Theoretical Overview on Neutrino Physics

周顺
中科院高能所

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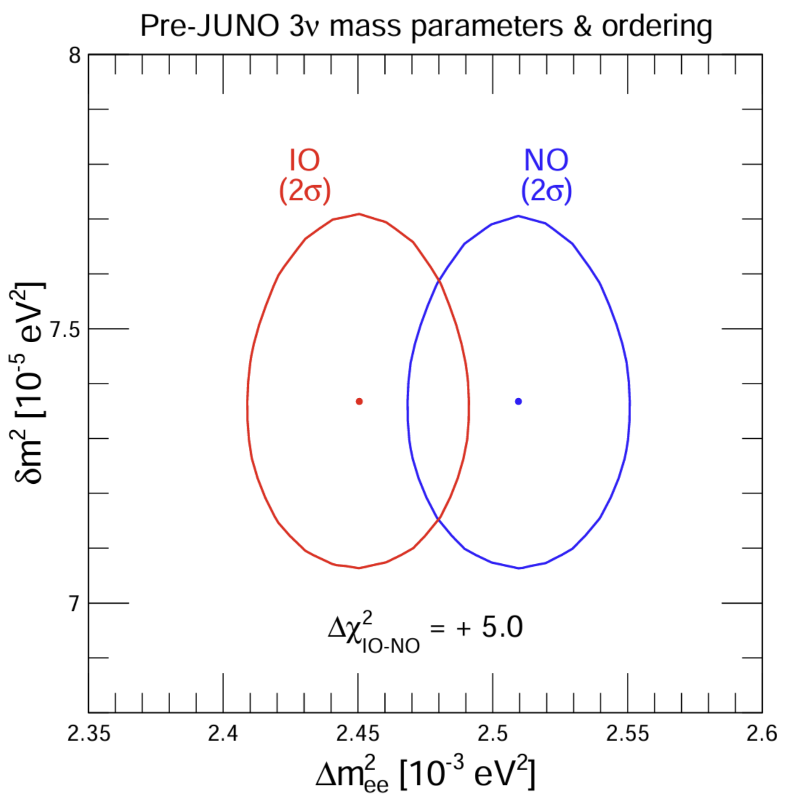
Current status of neutrino oscillations

Latest global-fit analysis of neutrino oscillation data

Esteban et al., 2410.05380 **NuFIT 6.0 (2024)**



IC24 with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
$\delta_{\text{CP}}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

PHYSICAL REVIEW D **111,** 093006 (2025)

2503.07752

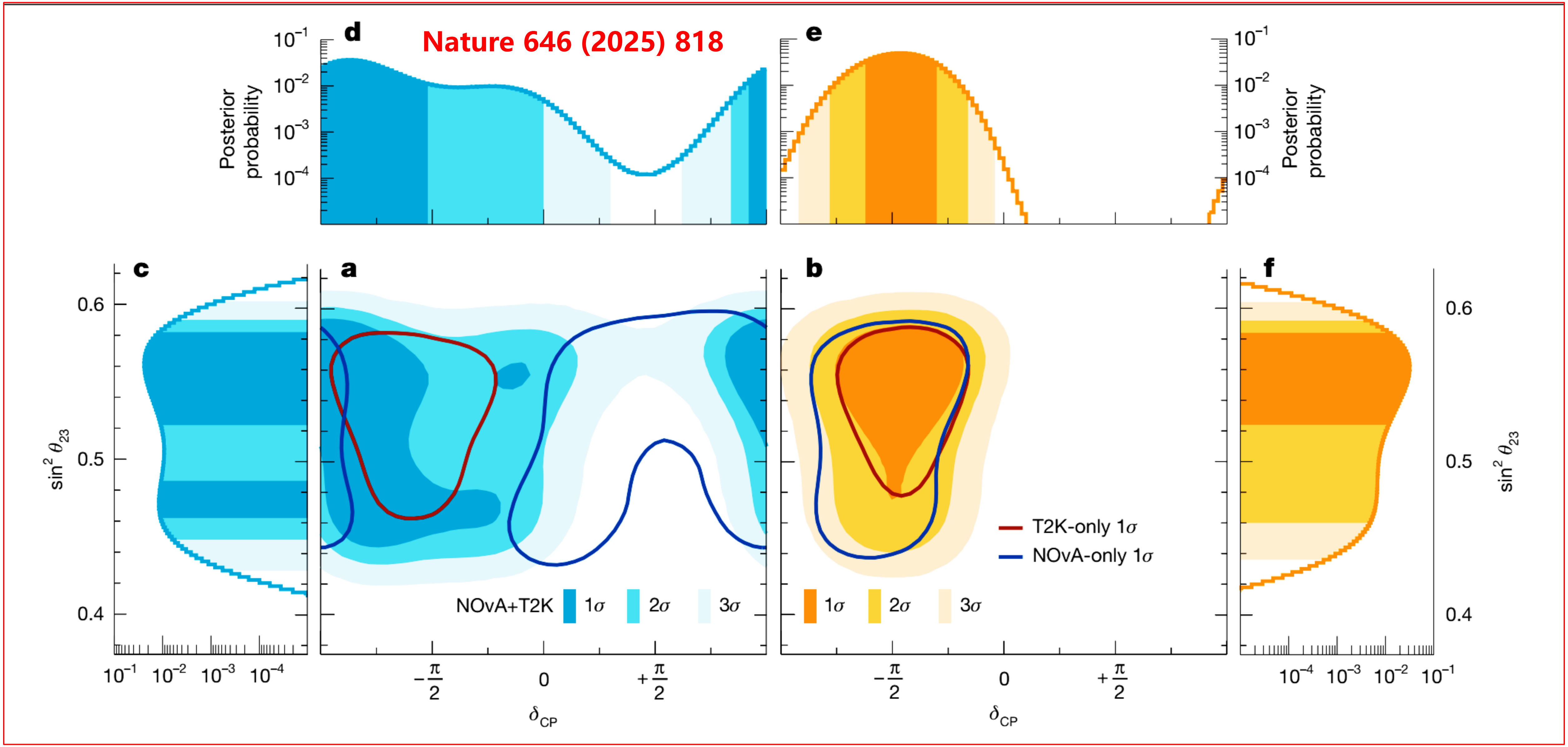
Neutrino masses and mixing: Entering the era of subpercent precision

Francesco Capozzi,[1,2] William Giarè,[3] Eligio Lisi,[4] Antonio Marrone,[5,4] Alessandro Melchiorri,[6,7] and Antonio Palazzo[5,4]

More challenging questions:

- Absolute neutrino masses
- Majorana nature of neutrinos
- Origin of neutrino masses

Latest results from T2K+NOvA

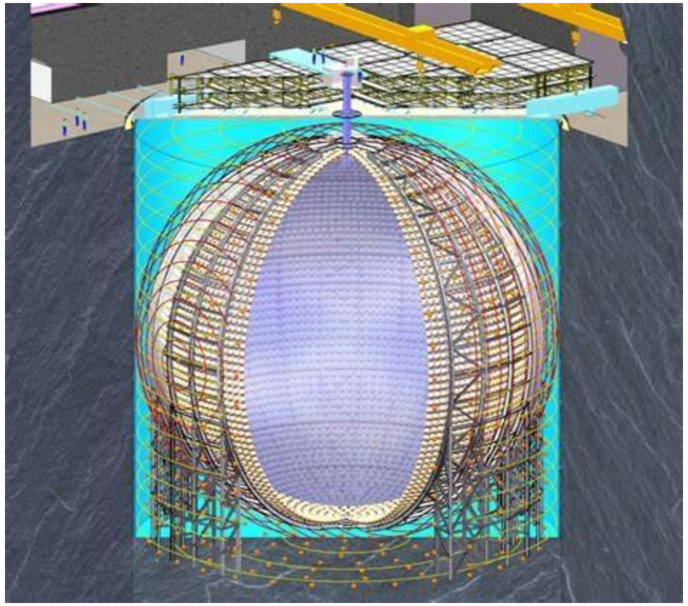


Normal Ordering

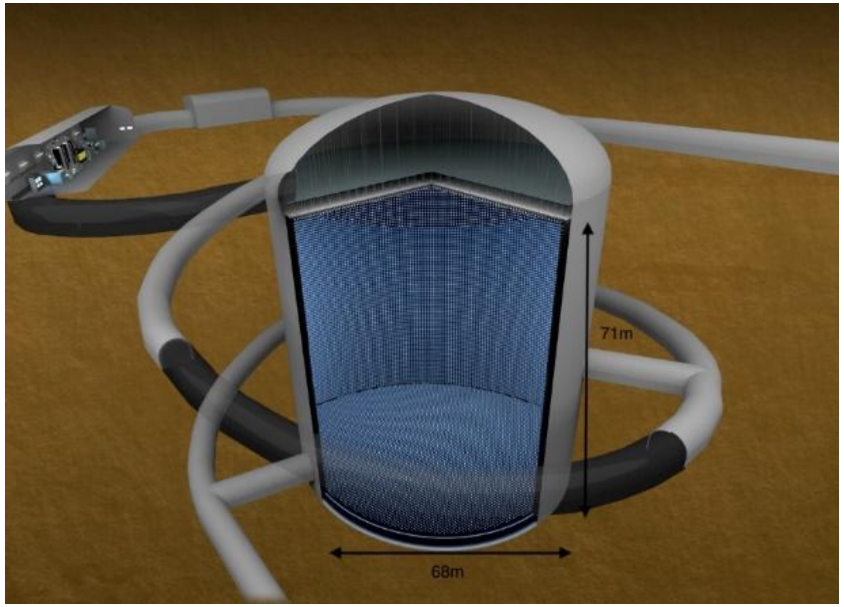
Inverted Ordering

Future neutrino oscillation experiments

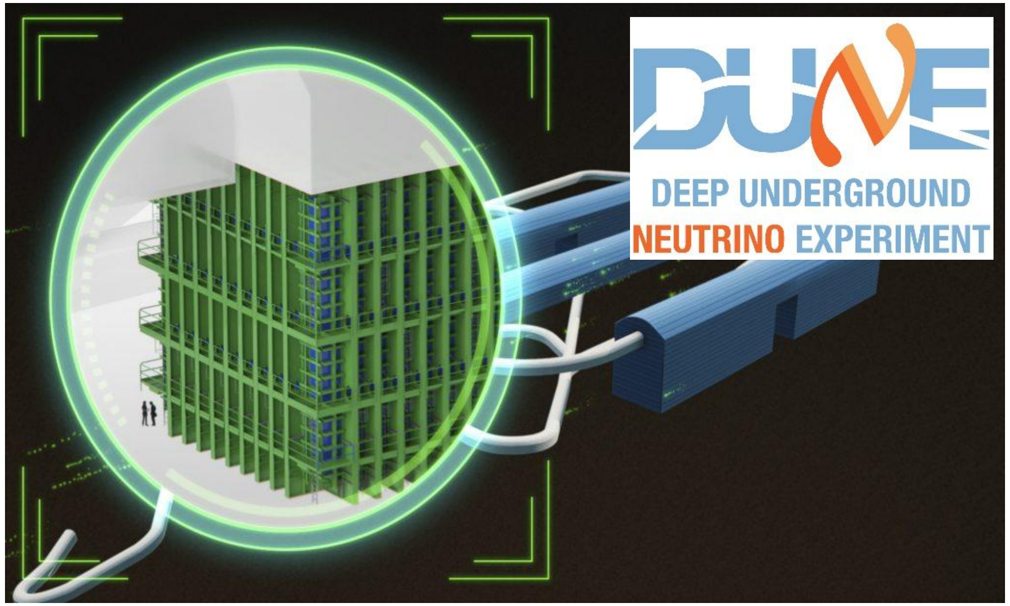
Precision era: 1. neutrino mass ordering; 2. mixing angles & mass-squared diff.s; 3. CP-violating phase



JUNO: 2025
20 kt liquid scintillator



Hyper-Kamiokande: 2027
260 kt water Cerenkov



DUNE: 2030
34 kt liquid Argon

JUNO collaboration, CPC 46 (2022) 123001

	Central Value	PDG2020	6 years
Δm^2_{31} ($\times 10^{-3}$ eV^2)	2.5283	± 0.034 (1.3%)	± 0.0047 (0.2%)
Δm^2_{21} ($\times 10^{-5}$ eV^2)	7.53	± 0.18 (2.4%)	± 0.024 (0.3%)
$\sin^2\theta_{12}$	0.307	± 0.013 (4.2%)	± 0.0016 (0.5%)

DUNE collaboration, 2002.03005

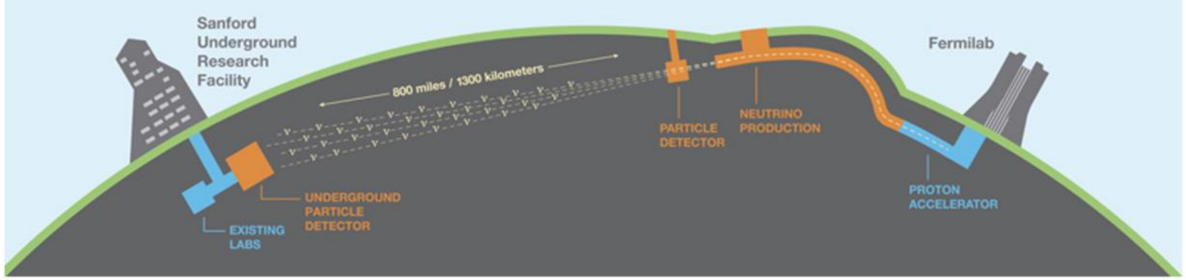
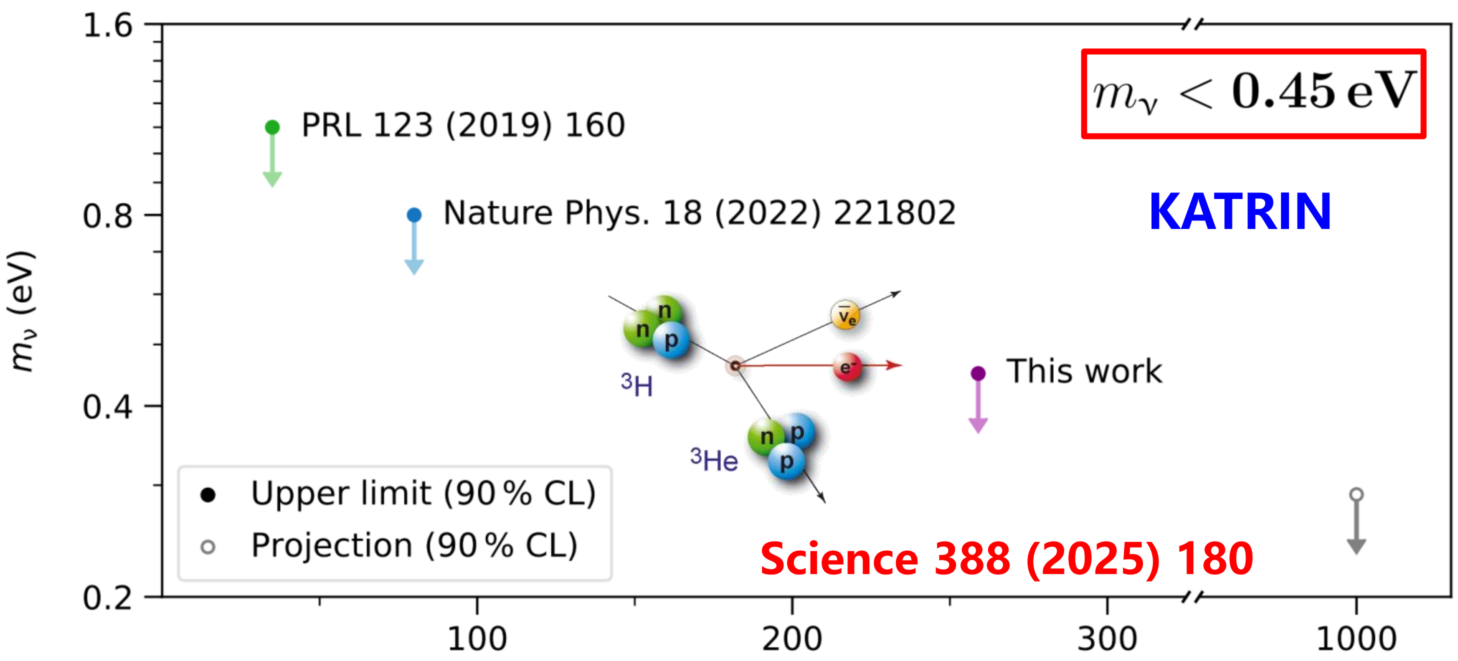
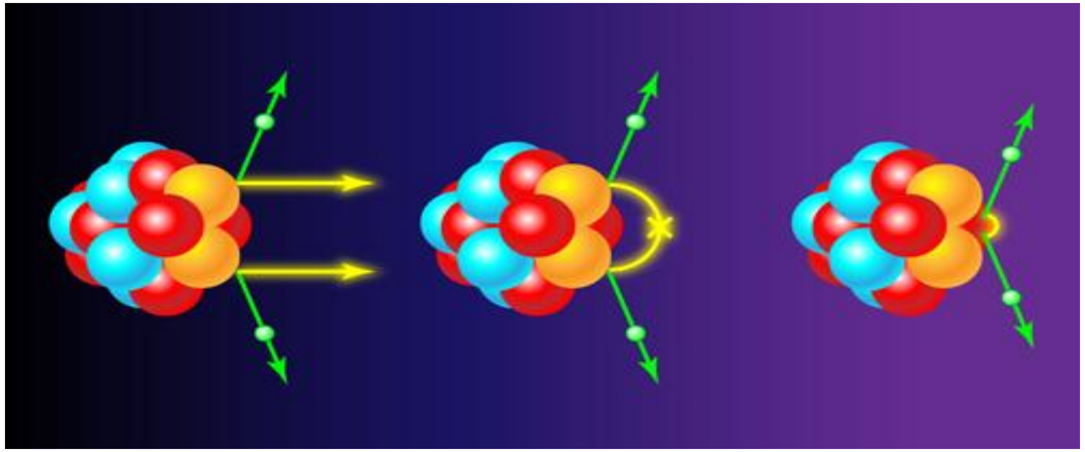
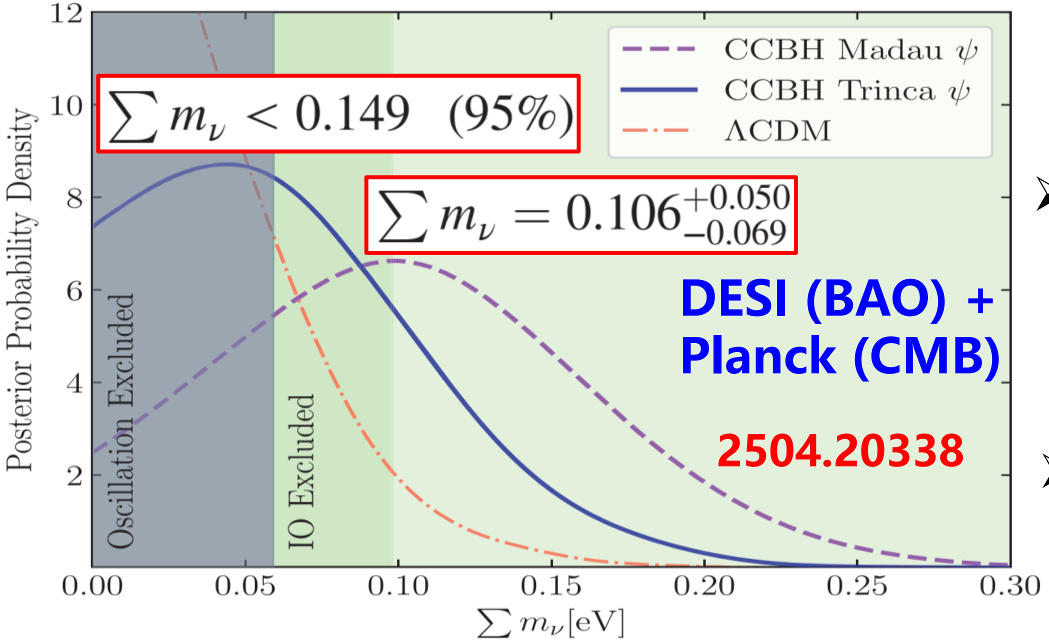


Figure 2.1: LBNF/DUNE project: beam from Illinois to South Dakota.

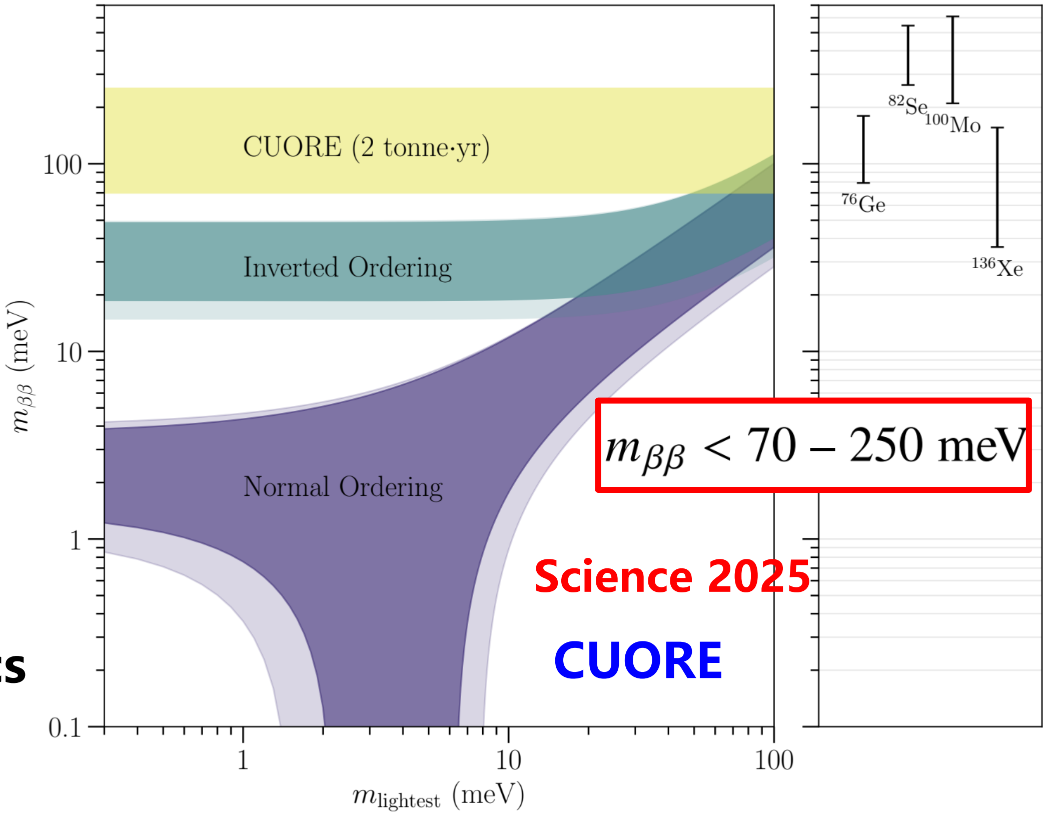
Neutrino mass measurements







- cosmology
- particle physics



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Neutrino oscillations in matter

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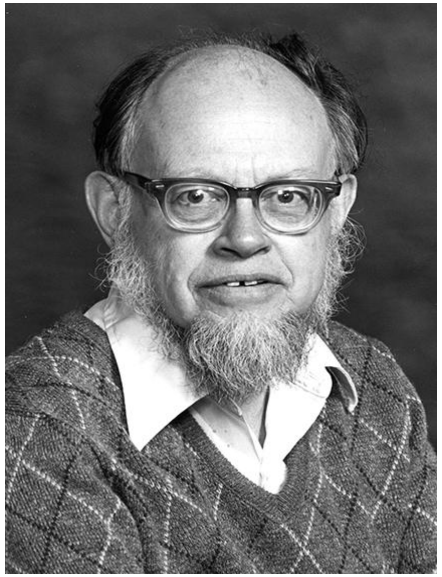
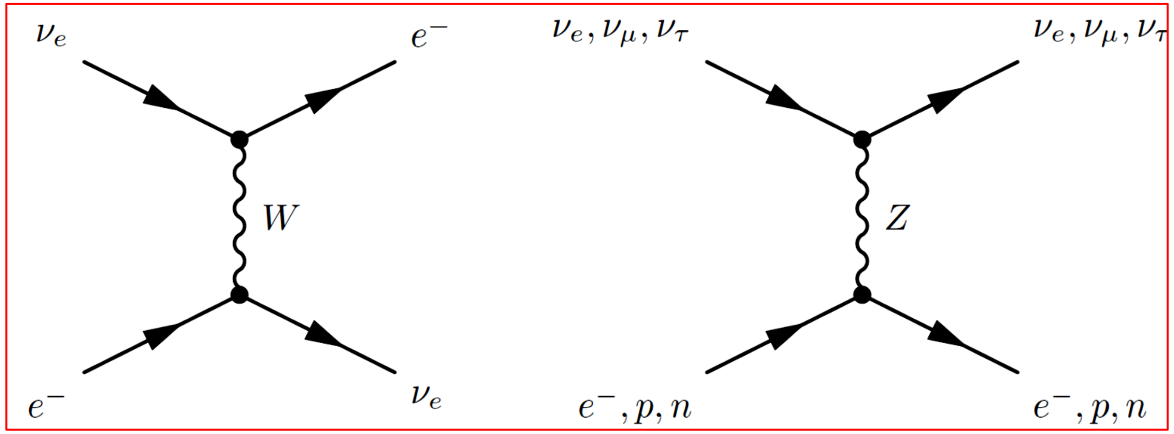
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.





Effective Hamiltonian for CC interactions

$$\boxed{c^e_{\mathrm{V,CC}} = c^e_{\mathrm{A,CC}} = 1}$$

$$\mathcal{H}^{\mathrm{CC}}_{\mathrm{eff}}(x) = \frac{G_\mu}{\sqrt{2}} \left[\overline{\nu_e(x)}\gamma^\mu \left(1-\gamma^5\right)\nu_e(x)\right]\left[\overline{e(x)}\gamma_\mu \left(c^e_{\mathrm{V,CC}} - c^e_{\mathrm{A,CC}}\gamma^5\right) e(x)\right]$$

Effective Hamiltonian for NC interactions

$$\mathcal{H}^{\mathrm{NC}}_{\mathrm{eff}}(x) = \frac{G_\mu}{\sqrt{2}} \left[\overline{\nu_\alpha(x)}\gamma^\mu \left(1-\gamma^5\right)\nu_\alpha(x)\right]\left[\overline{f(x)}\gamma_\mu \left(c^f_{\mathrm{V,NC}} - c^f_{\mathrm{A,NC}}\gamma^5\right) f(x)\right]$$

$$\mathcal{V}_{\mathrm{CC}} = \sqrt{2}G_\mu N_e c^e_{\mathrm{V,CC}}$$

$$\mathcal{V}_{\mathrm{NC}} = \sqrt{2}G_\mu N_f c^f_{\mathrm{V,NC}}$$

	$f=u$	$f=d$	$f=e$
$c^f_{\mathrm{V,NC}}$	$\frac{1}{2}-\frac{4}{3}s^2$	$-\frac{1}{2}+\frac{2}{3}s^2$	$-\frac{1}{2}+2s^2$
$c^f_{\mathrm{A,NC}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

CC-potential

$$\mathcal{V}_{\mathrm{CC}} = \sqrt{2}G_\mu N_e$$

NC-potential

$$\mathcal{V}_{\mathrm{NC}} = -\frac{G_\mu}{\sqrt{2}}\left[\left(1-4\sin^2\theta_{\mathrm{w}}\right)\left(N_e - N_p\right) + N_n\right]$$

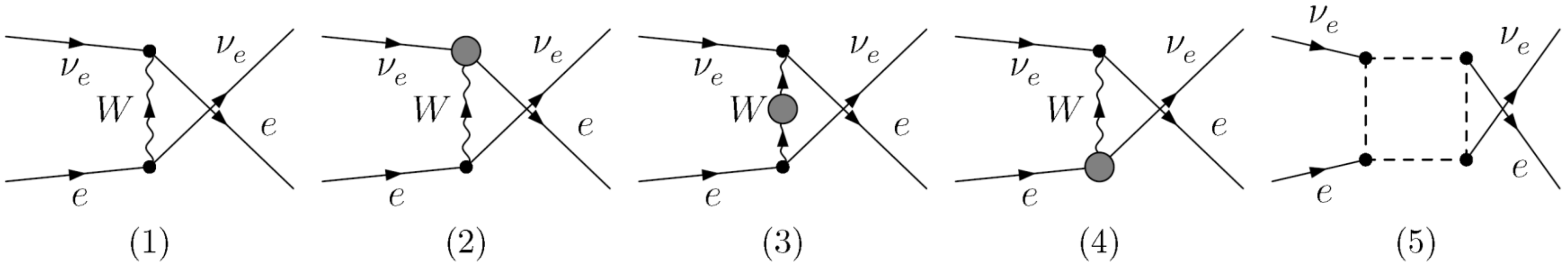
One-loop matter potential

Extract one-loop corrections to couplings from the renormalized amplitudes

$\Delta c^e_{\rm V,CC} \equiv \widehat{c}^e_{\rm V,CC} - c^e_{\rm V,CC}$

$\Delta c^e_{\rm V,CC}/c^e_{\rm V,CC}$

Tree & one-loop diagrams for the CC potential

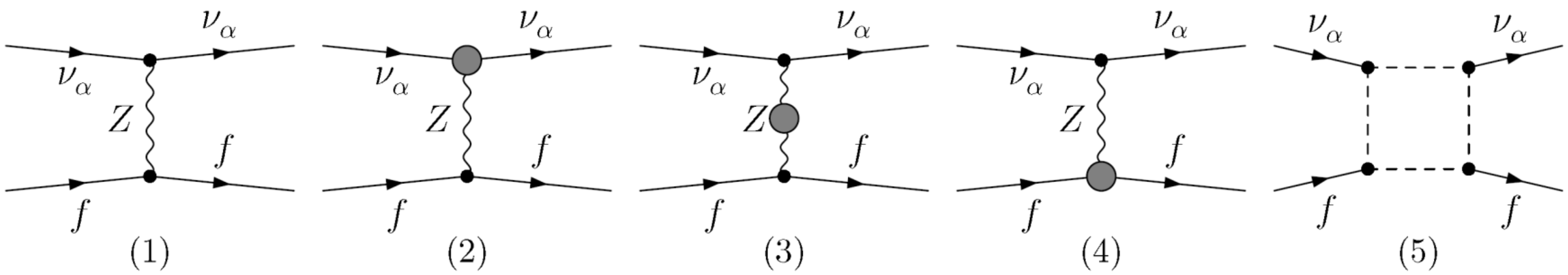


Finite corrections to CC couplings

$$\Delta c^e_{\rm V,CC} = \left(-\frac{\Sigma^{\rm r}_W}{m_W^2} + 2\times\sqrt{2}s\Gamma^{\rm r}_{\nu_e e W}\right) c^e_{\rm V,CC} - \frac{4m_W^2}{g^2}\mathcal{M}_{\rm CC}$$

Tree & one-loop diagrams for the NC potential

$\Delta c^f_{\rm V,NC} \equiv \widehat{c}^f_{\rm V,NC} - c^f_{\rm V,NC}$ $\quad \Delta c^f_{\rm V,NC}/c^f_{\rm V,NC}$



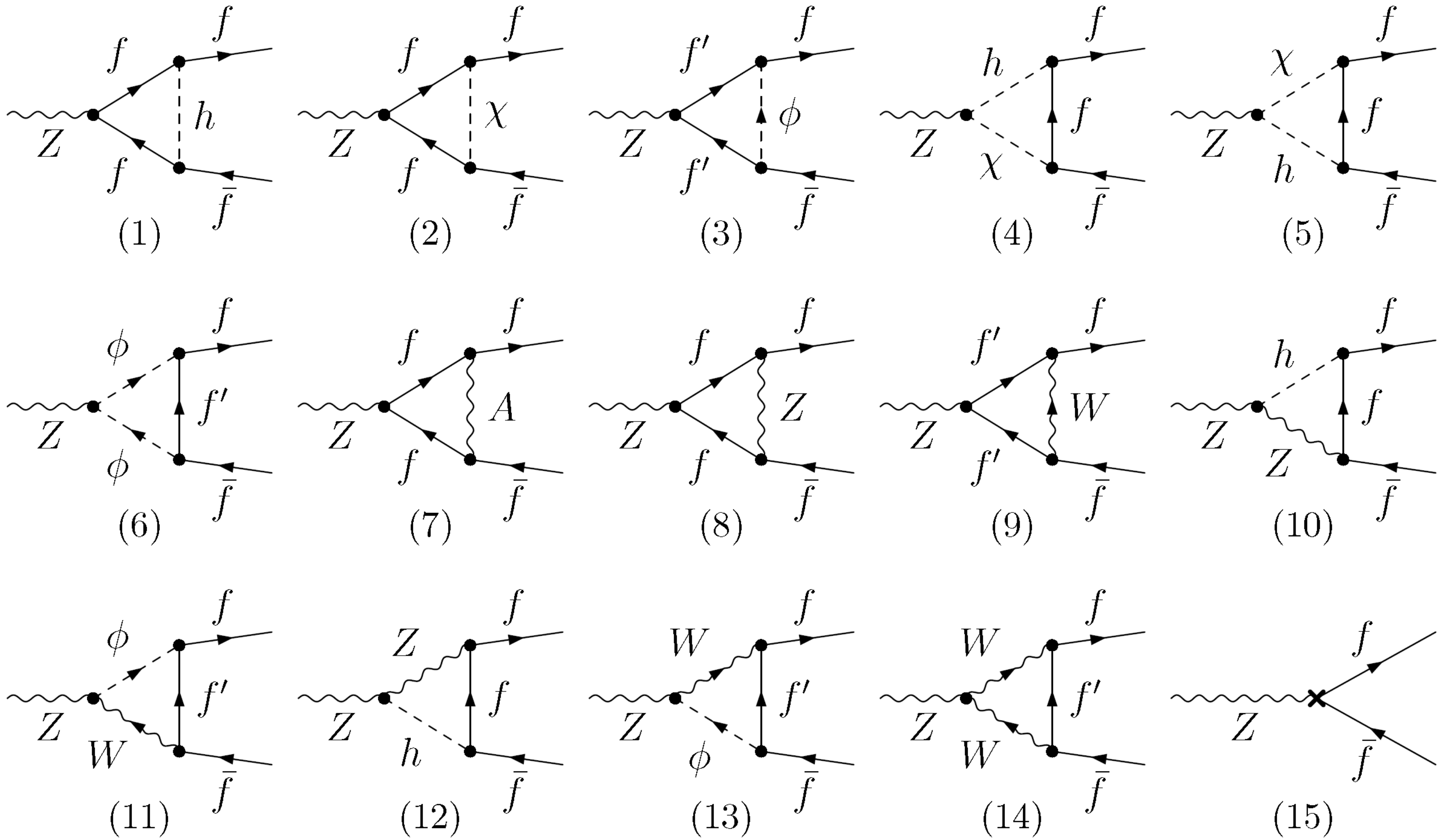
Finite corrections to NC couplings

$$\Delta c^f_{\rm V,NC} = \left(-\frac{\Sigma^{\rm r}_Z}{m_Z^2} + s_{\rm 2w}\Gamma^{\rm r}_{\nu_\alpha\nu_\alpha Z}\right) c^f_{\rm V,NC} + s_{\rm 2w}\Gamma^{\rm r}_{ffZ} - \frac{4m_W^2}{g^2}\mathcal{M}^f_{\rm NC}$$

J. Huang, S.Z., 2307.04685

On-shell scheme & input parameters

One example for the *f-f-Z* vertex



- **Dim. regularization & on-shell scheme**
- **Physical parameters: the EM fine-structure constant and particle masses**

$$\alpha \equiv e^2/(4\pi) = 1/137.035999084$$

$$m_W = 80.377 \text{ GeV}$$

$$m_Z = 91.1876 \text{ GeV}$$

$$m_h = 125.25 \text{ GeV}$$

$$m_e = 0.511 \text{ MeV},$$

$$m_\mu = 105.658 \text{ MeV},$$

$$m_\tau = 1.777 \text{ GeV}$$

$$m_u = 2.16 \text{ MeV}, \quad m_c = 1.67 \text{ GeV}, \quad m_t = 172.5 \text{ GeV}$$

$$m_d = 4.67 \text{ MeV}, \quad m_s = 93.4 \text{ MeV}, \quad m_b = 4.78 \text{ GeV}$$

Aoki et al., 82; Bohm, Spiesberger, Hollik, 86; Hollik, 90; Denner, 93

One-loop corrections

Corrections to the NC couplings

J. Huang, S.Z., 2307.04685

	Self-energy	ν_α-ν_α-Z	f-f-Z	Box diagrams	$\Delta c^f_{\rm V,NC}$
$f = u$	-2.1×10^{-3}	5.1×10^{-3} 1.5×10^{-6} (fd)	-6.0×10^{-3}	7.9×10^{-4} -4.2×10^{-6} (fd)	-2.2×10^{-3}
$f = d$	3.7×10^{-3}	-8.8×10^{-3} -2.6×10^{-6} (fd)	-3.3×10^{-3}	-6.1×10^{-3} 1.7×10^{-5} (fd)	-1.5×10^{-2}
$f = e$	5.6×10^{-4}	-1.4×10^{-3} -3.9×10^{-7} (fd)	15.3×10^{-3}	-5.3×10^{-3} 1.7×10^{-5} (fd)	9.2×10^{-3}

Estimate of one-loop correction

$$\frac{\Delta c_{\rm V,NC}}{c_{\rm V,NC}} = \frac{N_p\left(2\Delta c^u_{\rm V,NC} + \Delta c^d_{\rm V,NC} + \Delta c^e_{\rm V,NC}\right) + N_n\left(\Delta c^u_{\rm V,NC} + 2\Delta c^d_{\rm V,NC}\right)}{N_n\left(c^u_{\rm V,NC} + 2c^d_{\rm V,NC}\right)} \approx 0.062 + 0.02\frac{N_p}{N_n}$$

Corrections to the CC couplings

Self-energy	ν_e-e-W	box diagrams	$\Delta c^e_{\rm V,CC}$
-6.4×10^{-3}	4.5×10^{-2}	1.9×10^{-2}	5.8×10^{-2}

In conclusion, the one-loop correction to CC potential is 5.8%, while that for NC potential is 8.2%

Impact on DUNE

Tree-level potentials in term of on-shell parameters **Huang, Sampsa, Ohlsson, S.Z., 2504.15998**

$$\mathcal{V}_{\rm CC} = \frac{\pi\alpha m_Z^2}{m_W^2\left(m_Z^2 - m_W^2\right)} N_e c^e_{\rm V,CC} \qquad \mathcal{V}_{\rm NC} = \frac{\pi\alpha m_Z^2}{m_W^2\left(m_Z^2 - m_W^2\right)} N_f c^f_{\rm V,NC}$$

Tree-level relation between the Fermi constant and on-shell parameters

$$\sqrt{2}G_\mu = \frac{\pi\alpha m_Z^2}{m_W^2\left(m_Z^2 - m_W^2\right)}$$

$$G_\mu \approx 1.166 \times 10^{-5}~{\rm GeV}^{-2}$$

determined from the muon lifetime

Tree-level relation will be modified by radiative corrections

$$\sqrt{2}G_\mu = \frac{\pi\alpha m_Z^2}{m_W^2\left(m_Z^2 - m_W^2\right)}(1+\Delta r) \qquad \Delta r \approx 3.8\%$$

Denner, Dittmaier, 1912.06823

Tree-level matter potential with the Fermi constant already includes part of one-loop corrections

$$\widehat{\mathcal{V}}_{\rm CC} = \sqrt{2}G_\mu^{\rm LO} N_e \widehat{c}^e_{\rm V,CC} \simeq \sqrt{2}G_\mu^{\rm NLO} N_e c^e_{\rm V,CC}\left(1 + \boxed{\Delta c^e_{\rm V,CC} - \Delta r}\right) \longrightarrow$$

2.0%

In the presence of non-standard interactions (NSI)

Ohlsson, 1209.2710

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{1}{2E_\nu}\left[U\begin{pmatrix}0 & 0 & 0\\ 0 & \Delta m^2_{21} & 0\\ 0 & 0 & \Delta m^2_{31}\end{pmatrix}U^\dagger + A\begin{pmatrix}1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau}\\ \epsilon^*_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon^*_{e\tau} & \epsilon^*_{\mu\tau} & \epsilon_{\tau\tau}\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$A = 2\sqrt{2}G_\mu N_e E_\nu$$

$$\epsilon_{e\mu} = |\epsilon_{e\mu}|e^{i\phi_{e\mu}}$$

$$\Delta_{ij} \equiv \Delta m^2_{ij}L/(4E_\nu)$$

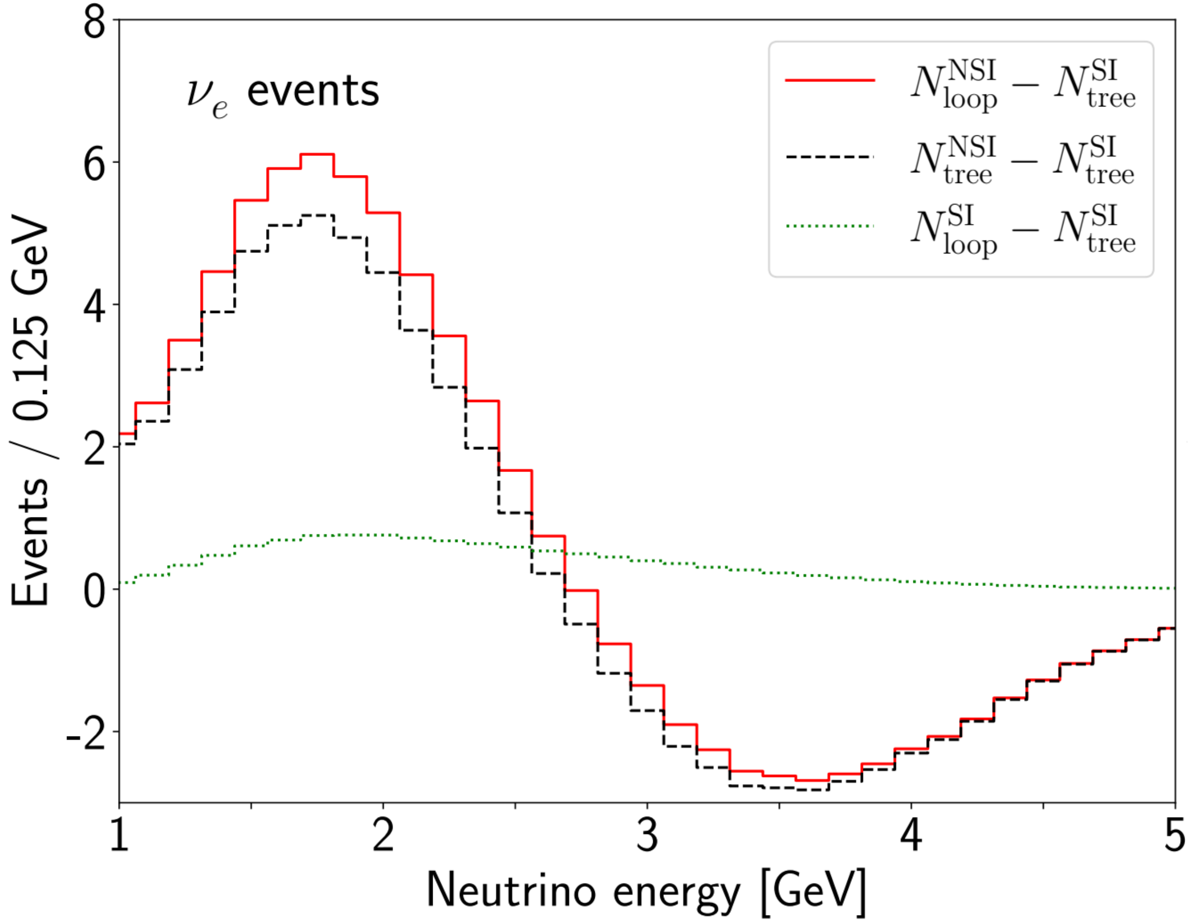
Extra contributions to oscillation probabilities (e.g., just one nonzero NSI parameter)

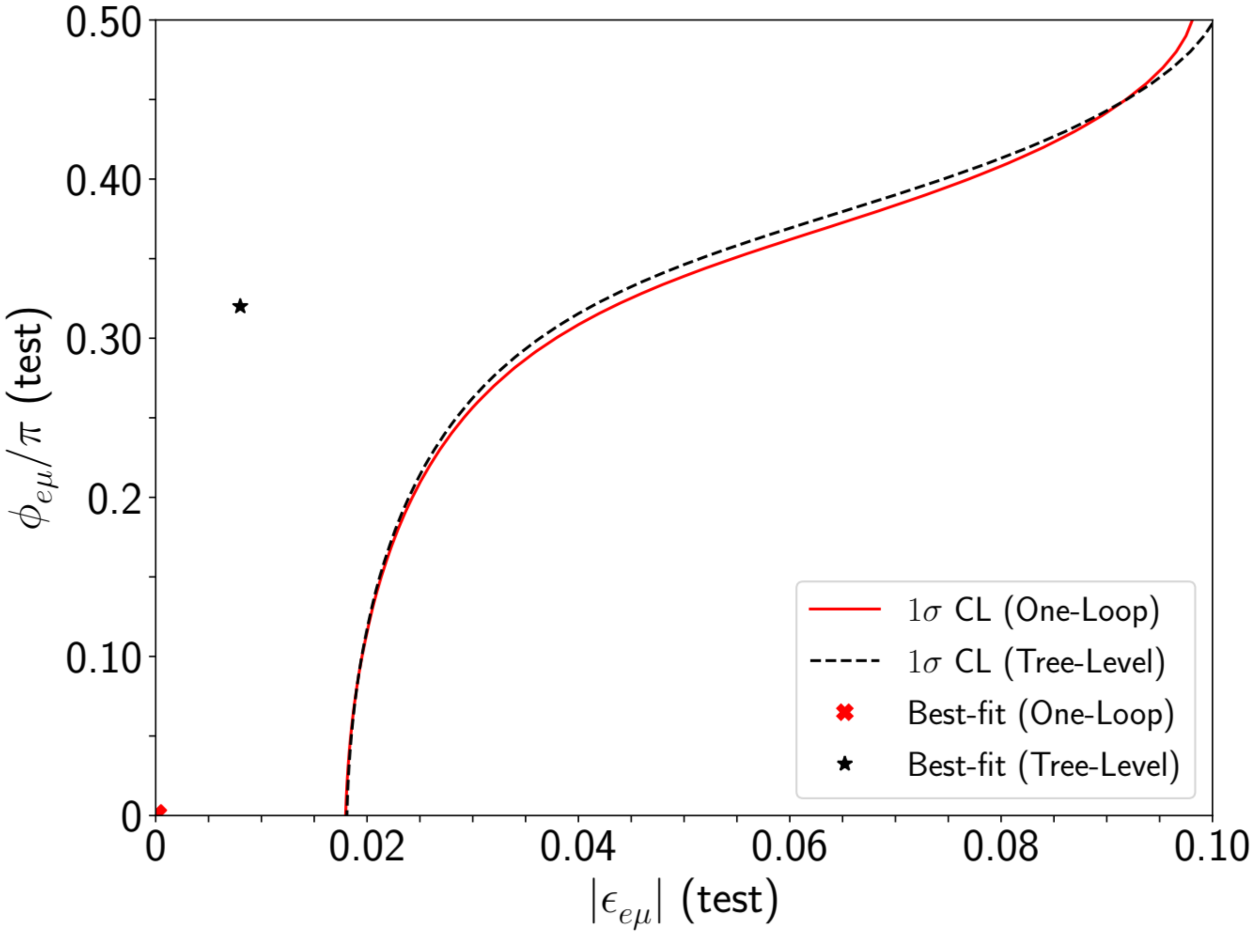
$$\begin{aligned}\Delta P^{\mathrm{NSI}}_{\mu e} \simeq\ & 4\left|\epsilon_{e\mu}\right|\cos^2\theta_{23}\sin 2\theta_{13}\sin\theta_{23}\sin(aL)\frac{\sin\left(\Delta_{31}-aL\right)}{\left(\Delta_{31}-aL\right)}\Delta_{31}\cos\left(\Delta_{31}+\delta_{\mathrm{CP}}+\phi_{e\mu}\right)\\ & +4\left|\epsilon_{e\mu}\right| aL\sin 2\theta_{13}\sin^3\theta_{23}\frac{\sin^2\left(\Delta_{31}-aL\right)}{\left(\Delta_{31}-aL\right)^2}\Delta_{31}\cos\left(\delta_{\mathrm{CP}}+\phi_{e\mu}\right)\\ & +4\left|\epsilon_{e\mu}\right|\cos^3\theta_{23}\sin 2\theta_{12}\frac{\sin^2(aL)}{aL}\Delta_{21}\cos\phi_{e\mu}\\ & +4\left|\epsilon_{e\mu}\right|\cos\theta_{23}\sin 2\theta_{12}\sin^2\theta_{23}\sin(aL)\frac{\sin\left(\Delta_{31}-aL\right)}{\Delta_{31}-aL}\Delta_{21}\cos\left(\Delta_{31}-\phi_{e\mu}\right)\ ,\end{aligned}$$

Example: non-standard interactions

Neutrino events at DUNE: SI=Standard Interaction

Huang, Sampsa, Ohlsson, S.Z., 2510.04841

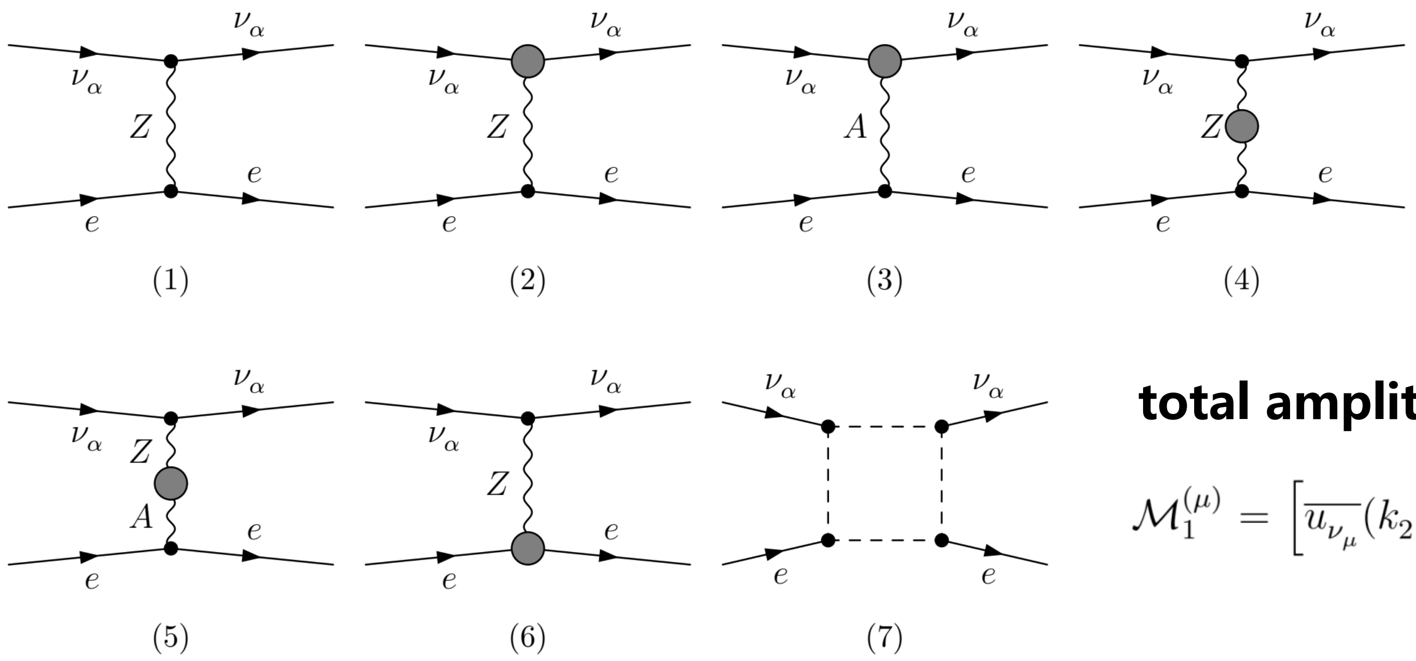




Warning! Do NOT mistake radiative corrections in the SM as the discovery of new physics!

Feynman diagrams for ν_α-e^- scattering

Sarantakos *et al.*, NPB, 83 **J. Huang, S.Z., 2412.17047**



$$A_{\rm NC} = A_Z + A_{AZ} + \underline{A_{\nu_\alpha\nu_\alpha Z} + A_{\nu_\alpha\nu_\alpha A} + A_{eeZ}} + A_{\square,{\rm NC}}$$

Z self-energy **Z-A mixing** **box-diag.**

total amplitude:

$$\mathcal{M}_1^{(\mu)} = \left[\overline{u_{\nu_\mu}}(k_2)\gamma_\mu P_{\rm L} u_{\nu_\mu}(k_1)\right] \times \overline{u_e}(p_2)\left[\gamma^\mu\left(A - B\gamma^5\right) + C\frac{(p_1+p_2)^\mu}{2m_e}\right]u_e(p_1)$$

differential cross section:

$$\frac{{\rm d}\sigma_1^{(\mu)}}{{\rm d}T_e} = \frac{g^4 m_e}{64\pi m_W^4}\left[\frac{m_e z}{E_\nu}\left(c_{\rm A}^2 - c_{\rm V}^2\right) + (1-z)^2(c_{\rm A}-c_{\rm V})^2 + (c_{\rm A}+c_{\rm V})^2\right]$$
$$+\frac{g^2 m_e}{8\pi m_W^2}\left\{\left[z(z-2)c_{\rm A} - \left(z^2-2z+2\right)c_{\rm V} + \frac{m_e z}{E_\nu}c_{\rm V}\right]A\right.$$
$$\left. + \left[z(z-2)c_{\rm V} - \left(z^2-2z+2\right)c_{\rm A} - \frac{m_e z}{E_\nu}c_{\rm A}\right]B + \left[2(z-1) + \frac{m_e z}{E_\nu}\right]c_{\rm V} C\right\}$$

recoil energy

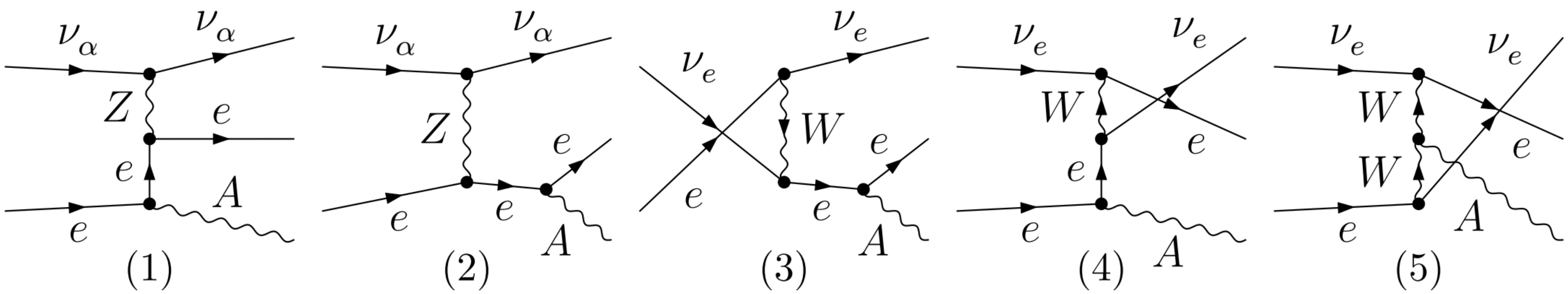
$$T_e \equiv E_e - m_e$$

$$z \equiv T_e/E_\nu$$

Cross sections at one-loop

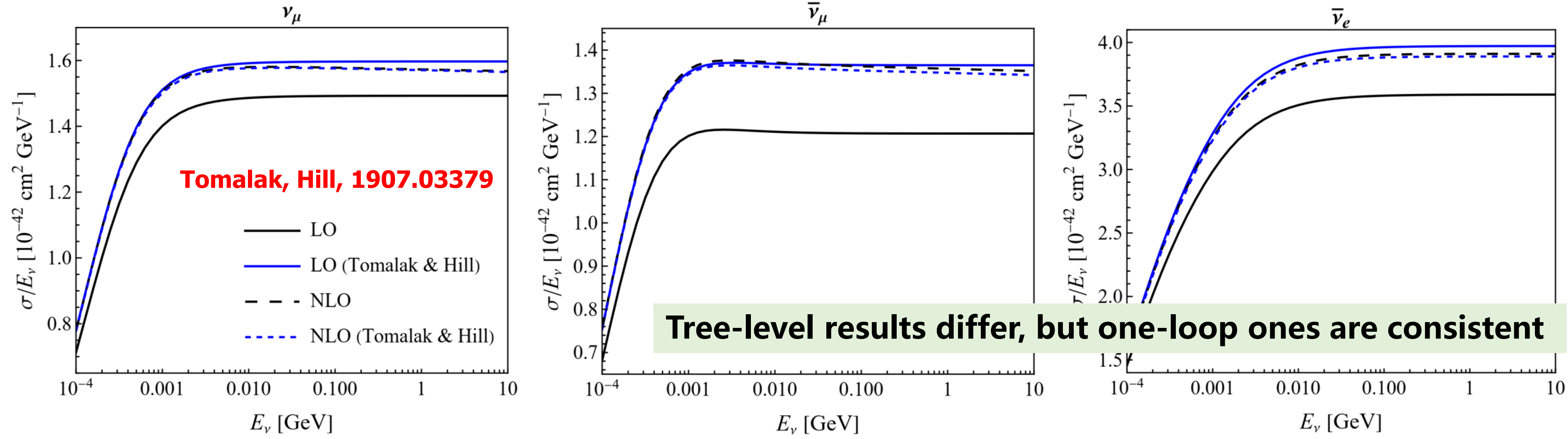
Infrared divergences removed by including soft-photon contributions

J. Huang, S.Z., 2412.17047



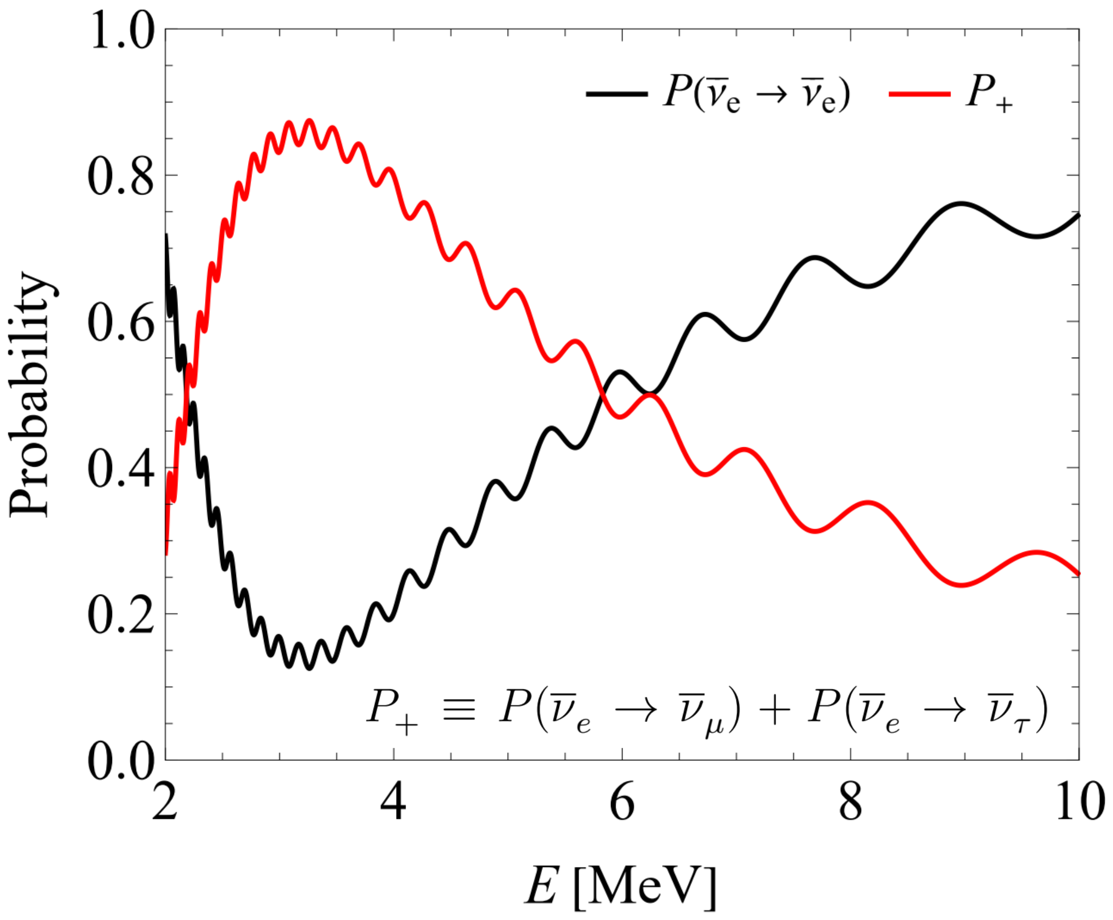
$$\frac{\mathrm{d}\sigma_\gamma^{(\mu)}}{\mathrm{d}T_e} = \frac{g^6 s^2 m_e}{256\pi^3 m_W^4}\left\{\hat{R}\left[\frac{m_e z}{E_\nu}\left(c_\mathrm{A}^2 - c_\mathrm{V}^2\right) + (c_\mathrm{A} + c_\mathrm{V})^2\right] + R(c_\mathrm{A} - c_\mathrm{V})^2\right\}$$

$$\boxed{\frac{\mathrm{d}\sigma^{(\alpha)}}{\mathrm{d}T_e} = \frac{\mathrm{d}\sigma_1^{(\alpha)}}{\mathrm{d}T_e} + \frac{\mathrm{d}\sigma_\gamma^{(\alpha)}}{\mathrm{d}T_e}}$$

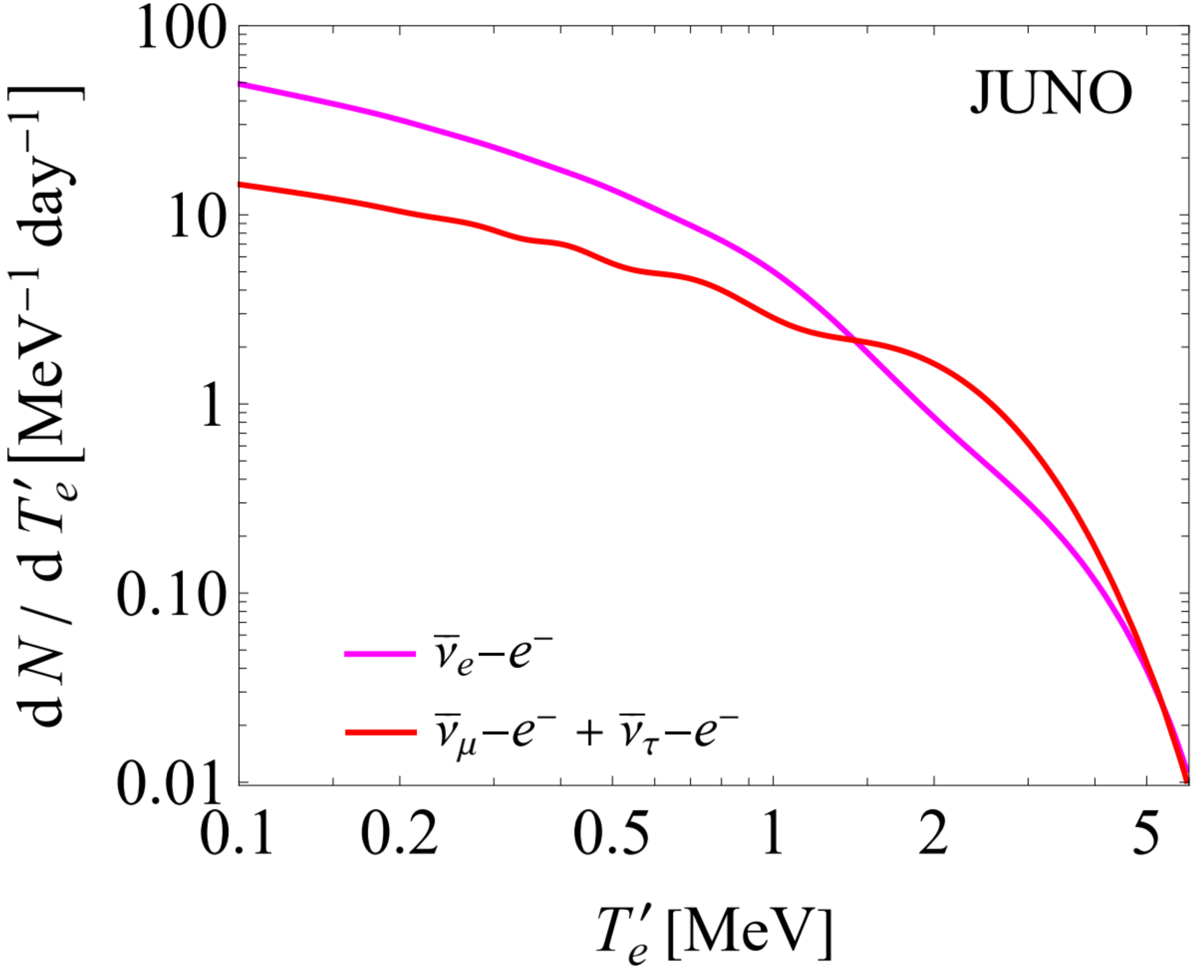


Tree-level results differ, but one-loop ones are consistent

Possible detection of $\overline{\nu}_\mu$ and $\overline{\nu}_\tau$ from reactor neutrino oscillations at JUNO Wang, Xing, S.Z., 2509.00422



Due to the oscillation maximum, there will be a large fraction of non-electron antineutrinos

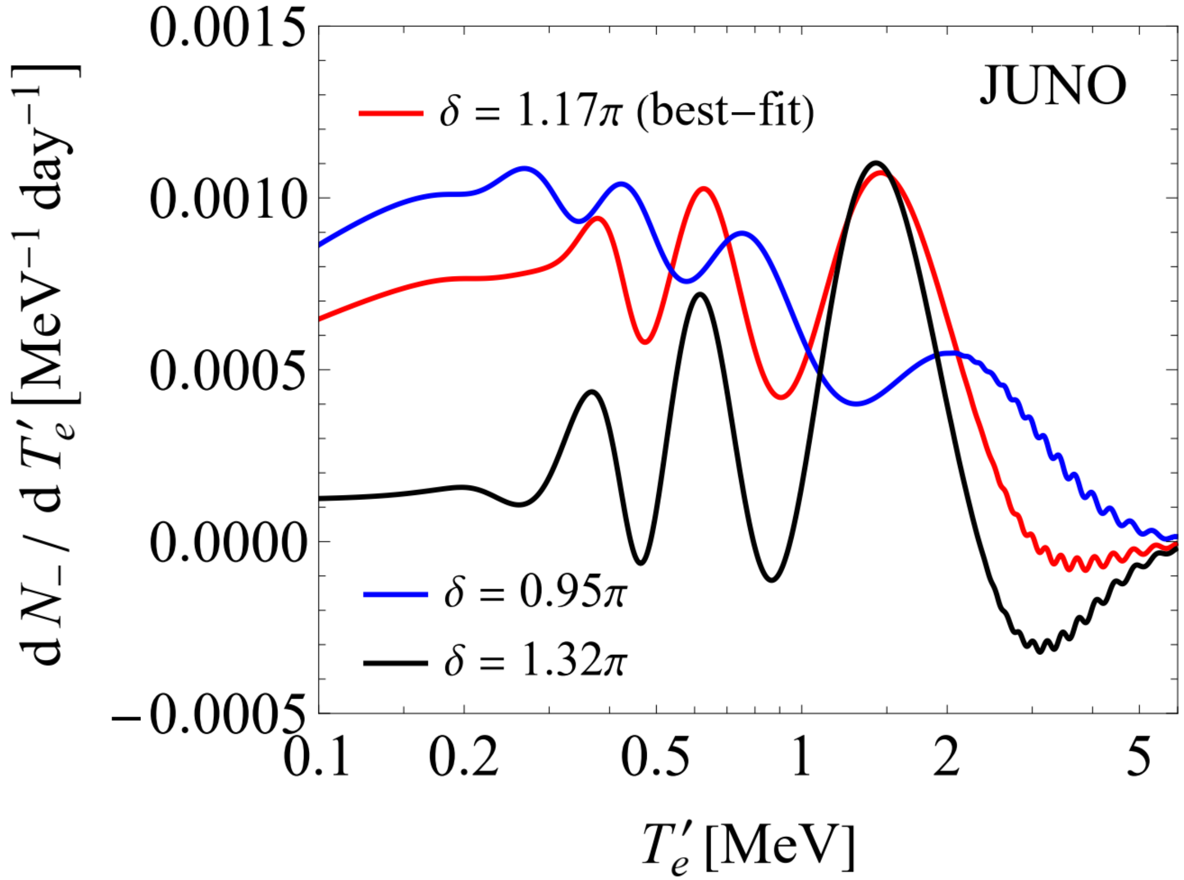


Event rates for elastic neutrino-electron scattering

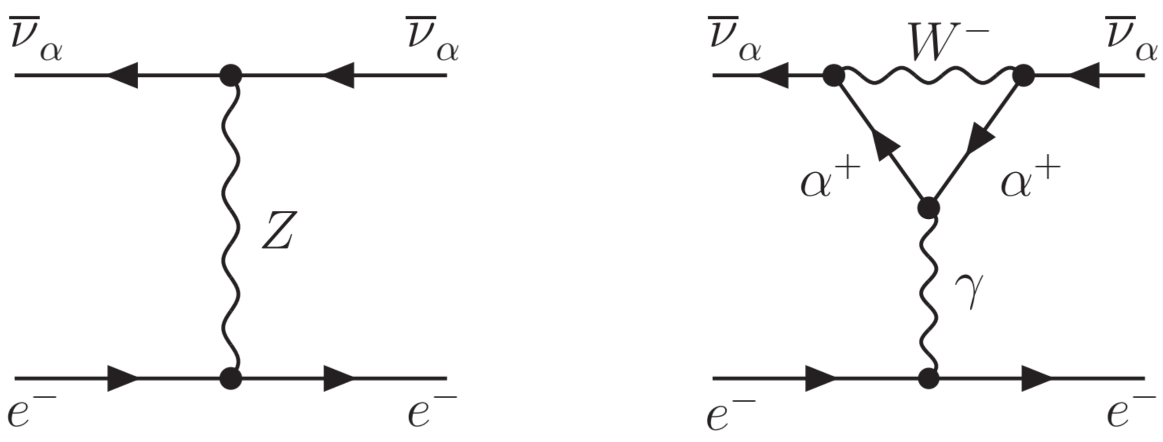
Example II: appearance channel at JUNO

Different cross sections for $\overline{\nu}_\mu$ and $\overline{\nu}_\tau$, but arising at one-loop level: only one percent

$$\sum_\alpha \frac{\mathrm{d}\sigma_\alpha}{\mathrm{d}T_e}\phi_\alpha = \frac{1}{4\pi L^2}\cdot\frac{\mathrm{d}N'_e}{\mathrm{d}E}\left[P(\overline{\nu}_e\to\overline{\nu}_e)\frac{\mathrm{d}\sigma_e}{\mathrm{d}T_e}+\frac{P_+}{2}\left(\frac{\mathrm{d}\sigma_\mu}{\mathrm{d}T_e}+\frac{\mathrm{d}\sigma_\tau}{\mathrm{d}T_e}\right)+\frac{P_-}{2}\left(\frac{\mathrm{d}\sigma_\mu}{\mathrm{d}T_e}-\frac{\mathrm{d}\sigma_\tau}{\mathrm{d}T_e}\right)\right]$$

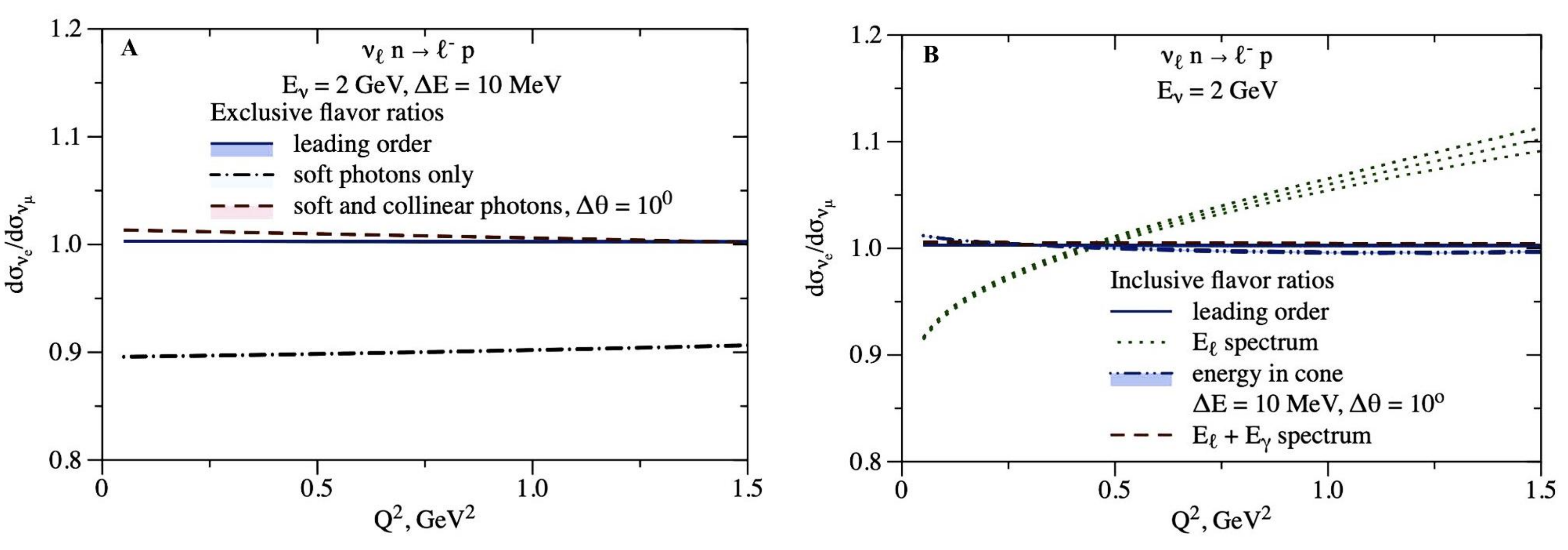


$$P_- \equiv P(\overline{\nu}_e\to\overline{\nu}_\mu) - P(\overline{\nu}_e\to\overline{\nu}_\tau)$$



- **The event rate *does* depend on the leptonic CP-violating phase**
- **However, too small to be observed at JUNO**

Neutrino-nucleon/nucleus interactions

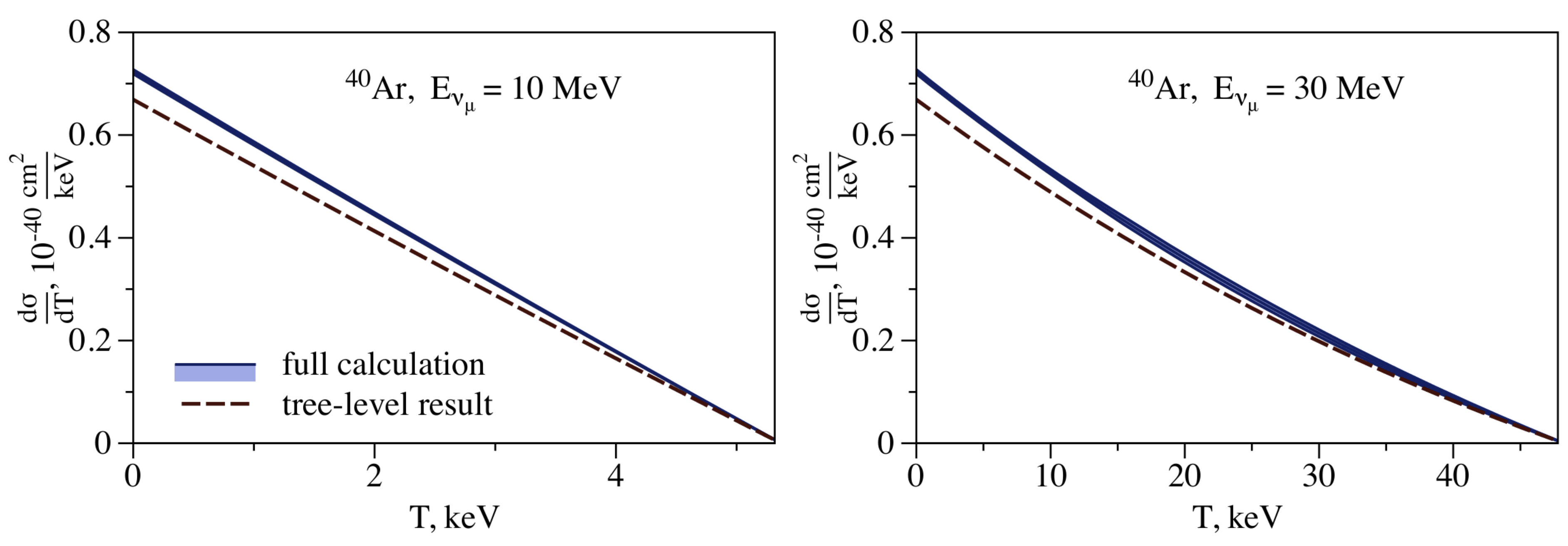


Tomalak et al., 2011.05960

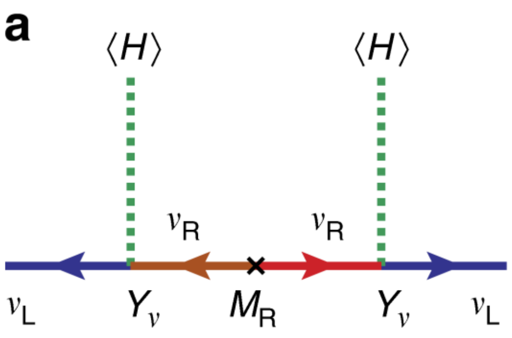
radiative corrections to coherent ν-A scattering

radiative corrections to ν-N scattering at GeV

Tomalak et al., 2105.07939, 2204.11379

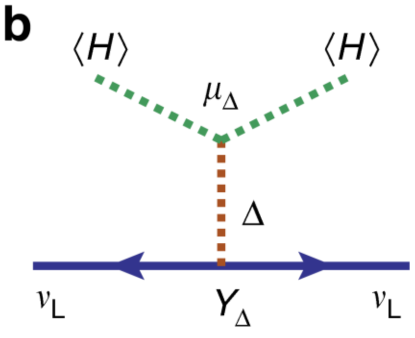


Minkowski, 77



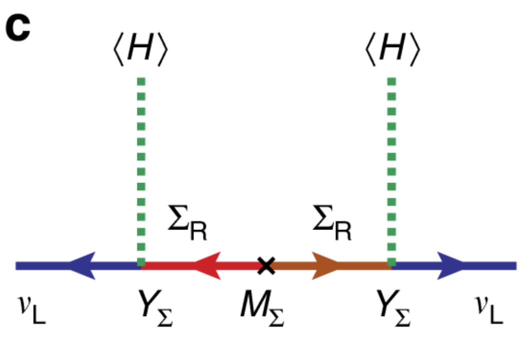
$$M_\nu = -\langle H\rangle^2 Y_\nu M_R^{-1} Y_\nu^T$$

Konetschny, Kummer, 77



$$M_\nu = \langle H\rangle^2 Y_\Delta \mu_\Delta / M_\Delta^2$$

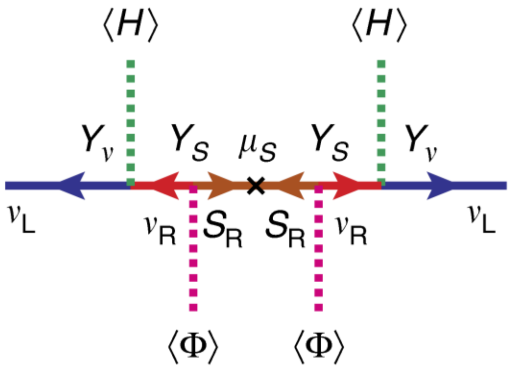
Foot, Lew, He, Joshi, 89



$$M_\nu = -\langle H\rangle^2 Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T$$

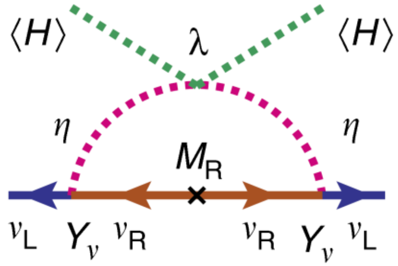
d Inverse seesaw model

Mohapatra, Valle, 87



$$M_\nu = F\mu_S F^T$$

e The scotogenic model



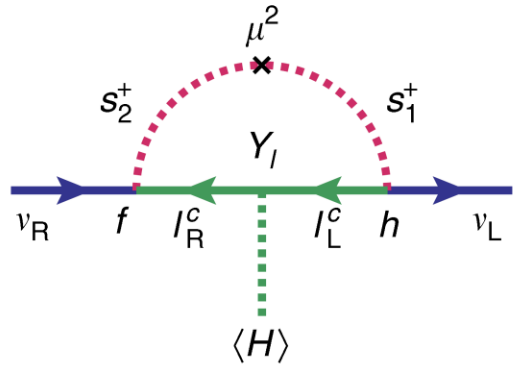
Z. Tao, 96; E. Ma, 06

Zee, 80, 86; Babu, 88

$$M_\nu = -\lambda \frac{\langle H\rangle^2}{16\pi^2} Y_\nu M_R^{-1} Y_\nu^T$$

f Radiative Dirac model

Kanemura et al, 11



$$M_\nu = \frac{h Y_l f}{16\pi^2} \langle H\rangle\, I(\mu^2, M_{s_1}^2, M_{s_2}^2)$$

- **The SM with three right-handed neutrinos: neutrino masses & baryon number asymmetry**

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}} {\rm i}\partial\!\!\!/ N_{\rm R} - \left[\frac{1}{2}\overline{N^{\rm C}_{\rm R}} \mathbf{m}_{\rm R} N_{\rm R} + \overline{\ell_{\rm L}}\widetilde{H}\mathbf{y}_\nu N_{\rm R} + {\rm h.c.}\right]$$

After the gauge symmetry breaking

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}\overline{\begin{pmatrix}\nu_{\rm L} & N^{\rm C}_{\rm R}\end{pmatrix}} \begin{pmatrix} \mathbf{0} & \mathbf{m}_{\rm D} \\ \mathbf{m}^{\rm T}_{\rm D} & \mathbf{m}_{\rm R}\end{pmatrix} \begin{pmatrix}\nu^{\rm C}_{\rm L} \\ N_{\rm R}\end{pmatrix} + {\rm h.c.}$$

Diagonalization of the 6x6 neutrino mass matrix

$$\begin{pmatrix}\mathbf{V} & \mathbf{R} \\ \mathbf{S} & \mathbf{U}\end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & \mathbf{m}_{\rm D} \\ \mathbf{m}^{\rm T}_{\rm D} & \mathbf{m}_{\rm R}\end{pmatrix} \begin{pmatrix}\mathbf{V} & \mathbf{R} \\ \mathbf{S} & \mathbf{U}\end{pmatrix}^* = \begin{pmatrix}\widehat{\mathbf{m}} & \mathbf{0} \\ \mathbf{0} & \widehat{\mathbf{M}}\end{pmatrix}$$

In the basis where the charged-lepton mass matrix is diagonal: CC, NC and Yukawa interactions

$$\mathcal{L}_{\rm cc} = \frac{g}{2\sqrt{2}}\left[\overline{l}\gamma^\mu(1-\gamma_5)\mathbf{V}\widehat{\nu}W^-_\mu + \overline{l}\gamma^\mu(1-\gamma_5)\mathbf{R}\widehat{N}W^-_\mu\right] + {\rm h.c.}\ ,$$

$$\mathcal{L}_{\rm nc} = \frac{g}{4\cos\theta_{\rm W}}\left[\overline{\widehat{\nu}}\gamma^\mu(1-\gamma_5)\mathbf{V}^\dagger\mathbf{V}\widehat{\nu}Z_\mu + \overline{\widehat{\nu}}\gamma^\mu(1-\gamma_5)\mathbf{V}^\dagger\mathbf{R}\widehat{N}Z_\mu + \overline{\widehat{N}}\gamma^\mu(1-\gamma_5)\mathbf{R}^\dagger\mathbf{V}\widehat{\nu}Z_\mu + \overline{\widehat{N}}\gamma^\mu(1-\gamma_5)\mathbf{R}^\dagger\mathbf{R}\widehat{N}Z_\mu\right]$$

$$\mathcal{L}_h = -\frac{g}{4M_W}h\left[\overline{\widehat{\nu}}\mathbf{V}^\dagger\mathbf{V}\widehat{\mathbf{m}}(1+\gamma_5)\widehat{\nu} + \overline{\widehat{\nu}}\mathbf{V}^\dagger\mathbf{R}\widehat{\mathbf{M}}(1+\gamma_5)\widehat{N} + \overline{\widehat{N}}\mathbf{R}^\dagger\mathbf{V}\widehat{\mathbf{m}}(1+\gamma_5)\widehat{\nu} + \overline{\widehat{N}}\mathbf{R}^\dagger\mathbf{R}\widehat{\mathbf{M}}(1+\gamma_5)\widehat{N}\right] + {\rm h.c.}$$

$\Longrightarrow$ **Calculate all the observables in expts.**

It is evident that only V and R in the first three rows of the 6x6 unitary matrix are physical!

- **All parametrizations are equivalent, but one can be more advantageous than another**

Xing, PLB, 2008; PRD, 2012

$$\mathcal{U} \equiv \begin{pmatrix} \mathrm{V} & \mathrm{R} \\ \mathrm{S} & \mathrm{U} \end{pmatrix} \Rightarrow \mathcal{U} = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{U}_0 \end{pmatrix} \cdot \begin{pmatrix} \mathrm{A} & \mathrm{R} \\ \mathrm{D} & \mathrm{B} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{V}_0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{O}_{ij} \equiv \begin{pmatrix} c_{ij} & \cdots & \widetilde{s}^*_{ij} \\ \vdots & \ddots & \vdots \\ -\widetilde{s}_{ij} & \cdots & c_{ij} \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{A} & \mathrm{R} \\ \mathrm{D} & \mathrm{B} \end{pmatrix} = \mathcal{O}_{36} \cdot \mathcal{O}_{26} \cdot \mathcal{O}_{16} \cdot \mathcal{O}_{35} \cdot \mathcal{O}_{25} \cdot \mathcal{O}_{15} \cdot \mathcal{O}_{34} \cdot \mathcal{O}_{24} \cdot \mathcal{O}_{14}$$

heavy sector

$$\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{U}_0 \end{pmatrix} = \mathcal{O}_{56} \cdot \mathcal{O}_{46} \cdot \mathcal{O}_{45}$$

6 = 3 angles + 3 phases

Interplay between heavy and light sectors

9 angles + 9 phases = 18

$$\boxed{\mathrm{V}\widehat{\mathrm{m}}\mathrm{V}^{\mathrm{T}} + \mathrm{R}\widehat{\mathrm{M}}\mathrm{R}^{\mathrm{T}} = 0}$$

$$\mathrm{V} = \mathrm{A} \cdot \mathrm{V}_0$$

light sector

$$\begin{pmatrix} \mathrm{V}_0 & 0 \\ 0 & 1 \end{pmatrix} = \mathcal{O}_{23} \cdot \mathcal{O}_{13} \cdot \mathcal{O}_{12}$$

6 = 3 angles + 3 phases

12 angles + 12 phases + 6 masses – 12 constraints = 18 parameters

Choosing 18 physical parameters: 3 heavy Majorana neutrino masses + 9 angles + 6 phases, one can in principle calculate all observables and determine parameters from experiments

- **Fully renormalize the canonical seesaw model the MS-bar scheme**

Huang, S.Z., 2507.21691

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\overline{l_\alpha}\gamma^\mu P_{\rm L}\mathcal{B}_{\alpha i}\chi_i W_\mu^- + {\rm h.c.}\ ,$$
$$\mathcal{L}_{\rm NC} = \frac{g}{4c}\overline{\chi_i}\gamma^\mu\left(P_{\rm L}\mathcal{C}_{ij} - P_{\rm R}\mathcal{C}_{ij}^*\right)\chi_j Z_\mu\ ,$$
$$\mathcal{L}_h = -\frac{g}{4m_W}h\overline{\chi_i}\left[\left(\widehat{m}_j P_{\rm R} + \widehat{m}_i P_{\rm L}\right)\mathcal{C}_{ij} + \left(\widehat{m}_i P_{\rm R} + \widehat{m}_j P_{\rm L}\right)\mathcal{C}_{ij}^*\right]\chi_j\ ,$$
$$\mathcal{L}_{\phi^\pm} = \frac{g}{\sqrt{2}m_W}\overline{l_\alpha}\left(\widehat{m}_i P_{\rm R} - m_\alpha P_{\rm L}\right)\mathcal{B}_{\alpha i}\chi_i\phi^- + {\rm h.c.}\ ,$$
$$\mathcal{L}_{\phi^0} = \frac{{\rm i}g}{4m_W}\phi^0\overline{\chi_i}\left[\left(\widehat{m}_j P_{\rm R} - \widehat{m}_i P_{\rm L}\right)\mathcal{C}_{ij} + \left(\widehat{m}_i P_{\rm R} - \widehat{m}_j P_{\rm L}\right)\mathcal{C}_{ij}^*\right]\chi_j\ ,$$

Diagonalization of ν mass matrices

$$\begin{pmatrix} \mathbf{V} & \mathbf{R} \\ \mathbf{S} & \mathbf{U}\end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & \mathbf{m}_{\rm D} \\ \mathbf{m}_{\rm D}^{\rm T} & \mathbf{m}_{\rm R}\end{pmatrix}\begin{pmatrix} \mathbf{V} & \mathbf{R} \\ \mathbf{S} & \mathbf{U}\end{pmatrix}^* = \begin{pmatrix} \widehat{\mathbf{m}} & \mathbf{0} \\ \mathbf{0} & \widehat{\mathbf{M}}\end{pmatrix}$$

Lepton flavor mixing matrix

$$\mathcal{B} \equiv \begin{pmatrix} \mathbf{V} & \mathbf{R}\end{pmatrix}\ , \quad \mathcal{C} \equiv \mathcal{B}^\dagger\mathcal{B} = \begin{pmatrix} \mathbf{V}^\dagger\mathbf{V} & \mathbf{V}^\dagger\mathbf{R} \\ \mathbf{R}^\dagger\mathbf{V} & \mathbf{R}^\dagger\mathbf{R}\end{pmatrix}$$

$$\boxed{\begin{aligned}16\pi^2\dot{m}_Z = -g^2 m_Z\Bigg\{&\frac{84c^4 - 14c^2 - 11}{12c^2} + 3\frac{m_h^4 + 4m_W^4 + 2m_Z^4}{4m_W^2 m_h^2} \\ &- \sum_{f=q,l}\left[\frac{\left(c_{\rm A}^f\right)^2 + \left(c_{\rm V}^f\right)^2}{3c^2} - \frac{2\left(c_{\rm A}^f\right)^2 m_f^2}{m_W^2} + \frac{2m_f^4}{m_W^2 m_h^2}\right] \\ &+ \sum_{i,j}\frac{3\left(\mathcal{C}_{ij}^2 + \mathcal{C}_{ij}^{*2}\right)\widehat{m}_i\widehat{m}_j + \left|\mathcal{C}_{ij}\right|^2\left(3\widehat{m}_i^2 + 3\widehat{m}_j^2 - 2m_Z^2\right)}{12m_W^2} - 2\sum_k\frac{\mathcal{C}_{kk}\widehat{m}_k^4}{m_h^2 m_W^2}\Bigg\}\end{aligned}}$$

- **extra contributions to the SM results (e.g., the running Z-boson mass)**
- **Complete one-loop RGEs for all parameters**

- **Fully renormalize the canonical seesaw model the on-shell scheme**

Huang, S.Z., 2509.12844

Naïve On-shell renormalization of the CKM matrix leads to gauge-dependent results

$$\widetilde{\delta}\mathbf{V}^{\mathrm{CKM}} = -\frac{1}{4}\left[\mathbf{V}^{\mathrm{CKM}}\left(\delta Z^{d,\mathrm{L}} - \delta Z^{d,\mathrm{L}\dagger}\right) + \left(\delta Z^{u,\mathrm{L}\dagger} - \delta Z^{u,\mathrm{L}}\right)\mathbf{V}^{\mathrm{CKM}}\right]$$

Denner, Sack, NPB, 1990

Gambino et al., PLB, 1998

A practical scheme for gauge-independent counter terms

$$\delta\mathbf{V}^{\mathrm{CKM}} = \left[\widetilde{\delta}\mathbf{V}^{\mathrm{CKM}}\right]^{u,\mathrm{GI}} + \left[\widetilde{\delta}\mathbf{V}^{\mathrm{CKM}}\right]^{d,\mathrm{GI}}$$

Liao, PRD, 2004

Generalized to the lepton sector with some modifications:

$$\boxed{\delta\mathcal{B} = \widetilde{\delta}\mathcal{B}^{l,\mathrm{GI}} + \widetilde{\delta}\mathcal{B}^{\chi,\mathrm{GI}} + \left[\widetilde{\delta}\mathcal{B}^{\chi,\xi}\right]_{\mathrm{div}}}$$

$$\widetilde{\delta}\mathcal{B} = -\frac{1}{4}\left[\mathcal{B}\left(\delta Z^{\chi,\mathrm{L}} - \delta Z^{\chi,\mathrm{L}\dagger}\right) + \left(\delta Z^{l,\mathrm{L}\dagger} - \delta Z^{l,\mathrm{L}}\right)\mathcal{B}\right]$$

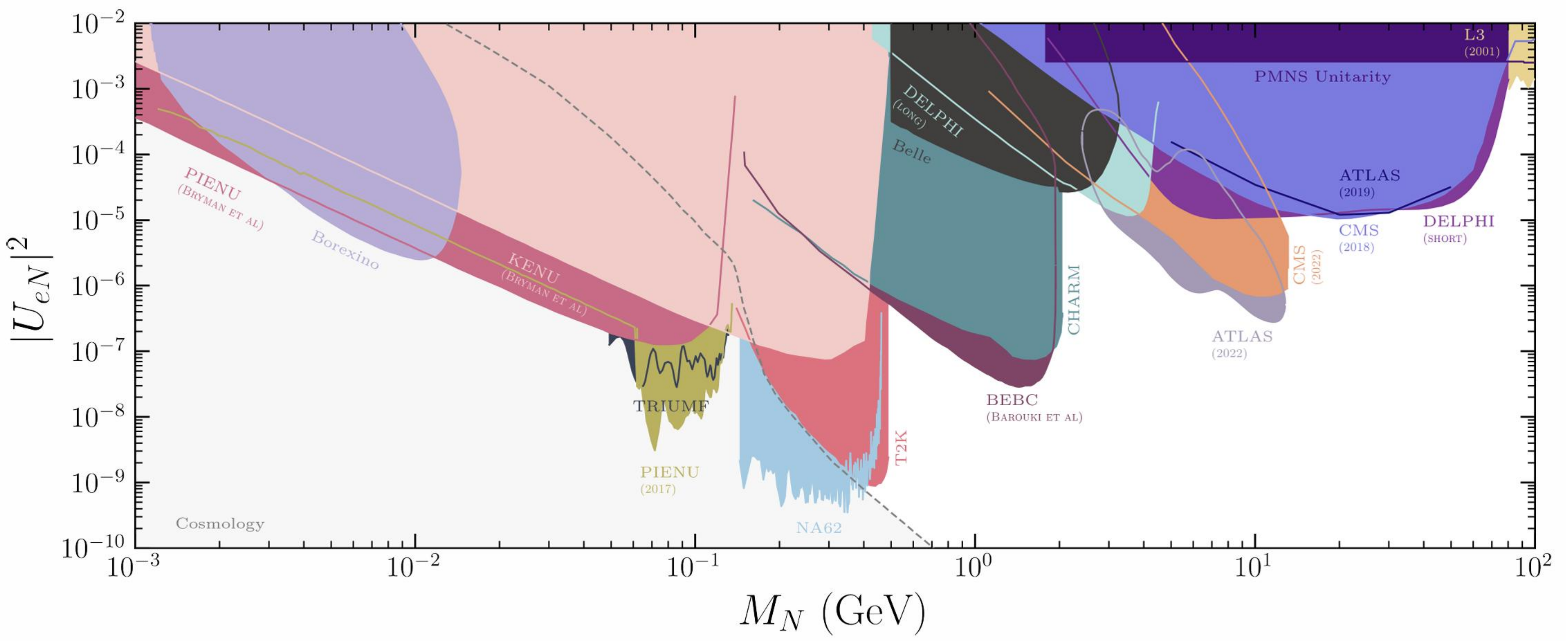
gauge-independent

GI/ξ terms named according to the scalar functions

$$\left[\widetilde{\delta}\mathcal{B}^{\chi,\xi}_{\alpha i}\right]_{\mathrm{div}} = \frac{\alpha\Delta}{64\pi s^2 m_W^2}\sum_{j\neq i}\mathcal{B}_{\alpha j}\left[3\mathcal{C}_{ji}\left(\widehat{m}_j^2 - \widehat{m}_i^2\right) + \sum_k \frac{\widehat{m}_k^3}{\widehat{m}_i\widehat{m}_j}\left(\mathcal{C}^*_{jk}\mathcal{C}_{ki}\widehat{m}_i - \mathcal{C}_{jk}\mathcal{C}^*_{ki}\widehat{m}_j\right)\right] + \frac{\alpha\Delta\mathcal{B}_{\alpha i}}{64\pi s^2 m_W^2}\sum_k \frac{\widehat{m}_k^3}{\widehat{m}_i}\left(\mathcal{C}^2_{ki} - \mathcal{C}^2_{ik}\right)$$

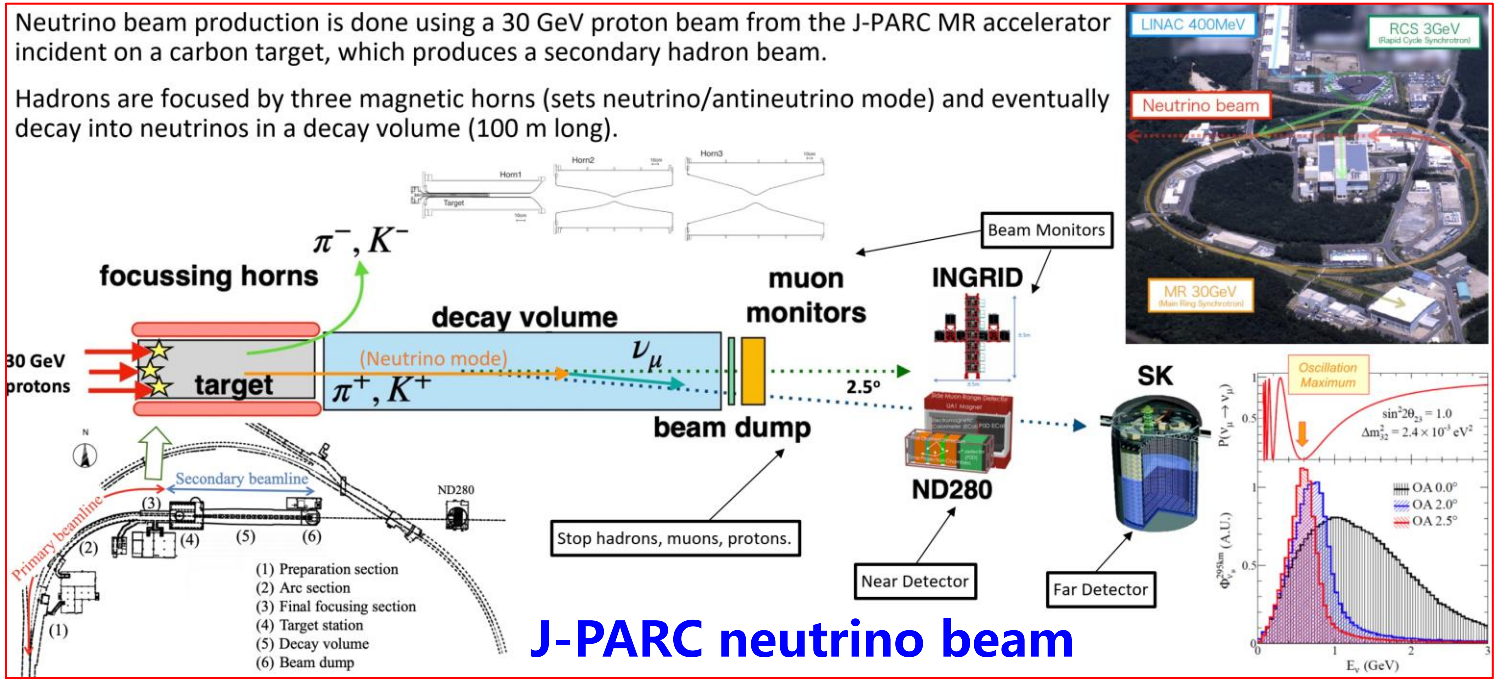
- **Take the seesaw model as the UV-complete theory at the electroweak scale**

Fernández-Martínez et al., 2304.06772

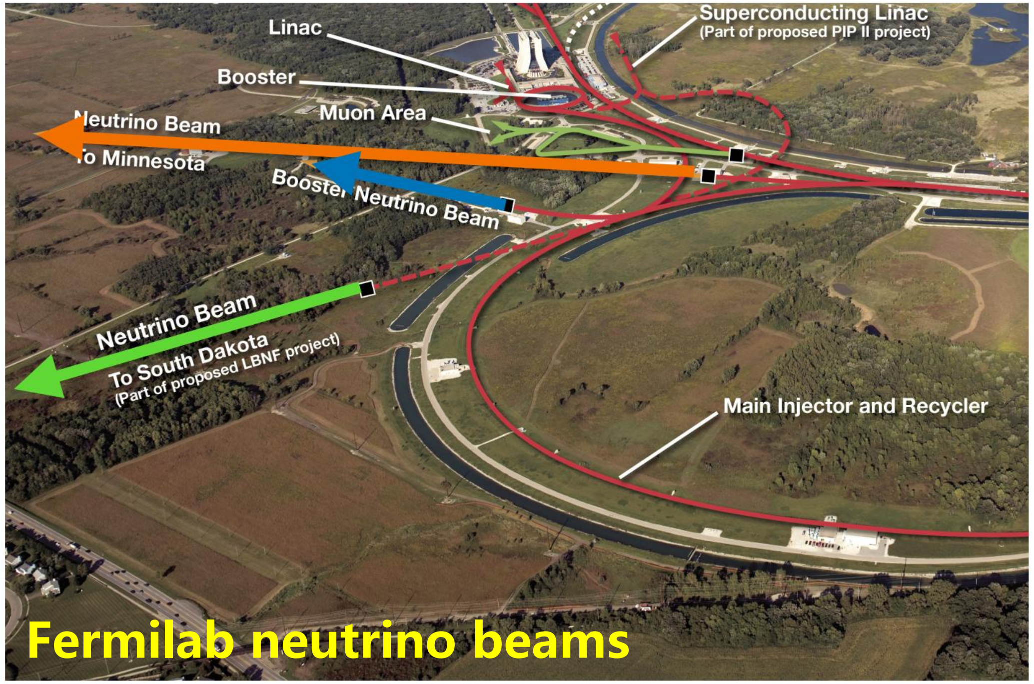


Precision tests of the SM require loop corrections, so calculate them for seesaw for self-consistency

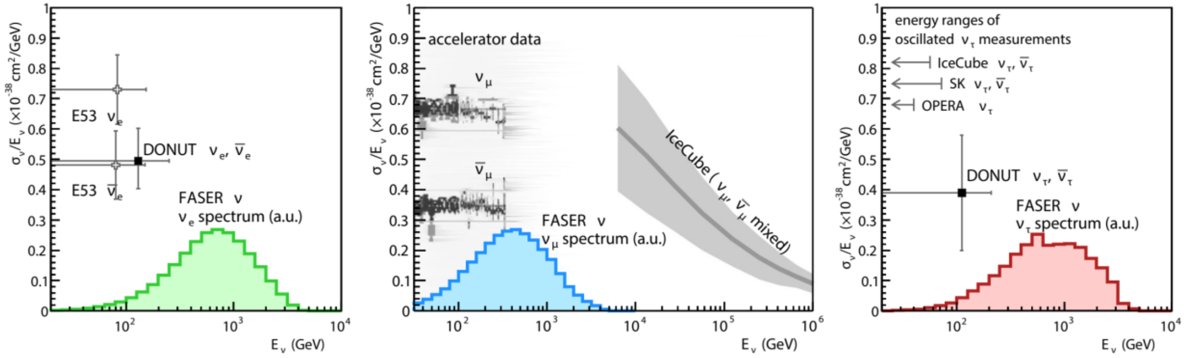
Summary & Outlook



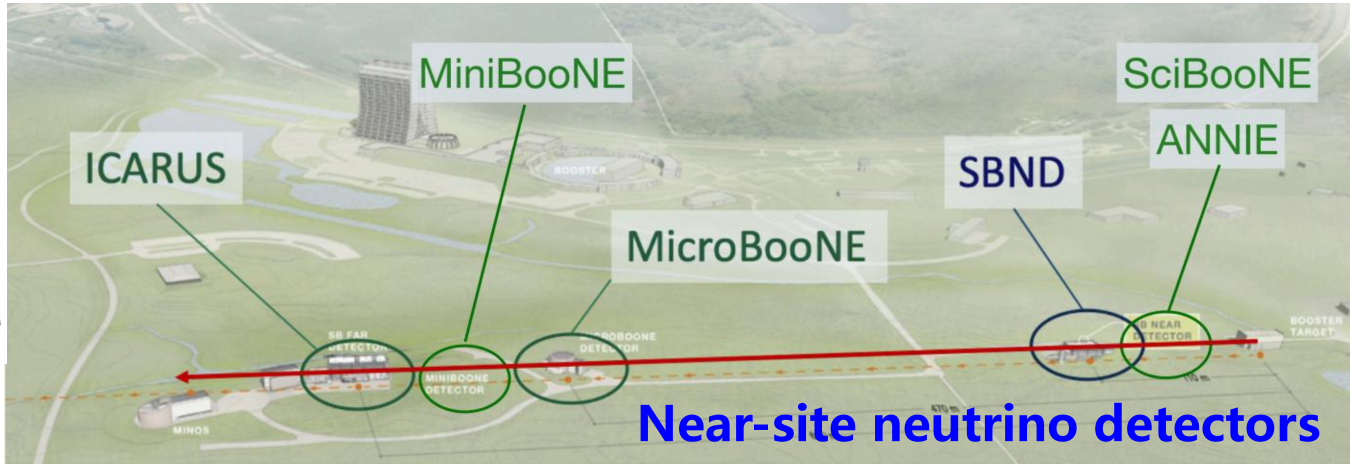
J-PARC neutrino beam



Fermilab neutrino beams



neutrino spectra @ FASERν



Near-site neutrino detectors

- **Precision measurements in neutrino physics are limited by complex neutrino-matter interactions**
- **Radiative corrections become relevant , more theoretical works needed to reduce TH uncertainties**

Summary & Outlook

■ Non-invertible symmetry

Kobayashi, Otsuka, 2408.13984

Weinberg operator $\frac{C_{W}{}^{ij}}{\Lambda}(L_i \varepsilon H) C (L_j \varepsilon H)$

Z_2 gauging of Z_M symmetry

$$\boxed{[g^k][g^{k'}] = [g^{k+k'}] + [g^{M-k+k'}]}$$

***M* = 3**
fusion rule

$$[g^0][g^0] = [g^0], \quad [g^0][g^1] = [g^1], \quad [g^1][g^1] = [g^0] + [g^1]$$

Flavor	Higgs $[g^0]$	Higgs $[g^1]$
(i) : $([g^0], [g^1], [g^1])$	$\begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$
(ii) : $([g^0], [g^0], [g^1])$	$\begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$

■ Gravitational waves

Murayama et al., 2506.15772