Quantum kinetic theories for QED and QCD

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SL, PRD 2022

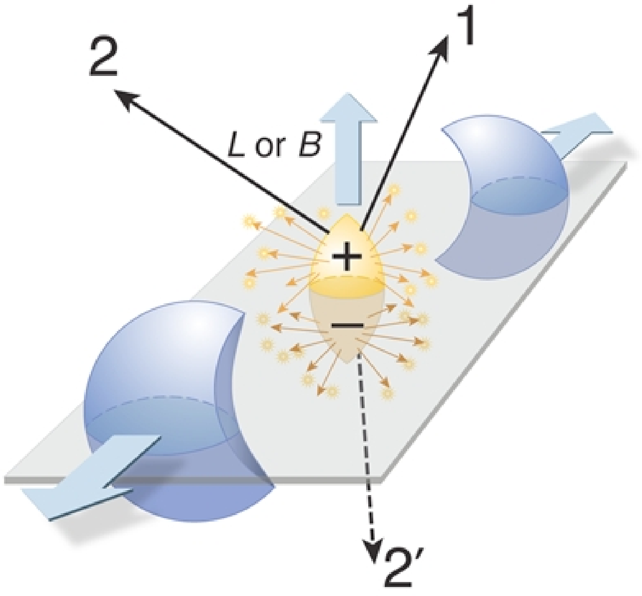
SL, Z.y. Wang, JHEP 2022

SL, to appear

Outline

- Motivation: Spin transport in heavy ion collisions
- Kinetic theory and relativistic hydrodynamics
- Quantum kinetic theories for QED & QCD
- Applications
- Summary and outlook

Spin phenomena in heavy ion collisions (HIC)



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

spin-orbital coupling via parton scattering

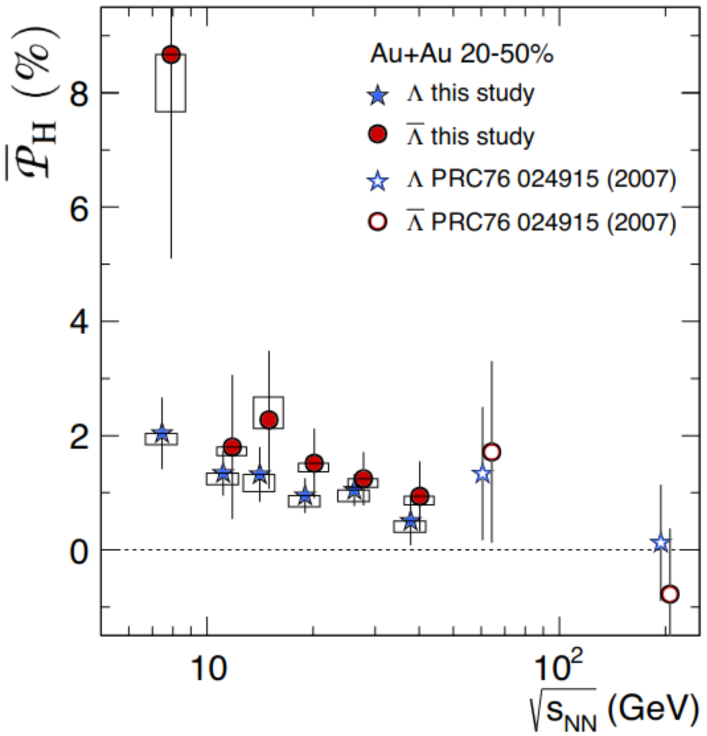
quark polarization P_q

recombination

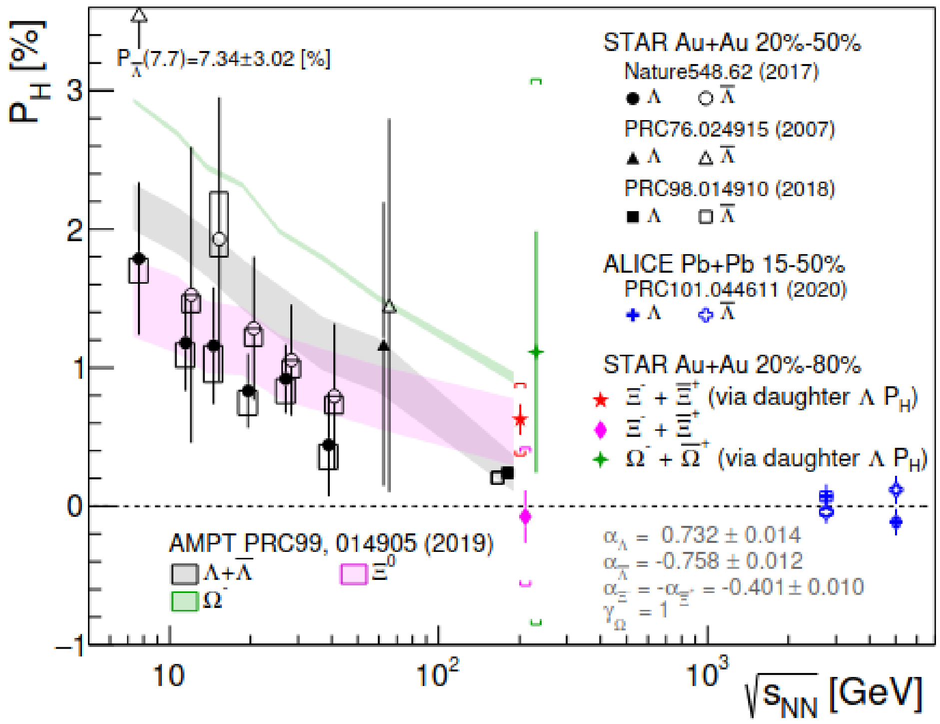
$qqq \longrightarrow B + X$ Baryon spin polarization

$q\bar{q} \longrightarrow V + X$ Vector meson spin alignment

Polarized baryons measured

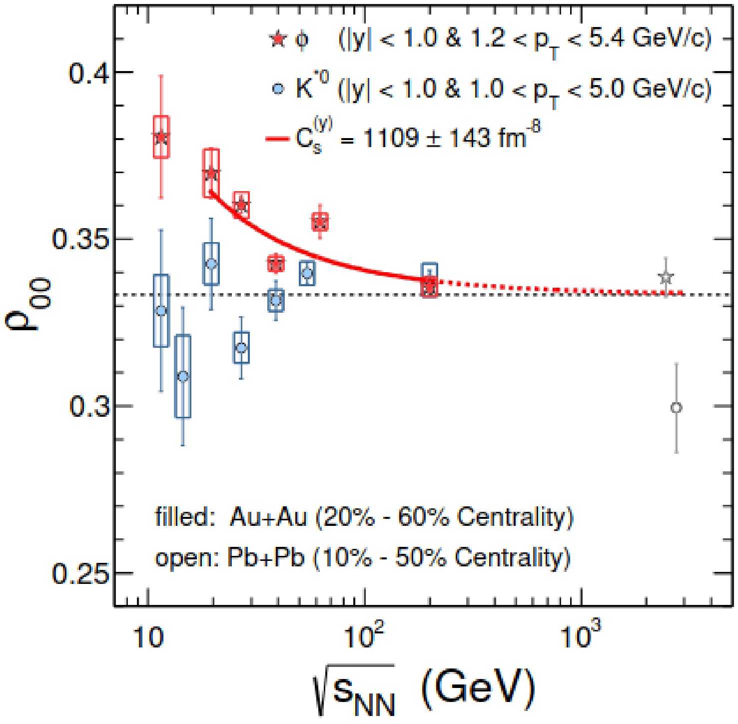


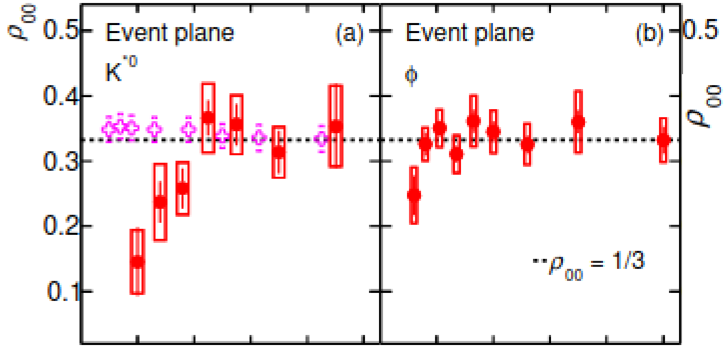
STAR, Nature 2017, PRL 2021

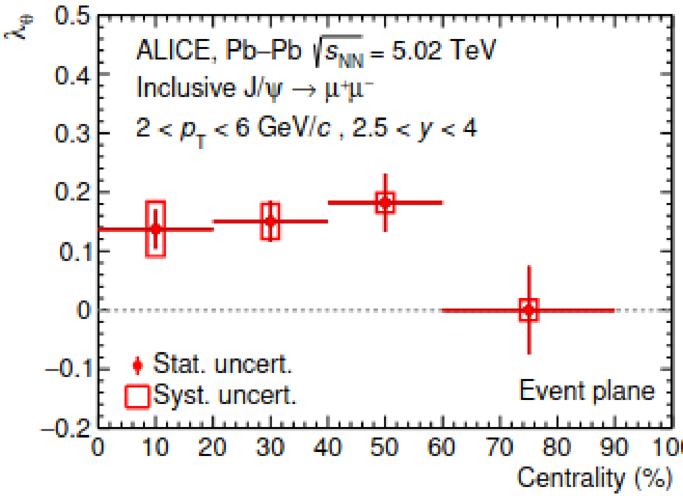


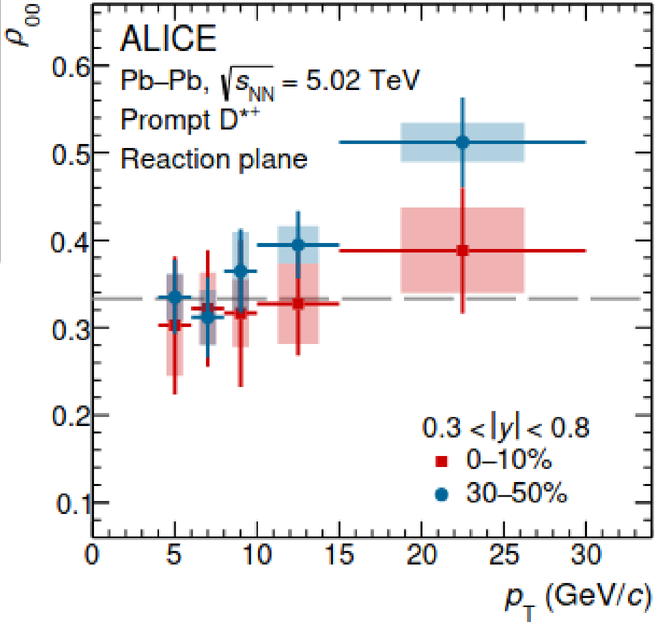
Λ, Ξ, Ω

Polarized vector meson measured









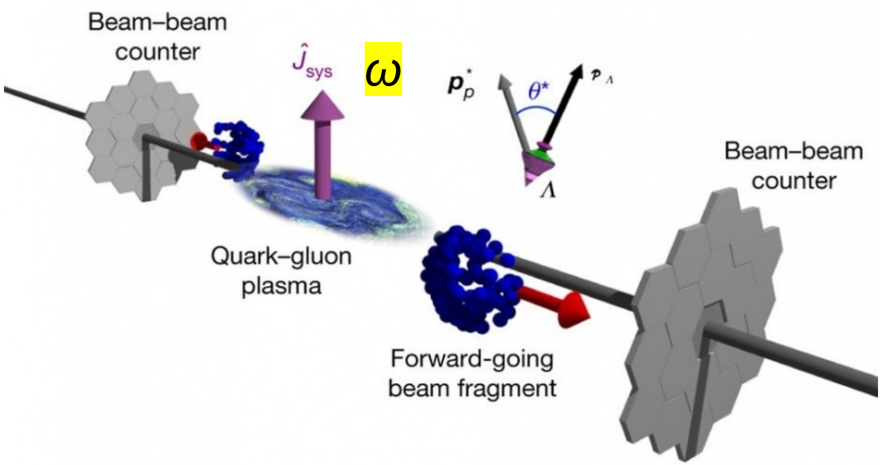
STAR, Nature 2023, ALICE PRL 2020, ALICE PRL 2023, JHEP 2025

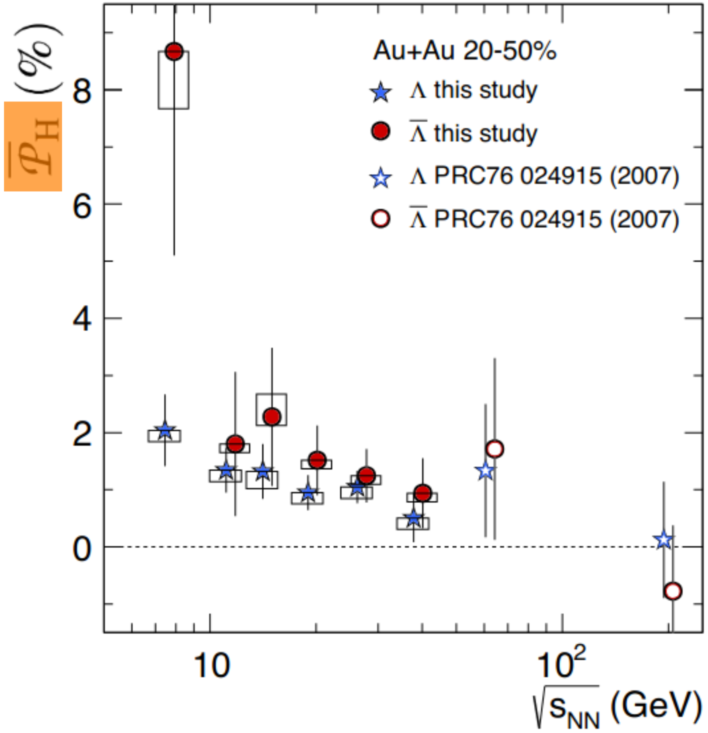
ϕ, K^{*0}, J/Ψ, D^{*+}

Polarization of Λ: spin response to hydrodynamic gradient

$\Lambda \to p + \pi^-$ weak decay

$$\frac{dN}{d\cos\theta^*} = \frac{1}{2}[1 + \alpha_H P_H \cos\theta^*]$$





STAR, Nature 2017

$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$

Becattini et al, PRC 2017

Spin couples to QGP vorticity

Strategy of spin polarization in HIC

- QGP: slow evolution — hydrodynamics
- Parton: fast polarization in hydrodynamic gradient — perturbative
- Hadron: polarization conversion from parton — non-perturbative

Need: a framework to describe how partons polarize in hydrodynamic QGP

Relativistic hydrodynamics as an EFT

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \cdots$$

$$T^{\mu\nu}_{(0)} = (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu}$$

$$T^{\mu\nu}_{(1)} = -\eta \sigma^{\mu\nu} - \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha$$

transport (Wilson) coefficients: how momentum density responds to hydrodynamic gradient

Wilson coefficients calculated with microscopic theory: kinetic theory

Kinetic theory for QCD

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{``}1\leftrightarrow 2\text{''}}[f],$$

Arnold, Moore, Yaffe, early 00s

$f_s(\boldsymbol{x}, \boldsymbol{p}, t)$: distributions of quarks and transverse gluons

$C_s^{2\leftrightarrow 2}[f]$: elastic collisions

$C_s^{\text{``}1\leftrightarrow 2\text{''}}[f]$: inelastic collisions

Spin-averaged, referred to as classical kinetic theory
widely used for calculation of non-spin transport coefficients

Kinetic theory vs off-equilibrium QFT

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f],$$

Simplifications of kinetic theory

- Track only on-shell quark/gluon
- Off-shell fields enslaved by on-shell DOF
- Efficient resummation of diagrams

$\eta \sim \sharp\, \frac{T^3}{g^4}$ dependence on coupling from $\delta f \sim \frac{\partial_x u}{g^4}$

direct field theory calculations exist only for QED

Gagnon, Jeon, PRD 2007

Routes toward a kinetic theory

1. One may begin with the full hierarchy of Schwinger-Dyson equations for (gauge-invariant) correlation functions in a weakly non-equilibrium state in the underlying quantum field theory. For weak coupling, one may systematically justify, and then in-

SL, PRD 2022

2. One may consider the diagrammatic expansion for the equilibrium correlator appearing in the Kubo relation (1.8) for some particular transport coefficient. After carefully

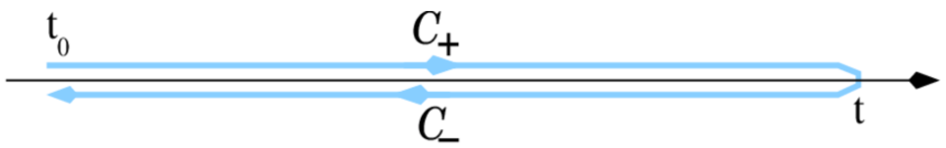
Gagnon, Jeon, PRD 2007

3. One may directly argue (by examining equilibrium finite temperature correlators) that, for sufficiently weak coupling, the underlying high temperature quantum field theory has well-defined quasi-particles, that these quasi-particles are weakly interacting with

Given the complexities of real-time, finite-temperature diagrammatic analysis in gauge theories (especially non-Abelian theories), we find the last approach to be the most physically transparent and compelling. But this is clearly a matter of taste.

Arnold, Moore, Yaffe, JHEP 2000

Kinetic theory: scalar particle



Chou, Su, Hao, Yu, Phys. Rept. 1985
Blaizot, Iancu, Phys. Rept. 2002

$$G^>(x,y) \equiv \mathrm{Tr}\left\{\mathcal{D}\,\phi(x)\phi(y)\right\} \quad G^<(x,y) \equiv \mathrm{Tr}\left\{\mathcal{D}\,\phi(y)\phi(x)\right\}$$

$$G^<(k,X) \equiv \int \mathrm{d}^4 s\, \mathrm{e}^{ik\cdot s}\, G^<\left(X+\frac{s}{2}, X-\frac{s}{2}\right) = \rho(\mathrm{X,k})\mathrm{f}(\mathrm{X,k})$$

X: coarse-grained hydro coordinate
k: particle momemtum

Kadanoff-Baym equation

$$\left(\partial_x^2 + m^2 + \Sigma^\delta(x)\right)G^<(x,y) = -\int_{-\infty}^{\infty} \mathrm{d}^4 z \left[\Sigma_R(x,z)\, G^<(z,y) + \Sigma^<(x,z)\, G_A(z,y)\right]$$

gradient expasion

$$\longrightarrow \quad \left(\partial_t + \hat{k}\cdot\nabla_x\right) f(X,\, k) = -C[f]$$

Kinetic theory: QED

$$\frac{i}{2}\partial\!\!\!/ S^< + (P\!\!\!/ - m)S^< = \frac{i}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right) - \frac{1}{4}\left(\{\Sigma^>, S^<\}_{\mathrm{PB}} - \{\Sigma^<, S^>\}_{\mathrm{PB}}\right)$$

$$S^< = S^{<(0)} + S^{<(1)} + \cdots$$

$$S^{<(0)}(X,P) = -2\pi\epsilon(P\cdot u)\delta(P^2 - m^2)(P\!\!\!/ + m)f_e(X,P)$$

$$\left[\cdots\right]D^<_{\nu\rho} = \frac{i}{2}\left(\Pi^{\mu\nu>}D^<_{\nu\rho} - \Pi^{\mu\nu<}D^>_{\nu\rho}\right) + \frac{1}{4}\{\Pi^{\mu\nu>}, D^<_{\nu\rho}\}_{\mathrm{PB}} - \frac{1}{4}\{\Pi^{\mu\nu<}, D^>_{\nu\rho}\}_{\mathrm{PB}},$$

$$D^<_{\mu\nu} = D^{<(0)}_{\mu\nu} + D^{<(1)}_{\mu\nu} + \cdots$$

$$D^{<(0)}_{\mu\nu}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2)P^T_{\mu\nu}f_\gamma(X,P)$$

u: fluid velocity

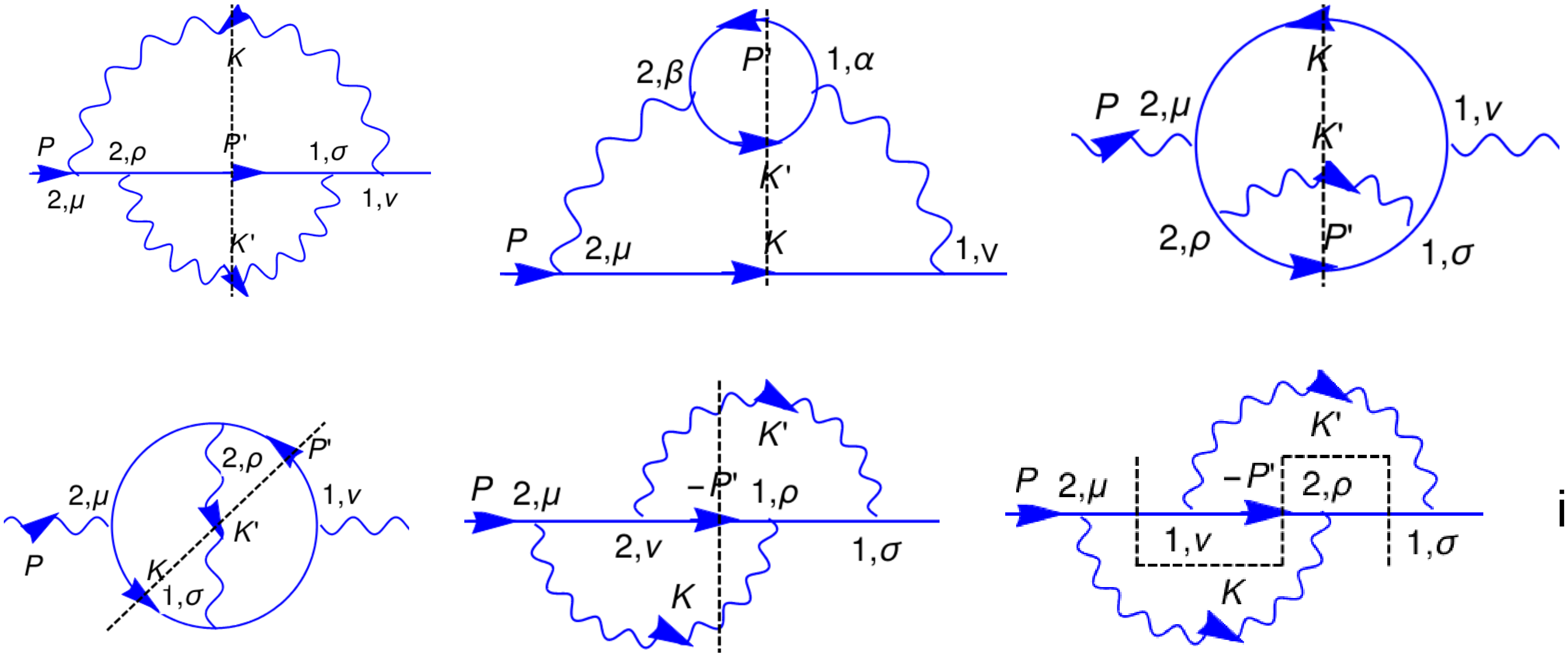
collision term

SL, PRD 2022

0th order: spin-averaged

1st order: spin polarization

Collision term: 2 ⟷ 2



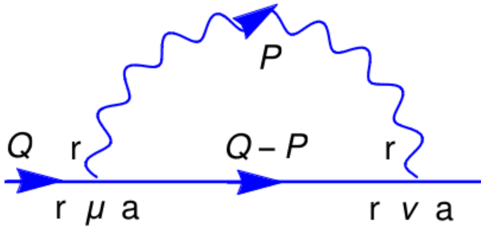
|amplitude|^2

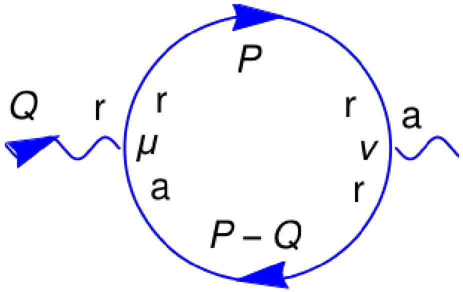
interference

self-energies $O(e^4)$

SL, PRD 2022

One-loop self-energies





- Correction to dispersions of on-shell DOF

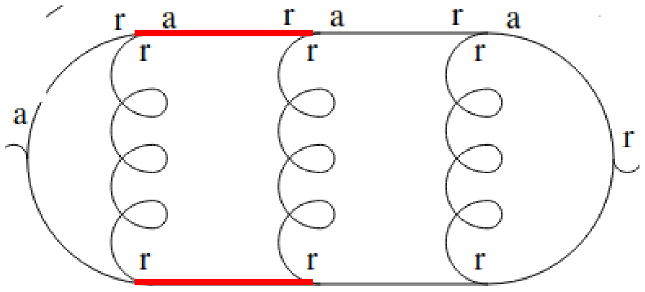
  thermal mass ~ O(eT), damping rate ~ O(e^2T)

  subleading for $2 \leftrightarrow 2$ collisions, crucial for $1 \leftrightarrow 2$ collisions

- Screening of long-range interaction by giving masses to exchanged photon/electron

SL, PRD 2022

Collision term: 1 ⟷ 2



$$\int \frac{dp^0}{2\pi}\, G_{ar}(K+P)\, G_{ra}(P)$$

$$\simeq \int \frac{dp^0}{2\pi} \frac{1}{\left[(p^0 + \frac{i}{2}\,\Gamma_{\mathbf{p}})^2 - E^2_{\mathbf{p}}\right]\left[(p^0 + k^0 - \frac{i}{2}\,\Gamma_{\mathbf{p+k}})^2 - E^2_{\mathbf{p+k}}\right]}$$

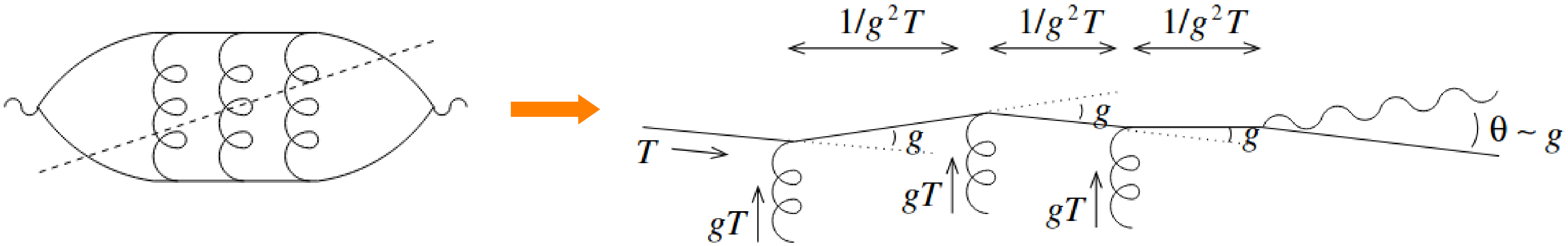
$$\simeq \frac{1}{4p_{\|}\,(p_{\|}+k)\,\left[\frac{1}{2}(\Gamma_{\mathbf{p}}+\Gamma_{\mathbf{p+k}}) + i\,\delta E\right]}$$

collinear kinematics: phase space suppression compensated by pinching mechanism, arbitrary gluon exchange at same order

self-energy $O(e^4)$

Aurenche, Gelis, Kobes, Petitgirard, PRD 1996
Arnold, Moore, Yaffe, JHEP 2001, 2002

Collision term: 1⟷2



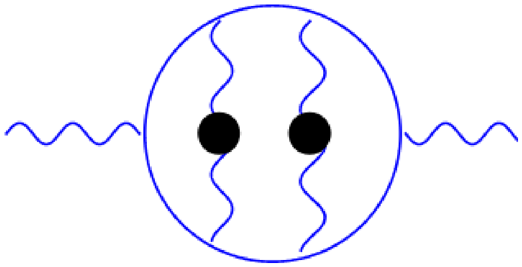
medium-induced collinear radiation:

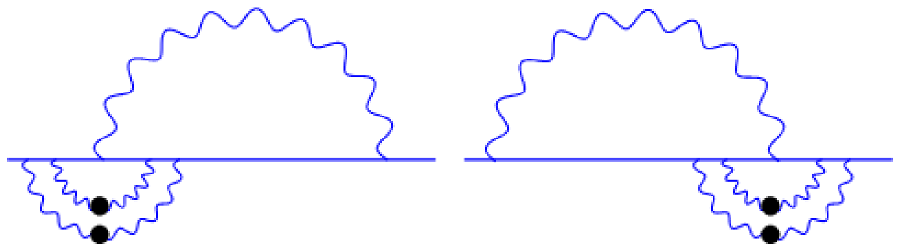
- mutiple soft kicks from medium particles, hard collinear radiation
- LPM effect (interference between kicks)

$$2\mathbf{p}_\perp = i\,\delta E\,\mathbf{f}(\mathbf{p}_\perp;p_\parallel,\mathbf{k}) + g^2 C_R \int_Q 2\pi\,\delta(q^0-q_\parallel)\,\left\langle\!\!\left\langle A^+(Q)\,[A^+(Q)]^* \right\rangle\!\!\right\rangle \times \left[\mathbf{f}(\mathbf{p}_\perp;p_\parallel,\mathbf{k}) - \mathbf{f}(\mathbf{p}_\perp-\mathbf{q}_\perp;p_\parallel,\mathbf{k})\right]$$

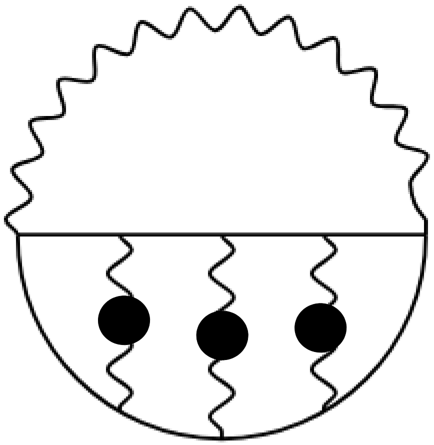
Arnold, Moore, Yaffe,
JHEP 2001, 2002

Collision term: 1 ⟷ 2





e/γ
collision
terms

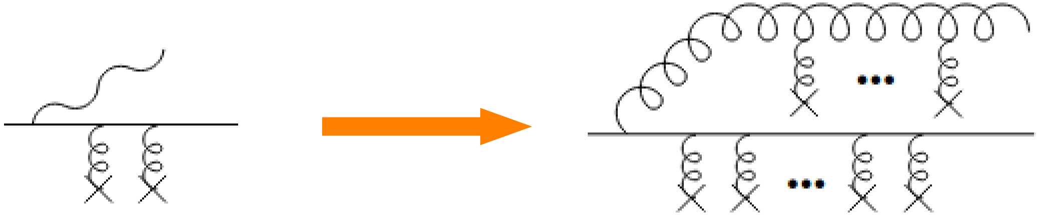


~ spectral density of “bound state”

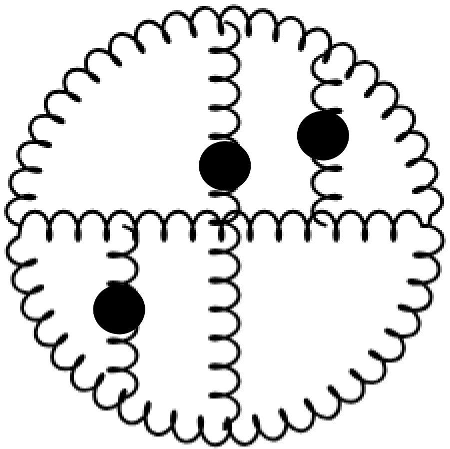
originated from electron
damping in medium

SL, PRD 2022
SL, to appear

From QED to QCD



gluon
collision
term



~ spectral density of “bound state”

originated from gluon
damping in medium

SL, to appear

Quantum kinetic theory

$$\frac{i}{2}\partial\!\!\!/ S^{<(0)} + (P\!\!\!/ - m)S^{<(1)} = \frac{i}{2}\left(\Sigma^{>(0)}S^{<(0)} - \Sigma^{<(0)}S^{>(0)}\right)$$

0^{th} order classical kinetic theory

1^{st} order $S^{<(1)}(P) = \gamma^5\gamma_\mu\mathcal{A}^\mu + \frac{i[\gamma_\mu,\gamma_\nu]}{4}\mathcal{S}^{\mu\nu},$

$$\mathcal{A}^\mu = -2\pi\hbar\epsilon(P\cdot u)\frac{\epsilon^{\mu\nu\rho\sigma}P_\rho u_\sigma \mathcal{D}_\nu f_e}{2(P\cdot u+m)}\delta(P^2-m^2)$$

electron spin polarization

$$D^{<(1)}_{\lambda\rho} = -2\pi\epsilon(P\cdot u)\delta(P^2)\frac{iP_{\lambda\alpha}P^{\nu\beta}P^\alpha\partial_\beta P^T_{\nu\rho}f_\gamma(P)}{2(-P^2+(P\cdot u)^2)} + 2\pi\epsilon(P\cdot u)\delta(P^2)P^T_{\nu\rho}\times$$
$$\frac{iu_\lambda u_\mu\left(\Pi^{\mu\nu>(0)}f_\gamma(P) - \Pi^{\mu\nu<(0)}(1+f_\gamma(P))\right)}{2(-P^2+(P\cdot u)^2)} - (\lambda\leftrightarrow\rho).$$

photon spin polarization

SL, PRD 2022

Application 1: shear induced polarization

$$\mathcal{A}^\mu = 2\pi\delta(P^2 - m^2)\left(a^\mu f_A + S^{\mu\nu}_{n,m}\mathcal{D}_\nu f\right)$$

$$\mathcal{D}_\nu = \partial_\nu - \boxed{\Sigma^>_\nu - \Sigma^<_\nu \frac{1-f_e}{f_e}}$$

collisional contribution

Naively, collision term $\Sigma \sim O(g^4)$

But compensated by $\delta f \sim \frac{\partial_x u}{g^4}$

steady state deviation from equilibrium

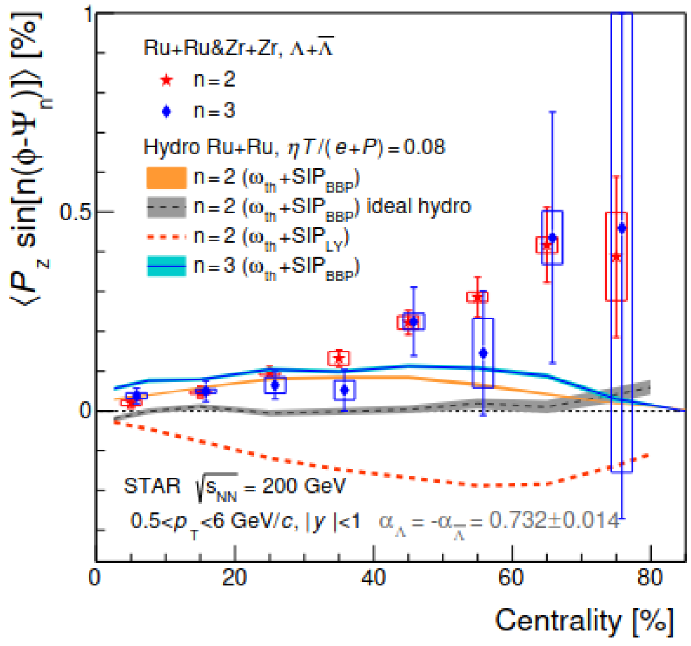
O(1) collisional contribution to shear induced polarization

reason for $\eta \sim \# \frac{T^3}{g^4}$

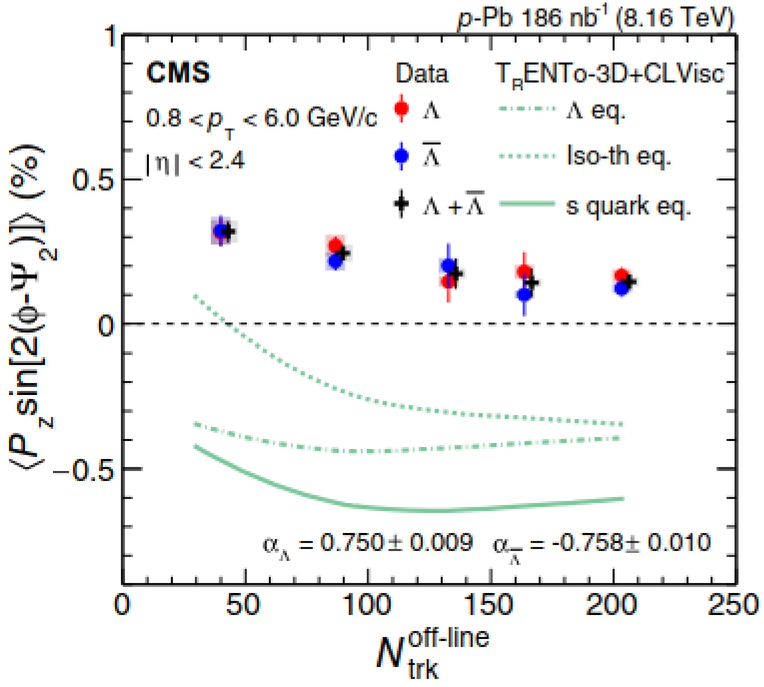
$$\frac{\mathcal{A}^a_\mu + \mathcal{A}^\partial_\mu + \boxed{\mathcal{A}^\Sigma_\mu}}{\mathcal{A}^\partial_\mu}\Big|_{p\gg m} = \frac{3}{2} + \frac{21T}{2p}$$

SL, Z.y. Wang, JHEP 2022, PRD 2025

Application 1: shear induced polarization



STAR, PRL 2023

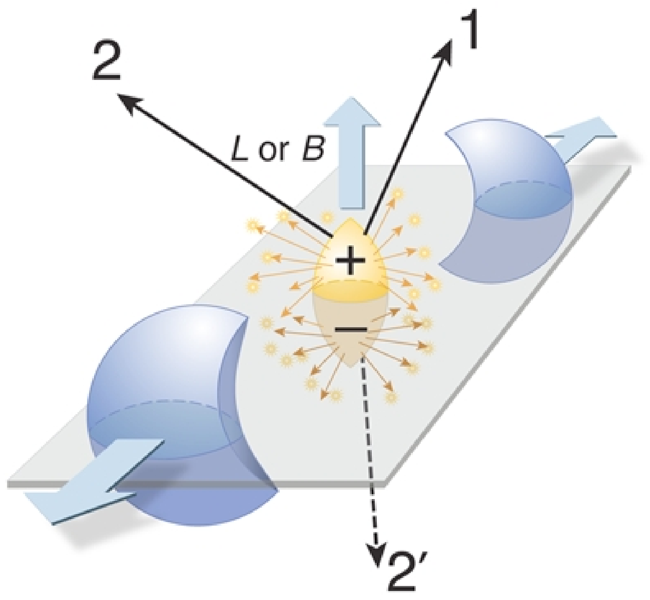


CMS, PRL 2025

Hydro: Yi-Wu-Zhu-Pu-Qin, PRCs 2025

enhanced shear induced polarization
help explain data?

Application 2: spin-orbit conversion



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

How OAM converts to particle spin polarization?

Nonlocal collision

Weickgenannt, Speranza, Sheng, Wang, Rischke, PRL 2021

$$f_s(\boldsymbol{x}, \boldsymbol{p}, t)$$

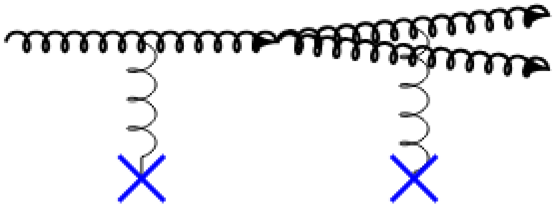
x: coarse-grained hydro coordinate
p: particle momemtum

However, doesn’t fit to realistic quantum kinetic theory

All collisions within hydro element: local

Application 2: spin-orbit conversion

1 $\longleftrightarrow$ 2
local collision



SL, to appear

Spin doesn’t conserve in gluon splitting
Accompanied by nonvanishing OAM in final state

Spin-orbit conversion rate ~ 1 $\longleftrightarrow$ 2 collision rate

Summary

- Derived quantum kinetic theory as a microscopic theory for spin transports
- Collisional contribution to shear induced polarization
- Spin-orbit AM conversion from 1 $\longleftrightarrow$ 2 collisions

Outlook

- Phenomenology implications for gluon polarization
- Quantum kinetic theory for mesons like J/Ψ etc

八秩仁者寿，桃李繁盛；

期颐仰高风，学脉传薪。

敬祝李重生老师华诞快乐！

谢谢！

Power countings

	QCD	QED
particle energy	T	T
thermal mass / electric scale	gT	eT
magnetic scale	g^2T	
inverse mean free time	g^4T	e^4T
hydro gradient	∂_x	∂_x