Quantum Field Theory and Particle Cosmology

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第五届量子场论及其应用研讨会

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w/ Yunjia Bao, Xingang Chen, Yanou Cui, Bingchu Fan, JiJi Fan, Soubhik Kumar, Yuanzhao Li, Haoyuan Liu, Tao Liu, Abraham Loeb, Qianshu Lu, Shiyun Lu, Zhehan Qin, Matthew Reece, Xi Tong, Lian-Tao Wang, Yi Wang, Jiayi Wu, Jiaju Zang, Hongyu Zhang, Yisong Zhang, Yiming Zhong

History of the Universe

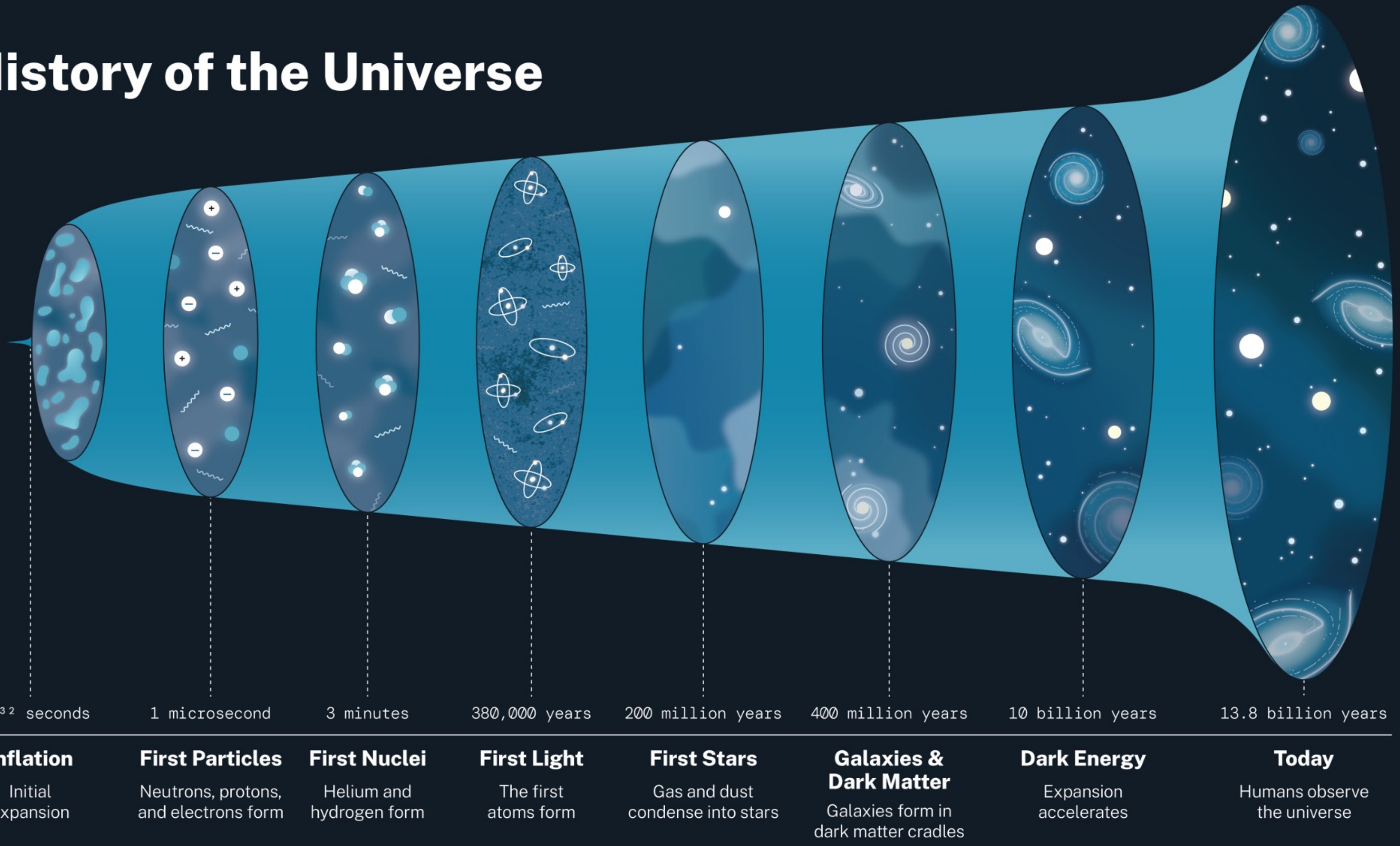
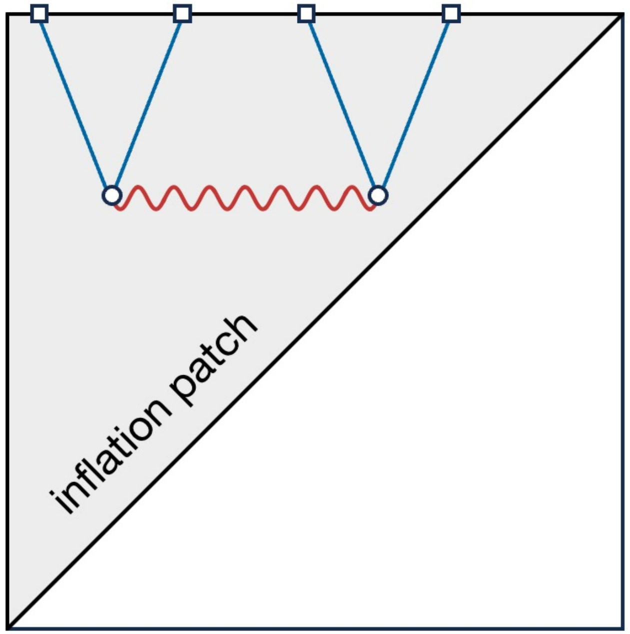


Image credit: NASA

Outline

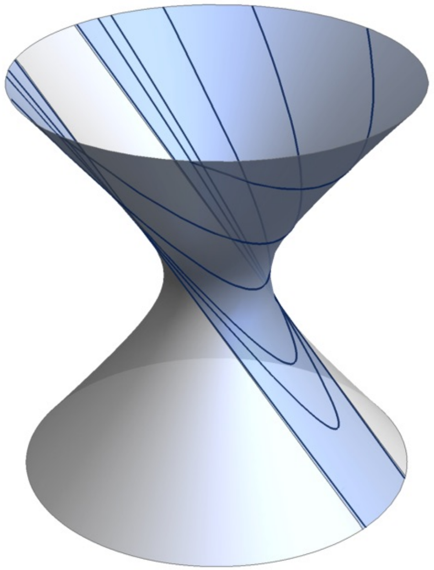


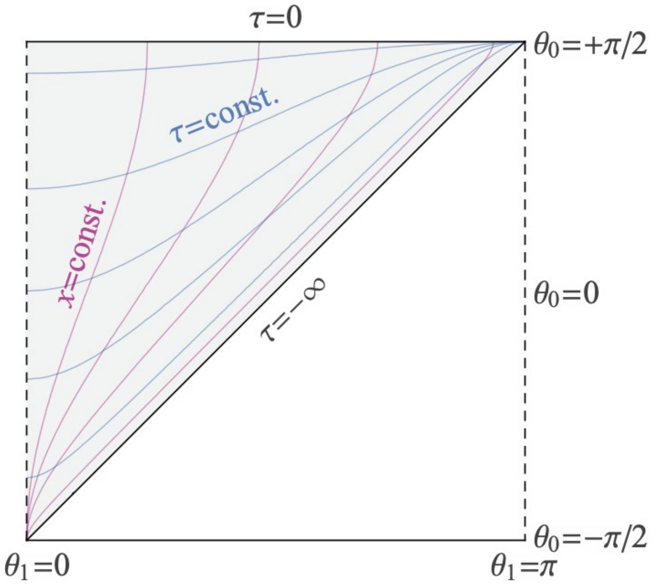
- QFT in an inflatinary spacetime (dS)
- Light fields: Stochastic EFT
- Heavy fields: Cosmological collider physics
- Cosmological amplitudes

de Sitter space

A maximally symmetric spacetime with positive cc; constant curvature $R = 12H^2$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2 = a^2(\tau)(-\mathrm{d}\tau^2 + \mathrm{d}\mathbf{x}^2) \qquad a(t) = e^{Ht} \qquad a(\tau) = -\frac{1}{H\tau}$$





Isometries: SO(4,1):

Space translations, rotations,
Dilatation, dS boosts

Two Casimirs:
Essentially mass and spin

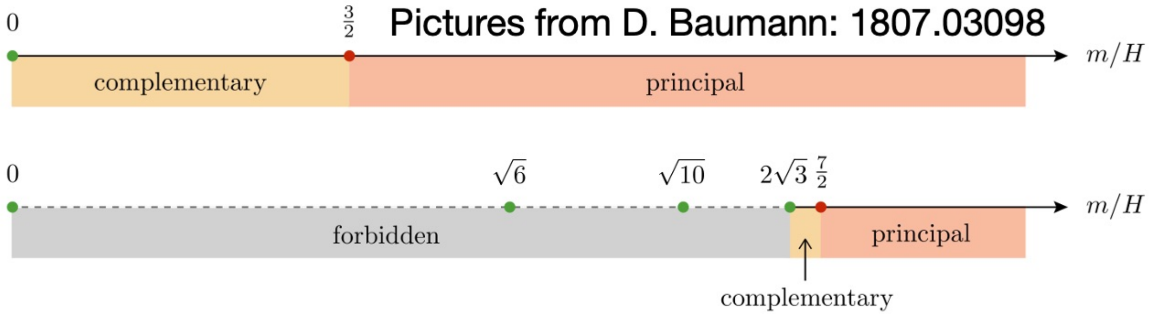
Pictures from ZX: *Lecture notes on particle physics in an inflationary universe*, unpublished

One-particle states as UIRs of SO(4,1)

$$\mathcal{C}_2 \equiv -\tfrac{1}{2}\mathcal{J}_{MN}\mathcal{J}^{MN} = m^2 - 2(s-1)(s+1)$$

$$\mathcal{C}_4 \equiv -\mathcal{W}_M\mathcal{W}^M = s(s+1)\left[m^2 - (s^2 + s - \tfrac{1}{2})\right]$$

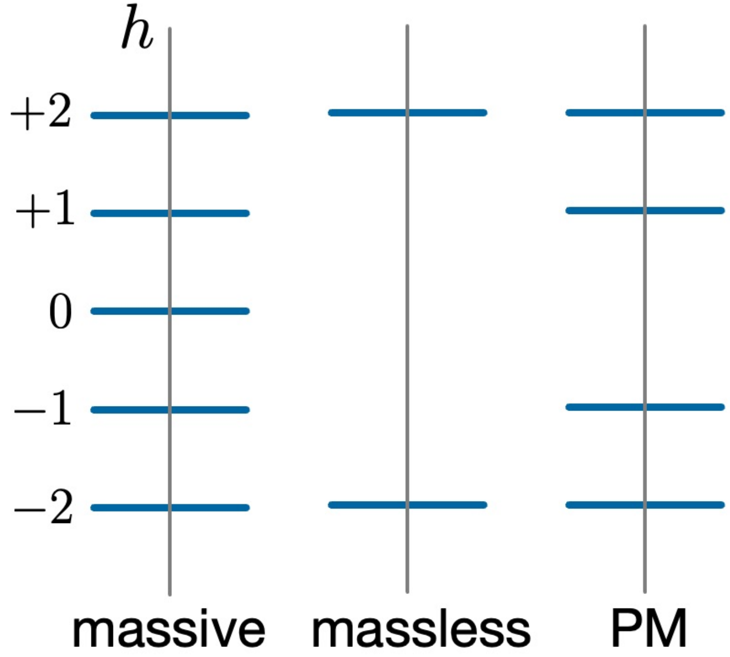
Pictures from D. Baumann: 1807.03098



	$s = 0:$	$s \geq 1:$
Principal	$m > \frac{3}{2}$	$m^2 > (s - 1/2)^2$
Complementary	$0 < m < \frac{3}{2}$	$s(s-1) < m^2 < (s-1/2)^2$
Discrete (Partially massless, PM)		$m^2 = s(s-1) - t(t+1)$ $t = 0, 1, \cdots, s-1$

More exotic reps also exists (exceptional series) which can be Goldstone bosons of enhanced Adler zeros

A spin-2 example:



Light fields vs heavy fields

Taking a scalar field ϕ as an example which satisfies the KG equation $(\Box - m^2)\phi = 0$

- A Massless scalar: $\phi_k(\tau) = \frac{1}{\sqrt{2k^3}}(1 + \mathrm{i}k\tau)e^{-\mathrm{i}k\tau}$

  -- approaching constant in the late-time limit $\tau \to 0$ (long-lived)
  -- the ultimate reason that dS QFT is relevant to cosmological observations

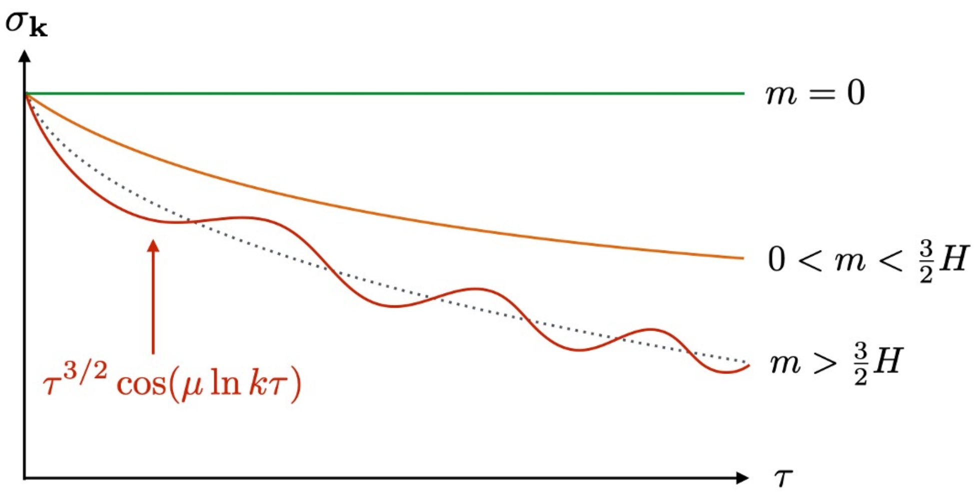
- A conformal scalar ($m = \sqrt{2}H$): $\phi_k(\tau) = \frac{1}{\sqrt{2k}}\tau e^{-\mathrm{i}k\tau}$

  -- like massless scalar in Mink, a good toy model

- A heavy scalar: $\nu = \sqrt{9/4 - m^2/H^2}$

$$\sigma(k,\tau) = \frac{\sqrt{\pi}}{2}e^{+\mathrm{i}\nu\pi/2}(-\tau)^{3/2}\mathrm{H}_\nu^{(1)}(-k\tau)$$
$$\to (-\tau)^{3/2\pm\nu}$$

  -- oscillatory in late-time limit, interesting pheno!



Pictures from D. Baumann: 1807.03098

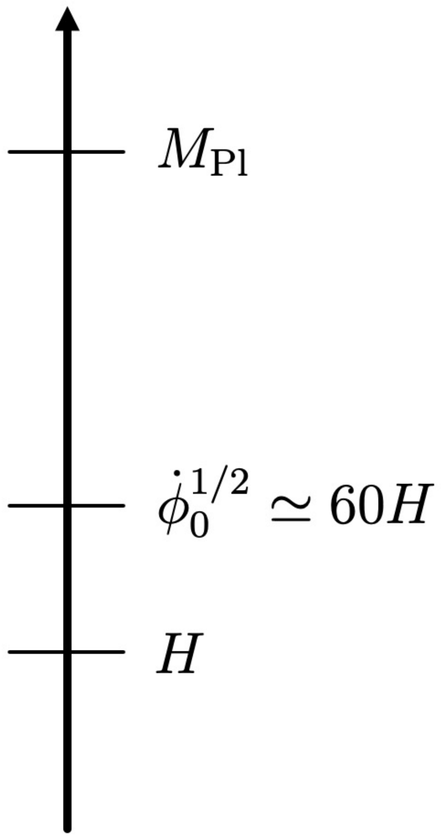
QFT (EFT) in dS

- Depending on the situation, we can consider an EFT with different cutoff scales:

- Lorentz-invariant EFTs: $\dot{\phi}_0^{1/2} < \Lambda < M_{\rm Pl}$ [Wang, ZX: 1910.12876]
  Most relevant for particle model building of CC physics
  Assuming a scalar degree (inflaton)
  Couple your NP model to the scalar degree with the power-counting scheme identitcal to a normal relativistic QFT.

- "EFT of Inflation" (~ChPT): $H < \Lambda < \dot{\phi}_0^{1/2}$ [Cheung et al: 0709.0293]
  Being ignorant about the inflaton; parameterize the fluctuation directly as a Goldstone of the broken time-diff

- "soft dS EFT" (~NRQED): $\Lambda < H$ [Cohen, Green: 2007.03693]
  Useful when there are peculiar infrared divergences
  (like in stochastic inflation)

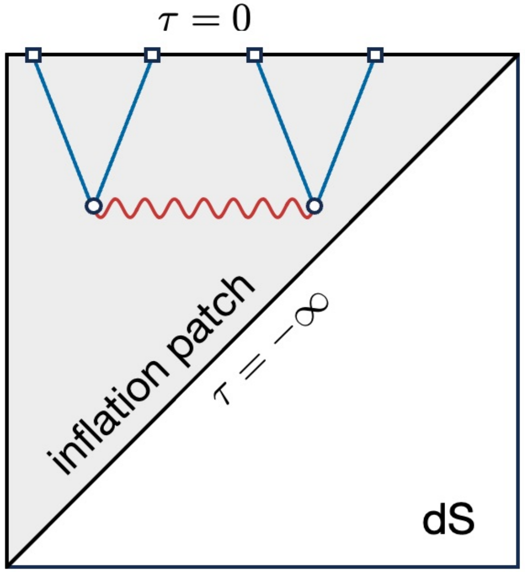


Correlators: Schwinger-Keldysh formalism

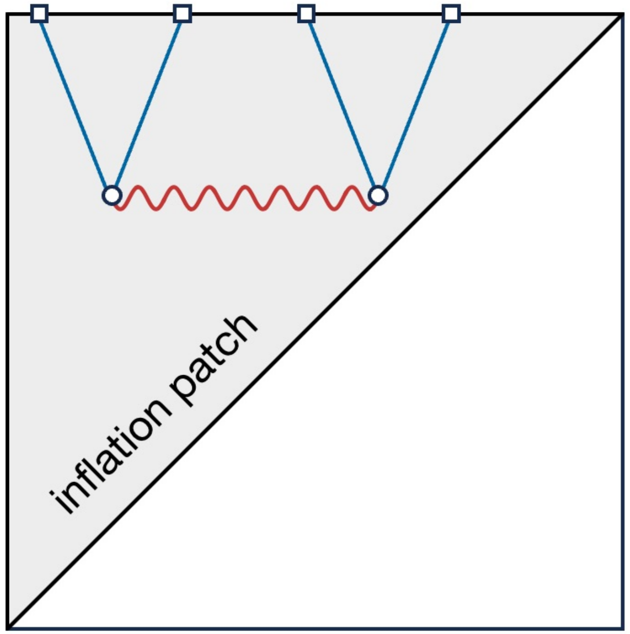
- Defining S matrix in dS is still a tricky issue, and the physical meaning is unclear
- Instead, correlation functions are well defined and measurable to us

$$\langle\Omega|\varphi(\tau_f,\mathbf{x}_1)\cdots\varphi(\tau_f,\mathbf{x}_n)|\Omega\rangle = \int \mathcal{D}\varphi_+\mathcal{D}\varphi_-\,\varphi_+(\tau_f,\mathbf{x}_1)\cdots\varphi_+(\tau_f,\mathbf{x}_n)$$

$$\times\exp\left\{\mathrm{i}\int_{\tau_0}^{\tau_f}\mathrm{d}^4x\left(\mathscr{L}[\varphi_+]-\mathscr{L}[\varphi_-]\right)\right\}\prod_{\mathbf{x}}\delta\Big[\varphi_+(\tau_f,\mathbf{x})-\varphi_-(\tau_f,\mathbf{x})\Big]$$



- The free initial state is usually chosen to be the BD vacuum state with good reasons and is SO(4,1) invariant
- On the contrary, interactions often break dS boosts
- Feynman rules similar to ordinary QFT [Chen, Wang, ZX, 1703.10166]
- A subtle issue of derivative couplings



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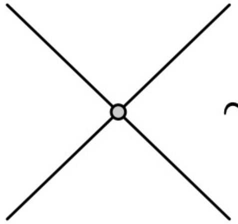
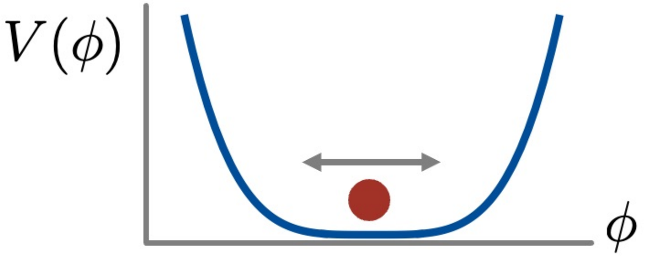
Self-interacting massless scalar

- A seemingly innocent model of $\lambda\phi^4$ becomes rather problematic in dS: all tree graphs are divergent in the late-time (IR) limit:



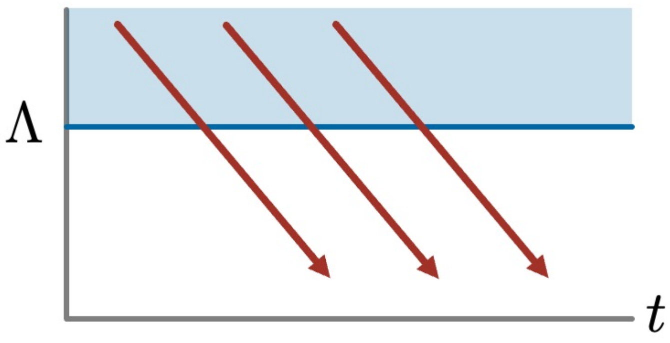
$$\sim \mathrm{Re}\int^{\tau_f} \frac{\mathrm{d}\tau}{\tau^4}\phi^4(\tau) \sim \log(-k_t\tau_f)$$

- Physically: dS vacuum is like a thermal state of Hawking temperature; Thermal fluctuations kicks the mean field so that $\Delta\phi \sim H$ in one Hubble time => Brownian motion

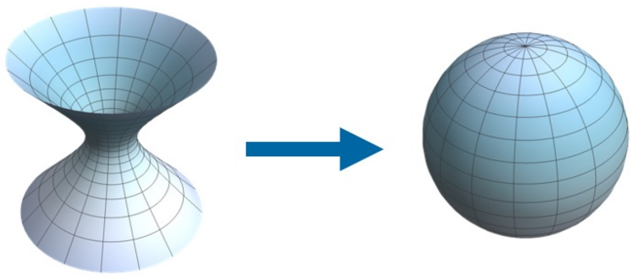


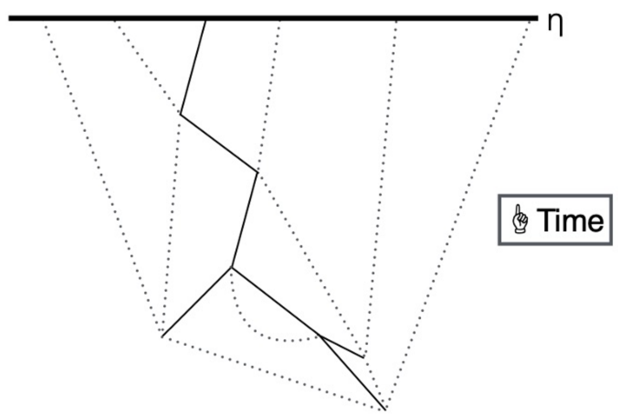
$$\lambda\phi^4 \sim H^4 \quad \Rightarrow \quad \langle\phi^4\rangle \sim H^4/\lambda$$

- In the spirit of Wilsonian EFT, the UV modes are constantly redshifted and thus crossing the horizon, resulting in a noisy EFT in the IR
=> Leading order EOM is a Langevin equation
[Starobinsky, Yokoyama, astro-ph/9407016]



Various approaches





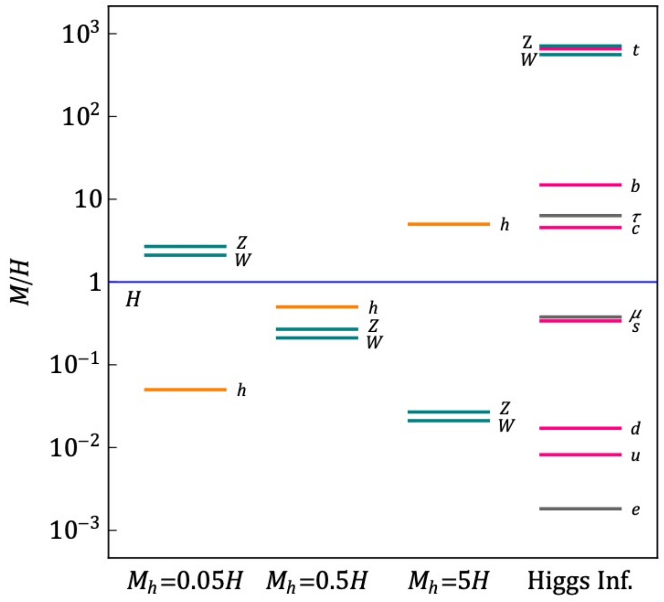
1. Euclidean [Rajaraman: 1008.1271]
IR div comes from strongly coupled zero mode, whose path integral is nonperturbatively computable

2. Resumming leading logs
[Baumgart, Sundrum, 1912.09502]
The most divergent graphs resum to a solution of Fokker-Planck eq (equiv. to Langevin eq)

3. Wavefunction approach
[Gorbenko, Senatore, 1911.00022]
The probability distribution of superhorizon modes are mod-square of the wavefunction that can be computed by solving the Schrödinger eq

4. EFT approach (Soft dS EFT)
[Cohen, Green, Premkumar, Ridgway, 2106.09728]
A systemmatic superhorizon EFT is a first-order system very similar to NRQED

Phenomenology

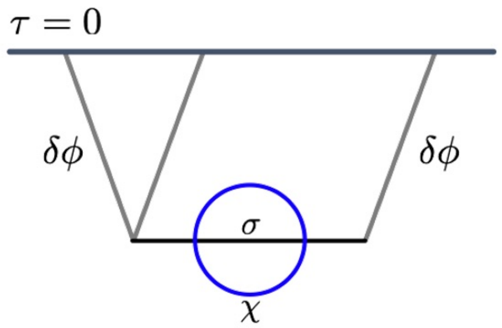


Chen, Wang, ZX, 1610.06597

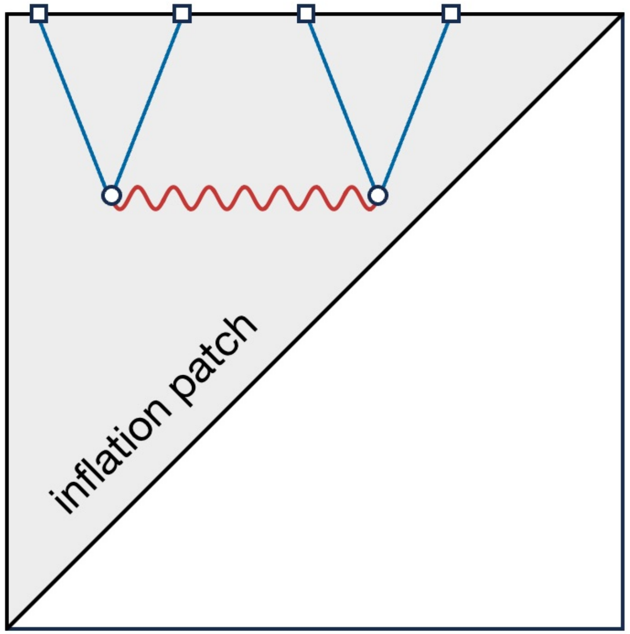
- The Standard Model mass spectrum is heavily distorted! [Chen, Wang, ZX, 1610.06597; 1612.08122]

$$g^2|H|^2A_\mu A^\mu \Rightarrow g^2\langle|H|^2\rangle A_\mu A^\mu \sim \frac{g^2H^2}{\sqrt{\lambda}}A_\mu A^\mu$$

- Telling dressed mass from intrinsic mass?
  – A Higgs can trigger additional decay channels
  [Lu, Reece, ZX, 2108.11385]



- Random walks of compact scalars
  [Chakraborty, Stout, 2311.09219; Chakraborty, 2501.07672]



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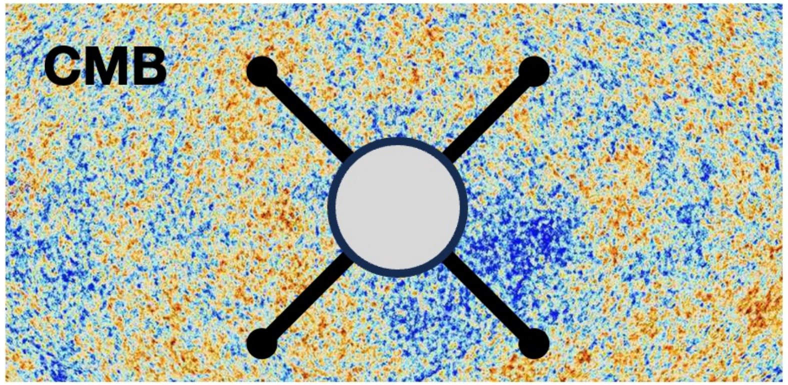
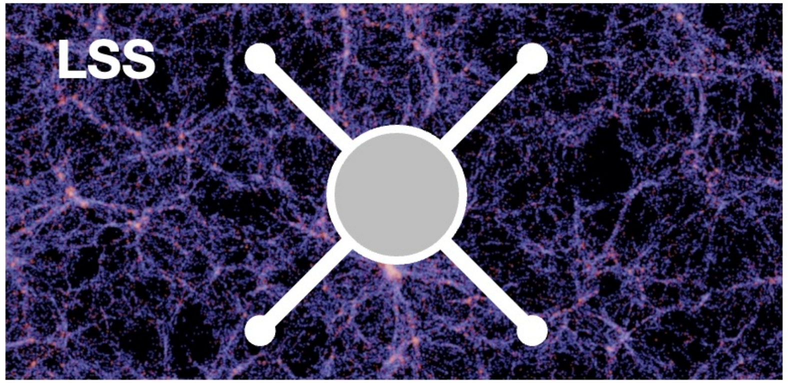
A Cosmological collider program

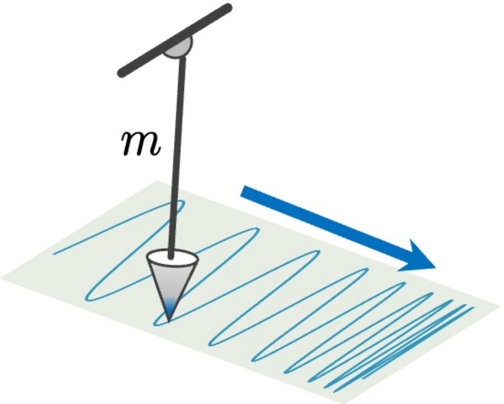
[Chen, Wang, 0911.3380; Arkani-Hamed, Maldacena, 1503.08043 and many more]

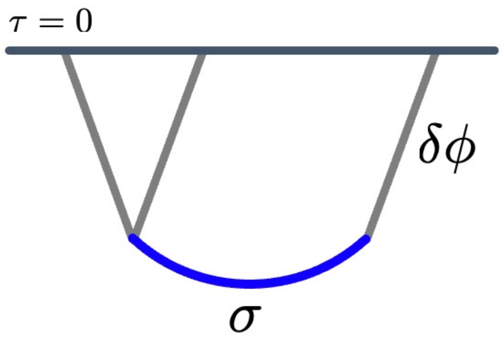


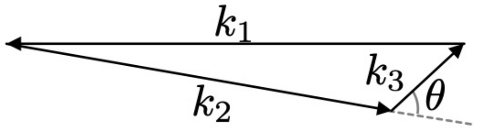


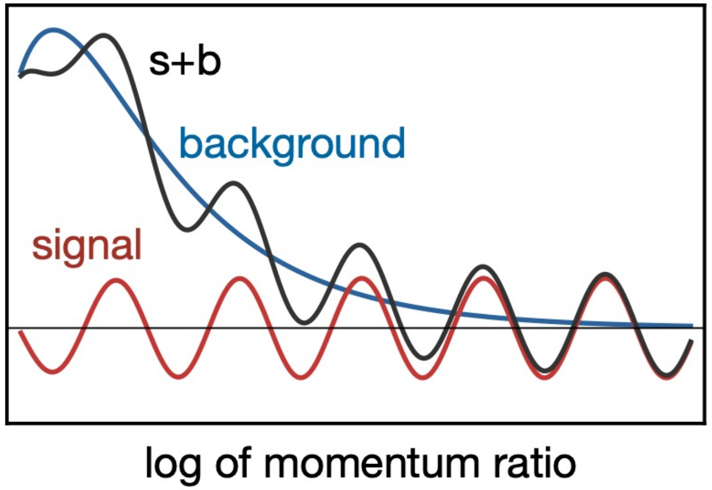
particle production

mass ~ 10^{14} GeV









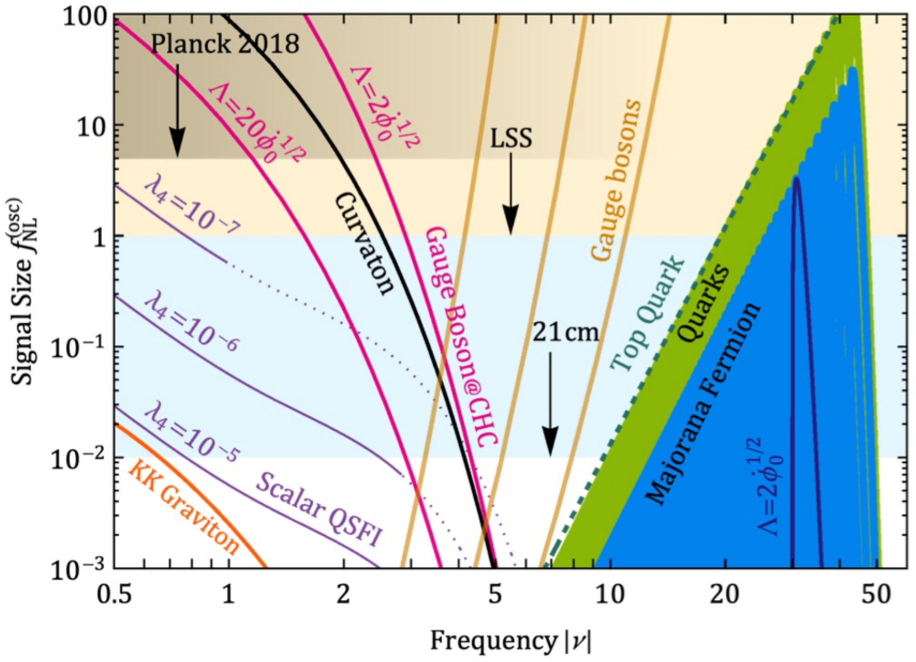
superhorizon resonance

mass, spin, coupling, etc

amplitude nonanalyticity

BSM phenomenology @ Cosmo Collider

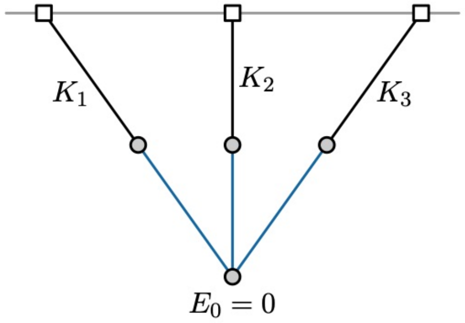
Upshot: not entirely trivial to get visibly large signals, but many natural models are identified over the years…



[Lian-Tao Wang, ZX, 1910.12876]

Several scenarios:

1. Quasi-single-field inflation



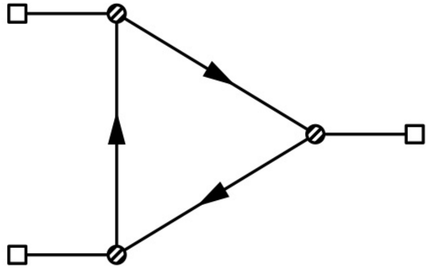
[Chen, Wang, 0911.3380]

triple massive exchange dominates the signal;

Strongly-coupled case

[An, McAneny, Ridgway, Wise, 1706.09971]

2. Chemical potential



[Chen, Wang, ZX, 1805.02656;
Wang, ZX, 1910.12876]

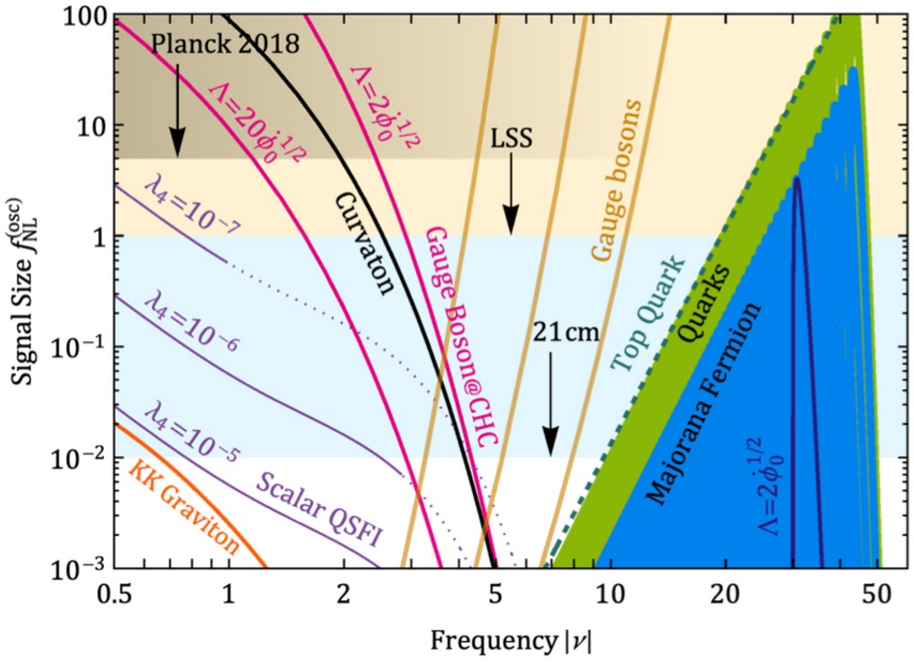
Lowest-dim operator when s>0:

$$\frac{1}{\Lambda}(\partial_\mu\phi)\mathcal{J}^\mu \quad => \quad \frac{1}{\Lambda}\dot\phi_0\mathcal{N}$$

=> Exponentially enhanced signal

BSM phenomenology @ Cosmo Collider

Upshot: not entirely trivial to get visibly large signals, but many natural models are identified over the years…



[Lian-Tao Wang, ZX, 1910.12876]

3. Evidence of parity violation from LSS?

[Hou, Slepian, Cahn: 2206.03625; Philcox: 2206.04227]

A no-go theorem: impossible from local contact => particle exchange important!

[Liu, Tong, Wang, ZX, 1909.01819]

4. Cosmological Higgs collider

[Lu, Wang, ZX, 1907.07390]

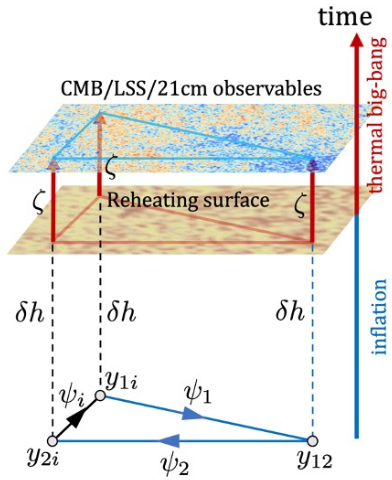
Using the SM Higgs to generate large-scale fluctuations

=> Higgs couplings

Neutrino physics

[Cui, ZX, 2112.10793]

[Han, He, Song, You, 2412.16033, 2412.21045]

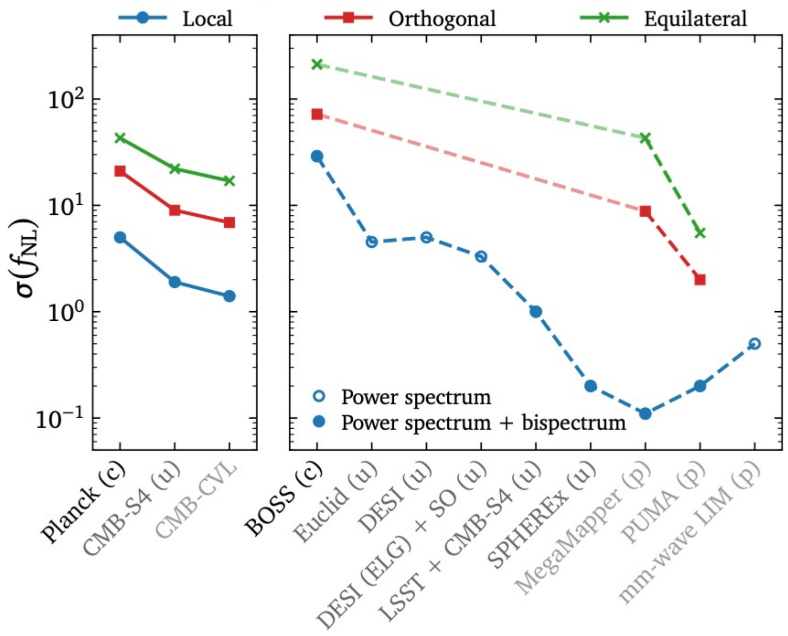


[Cui, ZX, 2112.10793]

Data are coming in!

- ~ 2 orders in near future; ~ 4 ultimately with 21cm

[Snowmass 2021: 2203.08128]

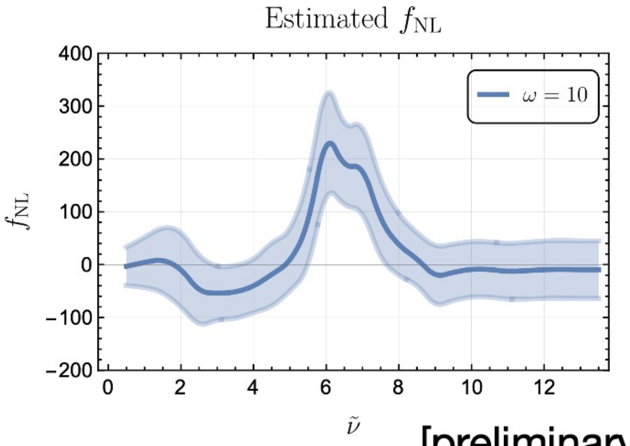


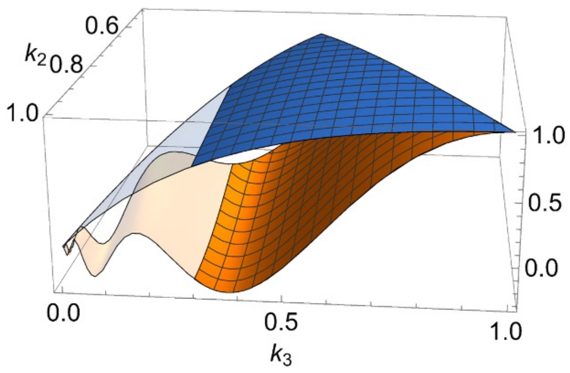
- Challenges for theorists:
Efficient and precise computation of massive cosmological correlators!

- Searches of single-change processes from CMB data [Sohn et al. 2404.07203] and LSS data [Cabass et al. 2404.01894]

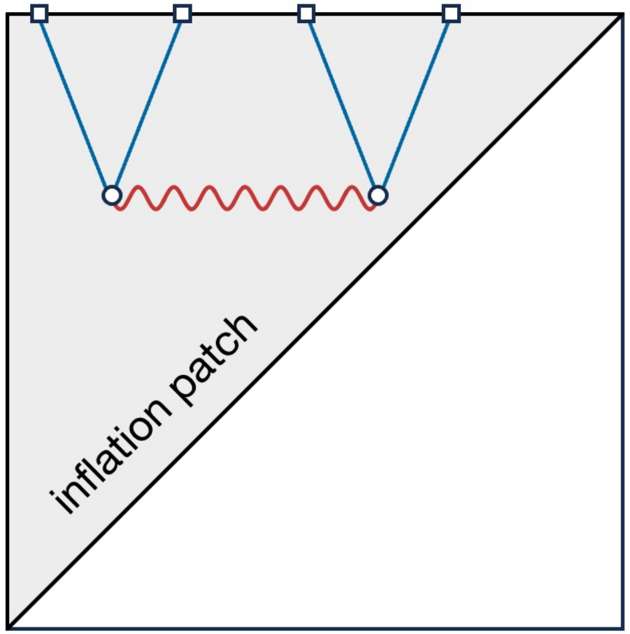
- Parity-violating particle models (chem potential) from LSS data [Bao, Wang, ZX, Zhong, 2504.02931]

- Many types of scalar-exchange models from Planck data [Kumar, Lu, ZX, Zhang, to appear]





[preliminary]



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Massive inflation correlators

[See Chen, Wang, ZX, 1703.10166 for a review]

- General comments:
  1. Perturbative in the bulk; boundary CFT unknown; purely kinematic info of conformal sym
  2. Large signals typically from boost-breaking or even dilatation-breaking interactions; crucial to have techniques indep of full dS isometries
  3. Tensor decomposition => Tensor structure (easy) $\times$ scalar integral (hard)

$$\mathcal{T}(\{\boldsymbol{k}\}) \sim \underbrace{\int \mathrm{d}\tau}_{\text{vertex int}} \underbrace{\int \mathrm{d}^d \boldsymbol{q}}_{\text{loop int}} \times \underbrace{(-\tau)^p \times e^{\mathrm{i}E\tau}}_{\text{ext line}} \times \underbrace{\mathrm{H}_{\mathrm{i}\widetilde{\nu}}\Big[-K(\boldsymbol{q},\boldsymbol{k})\tau\Big] \times \theta(\tau_i - \tau_j)}_{\text{bulk line}}$$

- Main challenges: no 4-momentum reps
  1. Special functions instead of plane waves; 2. time-ordered integrals

- Complexity increases with # of loops and # of vertices

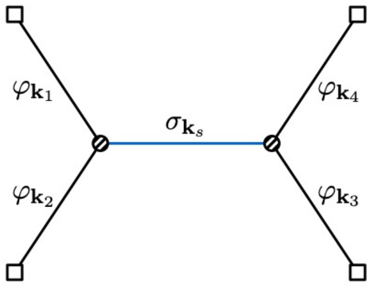
Recent progress

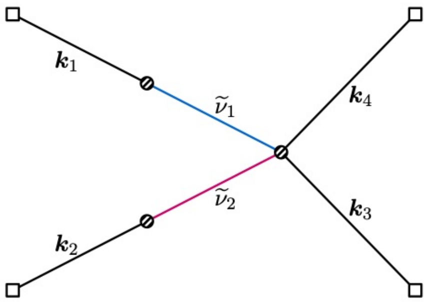
- “Cosmological bootstrap”
  Single exchange
  [Arkani-Hamed, Baumann, Lee, Pimentel, 1811.00024]
  Double exchange
  [Aoki, Pinol, Sano, Yamaguchi, Zhu, 2404.09547]
  Boost-breaking dispersions
  [Pimentel, Wang, 2205.00013,
  Qin, ZX, 2208.13790, 2301.07047,
  Jazayeri, Renaux-Petel, 2205.10340]

- Full Mellin-space reps [Sleight 1907.01143 etc]

- Conformal scalar amplitude: Kinematic-flow
  a set of diff eqs [Arkani-Hamed et al., 2312.05303]

- CosmoFlow: numerical approach
  [Werth, Pinol, Renaux-Petel, 2312.06559]





$$\sum_{\mathsf{a,b}=\pm}\mathcal{I}^{p_1p_2}_{\mathsf{ab}}(r_1,r_2)=\mathcal{I}^{p_1p_2}_{\mathrm{BG},>}(r_1,r_2)+\mathcal{I}^{p_1p_2}_{\mathrm{L},>}(r_1,r_2)+\mathcal{I}^{p_1p_2}_{\mathrm{NL}}(r_1,r_2)$$

$$\mathcal{I}^{p_1p_2}_{\mathrm{BG},>}(r_1,r_2)=\frac{1}{\mathrm{i}\widetilde{\nu}}\cos\left[\tfrac{\pi}{2}(p_1+p_2)\right]r_1^{5+p_1+p_2}\sum_{n_1,n_2=0}^{\infty}\frac{(-1)^{n_{12}}}{n_1!n_2!}\left(\frac{r_1}{2}\right)^{2n_{12}}(-\mathrm{i}\widetilde{\nu})_{-n_1}(\mathrm{i}\widetilde{\nu})_{-n_2}$$
$$\times\mathcal{F}\left[\begin{matrix}2n_2+p_2+\frac{5}{2}-\mathrm{i}\widetilde{\nu},2n_{12}+p_1+p_2+5\\2n_2+p_2+\frac{7}{2}-\mathrm{i}\widetilde{\nu}\end{matrix}\middle|-\frac{r_1}{r_2}\right]+\text{c.c.}$$

$$\mathcal{I}^{p_1p_2}_{\mathrm{L},>}(r_1,r_2)=\mathcal{C}^{p_1p_2}_{\widetilde{\nu}}\left(\frac{r_1}{r_2}\right)^{\mathrm{i}\widetilde{\nu}}\mathbf{F}^{p_1}_{\widetilde{\nu}}(r_1)\mathbf{F}^{p_2}_{-\widetilde{\nu}}(r_2)+\text{c.c.},$$

$$\mathcal{I}^{p_1p_2}_{\mathrm{NL}}(r_1,r_2)=\mathcal{C}^{p_1p_2}_{\widetilde{\nu}}\left(\frac{r_1r_2}{4}\right)^{\mathrm{i}\widetilde{\nu}}\mathbf{F}^{p_1}_{\widetilde{\nu}}(r_1)\mathbf{F}^{p_2}_{\widetilde{\nu}}(r_2)+\text{c.c.}$$

$$\mathbf{F}^{p}_{\widetilde{\nu}}(r)\equiv r^{5/2+p}\Gamma\left[\frac{5}{2}+p+\mathrm{i}\widetilde{\nu},-\mathrm{i}\widetilde{\nu}\right]{}_2\mathrm{F}_1\left[\begin{matrix}\frac{5}{4}+\frac{p}{2}+\frac{\mathrm{i}\widetilde{\nu}}{2},\frac{7}{4}+\frac{p}{2}+\frac{\mathrm{i}\widetilde{\nu}}{2}\\1+\mathrm{i}\widetilde{\nu}\end{matrix}\middle|r^2\right]$$

$$\mathcal{C}^{p_1p_2}_{\widetilde{\nu}}\equiv\frac{1}{2\pi}\left[\cos\left(\tfrac{\pi}{2}(p_1-p_2)\right)+\sin\left(\tfrac{\pi}{2}(p_1+p_2+2\mathrm{i}\widetilde{\nu})\right)\right]$$

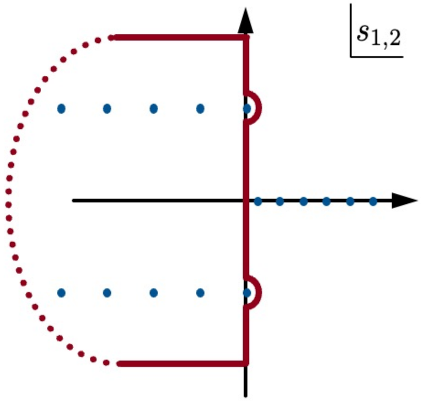
[Qin, ZX 2208.13790]

New techniques

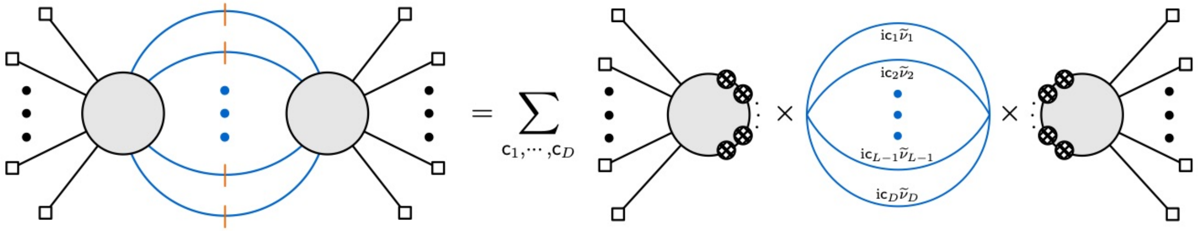
- Partial Mellin-Barnes Rep: Mellin transforming all *bulk* modes
  [Qin, ZX, 2205.01692, 2208.13790]

$$\mathcal{T}(\{\boldsymbol{k}\}) \sim \int \mathrm{d}s \times \mathcal{G}(s) \times \left[\int \mathrm{d}^d\boldsymbol{q} K(\boldsymbol{q},\boldsymbol{k})^\alpha\right] \times \left[\int \mathrm{d}\tau e^{\mathrm{i}E\tau} \times (-\tau)^\beta \times \theta(\tau_i - \tau_j)\right]$$

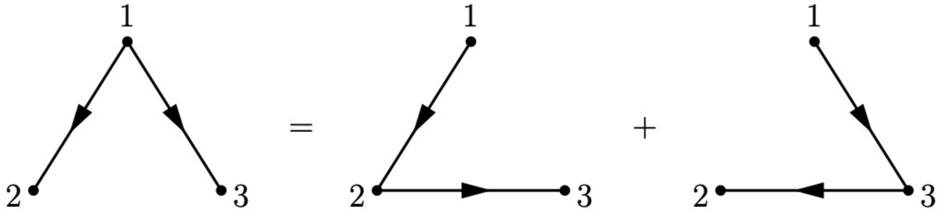
bulk lines loop int nested time int



- Factorization theorem and cutting rulls of nonlocal CC signals to all loop orders
  [Qin, ZX, 2304.13295; 2308.14802]

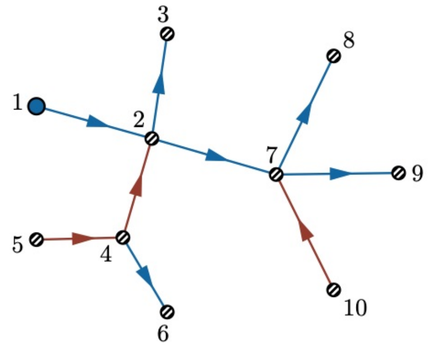


- Family-tree and family-chain decomposition
  [ZX, Zang, 2309.10849; Fan, ZX, 2403.07050; 2509.02684]
  MHFs of definite transcendental weight
  Polylogs in dS conformal scalar limit



New results

- Arbitrary tree correlators fully solved in multivariate hypergeometric functions
  => WYSIWYG solutions! [Liu, ZX, 2412.07843] Max cut part [Chen, Feng, Tao, 2411.03088]



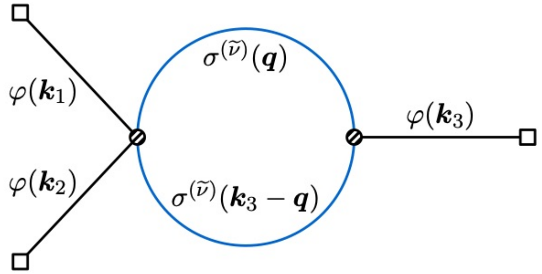
$$\boxed{\mathcal{G} = \sum_{i\in 2^{\{K\}}} \mathrm{Cut}_i\,[\mathcal{G}]}$$

$$[\![1\cdots V]\!] = \sum_{\{\ell,m\}} 2^V \cos(\pi p_{1\cdots V}/2)\Gamma(q_1+p_1+1) \times \prod_{i=2}^{V} \frac{(-1)^{\ell_i}}{\ell_i!\left(\frac{\ell_i+q_i+p_i}{2}+\frac{5}{4}\pm\frac{\mathrm{i}\widetilde{\nu}_i}{2}\right)_{m_i+1}}\left(\frac{K_i}{2E_1}\right)^{2m_i+3}\left(\frac{E_i}{E_1}\right)^{\ell_i+p_i+1}$$

- Likewise, general conformal-scalar amplitudes in FRW fully solved => Family tree structure
  [Fan, ZX, 2403.07050; He, et al., 2407.17715]

- A few loop processes solved completely with dS Källen-Lehman and dispersive techniques
  [ZX, Zhang, 2211.03810; Liu, Qin, ZX, 2407.12299; Qin, 2411.13636; Zhang, 2507.19318]

- Example: 3pt massive bubble:
  numerical [O(10^5) CPU hrs] vs. analytical [O(10s) @ laptop]
  [Wang, ZX, Zhong, 2109.14635] [Liu, Qin, ZX, 2407.12299]



Open questions & future directions

- Exotic dynamics of light or massless fields

  Stochastic scalars: EFT beyond the squeezed limit? Phenomenolgy of Higgsed matter?
  Charting out all consistent scalar and partially massless field theories?

- Cosmological collider physics

  Many BSM scenarios (SUSY, Extra dim, composites, strings) to be explored @ CC!
  Time to build template banks and test your models with data!

- Cosmological amplitudes

  Conjecture: Any graphic contribution to a renormalized massive cosmological correlator is a multivariate hypergeometric function with only power-law singularities (finite-deg poles or branch points)
  Analytical structures for all loops? analytical continuations? better numerical strategies? nonperturbative bootstrap?