On-shell Matrix Elements of EMT-trace and Heavy Quark Masses

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Take-home Messages of the Talk

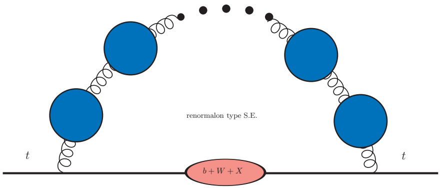
- In perturbative Quantum Gauge Theories, the relation

$$\boxed{\langle \vec{\mathbf{p}} | -\frac{\epsilon}{2} \left[F^a_{\rho\sigma} F^{a\,\rho\sigma} \right]_B + \sum \left[m_f \bar{\psi}_f \psi_f \right]_B | \vec{\mathbf{p}} \rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\mathrm{os}}\, u(\vec{\mathbf{p}})}$$

  is **proved** to hold as an *identity* for all elementary particles to any loops and to all orders in ϵ

  (without any ref. to prelaid *operator* renormalization conditions).

- The contribution from EMT trace anomaly is not only indispensable, but interestingly captures the entire leading IR-renormalon discovered in *pole* masses of **heavy quarks**.



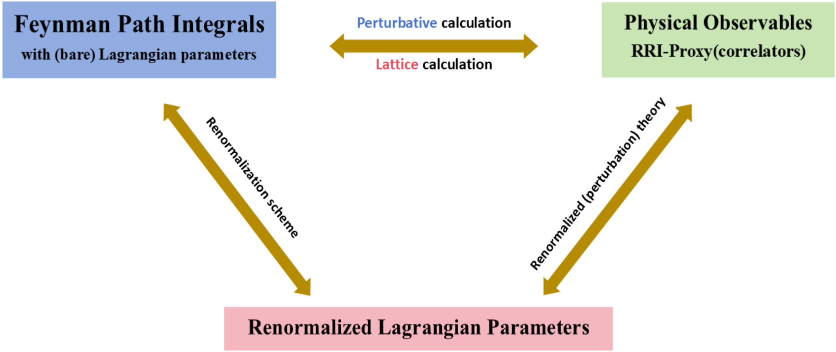
- A novel trace-anomaly-subtracted σ-mass definition for heavy quarks is introduced

  (which is scheme/scale and gauge invariant and free from the leading IR-renorm. ambiguity)

$$\boxed{\frac{m_\sigma}{m_{\mathrm{os}}} = \frac{1 + 2\beta \dfrac{\partial \ln \left(C_{\overline{m}} \left(\alpha_s , \mu/\overline{m} \right) \right)}{\partial \ln \left(\alpha_s \right)}}{1 - 2\gamma_m}}$$

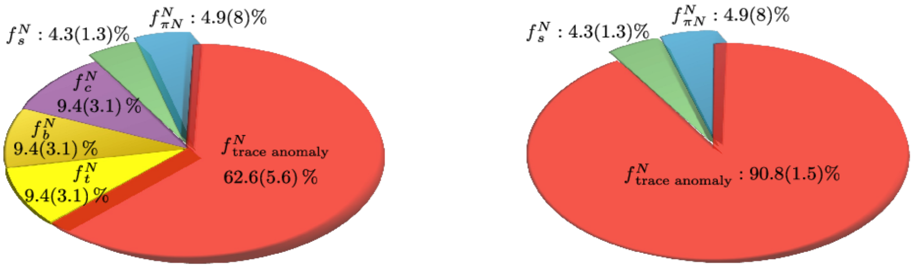
Quark Masses: Indispensable yet Intermediate Parameters

Quark masses are *fundamental* theoretical parameters of the Standard Model



- Find physical observable(s) sensitive to the quark-mass parameters in the *renormalized* Lagrangian (but insensitive to the unknown parameters or uncontrollable aspects)
- Determine the relations using perturbation techniques and/or Lattice-based approaches
- Different definitions for the **intermediate renormalized quark masses** exist (Pole, $\overline{\text{MS}}$, Kinetic, PS, 1S, (m)SMOM, ...)

Rest-mass(energy) Viewed from EMT-trace Formula



(plots taken from 2103.15768)

- Proton mass decomposition in terms of the nucleon σ-terms of different quark flavors and trace anomaly

- *Trace anomaly* does contribute a significant portion, about 2/3 to M_{proton} (not vanishing in the chiral limit)

- Continued interests in perturbation theories and Lattice-QCD [Adler et al. 77; Collins et al. 77; Nielsen 77; Shifman et al. 78; Kashiwa 79; Cheng 91; Ji 94; Polyakov 02; Hatta 18 ... Yang et al. 15,18 ......] , further renewed by the recent **EIC** and **EIcC** proposals

Masses from EMT Trace: *the role of anomaly?*

- Pole mass of a fermion (i.e. quark) may be claimed equal to its rest-mass/energy when isolated, but this is **NOT** its *definition* in QFT:

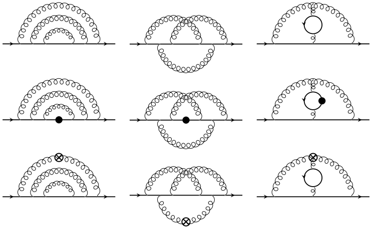
$$\not{p} - m_B - \Sigma_B(\not{p}, m_B)\Big|_{\not{p}=m_{os}} = 0\,.$$

- Why the forward form-factor c_1 of EMT [Pagels 66] is always 2 (to all-orders in QFT) ?

$$\langle \mathbf{p}, s|\,\Theta^{\mu\nu}(0)\,|\mathbf{p}, s\rangle\Big|_{r.n.} = c_1\, p^\mu p^\nu \;+\; c_2\, g^{\mu\nu}$$

  Is it **true** by (renormalization) definition (equation) or rather a theorem (identity) ?

- Feynman diagrams for $\Sigma_B(\not{p}, m_B)$ and on-shell matrix elements of $g_{\mu\nu}\Theta^{\mu\nu}$ are quite different from each other $\Longrightarrow$



- What is the role of the trace anomaly in all these considerations?

To promote a claim to a theorem requires more than hand-waving analogs, only a rigorous proof will suffice.

Masses from EMT Trace: *the role of anomaly*?

Could EMT trace anomaly lead to a **new** mass different from ***pole mass*** for any *elementary* particles?

- The physical (symmetric and conserved) EMT in QCD reads

$$\boxed{\Theta^{\mu\nu} \equiv -F^{a\,\mu}{}_{\rho} F^{a\,\nu\rho} + \frac{1}{4} g^{\mu\nu} F^{a}_{\rho\sigma}\, F^{a\,\rho\sigma} + \frac{i}{4} \sum_q \bar{q}\left(\gamma^{\mu} \overleftrightarrow{D}^{\nu} + \gamma^{\nu} \overleftrightarrow{D}^{\mu}\right) q\,,}$$

- The all-order **EMT-trace** formula: [Adler, Collins, Duncan, Joglekar 77; Nielsen 77]

$$\boxed{\left[\Theta^{\mu}_{\ \mu}\right]_{\mathrm{B}} = -\frac{\epsilon}{2}\left[F^{a}_{\rho\sigma}\, F^{a\,\rho\sigma}\right]_{B} + \sum_f \left[m_f \bar{\psi}_f \psi_f\right]_B = \frac{\beta}{2}\left[F^{a}_{\rho\sigma}\, F^{a\,\rho\sigma}\right]_{\mathrm{R}} + (1-2\gamma_m)\left[\sum_q m_q\, \bar{\psi}_q\, \psi_q\right]_{\mathrm{R}} .}$$

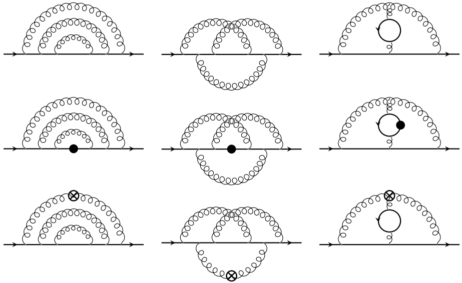
with $\frac{\mathrm{d}\ln a_s}{\mathrm{d}\ln\mu^2} \equiv -\epsilon + \beta\,,\ \frac{\mathrm{d}\ln m}{\mathrm{d}\ln\mu^2} \equiv \gamma_m\ .$

An operator subtraction scheme was specifically formulated [Adler, Collins, Duncan 77] to *define* $\left[\sum_q m_q\, \bar{\psi}_q\, \psi_q\right]_{\mathrm{R}}$ and $\left[F^{a}_{\rho\sigma}\, F^{a\,\rho\sigma}\right]_{\mathrm{R}}$ to **manually ensure** the condition $\langle e(\mathbf{p})|\,\Theta^{\mu}_{\ \mu}\,|e(\mathbf{p})\rangle\big|_{n.n.} = \mathbf{m_e}$ (and $\langle \gamma(\mathbf{p})|\,\Theta^{\mu}_{\ \mu}\,|\gamma(\mathbf{p})\rangle\big|_{n.n.} = 0$).

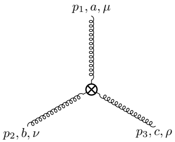
- In general, **UV-finite operator $\neq$ No quantum corrections to matrix elements** !

Could there be *regularization-dependent* remnant *artifact*, to be removed by additional **finite** renormalizations?

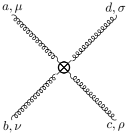
Explicit Verification up-to Three Loops in QCD



$$= i(-g^{\mu\nu}p^2 + p^\mu p^\nu)\delta_{ab} - \frac{i}{\xi}p^\mu p^\nu \delta_{ab}$$



$$= g f^{abc}\big[g^{\mu\nu}(p_1 - p_2)^\rho + g^{\nu\rho}(p_2 - p_3)^\mu + g^{\rho\mu}(p_3 - p_1)^\nu\big]$$



$$= -ig^2\big[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\big]$$

- We completed an explicit perturbative verification [arXiv: 2509.03580] of

$$\langle \vec{\mathbf{p}} | -\frac{\epsilon}{2}\left[F^a_{\rho\sigma}\, F^{a\,\rho\sigma}\right]_B + \sum_f \left[m_f \bar{\psi}_f \psi_f\right]_B |\vec{\mathbf{p}}\rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\mathrm{os}}\, u(\vec{\mathbf{p}})$$

  in both QCD and QED for quarks, electron, photon and gluons up-to three loops [LC, Li, Niggetiedt 25] .
- Trace-anomaly contribution is finite, non-zero, for massive fermions, but vanishing for gauge bosons.

A Direct All-order Diagrammatic Proof

Take-home messages: [LC, Li, Niggetiedt 25]

- A novel **diagrammatic proof of the *identity***

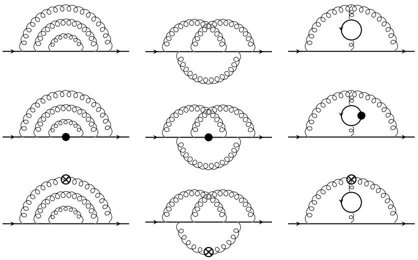
$$\langle\vec{\mathbf{p}}|-\frac{\epsilon}{2}\left[F^a_{\rho\sigma}\,F^{a\,\rho\sigma}\right]_B+\sum_f\left[m_f\bar{\psi}_f\psi_f\right]_B|\vec{\mathbf{p}}\rangle=\bar{u}(\vec{\mathbf{p}})\,m_{\mathrm{os}}\,u(\vec{\mathbf{p}})$$

to any loops in perturbative gauge theories (and all orders in ϵ) without any ref. to prelaid *operator* renormalization conditions.

- The proof is based on:
  - equation of mass-dimensional analysis in DR
  - topological properties of contributing diagrams
  - the pole-mass definition in OS-ren.

$$m_{\mathrm{os}}=Z_\psi\left(m_B+\Sigma_B(\not{p},m_B,\hat{\mu})-\not{p}\,\frac{\partial\,\Sigma_B(\not{p},m_B,\hat{\mu})}{\partial\,\not{p}}\right)\Big|_{\not{p}=m_{\mathrm{os}}}$$

[arXiv: 2509.03580]



$$\Sigma_B(\not{p},m_B,\hat{\mu})=\not{p}\,\frac{\partial\,\Sigma_B(\not{p},m_B,\hat{\mu})}{\partial\,\not{p}}+m_B\,\frac{\partial\,\Sigma_B(\not{p},m_B,\hat{\mu})}{\partial\,m_B}+\hat{\mu}\,\frac{\partial\,\Sigma_B(\not{p},m_B,\hat{\mu})}{\partial\,\hat{\mu}}$$

$$\bar{u}(\mathbf{p},s)\left(\hat{\mu}\,\frac{\partial\,\Sigma_B(\not{p},m_B,\hat{\mu})}{\partial\,\hat{\mu}}\right)u(\mathbf{p},s)=\sum_{L=1}^{\infty}2\epsilon\,L\,\hat{\alpha}^L\,\hat{\mu}^{2\epsilon\,L}\,\Sigma_B^{(L)}$$

$$=\langle\mathbf{p},s|\,2\epsilon\left[-\frac{1}{4}F_{\mu\nu}\,F^{\mu\nu}\right]_B|\mathbf{p},s\rangle\big|_{\mathrm{1PI}}$$

A Direct All-order Diagrammatic Proof

Take-home messages: [LC, Li, Niggetiedt 25]

- A novel **diagrammatic proof of the *identity***

$$\langle \vec{\mathbf{p}} | -\frac{\epsilon}{2} \left[F^{a}_{\rho\sigma} F^{a\,\rho\sigma} \right]_B + \sum_f \left[m_f \bar{\psi}_f \psi_f \right]_B | \vec{\mathbf{p}} \rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\mathrm{os}}\, u(\vec{\mathbf{p}})$$

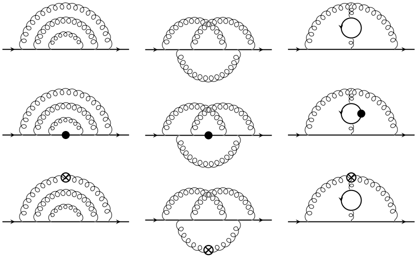
  to any loops in perturbative gauge theories (and all orders in ϵ) without any ref. to prelaid *operator* renormalization conditions.

- The proof is based on:
  - equation of mass-dimensional analysis in DR
  - topological properties of contributing diagrams
  - the pole-mass definition in OS-ren.

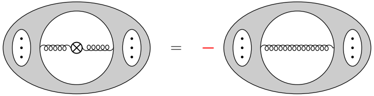
- The role of the trace anomaly is clarified

  (crucial for massive fermions, albeit vanishing for gauge bosons)

[arXiv: 2509.03580]



Reduction of diagrams with degree-2 vertex



For arbitrary vacuum diagrams in gauge theories:

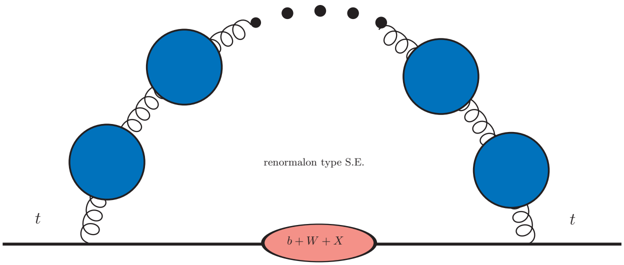
$$N_g^{\mathrm{eff}} = N_g - V_3 - V_4 = N_L - 1$$

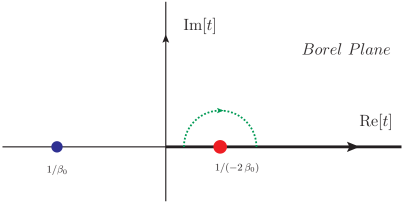
IR Renormalon in Pole Mass

The pole mass of a quark in p. QCD admits the following series result: [Bigi et al. 94; Beneke et al. 94]

$$m_{\text{os}} = \overline{m}(\mu)\, C_{\overline{m}}(\alpha_s,\, \mu/\overline{m}) = \overline{m}(\mu) + \sum_{n=0}^{\infty} r_n\, \alpha_s^{n+1}(\mu)$$

with the large-order behavior $r_n\big|_{n\to\infty} \longrightarrow C_F\, \frac{e^{5/6}}{\pi}\, \mu\, \left(-2\beta_0\right)^n n!$ (at the large n_f limit).





$$\sum_{n=0}^{\infty} n!\, x^{n+1} \sim \int_0^{\infty} \frac{x}{1-x\,t}\, e^{-t} \mathrm{d}t$$

The so-called leading IR-renormalon ambiguity: [Bigi et al. 94; Beneke et al. 94]

$$\boxed{\Lambda_{\text{LIR}} \equiv e^{5/6}\, \frac{C_F}{-\beta_0}\, \mu\, e^{\ln(\Lambda_{\text{QCD}}/\mu)} = e^{5/6}\, \frac{C_F}{-\beta_0}\, \Lambda_{\text{QCD}}\,.}$$

A Scheme/scale-invariant Trace-anomaly-subtracted σ-mass

Take-home messages: [**LC**, Li, Niggetiedt 25; **LC**, Zhao 25]

- **A discovery**:

  The leading IR-renormalon divergence in *pole-mass* of massive quarks **resides entirely** in the EMT-trace anomaly contribution!

  - $$\langle p,s \big| 2\epsilon \big[-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \big]_B \big| p,s \rangle \big|_{\text{ampu.}} = \bar{u}(p,s) \Big(\hat{\mu} \frac{\partial \Sigma_B(\not{p}, m_B, \hat{\mu})}{\partial \hat{\mu}} \Big) u(p,s) \,,$$

  - $$\hat{\mu} \frac{\partial \Sigma_B(\not{p}, m_B, \hat{\mu})}{\partial \hat{\mu}} \Big|_{\not{p} \to m_{\text{os}}} = \hat{\mu} \frac{\partial m_{\text{os}}(m_B, \hat{\mu})}{\partial \hat{\mu}} \text{ and } \mu \frac{\partial m_{\text{os}}(m_B, \mu)}{\partial \mu} \Big|_{\text{LIR}} = \overline{m}\, C_{\overline{m}}(\alpha_s, \mu/\overline{m}) \Big|_{\text{linear-}\mu} = m_{\text{os}}(m_B, \mu) \Big|_{\text{LIR}} \,.$$

- **A proposal**:

  A novel trace-anomaly-subtracted σ-mass definition for heavy quarks, scheme/scale and gauge invariant, and free from the leading IR-renorm. ambiguity. [arXiv:2509.03580, 2509.10786]

  (In other words, it naturally combines the merits of both pole and $\overline{\text{MS}}$ mass definitions!)

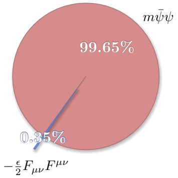
A Scheme/scale-invariant Trace-anomaly-subtracted σ-mass

The fraction Z_{TA} of the trace-anomaly contribution to m_{os} and m_σ of elementary fermions: [LC, Li, Niggetiedt 25]

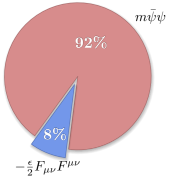
	electron	t-quark	b-quark	c-quark
Z_{TA}	0.347 %	7.9 %	20.4 %	34.3 %
m_σ	0.509 MeV	159.0 GeV	3.96 GeV	1.17 GeV

- Z_{TA} increase for lighter heavy quarks
- Z_{TA} will vanish in chiral limit $m_\sigma = 0$

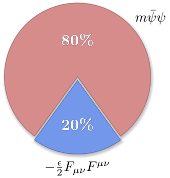
m_e^{os} -composition



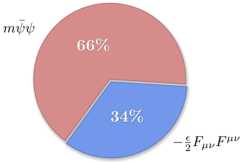
m_t^{os}-composition



m_b^{os}-composition



m_c^{os}-composition



Compact Formula and Results for m_σ in QCD

- A compact formula [LC, Zhao 25] [arXiv: 2509.10786]

$$\boxed{Z_\sigma = \frac{m_\sigma}{m_{\rm OS}} = \frac{1 + 2\beta \dfrac{\partial \ln\left(C_{\overline{m}}\left(\alpha_s, \mu/\overline{m}\right)\right)}{\partial \ln\left(\alpha_s\right)}}{1 - 2\gamma_m}}$$

i.t.o. anomalous dimensions of α_s and $\overline{m}$ and the pole-to-$\overline{\rm MS}$ mass ratio $C_{\overline{m}}\left(\alpha_s, \mu/\overline{m}\right)$.

  - **A nice feature**: Z_σ at $\mathcal{O}(\alpha_s^N)$ involves perturbatively $C_{\overline{m}}$ only up to $\mathcal{O}(\alpha_s^{N-1})$, i.e. one loop-order less!

- Result for the 5-loop relation to $m_{\rm OS}$ at $\mu = m_{\rm OS}$:

$$\begin{aligned} m_\sigma / m_{\rm OS} = 1 &+ \alpha_s\left(-0.636620\right) + \alpha_s^2\left(-1.11735 + 0.0731764\, n_l\right) \\ &+ \alpha_s^3\left(-4.98197 + 0.800055\, n_l - 0.0206485\, n_l^2\right) \\ &+ \alpha_s^4\left(-31.2996 + 6.70684\, n_l - 0.405322\, n_l^2 + 0.00658157\, n_l^3\right) \\ &+ \alpha_s^5\left(-243.76(11) + 68.515(5)\, n_l + 6.4963(2)\, n_l^2 + 0.240658\, n_l^3 - 0.00295411\, n_l^4\right) + \mathcal{O}(\alpha_s^6) \end{aligned}$$

- Result for the perturbative relation to $\overline{m}(\mu = \overline{m})$ at 4-loop:

$$\begin{aligned} m_\sigma / \overline{m} = 1 &+ \alpha_s\left(-0.212207\right) + \alpha_s^2\left(-0.0254365 - 0.0323361 n_l\right) \\ &+ \alpha_s^3\left(0.268010 + 0.00994659\, n_l + 0.000401805\, n_l^2\right) \\ &+ \alpha_s^4\left(1.162(17) - 0.29899(37)\, n_l + 0.0240154\, n_l^2 - 0.000380218\, n_l^3\right) + \mathcal{O}(\alpha_s^5) \end{aligned}$$

?

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Pole mass of the heavy quark: Perturbation theory and beyond

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The key quantity of the heavy quark theory is the quark mass m_Q. Since quarks are unobservable one can suggest different definitions of m_Q. One of the most popular choices is the pole quark mass routinely used in perturbative calculations and in some analyses based on heavy quark expansions. We show that no precise definition of the pole mass can be given in the full theory once nonperturbative effects are included. Any definition of this quantity suffers from an intrinsic uncertainty of order Λ_{QCD}/m_Q. This fact is succinctly described by the existence of an infrared renormalon generating a factorial divergence in the high-order coefficients of the α_s series; the corresponding singularity in the Borel plane is situated at $2\pi/b$. A peculiar feature is that this renormalon is not associated with the matrix element of a local operator. The difference $\overline{\Lambda} \equiv M_{H_Q} - m_Q^{\text{pole}}$ can still be defined by heavy quark effective theory, but only at the price of introducing an explicit dependence on a normalization point μ: $\overline{\Lambda}(\mu)$. Fortunately the pole mass $m_Q(0)$ *per se* does not appear in calculable observable quantities.

PACS number(s): 12.39.Hg, 12.38.Aw, 12.38.Lg, 13.20.He

Summary and Outlook

The masses of elementary fields, e. g. leptons and quarks, are fundamental parameters of SM, and a better understanding of them is of utmost importance.

- ☑ Presented a novel **diagrammatic all-order proof of the *identity***

$$\langle \vec{\mathbf{p}} | -\frac{\epsilon}{2} \left[F^a_{\rho\sigma} F^{a\,\rho\sigma} \right]_B + \sum_f \left[m_f \bar{\psi}_f \psi_f \right]_B | \vec{\mathbf{p}} \rangle = \bar{u}(\vec{\mathbf{p}})\, m_{\text{os}}\, u(\vec{\mathbf{p}})$$

  in perturbative gauge theories (to all orders in ϵ without any ref. to prelaid *operator* ren. conditions).

- ☑ Discovered that the leading IR-renormalon observed in *pole-mass* of heavy quarks **resides entirely** in the EMT-trace anomaly contribution!

- ☑ Proposed a novel trace-anomaly-subtracted σ-mass definition for heavy quarks (which is scheme/scale and gauge invariant and free from the leading IR-renorm. ambiguity).

- ☑ Derived a compact formula for the relation between σ-mass and pole-mass, exhibiting nice features!

$$\boxed{\frac{m_\sigma}{m_{\text{os}}} = \frac{1 + 2\beta \dfrac{\partial \ln \left(C_{\overline{m}} \left(\alpha_s, \mu/\overline{m} \right) \right)}{\partial \ln \left(\alpha_s \right)}}{1 - 2\gamma_m}}$$

- ☒ Investigating the effects of additional (virtual) massive quarks, QCD$\oplus$QED mixed corrections ... applications to Higgs and heavy-quark decays.

Thank you for listening!