Warmest congratulations and best wishes

Inaugural Symposium of the Central China Center for Nuclear Theory (C3NT) on Frontiers in Nuclear Theory

Recent progress in Femtoscopic studies

Li-Sheng Geng（耿立升）@ Beihang U.

Zhi-Wei Liu, Jun-Xu Lu, LSG*, PRD 107(2023)074019
Zhi-Wei Liu, Jun-Xu Lu, Ming-Zhu Liu, LSG*, PRD 108(2023)L031503
Zhi-Wei Liu, Jun-Xu Lu, Ming-Zhu Liu, LSG*, 2404.18607
Ming-Zhu Liu, Ya-Wen Pan, Zhi-Wei Liu, Tian-Wei Wu, Jun-Xu Lu, LSG*, Phys.Rept. 1108 (2025) 1-108

(Image: CERN)

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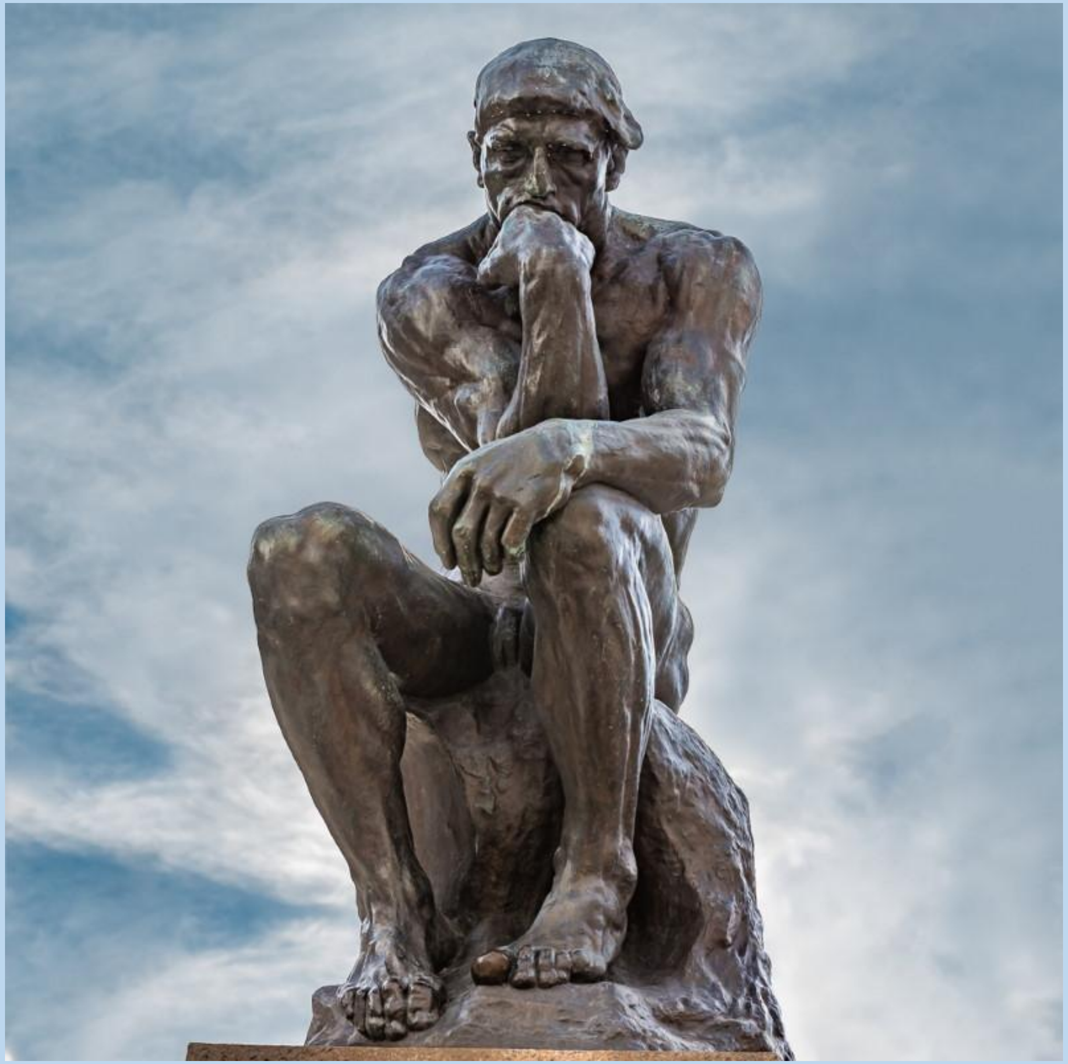
- ☞ **Summary and outlook**

One central theme in physics (Science)

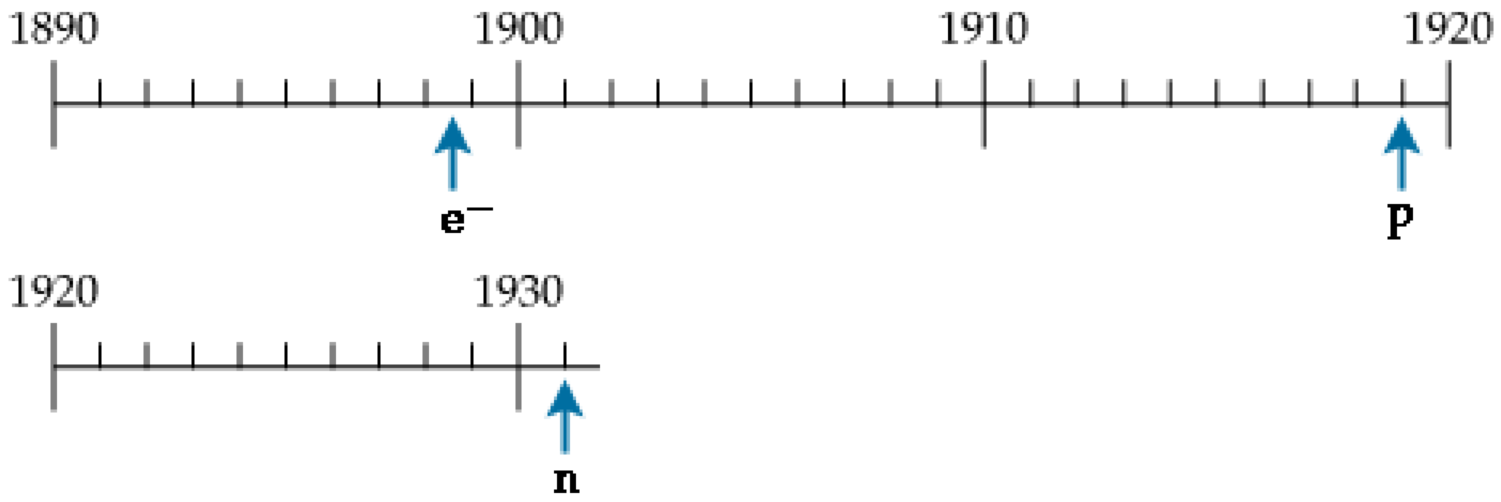


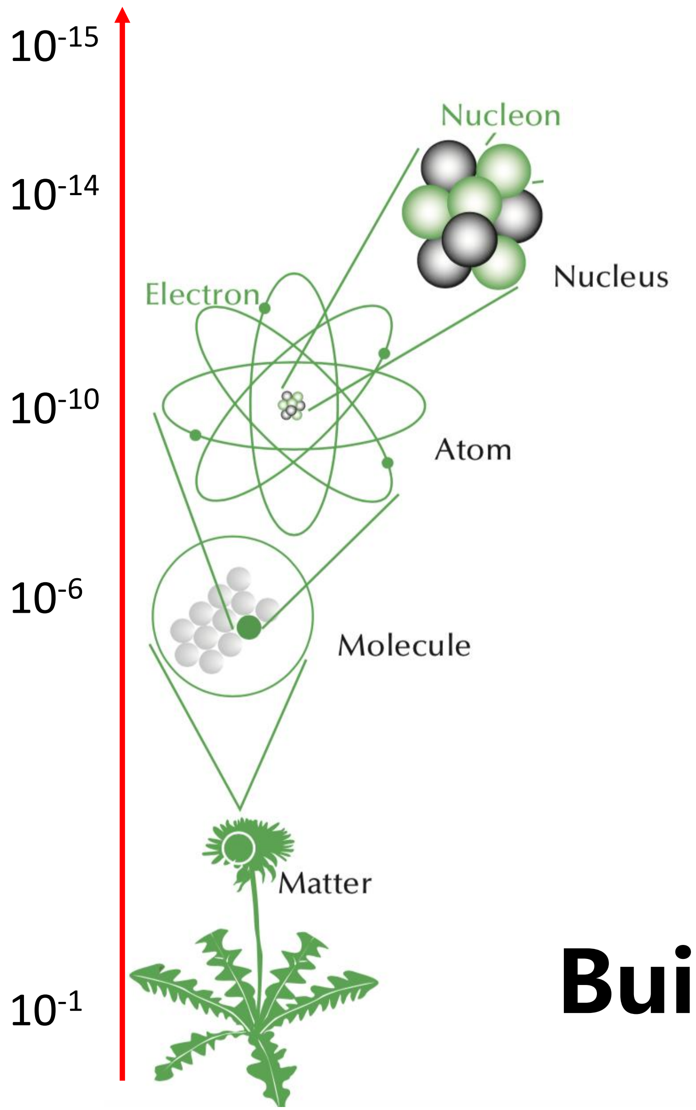
The Thinker by Auguste Rodin

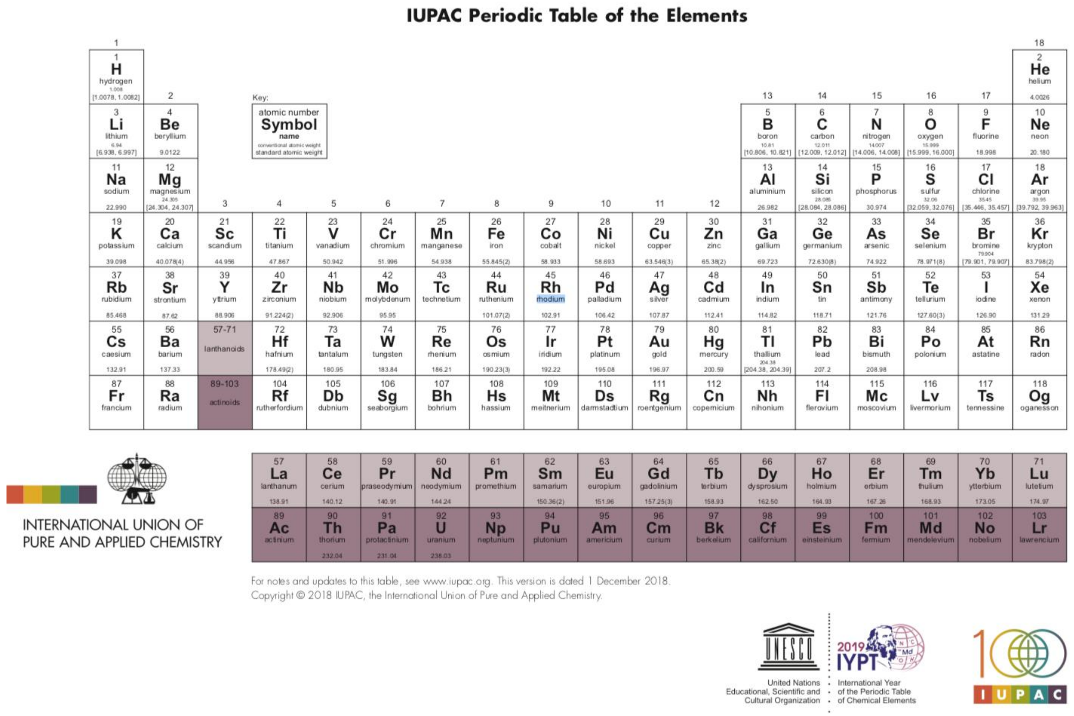
- What are the basic building blocks of NATURE?
- How do they interact with one another?

The world was once very simple

Particles discovered before 1932

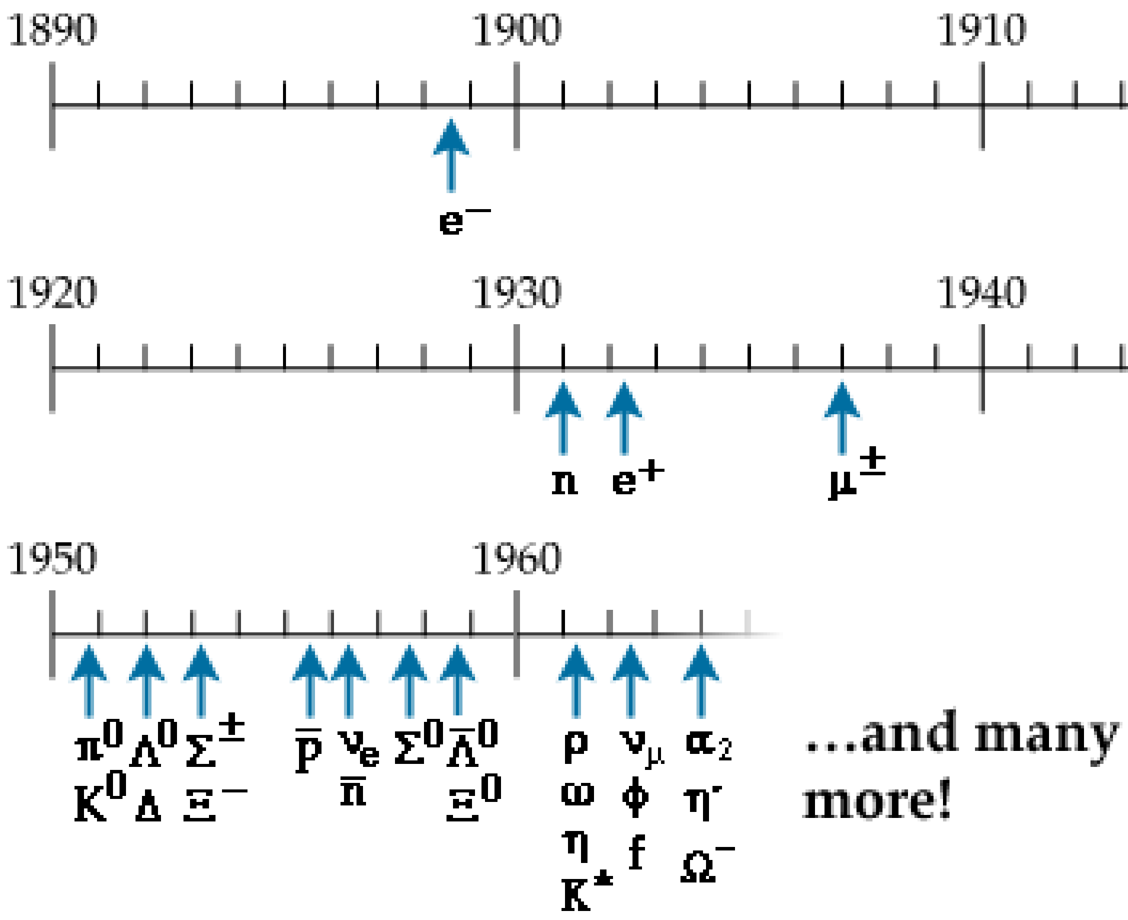






Building up the atomic world

Many particles observed in the 1950/60s

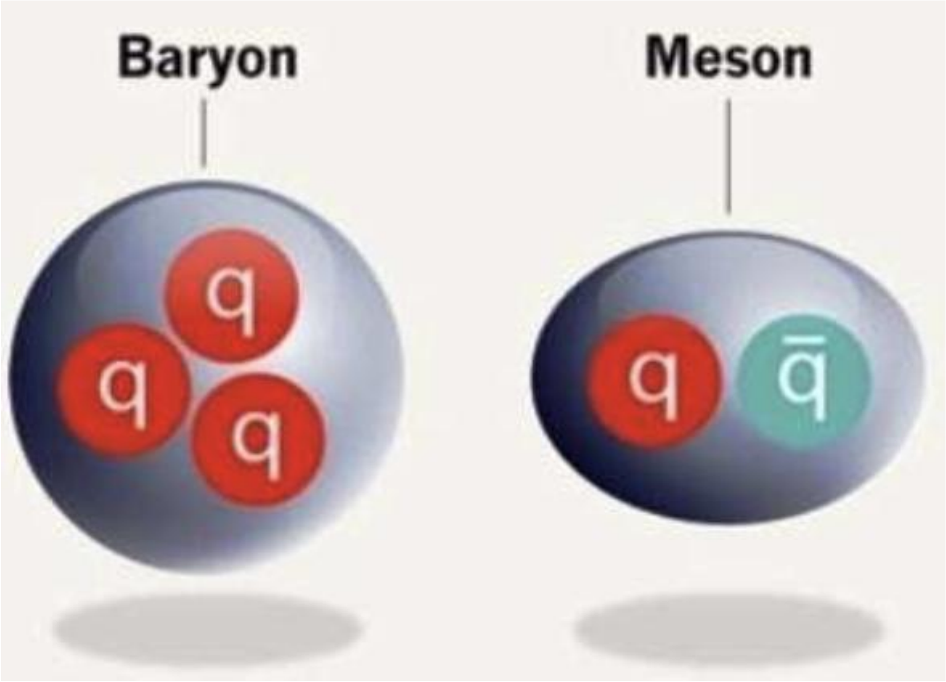


They cannot all be "elementary particles"!

Naive QM: hadron structure

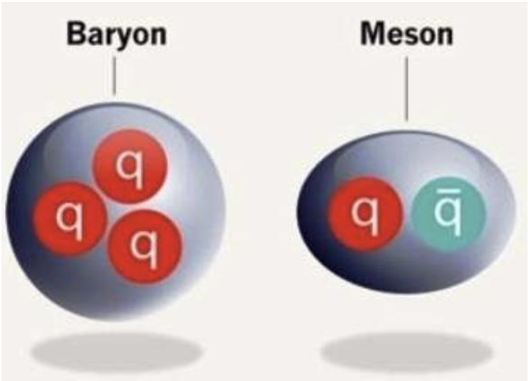
1964

Murray Gell-Mann



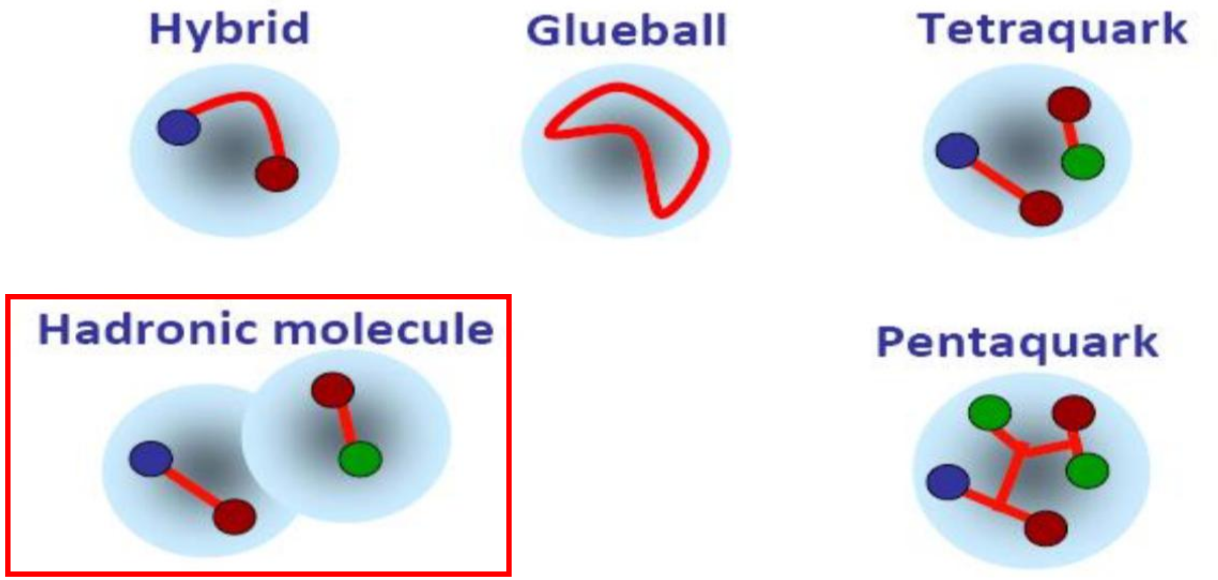
George Zweig

Beyond Naïve QM hadrons, more complicated structures allowed



Conventional Hadrons

Exotic Hadrons

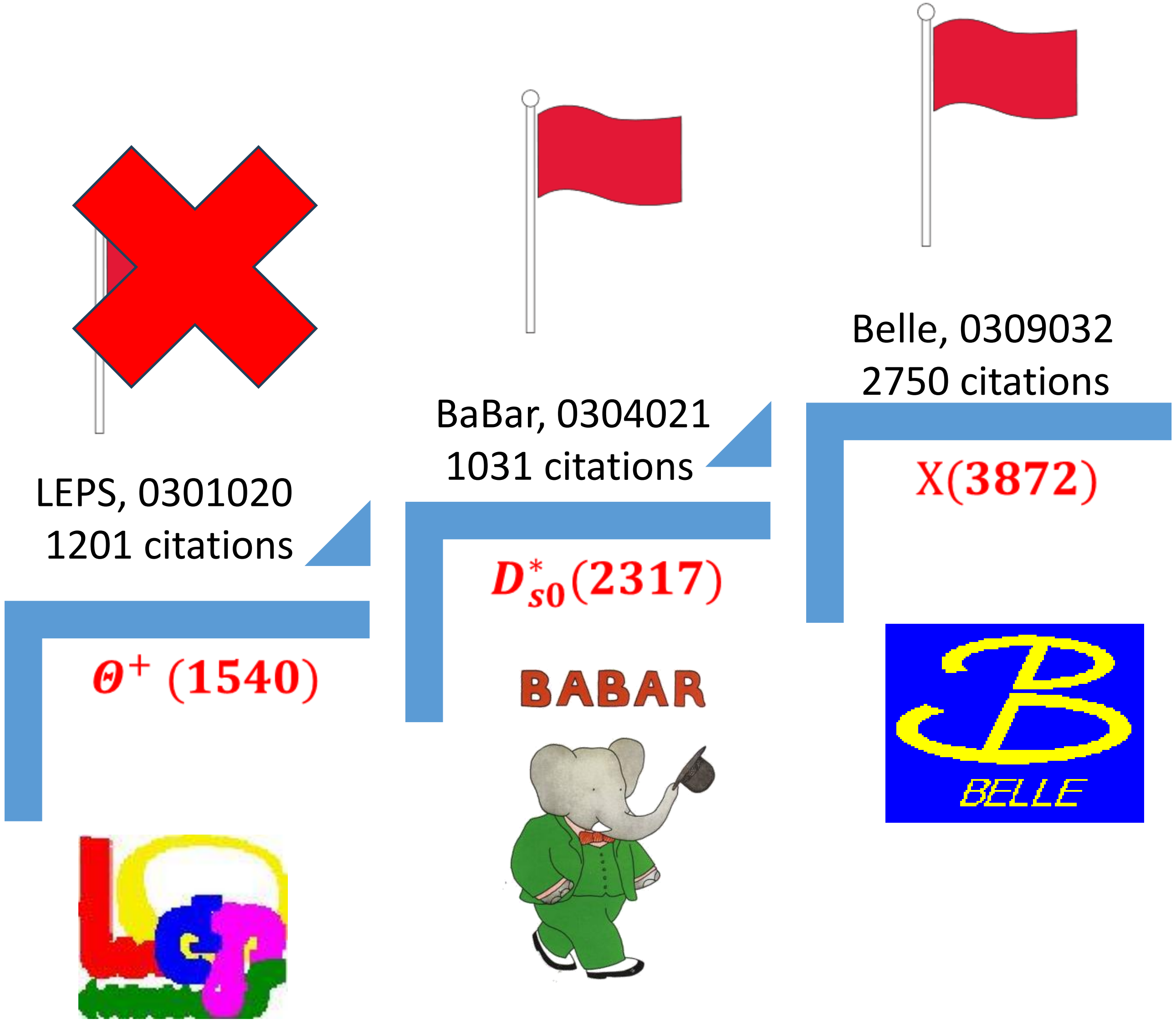


Naïve quark models more or less fine until 2003

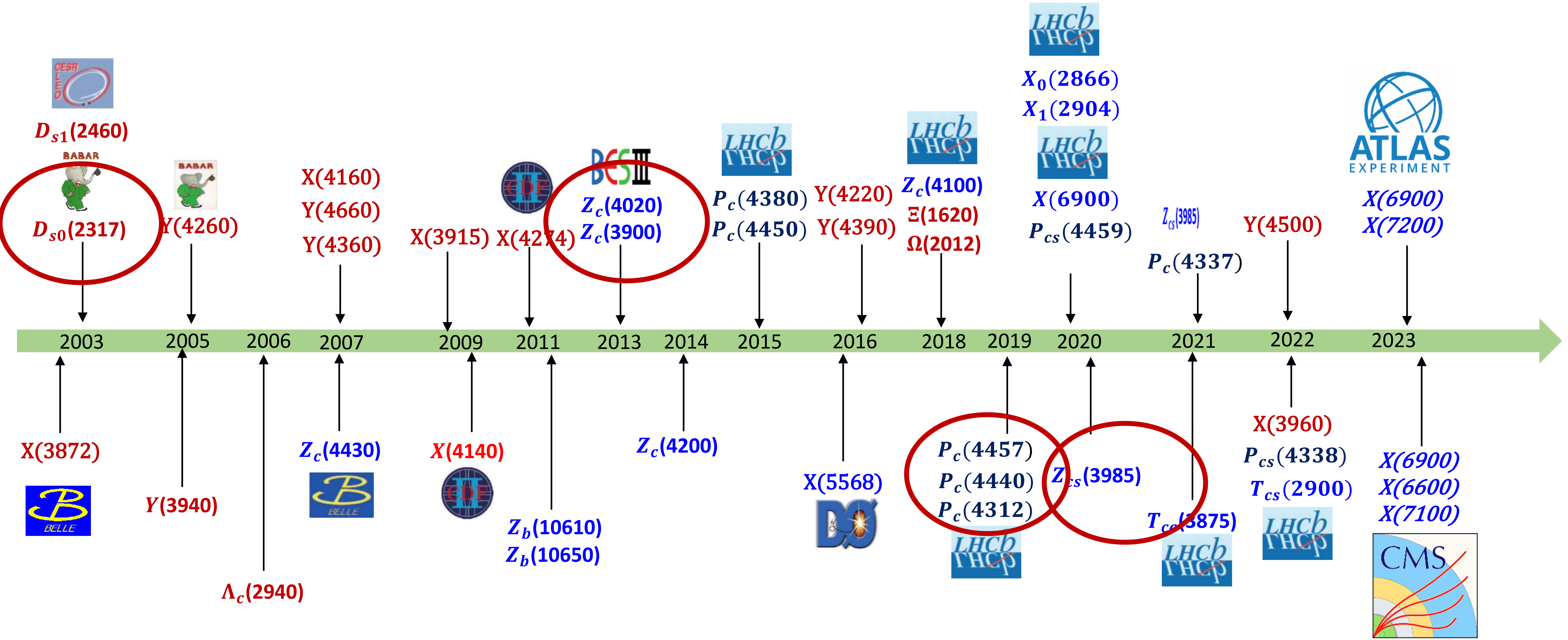
$\Lambda(1405)$， $N^*(1535)$,...
$f_0(500)$, $f_0(980)$, $a_0(980)$, ...

2003—the beginning of a new era

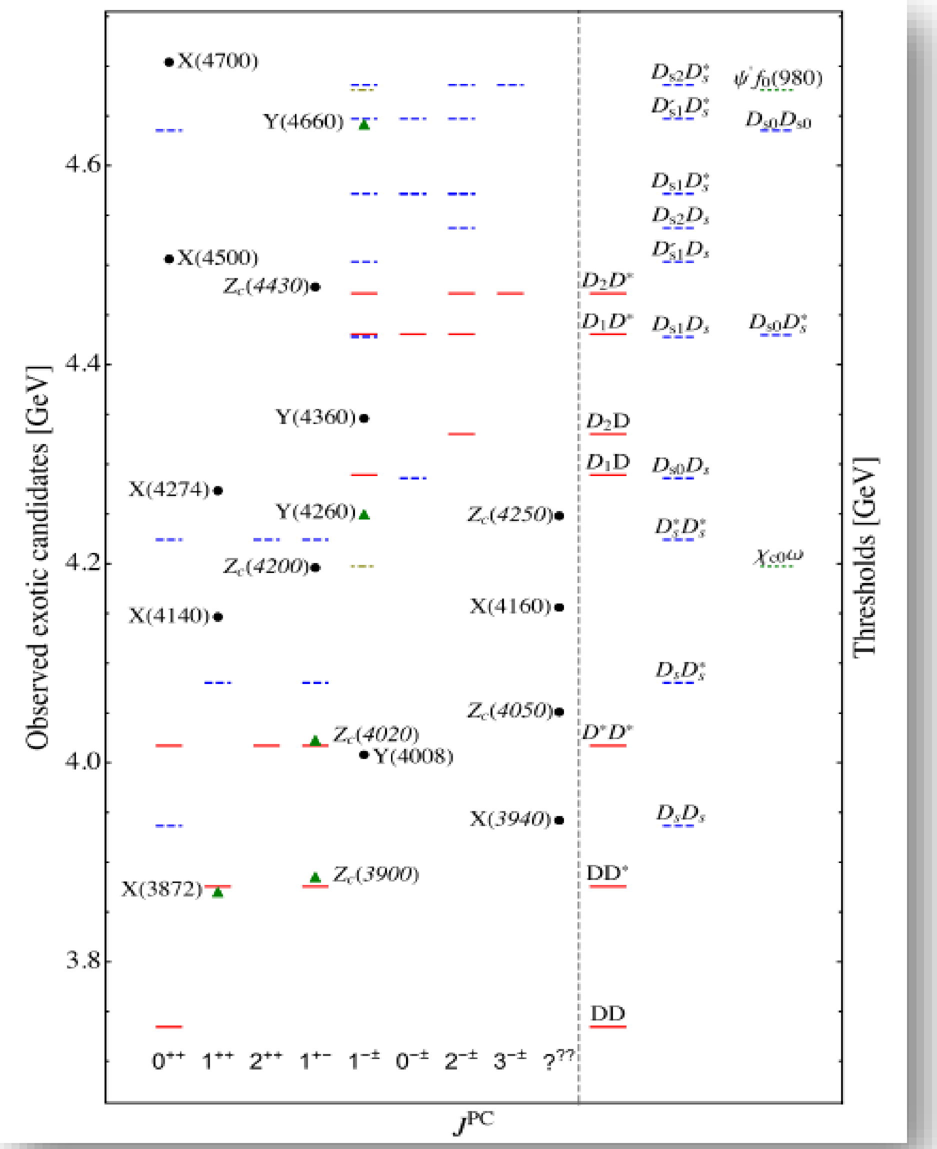
as of 2025.05.17



Many more exotic hadrons discovered



Many (if not all) of them close to thresholds—molecules



Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qiang Wang, Qiang Zhao, Bing-Song Zou. Rev.Mod.Phys. 90 (2018) 015004

Richard F. Lebed, Ryan E. Mitchell, Eric S. Swanson, Prog.Part.Nucl.Phys. 93 (2017) 143

Atsushi Hosaka, Toru Iijima, Kenkichi Miyabayashi, Yoshihide Sakai , Shigehiro Yasui, PTEP 2016 (2016) 062C01

Hua-Xing Chen, Wei Chen, Xiang Liu Shi-Lin Zhu, Phys. Rept.639 (2016) 1

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Physics Reports 1108 (2025) 1–108

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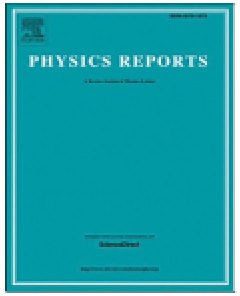
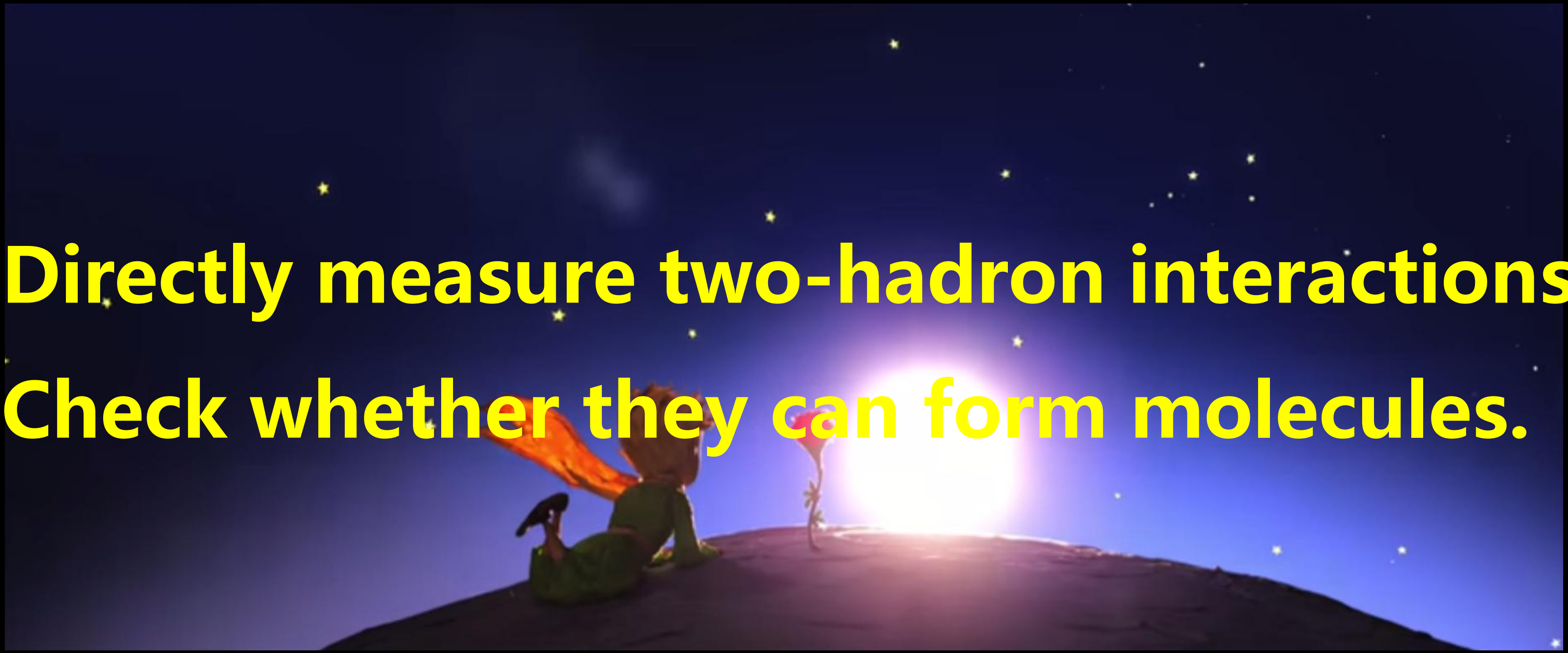


Three ways to decipher the nature of exotic hadrons: Multiplets, three-body hadronic molecules, and correlation functions

Ming-Zhu Liu [a,b], Ya-Wen Pan [c], Zhi-Wei Liu [c], Tian-Wei Wu [d], Jun-Xu Lu [c], Li-Sheng Geng [c,e,f,g,h,*]

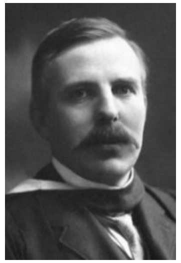
[a] *Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, China*
[b] *School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China*
[c] *School of Physics, Beihang University, Beijing 102206, China*
[d] *School of Science, Shenzhen Campus of Sun Yat-sen University, Shenzhen 518107, China*
[e] *Centrale Pekin, Beihang University, Beijing 100191, China*
[f] *Peng Huanwu Collaborative Center for Research and Education, Beihang University, Beijing 100191, China*
[g] *Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[h] *Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, China*

How to check the molecular picture?

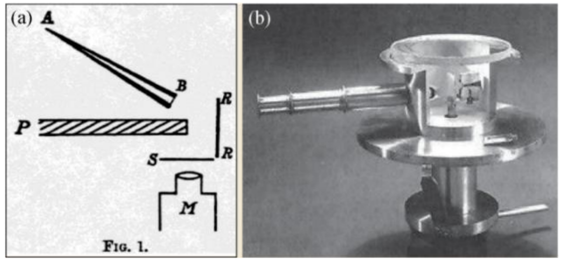


New probe—femtoscopic correlation functions

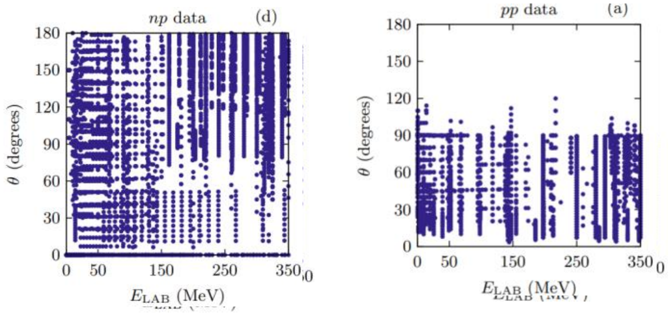
◻ For stable hadrons, scattering experiments are extremely valuable in extracting their interactions



Ernest Rutherford



Rutherford Scattering Experiment



- **NN scattering，8125 data**
- Foundation of high-precision nuclear force

◻ For unstable particles, direct scattering experiments are impossible!

- ➢ **Difficult to get large quantity of beam particles**
- ➢ **No fixed targets available**

$\Lambda p \to \Lambda p$	$\Sigma^- p \to \Lambda n$	$\Sigma^+ p \to \Sigma^+ p$	$\Sigma^- p \to \Sigma^- p$	$\Sigma^- p \to \Sigma^0 n$
12	6	4	7	6

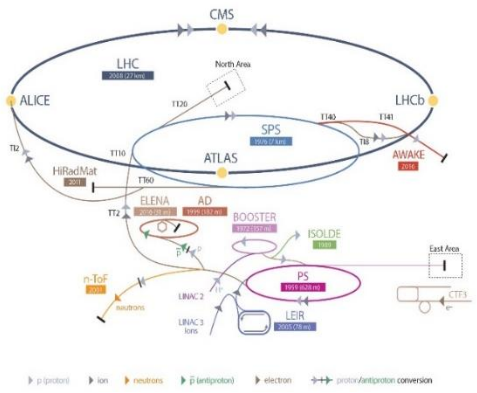
- **Hyperon-nucleon low energy scattering, 35 data**
- Hindering hyper-nuclear physics and neutron star studies

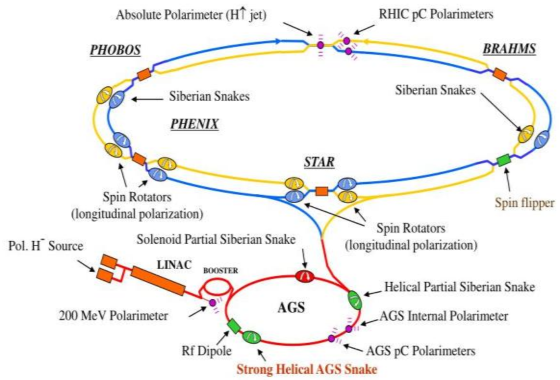
New probe—femtoscopic correlation functions

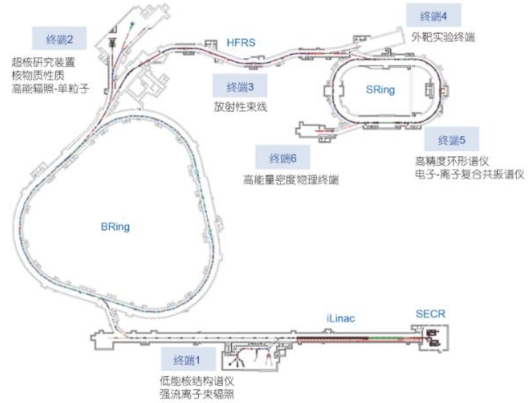
The Large Hadron Collider

Brookhaven National Laboratory — RHIC Relativistic Heavy Ion Collider

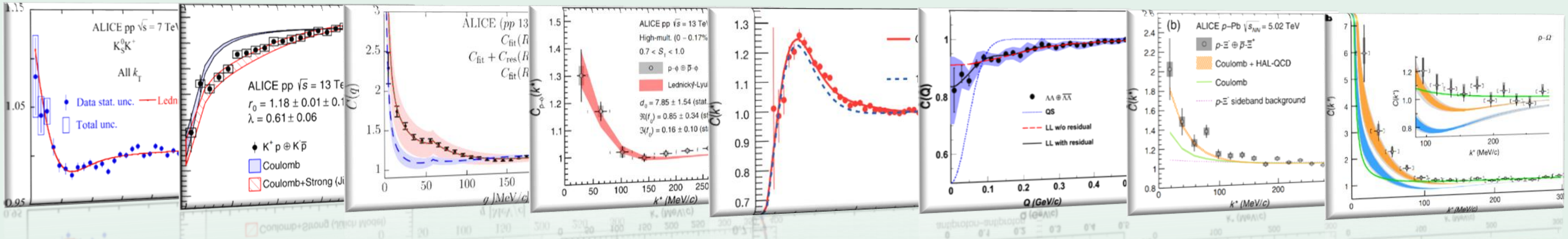
HIAF 强流重离子加速器装置 High Intensity heavy-ion Accelerator Facility







$K^0_S K^\pm$	$K^\pm p$	$K^- p$	ϕp	$\overline{p}\overline{p}$	$\Lambda\Lambda$	$\Xi^- p$	$\Omega^- p$



PLB 2019	PRL 2020	PRL 2020	PRL 2021	Nature 2015	PRL 2015	PRL 2019	Nature 2020

$K^- d$

$\Sigma^0 p$

$\Lambda\Xi$

$\Xi\Xi$

$\Omega\Omega$

pD

ppp

$pp\Lambda$

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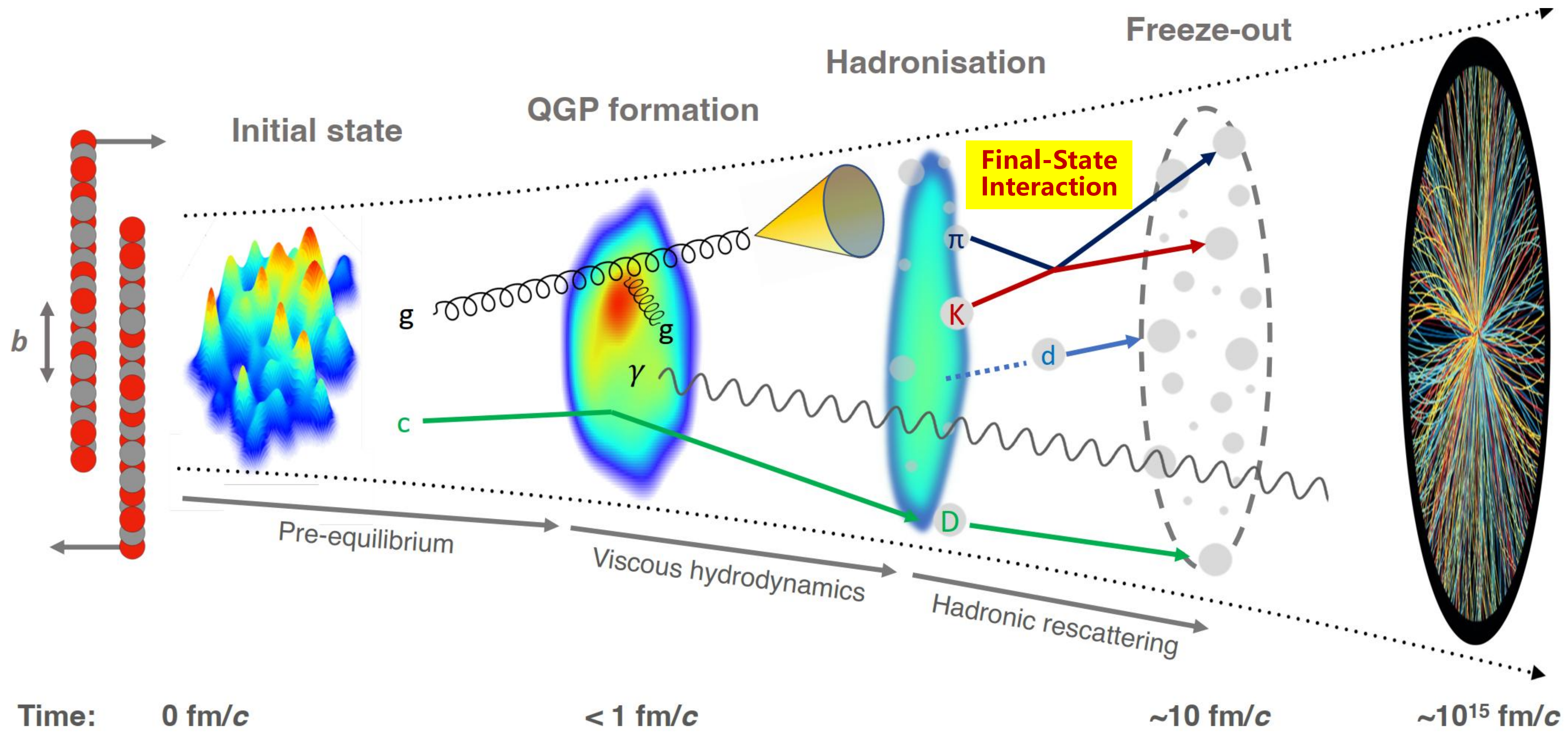
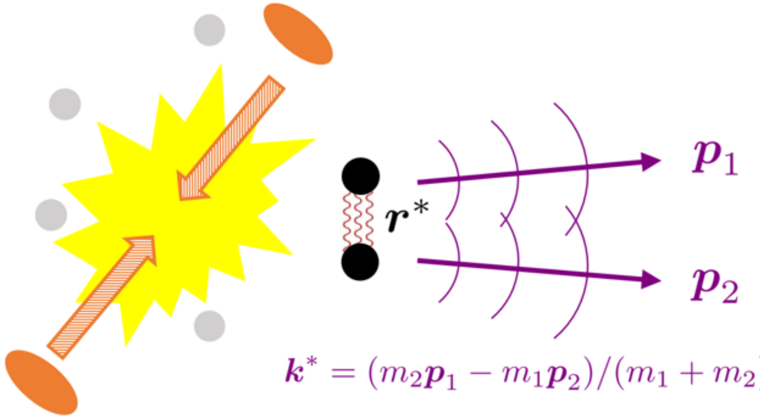
  - ➢ **Double J/Ψ**
- ☞ **Summary and outlook**

Abundant particles produced in AA, pA, and pp collisions



Femtoscopic correlation functions (CFs)

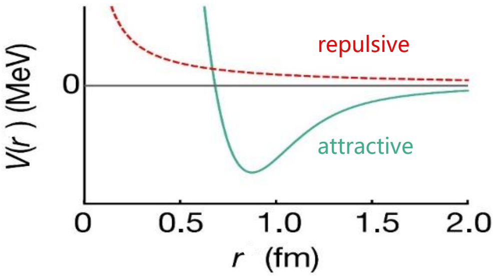
$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{\mathrm{P}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{\mathrm{P}(\boldsymbol{p}_1)\cdot\mathrm{P}(\boldsymbol{p}_2)}$$



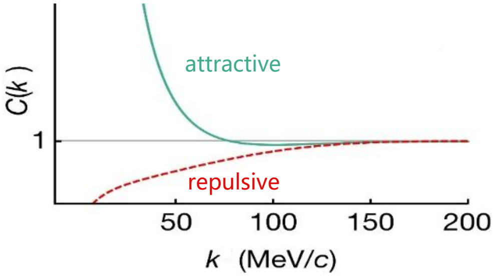
$$\boldsymbol{k}^* = (m_2\boldsymbol{p}_1 - m_1\boldsymbol{p}_2)/(m_1+m_2)$$

Emission source $S_{12}(\boldsymbol{r}^*)$ Two-particle wavefunction $\psi(\boldsymbol{k}^*,\boldsymbol{r}^*)$

Interacting potential



Correlation function



Schrödinger equation

Two-particle wavefunction $\psi(k\ ,r\ )$

Exp. measurement

mixed-event technique

$$C(k) = \xi(k)\frac{N_{\mathrm{same}}(k)}{N_{\mathrm{mixed}}(k)}$$

N_{same}: the same event distributions

N_{mixed}: the mixed event distributions

ξ: the corrections for experimental effects

Theo. description

Koonin–Pratt formula

$$C(k) = \int S_{12}(\boldsymbol{r})\ |\psi(\boldsymbol{k},\boldsymbol{r})|^2\ \mathrm{d}\boldsymbol{r}$$

spacial structure

final-state interactions

quantum statistics effects

coupled-channel effects

Basic Properties

$C(k)$:
- >1 if the interaction is attractive
- =1 if there is no interaction
- <1 if the interaction is repulsive

Femtoscopic correlation functions (CFs)

Koonin–Pratt (KP) formula

S. E. Koonin, Phys. Lett. B **70** (1) (1977) 43
A. Ohnishi, Nucl. Phys. A **954** (2016) 294

$$C(k) = \int S_{12}(r)\ |\Psi(\boldsymbol{r},\boldsymbol{k})|^2\ \mathrm{d}\boldsymbol{r}$$

Only S-waves

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2 \mathrm{d}r\ S_{12}(r)\ \left[|\psi_0(r,k)|^2 - |j_0(kr)|^2\right]$$

- **Common static and spherical Gaussian source**

$$S_{12}(r) = \exp[-r^2/(4R^2)]/(2\sqrt{\pi}R)^3$$

- **Scattering wave function**

  - **the Schrödinger equation**

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V\psi = E\psi$$

  - **the Lippmann-Schwinger equation**

$$T = V + VGT \quad \Longrightarrow \quad |\psi\rangle = |\phi\rangle + GT|\phi\rangle$$

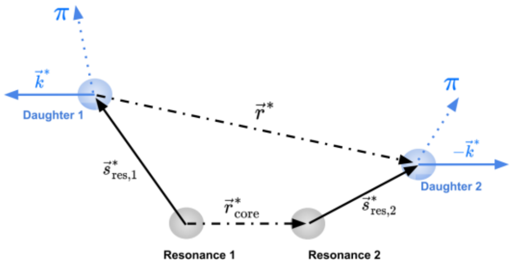
Constraining the source function

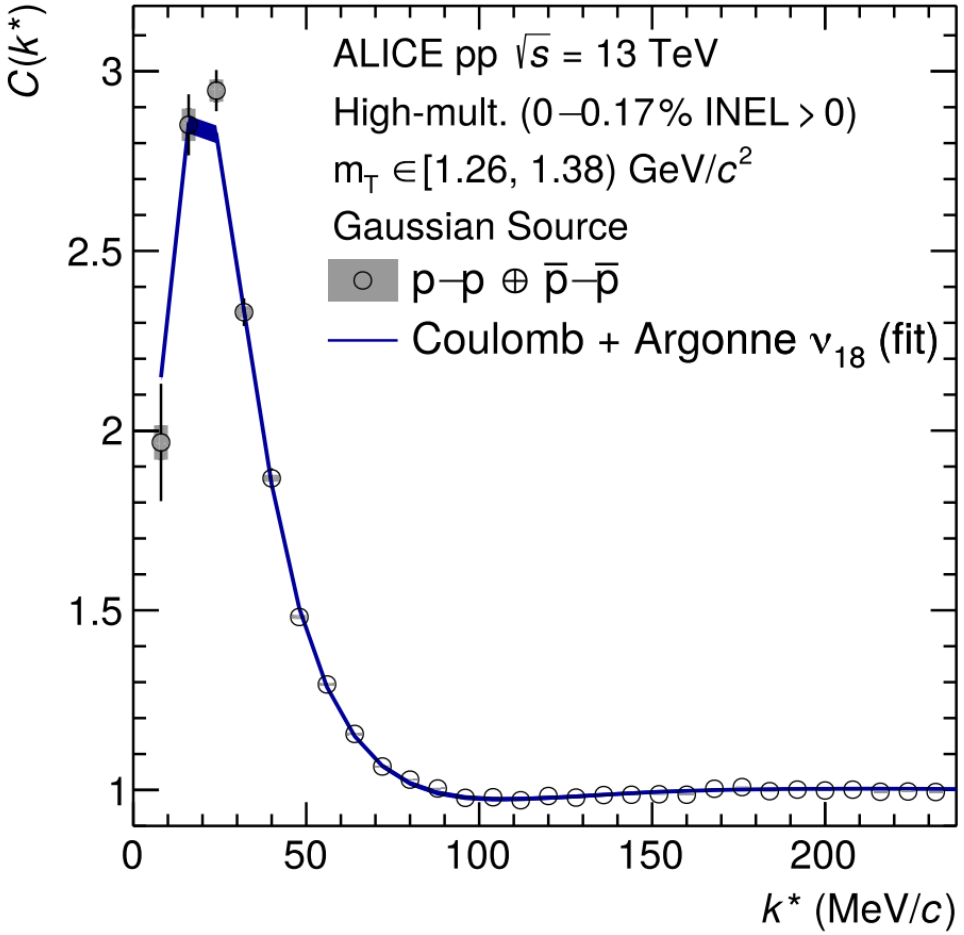
- **Using the well-known proton-proton interaction to calibrate the source**

$$C(k^*) = \int S(\vec{r}^*) \left|\psi\left(\vec{k}^*, \vec{r}^*\right)\right|^2 d^3\vec{r}^*$$

- **For CFs involving short-lived resonances ($c\tau \sim 1\text{fm}$)—Resonance Source Model**

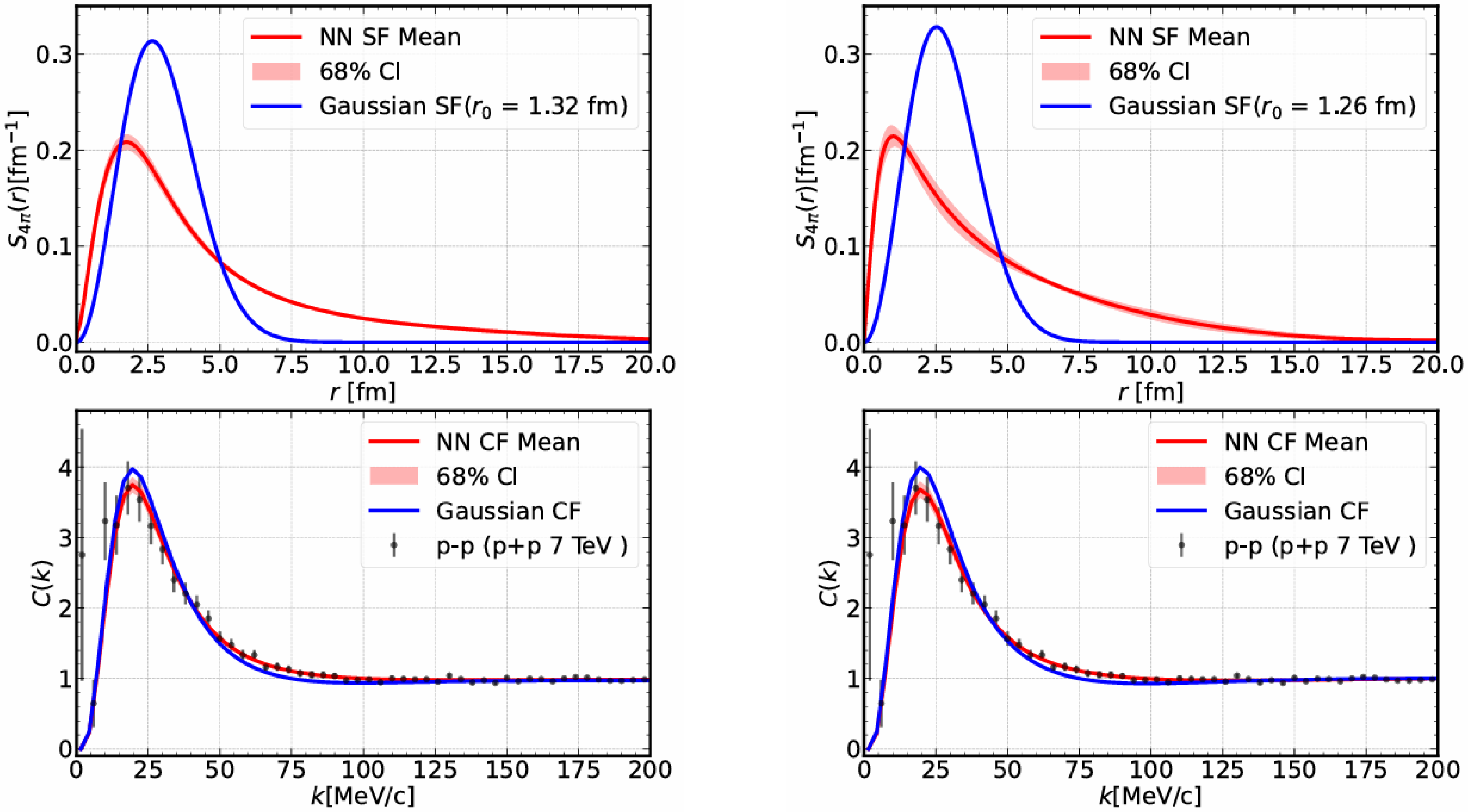
$$S(r^*) = \frac{1}{\left(4\pi r_{\text{core}}^2\right)^{3/2}} \exp\left(-\frac{r^{*2}}{4r_{\text{core}}^2}\right) \times \text{SLR}$$





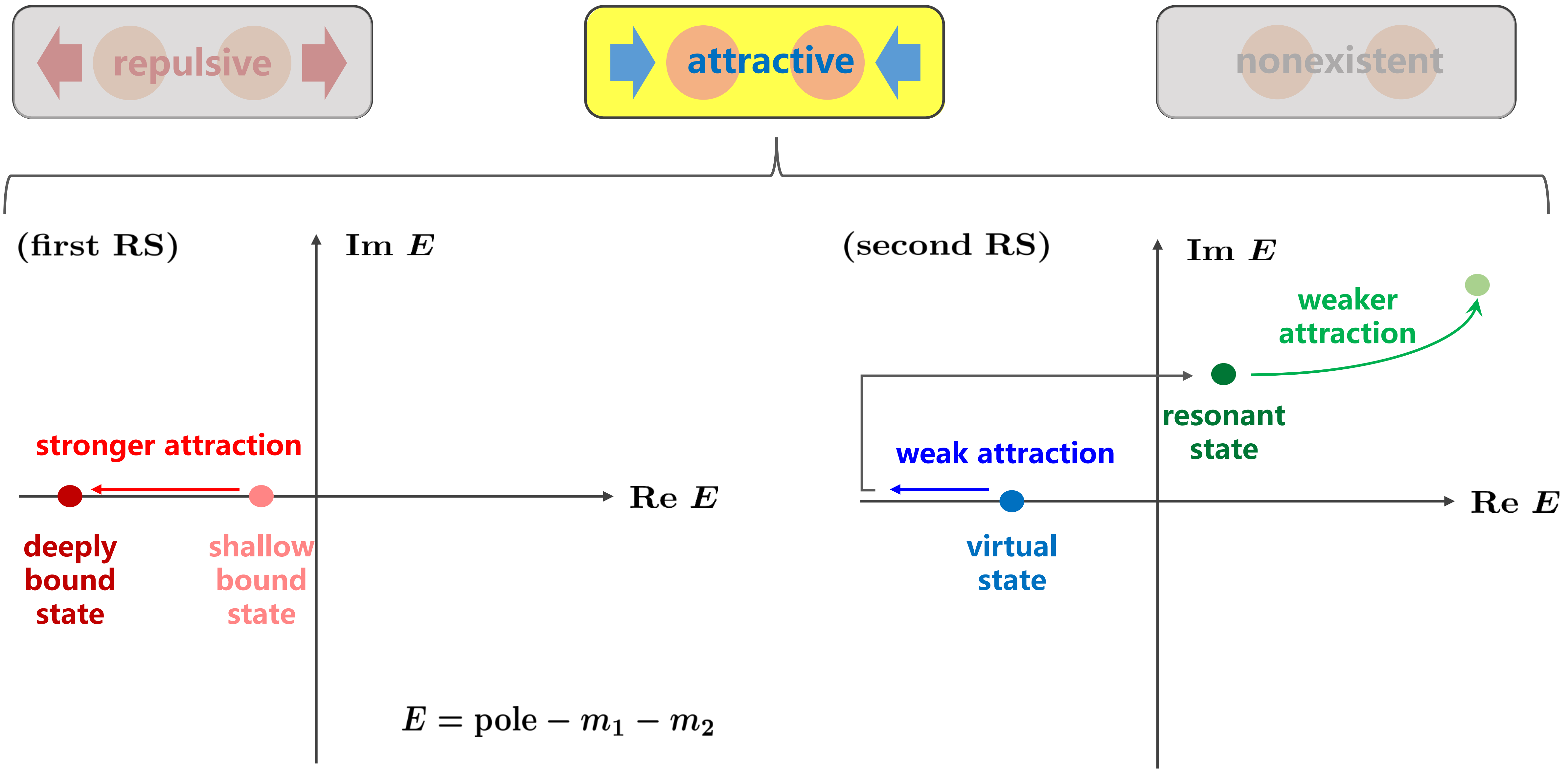
ALICE, PLB, 811 (2020) 135849

Constraining the source function



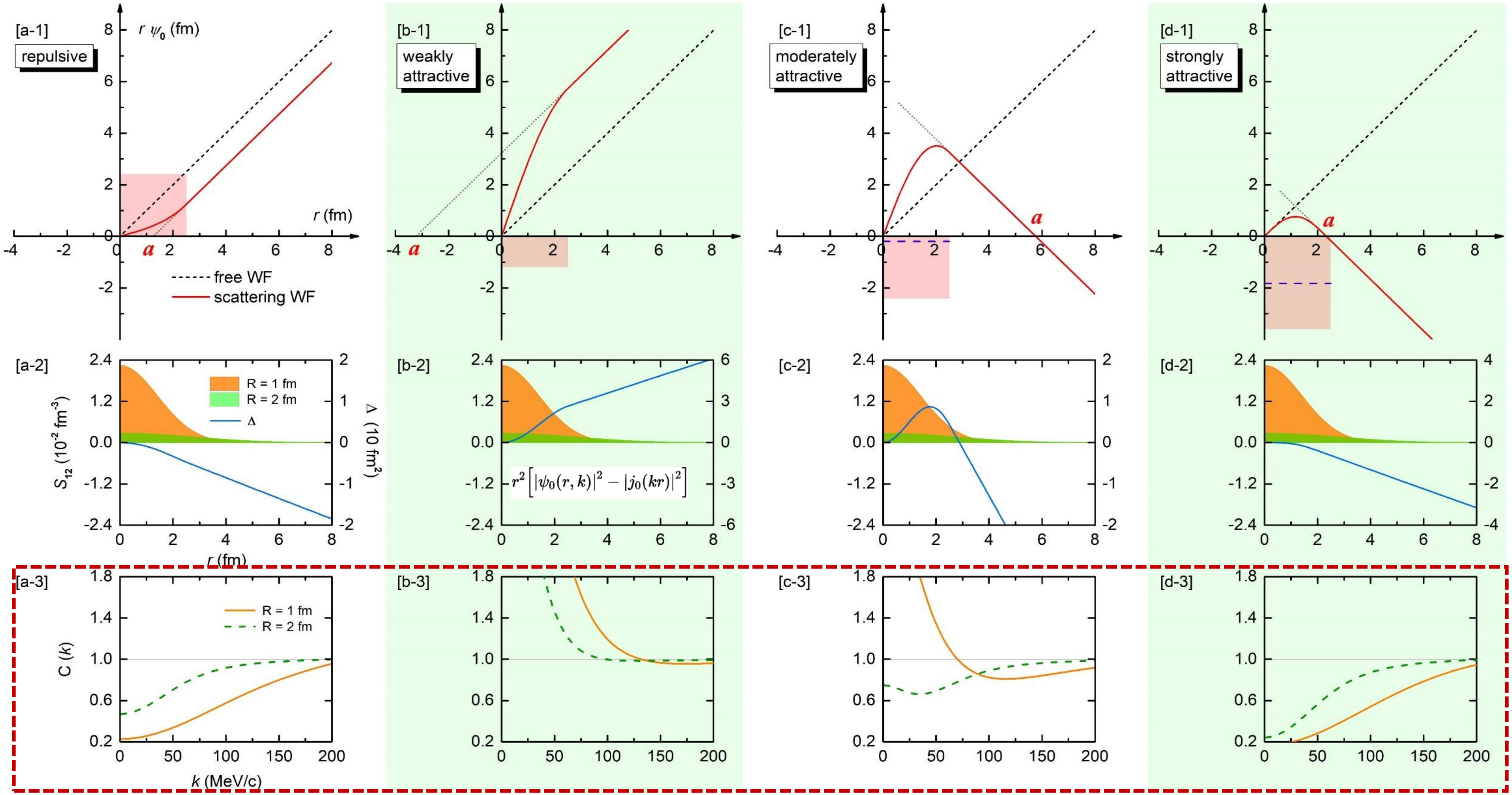
Lingxiao Wang and Jiaxing Zhao, 2411.16343

Classification of hadron-hadron interactions



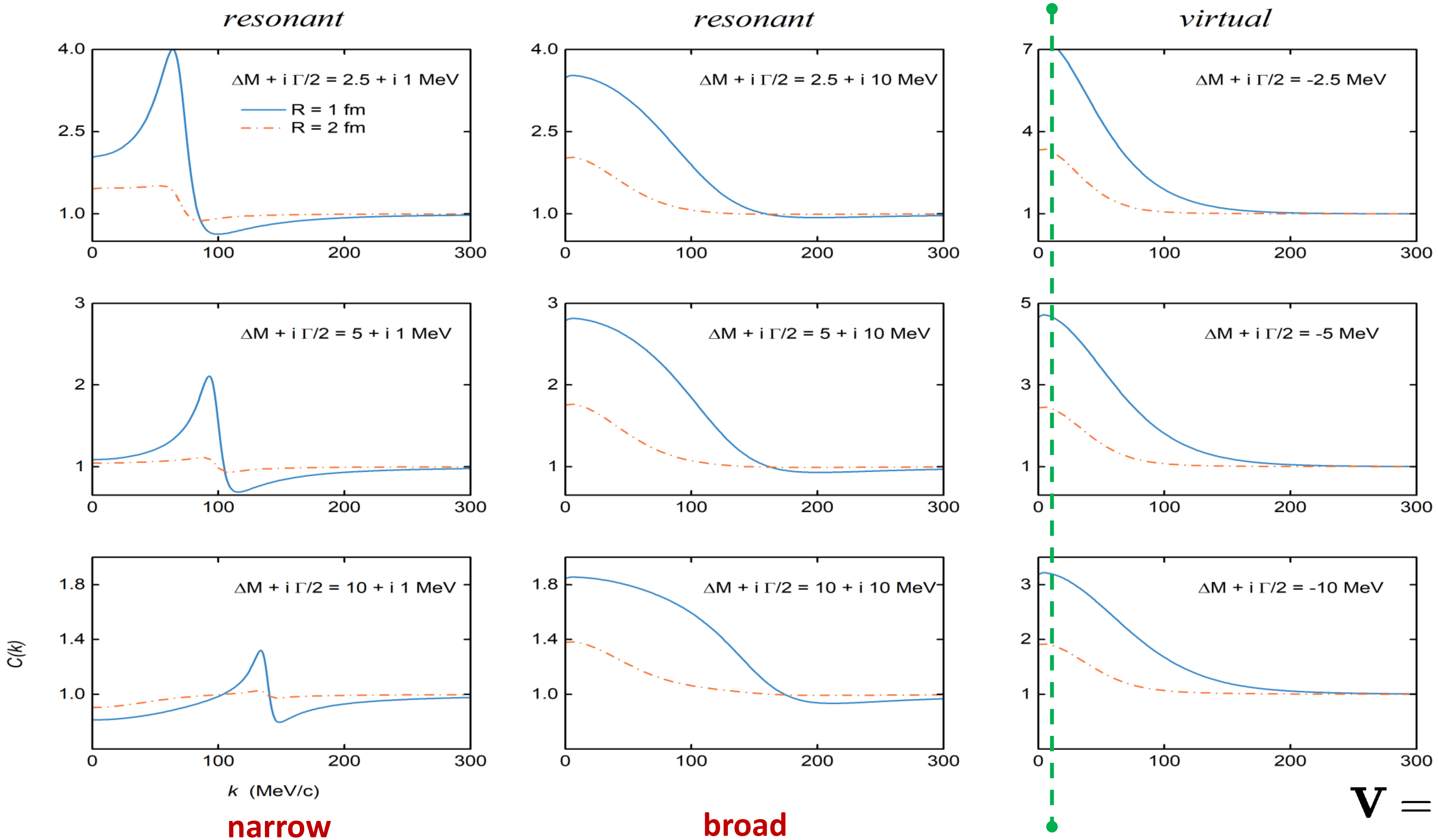
CFs in the presence of bound states

*Zhi-Wei Liu, Jun-Xu Lu and **LSG***, PRD 107, 074019 (2023)*



CFs in the presence of resonant and virtual states

Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG***, 2404.18607



$$\mathbf{V} = \mathbf{a} + \mathbf{b} \cdot \mathbf{k}^2$$

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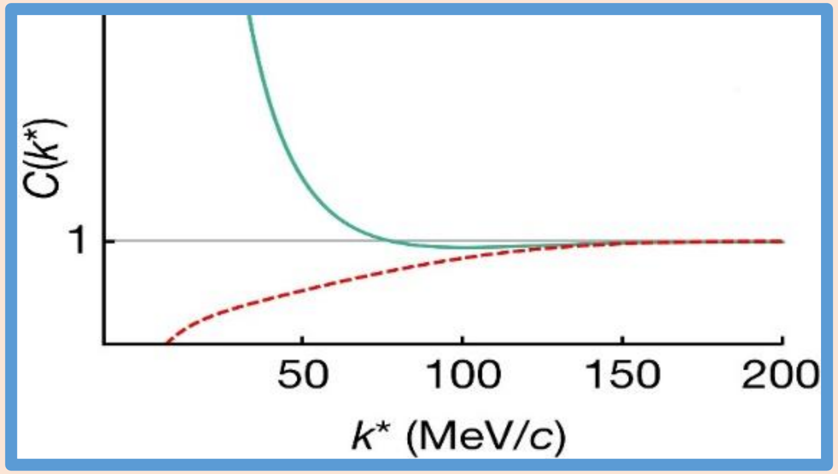
- ☞ **Summary and outlook**

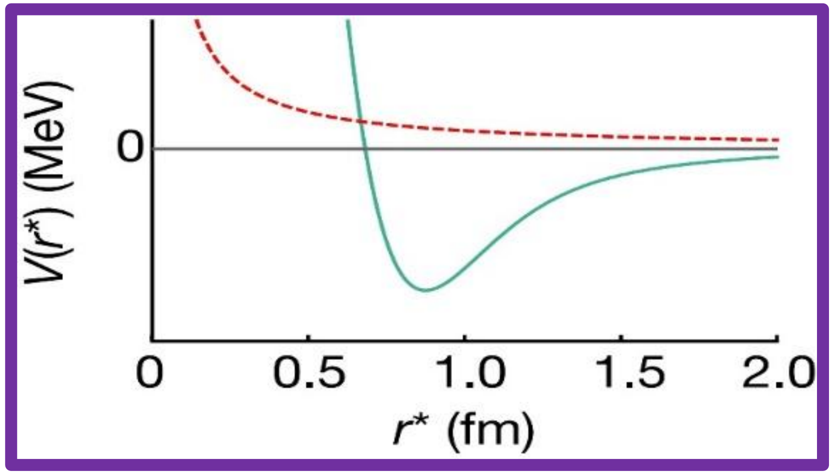
Basic philosophy

The interaction is known from, e.g., experiments, lattice QCD, or effective field theories

Direct

Predict





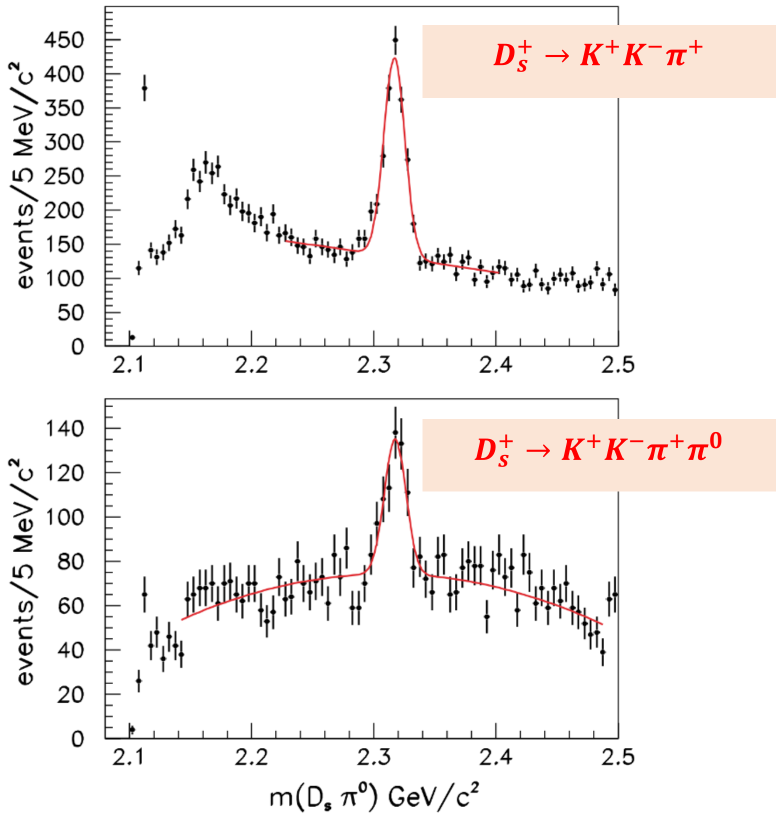
Inverse

Extract

Experiments measure the correlation functions

Mysterious exotic hadron $D_{s0}^*(2317)$

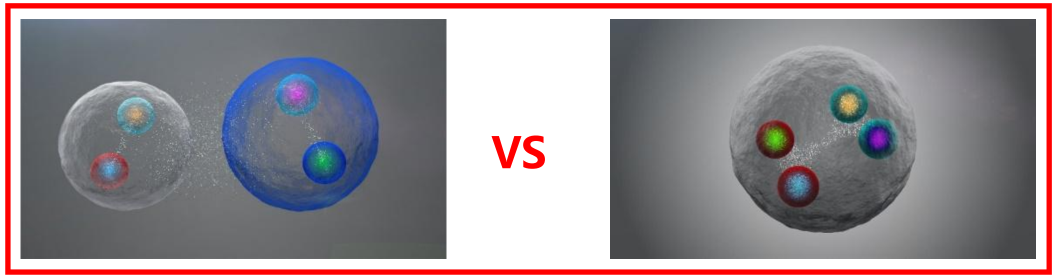
$\mathrm{M} = 2317.8 \pm 0.6$ and $\Gamma < 3.8$ MeV



BABAR, PRL90 (2003) 242001

- **160 MeV lower than the quark model predictions – difficult to understand as a conventional charm-strange meson**

- **It could be a DK bound state**
  - ✓ E. E. Kolomeitsev 2004
  - ✓ F. K. Guo 2006
  - ✓ D. Gamermann 2007



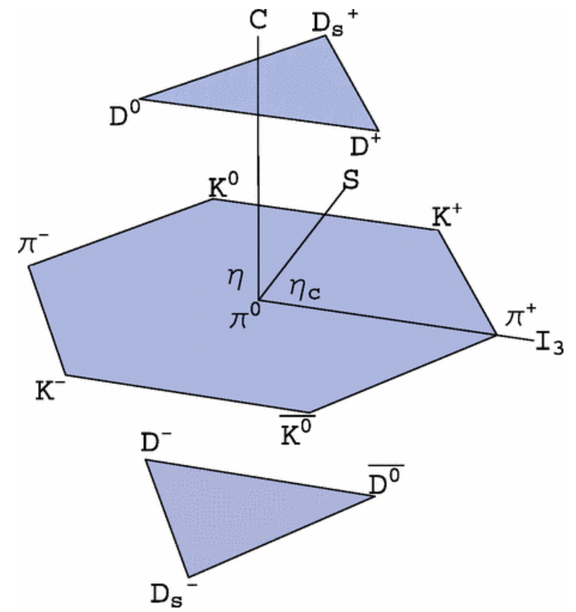
Weinberg-Tomozawa Interaction (leading order)

- **LO interaction between a NGB and a heavy pseudoscalar boson**

$$\mathcal{L} = \frac{1}{4f_\pi^2}\left(\partial^\mu P[\Phi, \partial_\mu \Phi]P^\dagger - P[\Phi, \partial_\mu \Phi]\partial^\mu P^\dagger\right)$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$P = \left(D^0, D^+, D_s^+\right)$$

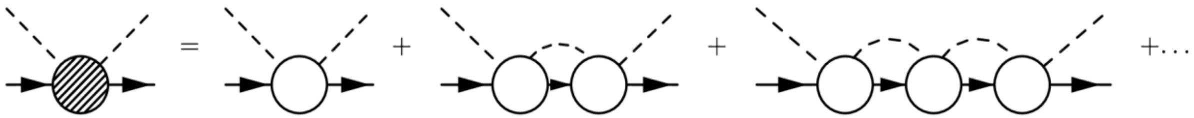


- **Weinberg-Tomozawa (WT) potential – parameter free**

$$V_{\nu'\nu} = \frac{C_{\nu'\nu}}{4f_0^2}\left[(p_1+p_2)^2 - (p_1-p_4)\right] \qquad p_{1(3)} = (E_{1(3)}, \boldsymbol{p}^{(\prime)}), \quad p_{2(4)} = (\sqrt{s} - E_{1(3)}, -\boldsymbol{p}^{(\prime)})$$

Scattering wave function

☐ Coupled-channel scat. eq.



$$T_{\nu'\nu}(k',k) = V_{\nu'\nu}\cdot f_{\Lambda_F}(k',k) + \sum_{\nu''}\int_0^\infty \frac{\mathrm{d}k''k''^2}{8\pi^2}\frac{V_{\nu'\nu''}\cdot f_{\Lambda_F}(k',k'')\cdot T_{\nu''\nu}(k'',k)}{E_{P,\nu''}E_{\Phi,\nu''}(\sqrt{s}-E_{P,\nu''}-E_{\Phi,\nu''}+i\epsilon)}$$

$$f_{\Lambda_F}(k',k) = \exp\left[-\left(\frac{k'}{\Lambda_F}\right)^2 - \left(\frac{k}{\Lambda_F}\right)^2\right]$$

$M_{D_{s0}^*} = 2317.8\ \mathrm{MeV} \rightarrow \Lambda_F = 1107\ \mathrm{MeV}$

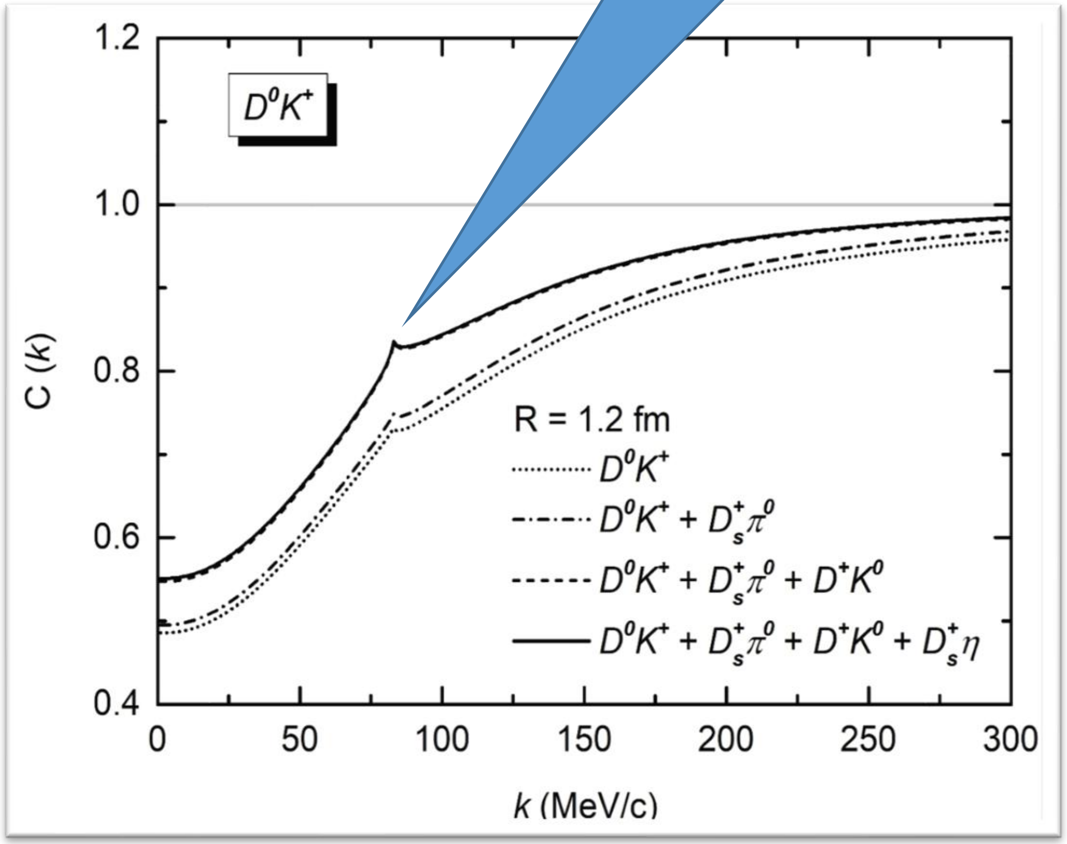
☐ S-wave scattering wave function (including off-shell effect)

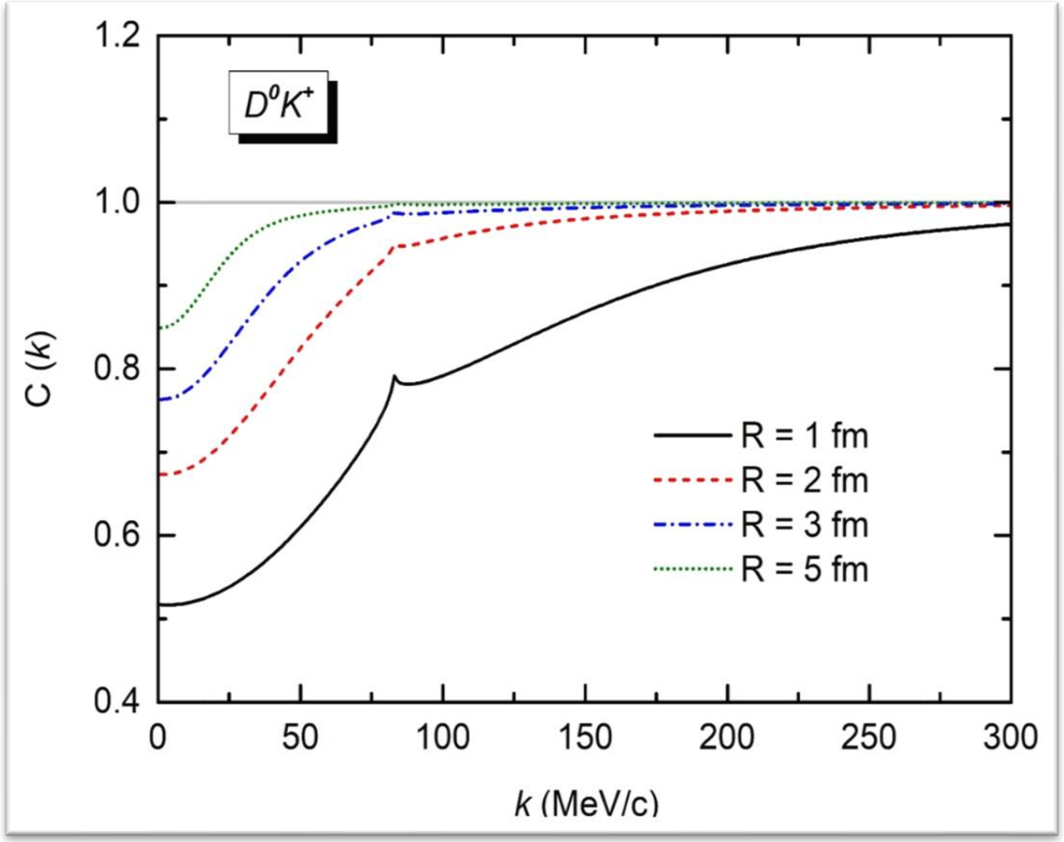
$$\psi_{\nu'\nu}(k,r) = \delta_{\nu'\nu}j_0(kr) + \int_0^\infty \frac{\mathrm{d}k'k'^2}{8\pi^2}\frac{T_{\nu'\nu}(k',k)\cdot j_0(k'r)}{E_{P,\nu'}E_{\Phi,\nu'}(\sqrt{s}-E_{P,\nu'}-E_{\Phi,\nu'}+i\epsilon)}$$

DK CFs and its source size dependence

$D^+K^0 \rightarrow D^0K^+$

Typical feature of deeply bound states





Zhi-Wei Liu, Jun-Xu Lu and ***LSG*****, PRD107(2023)074019*

Confirmed by two subsequent studies

PHYSICAL REVIEW D **108,** 014020 (2023)

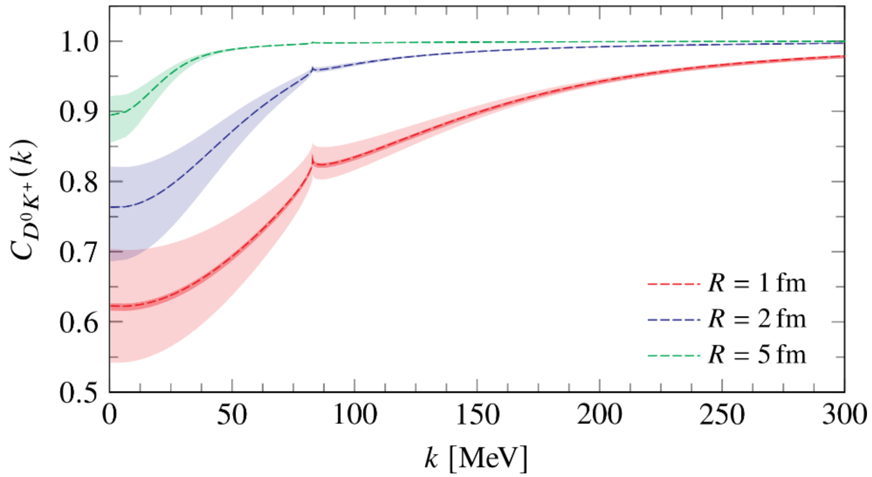
Femtoscopic signatures of the lightest *S*-wave scalar open-charm mesons

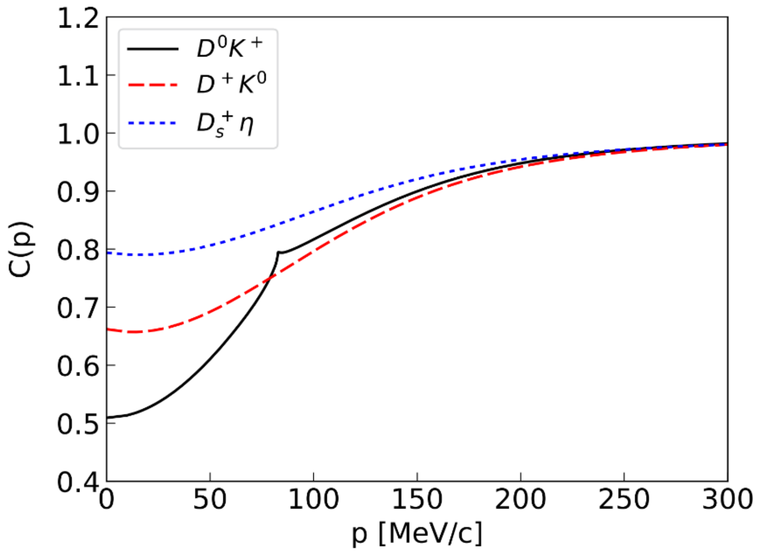
M. Albaladejo,[1,*] J. Nieves,[1,†] and E. Ruiz Arriola[2,‡]

[1] *Instituto de Física Corpuscular (centro mixto CSIC-UV), Institutos de Investigación de Paterna, C/Catedrático José Beltrán 2, E-46980 Paterna, Valencia, Spain*

[2] *Departamento de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, E-18071, Granada, Spain*

(Received 18 April 2023; accepted 26 June 2023; published 19 July 2023)





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Letter

Model independent analysis of femtoscopic correlation functions: An application to the $D^*_{s0}(2317)$

Natsumi Ikeno[a,b,*], Genaro Toledo[c], Eulogio Oset[d]

[a] Department of Agricultural, Life and Environmental Sciences, Tottori University, Tottori 680-8551, Japan
[b] Cyclotron Institute, Texas A&M University, College Station, TX 77843, USA
[c] Instituto de Fisica, Universidad Nacional Autonoma de Mexico, AP20-364, Ciudad de Mexico 01000, Mexico
[d] Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia - CSIC, Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain

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- ☞ **Femtoscopic correlation functions (CFs)—general features**
- ☞ **Recent applications**
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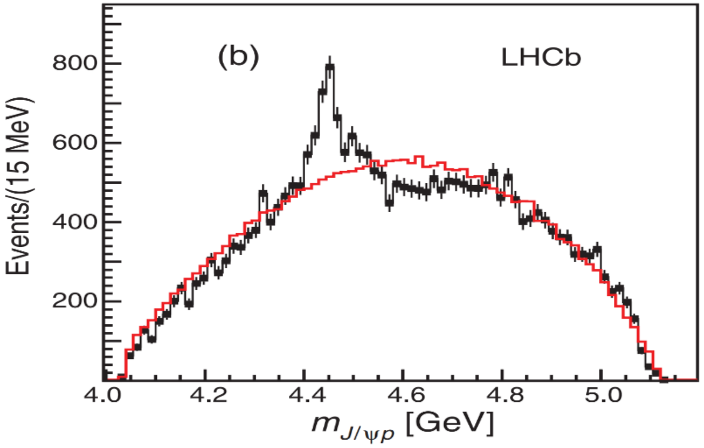
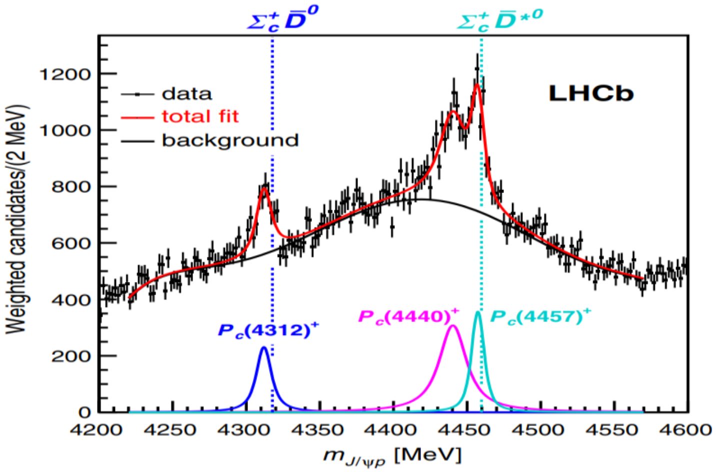
- ☞ **Summary and outlook**

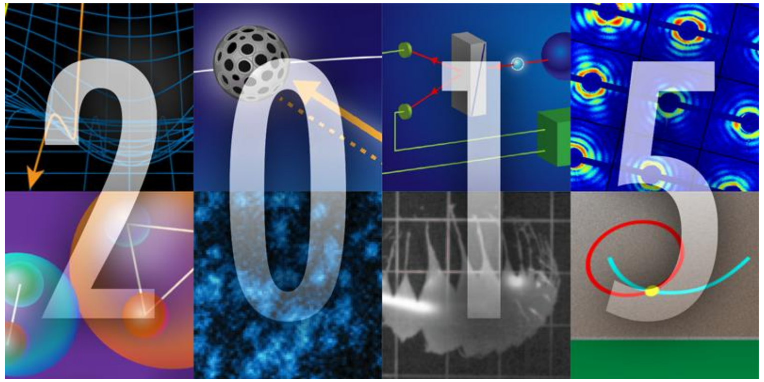
Pentaquark states $P_c(4440)$ & $P_c(4457)$



LHCb, PRL115 (2015) 072001



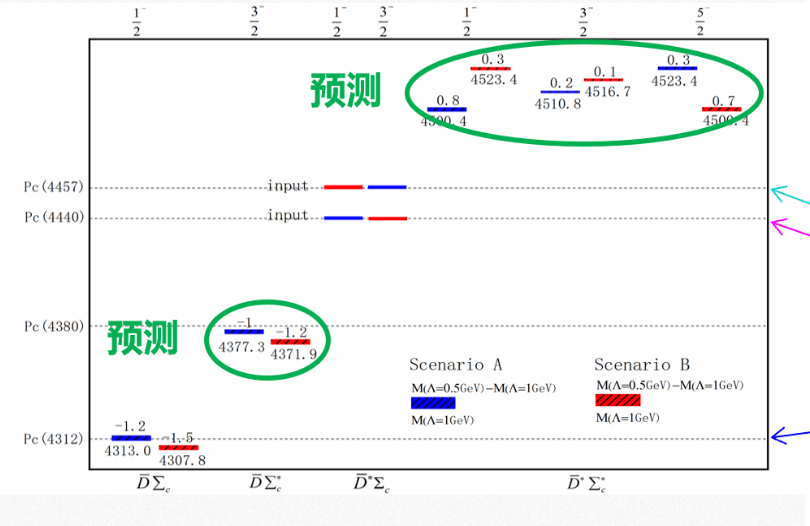
LHCb, PRL122 (2019) 222001

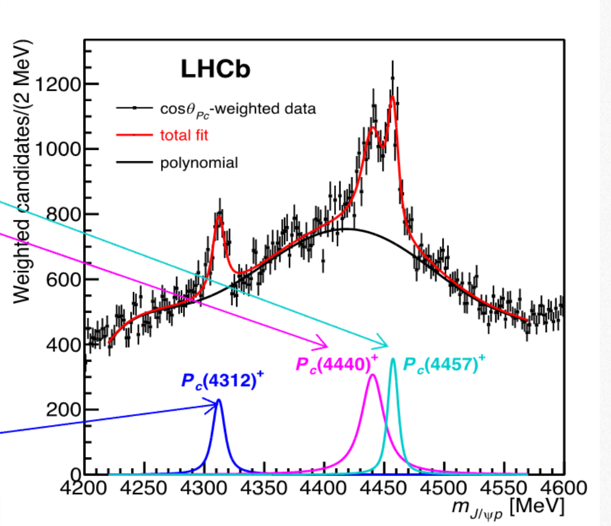


Pentaquark states • 2015 APS Highlights

- **They can be nicely arranged into a $\Sigma_c^{(*)}\bar{D}^{(*)}$ multiplet**

M. Z. Liu, Y. W. Pan, F. Z. Peng, M. Sánchez S, LSG, A. Hosaka, M. P. Valderrama, PR122 (2019) 242001*





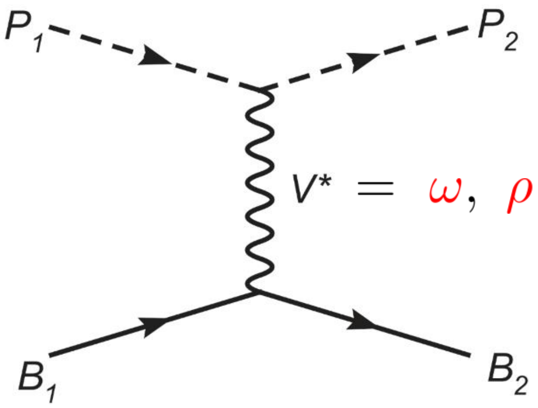
How to distinguish the spins of $P_c(4440)$ and $P_c(4457)$?

Light vector meson exchange interactions

- Interactions in The hidden local symmetry approach – **parameter free**

$$V^{I=\frac{3}{2}}_{\Sigma_c\bar{D}^{(*)}} = 2M_{\Sigma_c}M_{\bar{D}^{(*)}}\widetilde{\beta}_1\widetilde{\beta}_2 g_V^2\left(\frac{1}{m_\omega^2}+\frac{1}{m_\rho^2}\right)$$

$$V^{I=\frac{1}{2}}_{\Sigma_c\bar{D}^{(*)}} = 2M_{\Sigma_c}M_{\bar{D}^{(*)}}\widetilde{\beta}_1\widetilde{\beta}_2 g_V^2\left(\frac{1}{m_\omega^2}-\frac{2}{m_\rho^2}\right)$$



Isospin basis

↕

Charge basis

$$\left|\Sigma_c\bar{D}^{(*)}, I=\frac{3}{2}, I_3=\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\Sigma_c^{++}D^{(*)-}\right\rangle + \sqrt{\frac{2}{3}}\left|\Sigma_c^{+}\bar{D}^{(*)0}\right\rangle$$

$$\left|\Sigma_c\bar{D}^{(*)}, I=\frac{1}{2}, I_3=\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\Sigma_c^{++}D^{(*)-}\right\rangle - \sqrt{\frac{1}{3}}\left|\Sigma_c^{+}\bar{D}^{(*)0}\right\rangle$$

Two different spin assignments

◻ Interaction strengths

$$f_{\Lambda_F}(k', k) = \exp\left[-\left(\frac{k'}{\Lambda_F}\right)^2 - \left(\frac{k}{\Lambda_F}\right)^2\right]$$

$\Lambda_F = 1067\ \mathrm{MeV}$

deep bound state of $\Sigma_c\bar{D}^*$

.itation: R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 upda

$P_c(4440)^+$ Status: *

$P_c(4440)^+$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$4440.3 \pm 1.3^{+4.1}_{-4.7}$	AAIJ	19W	LHCB	pp at 7, 8, 13 TeV

$\Lambda_F = 860\ \mathrm{MeV}$

shallow bound state of $\Sigma_c\bar{D}^*$

$P_c(4457)^+$ Status: *

$P_c(4457)^+$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$4457.3 \pm 0.6^{+4.1}_{-1.7}$	AAIJ	19W	LHCB	pp at 7, 8, 13 TeV
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$4449.8 \pm 1.7 \pm 2.5$	[1] AAIJ	15P	LHCB	Repl. by AAIJ 19W

[1] Considering $P_c(4440)$ and $P_c(4457)$ as a single resonance.

CF for the shallow bound state is **significantly larger** than that for the deep bound

◻ Experimental CFs – spin-averaged

Scenarios A

$\mathrm{P_c(4440) : J^P = (1/2)^-}$ $\quad$ $\mathrm{P_c(4457) : J^P = (3/2)^-}$

$$\bar{\mathrm{C}} = 1/3 \cdot \mathrm{C_{deep}} + 2/3 \cdot \mathrm{C_{shallow}}$$

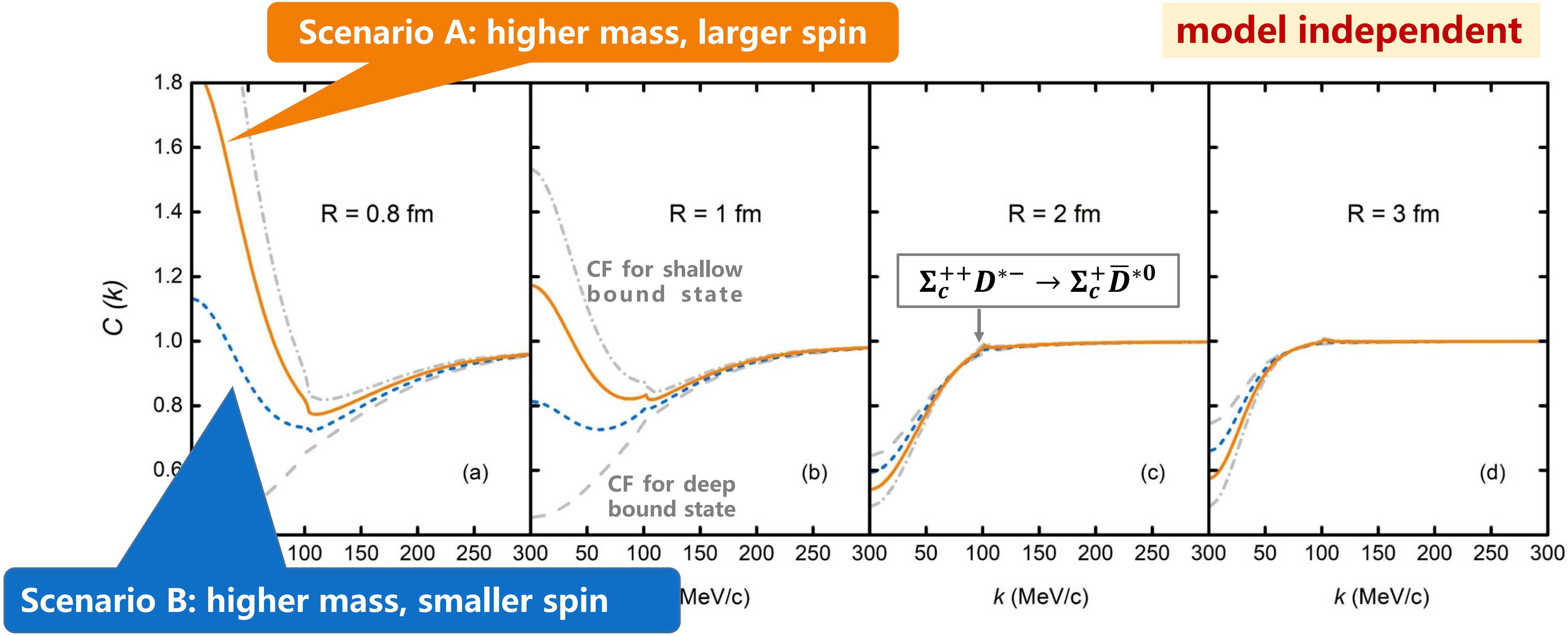
Scenarios B

$\mathrm{P_c(4440) : J^P = (3/2)^-}$ $\quad$ $\mathrm{P_c(4457) : J^P = (1/2)^-}$

$$\bar{\mathrm{C}} = 2/3 \cdot \mathrm{C_{deep}} + 1/3 \cdot \mathrm{C_{shallow}}$$

Spin-averaged $\Sigma_c\bar{D}^*$ CFs

model independent



*Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG***, PRD108(2023)L031503*

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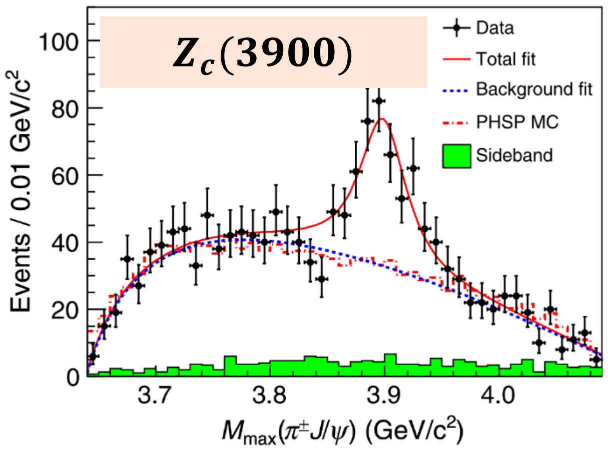
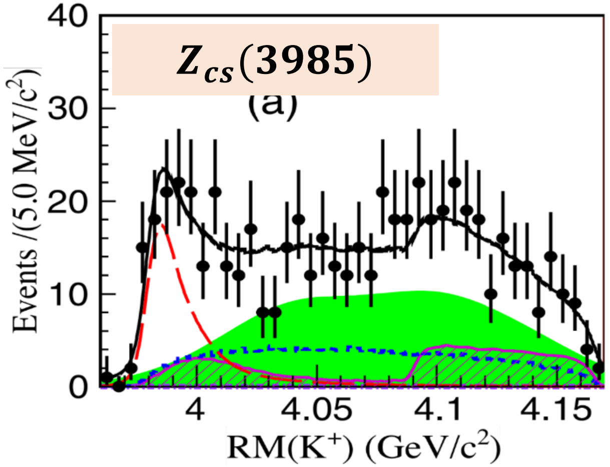
- ☞ **Summary and outlook**

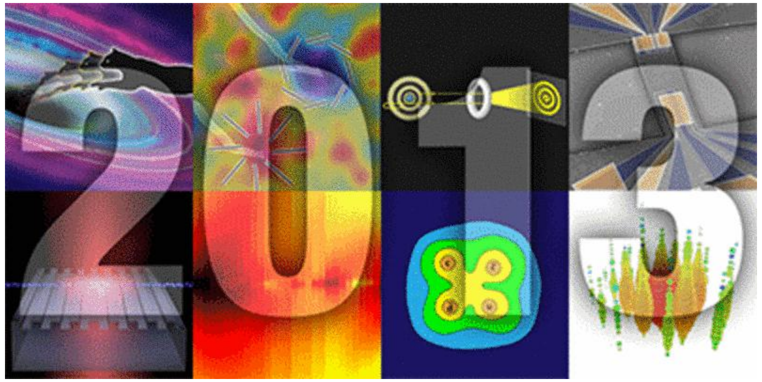
Tetraquark states $Z_c(3900)$ & $Z_{cs}(3985)$



BESIII, PRL110 (2013) 252001



BESIII, PRL126 (2021) 102001



Tetraquark states • 2013 APS Highlights

$Z_c(3900)$ & $Z_{cs}(3985)$: Resonant VS Virtual states

Particle Data Group, PTEP 2022 (2022) 083C01

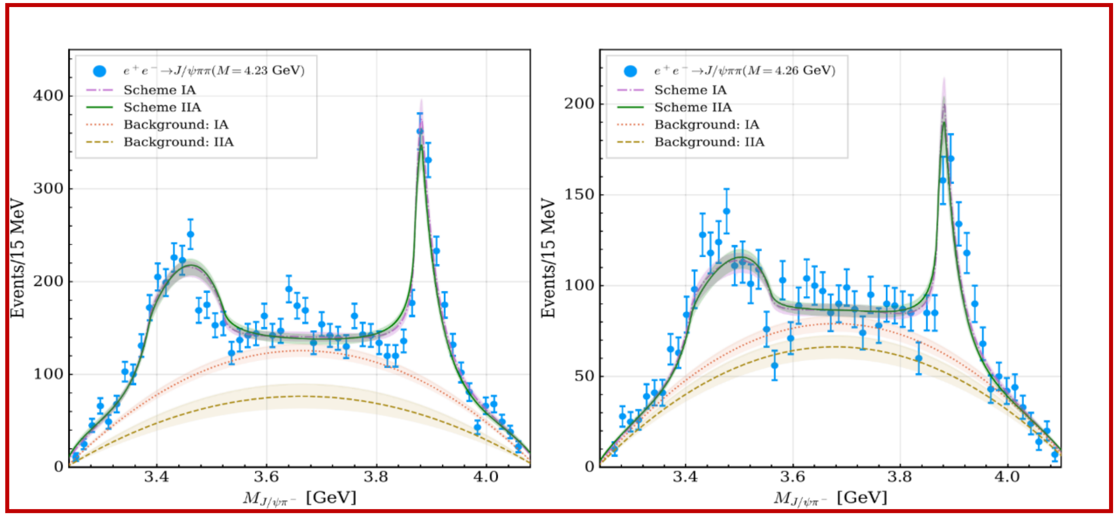
M.-L. Du, M. Albaladejo, F.-K. Guo and J. Nieves, PRD 105 (2022) 074018

T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, F.-K. Guo and J. Nieves, PRD106 (2022) 094002

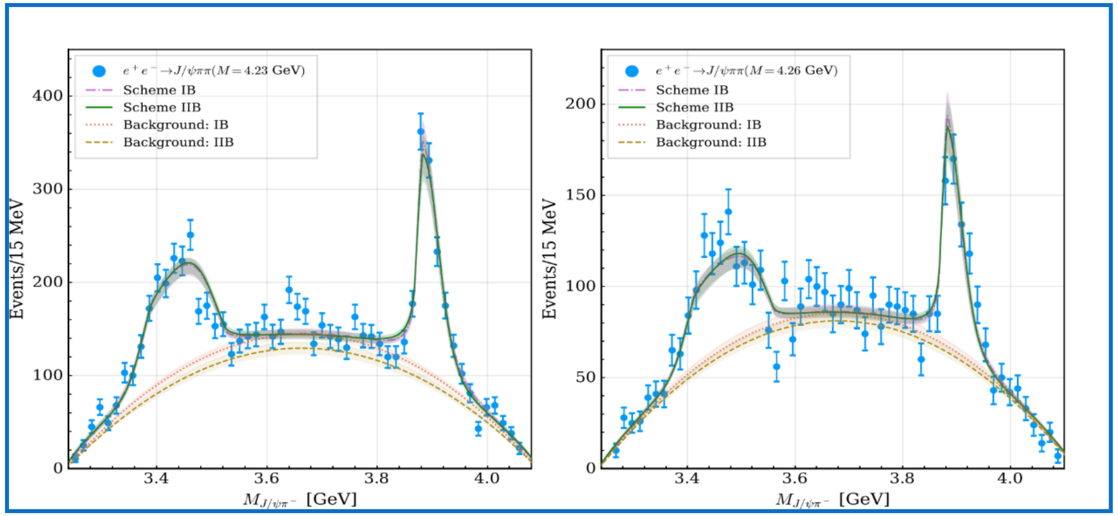
L.-W. Yan, Z.-H. Guo, F.-K. Guo, D.-L. Yao and Z.-Y. Zhou, PRD109 (2024) 014026

Invariant mass distributions fail to distinguish vir. or res.

Virtual state scenario



Resonant state scenario

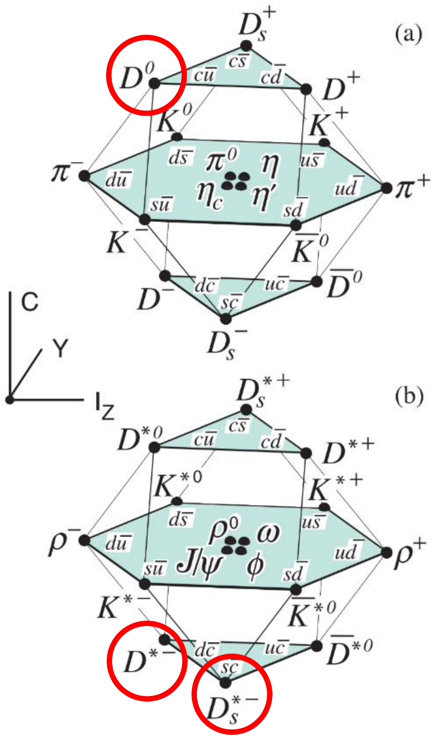


M.-L. Du, M. Albaladejo, F.-K. Guo, and J. Nieves, PRD105(2022)074018

Data are compatible with either a resonant or virtual state.

How to tell which is reality?

General potential from EFTs



◻ Interaction between heavy pseudoscalar bosons

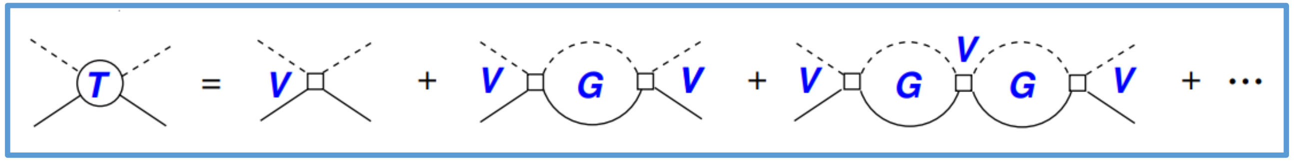
$$V = a + b \cdot k^2, \qquad k = \sqrt{[s-(m_1+m+2)^2][s-(m_1-m+2)^2]}/2\sqrt{s}$$

- ➢ energy-dependent potential —→ resonant state
- ➢ contact-range potential —→ bound or virtual state

◻ Scattering equation – unitarity

$$T = V + VGT$$

⟷



Loop function G with cutoff regularization

$$G(\sqrt{s}) = \int_0^{|q|<q_{max}} \frac{d^3k'}{(2\pi)^3} \frac{E_1(k')+E_2(k')}{2E_1(k')E_2(k')} \frac{1}{\sqrt{s}^2 - [E_1(k')+E_2(k')]^2 + i\varepsilon}, \qquad q_{max} \in [0.8, 1.2]\ \mathrm{GeV}$$

Interaction strengths determined by fitting to data

	Scenario	M [MeV]	Γ [MeV]	m_1+m_2 [MeV]	a	b [MeV^{-2}]
$Z_c(3900)$	Res. [95]	3887.1	28.4	$D^0 D^{*-}$ (3875.1)	-101.68	-1380.60
	Vir. [27]	3796	0	$D^0 D^{*-}$ (3875.1)	-87.36	0
$Z_{cs}(3985)$	Res. [95]	3988	13	$D^0 D_s^{*-}$ (3977.04)	-84.17	-2894.16
	Vir. [27]	3967	0	$D^0 D_s^{*-}$ (3977.04)	-130.21	0

【95】 Particle Data Group, PTEP 2022,(2022)083C01
【27】 M.-L. Du, M. Albaladejo, F.-K. Guo, and J. Nieves, PRD105(2022)074018

☐ Correlation functions with on-shell approximation

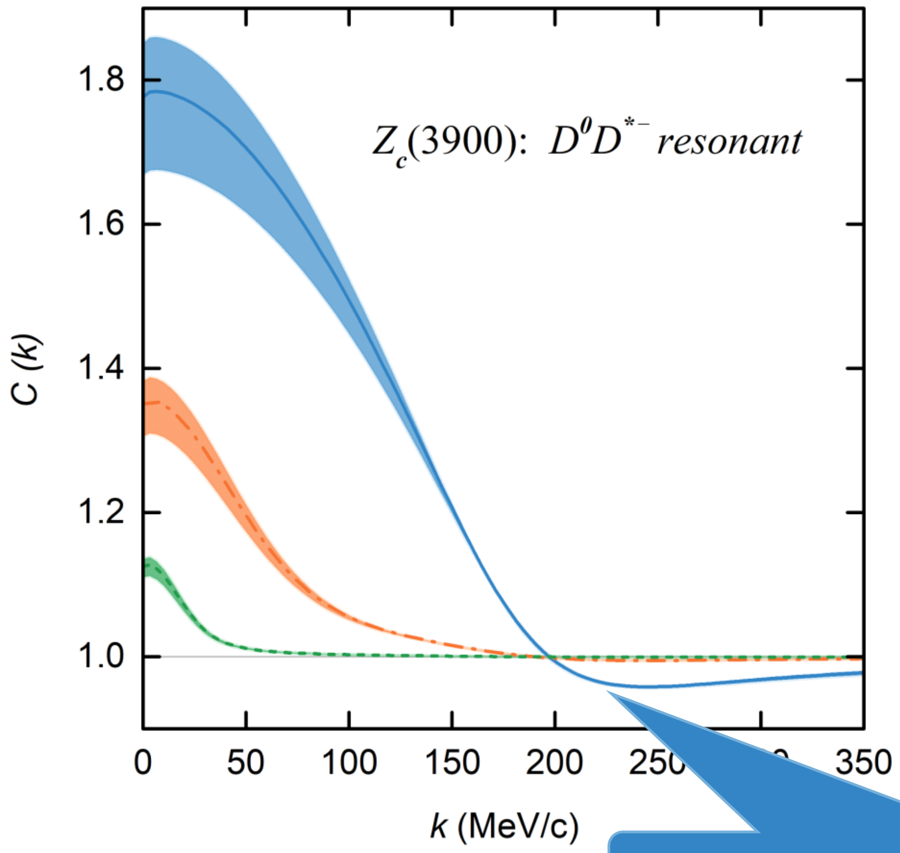
$$\mathbf{C(k)=1+\int_0^\infty 4\pi r^2 dr\; S_{12}(r)\; \theta(q_{max}-k)\left[\left|j_0(kr)+T(\sqrt{s})\;\widetilde{G}(r,\sqrt{s})\right|^2-|j_0(kr)|^2\right]}$$

$$\mathbf{\widetilde{G}(r,\sqrt{s})=\int_0^{|q|<q_{max}}\frac{d^3k'}{(2\pi)^3}\frac{E_1(k')+E_2(k')}{2E_1(k')E_2(k')}\frac{j_0(k'r)}{\sqrt{s}^2-[E_1(k')+E_2(k')]^2+i\varepsilon}}$$

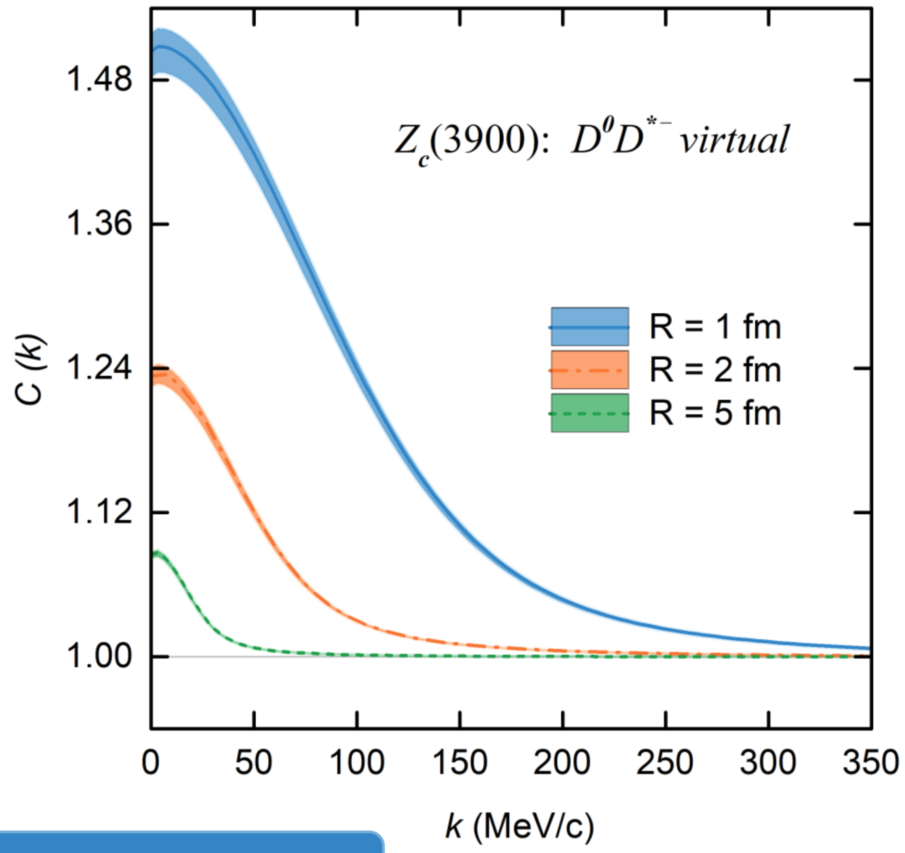
$D^0 D^{*-}$ CFs for $Z_c(3900)$

*Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG***, 2404.18607*

Resonant state scenario



Virtual state scenario

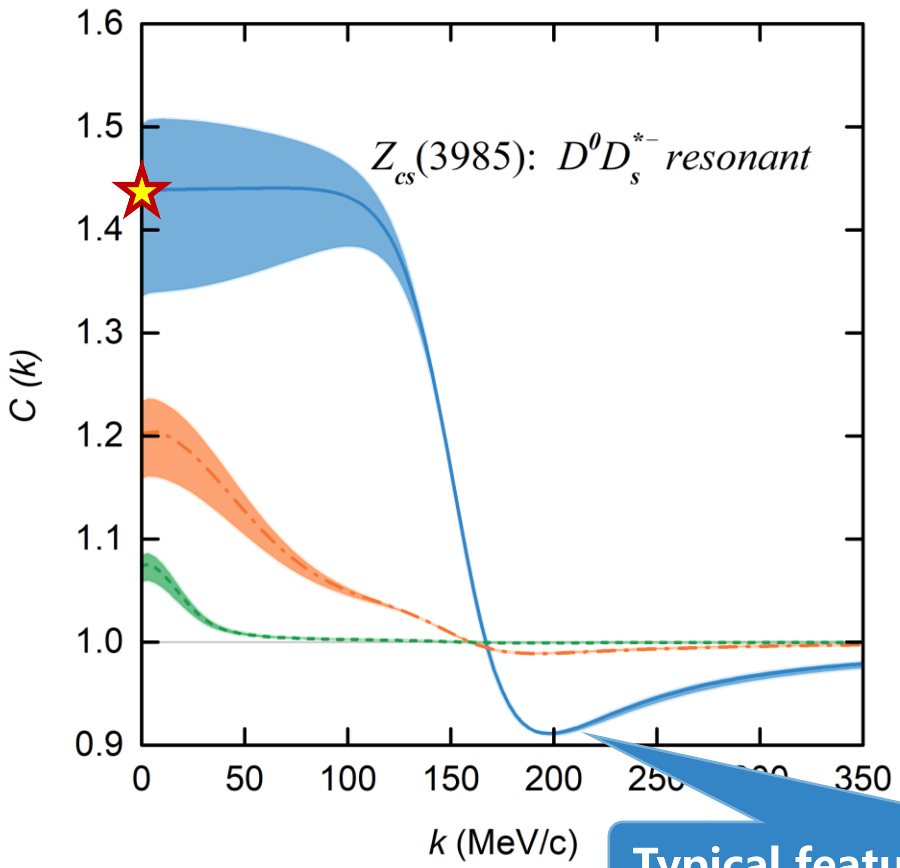


Typical feature of broad resonant state

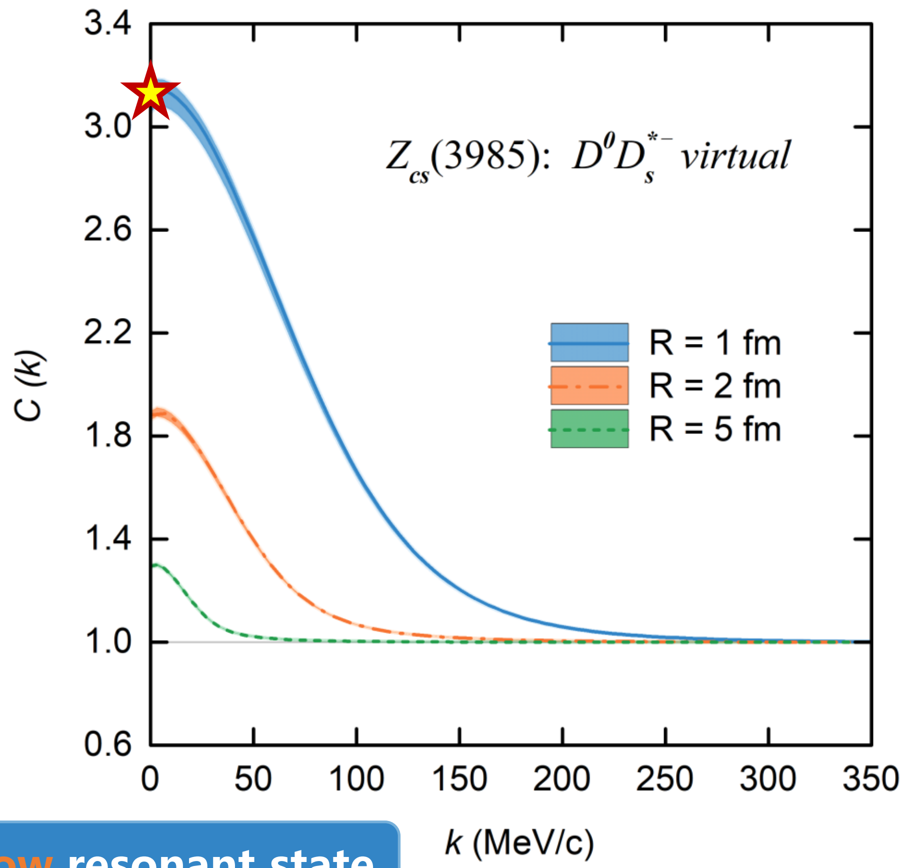
$D^0 D_s^{*-}$ CFs $Z_{cs}(3985)$

Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG***, 2404.18607

Resonant state scenario



Virtual state scenario



Typical feature of narrow resonant state

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Summary and outlook

- Femtoscopy offers high-precision tests of the strong interaction between pairs of (un)stable particles and can be valuable to decipher the nature of the many exotic hadrons discovered so far.

✓ $D_{s0}^*(2317)$,$P_c(4440)$ and $P_c(4457)$, $Z_c(3900)/Z_{cs}(3985)$

- More recent studies not covered in this talk

✓ J/Ψ-N and η_c-N correlation functions with lattice QCD phaseshifts—in relation to the tetra-charm X(6200), 2504.04853

✓ Deuteron-deuteron interactions in comparison with the STAT data—in relation to nuclear clusters, 2502.18872

✓ J/Ψ- J/Ψ correlation functions with EFT potential—in relation to the pentaquark states, in preparation

Summary and outlook

- ◻ With more data from LHC Run3/4/5, more two-hadron correlations involving s, c, b quarks, and even three-particle correlations can be studied

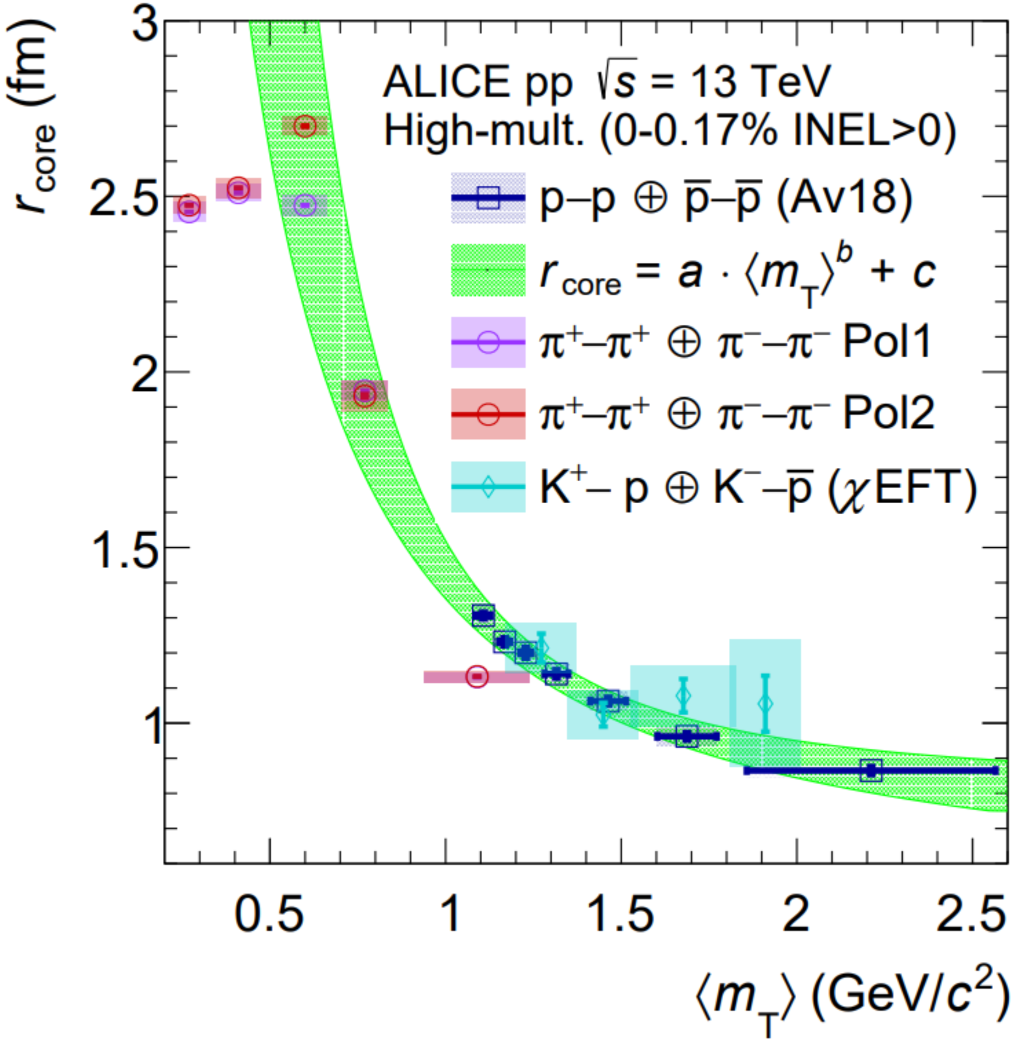
- ◻ Many questions remain unanswered, at least not satisfactorily
- ✓ Is the factorization of the KP formula well justified
- ✓ Can one define a universal source function
- ✓ How to study three-body correlation functions
- ✓ …

Thanks a lot for your attention!

(Image: CERN)

Universal source function

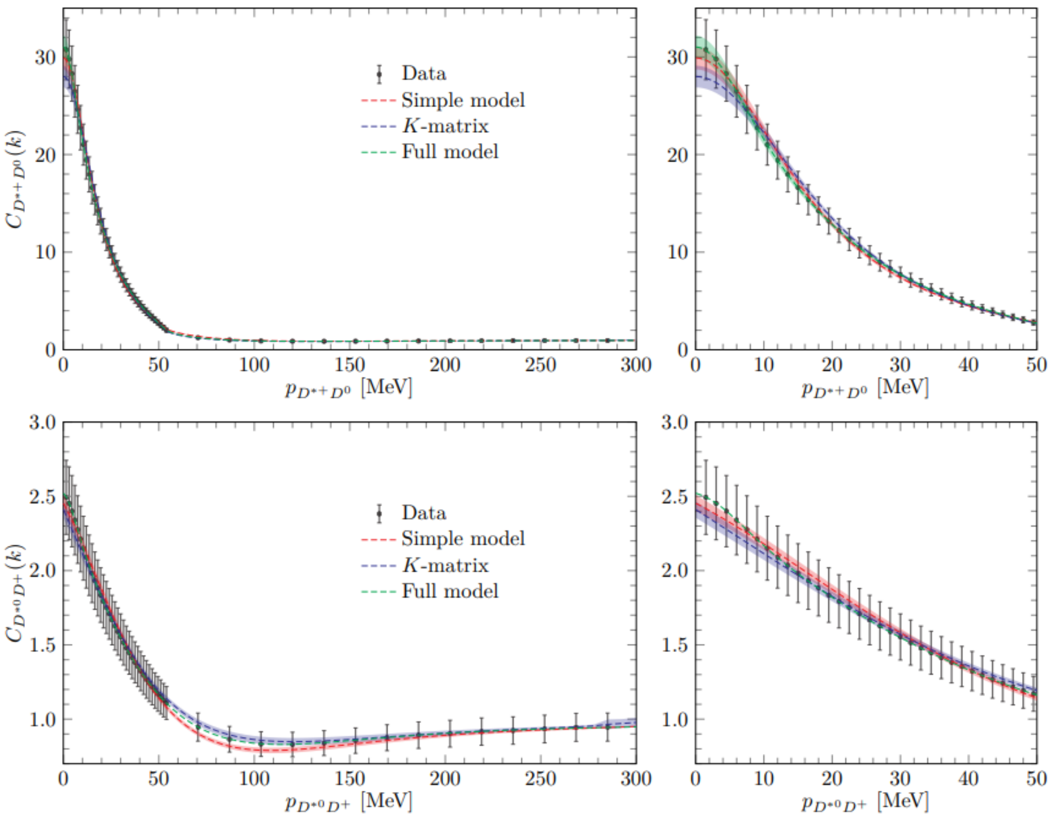
2311.14527v1.



… "providing compelling evidence for the presence of a common emission source for all hadrons in small collision systems at the LHC …"

Inverse method

- **One can also perform inverse studies and extract hadron-hadron interaction from the exp. CF data**



Inverse problem in femtoscopic correlation functions: The Tcc(3875)+ state,
Albaladejo , Feijoo , Vidaña , Nieves , and Oset, 2307.09873

$$C_{D^0D^{*+}}(p_{D^0}) = 1 + 4\pi\,\theta(\Lambda - p_{D^0}) \int_0^\infty drr^2 S_{12}(r) \times \left\{ \left| j_0(p_{D^0}r) + T_{11}(s)\widetilde{G}_1(r;s) \right|^2 + \left| T_{12}(s)\widetilde{G}_2(r;s) \right|^2 - j_0^2(p_{D^0}r) \right\} , \quad (1)$$

$$C_{D^+D^{*0}}(p_{D^+}) = 1 + 4\pi\,\theta(\Lambda - p_{D^+}) \int_0^\infty drr^2 S_{12}(r) \times \left\{ \left| j_0(p_{D^+}r) + T_{22}(s)\widetilde{G}_2(r;s) \right|^2 + \left| T_{12}(s)\widetilde{G}_1(r;s) \right|^2 - j_0^2(p_{D^+}r) \right\} , \quad (2)$$