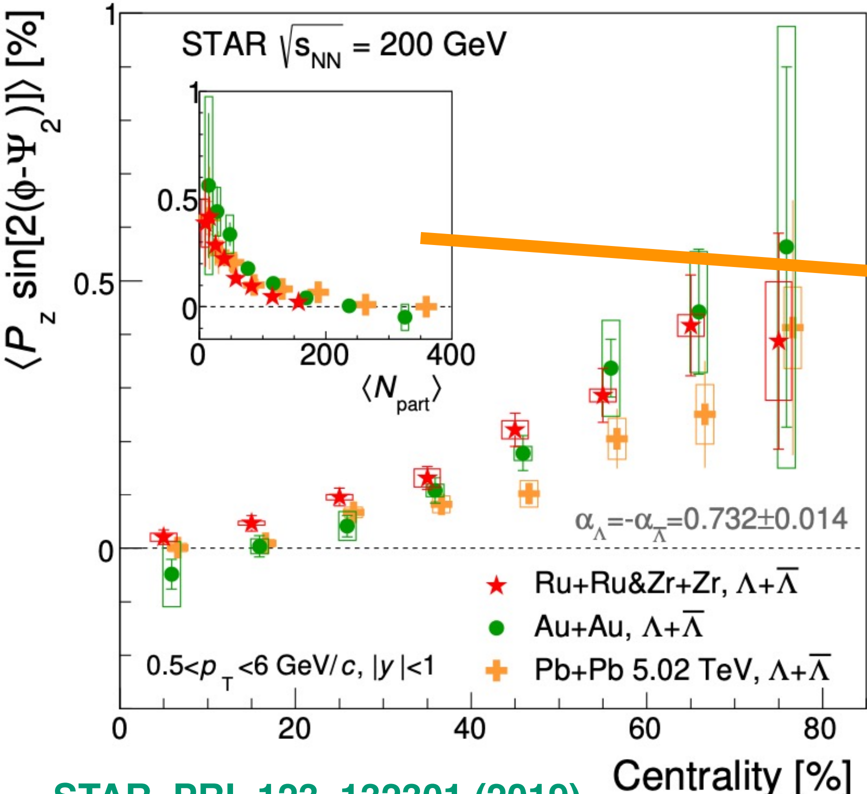
Palermo, Grossi, Karpenko, Becattini, EPJC (2024)



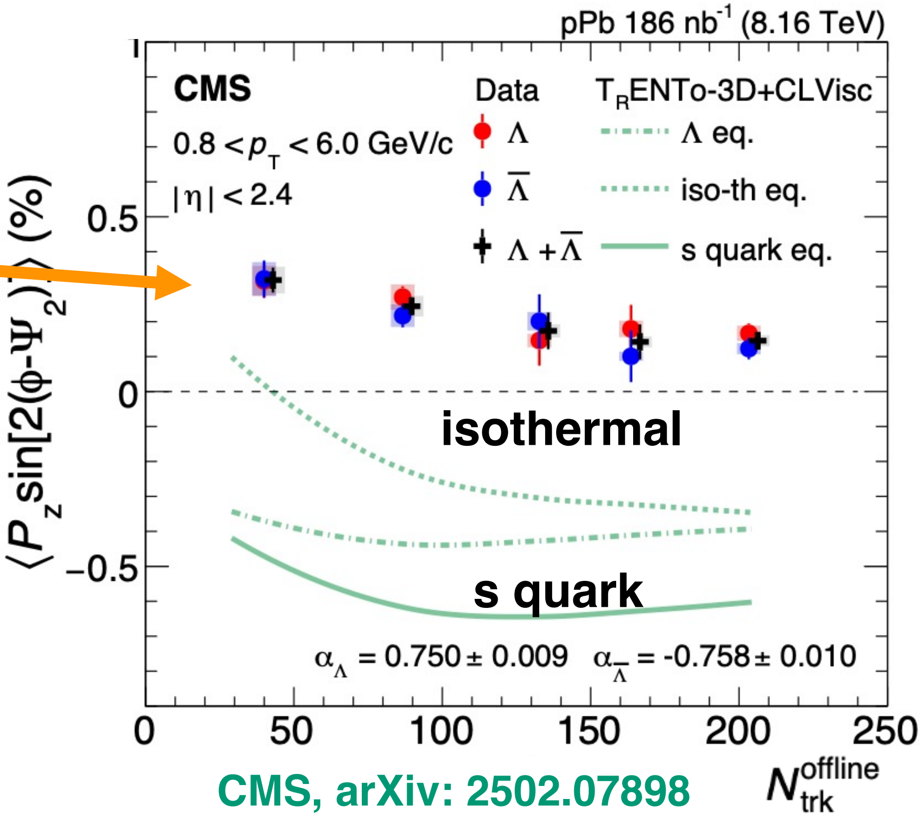
- **Energy and system dependence is not obvious.**
- **Hydrodynamics can describe the date at high energy collisions.**

Yi, et. al. (in preparation)
Also see:
Singh, Ryblewski, Florkowski, PRC (2025)

Puzzle: Local polarization in pPb systems



STAR, PRL 123, 132301 (2019)
ALICE, PRL 128, 172005 (2022)



CMS, arXiv: 2502.07898

Prediction: Yi, Wu, Zhu, SP, Qin, PRC(2025)

- **P_z in pA systems is closely aligned with that observed in AA systems.**
  Weak system dependence?
- **Hydrodynamic simulations across three scenarios fail to describe the data.**

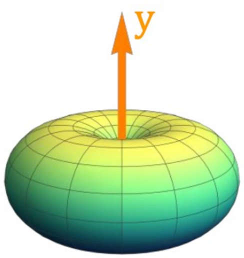
New spin polarization mechanism?

Spin alignment

Spin alignment of vector mesons

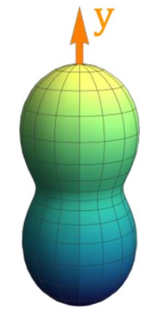
- **Spin density matrix**

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{1,0} & \rho_{1,-1} \\ \rho^*_{1,0} & \rho_{00} & \rho_{0,-1} \\ \rho^*_{1,-1} & \rho^*_{0,-1} & \rho_{-1,-1} \end{pmatrix}$$



$\rho_{00} < 1/3$

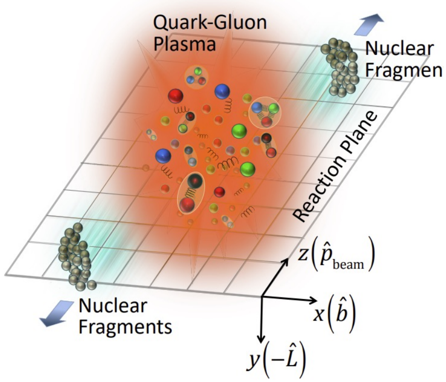
Transverse polarization

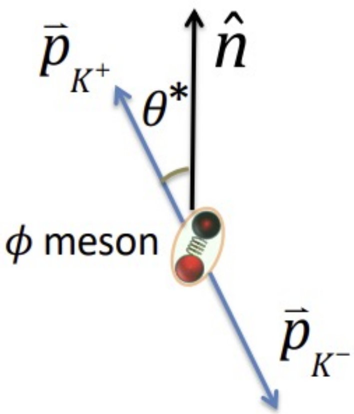


$\rho_{00} > 1/3$

Longitudinal polarization

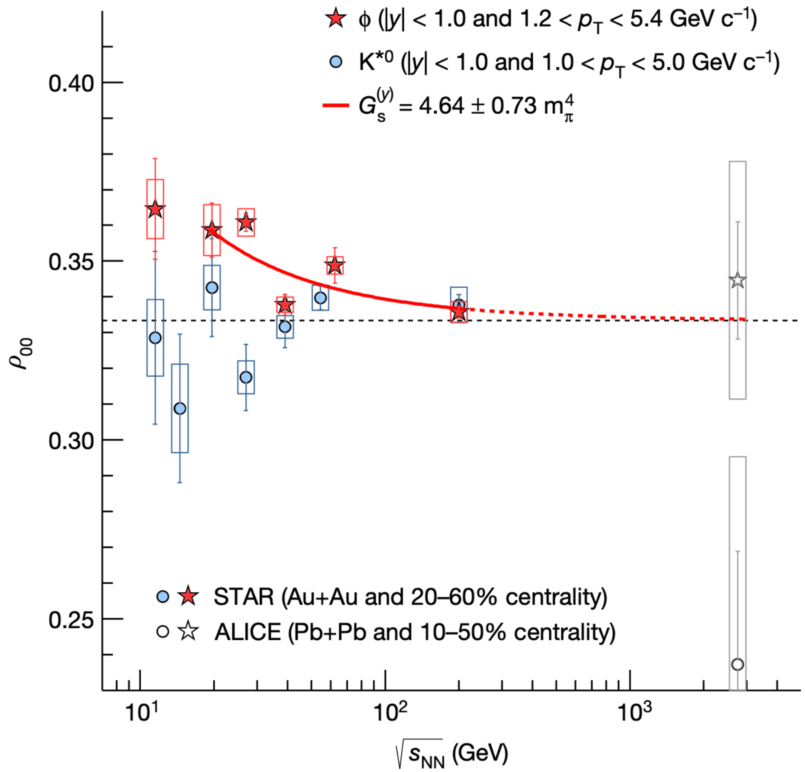
- **The cross section for the decay of spin-1 particles into two spin-0 particles is given by,**





$$\frac{dN}{(\cos\theta^*)} \propto (1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^*,$$

Spin alignment for ϕ meson



STAR, Nature 614, 244 (2023)

STAR has found a huge spin alignment of ϕ meson cannot be explained by vorticity effects.

$$\delta\rho_{00}(\omega) = \rho_{00}(\omega) - \frac{1}{3} < 0, \quad \sim 10^{-4}$$

Physics Mechanisms	(ρ_{00})
c_Λ: Quark coalescence vorticity & magnetic field[1]	< 1/3 (Negative ~ 10^{-5})
c_ε: Vorticity tensor[1]	< 1/3 (Negative ~ 10^{-4})
c_E: Electric field[2]	> 1/3 (Positive ~ 10^{-5})
Fragmentation[3]	> or, < 1/3 (~ 10^{-5})
Local spin alignment and helicity[4]	< 1/3
Turbulent color field[5]	< 1/3
c_ϕ: Vector meson strong force field[6]	> 1/3

[1] Liang, Wang, PLB (2005); Yang et al., PRC (2018); Xia et al., PLB (2021); Beccattini et al., PRC (2013)
[2] Sheng et al., PRC (2020); Yang et al., PRC (2018)
[3] Liang, Wang, PLB (2005);
[4] Xia et al., PLB (2021); Gao, PRD (2021)
[5] Muller, Yang, PRD (2022)
[6] Sheng et al., PRD (2020); PRD (2020)

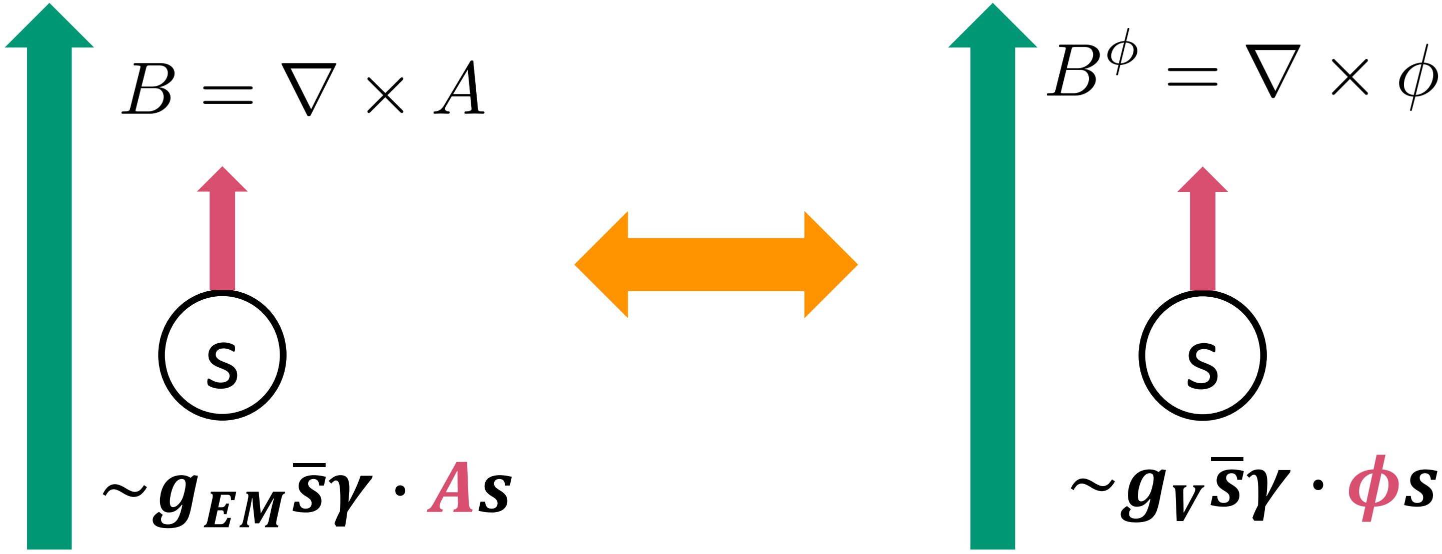
Vector meson strong force field

- **From chiral quark model, one can consider that $s, \bar{s}$ quarks are living in an effective strong force of ϕ fields,**

$$\mathcal{L} = \overline{\psi}(i\gamma \cdot \partial - \underline{\gamma \cdot g_v \phi_v} - M)\psi, \qquad \psi = (u, d, \underline{s})^T$$

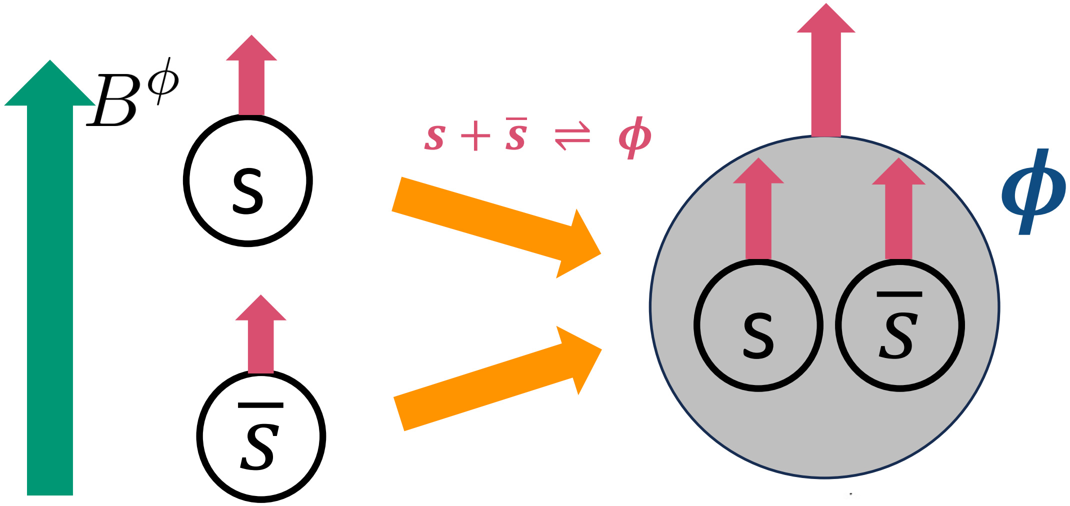
A. Manohar, H. Georgi, NPB (1984). **Csernai, Kapusta, Welle, PRC (2019)**

- **The coupling between $s, \bar{s}$ and ϕ fields is similar to EM coupling,**



$s, \bar{s}$ quarks spin polarization induced by vector meson fields.

Relativistic quark coalescence model



$$\rho_{00}(x,\mathbf{p}) = \frac{1}{3} - \frac{4g_\phi^2}{m_\phi^2 T_\mathrm{h}^2} C_1 \left[\frac{1}{3}\mathbf{B}'_\phi \cdot \mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0 \cdot \mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_\mathrm{h}^2} C_1 \left[\frac{1}{3}\mathbf{E}'_\phi \cdot \mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0 \cdot \mathbf{E}'_\phi)^2\right]$$

B'_ϕ, E'_ϕ: EM parts of the ϕ field in the meson's rest frame

After the space time average <…>, one assume that

$$< B_\phi > = < E_\phi > = 0$$

B_ϕ, E_ϕ: EM parts of the ϕ strong force field in the laboratory frame

Therefore, the vector meson strong force will NOT contribute to spin polarization of Λ and $\bar{\Lambda}$ hyperons. But, the fluctuations can survive

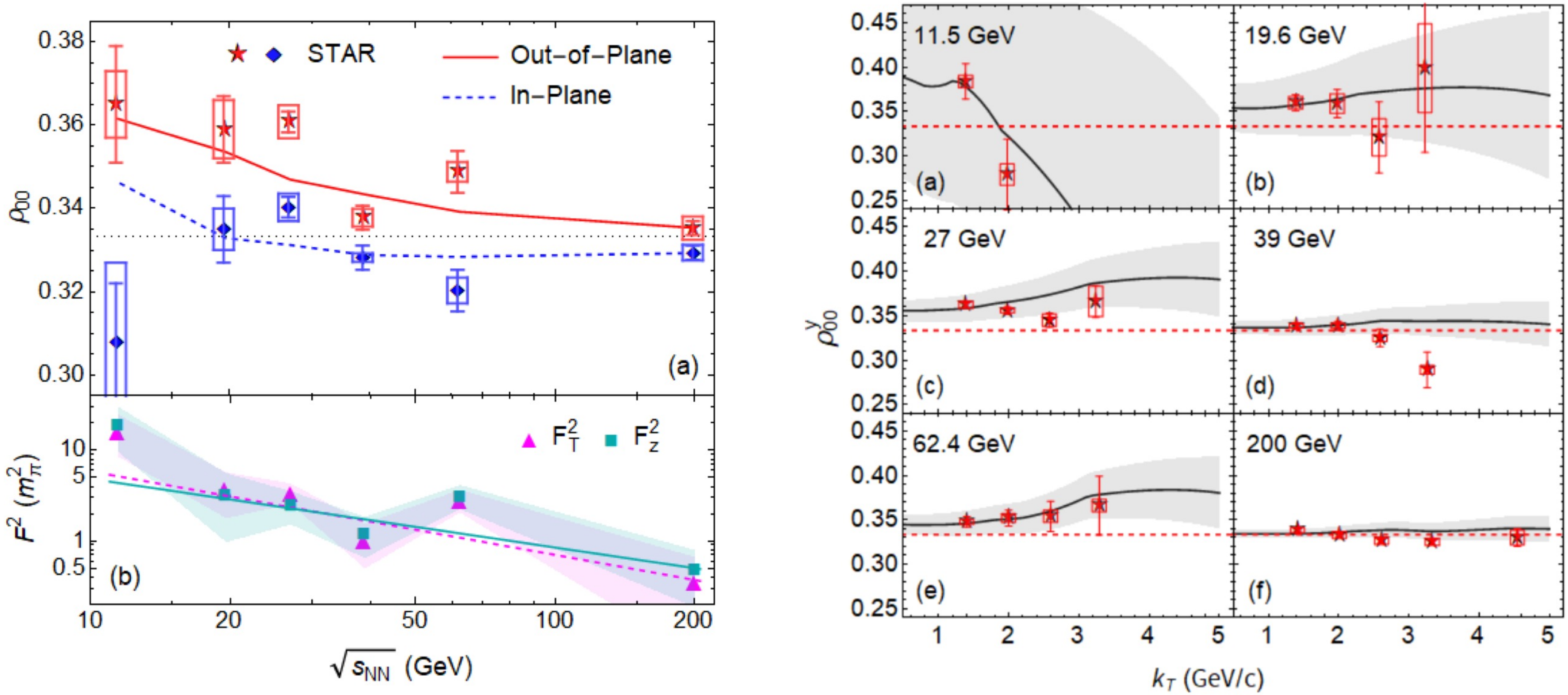
$$< B_\phi^2 >, < E_\phi^2 > \neq 0,$$

and will lead to the spin alignment of ϕ meson.

Sheng, Oliva, Liang, Wang, Wang, PRD (2022); PRL (2023); Sheng, SP, Wang, PRC (2023)

Global spin alignment for ϕ meson

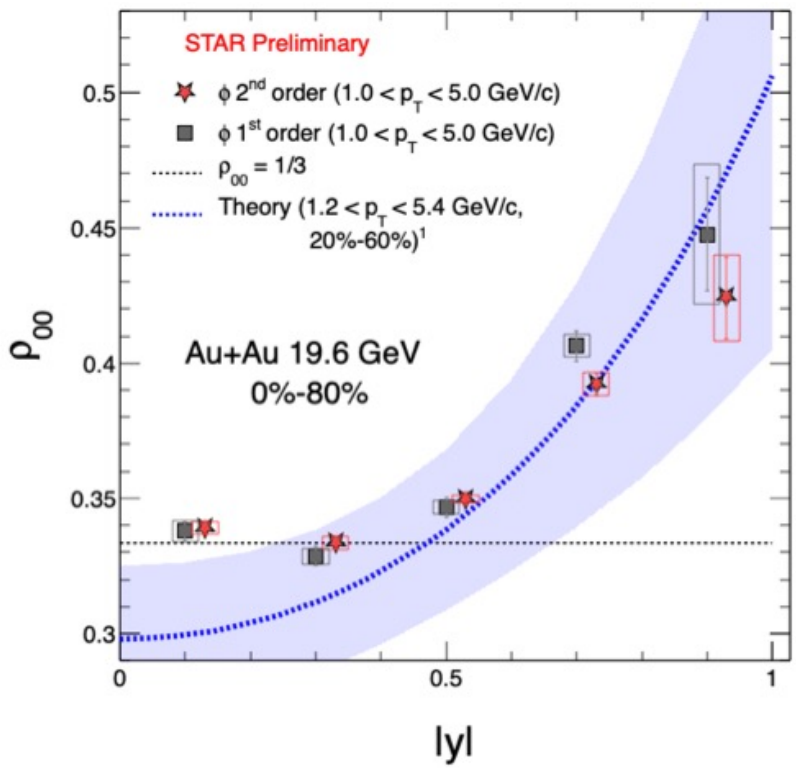
- The value of $< B_\phi^2 >, < E_\phi^2 >$ in QGP is unknown. By fitting them from global spin alignment for ϕ meson, it can describe or predict the local spin alignment.
- In heavy ion collisions, anisotropy comes from the beam direction (z-direction). Naturally, one can assume that transverse and longitudinal fluctuations are different.

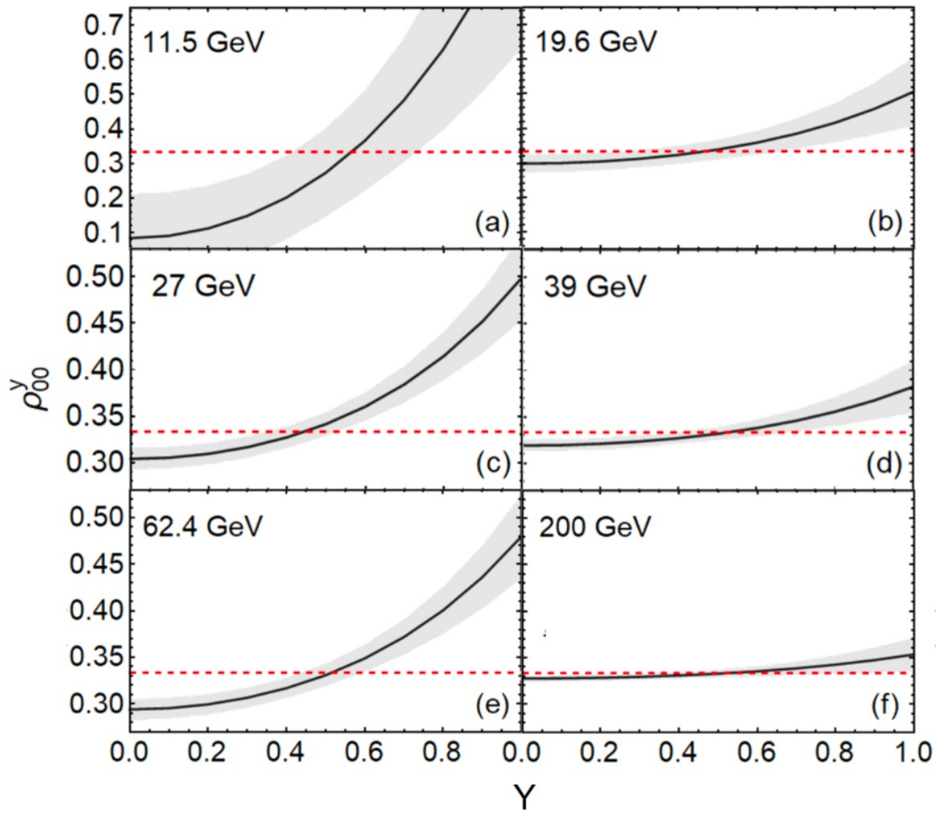


Sheng, Oliva, Liang, Wang, Wang, PRL (2023)

Rapidity dependence

With the same parameters chosen for global spin alignment, the predication for rapidity dependence agrees with the measurement.

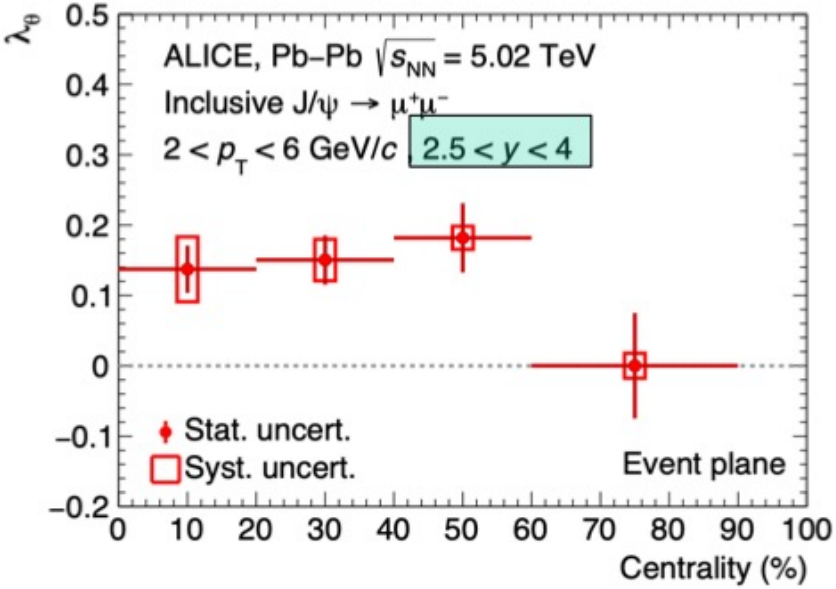


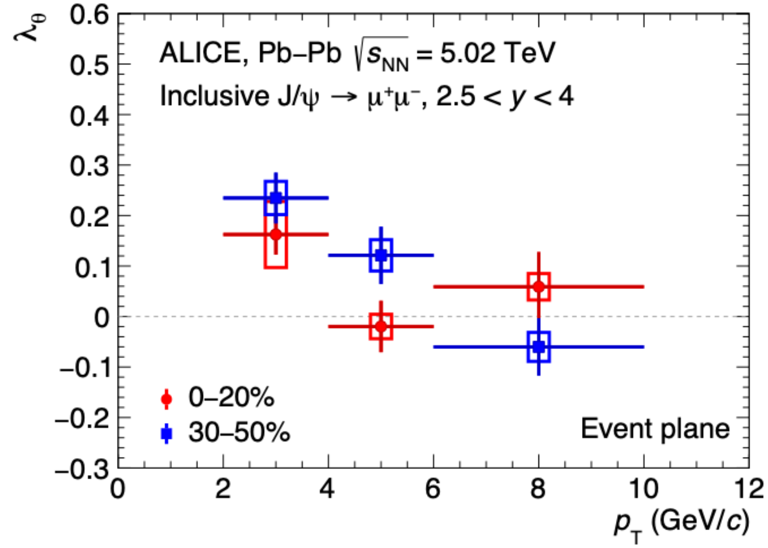


Ai-Hong Tang @ QM2023

Prediction: Sheng, Pu, Wang, PRC 2023

Spin alignment for J/Ψ





ALICE, PRL 131 042303 (2023)

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} \qquad \lambda_\theta > 0, \quad \rightarrow \boxed{\rho_{00}^{J/\Psi} < 1/3}$$

Faccioli, Lourenco, Seixas, Woehri, EPJC (2010)

So far, it seems that spin alignment for J/Ψ cannot be fully understood within the framework of effective strong force models due to the complexities in the behavior of heavy quarks.

Hyper spin correlations VS spin alignment

$$P_\Lambda \sim \langle P_s \rangle, \quad P_{\bar{\Lambda}} \sim \langle P_{\bar{s}} \rangle$$

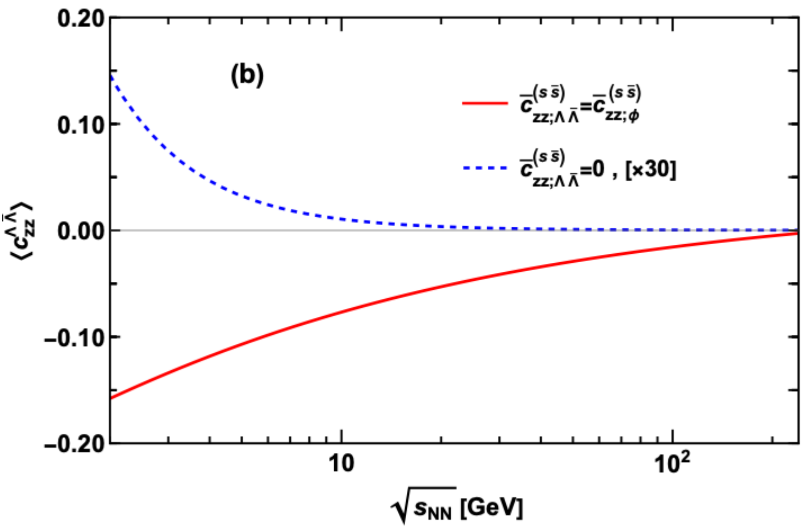
$$\rho_{00}^{\phi} - \frac{1}{3} \sim \langle P_s P_{\bar{s}} \rangle \neq \langle P_s \rangle \langle P_{\bar{s}} \rangle \sim P_\Lambda P_{\bar{\Lambda}}$$

$$\langle \rho_{00}^{\phi} \rangle \sim \frac{1 - \bar{c}_{zz;\phi}^{(s\bar{s})} - \cancel{\langle P_s \rangle^2}}{3 + \bar{c}_{zz;\phi}^{(s\bar{s})} + \cancel{\langle P_s \rangle^2}},$$

One can obtain the spin correlation of s and $\bar{s}$, $\bar{C}_{\phi}^{(s\bar{s})}$ from spin alignment of ϕ meson. Since $P_{\Lambda/\bar{\Lambda}} \sim P_{s/\bar{s}}$, one can further estimate the spin correlation of Λ and $\bar{\Lambda}$,

$$\langle c_{zz}^{\Lambda\bar{\Lambda}} \rangle \sim \bar{c}_{zz;\Lambda\bar{\Lambda}}^{(s\bar{s})} + \langle P_s \rangle^2,$$

$$c_{nn}^{H_1\bar{H}_2} = \frac{f_{++}^{H_1\bar{H}_2} + f_{--}^{H_1\bar{H}_2} - f_{+-}^{H_1\bar{H}_2} - f_{-+}^{H_1\bar{H}_2}}{f_{++}^{H_1\bar{H}_2} + f_{--}^{H_1\bar{H}_2} + f_{+-}^{H_1\bar{H}_2} + f_{-+}^{H_1\bar{H}_2}},$$



Lv, Yu, Liang, Wang, Wang PRD (2024)
Sheng, Wu, Wang, (in preparation)

Also see the spin correlation estimated by vorticity effects:
Pang, Petersen, Wang, Wang, PRL (2016)

Recent development

Non-local interactions for spin evolution

Boltzmann equation for spin-1/2 particles with nonlocal collision term

$$k \cdot \partial f(x, k, \mathfrak{s}) = \mathfrak{C}[f]$$

Massive fermions

$$\mathfrak{C}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' d\bar{S}(k)\, \mathcal{W}_{\mathbf{kk}' \to \mathbf{k}_1\mathbf{k}_2}^{\bar{\mathfrak{s}}\mathfrak{s}' \to \mathfrak{s}_1\mathfrak{s}_2} (2\pi\hbar)^4 \delta^{(4)}(k + k' - k_1 - k_2)$$
$$\times \left[f(x + \Delta_1 - \Delta, k_1, \mathfrak{s}_1) f(x + \Delta_2 - \Delta, k_2, \mathfrak{s}_2) - f(x, k, \bar{\mathfrak{s}}) f(x + \Delta' - \Delta, k', \mathfrak{s}') \right]$$

Nonlocal position shift corresponds to the side jump in condensed matter.
Weickgenannt, Speranza, Sheng, Wang, Rischke, PRL (2021); PRD (2021); PRD (2019); Wagner, Weickgenannt, Rischke, PRD (2022);
Also see: the relevant works from different approaches, by D.L. Yang, S. Fang, SP, X.G. Huang, P.F. Zhuang, S. Lin, K. Hattori, Z.Y. Wang, H. Yee, …

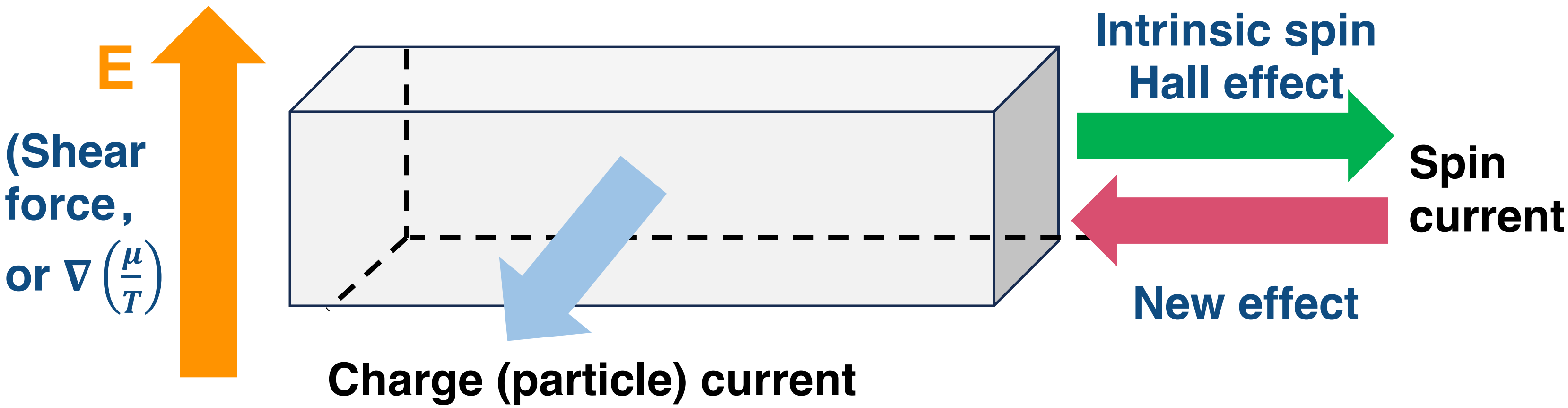
It can lead to novel spin polarization induced by shear tensor.

$$S^\mu(p) \sim -\int d\Sigma \cdot p \frac{f_{0p}}{2\mathcal{N}} \chi_q \tau_q h_\tau^{(2)} \boxed{\sigma_\rho^{\langle\alpha}} \epsilon^{\beta\rangle\nu\sigma\rho} u_\sigma p_{\langle\alpha} p_{\beta\rangle}$$

Shear tensor

Such kinds of corrections come from interactions but does not depend on coupling constant. Weickgenannt, Wagner, Speranza, Rischke PRD (2022); but it depends on the type of interaction (scalar, vector, etc.), Sapna, Singh, Wagner, arXiv: 2503.22552

Novel spin polarization effects from side jump



By replacing the electric force with shear force or gradient of baryon chemical potential, our results align consistently with findings in condensed matter physics.
Valet, Raimondi, PRB Lett. (2024)

$$\delta\mathcal{S}^{\mu}_{(\mathrm{I}),\mathrm{shear}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 \boxed{g_2(E_{\mathbf{p}})} \epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \sigma_{\nu\alpha}p^\alpha,$$

$$\delta\mathcal{S}^{\mu}_{(\mathrm{I}),\mathrm{chem}} = -\frac{\hbar^2}{4N}\int_{\Sigma}\frac{\mathrm{d}\Sigma\cdot p}{E_{\mathbf{p}}}\beta_0 \boxed{g_1(E_{\mathbf{p}})} \epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma \nabla_\nu\left(\frac{\mu}{T}\right).$$

g1 and g2 come from scatterings but do NOT depend on coupling constant explicitly. They correspond to anomalous (spin) Hall effects in condensed matter.
Fang, SP, PRD (2024) for QED type scattering in Hard-Thermal-Loop approximation.

Can we observe the anomalous (spin) Hall effect in HIC?

Spin evolution equation

- **Bargmann-Michel-Telegdi (BMT) equation**

$$\frac{ds^\mu}{d\tau} = \gamma(F^{\mu\nu}s_\nu + u^\mu s_\rho F^{\rho\sigma}u_\sigma) - u^\mu s^\alpha \frac{du_\alpha}{d\tau}$$

Spin EM fields coupling Thomas procession

- **We derive the extended BMT equation for spin density in a rotating system from entropy principle:**

$$\begin{aligned}\frac{ds_\alpha}{d\tau} = & -u_\alpha s^\beta \frac{du_\beta}{d\tau} \\ & +(\varepsilon_{\alpha\beta\rho\sigma}s^\beta u^\rho + 2\beta\gamma g_{\alpha\sigma})(\Omega^\sigma + 2\omega^\sigma) \\ & -s^\beta \partial_{<\alpha}u_{\beta>} - \left(\frac{1}{3} + 2\chi_e\right) s_\alpha(\partial \cdot u).\end{aligned}$$

Thomas procession

EM fields -> vorticity

Coupling to dissipative effects

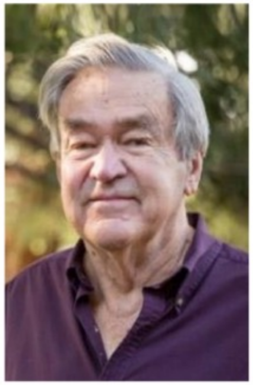
Fang, Fukushima, Pu, Wang, in preparation

Summary and outlook

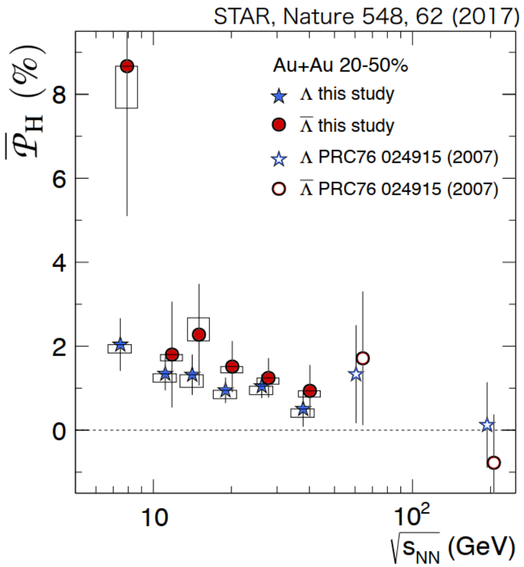
Summary

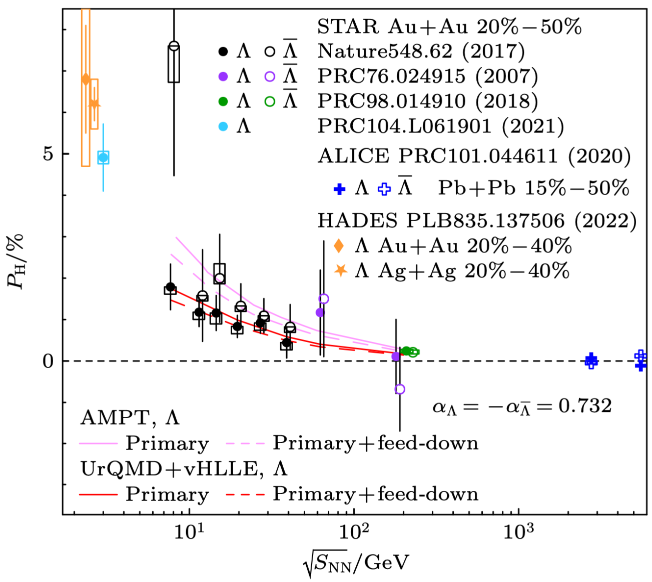
Polarization data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection.

----- J.D. Bjorken



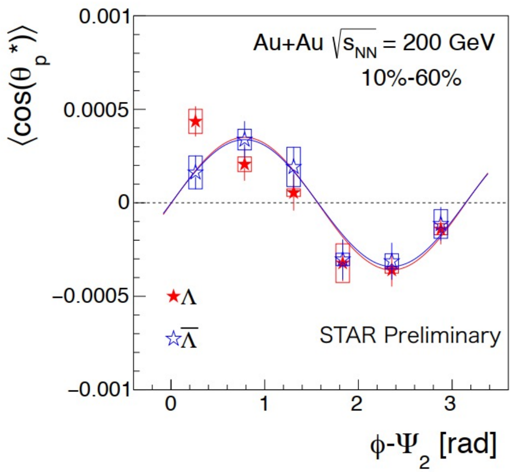
- **In the field of spin polarization in HIC, many theoretical predictions have been confirmed by the experiments.**

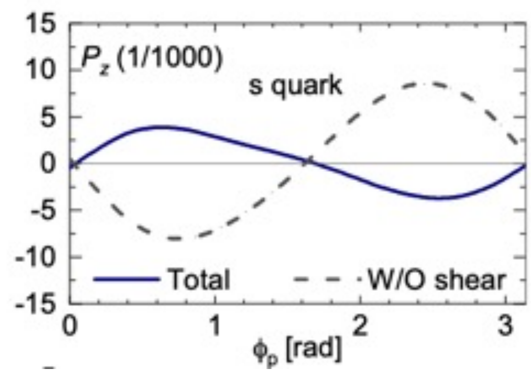


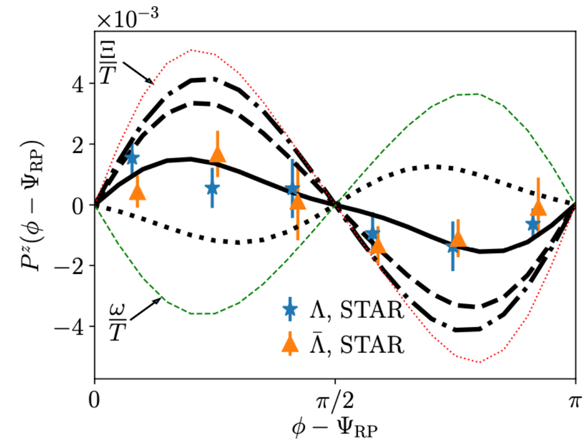


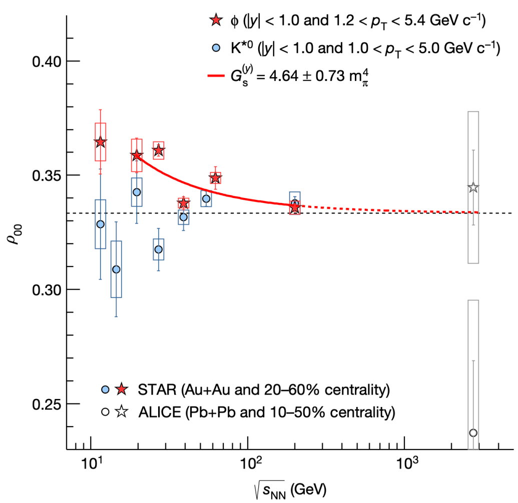
Summary

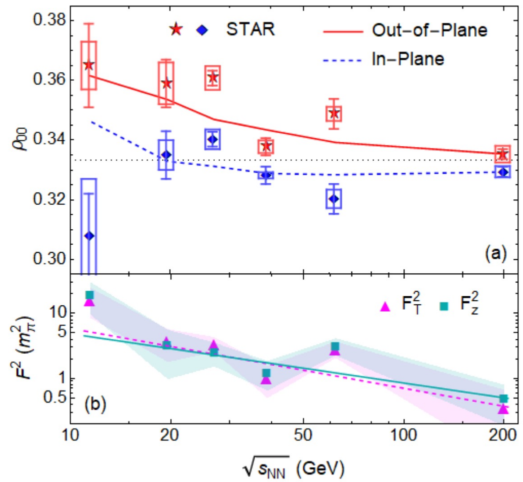
- **Some measurements present puzzles and challenge our understanding, but they ultimately lead us to a deeper comprehension of physics.**

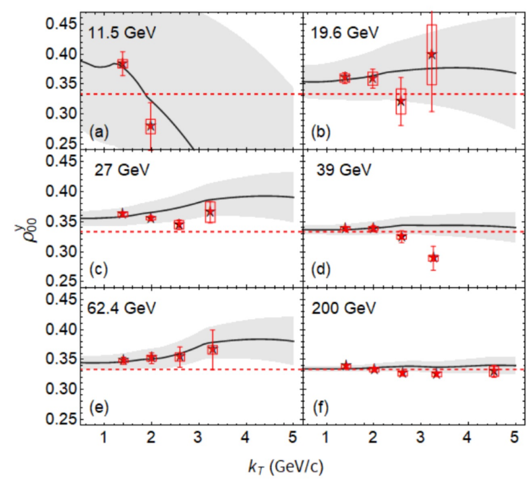






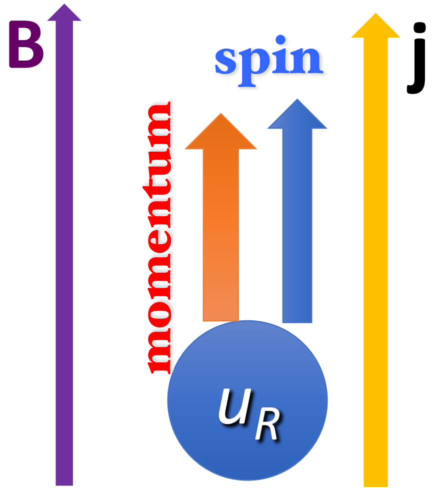


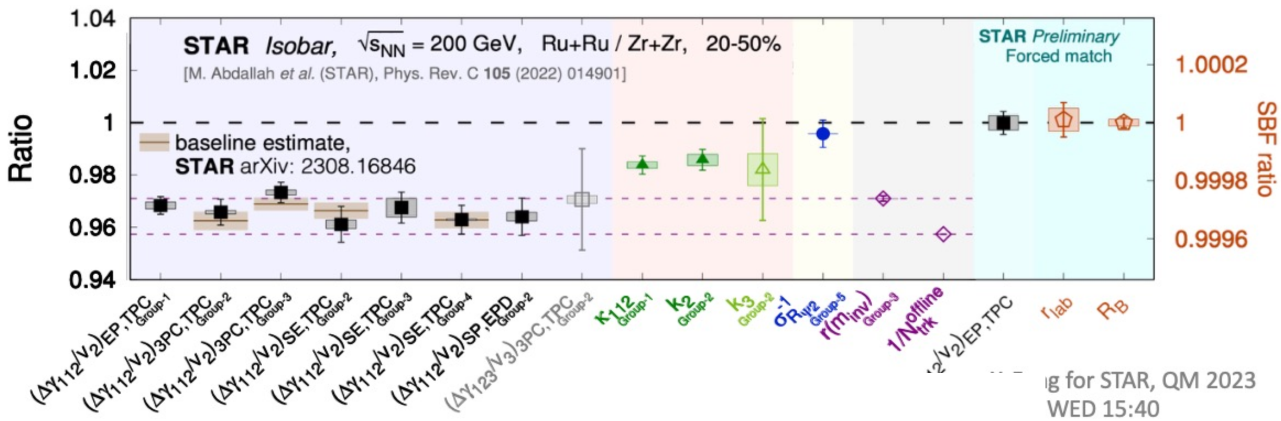


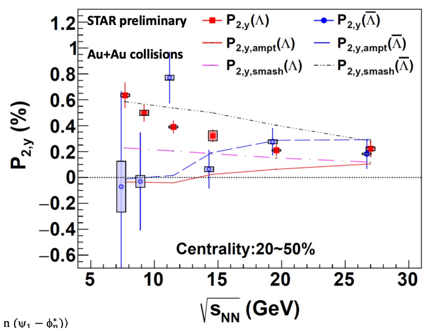


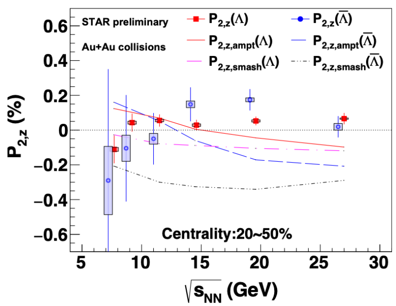
Summary

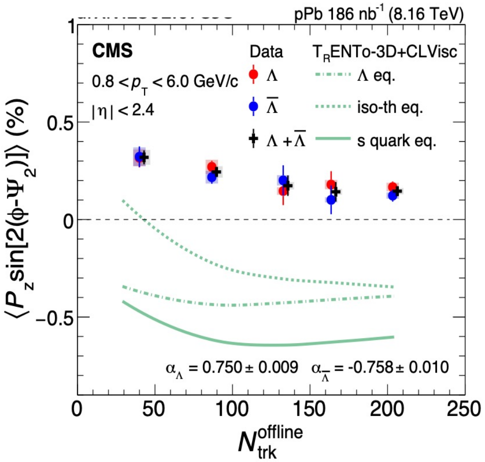
- The deeper we delve, the more puzzles arise.

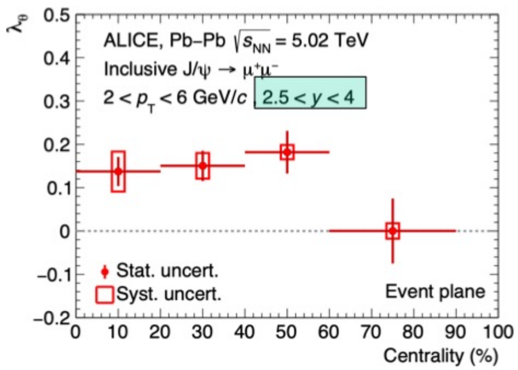












More exciting developments in physics await us.

Thank you for your time!

Any comments and suggestions are welcome!

Backup

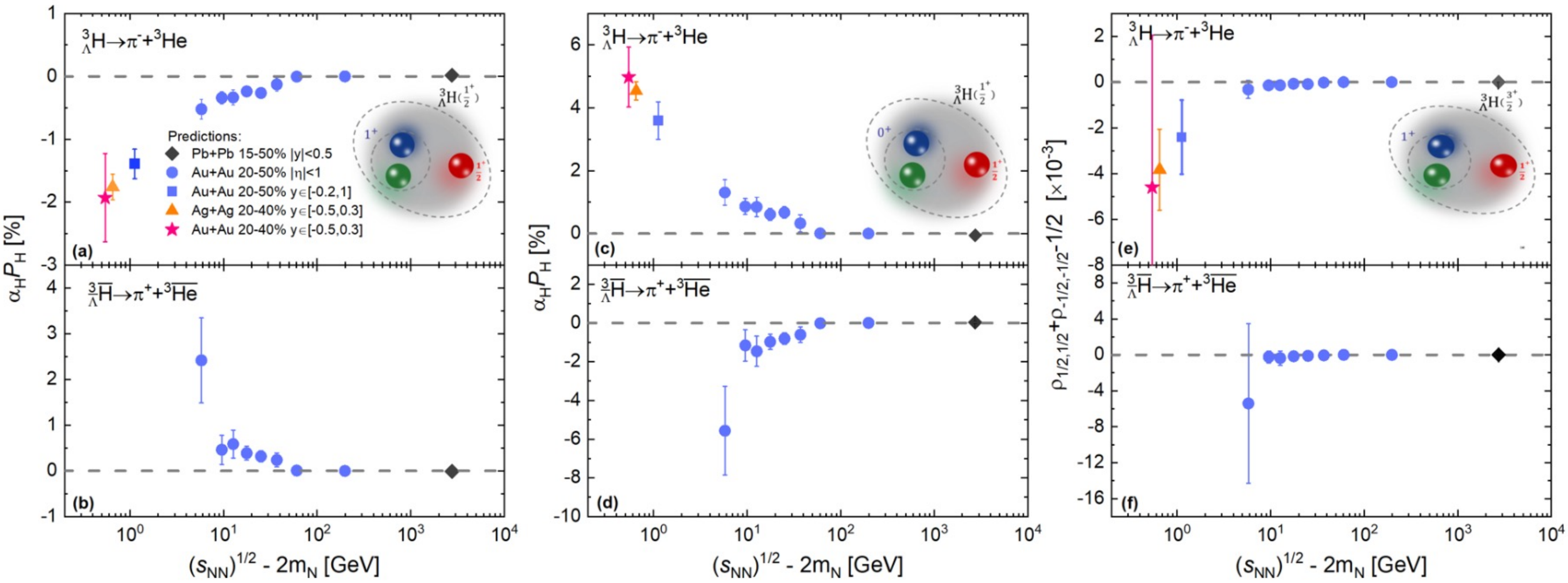
Global polarization

Global polarization for ${}^{3}_{\Lambda}\mathrm{H}$

The measurement of hypertriton polarization provides a novel method to uniquely determine its internal spin structure.

$\alpha_{{}^{3}_{\Lambda}\mathrm{H}} \approx -\frac{1}{2.58}\alpha_{\Lambda}$

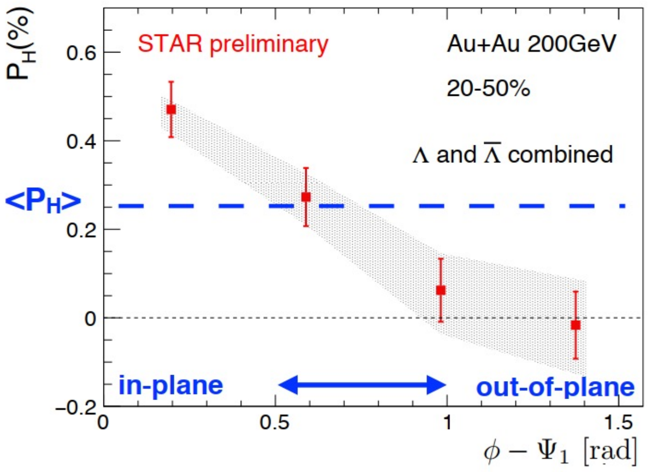
$\alpha_{{}^{3}_{\Lambda}\mathrm{H}} \approx \alpha_{\Lambda}$



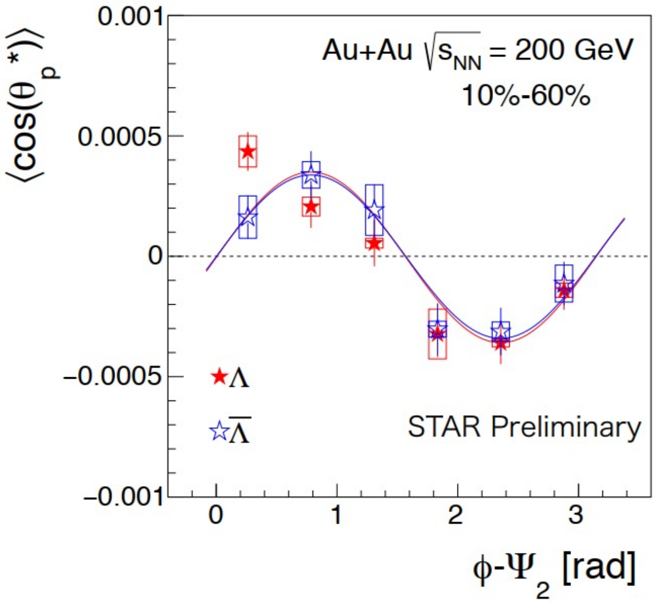
Prediction from Fudan's group: Sun, Liu, Zhen, Chen, Ko, Ma, PRL (2024)

Local polarization

$P_{2,y}$ and $P_{2,z}$ across BES

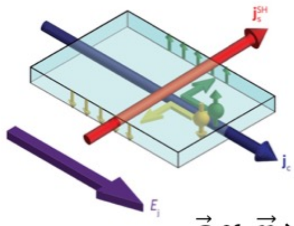


$$P_{2,y} \equiv \langle P_y \cos[2(\phi_\Lambda - \Psi_2)] \rangle$$



$$P_{2,z} \equiv \langle P_z \sin[2(\phi_\Lambda - \Psi_2)] \rangle$$

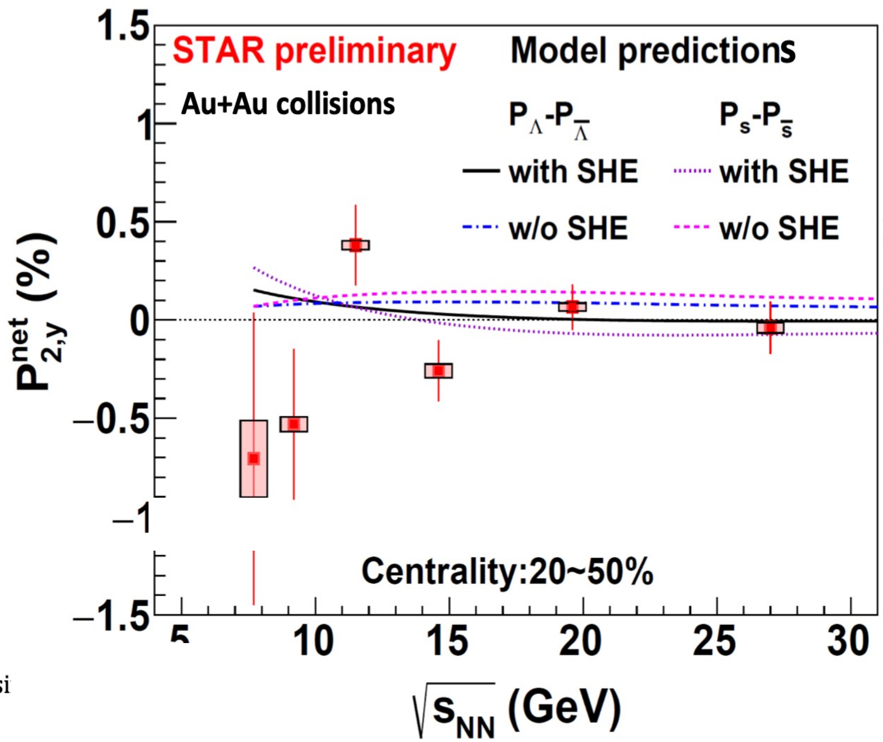
Local polarization and spin Hall effect

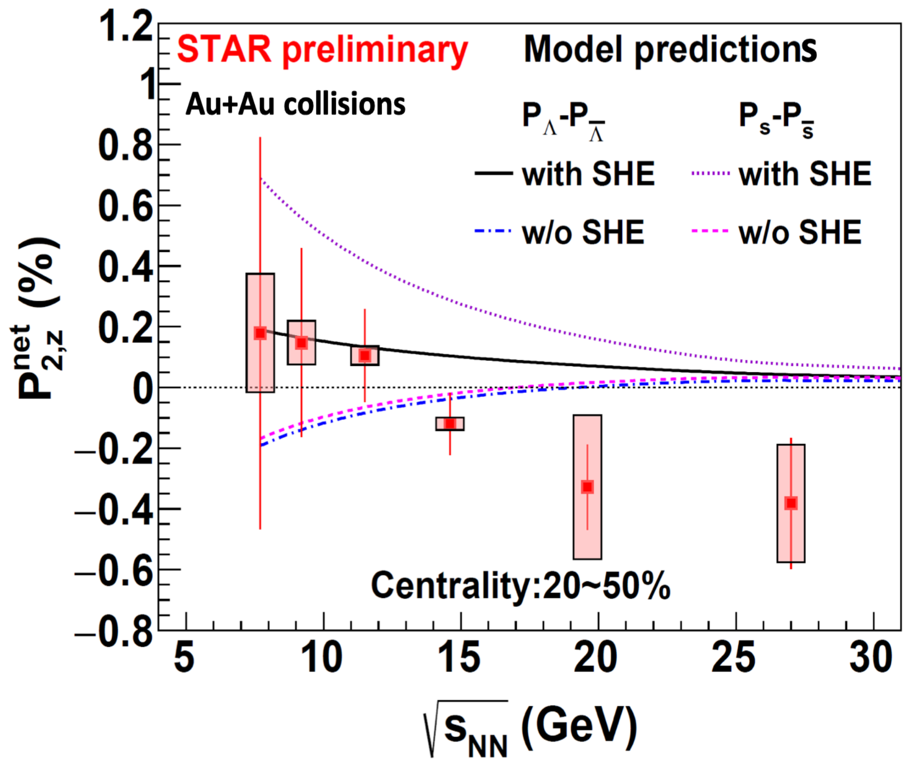


$\vec{s} \propto \vec{p} \times \vec{E}$

$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma \cdot p f_V^{(0)}(1-f_V^{(0)})\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{\partial_{\nu}\frac{\mu}{T}},$$

Similar to spin Hall effect in condensed matter, gradient of baryon chemical potential can also induce polarization.





Figures: Q. Hu @ SQM2024

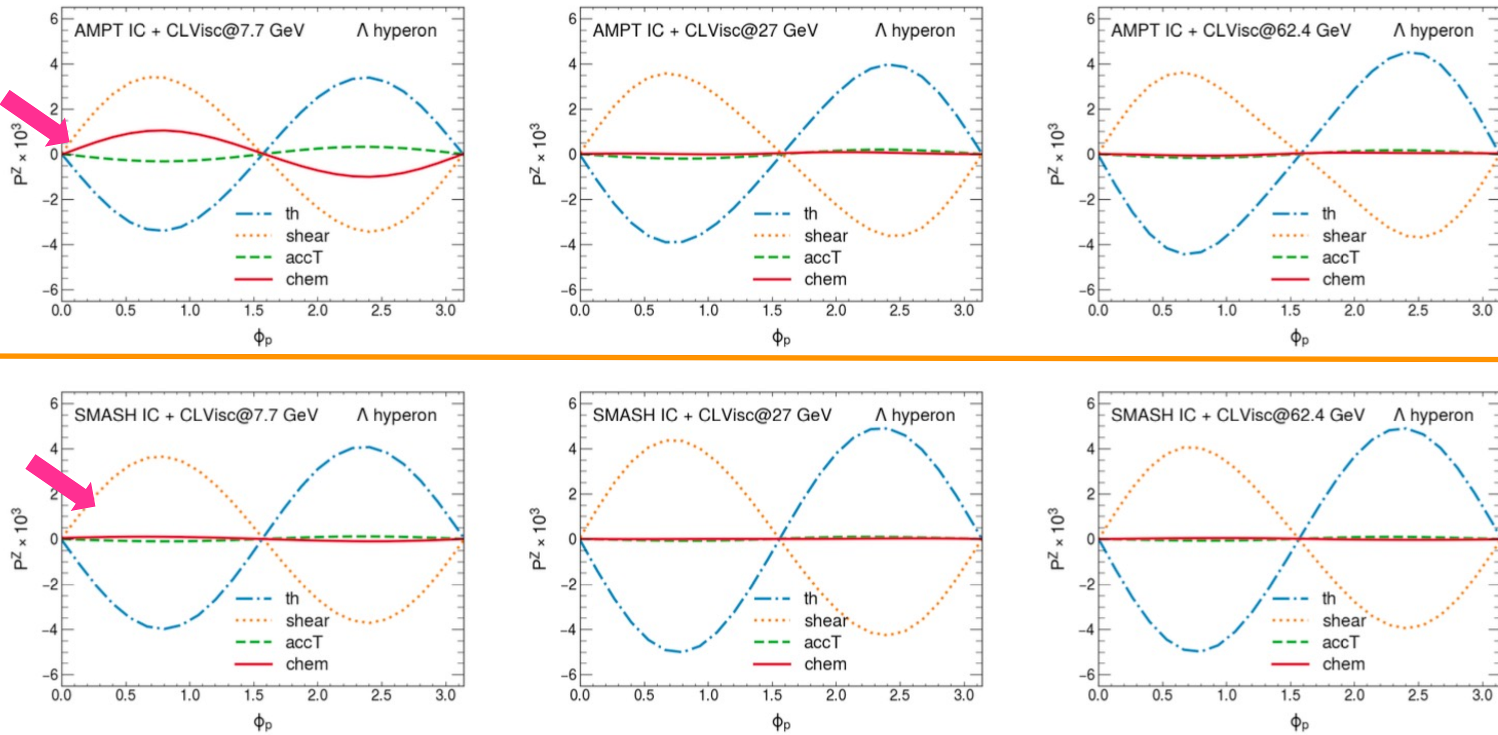
Prediction: Fu, Pang, Song, Yin, 2208.00430

Local polarization and spin Hall effect: SMASH VS AMPT

7.7 GeV | 27 GeV | 62.4 GeV

From AMPT Initial condition

From SMASH Initial condition



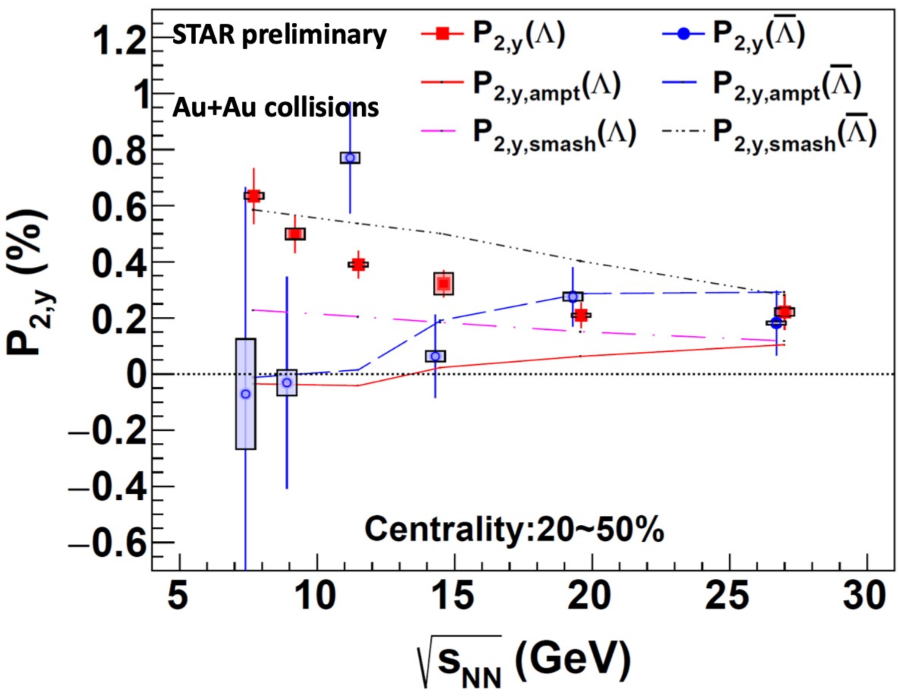
Red lines: contributions from spin Hall effect

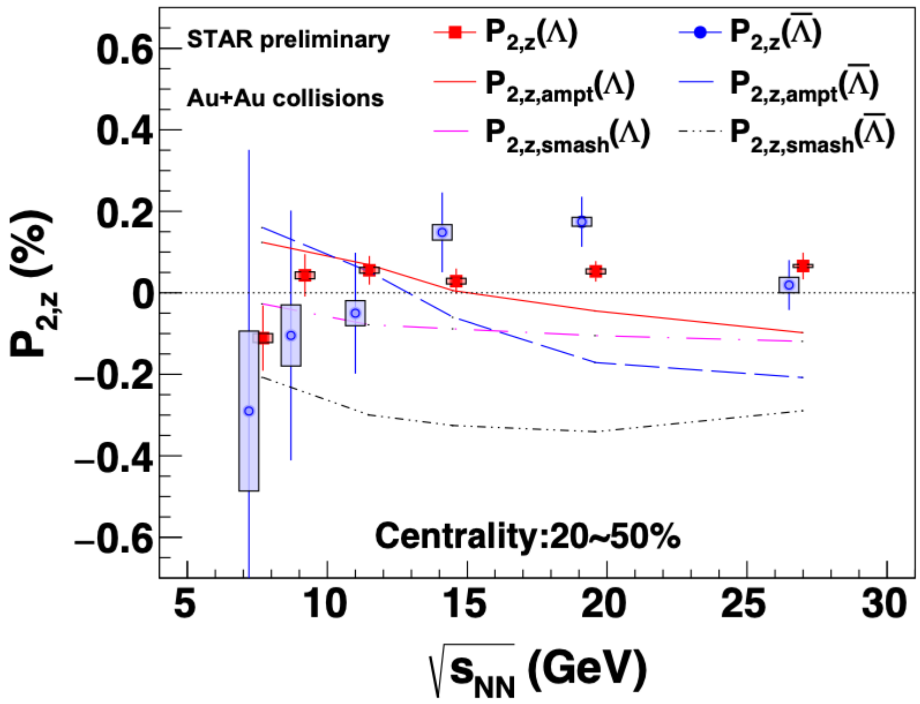
Polarization induced by SHE is almost zero at 27, 62.4GeV and it depends on the initial conditions. For SMASH, Pz is still almost vanishing at 7.7 GeV.

Wu, Yi, Qin, SP, PRC (2022)

Local polarization splitting

How can we understand the data in low energy collisions?

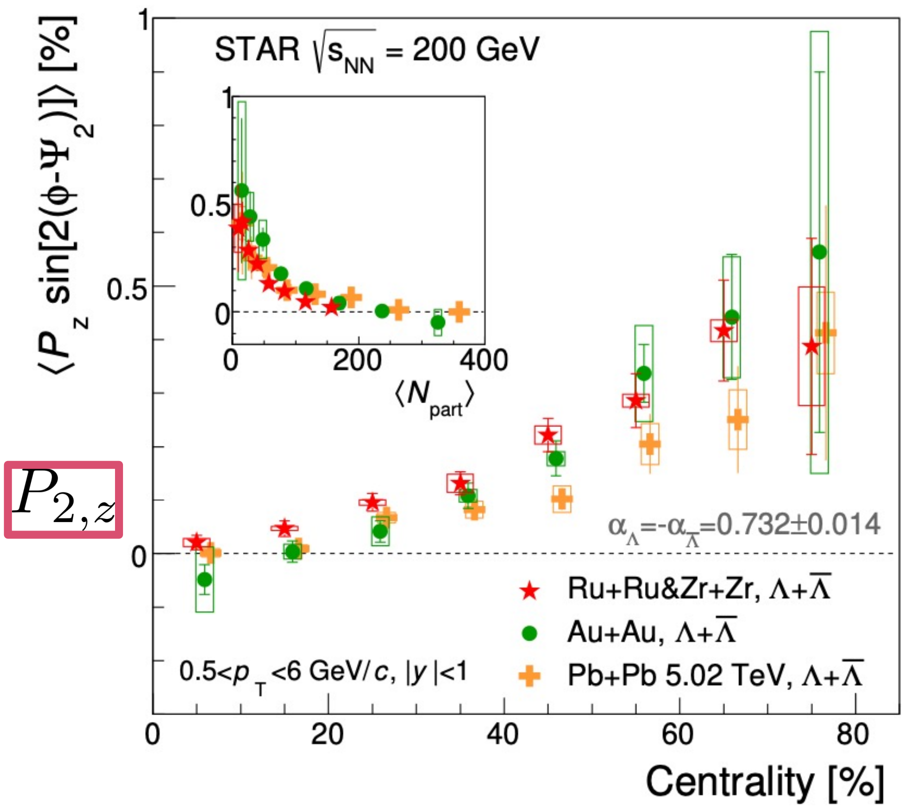


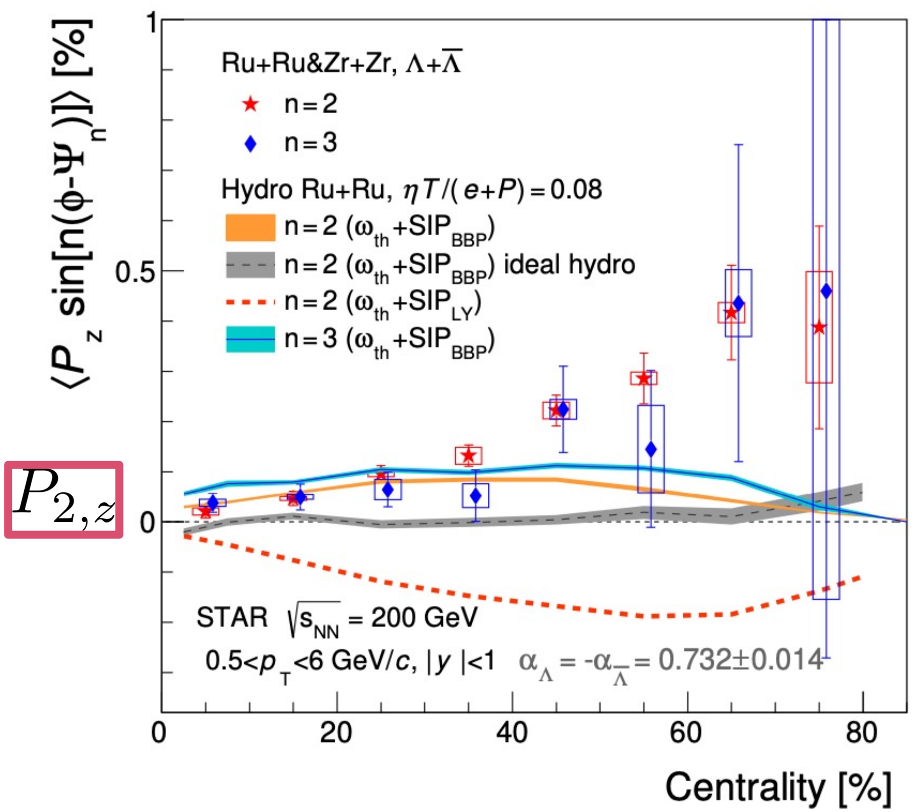


Figures: Q. Hu @ SQM2024

Model Predition: Wu, Yi, Qin, SP, PRC (2022)

Local polarization VS centrality





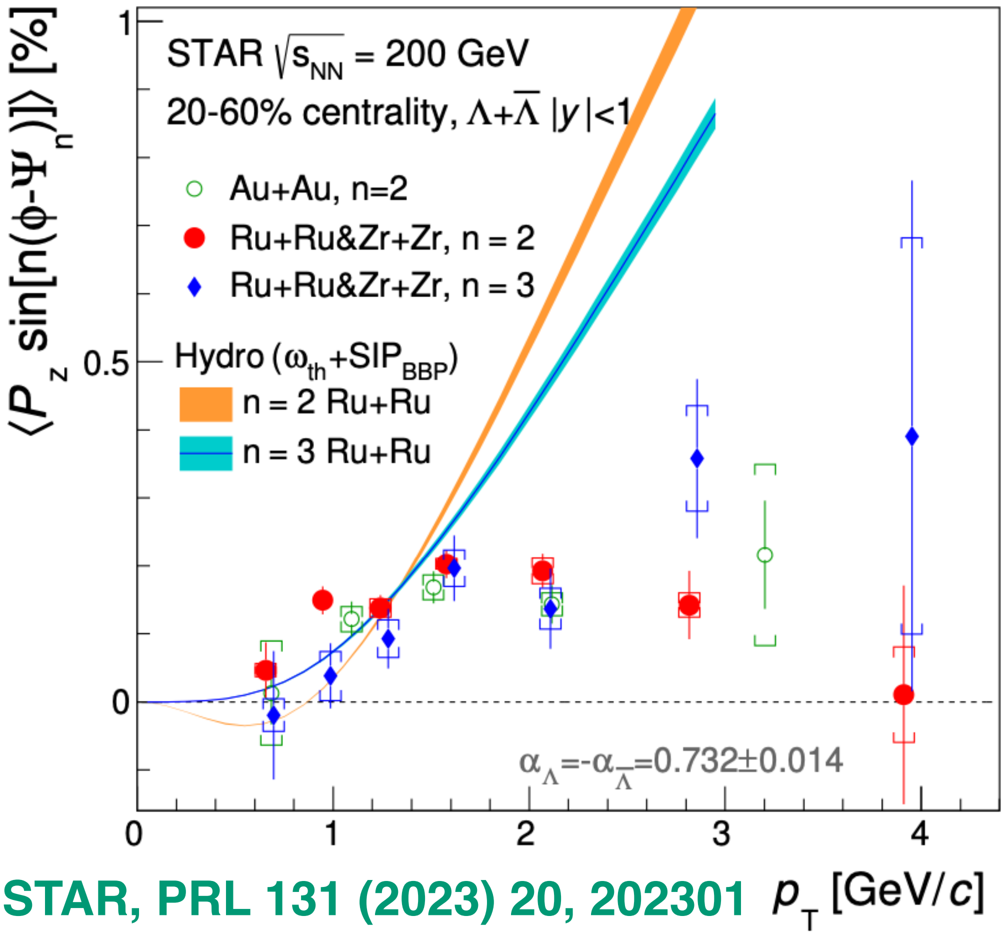
STAR, PRL 123, 132301 (2019)
ALICE, PRL 128, 172005 (2022)

STAR, PRL 131 (2023) 20, 202301

Energy dependence is not obvious.

Model Calculation:
BBP: isothermal equilibrium; LY: s quark equilibrium
Simulation: Alzhrani, Ryu, Shen, PRC 106, 014905 (2022)

Local polarization VS p_T



STAR, PRL 131 (2023) 20, 202301

Similar question arises:

How large will the local polarization become when p_T is infinte?

$$\mathcal{S}^{\mu}_{\rm thermal}(\mathbf{p}) = \frac{\hbar}{8m_{\Lambda}N}\int d\Sigma^{\sigma}p_{\sigma}f_V^{(0)}(1-f_V^{(0)})\epsilon^{\mu\nu\alpha\beta}p_{\nu}\partial_{\alpha}\frac{u_{\beta}}{T},$$

Reason:
For a given thermal vorticity, there is no suppressed factor proportional to p_T in denominator.

Model Calculation:
BBP: isothermal equilibrium; LY: s quark equilibrium
Simulation: Alzhrani, Ryu, Shen, PRC 106, 014905 (2022)

Setup for pA collisions

- **We implement the 3+1D CLVisc hydrodynamics model**
  Pang, Wang, Wang, PRC (2012); Wu, Qin, Pang, Wang, PRC (2022)
- **Initial condition: TRENTo-3D model**
  Soeder, Ke, Paquet, Bass, 2306.08665; Moreland, Bernhard, Bass, PRC (2015); PRC (2020);Ke, Moreland, Bernhard, Bass, PRC (2017)
- **p+Pb collisions at $\sqrt{s_{NN}} = 8.16$ TeV**
- **We follow the modified Cooper-Frye formula to compute the polarization pseudo-vector including the contribution from thermal vorticity and thermal shear tensor.**

$$\mathcal{S}^\mu(\mathbf{p}) \quad = \quad \mathcal{S}^\mu_{\text{thermal}}(\mathbf{p}) + \mathcal{S}^\mu_{\text{th-shear}}(\mathbf{p})$$

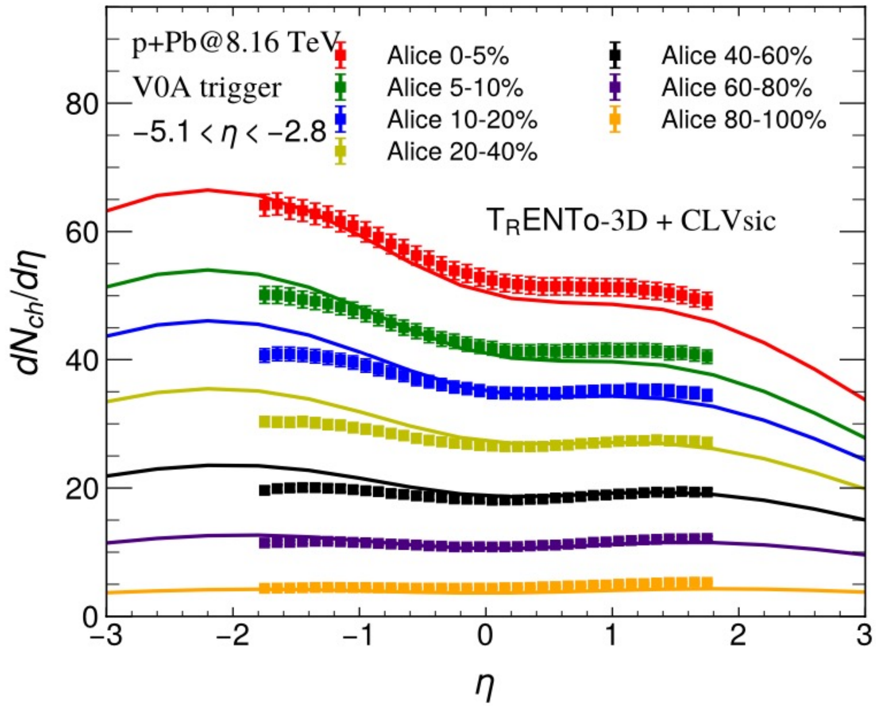
thermal vorticity

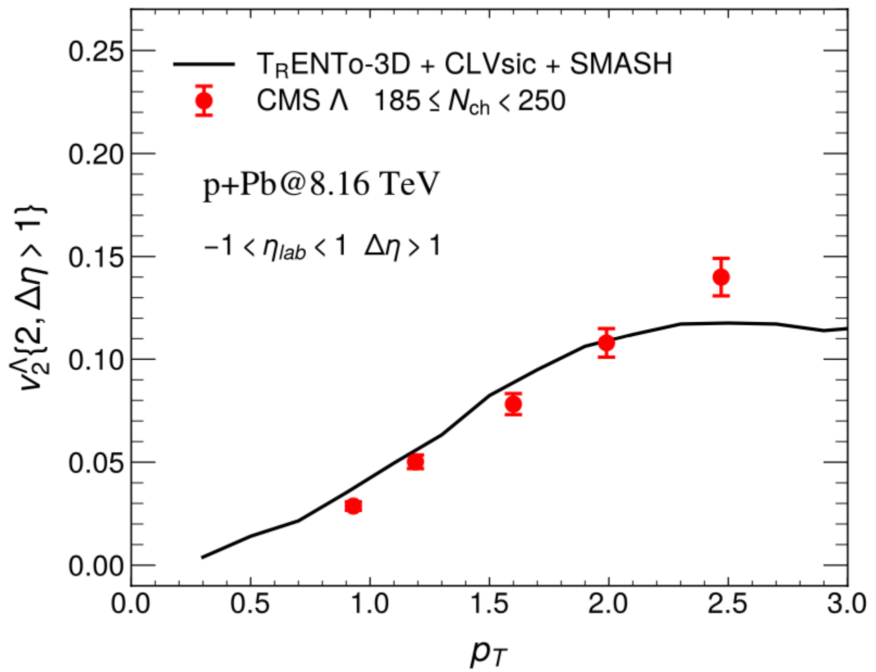
$$\mathcal{S}^\mu_{\text{thermal}}(\mathbf{p}) \quad = \quad \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \varpi_{\alpha\beta},$$

thermal shear tensor

$$\mathcal{S}^\mu_{\text{th-shear}}(\mathbf{p}) \quad = \quad \hbar \int d\Sigma \cdot \mathcal{N}_p \frac{\epsilon^{\mu\nu\alpha\beta} p_\nu n_\beta}{(n \cdot p)} p^\sigma \xi_{\sigma\alpha}$$

Fit parameters and test v2 of Λ

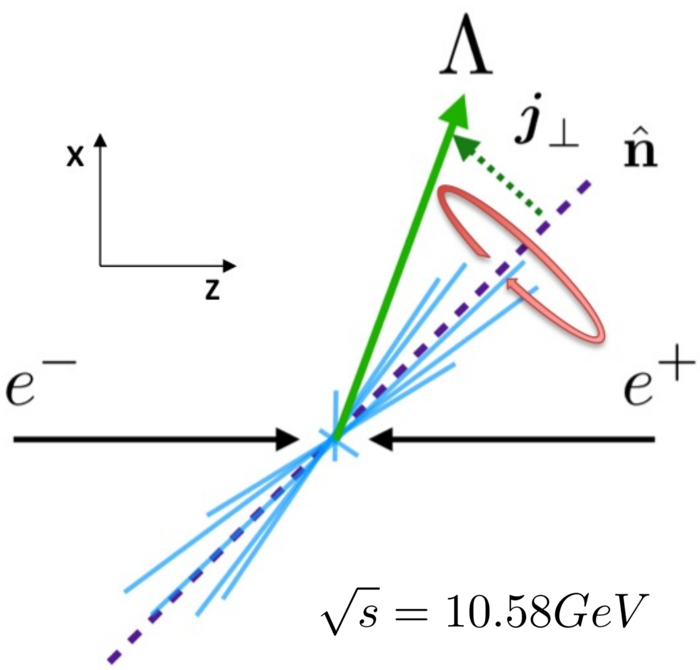




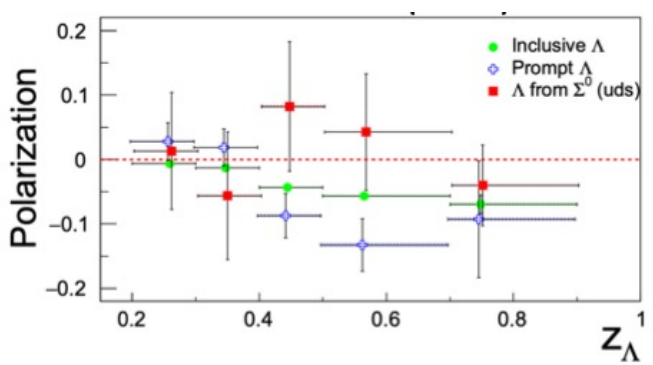
Multiplicity intervals	$\langle N_{\rm ch}\rangle_{\rm exp}$	$\langle N_{\rm ch}\rangle_{\rm CLVisc}$
[185,250)	203.3	204.2
[150,185)	163.6	164.5
[120,150)	132.7	133.57
[60,120)	86.7	87.7
[3,60)	40	29.3

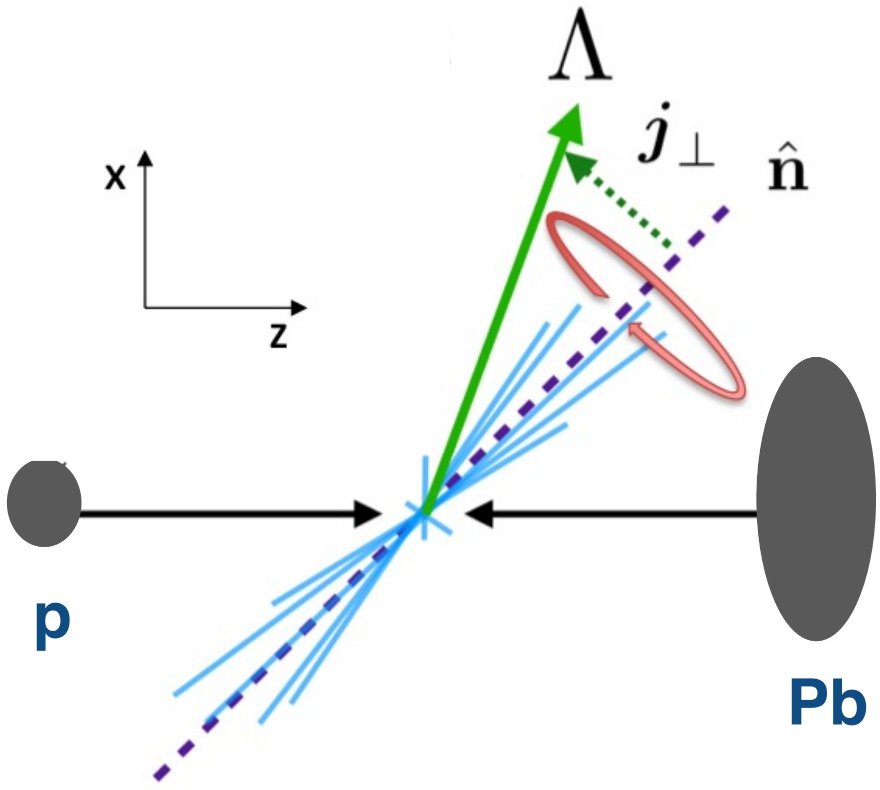
We have run 10^5 minimum bias events to divide the centrality. The centrality-dependent pseudo-rapidity distributions of charged hadrons and elliptic flow for Λ hyperons computed by our model are consistent with the experimental measurements.

Transverse hyperon polarization in jet ?



Belle, PRL 122, 042001 (2019)





Spin alignment

Original idea in pioneer works

Assuming that the spin of a vector meson originates from its two constituent quarks. The global polarization aligns with the y-direction (the direction of initial angular momentum), then we have

$$\rho_{00} = \frac{1 + P_x^q P_x^{\bar{q}} - P_y^q P_y^{\bar{q}} + P_z^q P_z^{\bar{q}}}{3 + \mathbf{P}^q \cdot \mathbf{P}^{\bar{q}}} \approx \frac{1 - P_y^q P_y^{\bar{q}}}{3 + P_y^q P_y^{\bar{q}}} \approx \frac{1}{3} - \frac{4}{9} P_y^q P_y^{\bar{q}}$$

Since the spin of Λ or $\overline{\Lambda}$ arises from s or $\bar{s}$, respectively, then

$$\delta\rho_{00}^{\phi} \equiv \rho_{00}^{\phi} - \frac{1}{3} \simeq -\frac{4}{9}(P_s^y)^2 < 0, \qquad |\delta\rho_{00}^{\phi}| \sim 10^{-4}$$

Liang, Wang, PLB (2005); Xia, Li, Huang, Huang, PLB (2021)

Large global spin alignment is NOT induced by the vorticity effects!

Green function for spin-1 particles

- **From the Lagrangian of spin-1 particles,**

$$\mathcal{L} = -\frac{1}{4}F_V^2 + \frac{1}{2}m_V^2 A_V^2 - A_V \cdot j,$$

one can define the two point Green functions

$$G^{\mu\nu}(x_1, x_2) = \langle T_C A_V^{\mu}(x_1) A_V^{\nu\dagger}(x_2)\rangle = \begin{pmatrix} G^{++} & G^{+-} \\ G^{-+} & G^{--} \end{pmatrix} = \begin{pmatrix} G^{F} & G^{<} \\ G^{>} & G^{\overline{F}} \end{pmatrix}.$$

- **We also know that**

$$A_V^{\mu}(x) = \sum_{\lambda=0,\pm1} \int \frac{d^3k}{(2\pi\hbar)^3}\frac{1}{2E_k^V}\left[\underline{\epsilon^{\mu}(\lambda,\mathbf{k})} a_V(\lambda,\mathbf{k}) e^{-ik\cdot x/\hbar} + \underline{\epsilon^{\mu*}(\lambda,\mathbf{k})} a_V^{\dagger}(\lambda,\mathbf{k}) e^{+ik\cdot x/\hbar}\right],$$

polarization vector

If choosing y direction as the spin quantization direction

$$\epsilon^{\mu}(\lambda,\mathbf{p}) = \left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{m_{\phi}}, \boldsymbol{\epsilon}_{\lambda} + \frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{m_{\phi}(E_p^{\phi}+m_{\phi})}\mathbf{p}\right)$$

$$\begin{aligned}\boldsymbol{\epsilon}_0 &= (0,1,0),\\ \boldsymbol{\epsilon}_{+1} &= -\frac{1}{\sqrt{2}}(i,0,1),\\ \boldsymbol{\epsilon}_{-1} &= \frac{1}{\sqrt{2}}(-i,0,1).\end{aligned}$$

Wigner function related to spin density matrix

- **Wigner function is defined as,**

$$G^{<}_{\mu\nu}(x,p)=\int d^4y e^{ip\cdot y/\hbar}G^{<}_{\mu\nu}(x_1,x_2)=\int d^4y e^{ip\cdot y/\hbar}\langle A^{V\dagger}_{\nu}(x_2)A^{V}_{\mu}(x_1)\rangle,$$

- **In gradient expansion, we get**

$$\begin{aligned}G^{<}_{\mu\nu}(x,p)&=\int d^4y e^{ip\cdot y/\hbar}\langle A^{V\dagger}_{\nu}(x_2)A^{V}_{\mu}(x_1)\rangle\\&=(2\pi\hbar)\delta(p^2-m^2)\sum_{\lambda,\lambda'}\{\theta(p^0)\epsilon_\mu(\lambda,\mathbf{p})\underline{\epsilon^*_\nu(\lambda',\mathbf{p})}\,\underline{f_{\lambda\lambda'}(\mathbf{p})}\\&\quad+\theta(-p^0)\epsilon^*_\mu(\lambda,-\mathbf{p})\underline{\epsilon_\nu(\lambda',-\mathbf{p})}[\delta_{\lambda\lambda'}+\underline{f_{\lambda'\lambda}(-\mathbf{p})}]\}.\end{aligned}$$

where matrix-valued spin dependent distribution (MVSD) in phase space is

$$f_{\lambda\lambda'}(\mathbf{p})=\int\frac{d^4u}{2(2\pi\hbar)^3}\delta(u\cdot p)e^{-iu\cdot x/\hbar}\langle a^\dagger_V(\lambda',\mathbf{p}-\mathbf{u}/2)a_V(\lambda,\mathbf{p}+\mathbf{u}/2)\rangle,$$

Spin density matrix $\rho_{\lambda\lambda'}$ $\sim$ normalized MVSD

Vector meson strong force (I)

- **From chiral quark model, we can consider that $s, \bar{s}$ quarks are living in an effective strong force of ϕ fields**

Csernai, Kapusta, Welle, PRC (2019)

$$\mathcal{L} = \overline{\psi}(i\gamma \cdot \partial - \underline{\gamma \cdot g_v \phi_v} - M)\psi, \qquad \psi = (u, d, \underline{s})^T$$

$$\phi_{v,\mu} = \begin{pmatrix} \frac{\rho^0+\omega}{\sqrt{2}} & \rho^+ & K^{*,+} \\ \rho^- & \frac{-\rho^0+\omega}{\sqrt{2}} & K^{*,0} \\ K^{*,-} & \overline{K}^{*,0} & \underline{\phi} \end{pmatrix}_\mu \qquad M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

$$\sim g_V \bar{s}\gamma \cdot \phi s$$

A. Manohar, H. Georgi, NPB (1984).

Vector meson strong force (II)

- The coupling between $s, \bar{s}$ and ϕ fields are similar to electromagnetic coupling,

$$\sim g_V \bar{s} \gamma \cdot \phi s \qquad\qquad \sim g_{EM} \bar{s} \gamma \cdot A s$$

- Recalling spin polarization induced by EM fields

$$\mathcal{S}^{\mu}_{\mathrm{EB}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N} \int d\Sigma \cdot p f_V^{(0)} (1 - f_V^{(0)}) \left(\frac{1}{(u \cdot p)T} \epsilon^{\mu\nu\alpha\beta} p_{\alpha} u_{\beta} E_{\nu} + \frac{B^{\mu}}{T} \right)$$

we replace EM fields by vector meson fields

$$F^{\mu\nu} \to F^{\mu\nu}_{\phi}, \quad T \to T_{\mathrm{eff}}$$

$s, \bar{s}$ quarks spin polarization induced by vector meson fields,

$$P^{\mu}_{s}(x, \mathbf{p}) = \frac{1}{4m_s} \epsilon^{\mu\nu\rho\sigma} \left(\frac{g_{\phi}}{E^s_p T} F^{\phi}_{\rho\sigma} \right) p_{\nu} [1 - f_s(x, \mathbf{p})],$$

$$P^{\mu}_{\bar{s}}(x, \mathbf{p}) = \frac{1}{4m_s} \epsilon^{\mu\nu\rho\sigma} \left(-\frac{g_{\phi}}{E^s_p T} F^{\phi}_{\rho\sigma} \right) p_{\nu} [1 - f_{\bar{s}}(x, \mathbf{p})],$$

Relativistic quark coalescence model

- **Recalling** $$G^{\mu\nu} \sim \epsilon^{\mu}(\lambda)\epsilon^{*\nu}(\lambda')\rho_{\lambda\lambda'}$$

Wigner function for spin-1 $\sim$ **polarization vector** $\times$ **spin density matrix**

If we only consider the coalescence process $s+\bar{s} \rightleftharpoons \phi$, then the component $\lambda_1\lambda_2$ of (unnormalized) spin density matrix are

$$\begin{aligned}\rho^{\phi}_{\lambda_1\lambda_2}(x,\mathbf{p}) \propto & \frac{\Delta t}{32}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\bar{s}}_{p'}E^{s}_{\mathbf{p}-\mathbf{p}'}E^{\phi}_{p}} f_{\bar{s}}(x,\mathbf{p}')f_{s}(x,\mathbf{p}-\mathbf{p}') \\ & \times 2\pi\hbar\delta\left(E^{\phi}_{p}-E^{\bar{s}}_{p'}-E^{s}_{\mathbf{p}-\mathbf{p}'}\right)\epsilon^{*}_{\alpha}(\lambda_1,\mathbf{p})\epsilon_{\beta}(\lambda_2,\mathbf{p}) \\ & \times \mathrm{Tr}\left\{\Gamma^{\beta}(p'\cdot\gamma-m_{\bar{s}})\left[1+\gamma_5\gamma\cdot\underline{P^{\bar{s}}(x,\mathbf{p}')}\right]\Gamma^{\alpha}\right. \\ & \left.\times\left[(p-p')\cdot\gamma+m_s\right]\left[1+\gamma_5\gamma\cdot\underline{P^{s}(x,\mathbf{p}-\mathbf{p}')}\right]\right\},\end{aligned}$$

$P^{s,\bar{s}}$ denotes spin polarization of $s,\bar{s}$ quarks

Sheng, Oliva, Liang, Wang, Wang, PRD (2022); PRL (2023); Sheng, SP, Wang, PRC 2023

Relativistic quark coalescence model

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$P^{s,\bar{s}}$ denotes spin polarization of $s,\bar{s}$ quarks

- **Inserting the polarization induced by vector meson strong force**

$$\rho_{00}(x,\mathbf{p}) = \frac{1}{3}-\frac{4g_{\phi}^{2}}{m_{\phi}^{2}T_{\mathrm{h}}^{2}}C_1\left[\frac{1}{3}\mathbf{B}'_{\phi}\cdot\mathbf{B}'_{\phi}-(\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_{\phi})^2\right] -\frac{4g_{\phi}^{2}}{m_{\phi}^{2}T_{\mathrm{h}}^{2}}C_1\left[\frac{1}{3}\mathbf{E}'_{\phi}\cdot\mathbf{E}'_{\phi}-(\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_{\phi})^2\right]$$

C_1, C_2: functions of masses of s quark and ϕ meson
B'_{ϕ}, E'_{ϕ}: electric and magnetic parts of the ϕ field in the meson's rest frame

Sheng, Oliva, Liang, Wang, Wang, PRD (2022); PRL (2023); Sheng, SP, Wang, PRC (2023)

Local ρ_{00} can be smaller than 1/3

One can assume that B_ϕ, E_ϕ are homogenous in lab frame.

$$\left\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{B}_\phi^j / T_\mathrm{h}^2 \right\rangle = \left\langle g_\phi^2 \mathbf{E}_\phi^i \mathbf{E}_\phi^j / T_\mathrm{h}^2 \right\rangle = F^2 \delta^{ij}.$$

But, after Lorentz transformation, B'_ϕ, E'_ϕ in meson's rest frame is inhomogeneous.

$$\rho_{00}(x, \mathbf{p}) - \frac{1}{3} = \frac{8}{3m_\phi^4}(C_1 + C_2)F^2 \left(\frac{p_x^2 + p_z^2}{2} - p_y^2 \right)$$

$(C_1 + C_2)F^2$ are always positive.

If $p_y^2 > (p_x^2 + p_z^2)/2$, $\rho_{00} < \frac{1}{3}$;

If $p_y^2 < (p_x^2 + p_z^2)/2$, $\rho_{00} > \frac{1}{3}$;

Shear induced spin alignment

- **Let us say:**

$$G^{\mu\nu} \sim \underline{\epsilon^{\mu}(\lambda)\epsilon^{*\nu}(\lambda')}\,\underline{\rho_{\lambda\lambda'}}$$

Wigner function for spin-1 $\sim$ polarization vector $\times$ spin density matrix

- **We can also take a tensor decomposition to G,**

$$\begin{aligned} G^{\mu\nu} \sim\; & Au^{\mu}u^{\nu} + B(g^{\mu\nu} - u^{\mu}u^{\nu}) && \text{Leading order} \\ & +\underbrace{a\pi^{\mu\nu}}_{\text{Shear tensor}} + b(u^{\mu}h^{\nu} + u^{\nu}h^{\mu}) + \ldots && \text{1st order} \\ & +\text{2nd order terms} && \text{2nd order} \end{aligned}$$

$$h^{\mu} = \{\partial^{\mu}\mu, \partial^{\mu}T, \omega^{\mu}, Du^{\mu}, \ldots\}$$

All of these term are allowed by symmetry, but the coefficients can be zero or negligible! Shear tensor may also induce the spin alignment. Li, Liu, arXiv: 2206.11890

Shear induced spin alignment

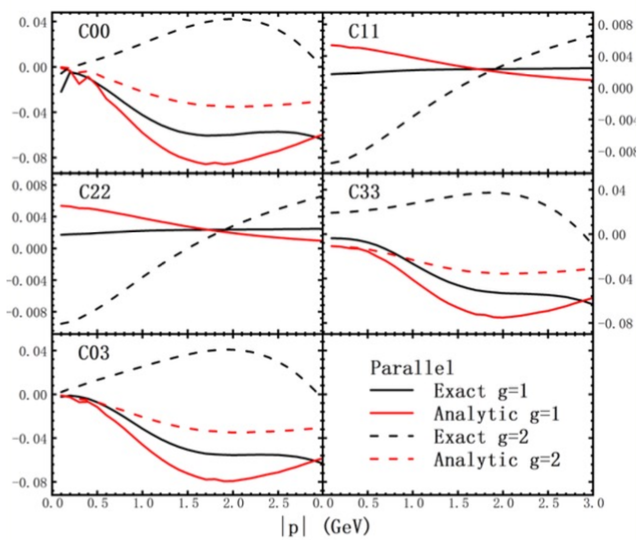
- **Shear tensor may also induce the spin alignment.**

Li, Liu, arXiv: 2206.11890； Wagner, Weickgenannt, Speranza, PRR (2023)

- **It is vanishing at local equilibrium due to symmetries,**

$$\rho_{sr}^{(1)}(x,k) = -(-1)^{r+s}\rho_{-r,-s}^{(1)}(x,k) \;\;\rightarrow\;\; \rho_{00}^{(1)} = 0$$

but can have nonzero corrections from interactions,



$$\rho_{00} = C^{\mu\nu}\pi_{\mu\nu} \quad C^{\mu\nu}\sim 10^{-2} - 10^{-3}$$

Shear tensor $\pi_{\mu\nu}\sim 10^{-2}$ from hydro simulations

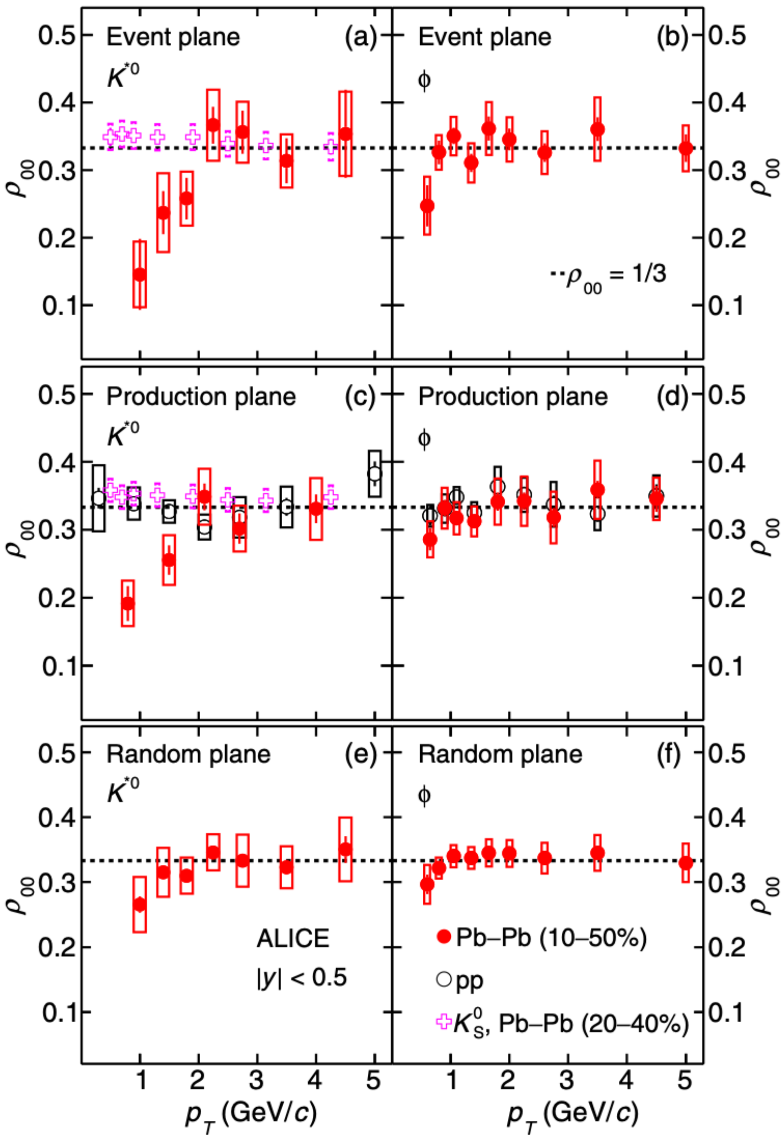
Total contribution is $10^{-4} - 10^{-5}$.

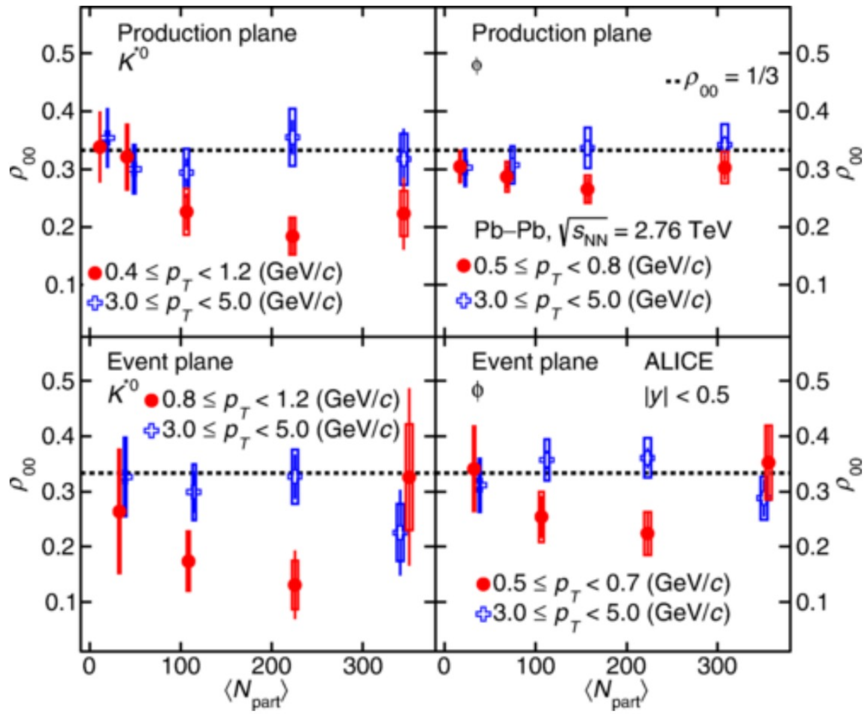
Dong, Yin, Sheng, Yang, Wang, PRD (2024)

- **Moreover, the second order corrections are given by Zubarev's approaches,**

Zhang, Huang, Becattini, Sheng, arXiv: 2412.19416； Yang, Gao, SP, Wang, arXiv: 2412.19400

Spin alignment of meson in ALICE



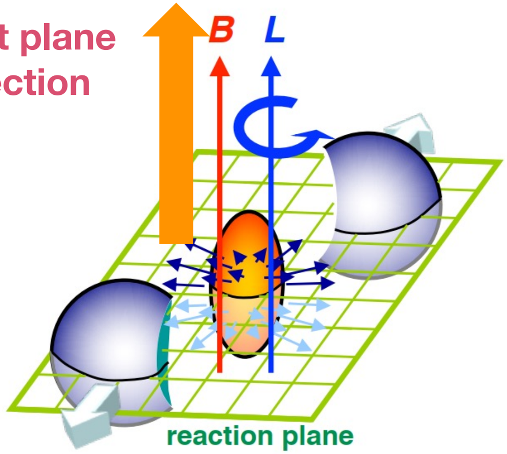


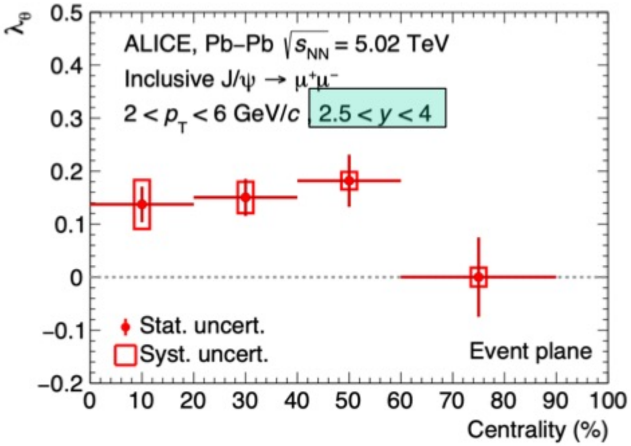
ALICE, 2.76 TeV Pb-Pb collisions

PRL 125, 012301 (2020)

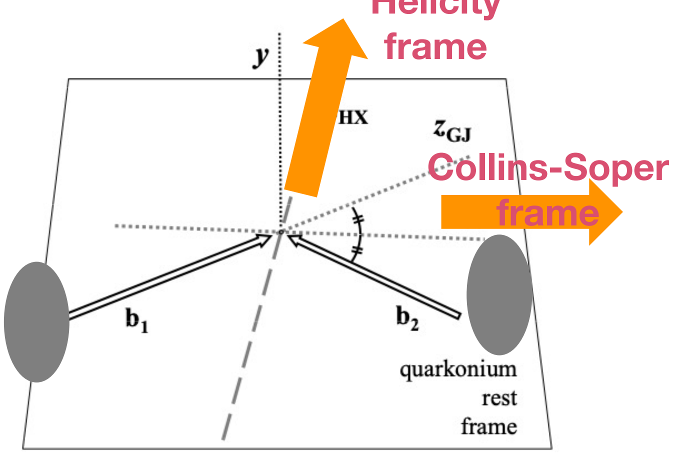
Spin alignment for J/Ψ

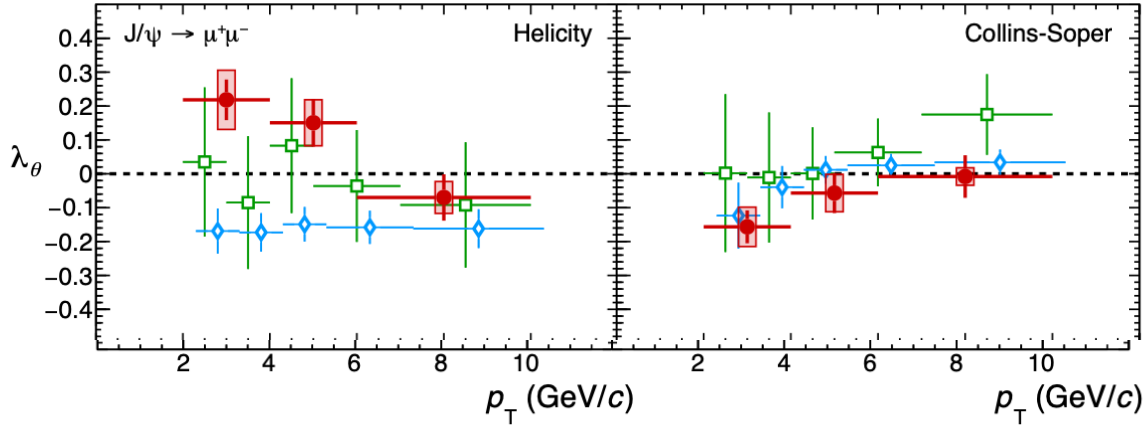
Choosing a different 'frame' means selecting different polarization axes.





ALICE, PLB 815 (2021) 136146





Faccioli, Lourenco, Seixas, Woehri, EPJC (2010)

ALICE, PRL 131 042303 (2023)

Open questions

Novel interaction corrections to spin Boltzmann eq.

- **We derive the spin Boltzmann equation incorporating Møller scattering process using hard thermal loop approximations.**

$$p^\mu \partial_\mu f_{\mathrm{A}}^{<}(p) + \hbar \partial_\mu S^{(u),\mu\alpha}(p) \partial_\alpha f_{\mathrm{V}}^{<}(p) \quad = \quad \mathcal{C}_{\mathrm{A}} + \hbar \partial_\mu \left(S^{\mu\alpha}_{(u)} C_{\mathrm{V},\alpha}[f_{\mathrm{V}}^{<}] \right)$$

Fang, SP, Yang, PRD (2022)

- **Scenario: particle distribution function is off equilibrium**

$$\partial \sim \lambda^{-1} \mathrm{Kn} \ll \lambda^{-1}$$

Kn: Knudsen number
λ: mean free path

- **Consider perturbations near equilibrium**

$$f = f_{eq} + \delta f, \rightarrow \delta f \sim 1/\mathcal{C} \sim 1/g^4$$

then the corrections to spin tensor is

$$\delta S^\mu(p) \sim \int_{p',k,k'} \mathcal{C}[\delta f] \Delta^\mu \propto g^4 \delta f \sim \cancel{g^4} \times \frac{1}{\cancel{g^4}} \sim \mathcal{O}(g^0)$$

Fang, SP, PRD (2024);

- **Leading order in gradient expansion!**
- **Corrections from scatterings but do NOT depend on coupling constant !**

(b1) Master equation for Wigner functions

$$\left[\frac{i\hbar}{2}\gamma^\mu\nabla_\mu + \gamma^\mu\Pi_\mu - m + \underline{\overline{\Sigma}_{\mathrm{g}}\star}\right] S^<(q,X) \;=\; -\frac{i\hbar}{2}(\Sigma^>_{\mathrm{g}}\star S^< - \Sigma^<_{\mathrm{g}}\star S^>),$$

$$S^<\left(-\frac{i\hbar}{2}\gamma^\mu\overleftarrow{\nabla}_\mu + \gamma^\mu\overleftarrow{\Pi}_\mu - m\right) + S^<\underline{\star\overline{\Sigma}_{\mathrm{g}}} \;=\; -\frac{i\hbar}{2}(S^>\star\Sigma^<_{\mathrm{g}} - S^<\star\Sigma^>_{\mathrm{g}}),$$

$\star$ denotes the Moyal product

$S^<$: Wigner function

$$\overline{\Sigma}_{\mathrm{g}}(q,X) \;=\; \Sigma^\delta(X) + \mathrm{Re}\Sigma^{\mathrm{r}}_{\mathrm{g}}$$

For a long time, we always neglect the self-energy terms for simplicity. Now, we consider the contributions from them carefully.

Applications to spin polarization

- **We consider effects from the thermal QCD background. After a heavy calculation, we get the corrections to polarization vectors from self-energies:**

Corrections to polarization induced by:

$$\delta\mathcal{P}^{\mu}_{\rm therm}(t,\mathbf{q}) = -\frac{\hbar^2}{2mN}\int_{\Sigma} q\cdot {\rm d}\sigma G_{\rm T}(E_q,\mathbf{q})\frac{m_f^2 T}{E_q^3}\epsilon^{\mu\nu\alpha\beta}q_\nu \partial_\alpha \underline{\left(\frac{u_\beta}{T}\right)},$$

⟷ Thermal vorticity

$$\delta\mathcal{P}^{\mu}_{\rm shear}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot {\rm d}\sigma G_{\omega_1}(E_q,\mathbf{q})\frac{m_f^2}{E_q^3}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q} q^\gamma \underline{\sigma_{\nu\gamma}},$$

⟷ Shear tensor

$$\delta\mathcal{P}^{\mu}_{\rm chem}(t,\mathbf{q}) = -\frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot {\rm d}\sigma G_{\rm T}(E_q,\mathbf{q})\frac{C_{\rm F} g^2 \mu T}{4\pi^2 E_q^2}\frac{\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma}{E_q} \underline{\nabla_\nu \left(\frac{\mu}{T}\right)},$$

⟷ Gradient of chemical Potential over temperature

$$\delta\mathcal{P}^{\mu}_{\rm acc}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot {\rm d}\sigma G_{\rm T}(E_q,\mathbf{q})\frac{3m_f^2}{E_q^3}\epsilon^{\mu\nu\rho\sigma}q_\rho u_\sigma \underline{Du_\nu},$$

⟷ Fluid acceleration

$$\begin{aligned}\delta\mathcal{P}^{\mu}_{\rm vor}(t,\mathbf{q}) = \frac{\hbar^2}{4mN}\int_{\Sigma} q\cdot {\rm d}\sigma \frac{m_f^2}{E_q^2}\Bigg[&\underline{\omega^\mu}\left(4G_{\rm T}(E_q,\mathbf{q}) - \frac{|q_\perp|^2}{E_q^2}G_{\omega_1}(E_q,\mathbf{q}) + 2G_{\omega_2}(E_q,\mathbf{q})\right) \\ &-\frac{(\underline{\omega}\cdot q)}{E_q}\left(6u^\mu G_{\rm T}(E_q,\mathbf{q}) + \frac{q_\perp^\mu}{E_q}G_{\omega_1}(E_q,\mathbf{q})\right)\Bigg],\end{aligned}$$

⟷ Kinetic vorticity

Shuo Fang, Shi Pu, Di-Lun Yang, PRD (2024), arXiv: 2311.15197

Other second order corrections

- **Scenario (II):**

$$
\begin{aligned}
\delta\mathcal{P}^{\mu}_{\rm (II)}(\mathbf{p}) \quad = \quad & \mathcal{P}^{\mu}_{{\rm (II)},\omega-\nabla T}+\mathcal{P}^{\mu}_{{\rm (II)},\nabla\omega}+\mathcal{P}^{\mu}_{{\rm (II)},\omega-{\rm chem}}+\mathcal{P}^{\mu}_{{\rm (II)},\omega-{\rm shear}} \\
& +\mathcal{P}^{\mu}_{{\rm (II)},{\rm chem}-\nabla {\rm T}}+\mathcal{P}^{\mu}_{{\rm (II)},{\rm shear}-\nabla {\rm T}}+\mathcal{P}^{\mu}_{{\rm (II)},{\rm shear}-{\rm chem}}+\mathcal{O}(\hbar^2\partial^3)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}^{\mu}_{{\rm (II)},\omega-\nabla T} &= -\hbar^2\int_\Sigma {\rm d}\Sigma\cdot p a^{\mu}_{\rm (II)}\left[d_2\left(E_{\mathbf{p}}-d_1\frac{1}{\beta_0}\right)\omega^\alpha\nabla_\alpha\beta_0-d_6\beta_0 p_{\langle\alpha}p_{\rho\rangle}\omega^\alpha\nabla^\rho\beta_0\right], \\
\mathcal{P}^{\mu}_{{\rm (II)},\nabla\omega} &= \hbar^2\int_\Sigma {\rm d}\Sigma\cdot p a^{\mu}_{\rm (II)}\left[\left(E_{\mathbf{p}}-d_1\frac{1}{\beta_0}\right)d_2\beta_0\nabla^\alpha\omega_\alpha+d_6\frac{1}{2}\beta_0^2\nabla^\alpha\omega^\rho p_{\langle\alpha}p_{\rho\rangle}\right], \\
\mathcal{P}^{\mu}_{{\rm (II)},\omega-{\rm chem}} &= \hbar^2\int_\Sigma {\rm d}\Sigma\cdot p a^{\mu}_{\rm (II)}\left[\left(E_{\mathbf{p}}-d_1\frac{1}{\beta_0}\right)d_3\beta_0\omega^\alpha\nabla_\alpha\alpha_0-d_8\beta_0^2\omega^\alpha\nabla^\rho\alpha_0 p_{\langle\alpha}p_{\rho\rangle}\right], \\
\mathcal{P}^{\mu}_{{\rm (II)},\omega-{\rm shear}} &= -\hbar^2\beta_0\int_\Sigma {\rm d}\Sigma\cdot p a^{\mu}_{\rm (II)}\left[-d_4\omega^\rho\sigma^\alpha_\rho p_{\langle\alpha\rangle}+d_9\beta_0^2\omega^\beta\sigma^{\alpha\lambda}p_{\langle\beta}p_\alpha p_{\lambda\rangle}\right] \\
\mathcal{P}^{\mu}_{{\rm (II)},{\rm chem}-\nabla{\rm T}} &= \hbar^2\int_\Sigma {\rm d}\Sigma\cdot p a^{\mu}_{\rm (II)}d_5\epsilon^{\rho\nu\alpha\beta}u_\beta\nabla_\nu\alpha_0\nabla_\rho\beta_0 p_{\langle\alpha\rangle}, \\
\mathcal{P}^{\mu}_{{\rm (II)},{\rm shear}-\nabla{\rm T}} &= \hbar^2\beta_0\int_\Sigma {\rm d}\Sigma\cdot p a^{\mu}_{\rm (II)}d_6\epsilon^{\beta\nu\sigma\rho}\sigma^\alpha_\beta u_\sigma\nabla_\nu\beta_0 p_{\langle\alpha}p_{\rho\rangle}, \\
\mathcal{P}^{\mu}_{{\rm (II)},{\rm shear}-{\rm chem}} &= -\hbar^2\beta_0^2\int_\Sigma {\rm d}\Sigma\cdot p a^{\mu}_{\rm (II)}d_7\epsilon^{\mu\nu\sigma\rho}\sigma^\alpha_\mu u_\sigma\nabla_\nu\alpha_0 p_{\langle\alpha}p_{\rho\rangle},
\end{aligned}
$$

Fang, SP, PRD (2024)

Theoretical developments

- **Spin hydrodynamics (macroscopic approach)**
  - Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  - Montenegro, Tinti, Torrieri (2017-2019);
  - Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  - Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022); …
  - S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  - D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060
  - Weickgenannt, Wanger, Speranze, Rischke, PRD 2022; PRD 2022; Weickgennatt, Wanger, Speranza, PRD 2022; arXiv:2306.05936;
- **Quantum kinetic theory with collisions (microscopic approach)**
  - Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  - Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  - Liu, Mameda, Huang, arXiv:2002.03753.
  - Gao, Liang, PRD 2019
  - Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019)； Z.Y. Wang, arXiv:2205.09334;
  - Li ,Yee, PRD100, 056022 (2019)
  - Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  - Fang, SP, Yang, PRD (2022)
- **Other approaches:**
  - Side-jump effect Liu, Sun, Ko PRL(2020)
  - Mesonic mean-field Csernai, Kapusta, Welle, PRC(2019)
  - Using different vorticity Wu, Pang, Huang, Wang, PRR (2019)
- **Recent reviews:**
  - Gao, Ma, SP, Wang, NST (2020)
  - Gao, Liang, Wang, IJMPA (2021)
  - Hidaka, SP, Yang, Wang, PPNP (2022)
  - F. Becattini, M. Buzzegoli, T. Niida, SP, A.H. Tang, Q. Wang, arXiv: 2402.04540

Puzzle : T-odd/T-even VS dissipative/non-dissipative

Are shear induced polarization or spin alignment non-dissipative?

Q1: If the coefficient is T-even, it is non-dissipative.

CME $\mathbf{j} \sim C\mathbf{B}$ —Taking T transformation→ $\mathbf{j} \rightarrow -\mathbf{j}$, $\mathbf{B} \rightarrow -\mathbf{B}$ C: T-even ✓

Spin polarization vector $\mathcal{S}^i \sim C^{ijk}(\partial_j u_k + \partial_k u_j)$ → $\mathcal{S}^i \rightarrow -\mathcal{S}^i$, $\partial_j u_k \rightarrow -\partial_j u_k$ C: T-even Non-dissipative?

Spin alignment $\epsilon^i(\lambda)\epsilon^{*j}(\lambda')\rho_{\lambda\lambda'} \sim a\pi^{ij}$ → $\rho_{\lambda\lambda'} \rightarrow (-1)^{\lambda+\lambda'}\rho_{\lambda\lambda'}$, $\epsilon_\mu^{s*}(\mathbf{p}) \rightarrow -(-1)^s \tilde{\delta}_\mu^\alpha \epsilon_\alpha^{-s*}(-\mathbf{p})$, $\pi^{ij} \rightarrow -\pi^{ij}$

For ρ_{00}, the coefficient "a" is T-odd. Dissipative?

In Zubarev approach, the non-dissipative means the results does NOT depend on hypersurface. But, shear tensor comes from local equilibrium operators and should always depends on hypersurface. So, shear induced something is always dissipative?