重庆大学
CHONGQING UNIVERSITY

Nonperturbative Expansion of QCD

Si-Xue Qin

Beijing

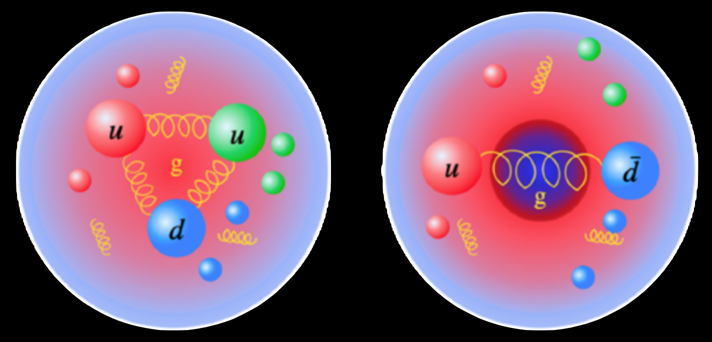
2025-11-02

恭贺李老师八十华诞

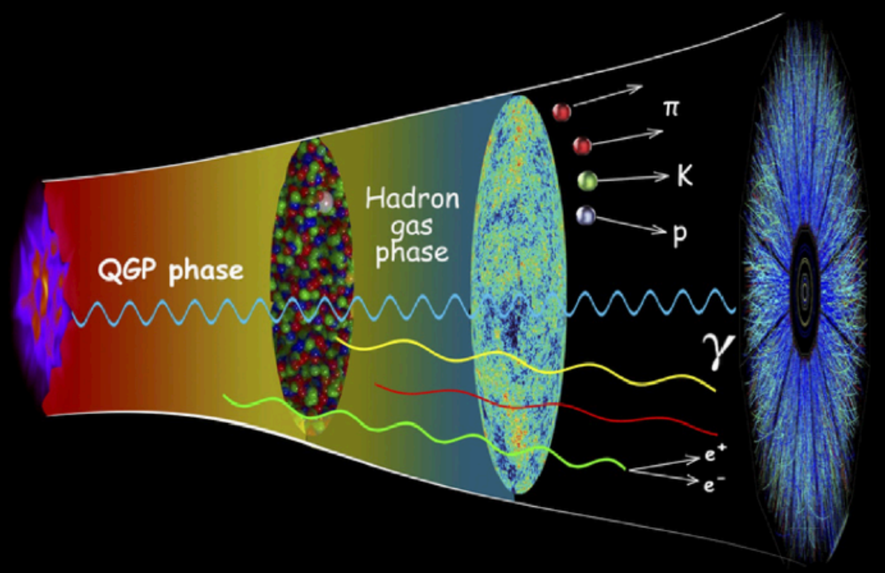
Introduction

Introduction: QCD frontiers

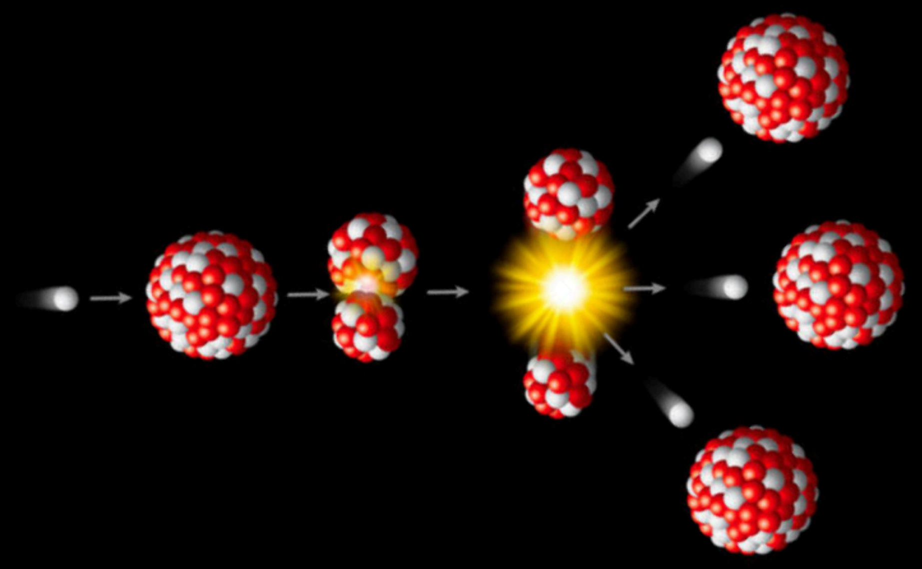
Hadron



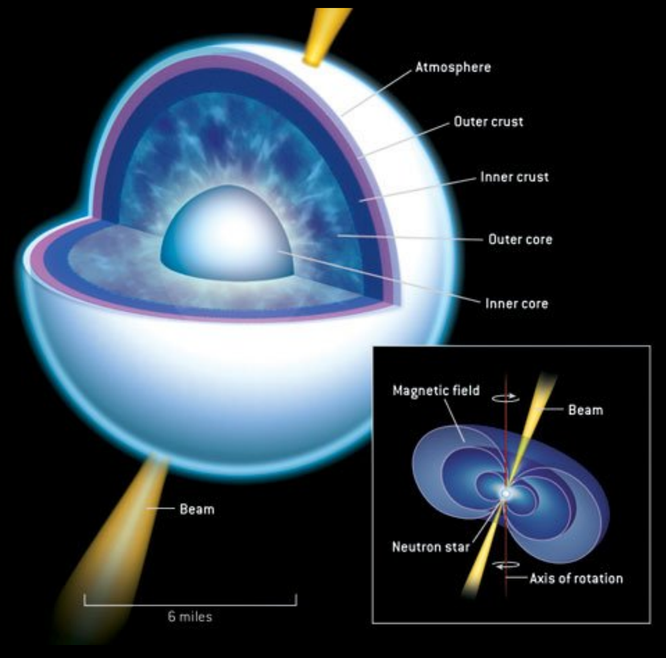
Hot Matter



Nucleus



Dense Matter



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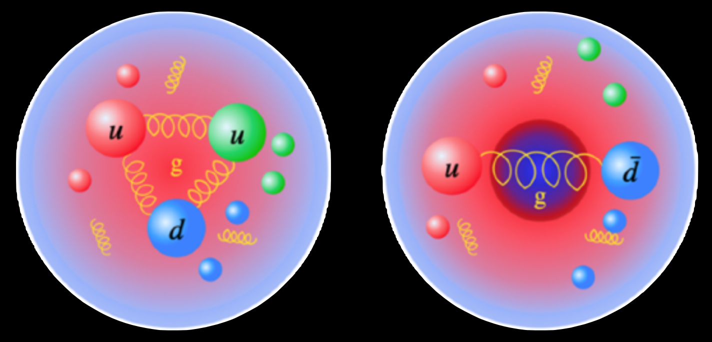
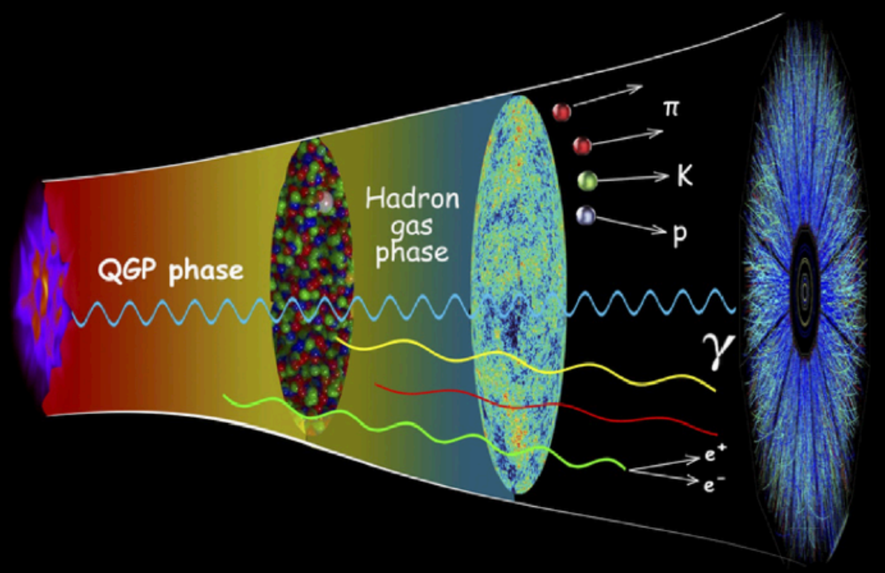
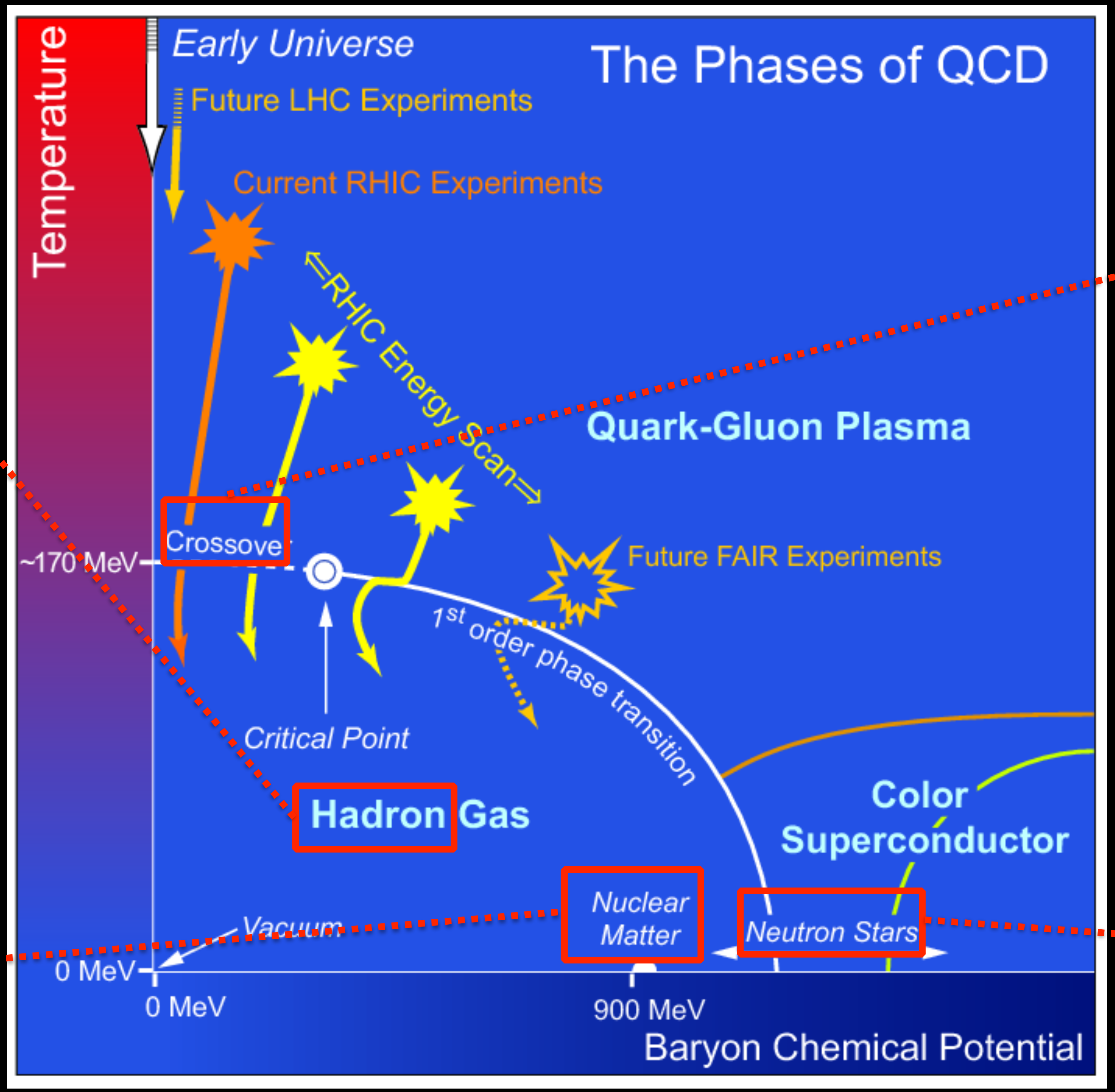


Hot Matter





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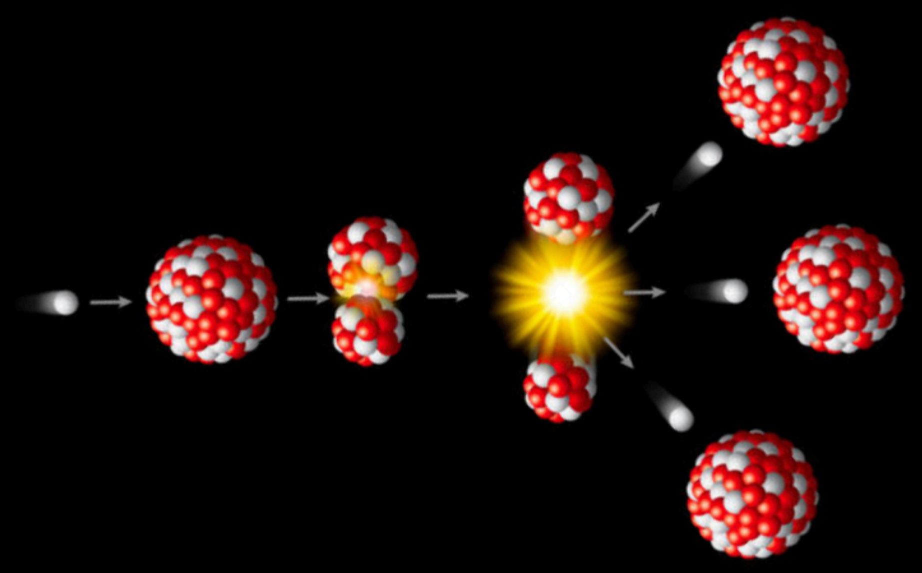

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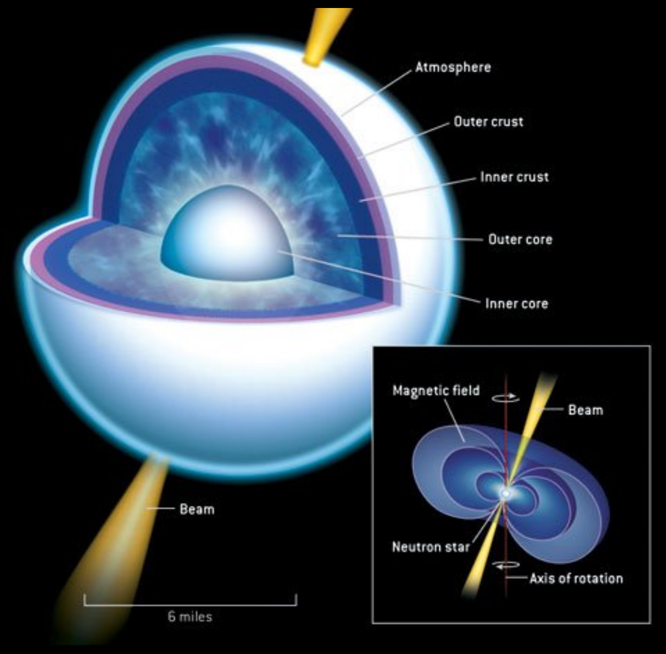
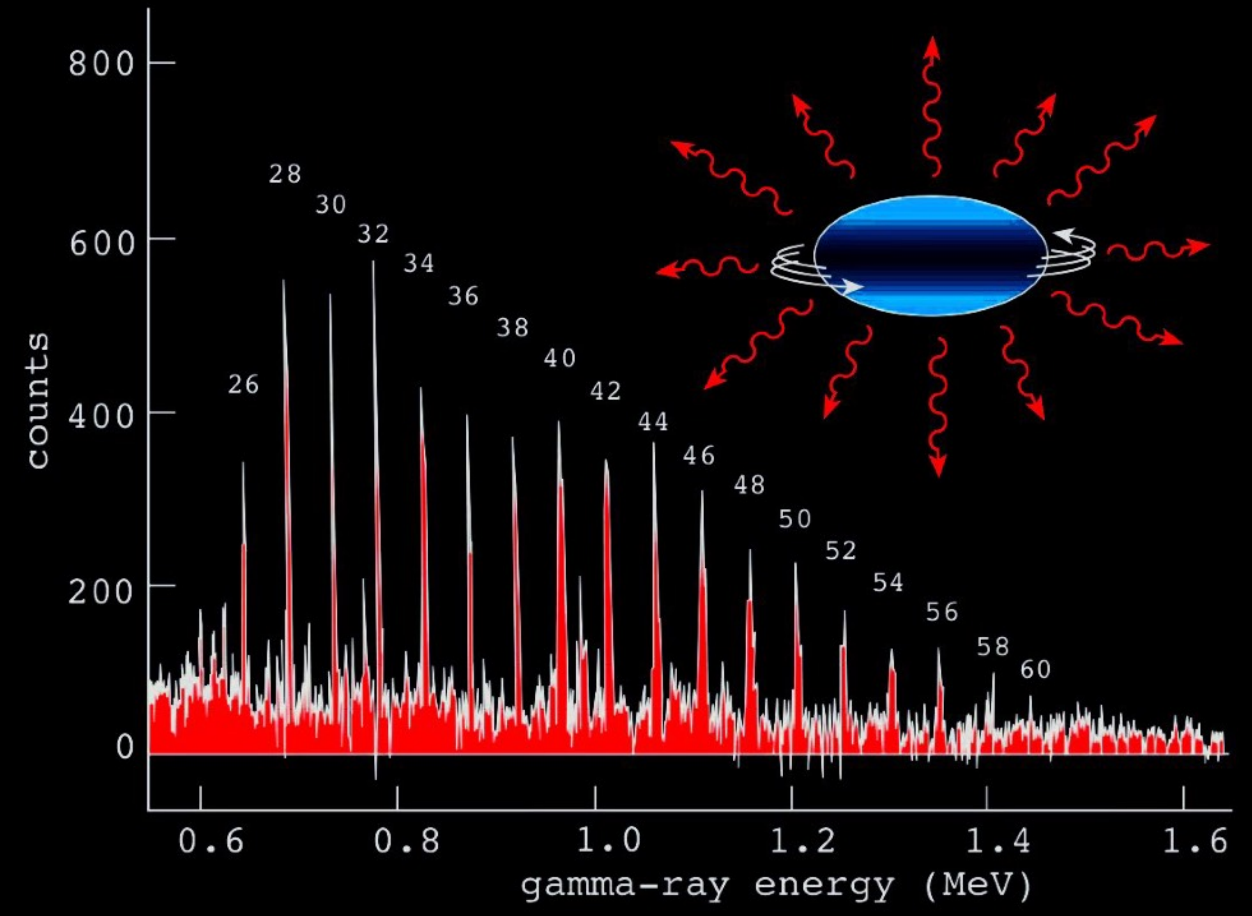
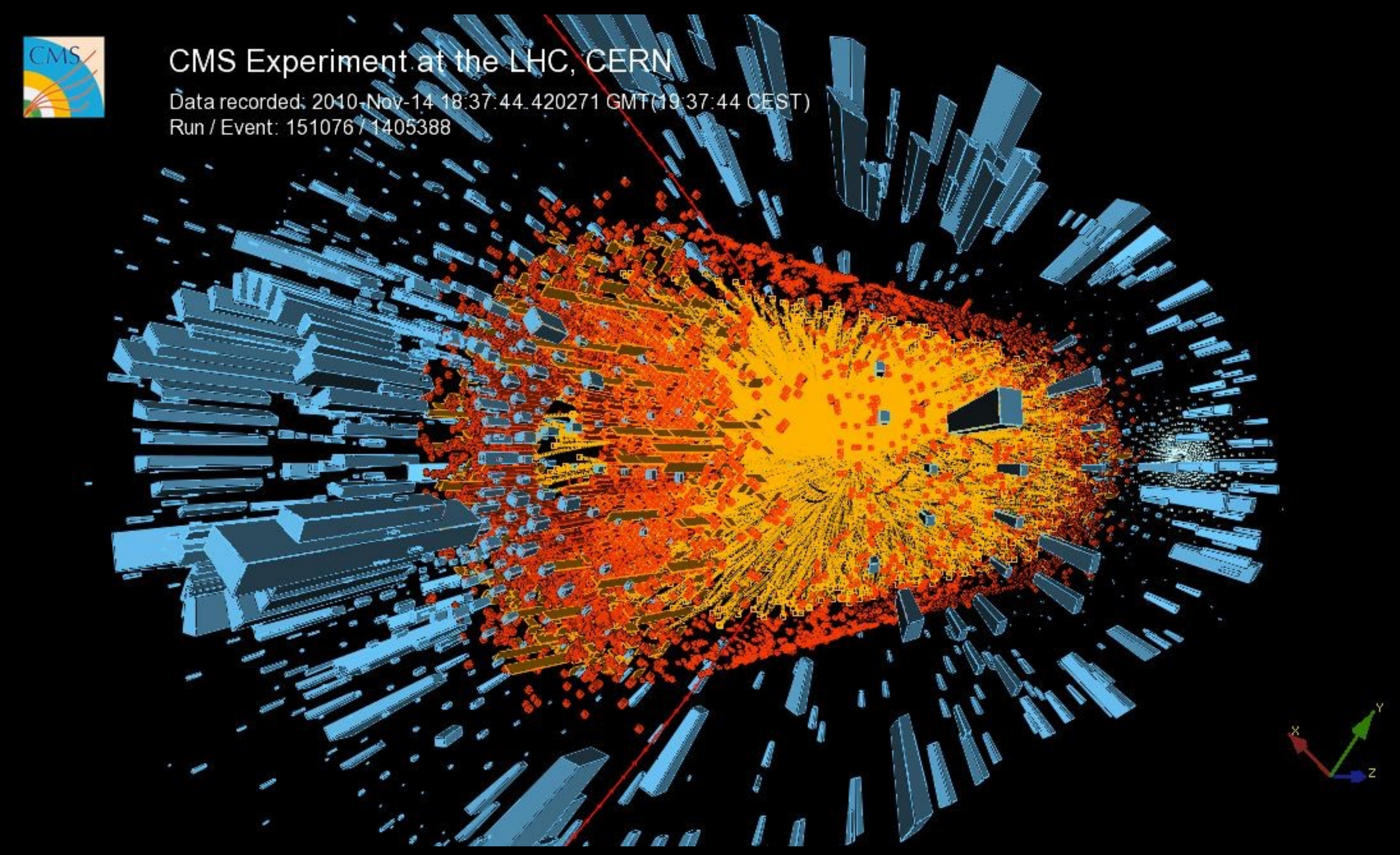


Introduction: QCD frontiers

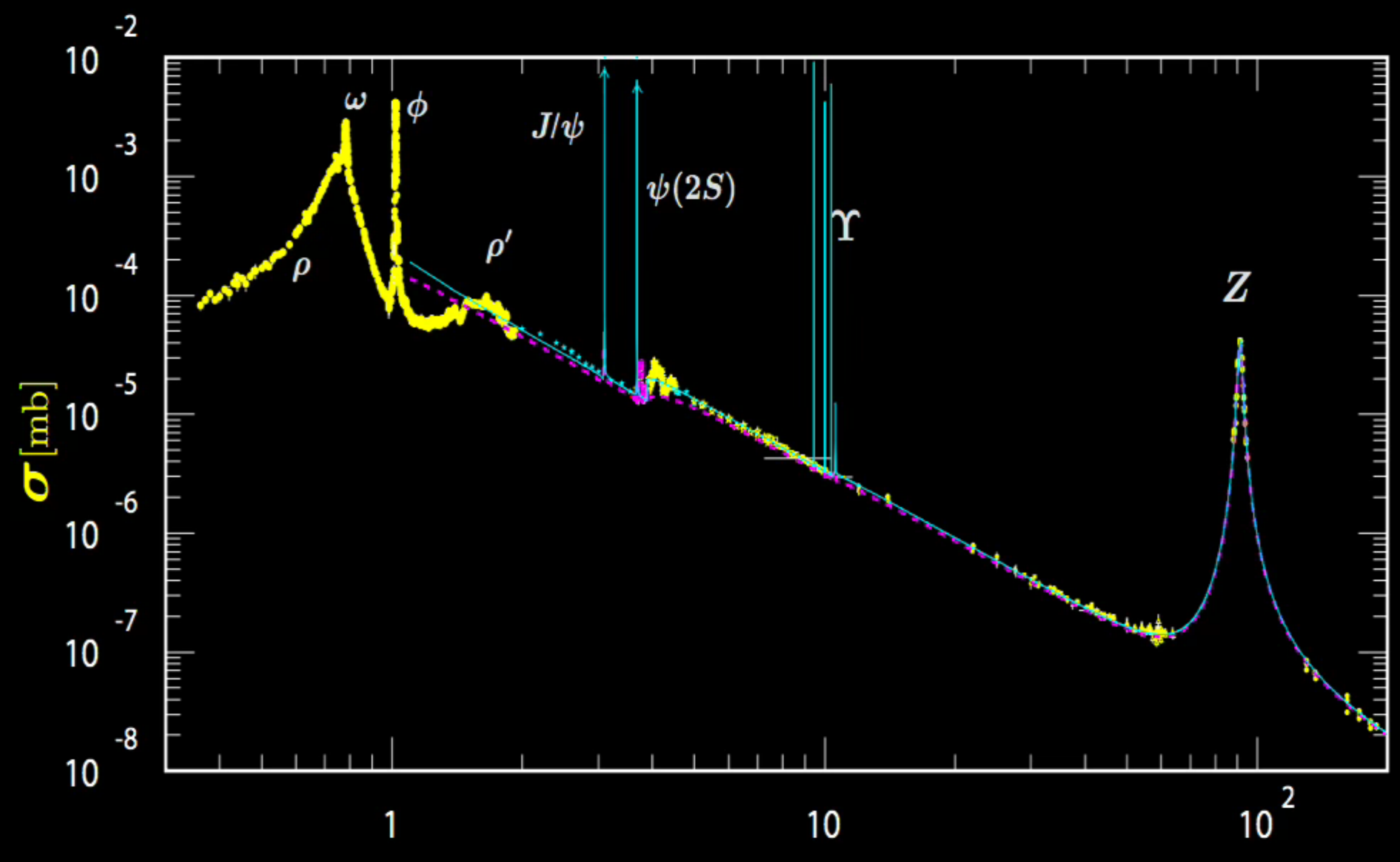
◉ Novel states of nuclei



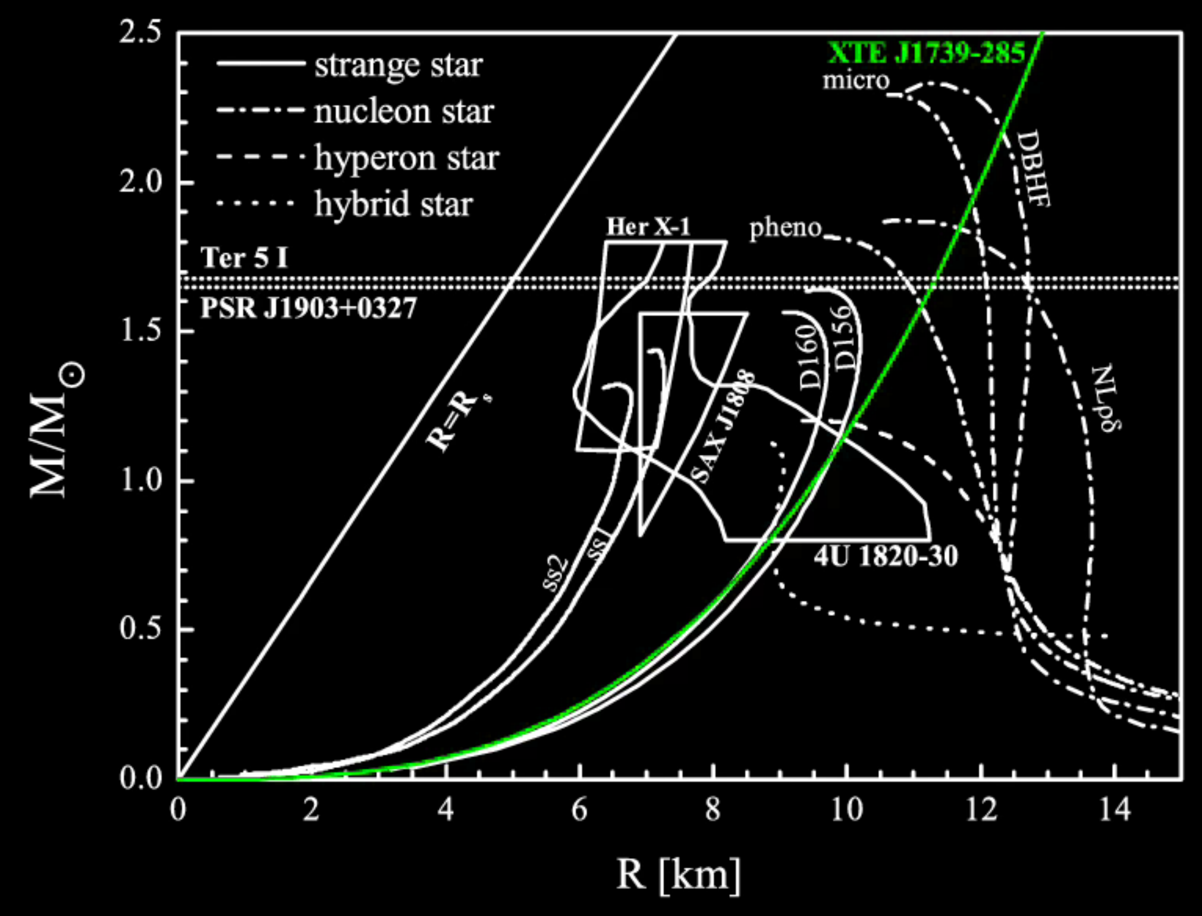
◉ Relativistic heavy-ion collision



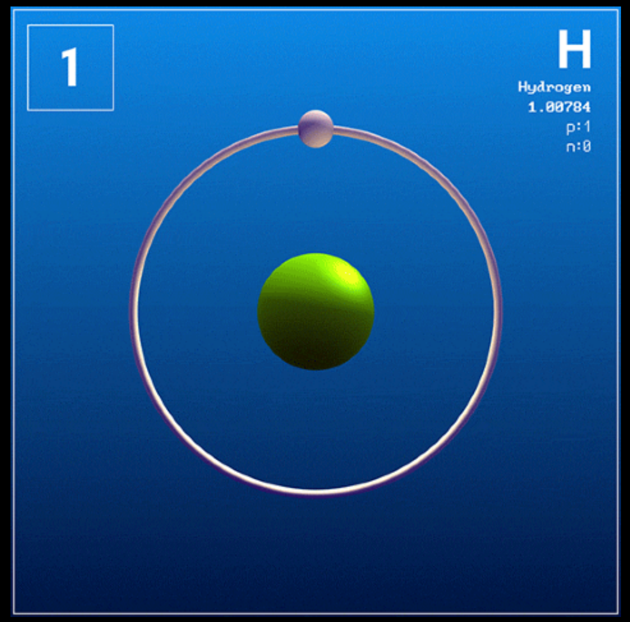
◉ e+e- hadronic annihilation



◉ Mass-radius relation of starts



Introduction: Lesson from QED



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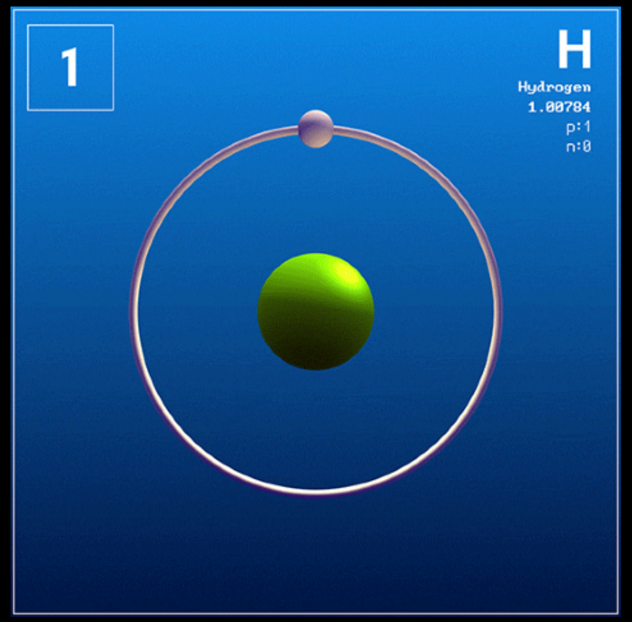

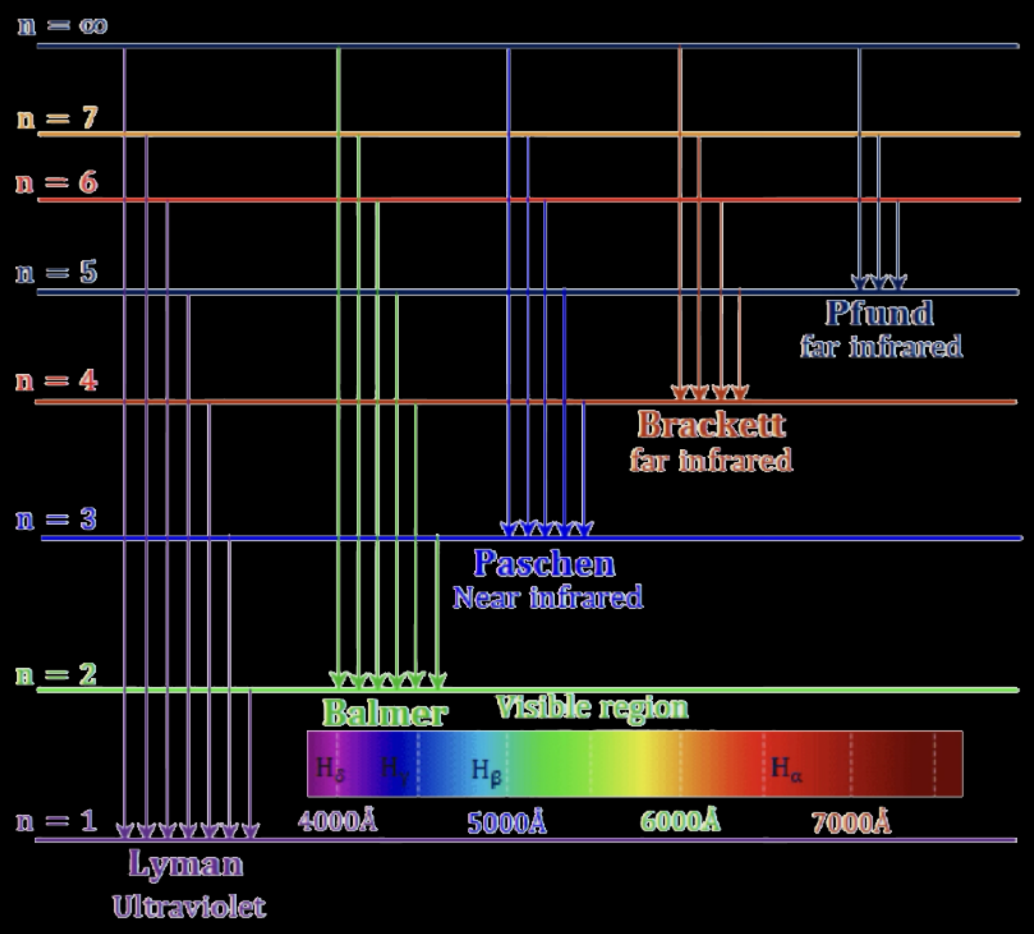

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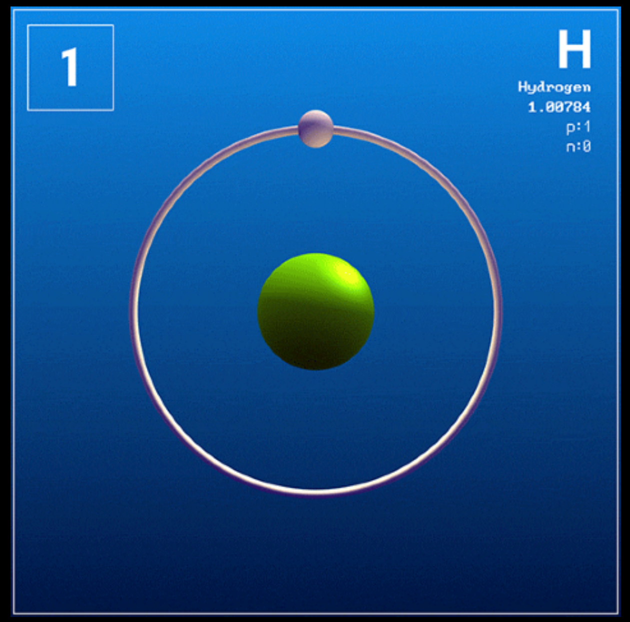

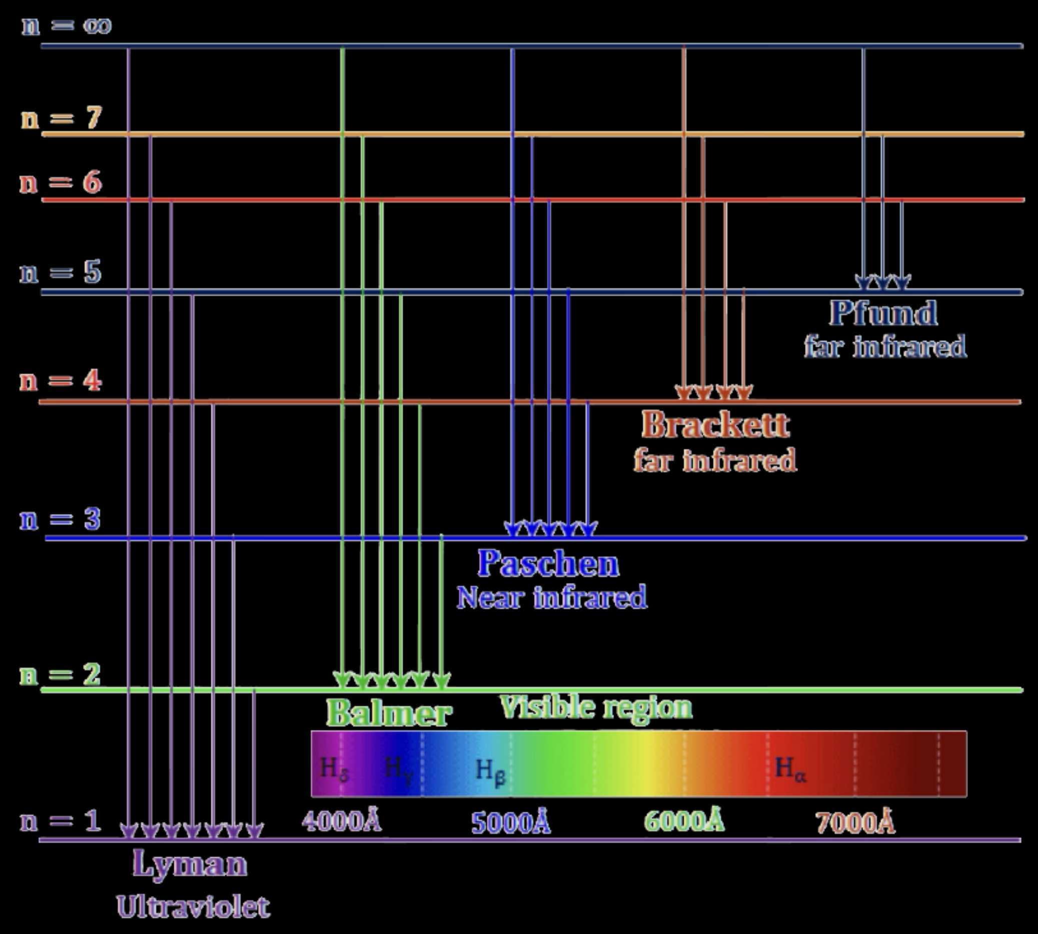

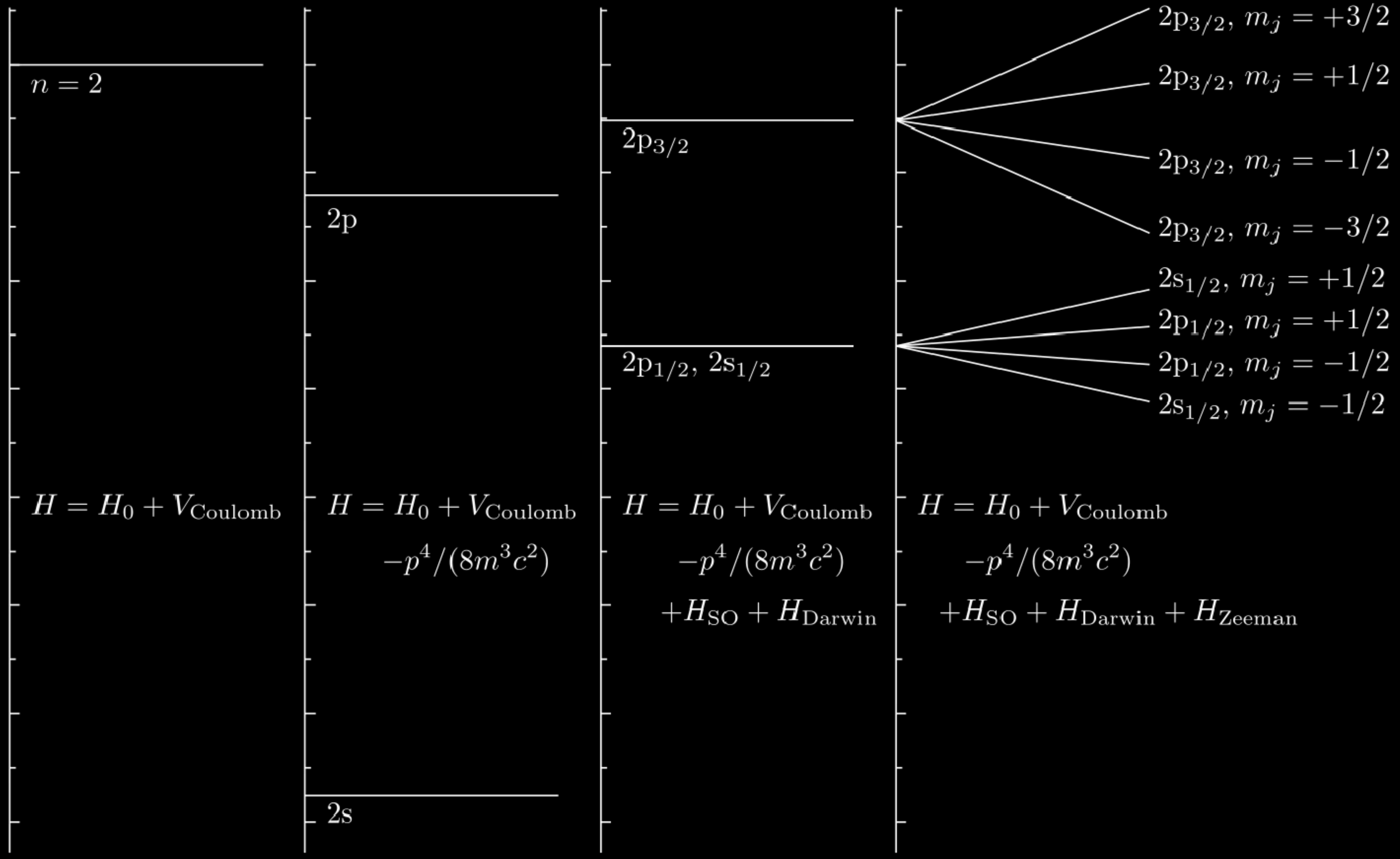

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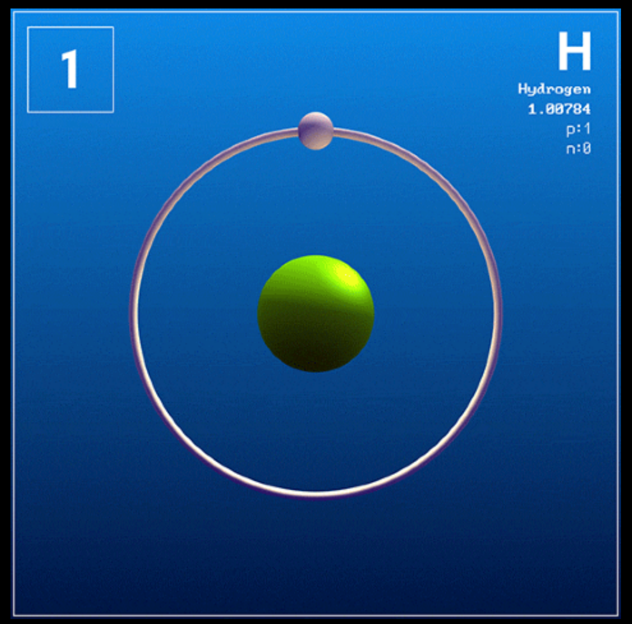

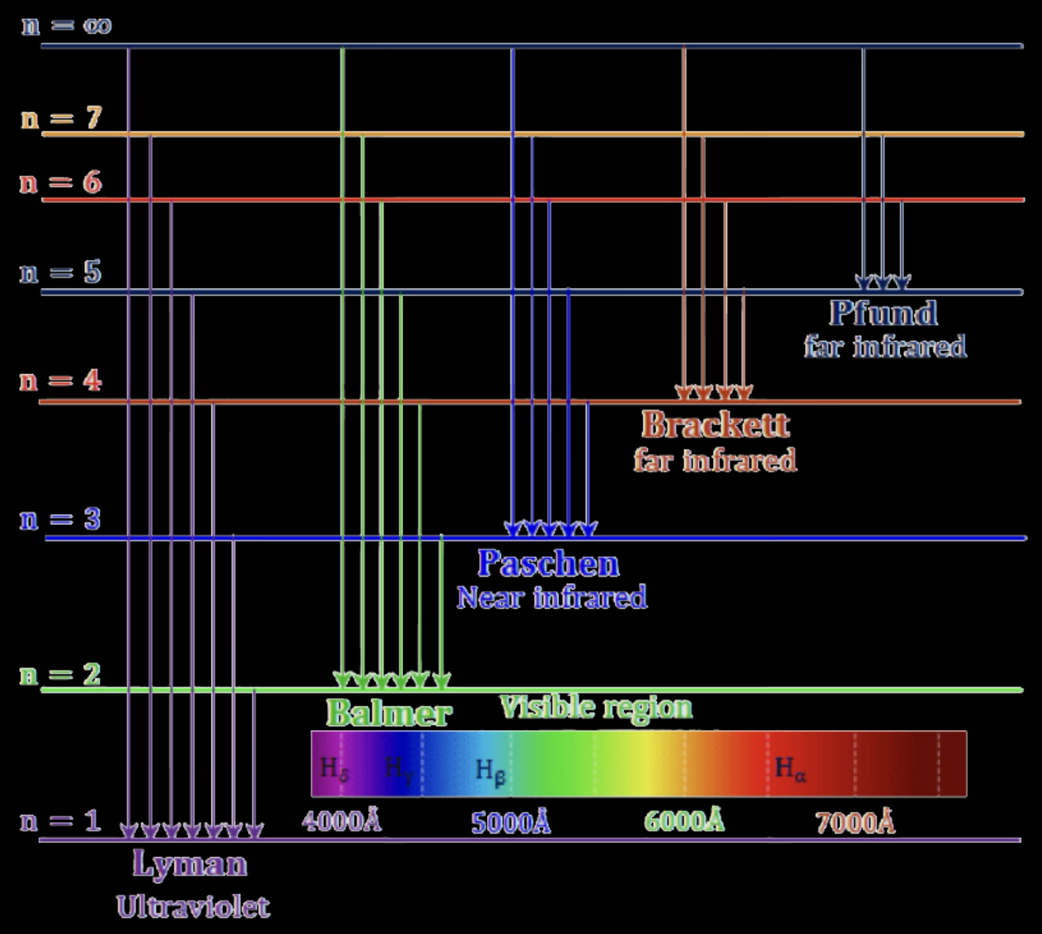

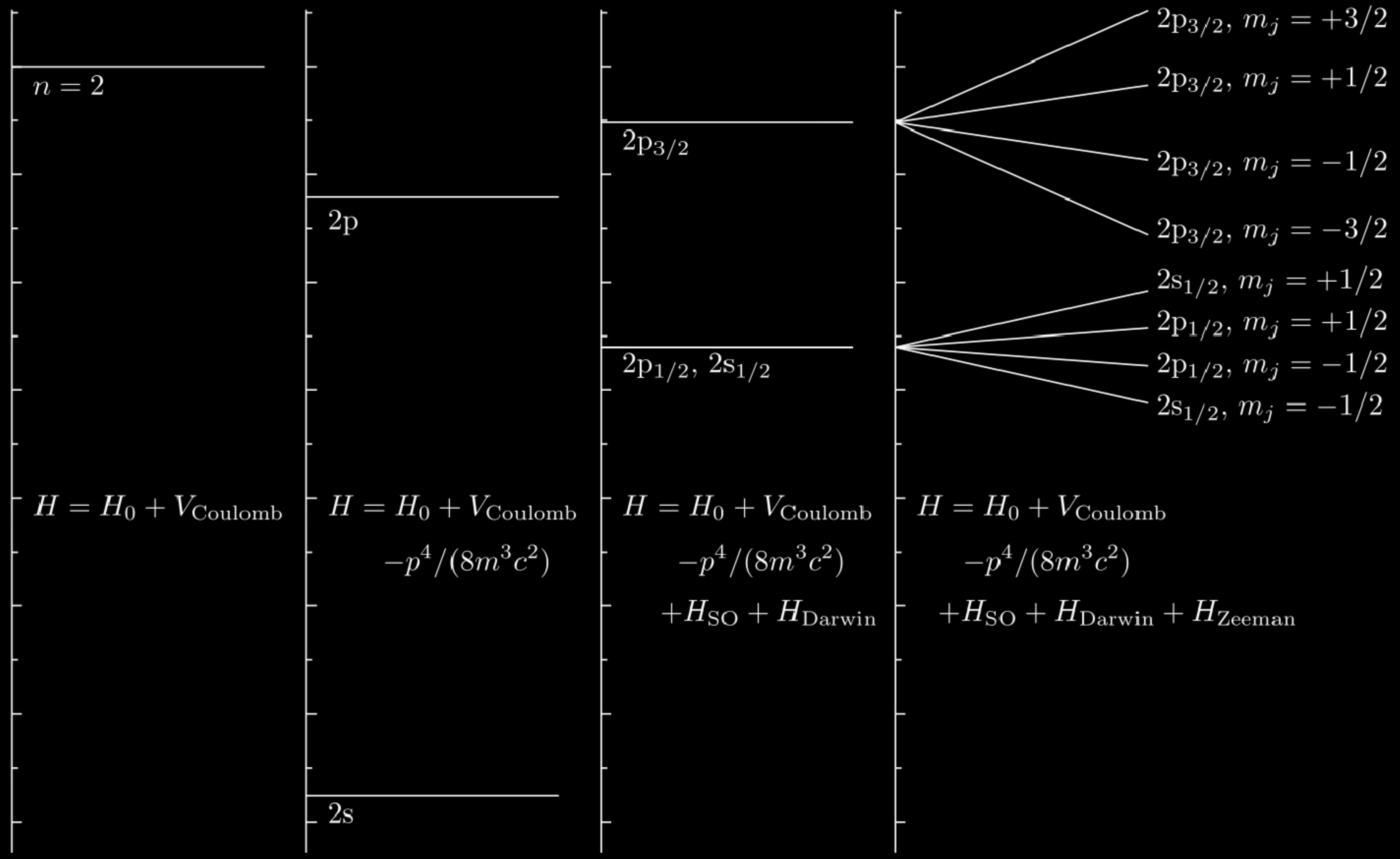

$$H = H_{\rm kinetic} + H_{\rm Coulomb} + H_{\rm spin-orbit} + H_{\rm relativistic} + H_{\rm vacuum}$$

Framework

Framework: Equation of Motion (DSE)

$$\mathscr{L}_{QCD} = \bar{\psi}_i \left[i \left(\gamma^\mu D_\mu \right)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

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Green functions of QCD

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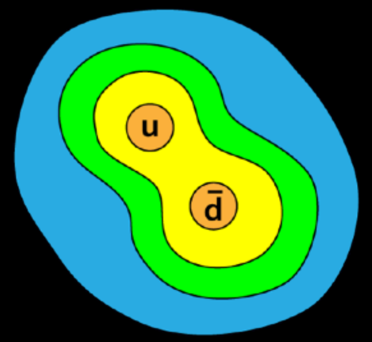
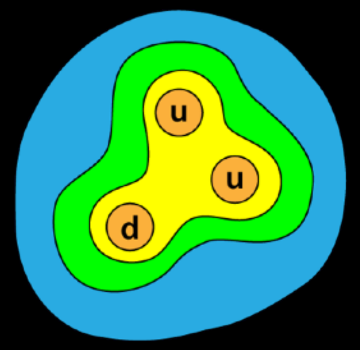
Meson



Baryon



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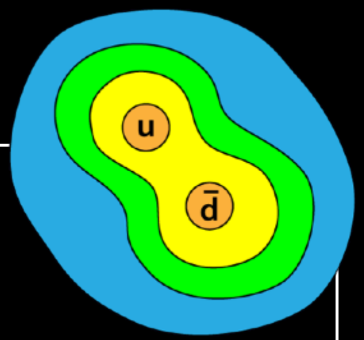
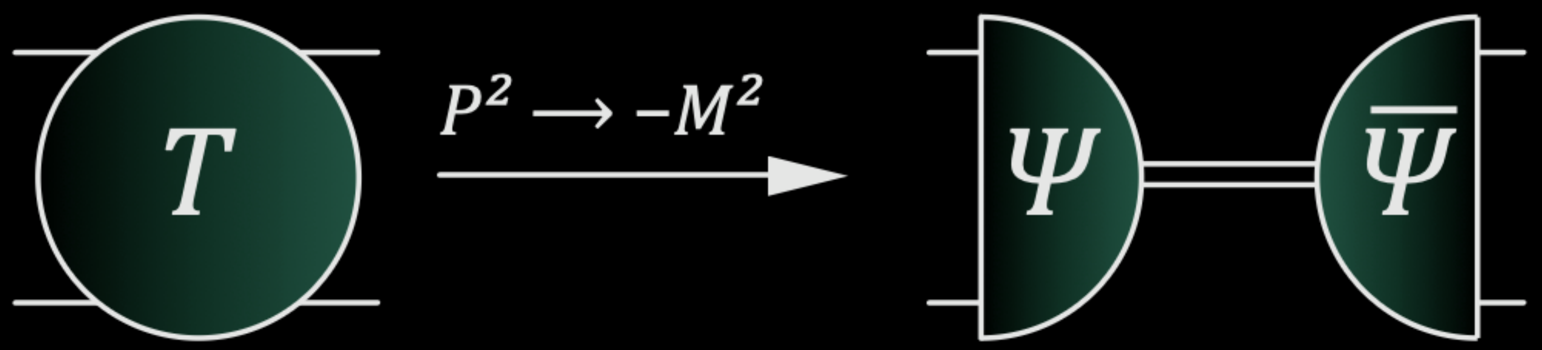
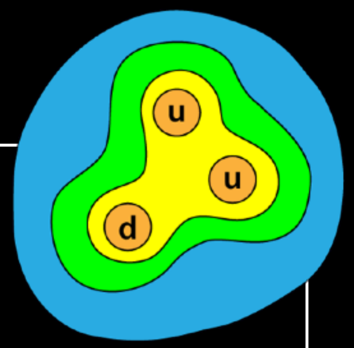
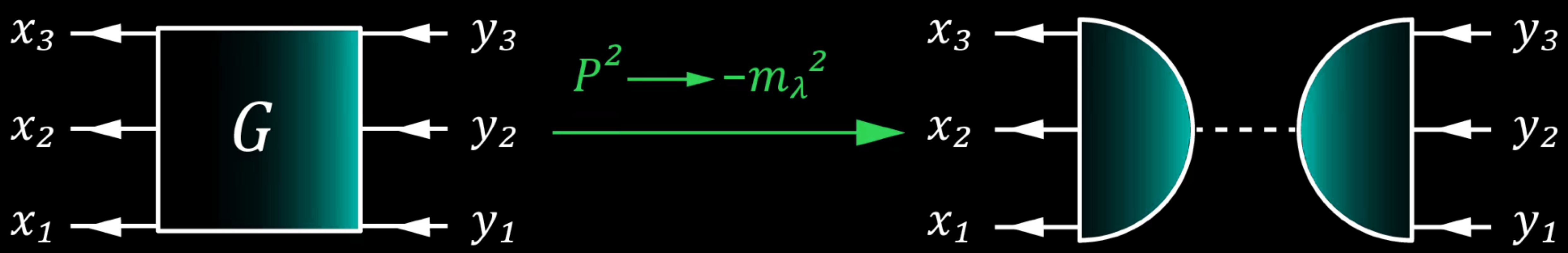




$$G^{(4)} = \langle \Omega \,|\, q(x_1)\bar{q}(x_2)q(y_1)\bar{q}(y_2) \,|\, \Omega \rangle$$

Baryon





$$G^{(6)} = \langle \Omega \,|\, q(x_1)q(x_2)q(x_3)q(y_1)q(y_2)q(y_3) \,|\, \Omega \rangle$$

Framework: Equation of Motion (DSE)

Classical Mechanics		Quantum Field Theory
	Degrees of freedom	
Generalized coord.		Fields on spacetime
	Principle of Stationary Action	
$\frac{\delta S[q]}{\delta q} = 0$		$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$
	Equations of Motion	
Euler-Lagrange Equation		Dyson-Schwinger Equations

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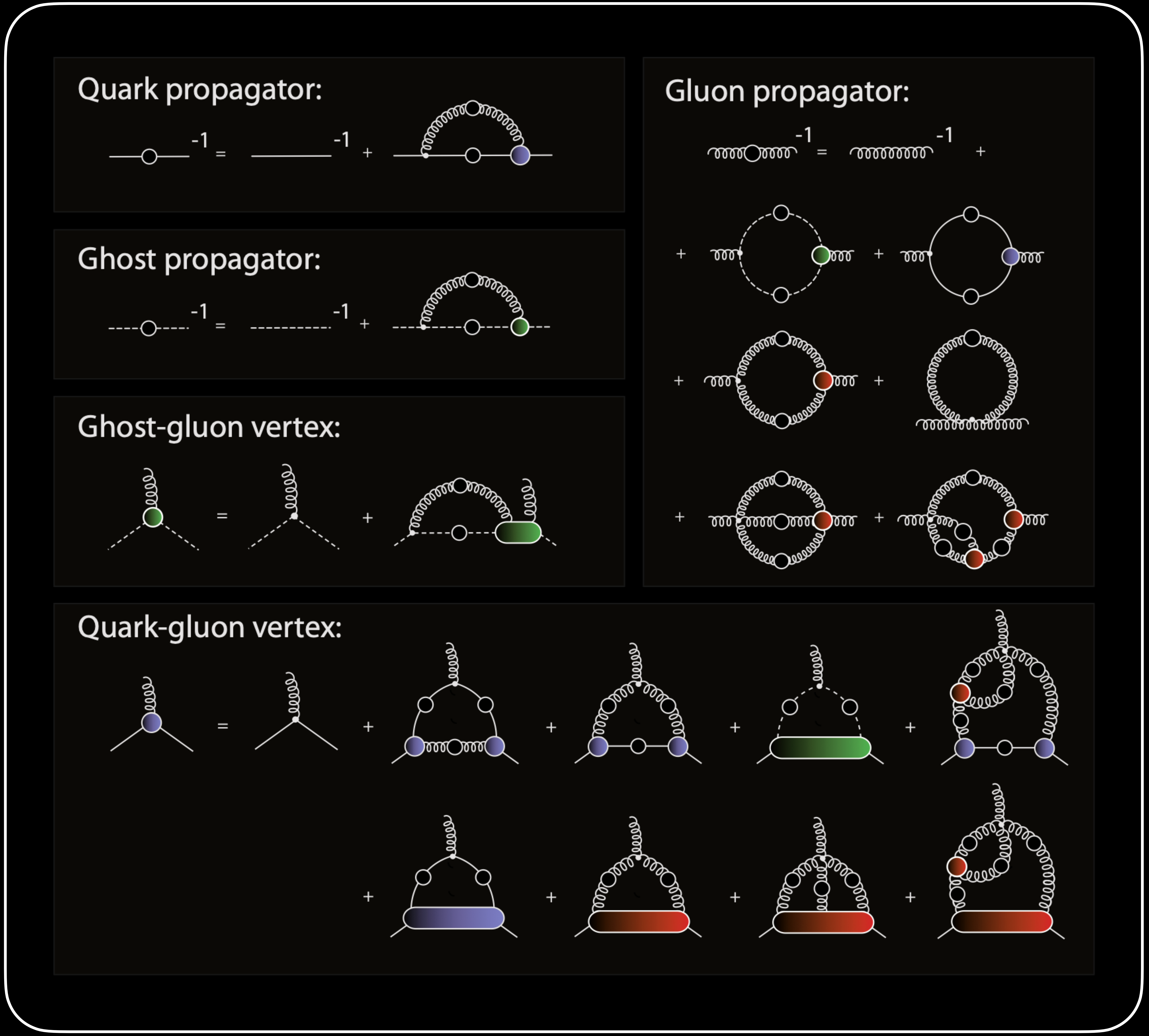

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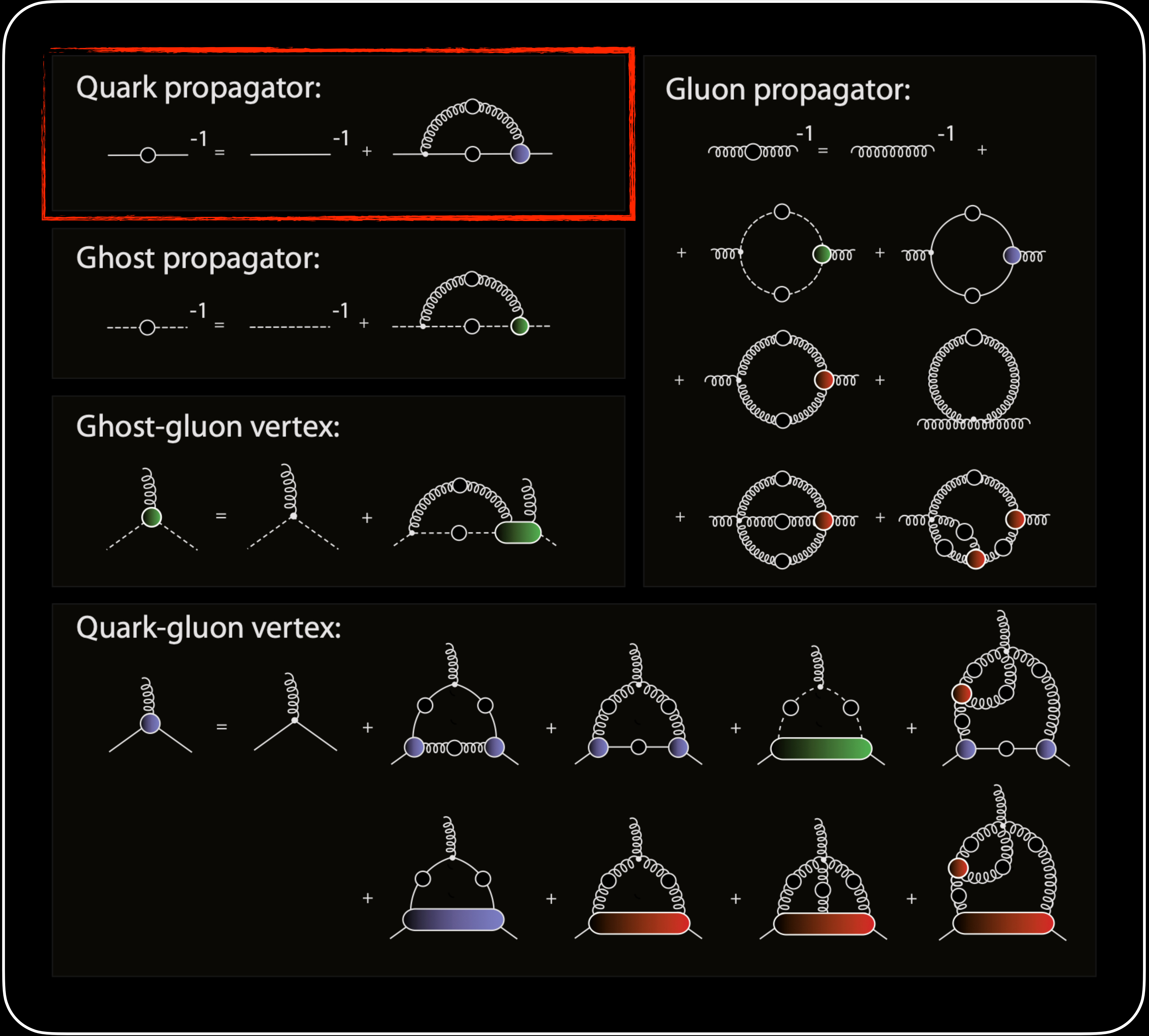
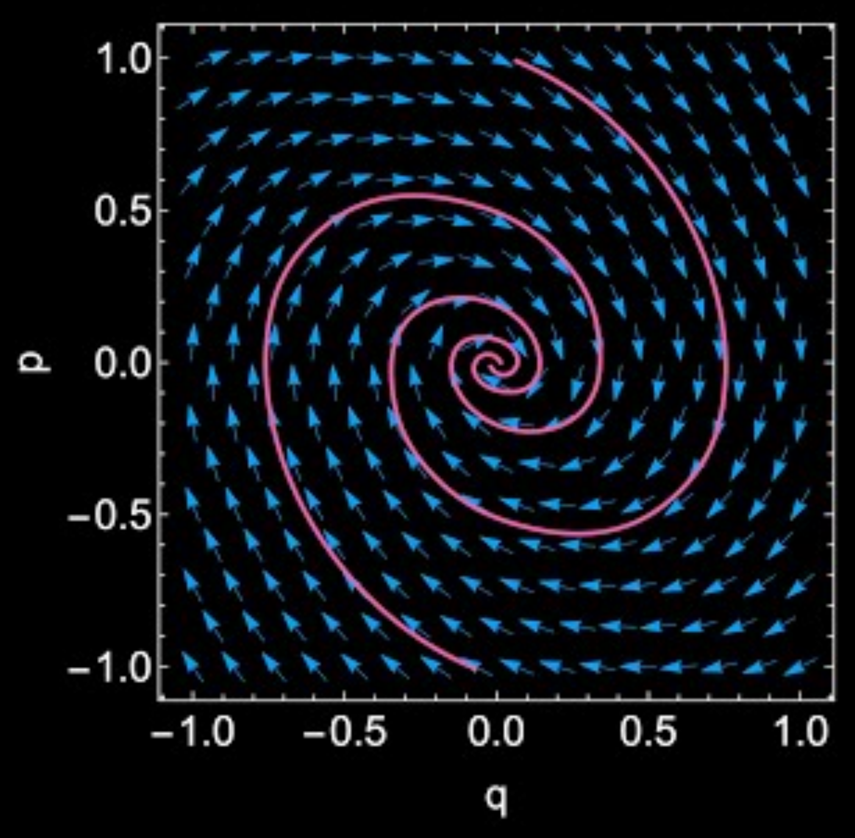
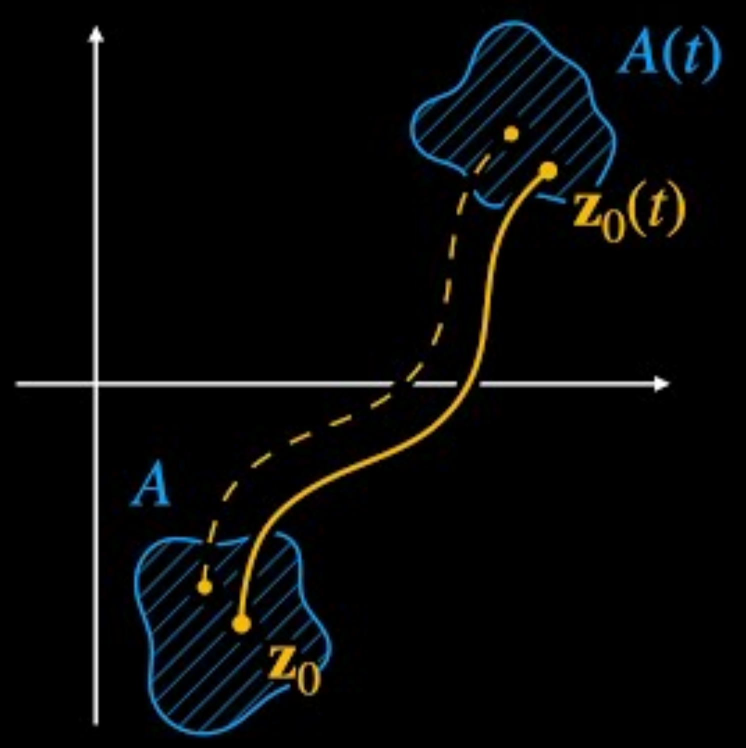


Framework: Equation of Motion (DSE)

Liouville equation

$$\frac{\partial \rho}{\partial t} + \{\rho, H\} = 0$$





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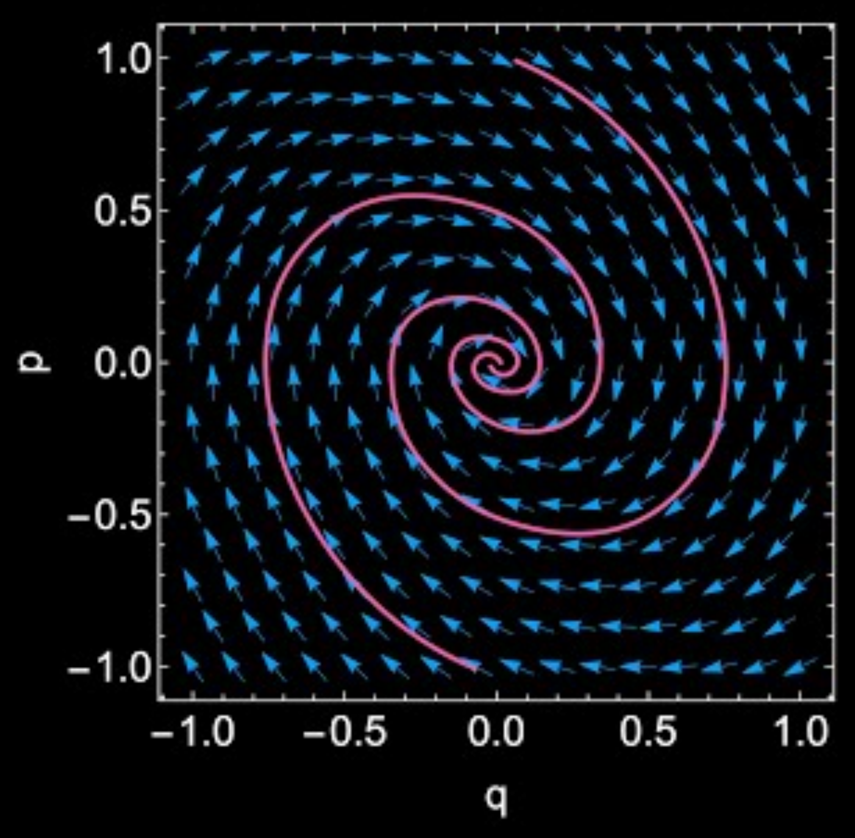
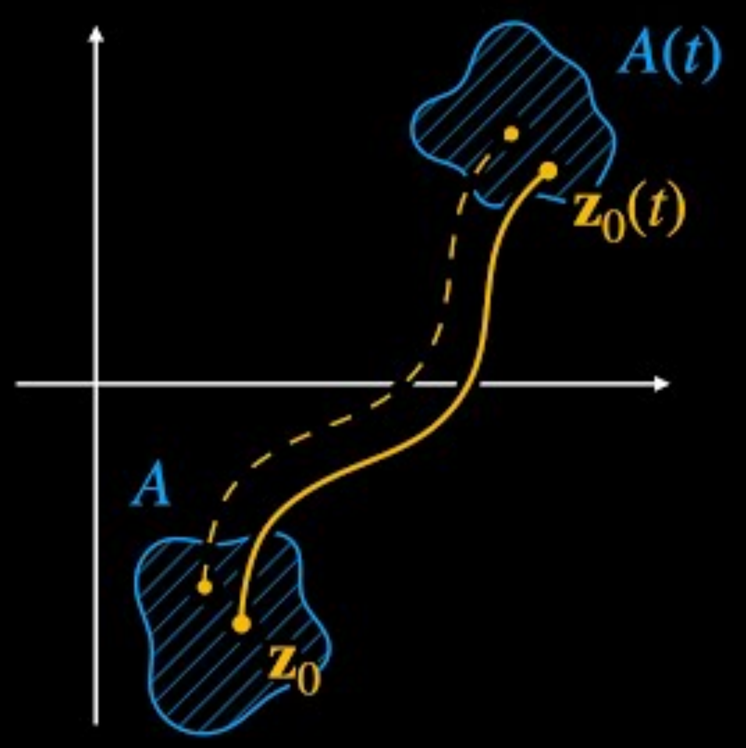
BBGKY hierarchy

$$\boxed{\frac{\partial \rho_n}{\partial t} = \{H_n, \rho_n\} + (N-n)\sum_{i=1}^{n}\int \frac{\partial \rho_{n+1}}{\partial \mathbf{p}_i}\cdot\frac{\partial \phi_{i\,n+1}}{\partial \mathbf{r}_i} dz_{n+1}}$$

DSE-like





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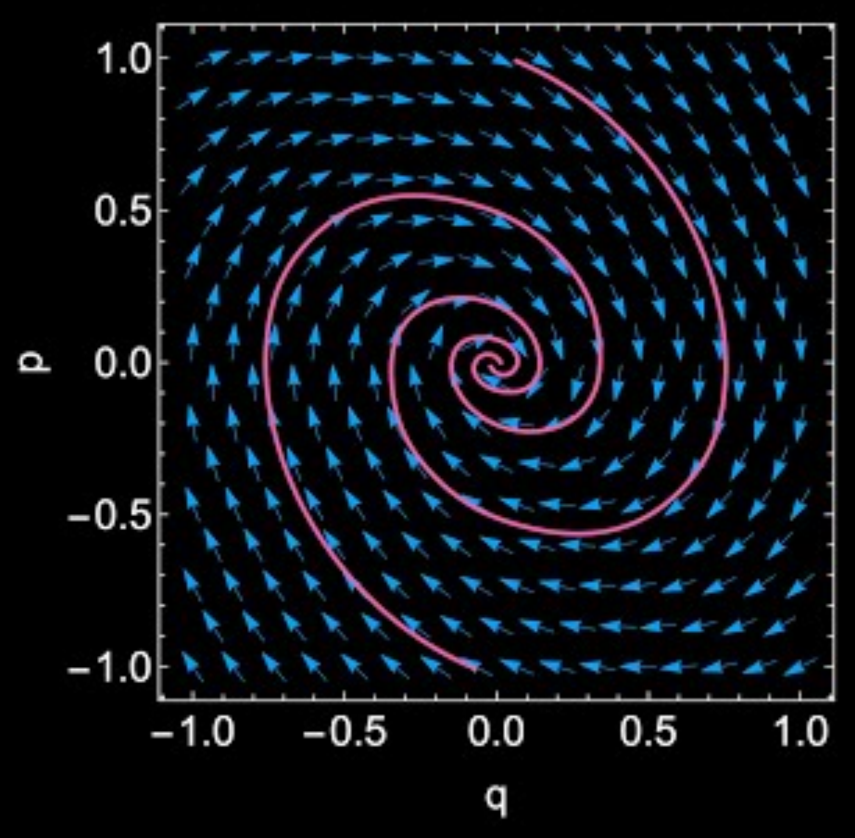

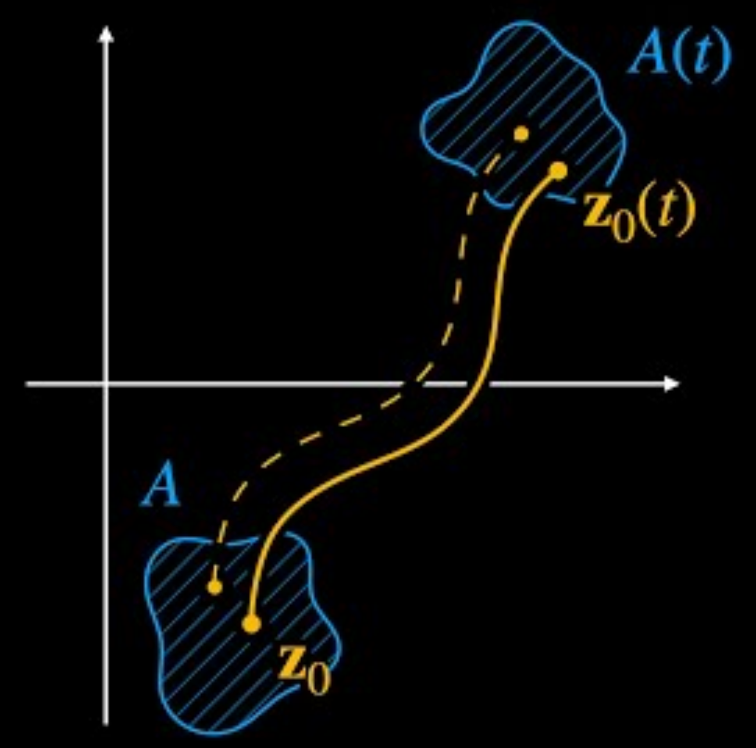

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DSE-like

One-body function

$$\rho_1(z_1) = \int_D \rho(Z,t)\, dz_2 \cdots dz_N$$

propagator-like

Framework: Equation of Motion (DSE)

Liouville equation

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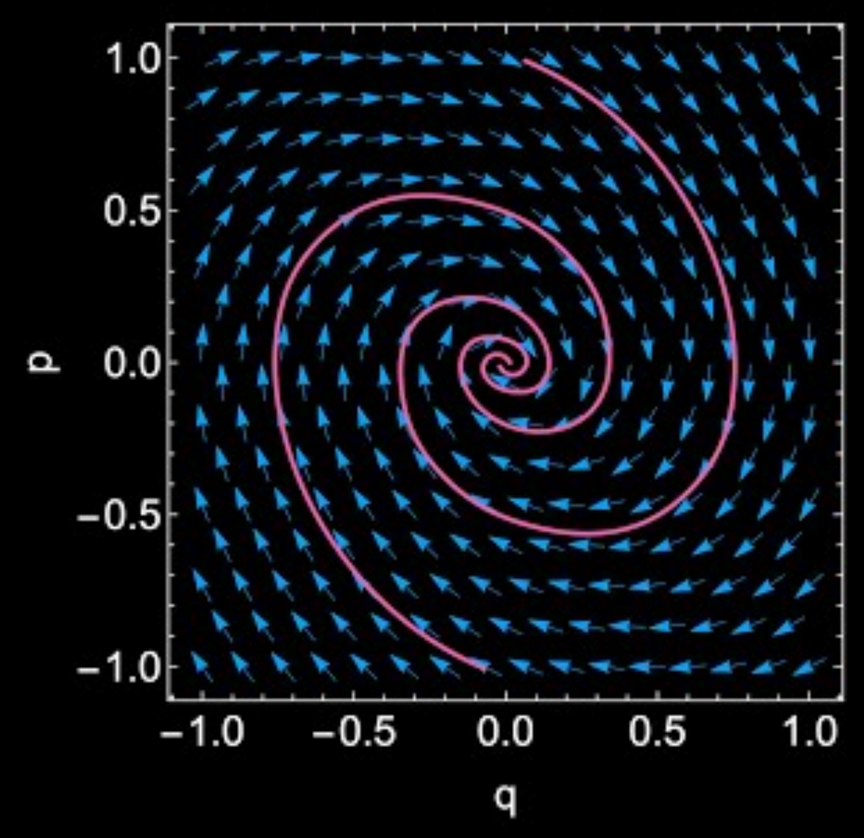

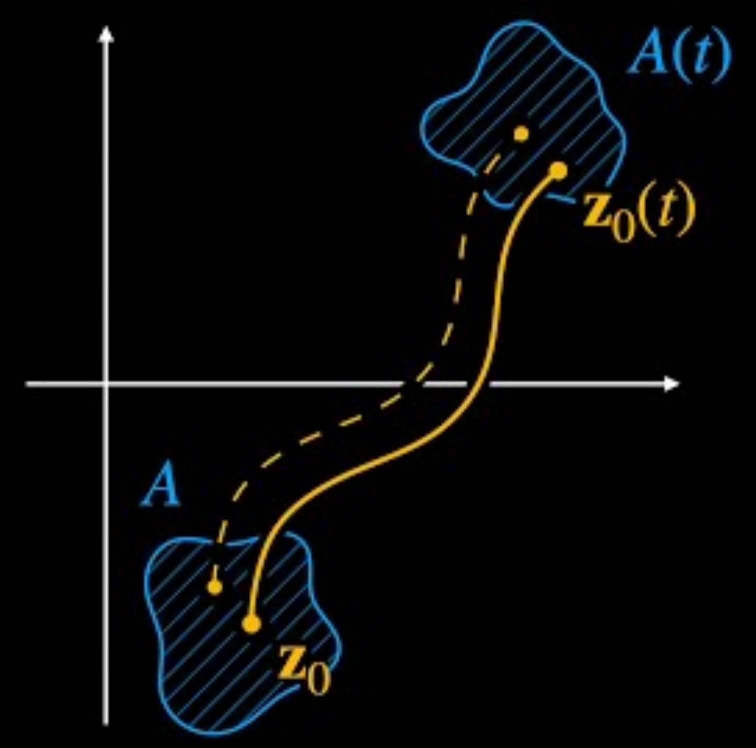

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DSE-like

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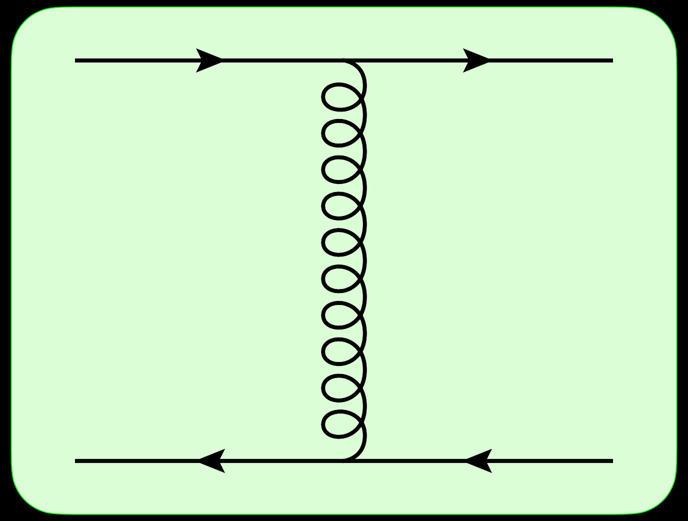
propagator-like

Boltzmann equation

$$\frac{\partial \rho_1}{\partial t} = \{H_1, \rho_1\} + \int \left[\rho_1(\mathbf{p}_1')\rho_1(\mathbf{p}_2') - \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)\right] N\nu\, d\sigma\, d^3\mathbf{p}_2$$

Framework: Equation of Motion (DSE)



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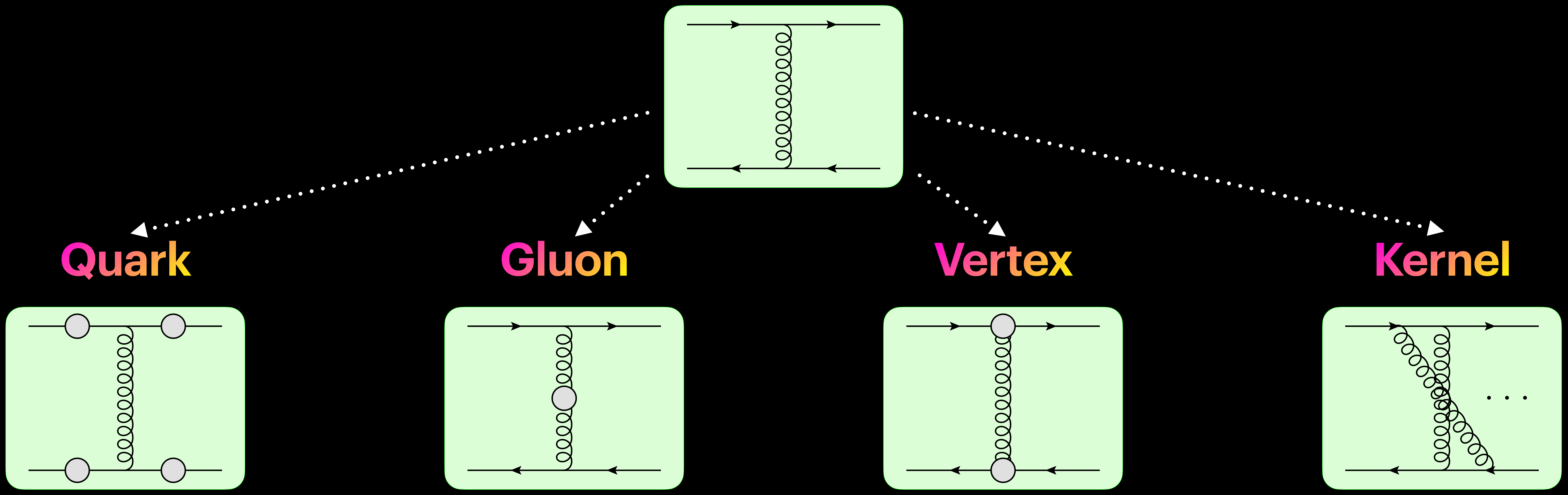

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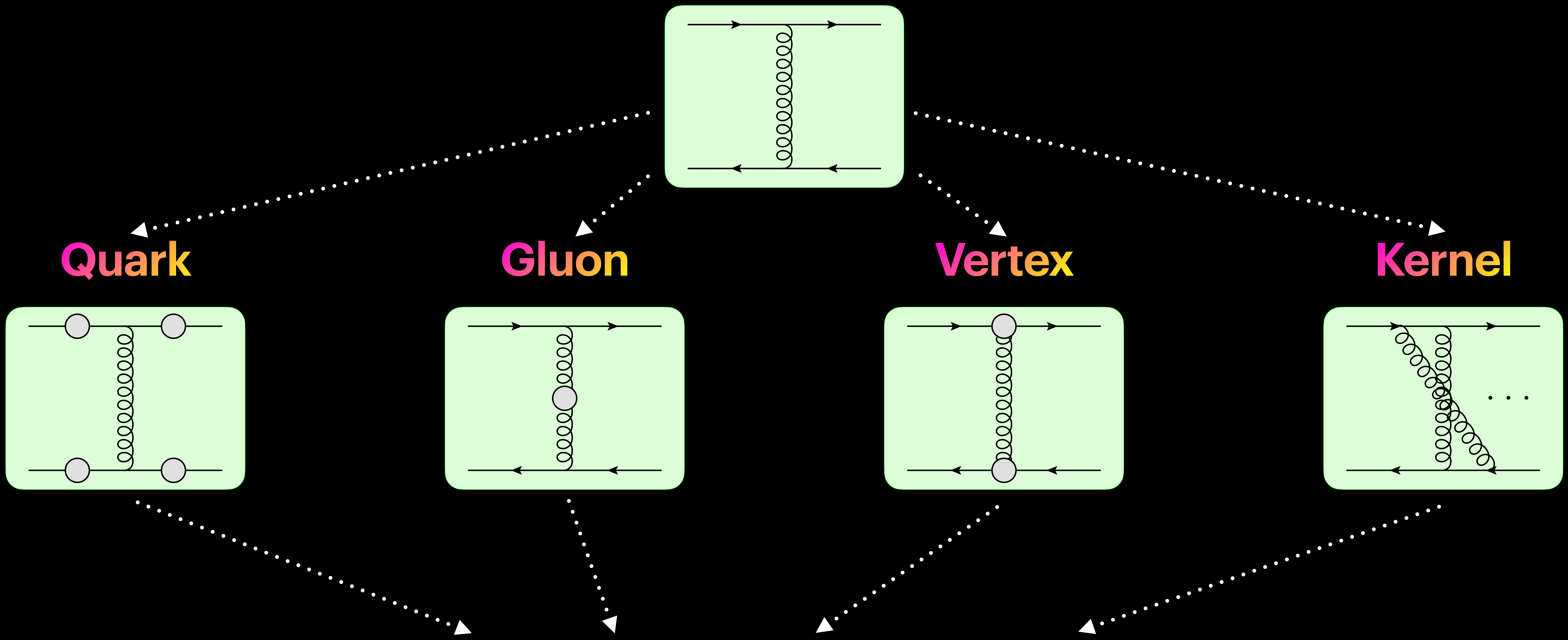
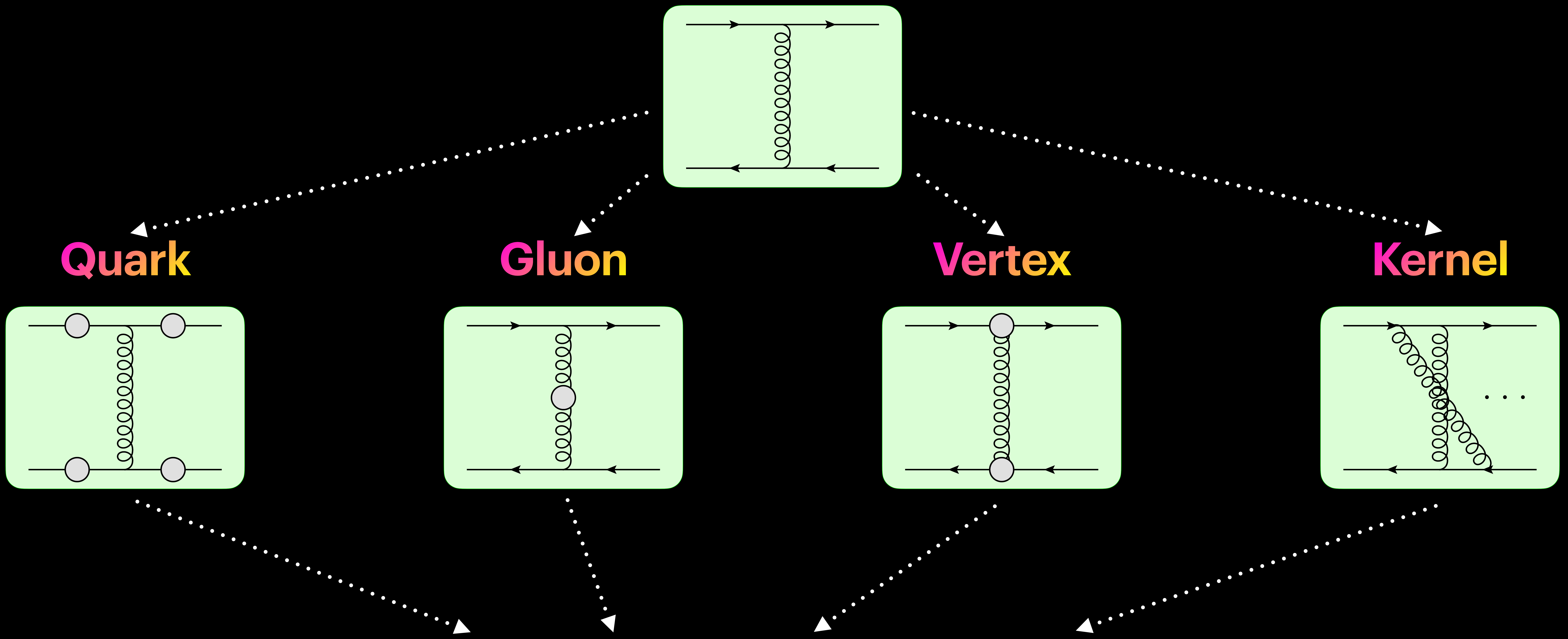


$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{vacuum}}$$

Framework: Equation of Motion (DSE)



Ground $\longrightarrow$ $H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{vacuum}}$ $\longrightarrow$ Excited

Solution

Solution: Quark properties

- **Quark propagator:**

$$\boxed{S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}}$$

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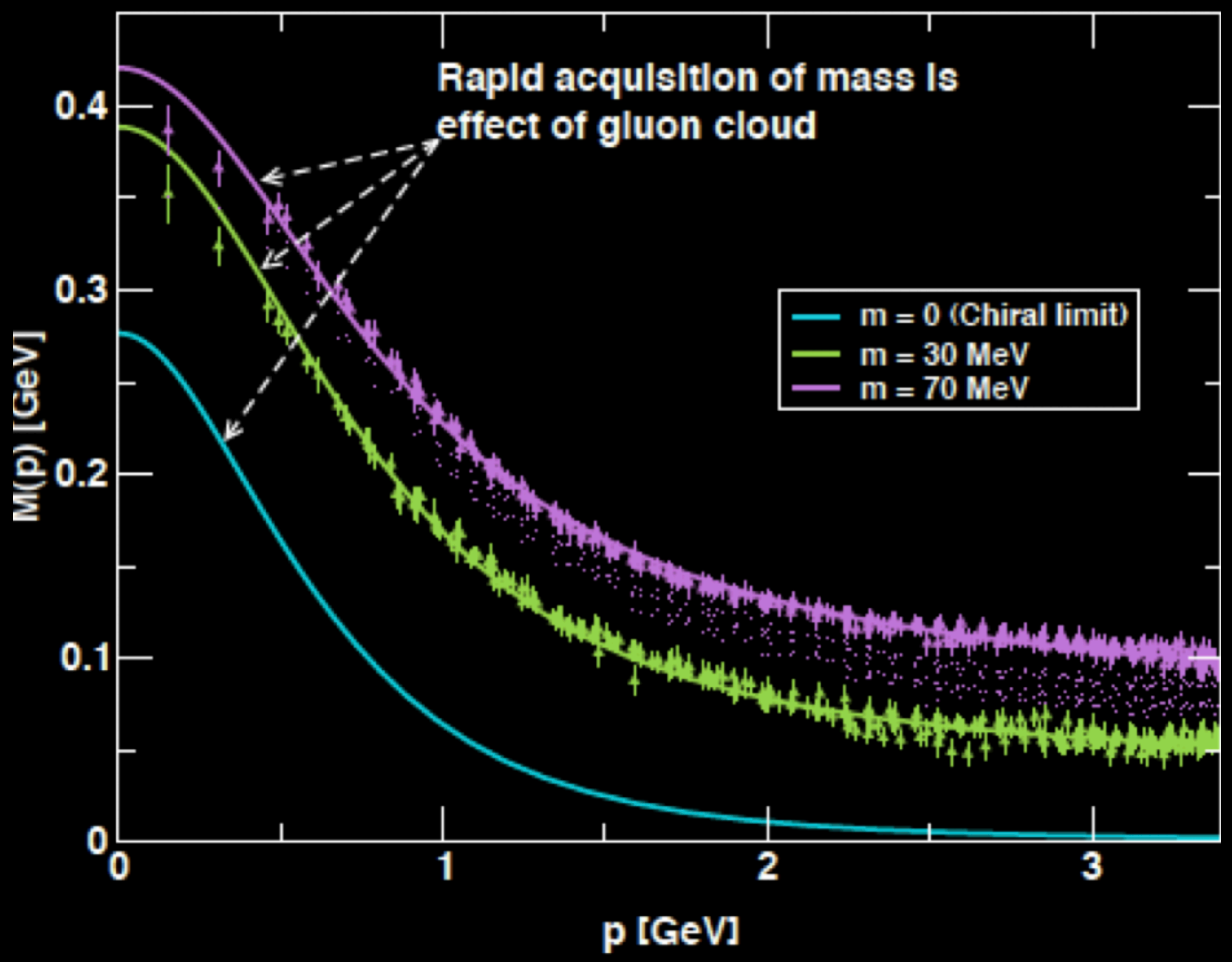

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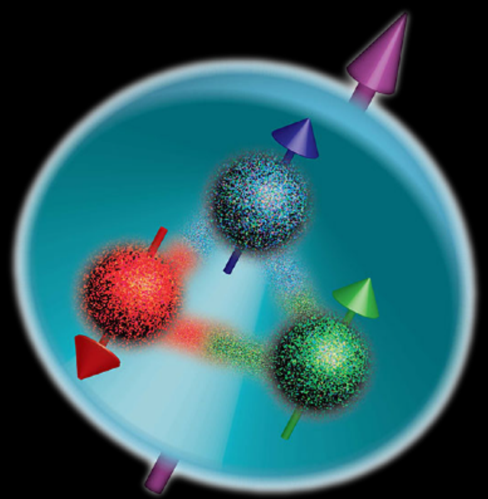
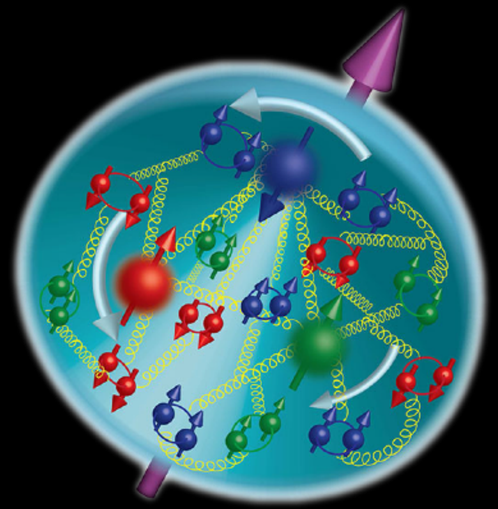
- **Quasi-quark：**

Low p  High p 

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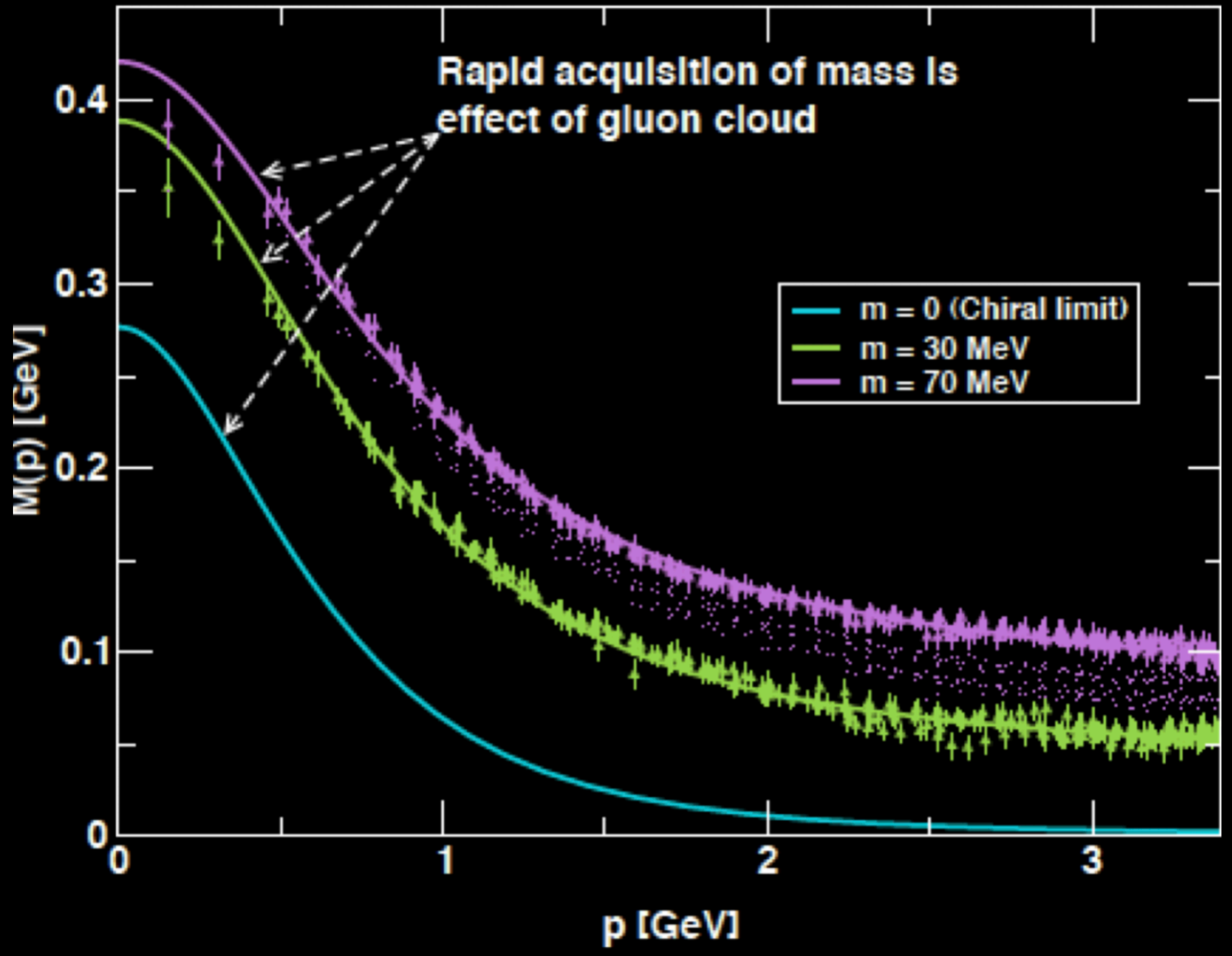

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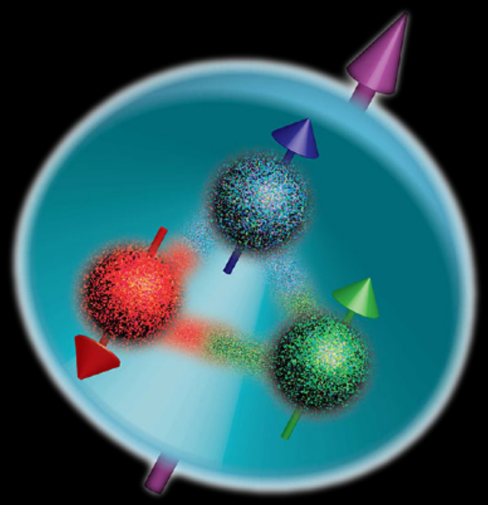
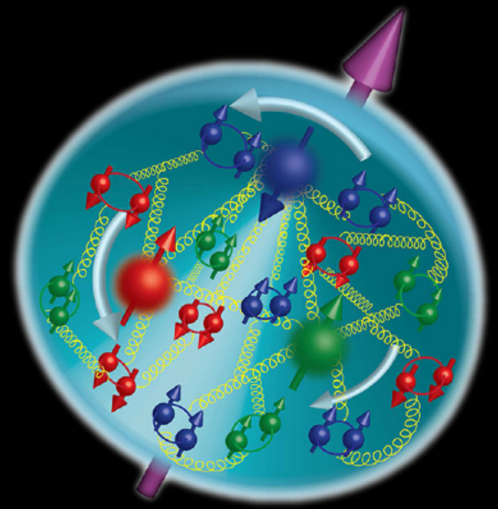
Low p  High p 

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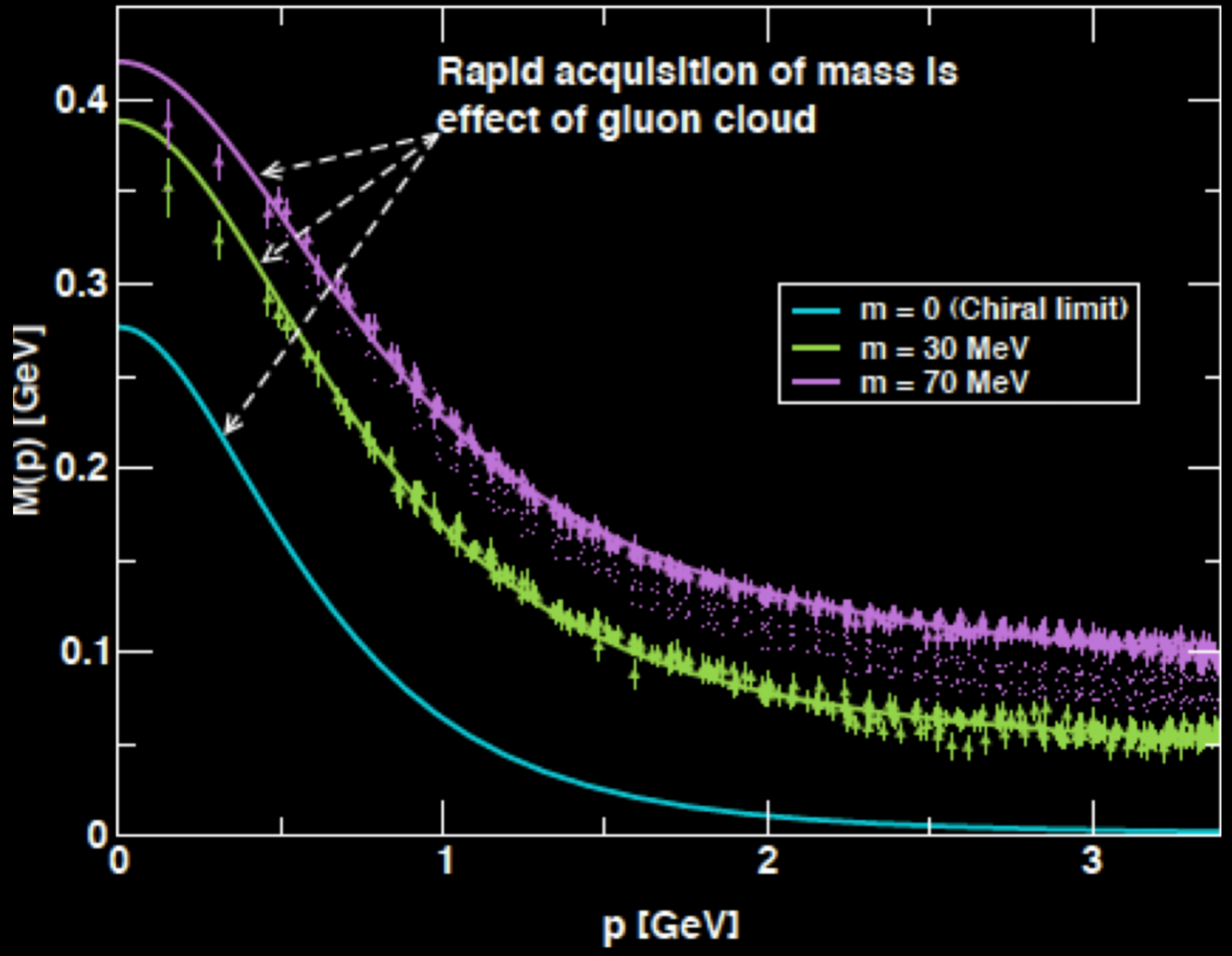

QCD vacuum — highly dispersive medium

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- **Gluon propagator**:

$$\boxed{\mathcal{G}(k^2) \approx \frac{4\pi\alpha_s(k^2)}{k^2 + m_g{}^2(k^2)}}$$

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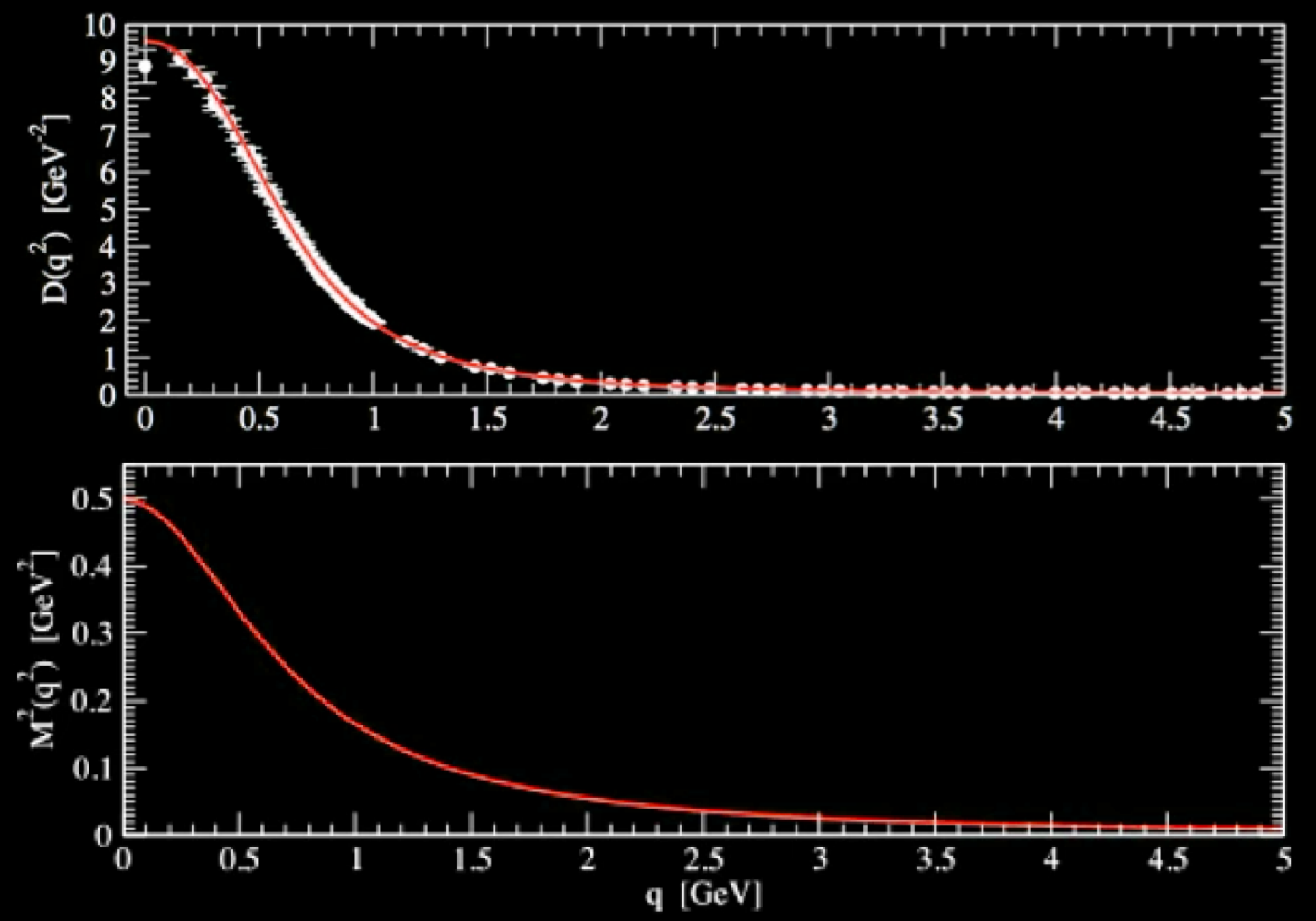

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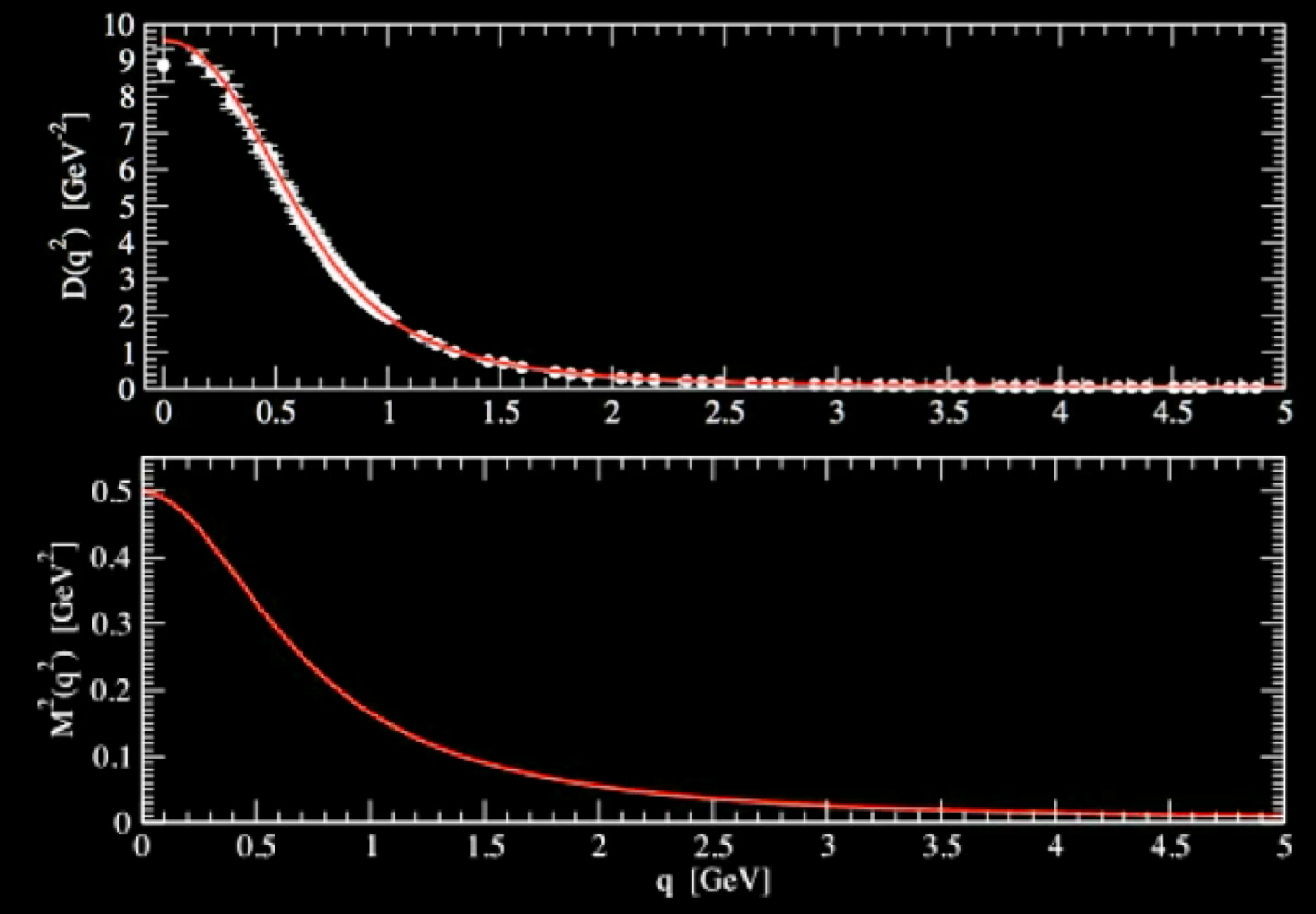
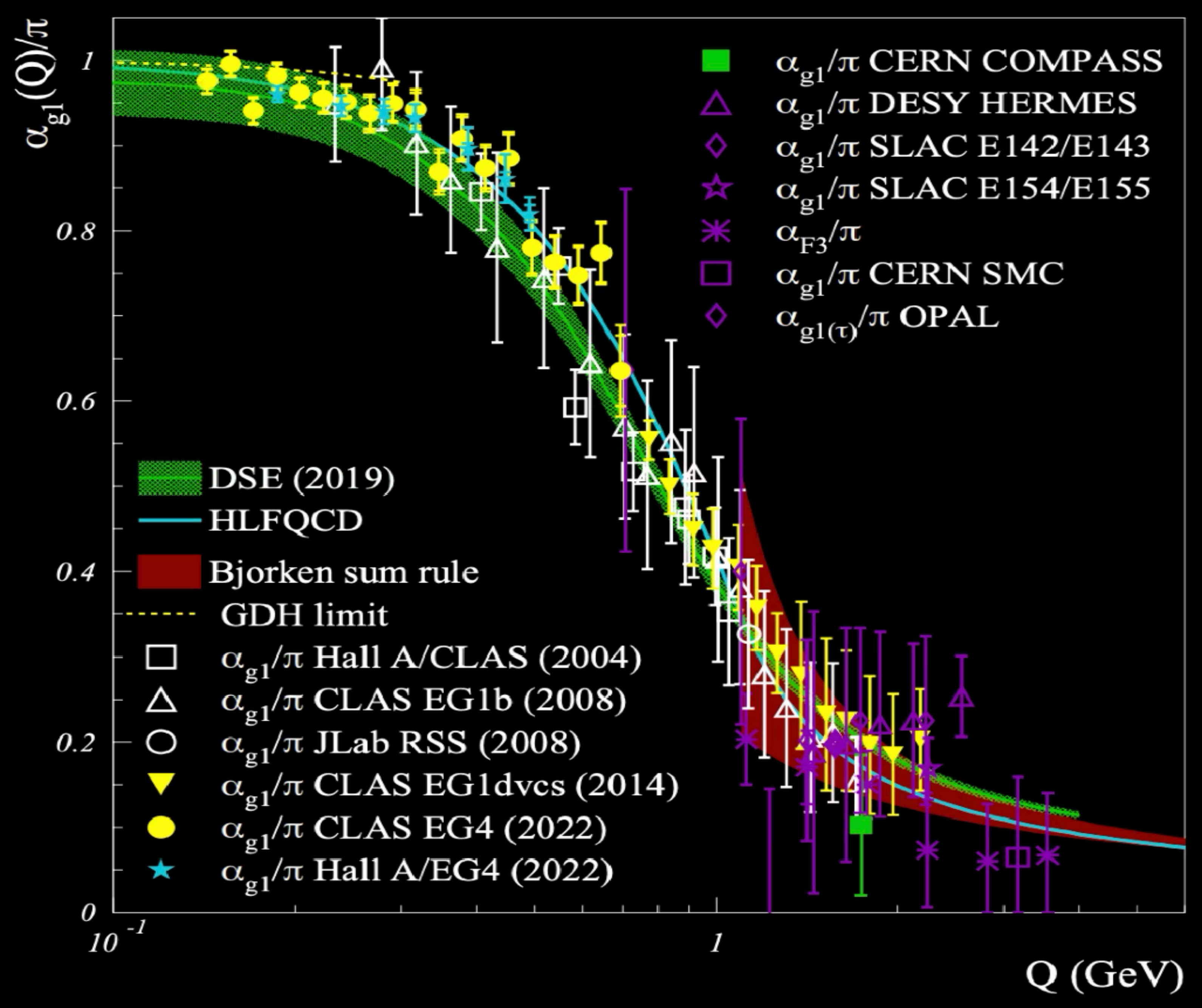


- **Running coupling:**



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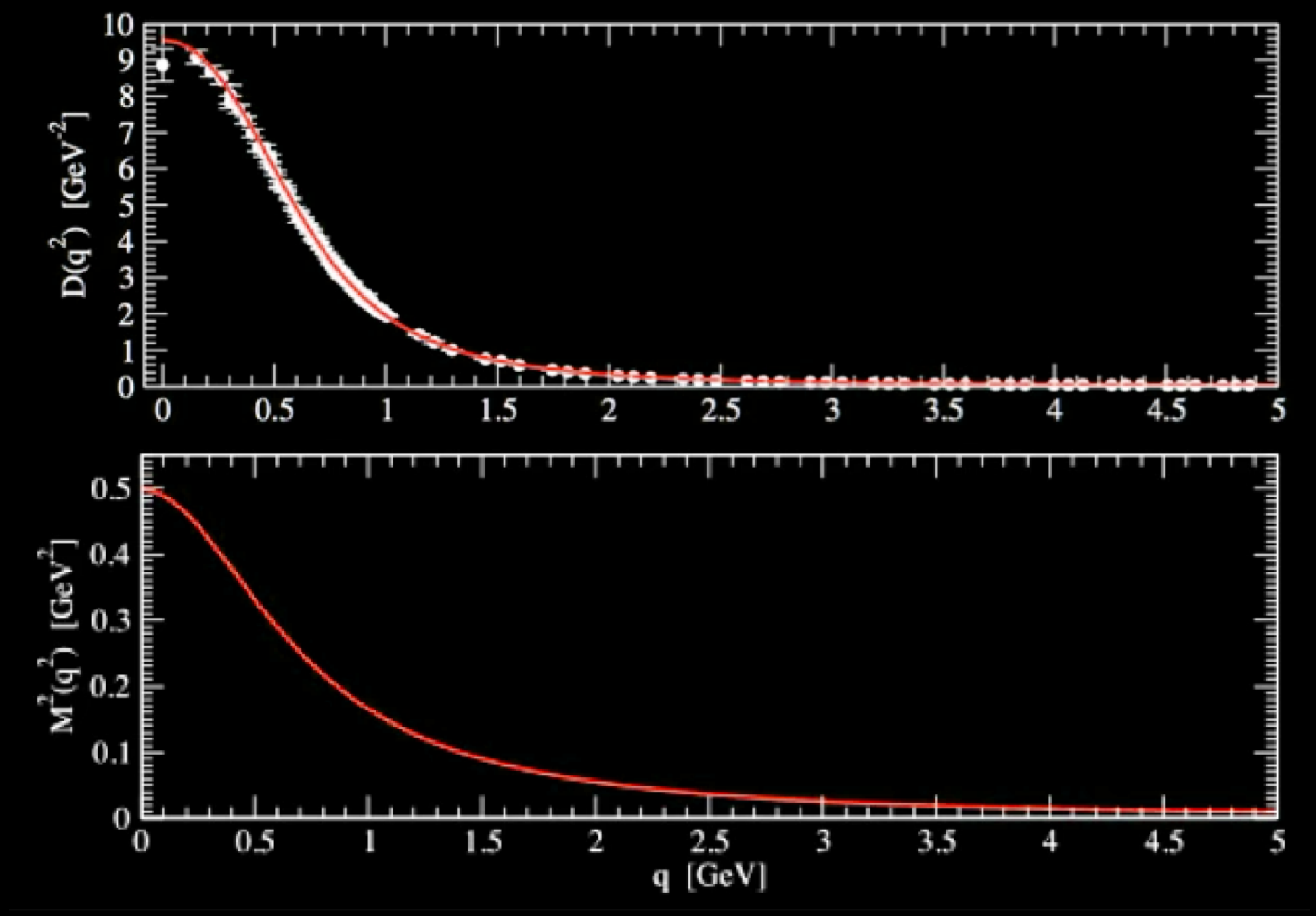
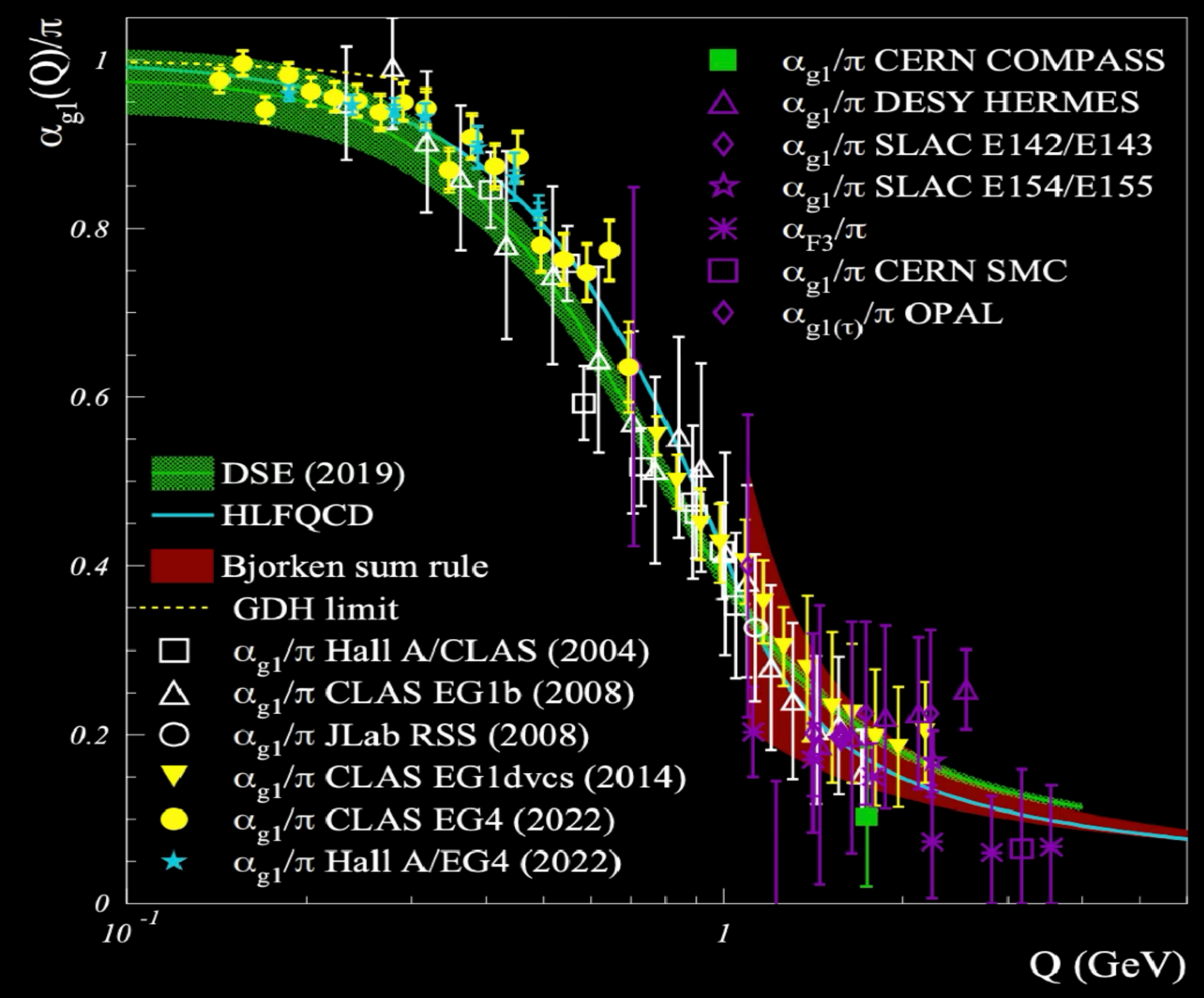
$$\alpha_s(0) \sim \pi$$

- **Quasi-gluon：**



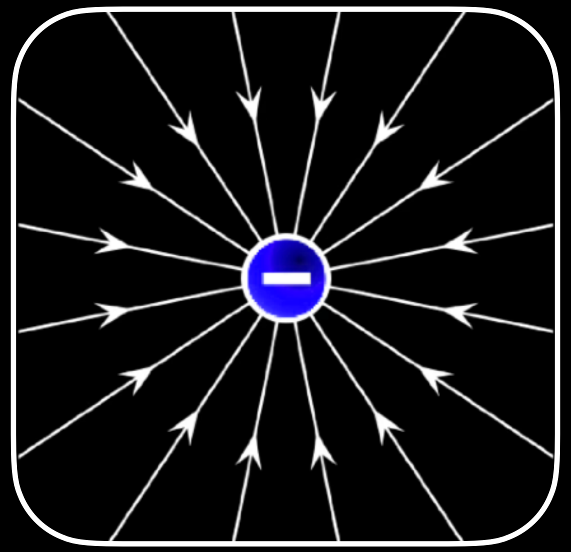
well-defined everywhere

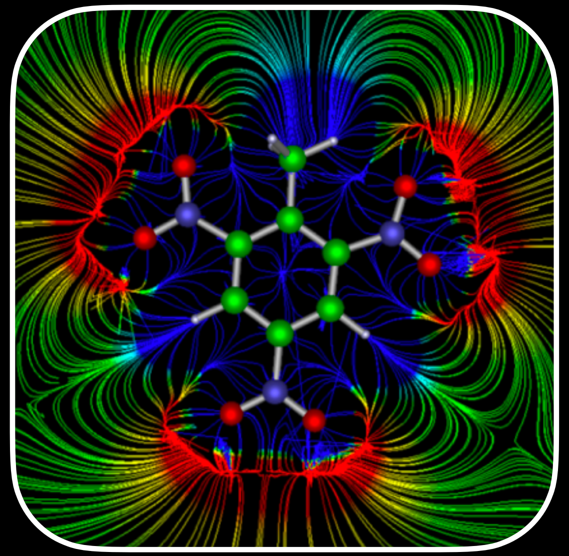
- **Running coupling：**



Solution: Vertex structures

- **Quark-gluon vertex：**





Solution: Vertex structures

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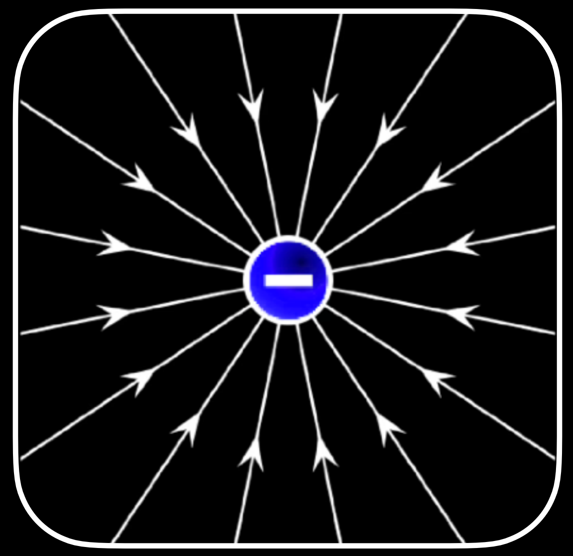
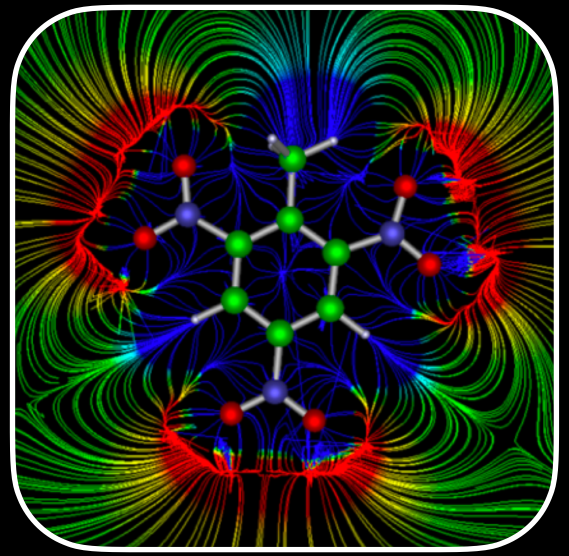
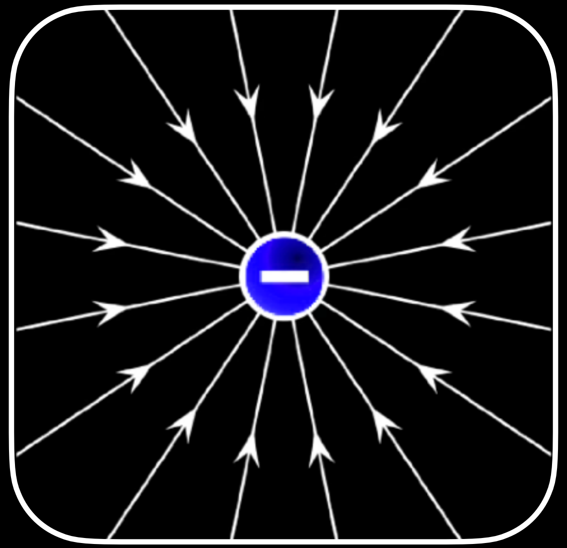


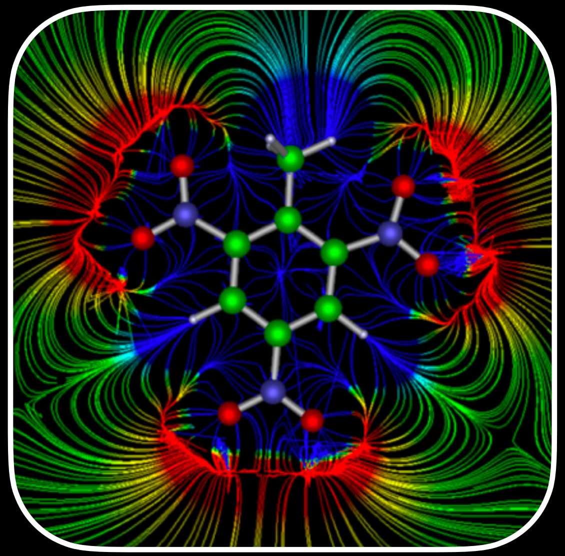


$$\Gamma^\mu(P',P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

Solution: Vertex structures

- **Quark-gluon vertex：**





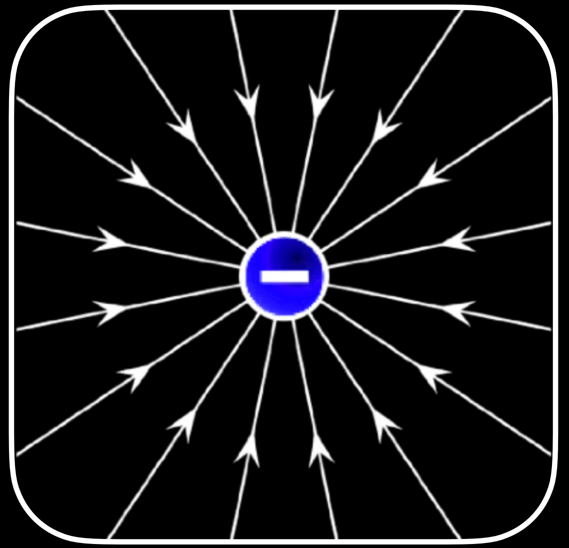
$$\Gamma^\mu(P',P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

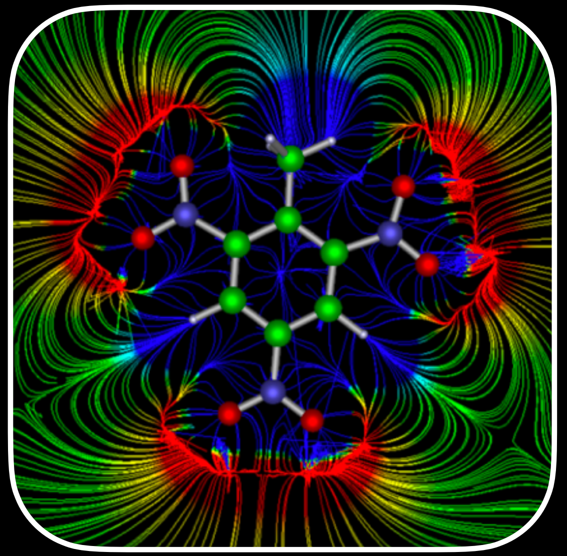
Dirac **Pauli**

$$S^{-1}(p) = i\gamma\cdot p\, A(p^2) \quad + \quad B(p^2)$$

Solution: Vertex structures

- **Quark-gluon vertex：**





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Dirac **Pauli**

$$S^{-1}(p) = i\gamma \cdot p\, A(p^2) \quad + \quad B(p^2)$$

- **DCSB feedback:**

Equation of Motion (DSEs)

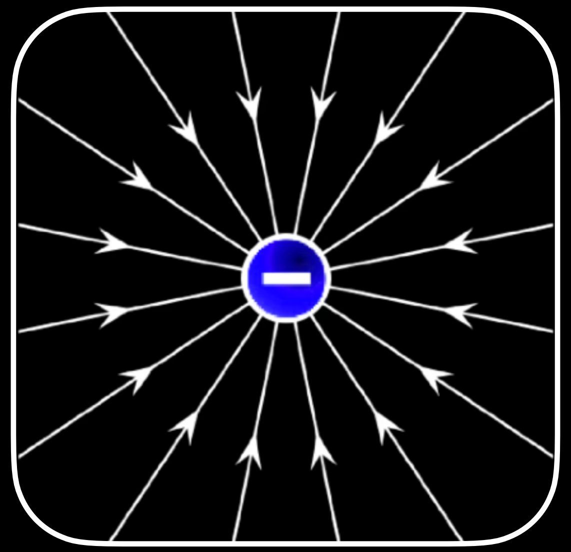
$$S = \frac{1}{i\gamma \cdot p A(p^2) \ + \ B(p^2)}$$

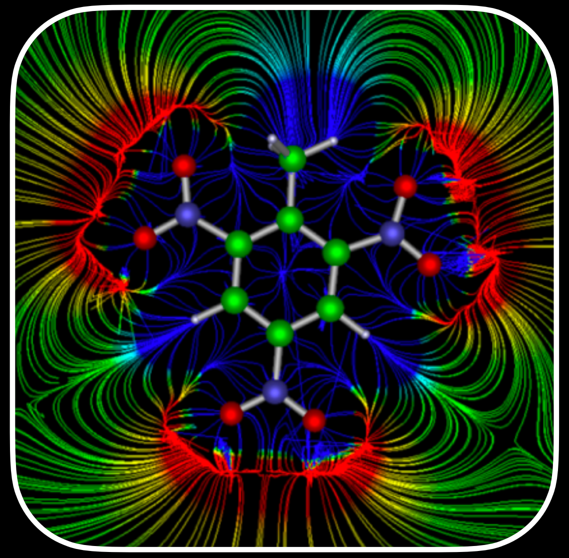
$$\Gamma^\mu \sim \gamma^\mu \Sigma_A \ + \ i\sigma_{\mu\nu} Q^\nu \Delta_B$$

Ward identities (Symmetries)

Solution: Vertex structures

- **Quark-gluon vertex：**





$$\boxed{\Gamma^\mu(P',P) = \gamma^\mu \underbrace{F_1(Q^2)}_{\text{Dirac}} + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu \underbrace{F_2(Q^2)}_{\text{Pauli}}}$$

Dirac **Pauli**

$$\boxed{S^{-1}(p) = i\gamma\cdot p\, A(p^2) \quad + \quad B(p^2)}$$

- **DCSB feedback:**

Equation of Motion (DSEs)

$$\boxed{S = \frac{1}{i\gamma\cdot p A(p^2) \; + \; B(p^2)}} \qquad \boxed{\Gamma^\mu \sim \gamma^\mu \Sigma_A \; + \; i\sigma_{\mu\nu} Q^\nu \Delta_B}$$

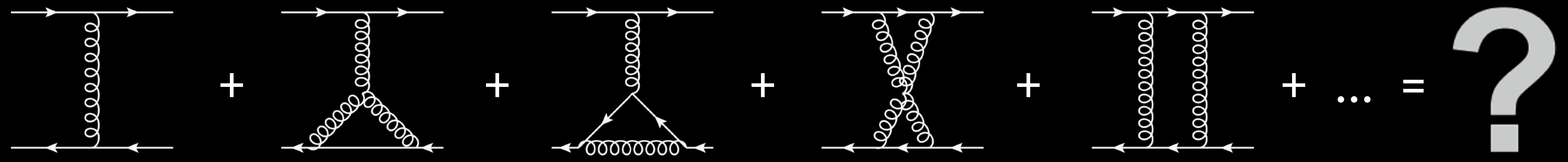
Ward identities (Symmetries)

- **Dressed vertex:**

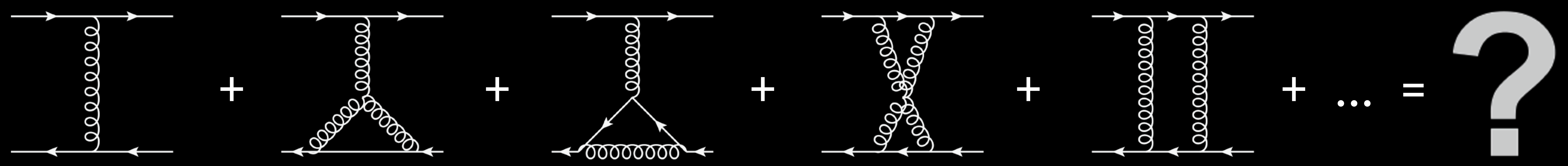
$$\boxed{\Gamma^\mu \sim \gamma^\mu + i\eta\sigma_{\mu\nu}Q^\nu\Delta_B}$$

DCSB rendered

Solution: Kernel decompositions



Solution: Kernel decompositions



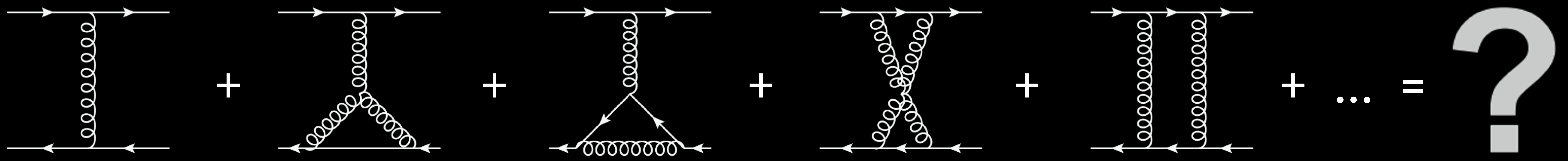
Poincaré symmetry

C-, P-, T-symmetry

Gauge symmetry

Chiral symmetry

Solution: Kernel decompositions



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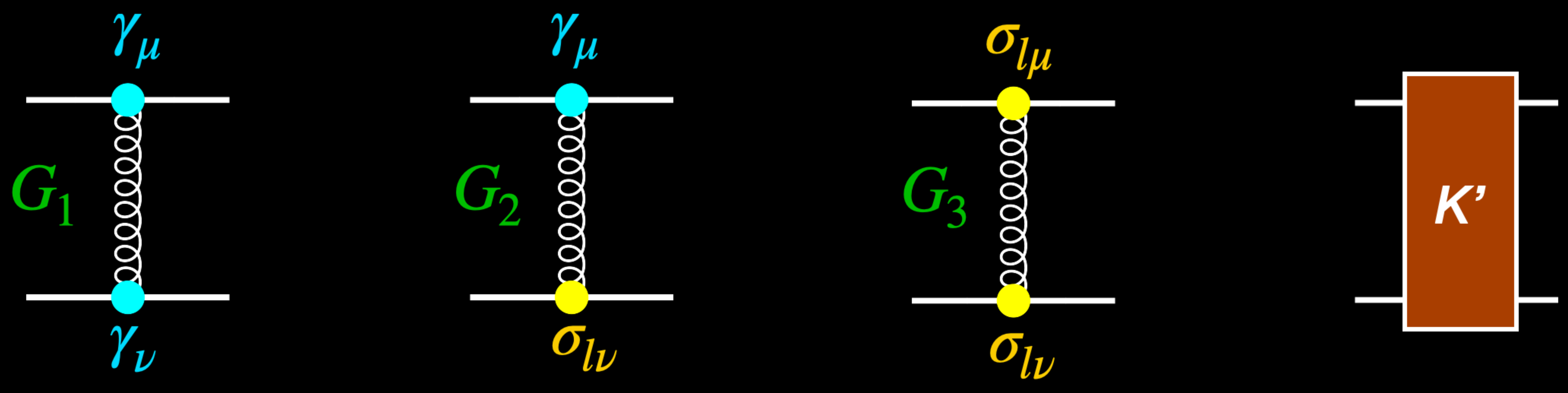

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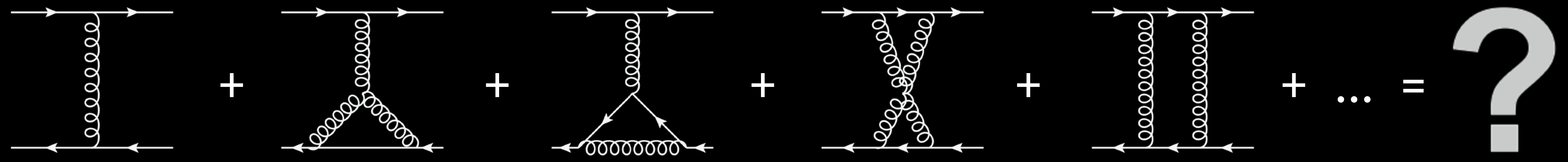

Poincaré symmetry

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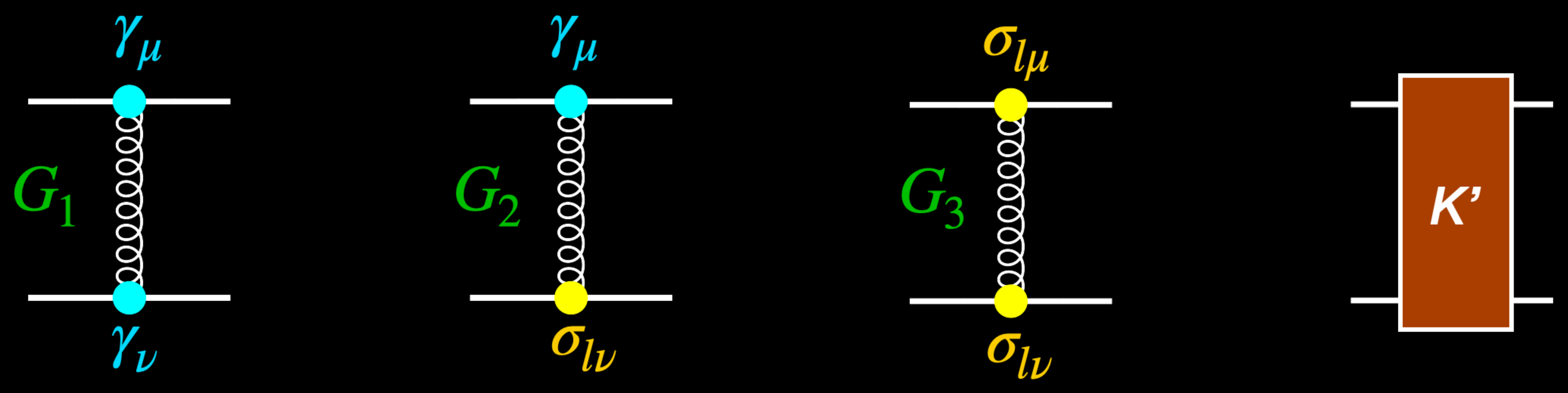
pion meson
twofold role



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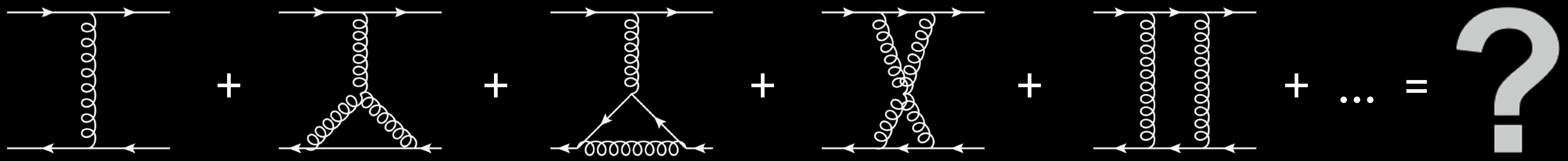

Poincaré symmetry

C-, P-, T-symmetry

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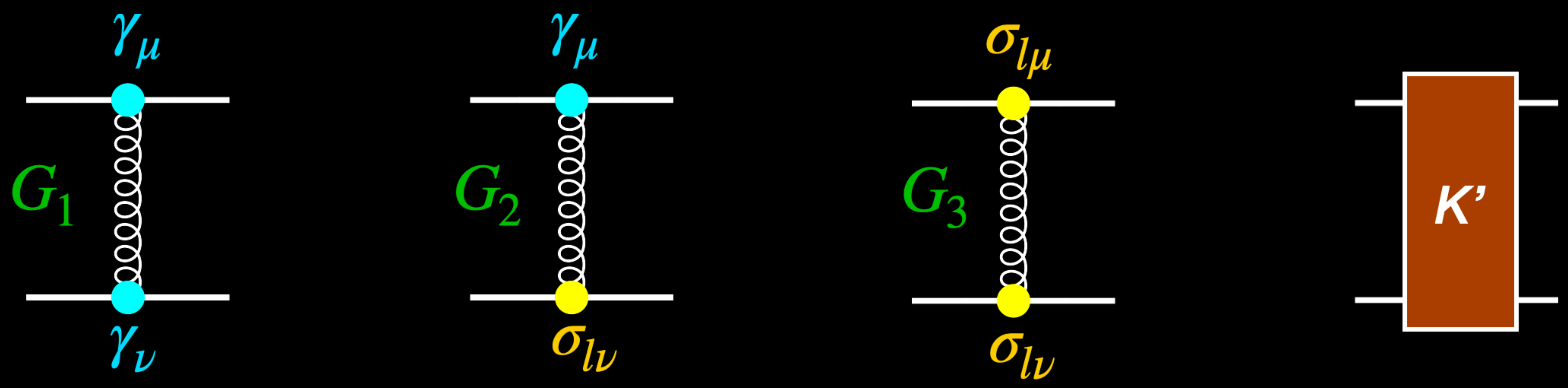
pion meson twofold role

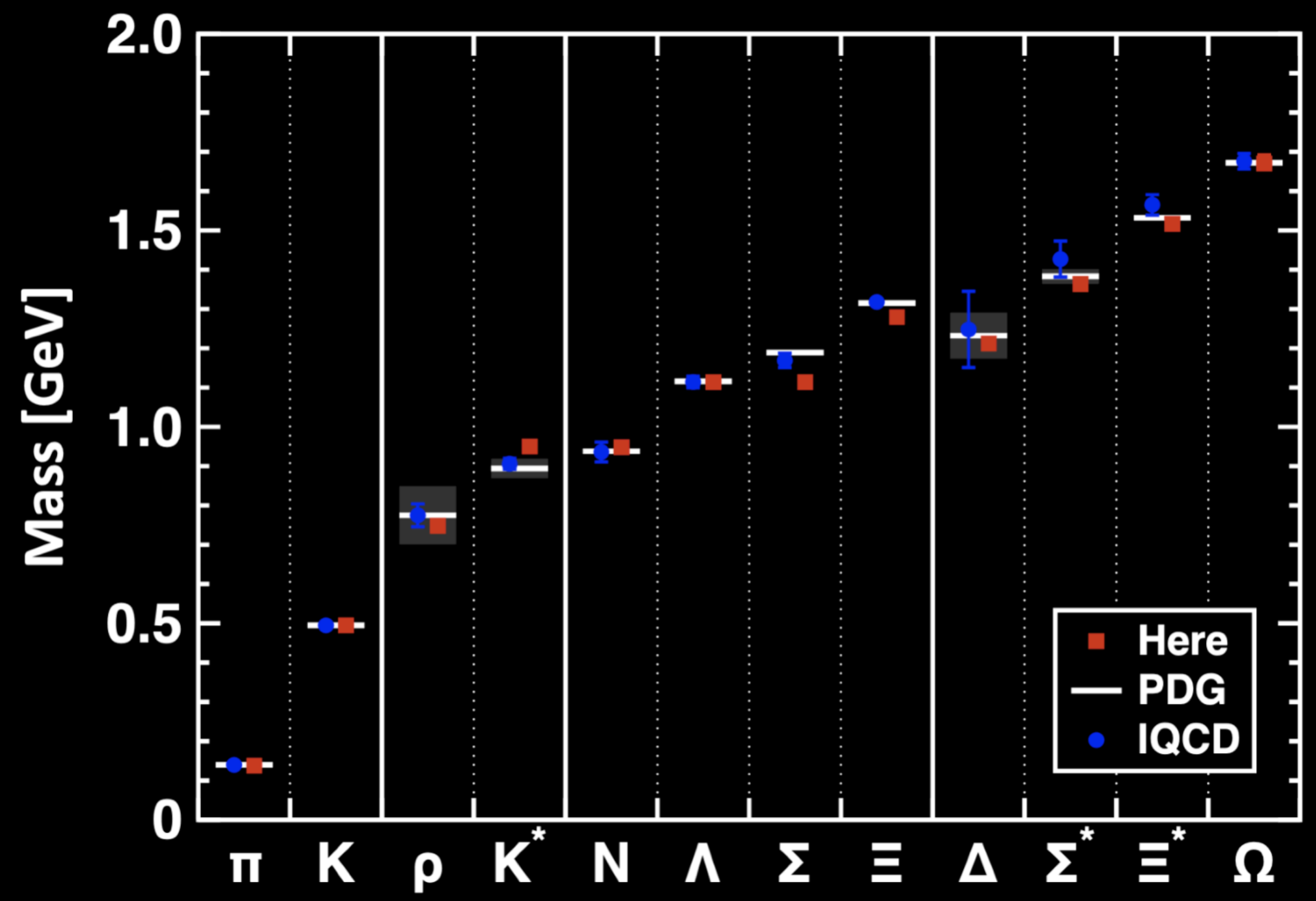


$G_{2,3} \sim DCSB$

Result

Result: Ground states

- **Hadron spectrum:**



Result: Ground states

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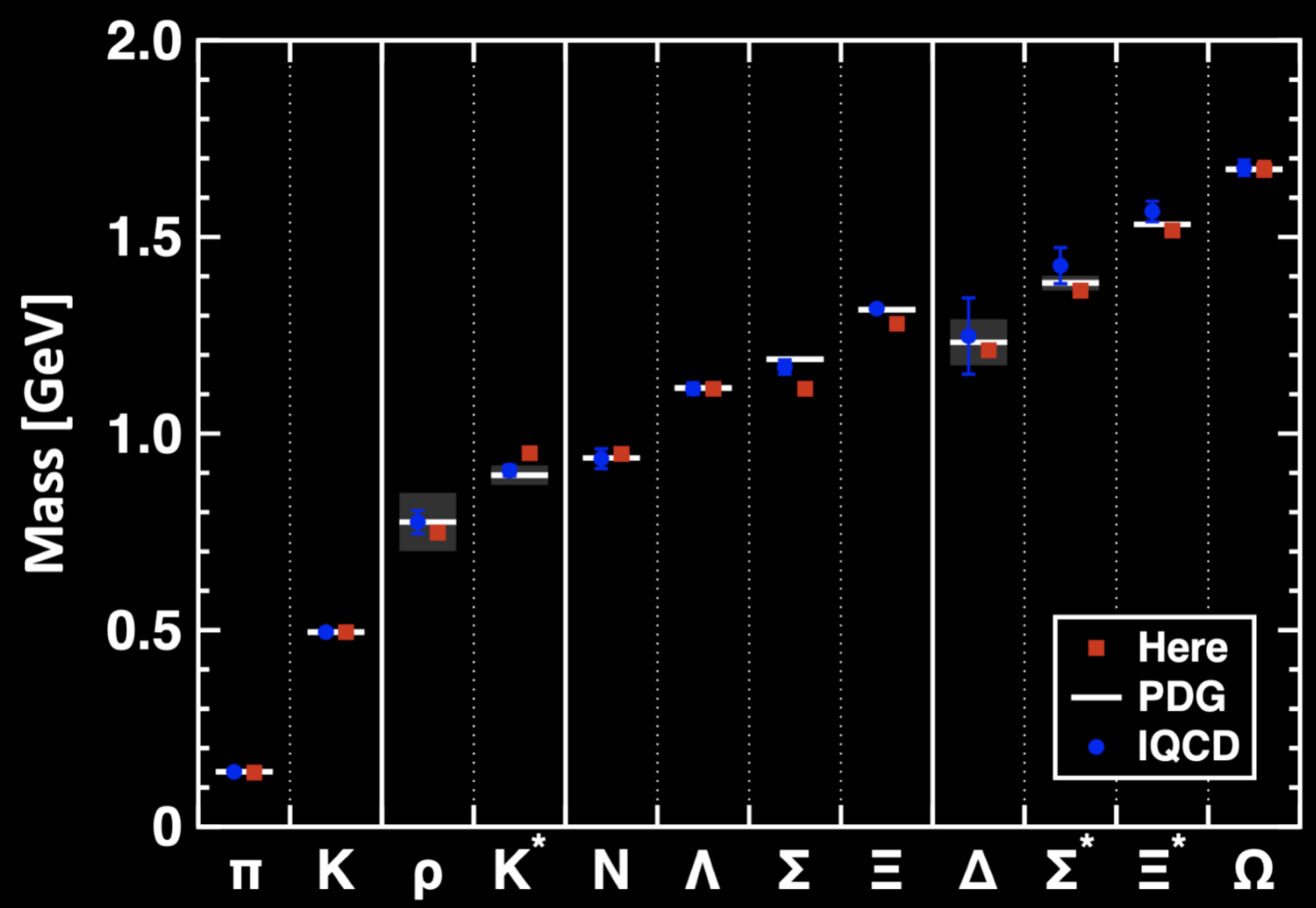
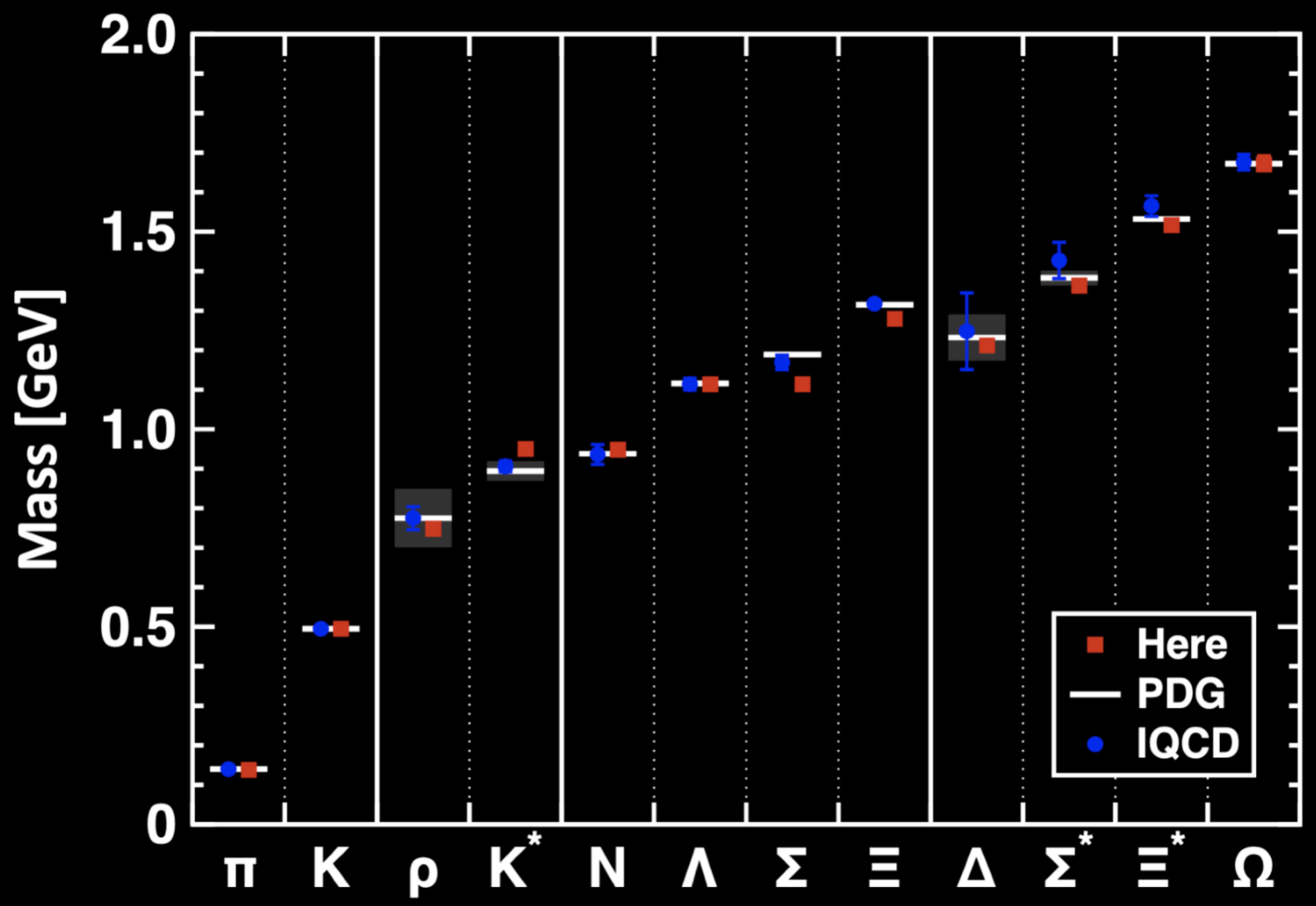


Effective strength

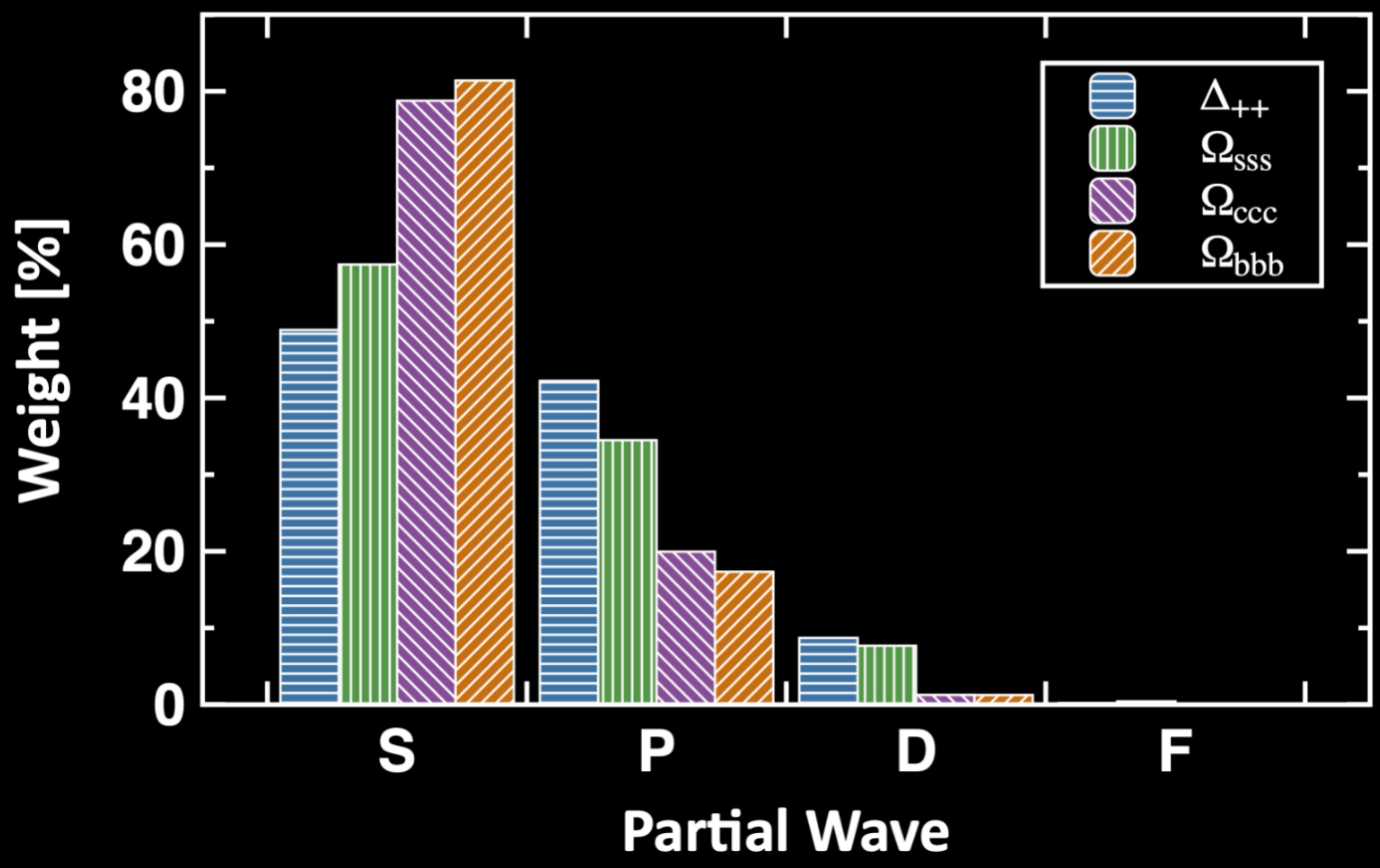
Result: Ground states

- **Hadron spectrum:**



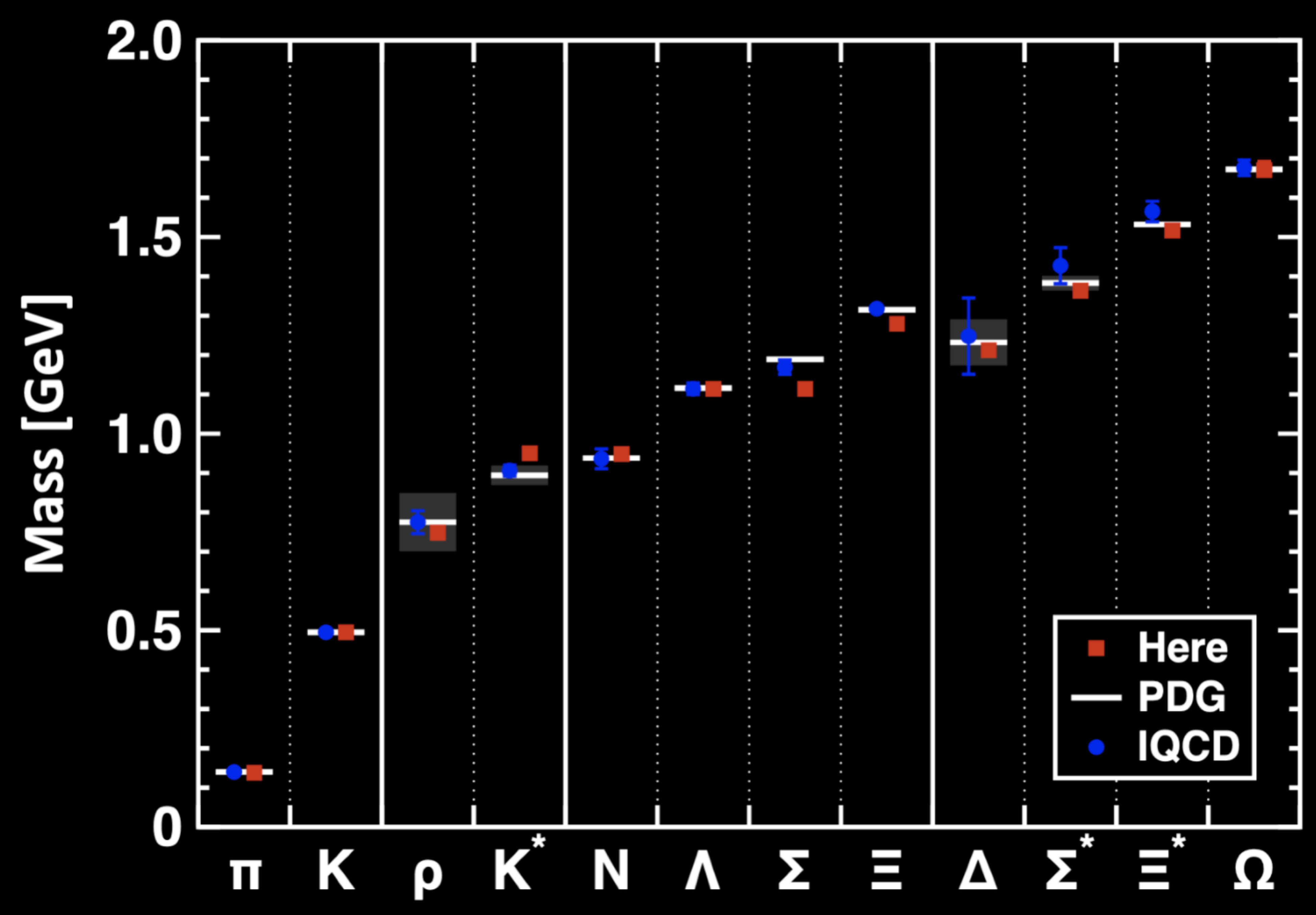
Effective strength

- **Hadron structure:**



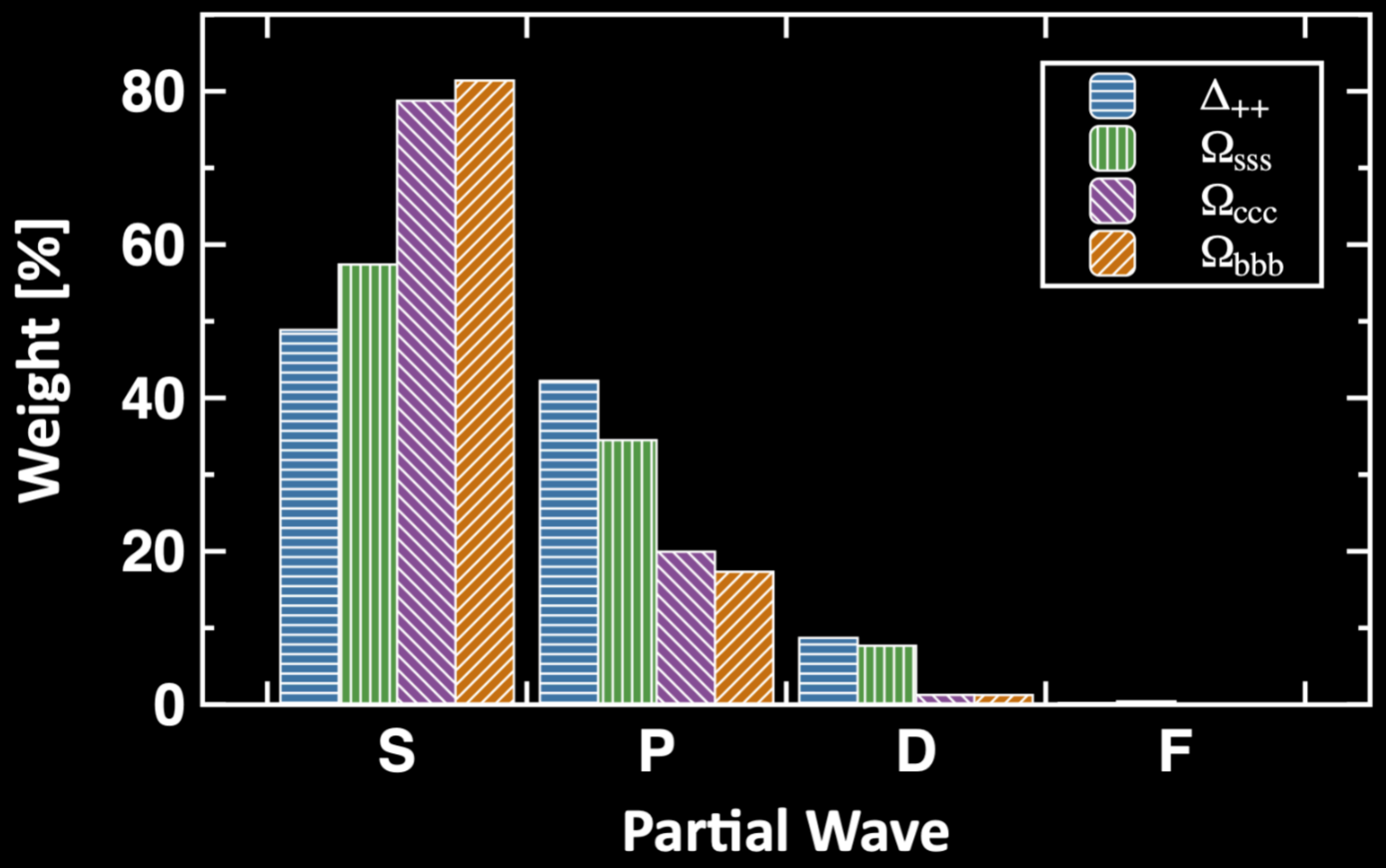
Result: Ground states

- **Hadron spectrum:**



Effective strength

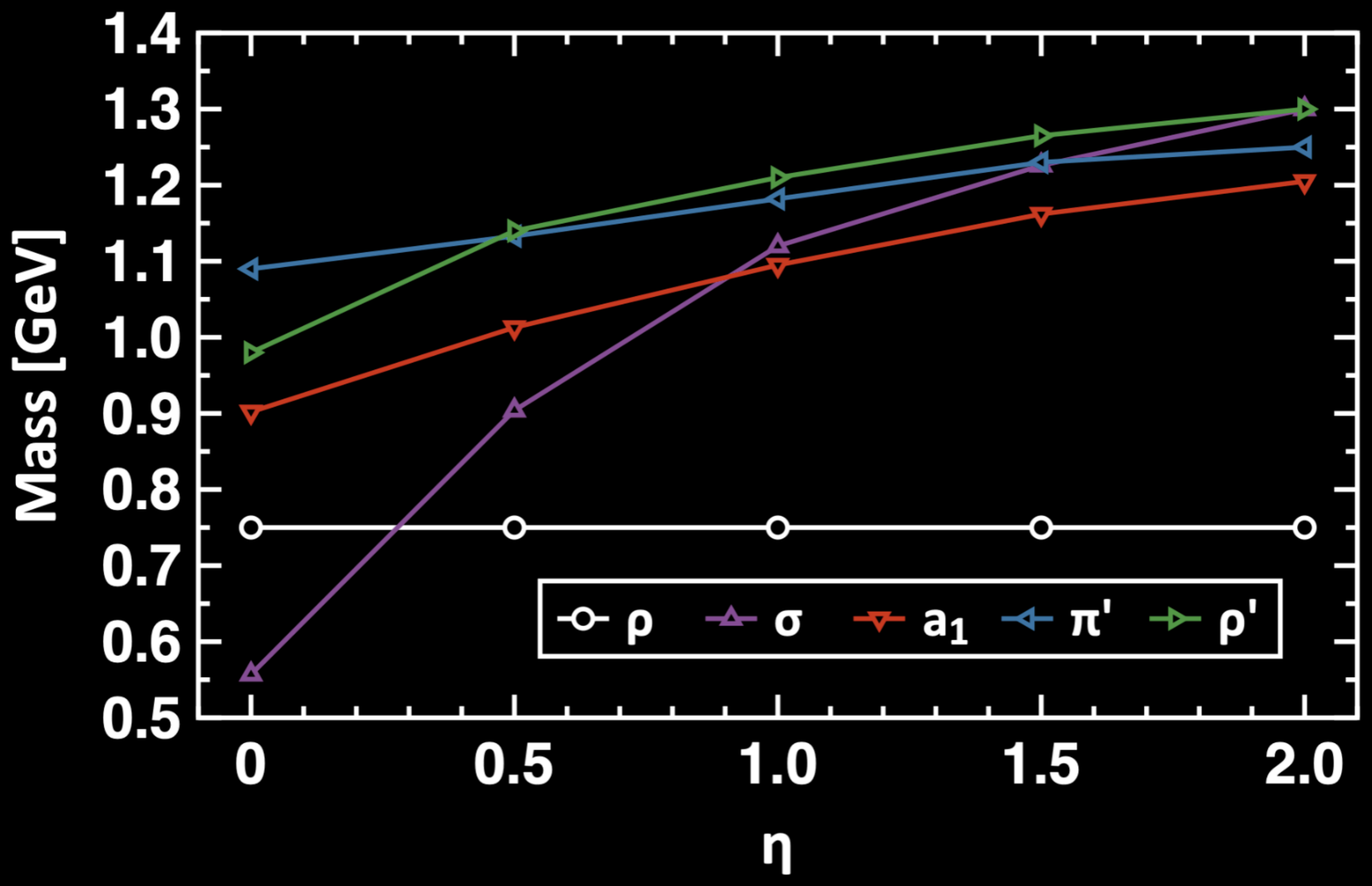
- **Hadron structure:**



NR potential models

Result: Excited states

- **Impact of Pauli term (DCSB):**



Result: Excited states

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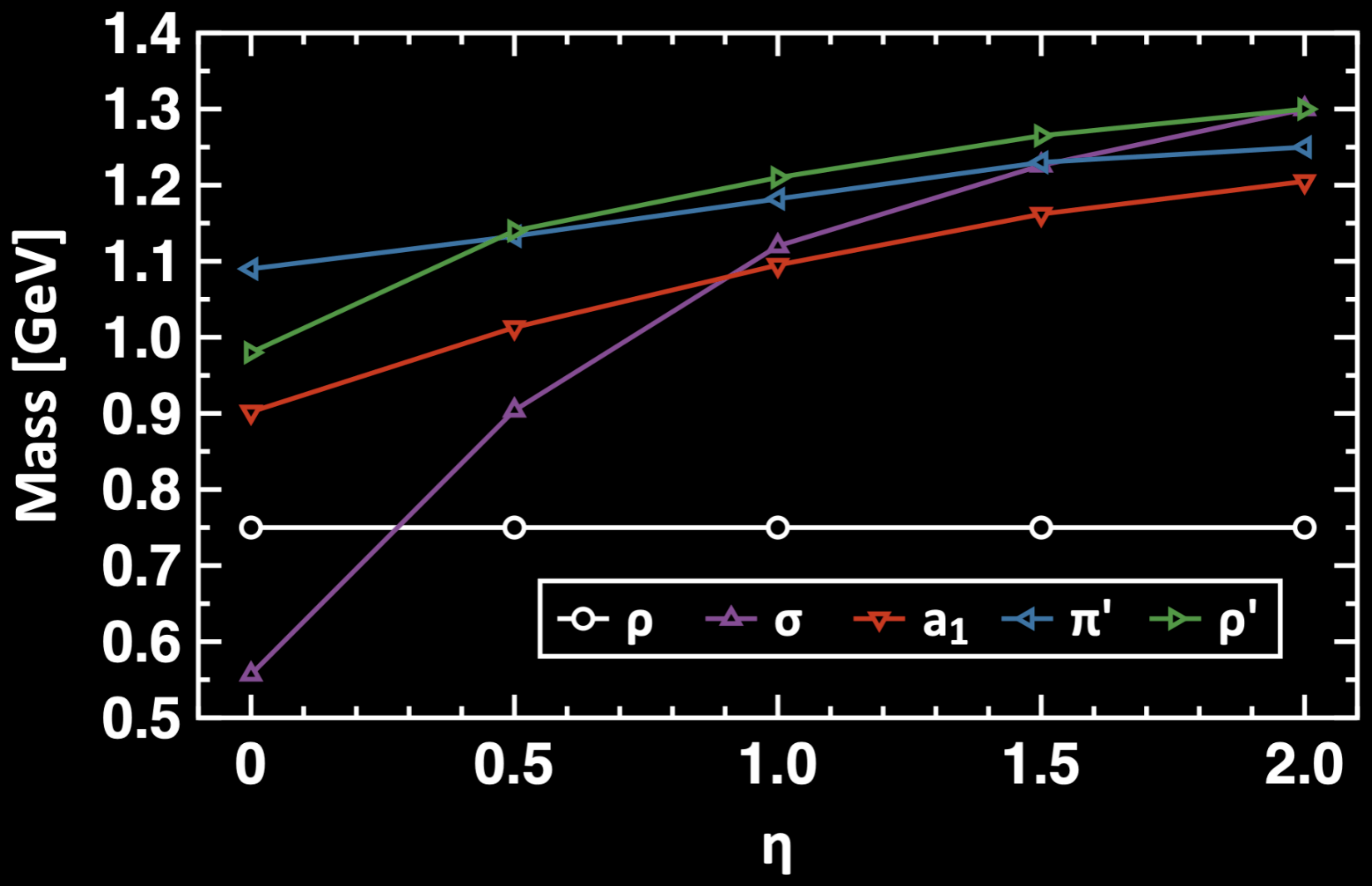
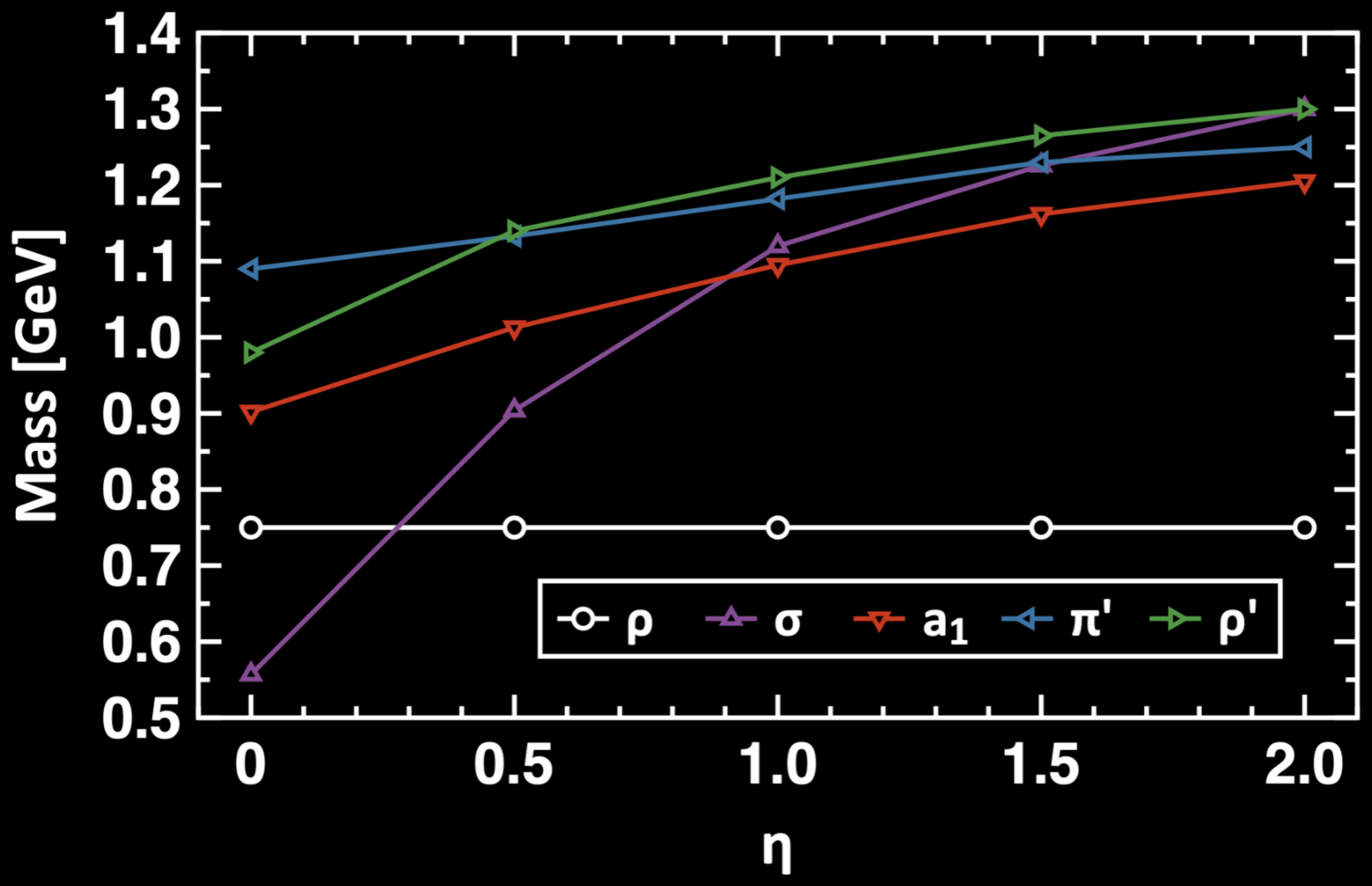


spin-orbit repulsion

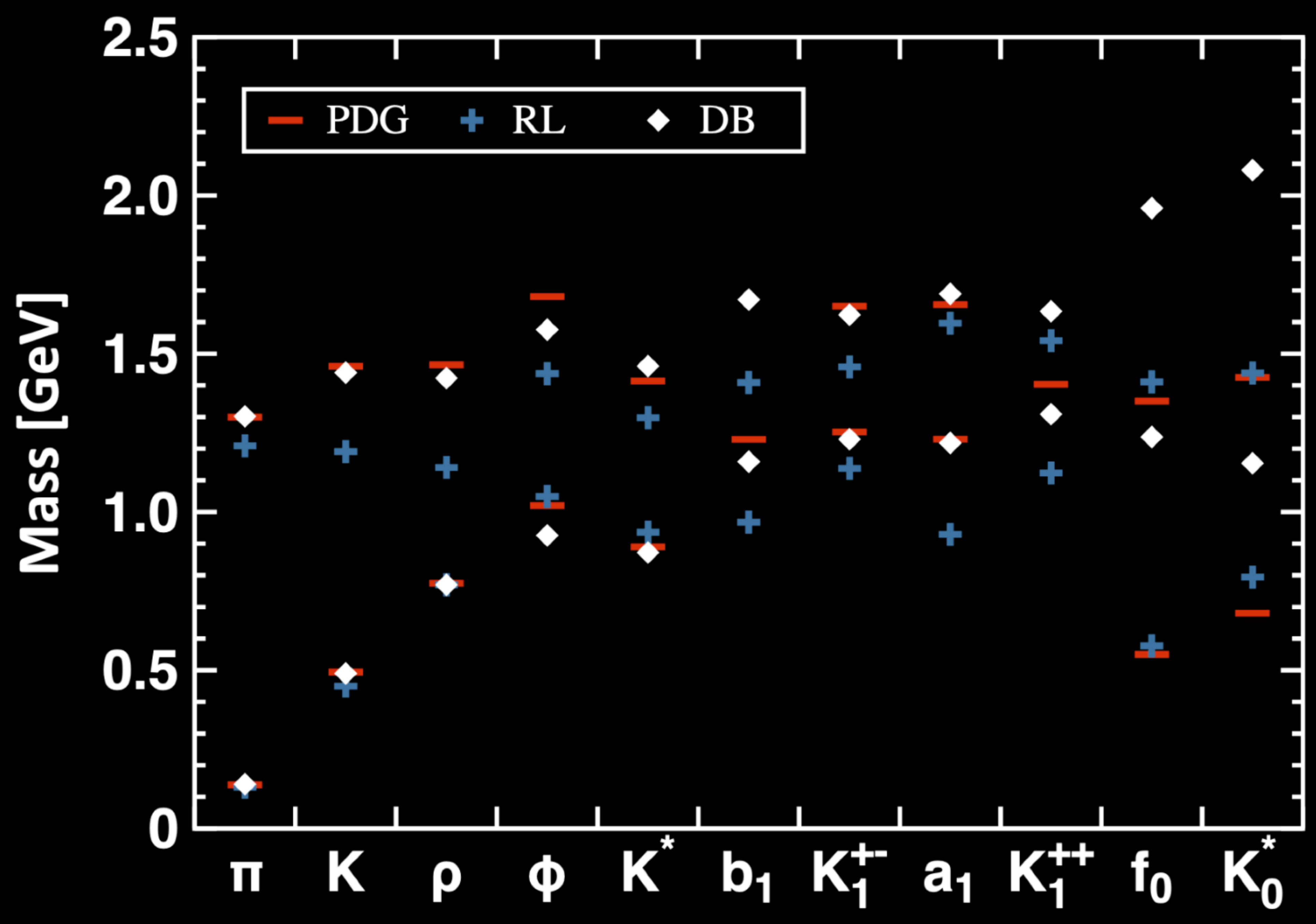
Result: Excited states

- **Impact of Pauli term (DCSB):**



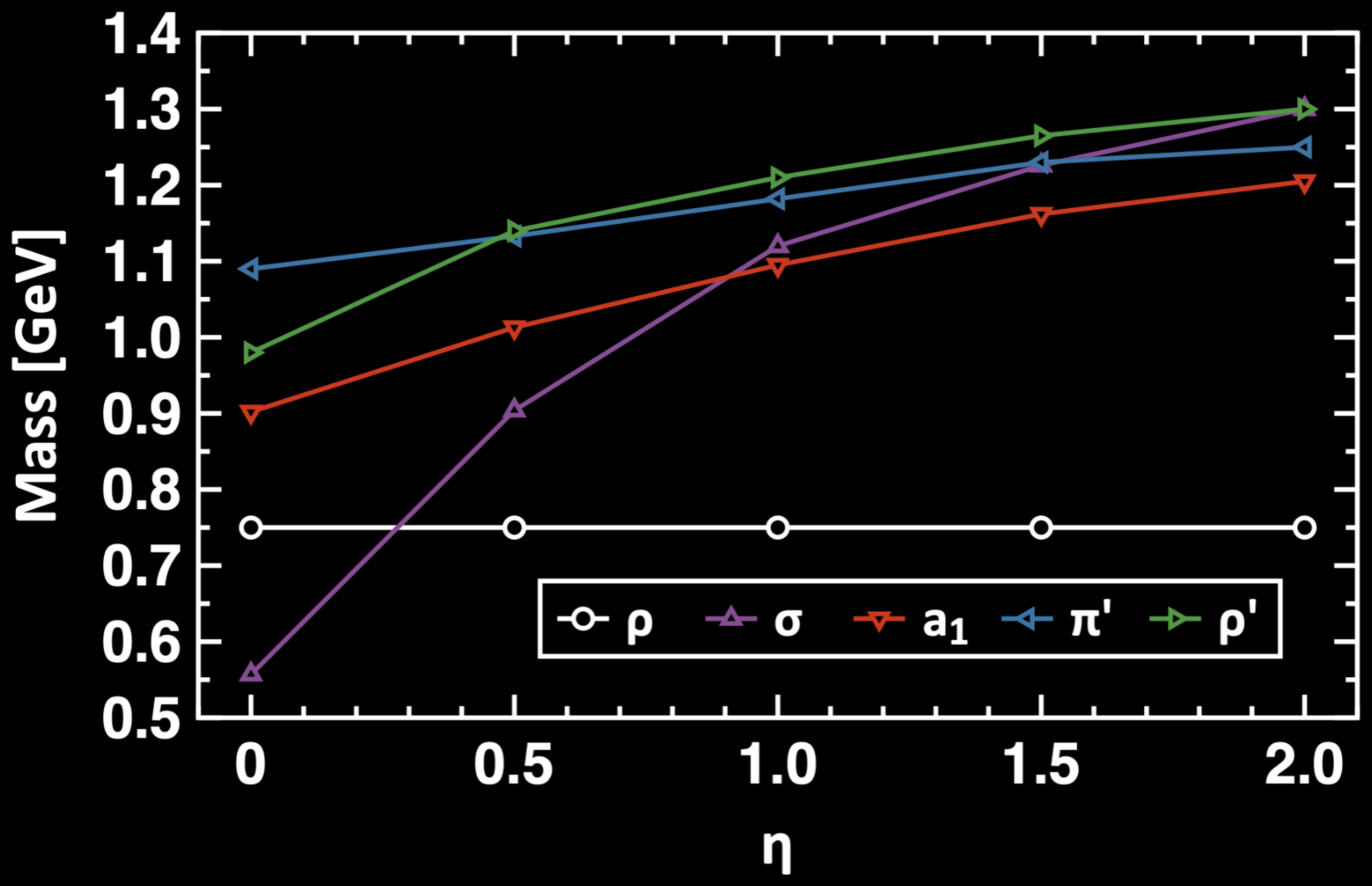
spin-orbit repulsion

- **DCSB-enhanced spectrum:**



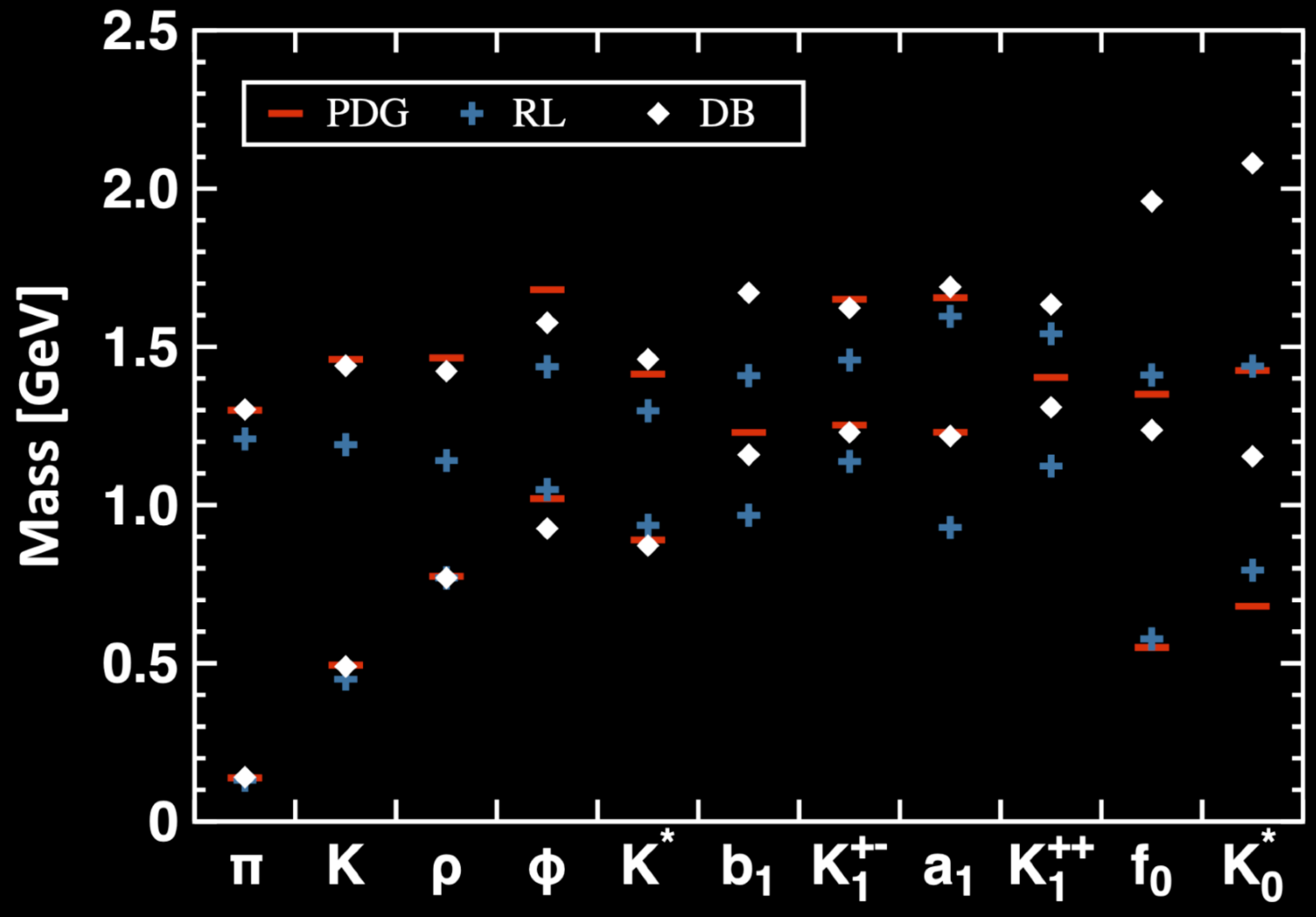
Result: Excited states

- **Impact of Pauli term (DCSB):**



spin-orbit repulsion

- **DCSB-enhanced spectrum:**



magnitude and ordering

Summary

Summary:

- The continuum framework of nonperturbative QCD, i.e., EoM (DSE), and its basics (quark, gluon, vertex, kernel) are introduced.
- Hadron properties are studied: **a**) ground states — full spectrum; **b**) excited states — partial waves, spin-orbit interaction, DCSB-rendered spectrum.

Summary:

- The continuum framework of nonperturbative QCD, i.e., EoM (DSE), and its basics (quark, gluon, vertex, kernel) are introduced.
- Hadron properties are studied: **a**) ground states — full spectrum; **b**) excited states — partial waves, spin-orbit interaction, DCSB-rendered spectrum.
- Use the DSE to a wider range of applications in baryon problems of QCD: transition form factors, parton distribution functions, and etc.
- Iterating with high precision experiments on hadrons, from spectroscopy to structures, we may provide a faithful path to understand QCD.

重庆大学
CHONGQING UNIVERSITY

Nonperturbative Expansion of QCD

Si-Xue Qin

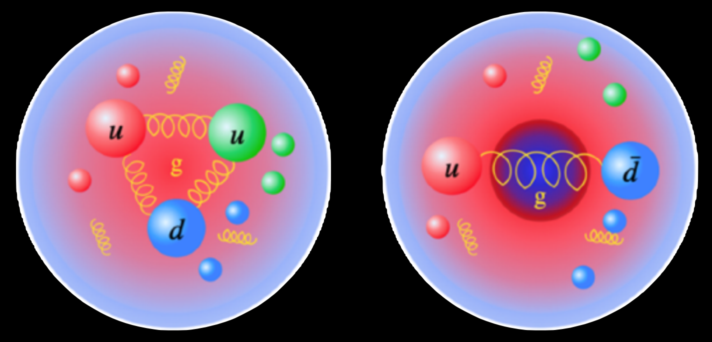
Beijing

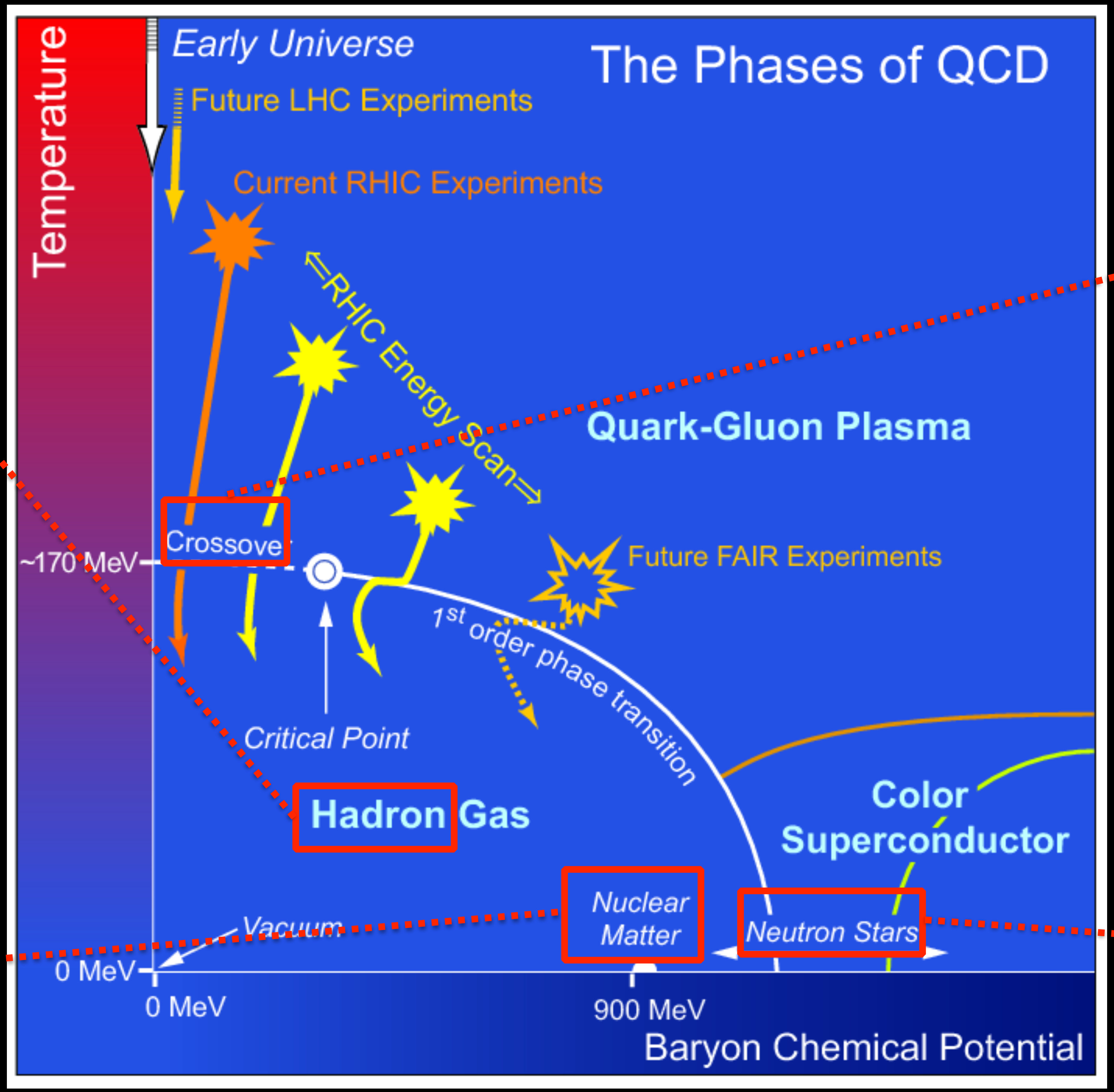
2025-11-02

Introduction

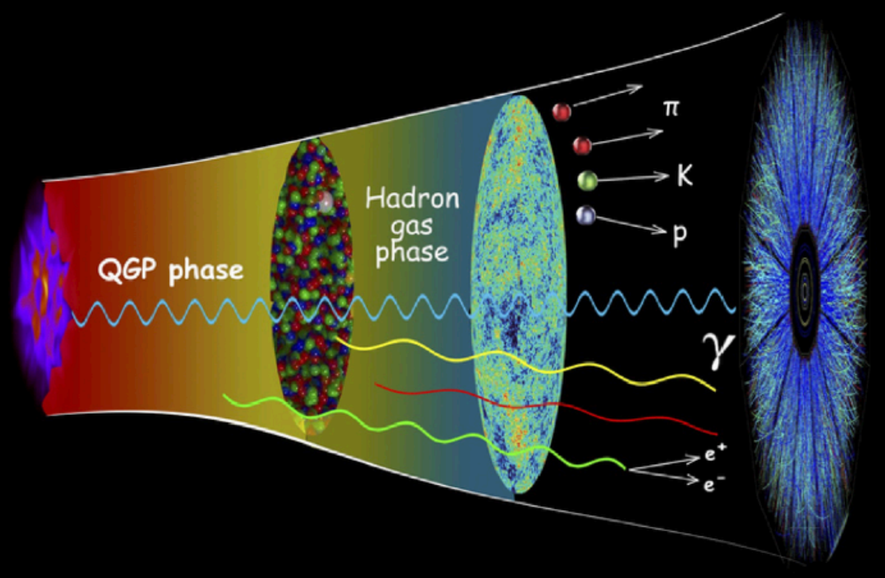
Introduction: QCD frontiers

Hadron

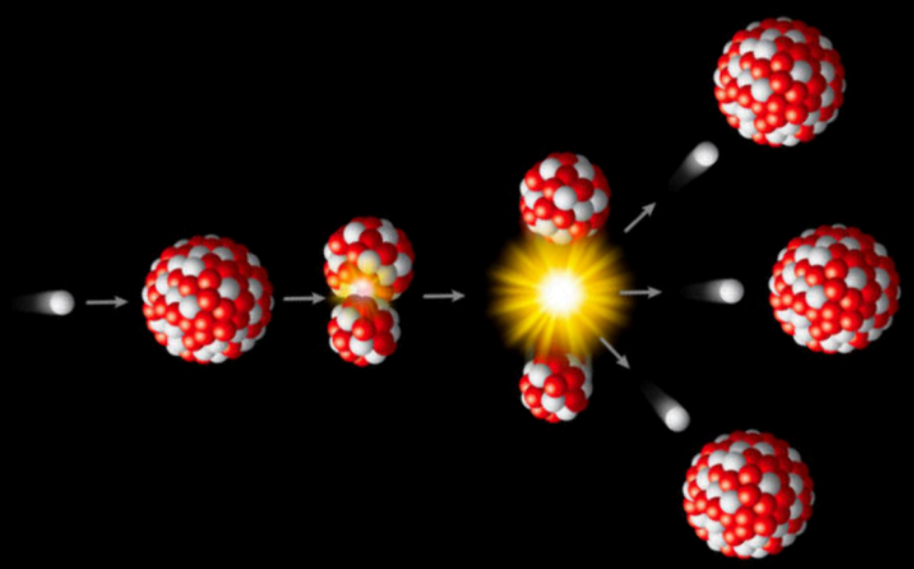




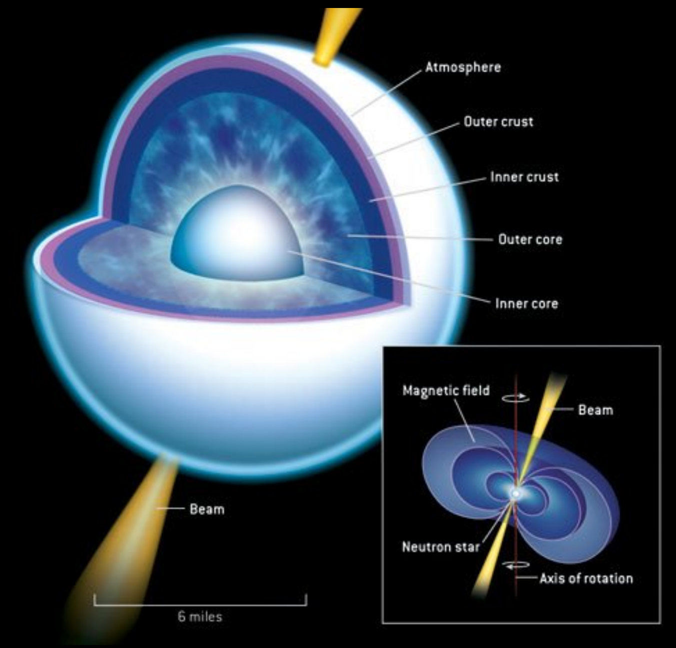
Hot Matter



Nucleus

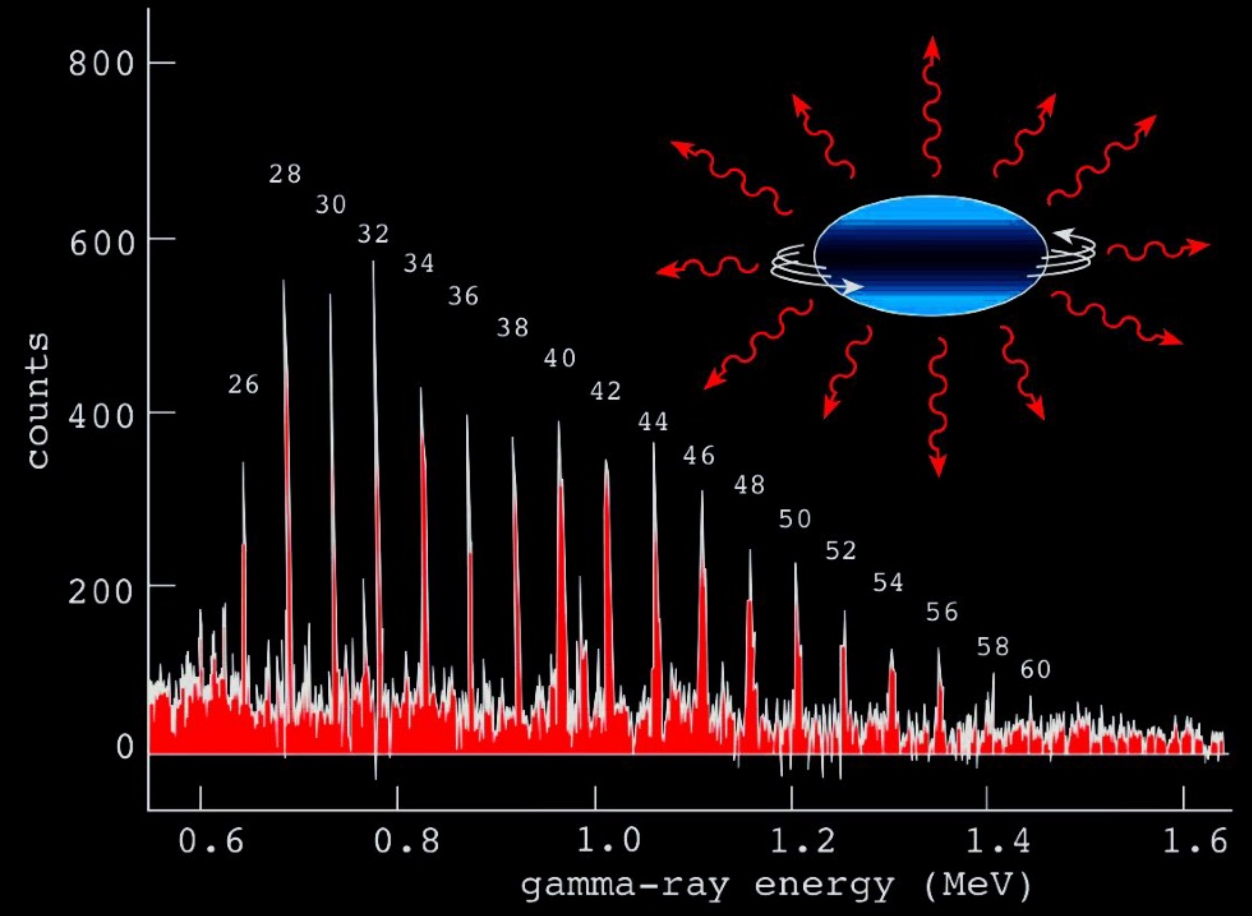


Dense Matter

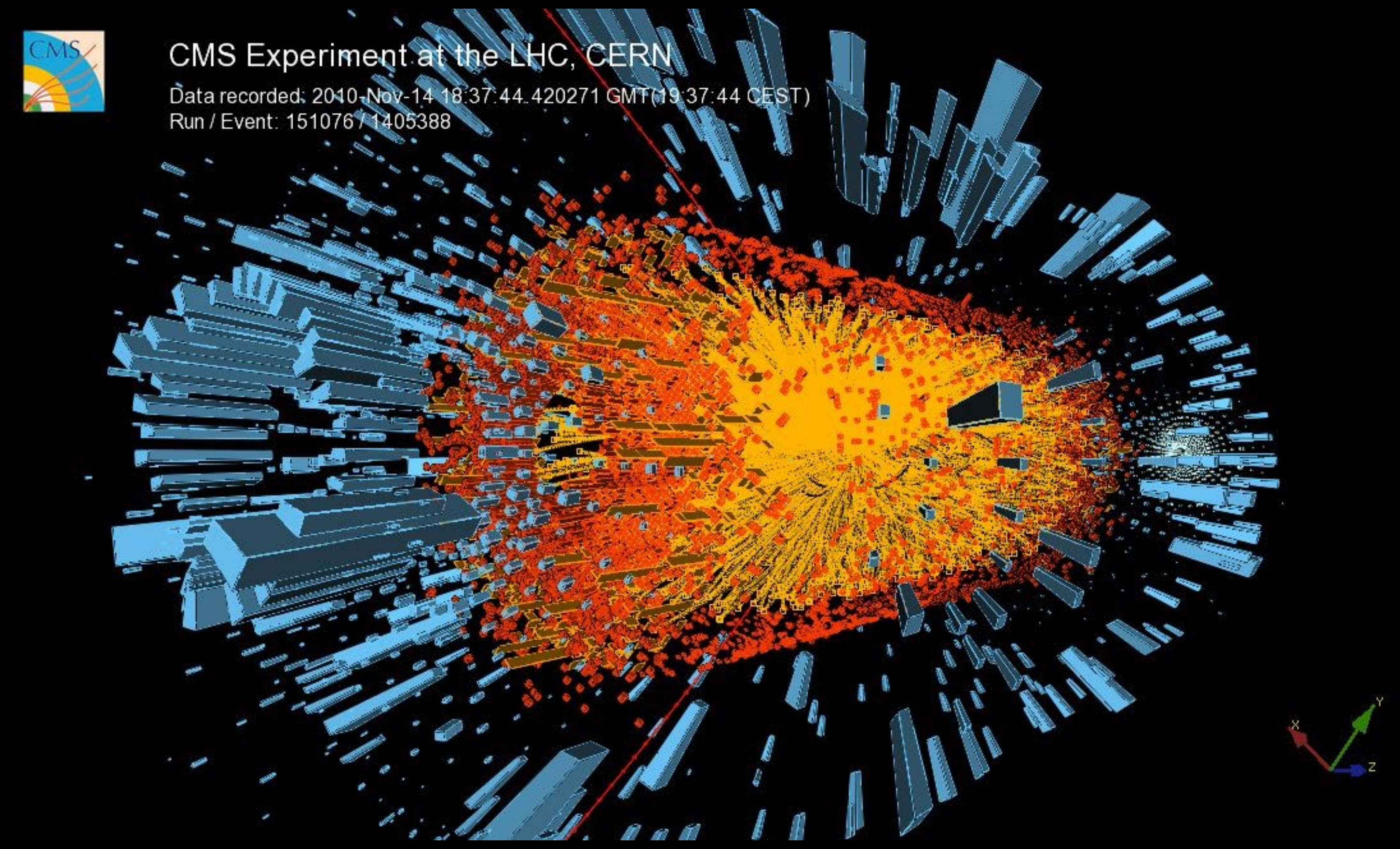


Introduction: QCD frontiers

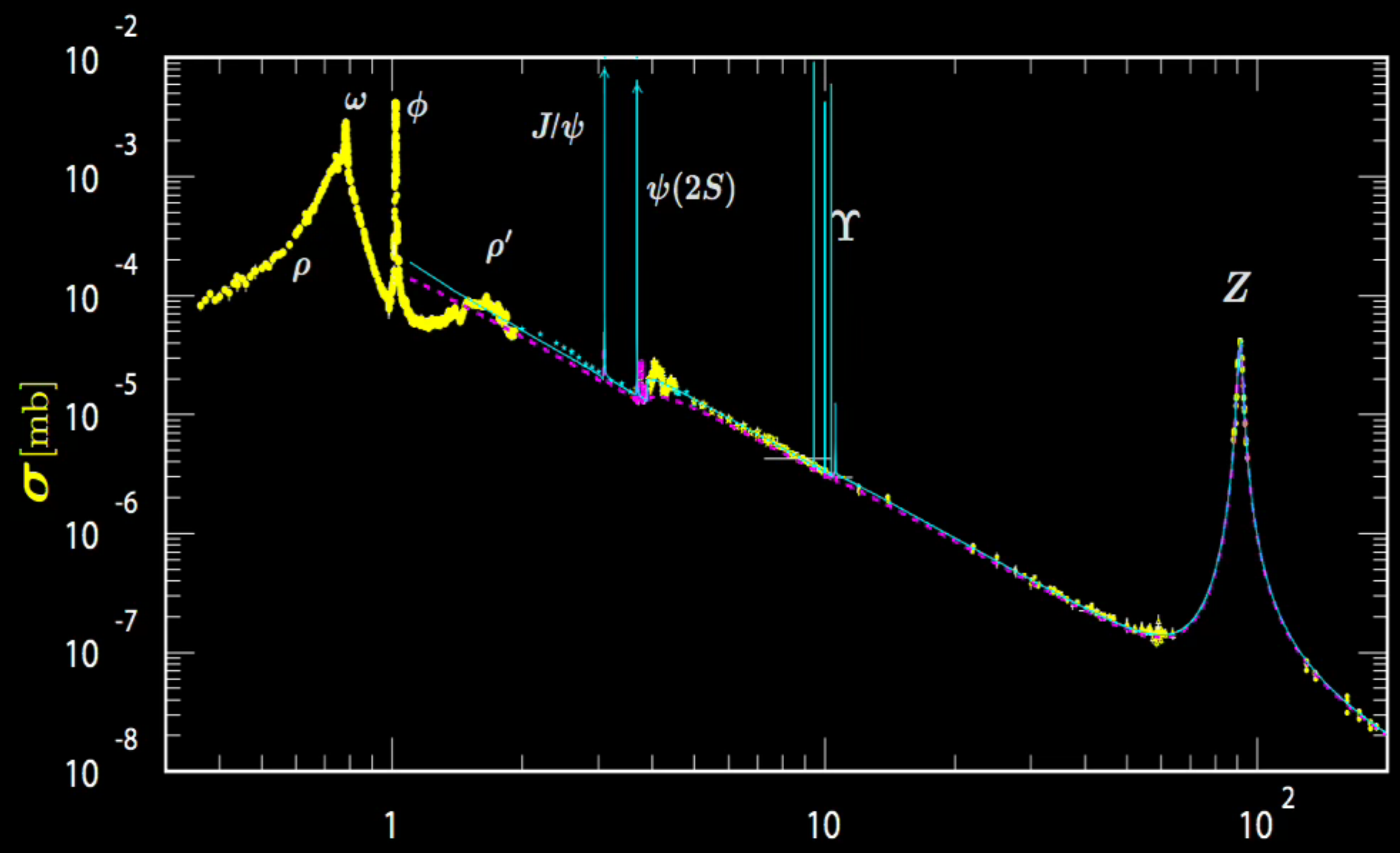
◉ Novel states of nuclei



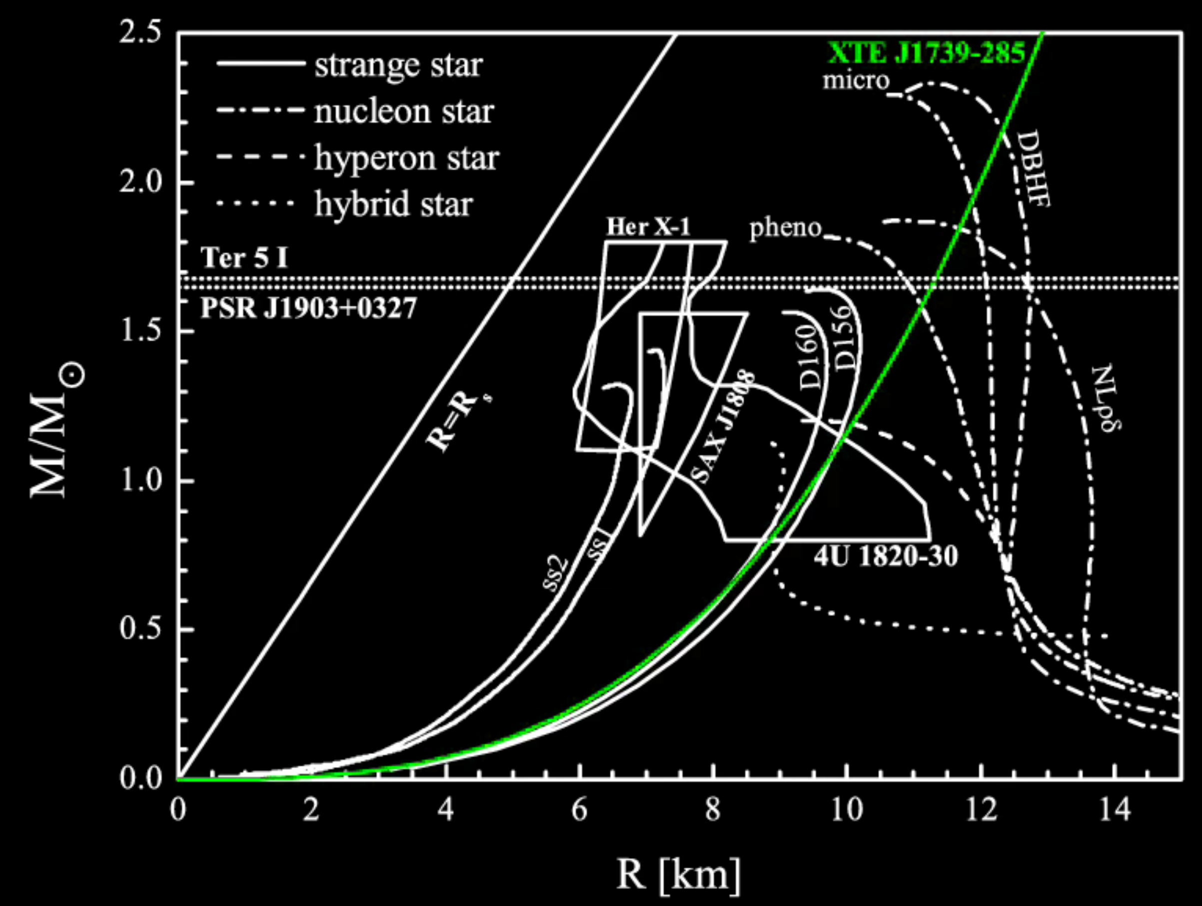
◉ Relativistic heavy-ion collision



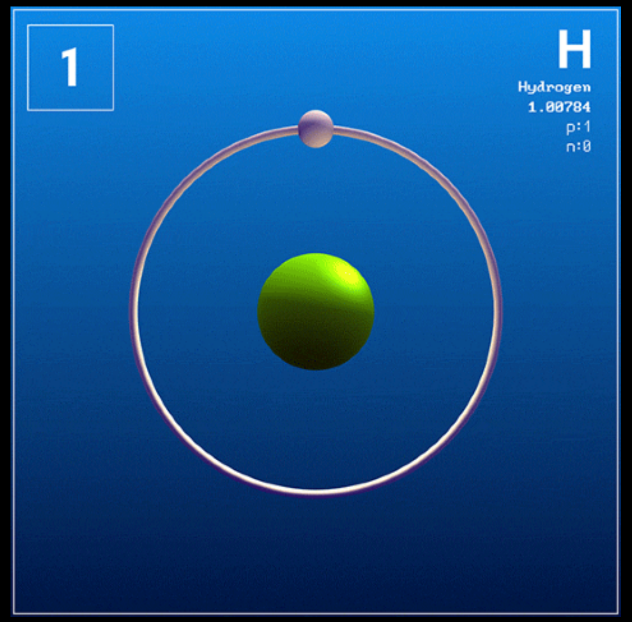
◉ e+e- hadronic annihilation

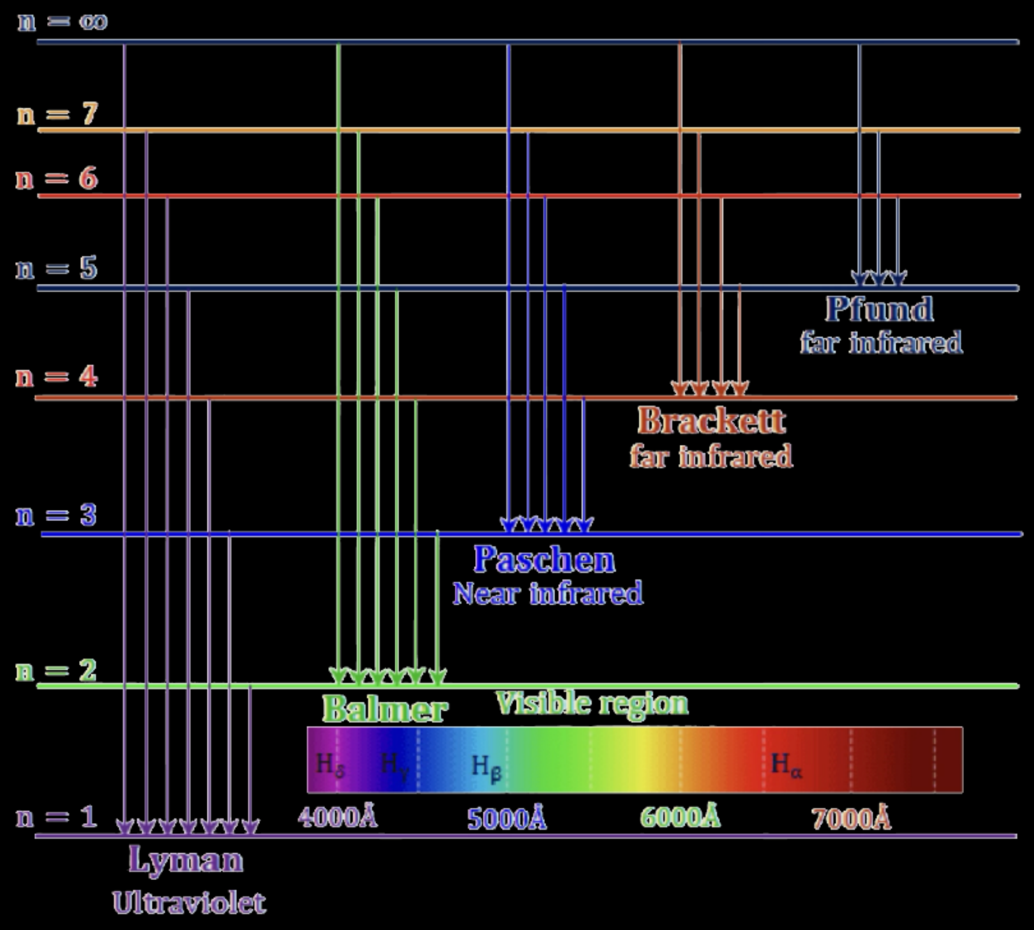


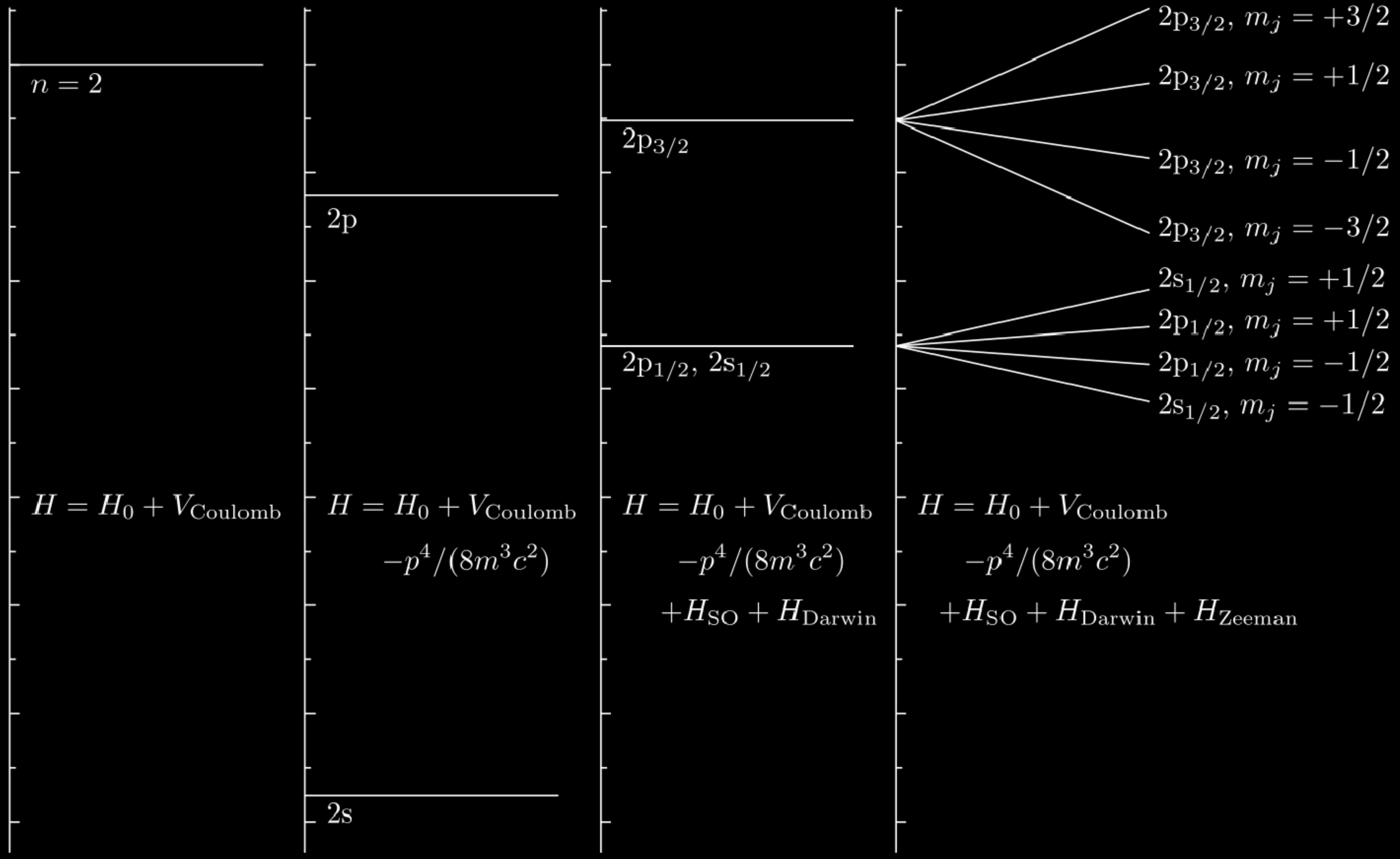
◉ Mass-radius relation of starts



Introduction: Lesson from QED







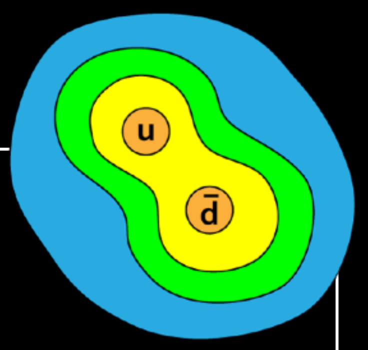
$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{vacuum}}$$

Framework

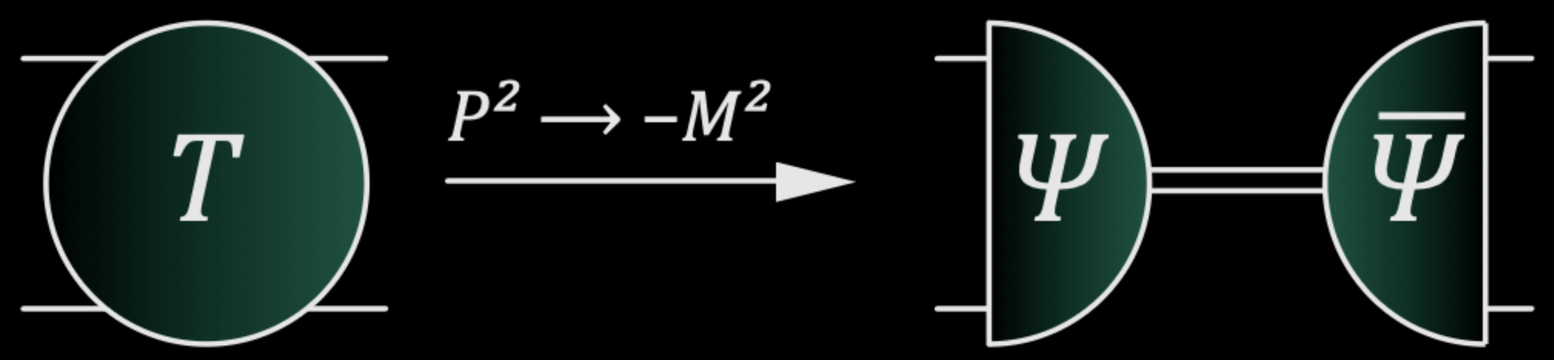
Framework: Equation of Motion (DSE)

$$\mathscr{L}_{QCD} = \bar{\psi}_i \left[i \left(\gamma^\mu D_\mu \right)_{ij} - m\delta_{ij} \right] \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

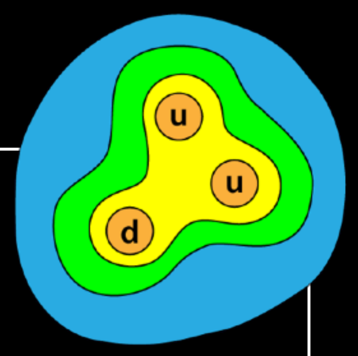
Green functions of QCD



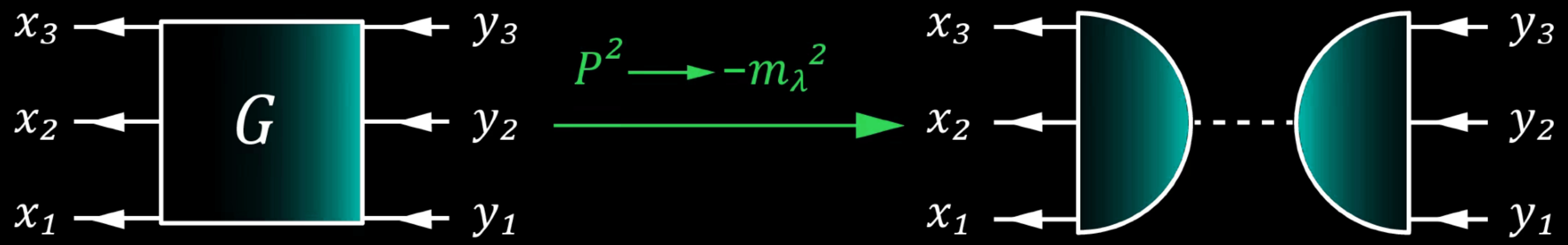
Meson



$$G^{(4)} = \langle \Omega \,|\, q(x_1)\bar{q}(x_2)q(y_1)\bar{q}(y_2) \,|\, \Omega \rangle$$



Baryon



$$G^{(6)} = \langle \Omega \,|\, q(x_1)q(x_2)q(x_3)q(y_1)q(y_2)q(y_3) \,|\, \Omega \rangle$$

Framework: Equation of Motion (DSE)

Classical Mechanics	Quantum Field Theory

Degrees of freedom

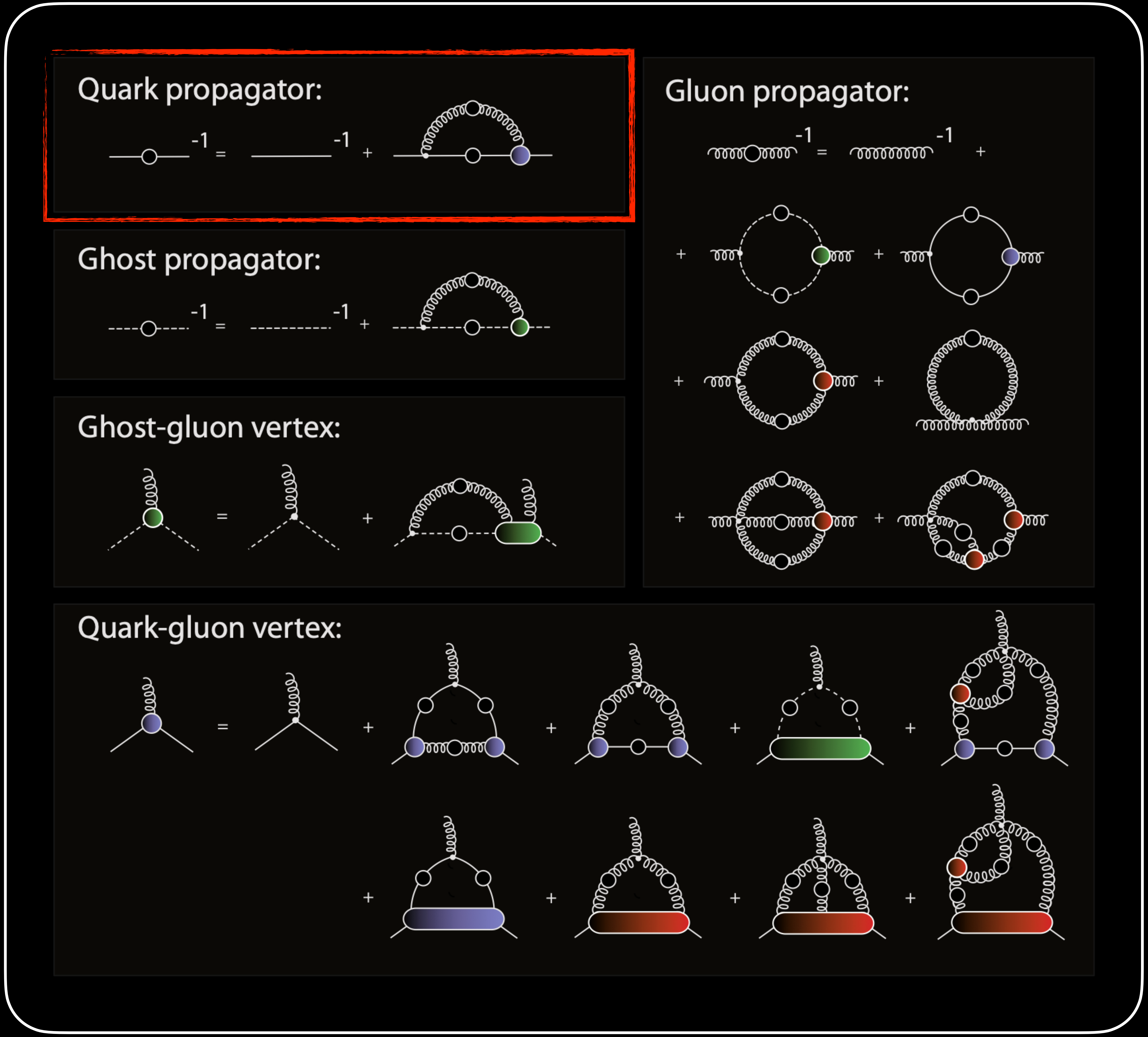
Generalized coord.	Fields on spacetime

Principle of Stationary Action

$$\frac{\delta S[q]}{\delta q} = 0 \qquad\qquad \left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Equations of Motion

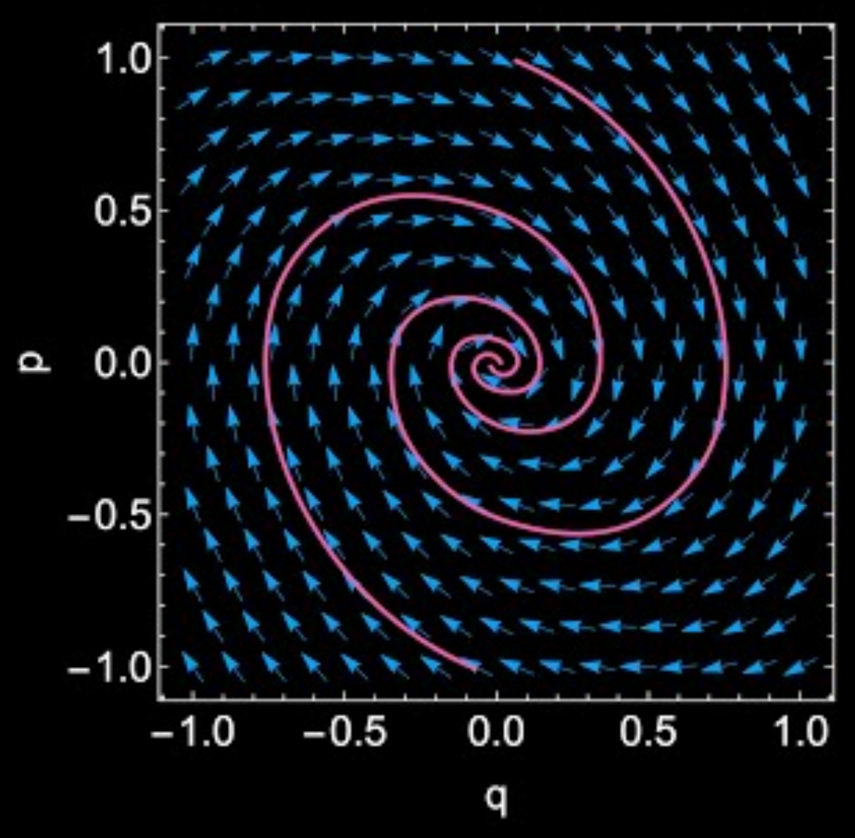
Euler-Lagrange Equation	Dyson-Schwinger Equations

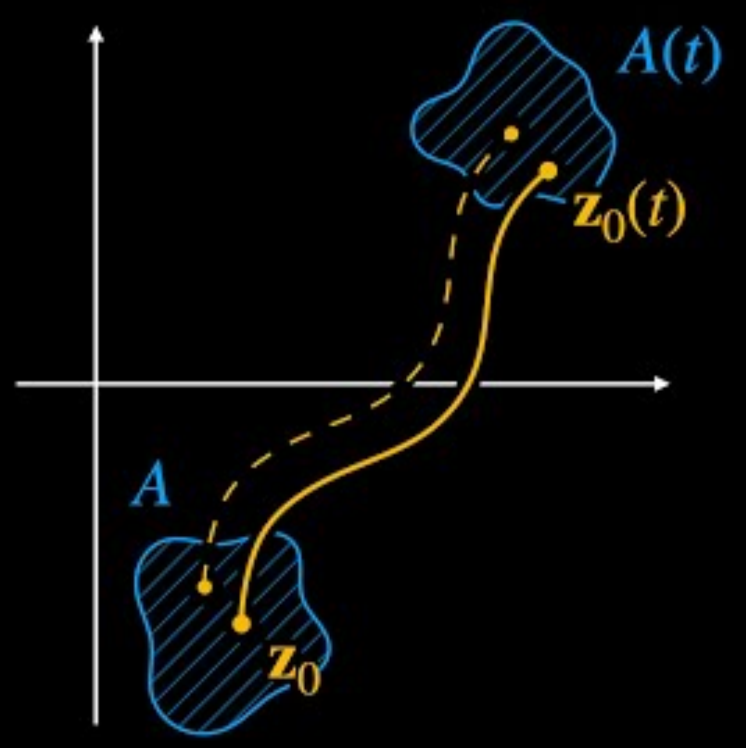


Framework: Equation of Motion (DSE)

Liouville equation

$$\frac{\partial \rho}{\partial t} + \{\rho, H\} = 0$$





BBGKY hierarchy

$$\frac{\partial \rho_n}{\partial t} = \{H_n, \rho_n\} + (N-n)\sum_{i=1}^{n}\int \frac{\partial \rho_{n+1}}{\partial \mathbf{p}_i}\cdot\frac{\partial \phi_{i\,n+1}}{\partial \mathbf{r}_i} dz_{n+1}$$

DSE-like

One-body function

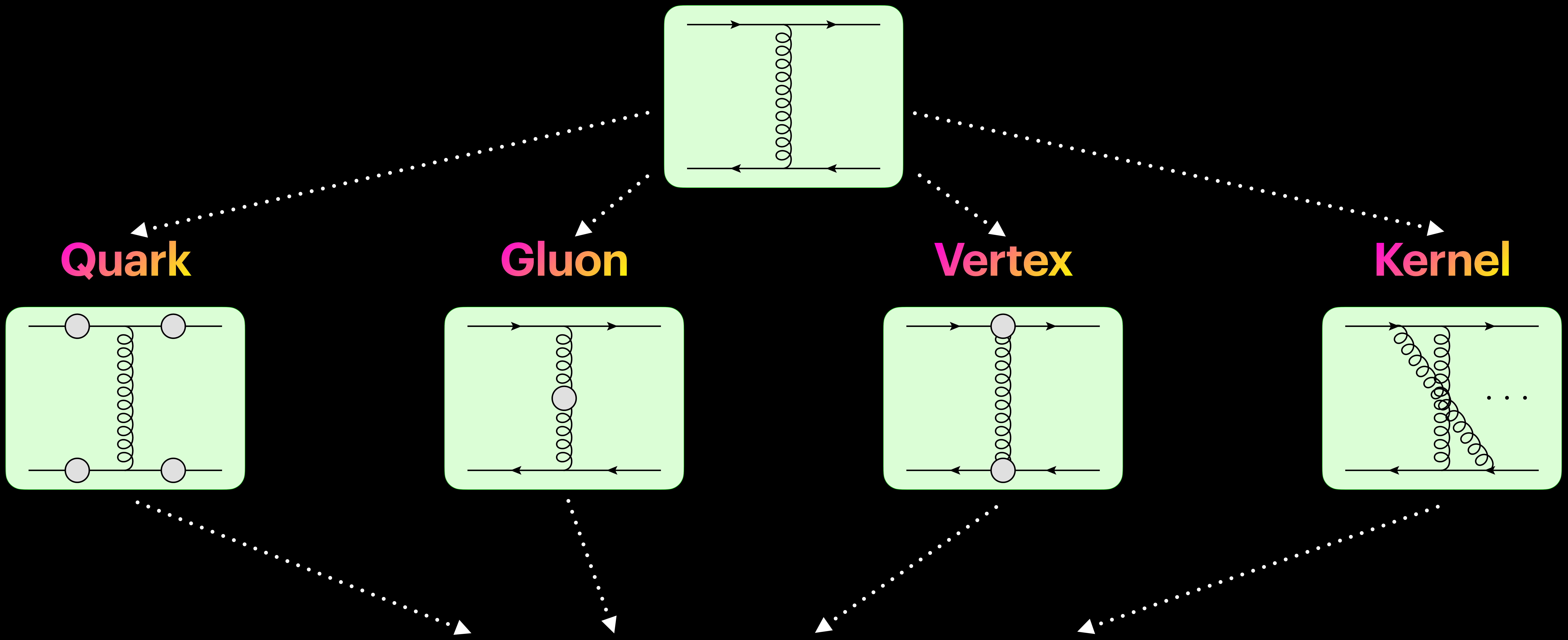
$$\rho_1(z_1) = \int_D \rho(Z,t)\, dz_2 \cdots dz_N$$

propagator-like

Boltzmann equation

$$\frac{\partial \rho_1}{\partial t} = \{H_1, \rho_1\} + \int \left[\rho_1(\mathbf{p}_1')\rho_1(\mathbf{p}_2') - \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2)\right] N\nu\, d\sigma\, d^3\mathbf{p}_2$$

Framework: Equation of Motion (DSE)



Ground $\longrightarrow$ $H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{vacuum}}$ $\longrightarrow$ Excited

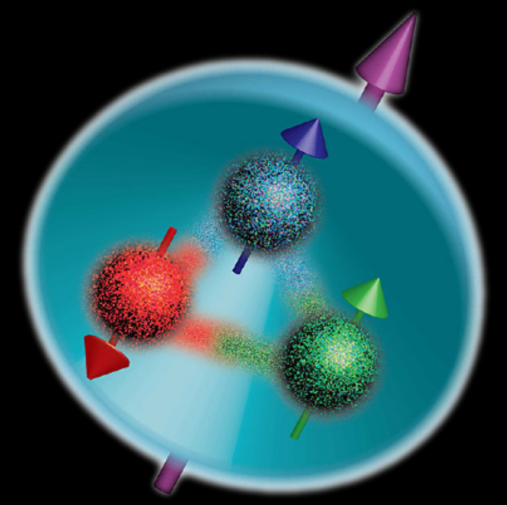
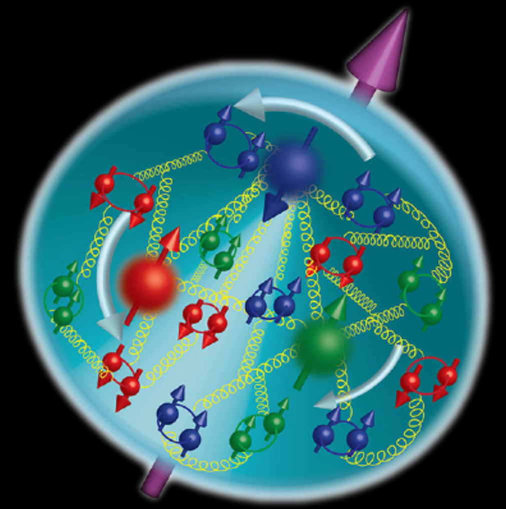
Solution

Solution: Quark properties

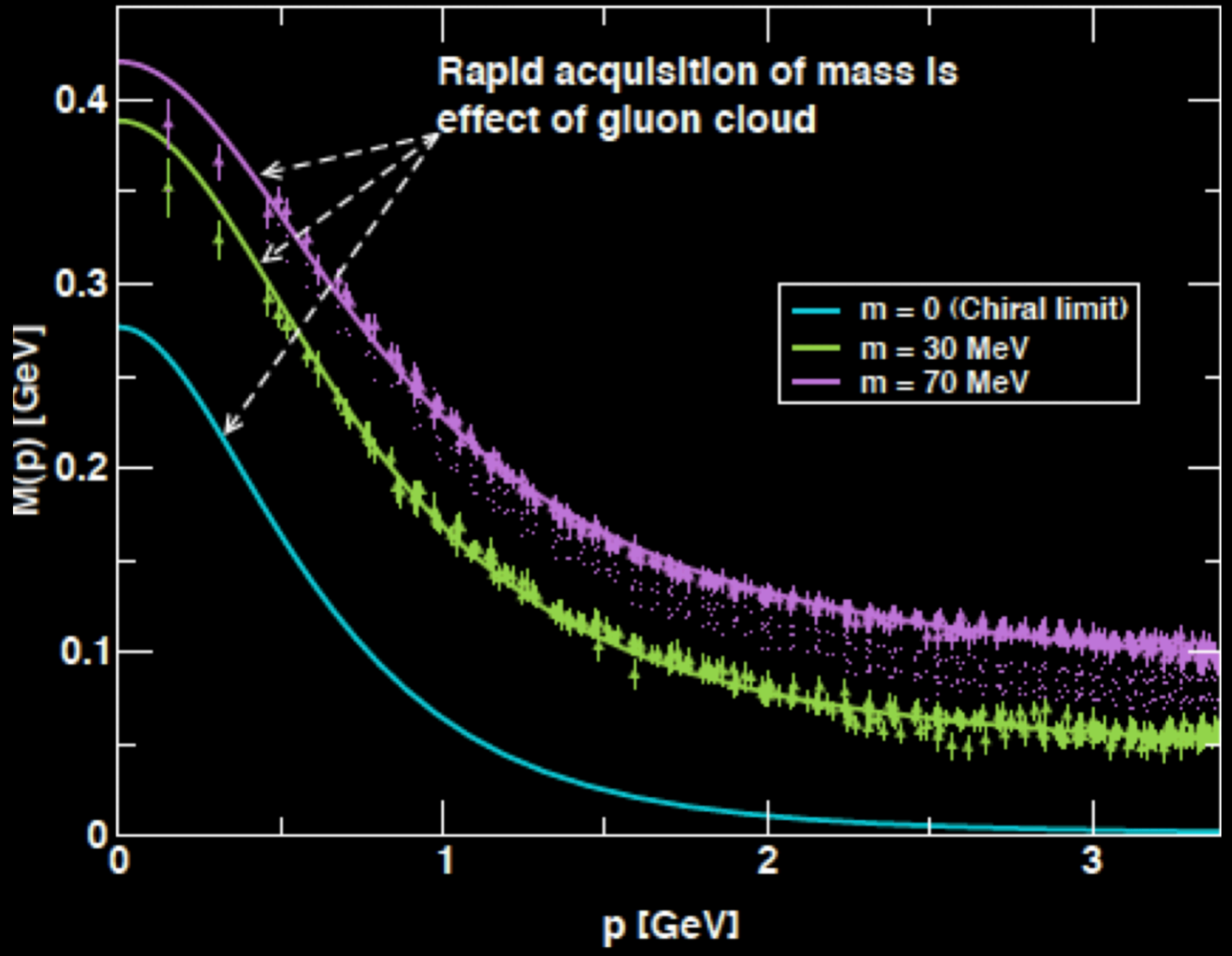
- Quark propagator：

$$S(p) = \frac{1}{i\gamma\cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

- Quasi-quark：

Low p  High p 

- Mass function：



QCD vacuum — highly dispersive medium

Solution: Gluon properties

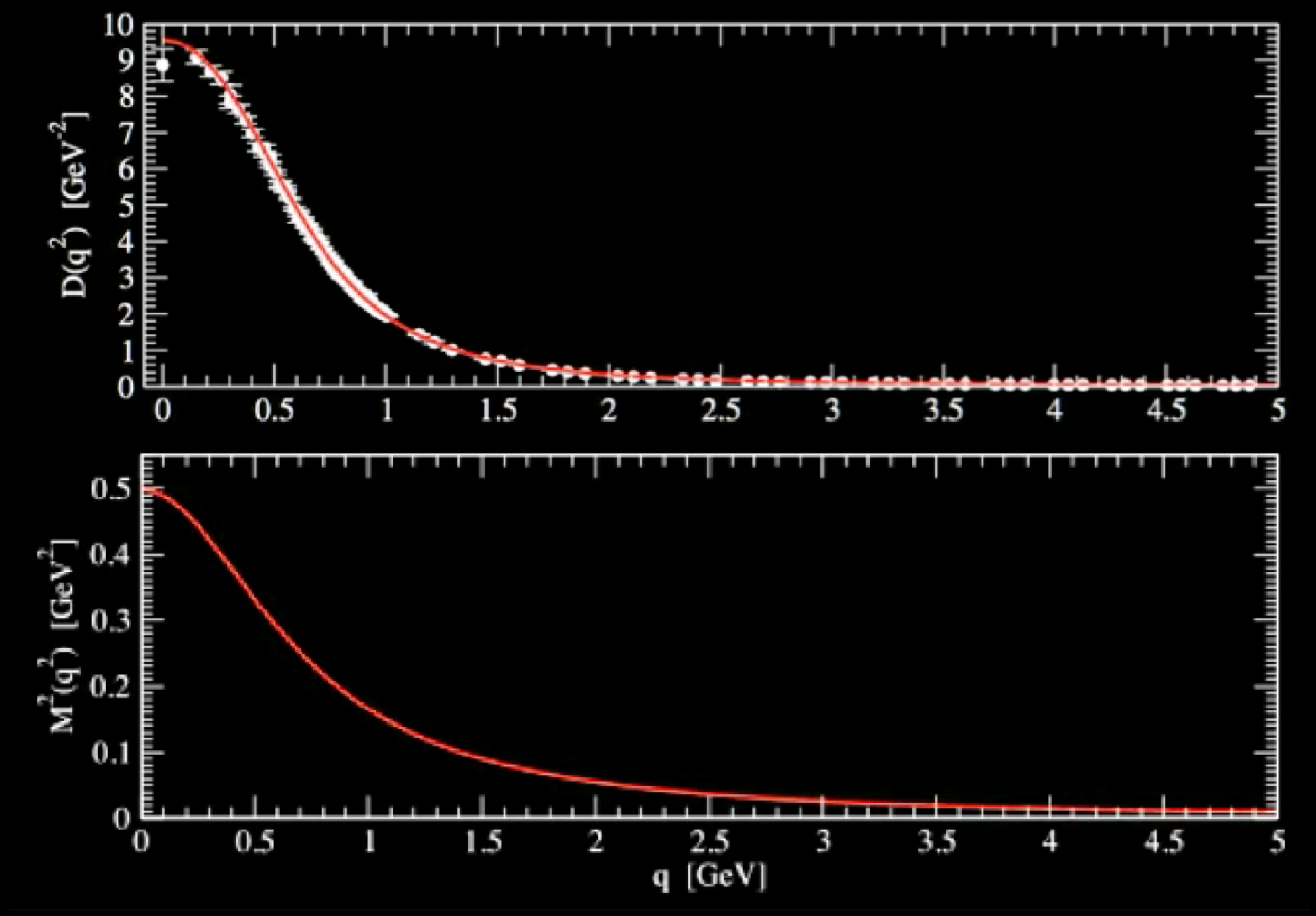
- **Gluon propagator:**

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_s(k^2)}{k^2 + m_g{}^2(k^2)}$$

$$m_g^2(k^2) = \frac{M_g^4}{M_g{}^2 + k^2}$$

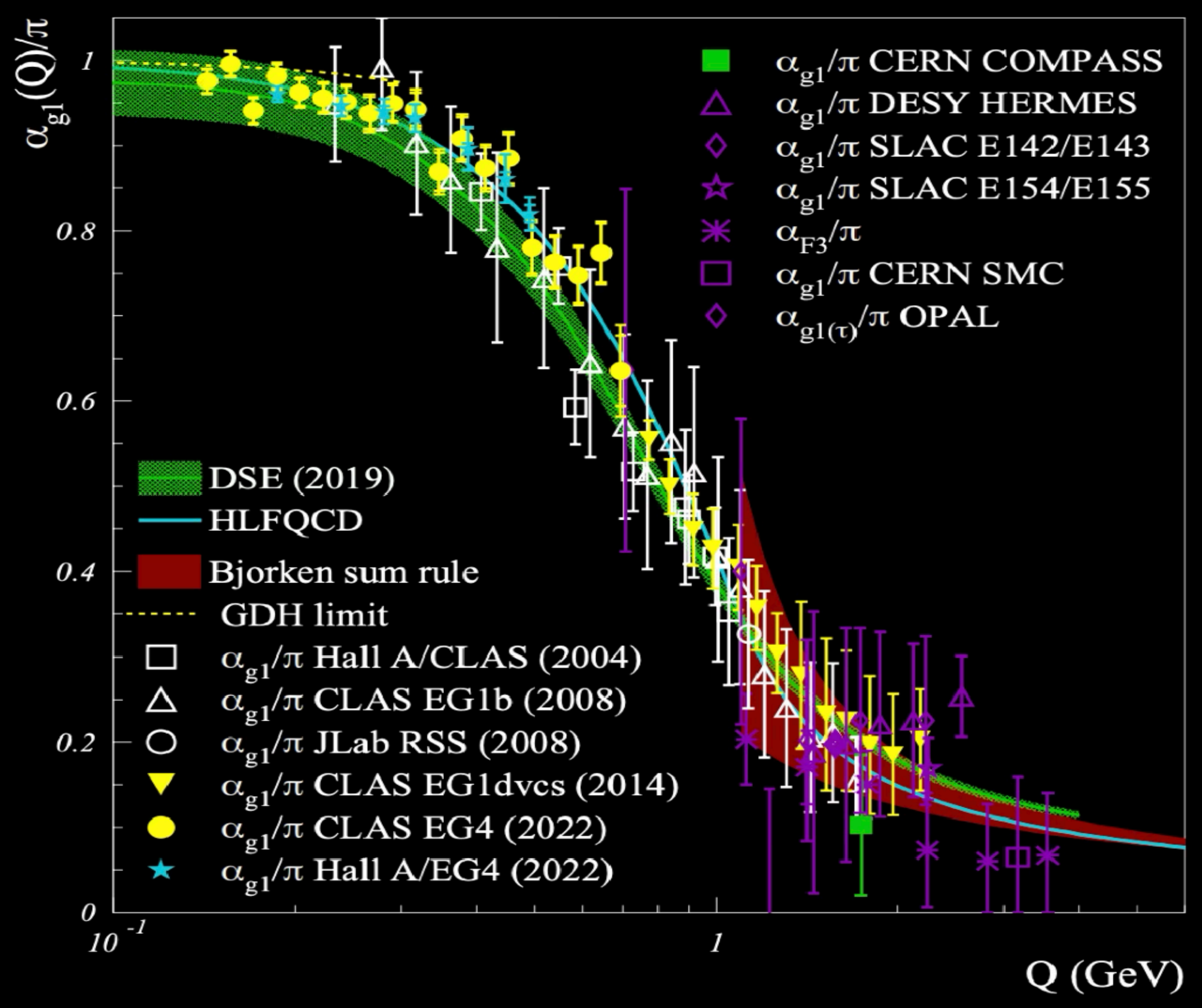
$$\alpha_s(0) \sim \pi$$

- **Quasi-gluon:**



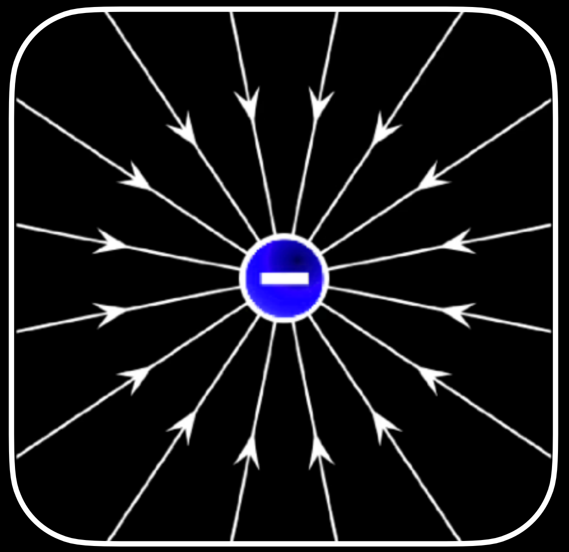
well-defined everywhere

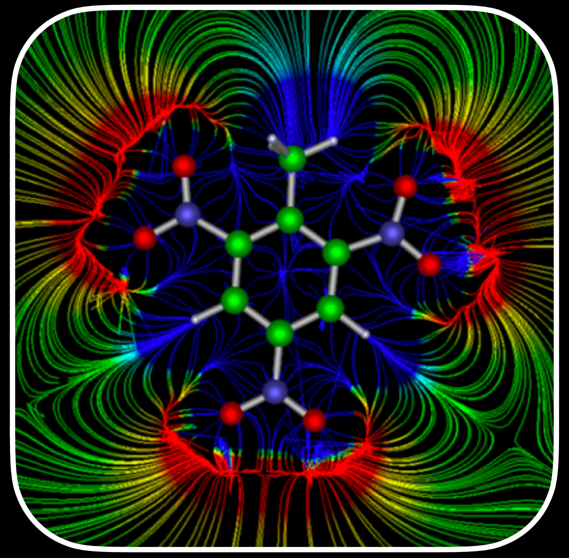
- **Running coupling:**



Solution: Vertex structures

- **Quark-gluon vertex：**





$$\Gamma^{\mu}(P', P) = \gamma^{\mu} F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^{\nu} F_2(Q^2)$$

Dirac

Pauli

$$S^{-1}(p) = i\gamma \cdot p\, A(p^2) + B(p^2)$$

- **DCSB feedback:**

Equation of Motion (DSEs)

$$S = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

$$\Gamma^{\mu} \sim \gamma^{\mu} \Sigma_A + i\sigma_{\mu\nu} Q^{\nu} \Delta_B$$

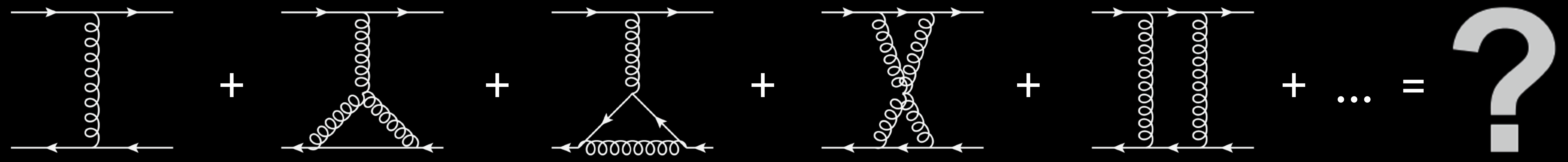
Ward identities (Symmetries)

- **Dressed vertex:**

$$\Gamma^{\mu} \sim \gamma^{\mu} + i\eta\sigma_{\mu\nu} Q^{\nu} \Delta_B$$

DCSB rendered

Solution: Kernel decompositions



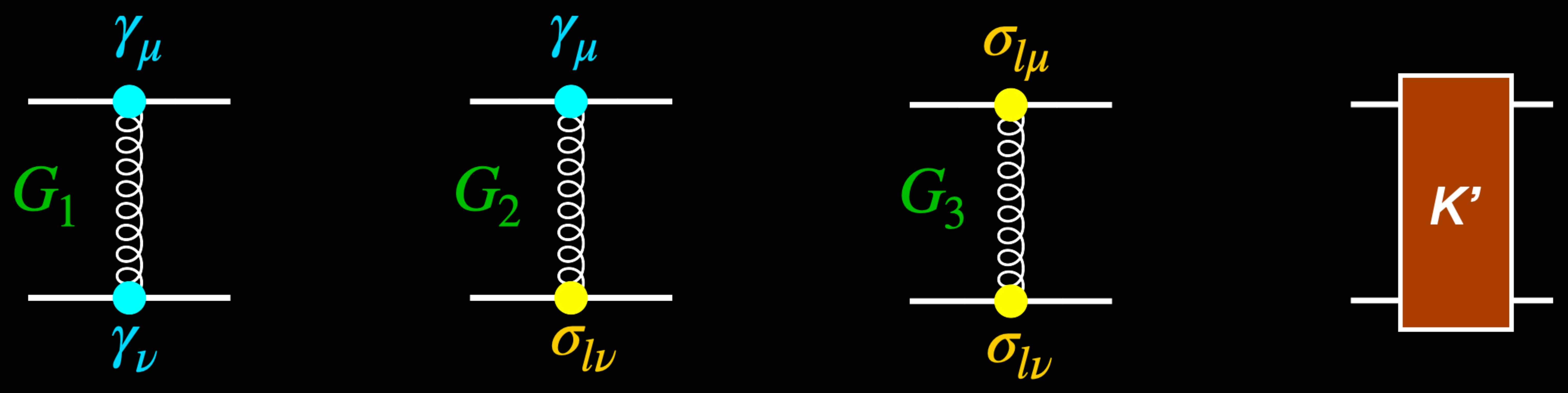
Poincaré symmetry

C-, P-, T-symmetry

Gauge symmetry

Chiral symmetry

pion meson twofold role

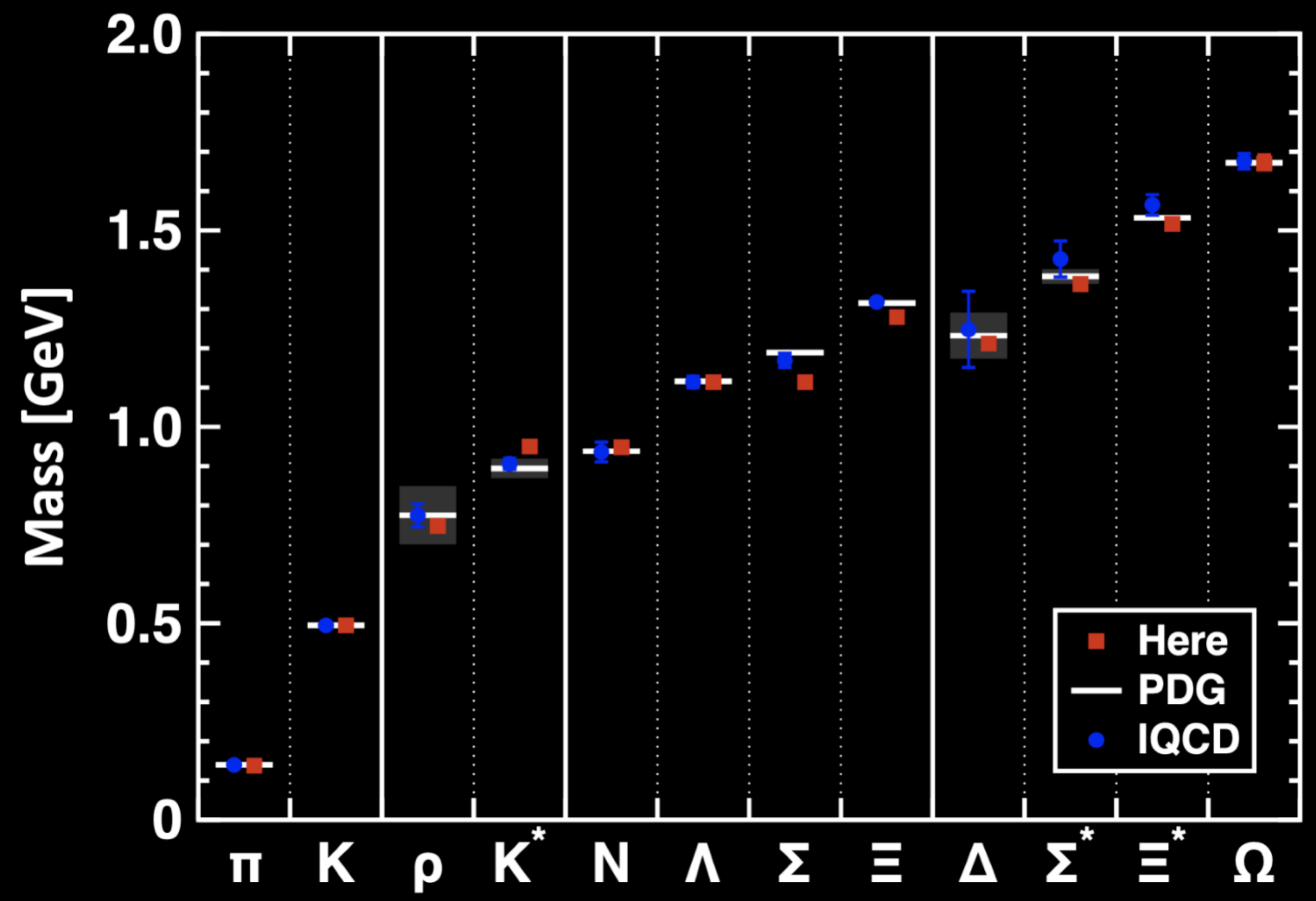


$G_{2,3} \sim DCSB$

Result

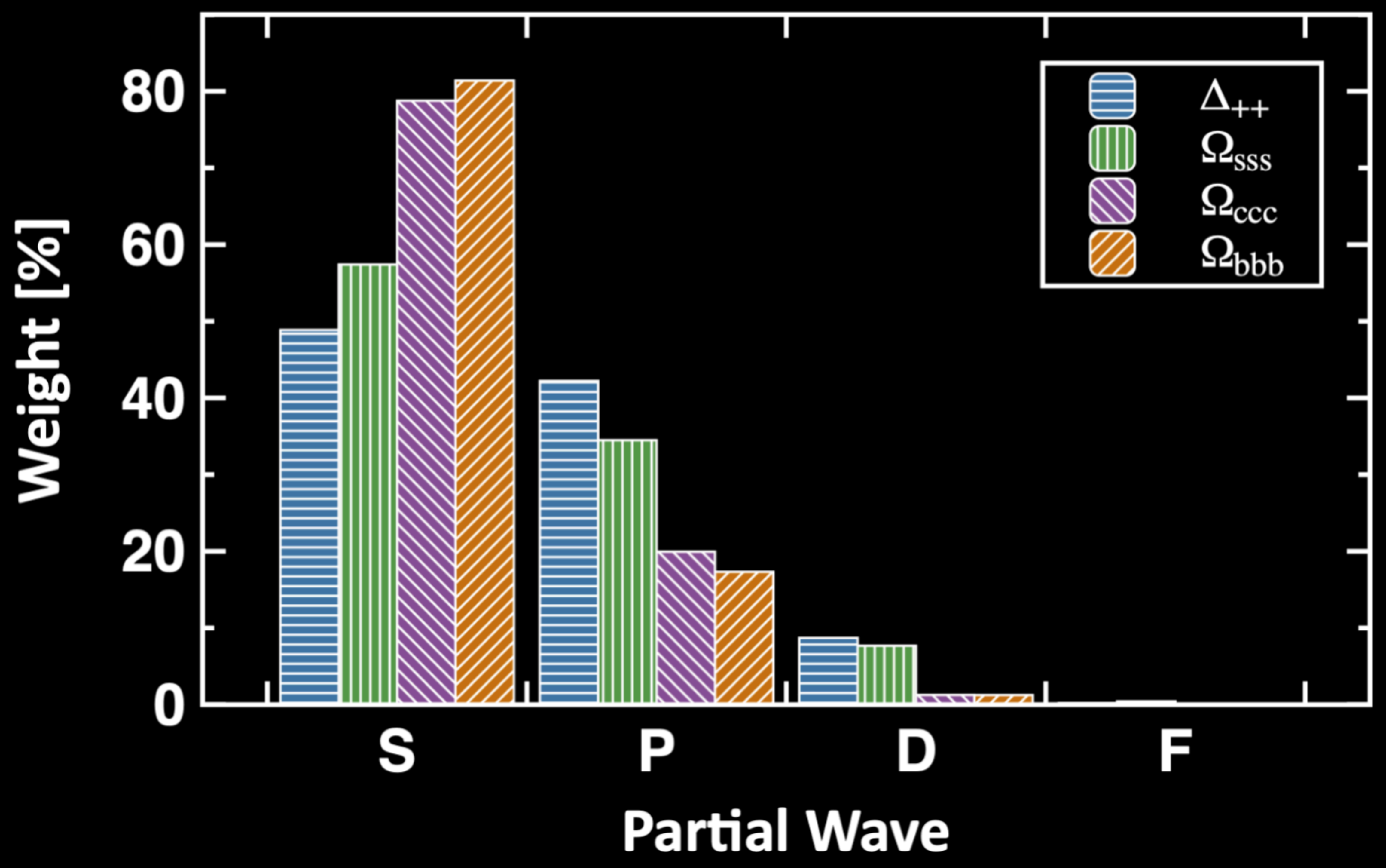
Result: Ground states

- **Hadron spectrum:**



Effective strength

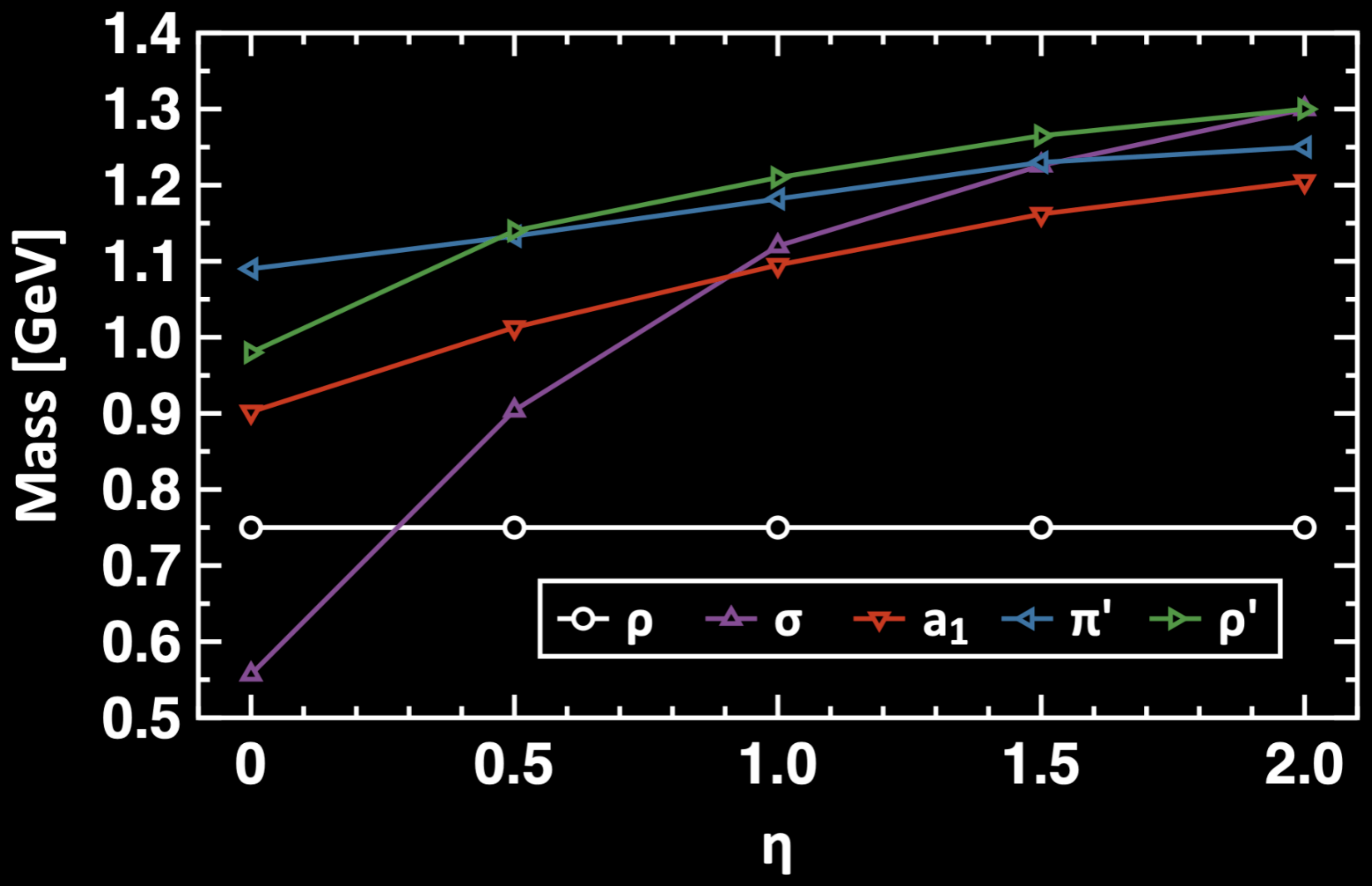
- **Hadron structure:**



NR potential models

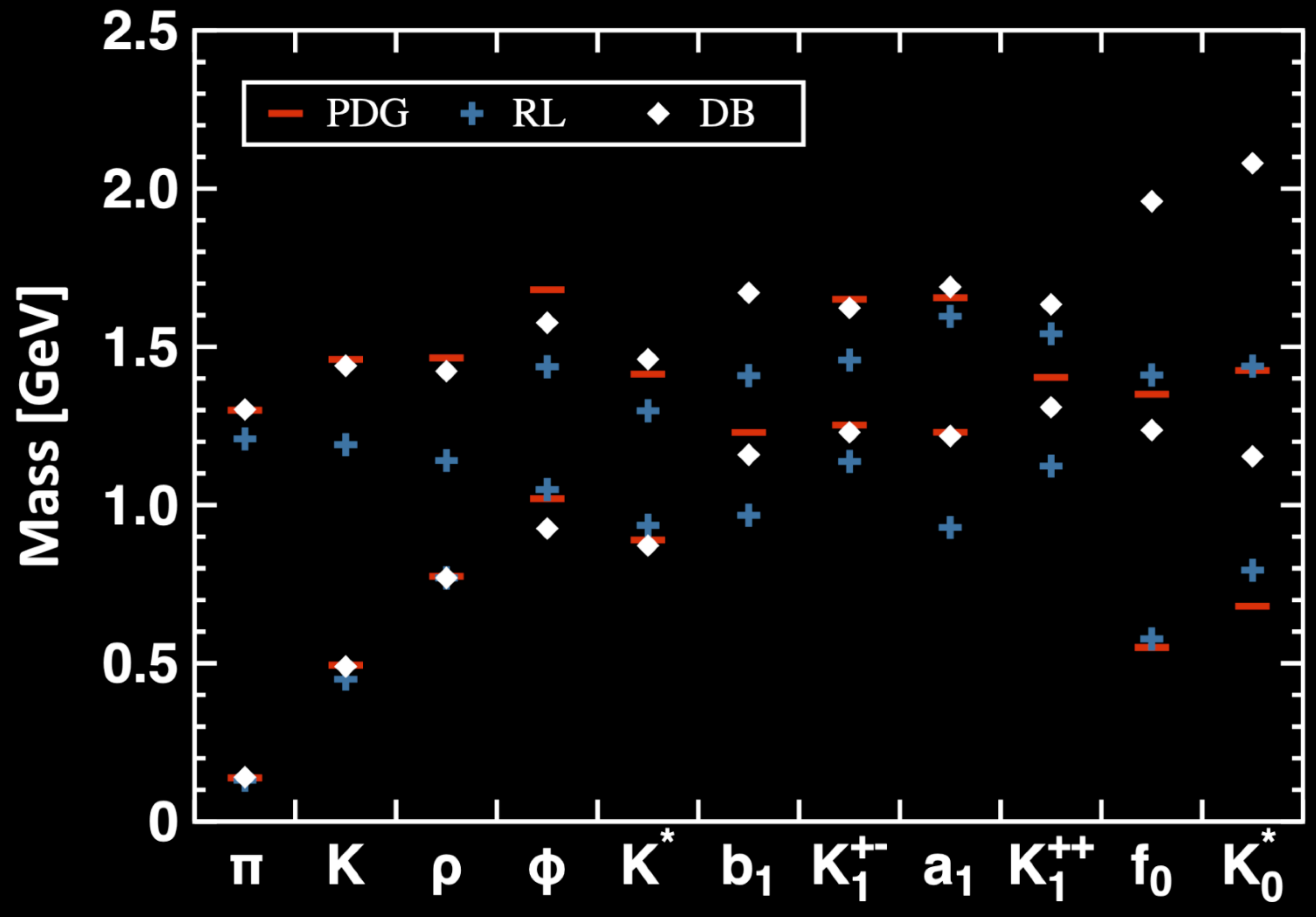
Result: Excited states

- **Impact of Pauli term (DCSB):**



spin-orbit repulsion

- **DCSB-enhanced spectrum:**



magnitude and ordering

Summary

Summary:

- The continuum framework of nonperturbative QCD, i.e., EoM (DSE), and its basics (quark, gluon, vertex, kernel) are introduced.
- Hadron properties are studied: **a**) ground states — full spectrum; **b**) excited states — partial waves, spin-orbit interaction, DCSB-rendered spectrum.
- Use the DSE to a wider range of applications in baryon problems of QCD: transition form factors, parton distribution functions, and etc.
- Iterating with high precision experiments on hadrons, from spectroscopy to structures, we may provide a faithful path to understand QCD.