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Emergence of locality and unitarity from hidden zeros

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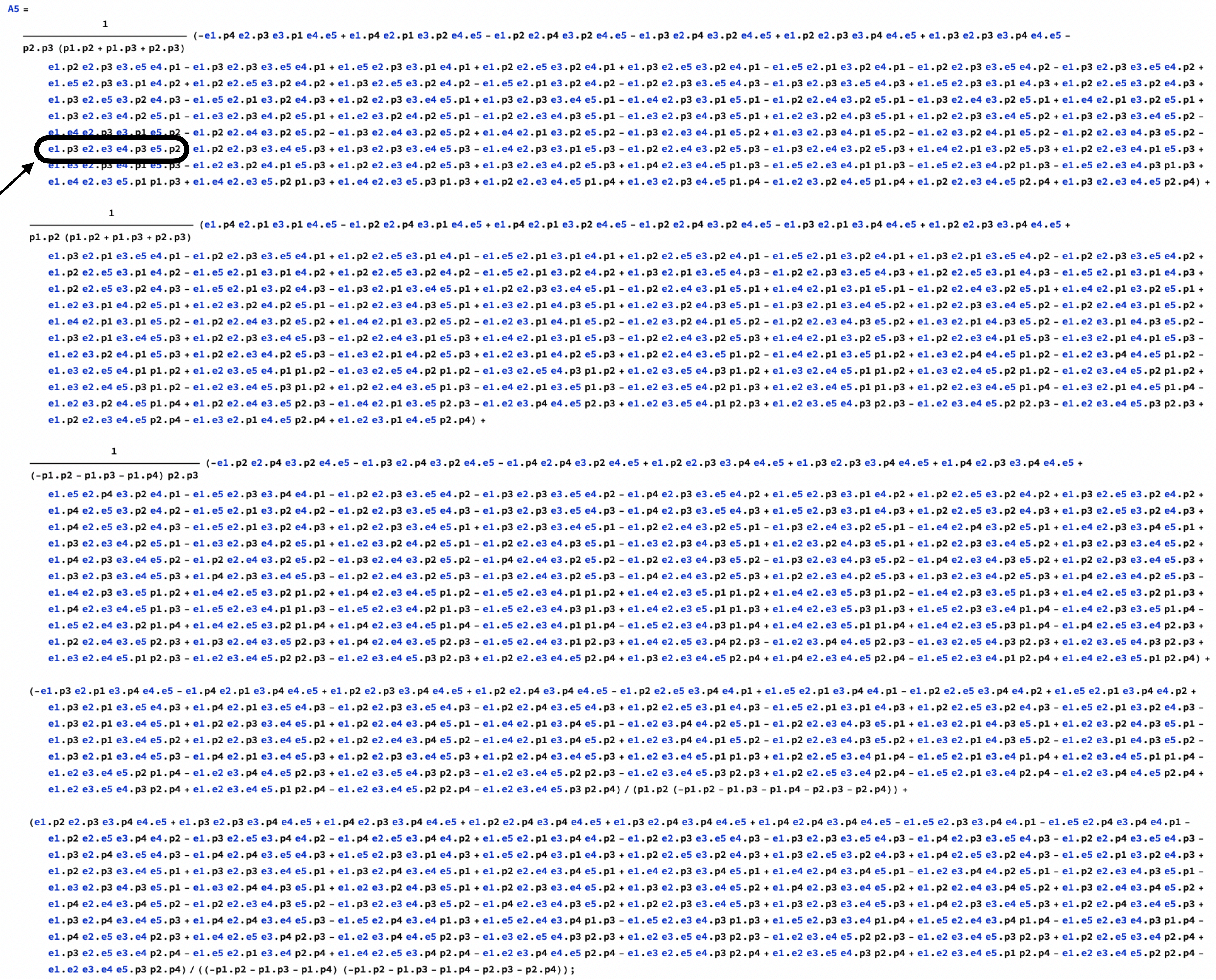
The unreasonable "generosity" of scattering amplitudes

- Simple enough to compute, but still highly non-trivial and rich (also useful)
- For some reason, reveal secrets way beyond the formalism we apply
- Like an advanced alien object given to us, that we're still probing and trying to understand
- Sometimes the hinted direction is totally orthogonal to our assumptions
- In this talk: unitarity and locality (fundamental principles in QFT) emerge from other properties at both tree and 1-loop level

First hint in Yang-Mills

- Gluon amplitude is naively complicated



$$\texttt{e1.p2 e2.p3 e3.p4 e4.e5}$$

- But in the right variables simplifies dramatically:

$$A_5 = \frac{1}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Zvi Bern

What determines an Amplitude?

- Recent methods have revealed very radical ways to compute amplitudes
- How do we know an amplitude is correct?
- What is the **minimal** data needed to fix it?
- Traditionally we rely on: locality, unitarity, gauge invariance, soft theorems, recursion, etc.
- In fact, these principles form an **overdetermined** system: Not all are independent!

Uniqueness and emergence of unitarity

	Gauge invariance	Adler zero	soft behavior	UV/BCFW scaling	BCJ relations	hidden zeros
YM	x		x	x		x
GR	x		x	x		
ϕ^3			x	x	x	x
NLSM		x	x	x	x	x
DBI		x	x	x		
sGal		x	x	x		
CHY		x		x		
cosmo wvf						?

Arkani-Hamed, R. Trnka [PRL 2016]
R. [JHEP 2016]
R. [JHEP 2016]
R. [PRL 2018]
Carrasco, R. [PRD 2019]
R. [PRD 2020]
R. [PRL 2024]
Backus, R. [PRL 2025] - loop level

De, Li, R. 2025?

In all cases, **unitarity** (factorization) is an emergent property!

In some cases, so is **locality**

BCFW/UV scaling and recursion

- Britto-Cachazo-Feng-Witten recursion: consider a shift $p_a \to p_a + zq, p_b \to p_b - zq$

  1. If A is unitary (factorizes on poles), and has scaling $\mathcal{O}(1/z)$ or better (non-trivial)

  2. Then via residue theorem A can be computed **recursively** from lower point amplitudes

- Revolutionary, but still mysterious: what is the origin of the "enhanced" UV scaling?

(see Arkani-Hamed, Kaplan 2008)

$$A_4(z) = A_s(z) + A_t(z) = \mathcal{O}(z) - \mathcal{O}(z) = \mathcal{O}(1/z)$$

Two big open questions:

1. Q: **Are the above principles in the “uniqueness table” truly independent?**
   In particular, how can both IR and UV constraints independently fix amplitude?
   And what is the origin of the enhanced UV scaling?

   $$\text{UV scaling} \overset{?}{\leftrightarrow} \text{IR scaling}$$

   A: Hidden zeros now finally clarify the relation:

   $$\boxed{\text{UV scaling} \Leftrightarrow \text{Hidden zeros} \Rightarrow \text{IR scaling}}$$

2. Q: **Can these ideas extend beyond tree level?**

   A: Surface kinematics extend emergence of locality and unitarity to loop level from hidden zeros

Hidden zeros at tree level

- Hidden zeros are a new surprising property of scattering amplitudes, originally in Trϕ^3, NLSM, Yang-Mills Arkani-Hamed, Cao, Dong, Figueiredo, He 2023

- Extended via double copy to other theories, Bartsch et al 2024, Li, Roest, Veldhuis 2024
and even to cosmological wavefunctions De, Paranjape, Pokraka, Spradlin, Volovich 2025

- What are they: Amplitudes vanish for particular configurations of external momenta
  - $A_4 = \dfrac{1}{p_1 \cdot p_2} + \dfrac{1}{p_1 \cdot p_4} = \dfrac{1}{p_1 \cdot p_2} - \dfrac{1}{p_1 \cdot p_2 + p_1 \cdot p_3} \to 0$ (when $p_1 \cdot p_3 = 0$)

- Furthermore, near the zeros, amplitudes factorize

$$\mathcal{A}_n^{\text{tree}}(c_\star \neq 0) = \left(\frac{c_\star}{X_{\text{B}} X_{\text{T}}}\right) \times \mathcal{A}_{\text{down}}^{\text{tree}} \times \mathcal{A}_{\text{up}}^{\text{tree}}$$

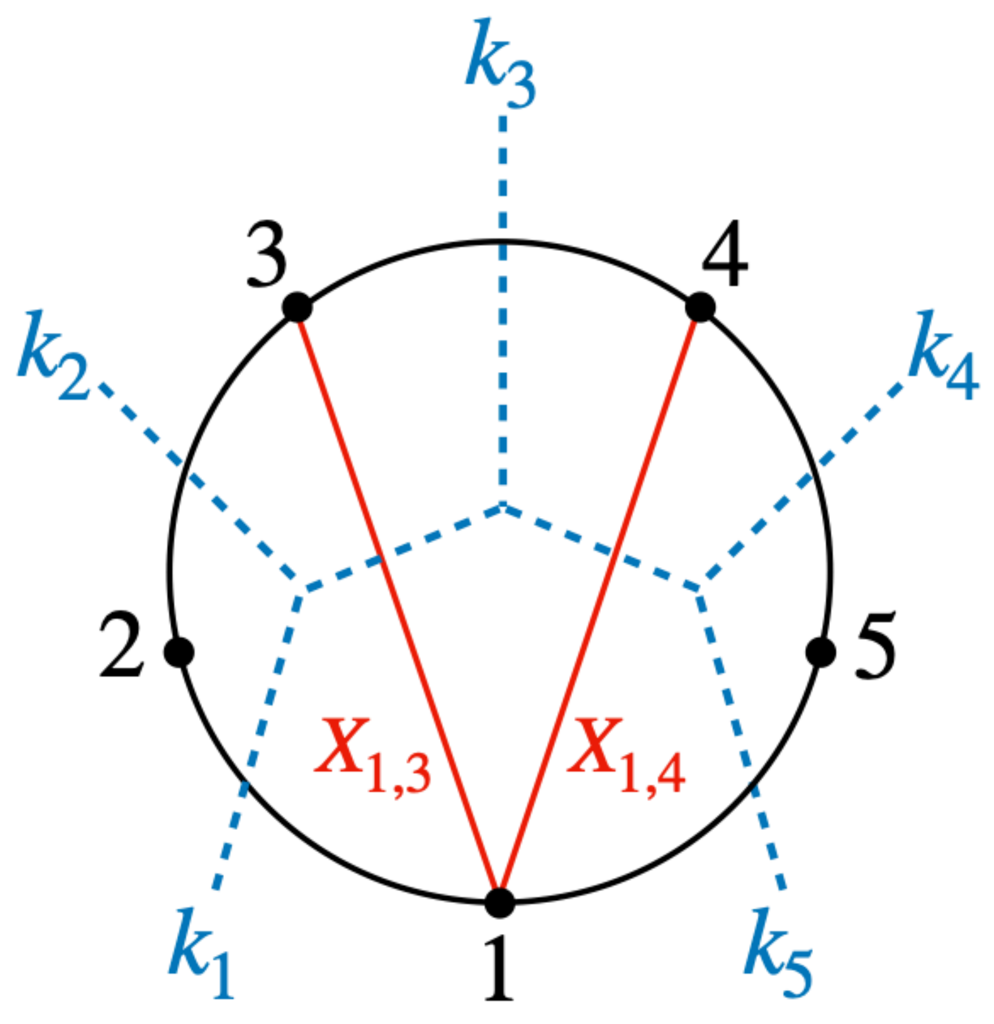
- No clear physical origin for either property!

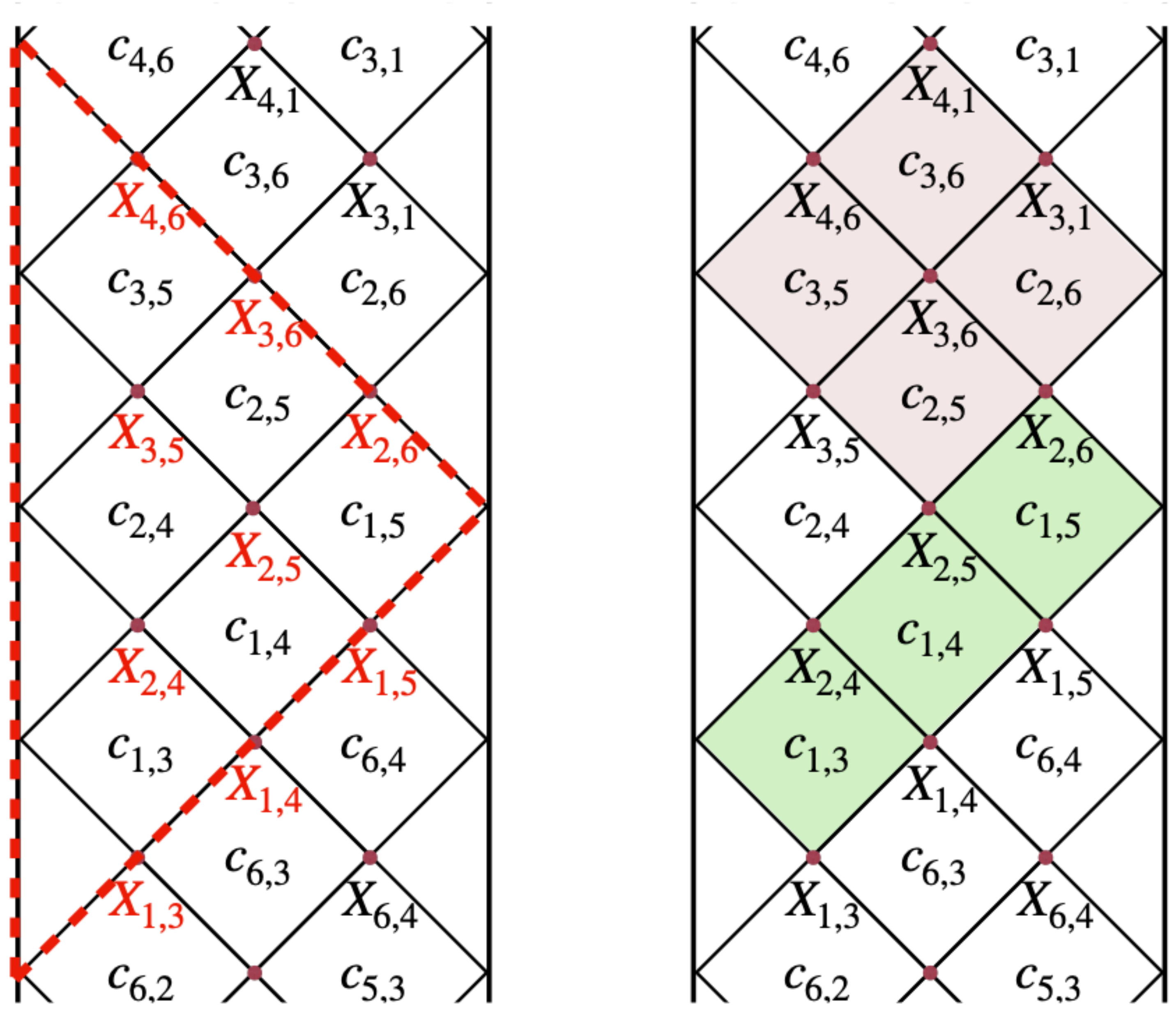
Uniqueness from hidden zeros

- Conjecture by Arkani-Hamed et el: **hidden zeros uniquely fix Tr ϕ^3, NLSM amplitudes**
- $$A_4 = \frac{1}{p_1 \cdot p_2} + \frac{1}{p_1 \cdot p_4} = \frac{1}{p_1 \cdot p_2} - \frac{1}{p_1 \cdot p_2 + p_1 \cdot p_3} \to 0 \text{ (when } p_1 \cdot p_3 = 0\text{)}$$
- **Local** ansatz $B_4 = \dfrac{a_1}{p_1 \cdot p_2} + \dfrac{a_2}{p_1 \cdot p_4}$
- Zero fixes $a_1 = a_2$
- Higher point: $B_5 = \dfrac{a_1}{p_1 \cdot p_2 \, p_3 \cdot p_4} + \ldots$

Surface kinematics

- Kinematic data: planar variables $X_{ij} = (k_i + k_{i+1} + \ldots + k_{j-1})^2$ form a basis
- Non-planar variables $c_{ij} = -k_i \cdot k_j = X_{ij} + X_{i+1,j+1} - X_{i,j+1} - X_{i+1,j}$
- $X_{i,j}$ are chords on the **kinematic disk**
- Feynman diagrams in **Tr** ϕ^3 are triangulations
- Data can be organized in a **kinematic mesh**
- Hidden zeros: $c_{ij} = 0$ in maximal rectangles





Hidden Zeros = secret non-adjacent BCFW scaling

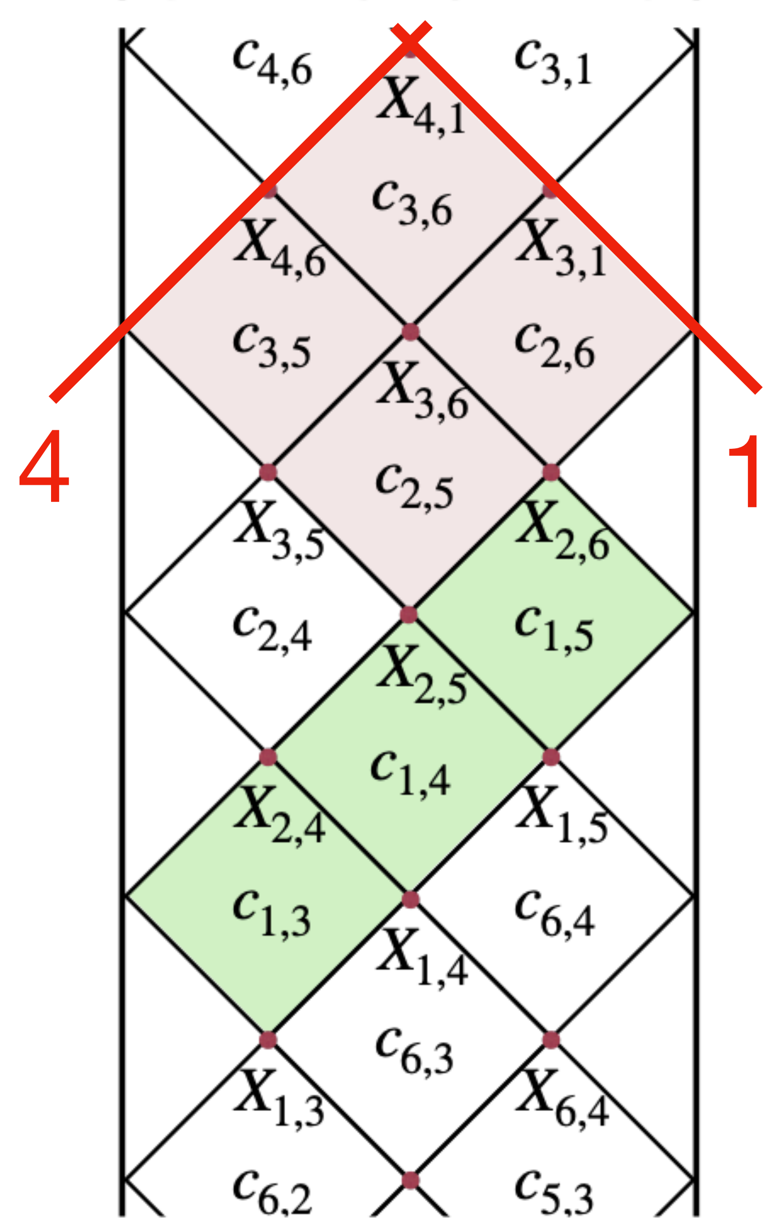
Rodina 2024

- Scalar theories (Tr(ϕ^3), NLSM, YMS, DBI, etc.) enjoy enhanced BCFW scaling under **non-adjacent** shifts $k_i \to k_i + zq,\ k_j \to k_j - zq$ Carrasco, R. 2019

- Mysterious origin for scalars (for YM, GR, can be traced to an "enhanced Lorentz spin symmetry") Arkani-Hamed, Kaplan 2008

- $\boxed{\text{In fact, for any rational function, the enhanced scaling is equivalent to hidden zeros}}$

- What shift vs what what zero? Easy: in any zero $c_{ij} = 0$, there are always 2 labels that never show up, giving the shifted legs (eg: $c_{13} = c_{14} = c_{15} = 0$: 2 and 6; $c_{25} = c_{26} = c_{35} = c_{36} = 0$: 1 and 4).



Uniqueness from zeros+locality

$$B_5 = \frac{a_1}{p_1 \cdot p_2\, p_4 \cdot p_5} + \frac{a_2}{p_2 \cdot p_3\, p_4 \cdot p_5} + \ldots$$

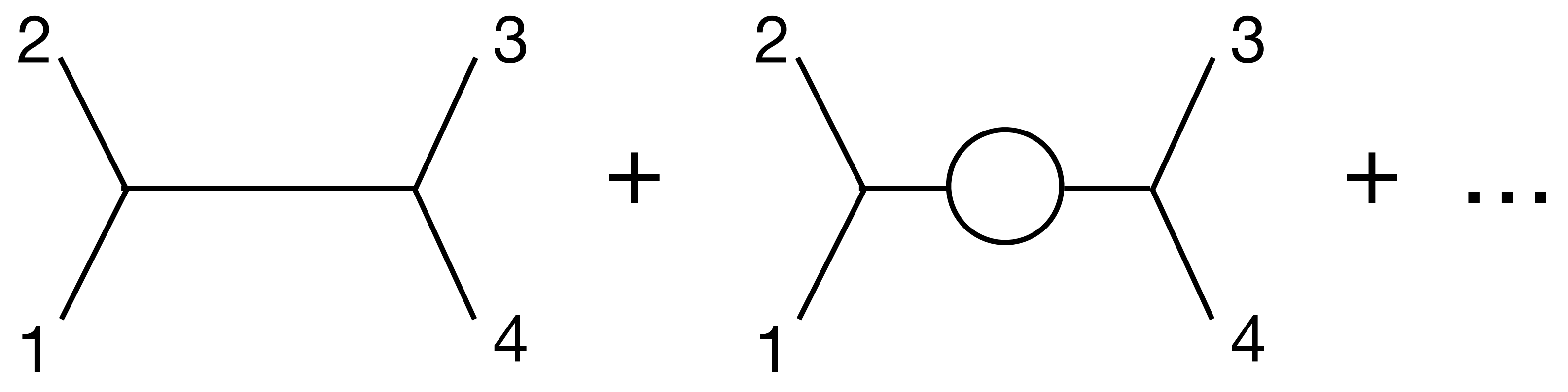
$$\text{Res}_{p_4 \cdot p_5}[B_5] = \frac{a_1}{p_1 \cdot p_2} + \frac{a_2}{p_2 \cdot p_3} = B_4$$

- $\text{Res}_{p_{n-1} \cdot p_n}[A_n] = A_{n-1}$
- Ansatz assuming locality: $\text{Res}_{p_{n-1} \cdot p_n}[B_n] = B_{n-1}$
- On the cut, **some** of the zeros act as lower point zeros, so by induction:

$$\text{Res}_{p_i \cdot p_{i+1}}[B_n] = a_i A_{n-1}$$

- The **remaining** zeros fix $a_i = a$, so $B_n = aA_n$
- So **Hidden zeros** + **Locality** uniquely fix the amplitude, **Unitarity** (factorization) is automatic

Loop level

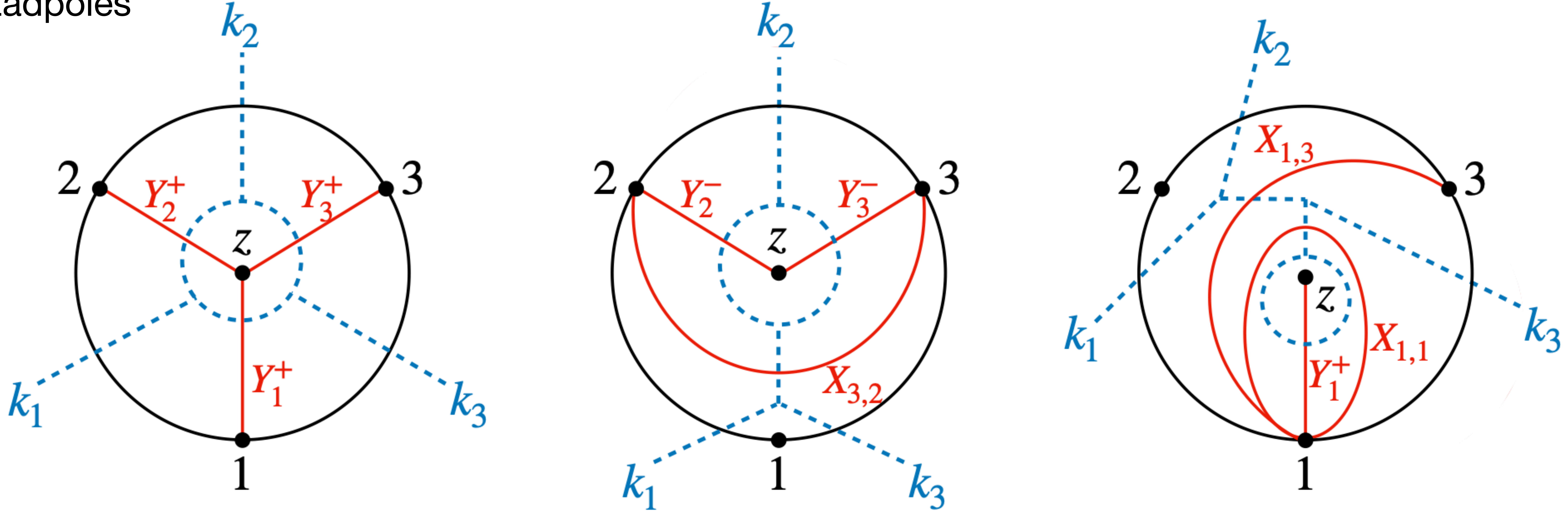


- A major roadblock has been extending uniqueness results beyond tree level
- If locality/unitarity are emergent, this should also be visible at loop level!
- Difficult to approach, and until recently ill defined: there is no unique loop integrand
- These problems are elegantly solved by "**surface kinematics**" *Arkani-Hamed, Frost, Salvatori, Plamondon, Thomas, 2023*
- The formalism gives canonical integrands, with well defined single loop cuts

Loop kinematic disk

Backus, LR 2025

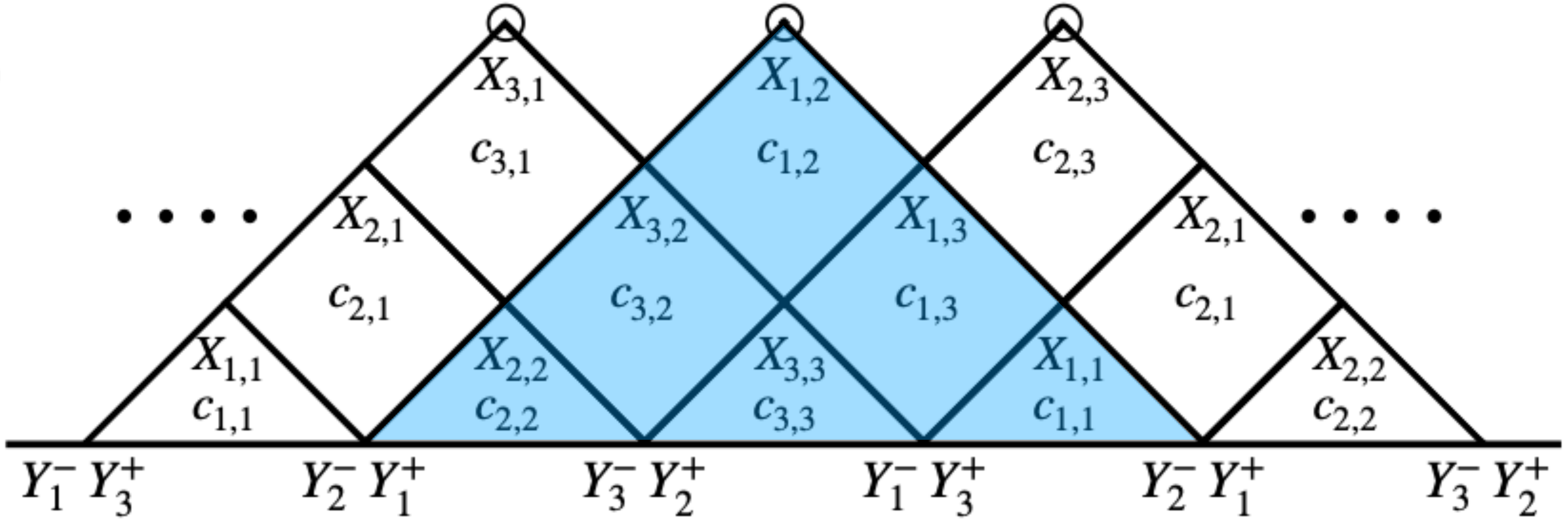
- Add a puncture to the disk: Y variables; allow $X_{ij} \neq X_{ji}$, and $X_{ii}, X_{i,i+1} \neq 0$ which keeps external bubbles and tadpoles



- The extra “unphysical” variables are needed to have well defined single loop cuts
- When assuming locality: $\textbf{Res}_Y[I_n] = B_{n+2}$

Loop mesh and loop zeros

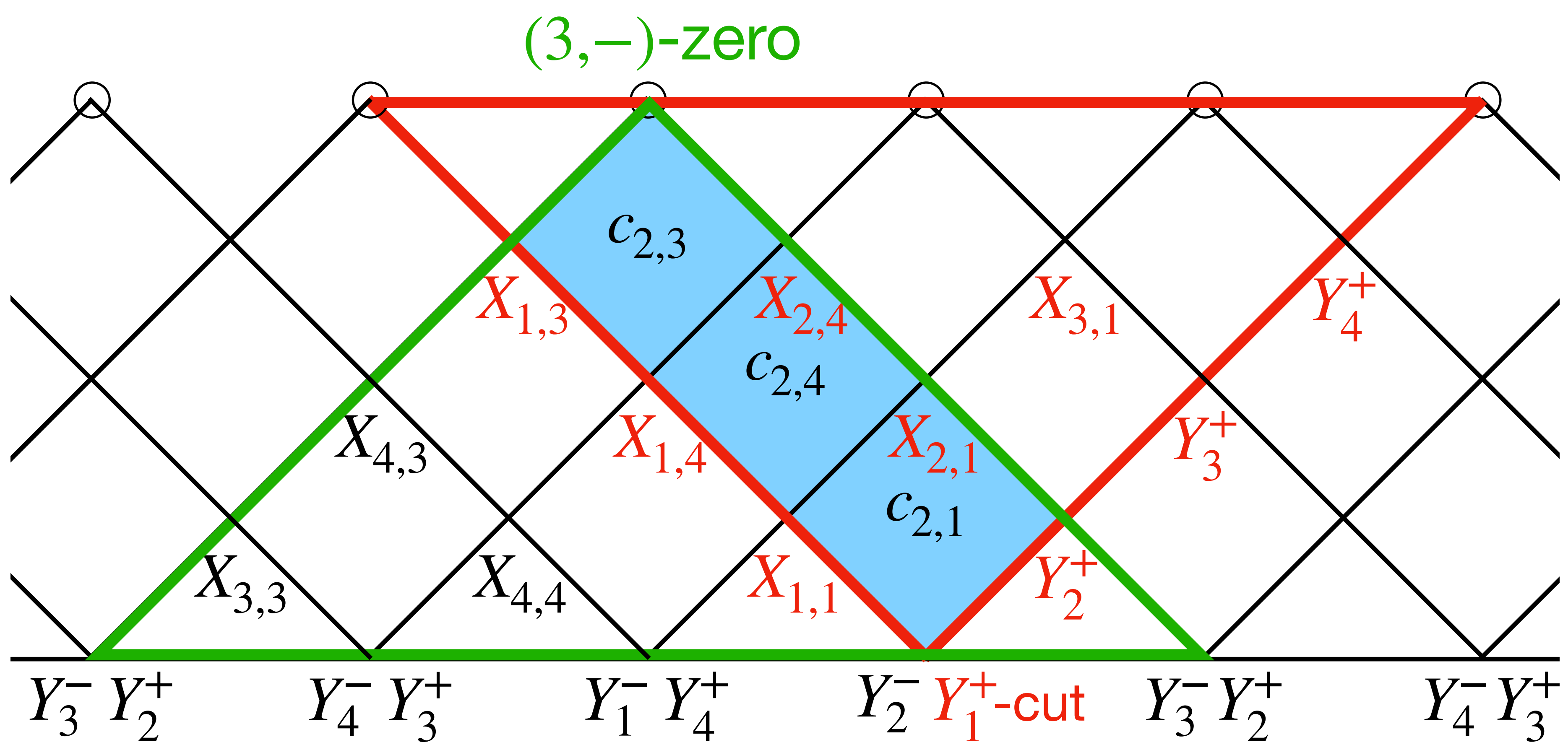
- New kinematic mesh; Loop zeros: $c_{ij}(X,Y)=0$ inside "big mountains"

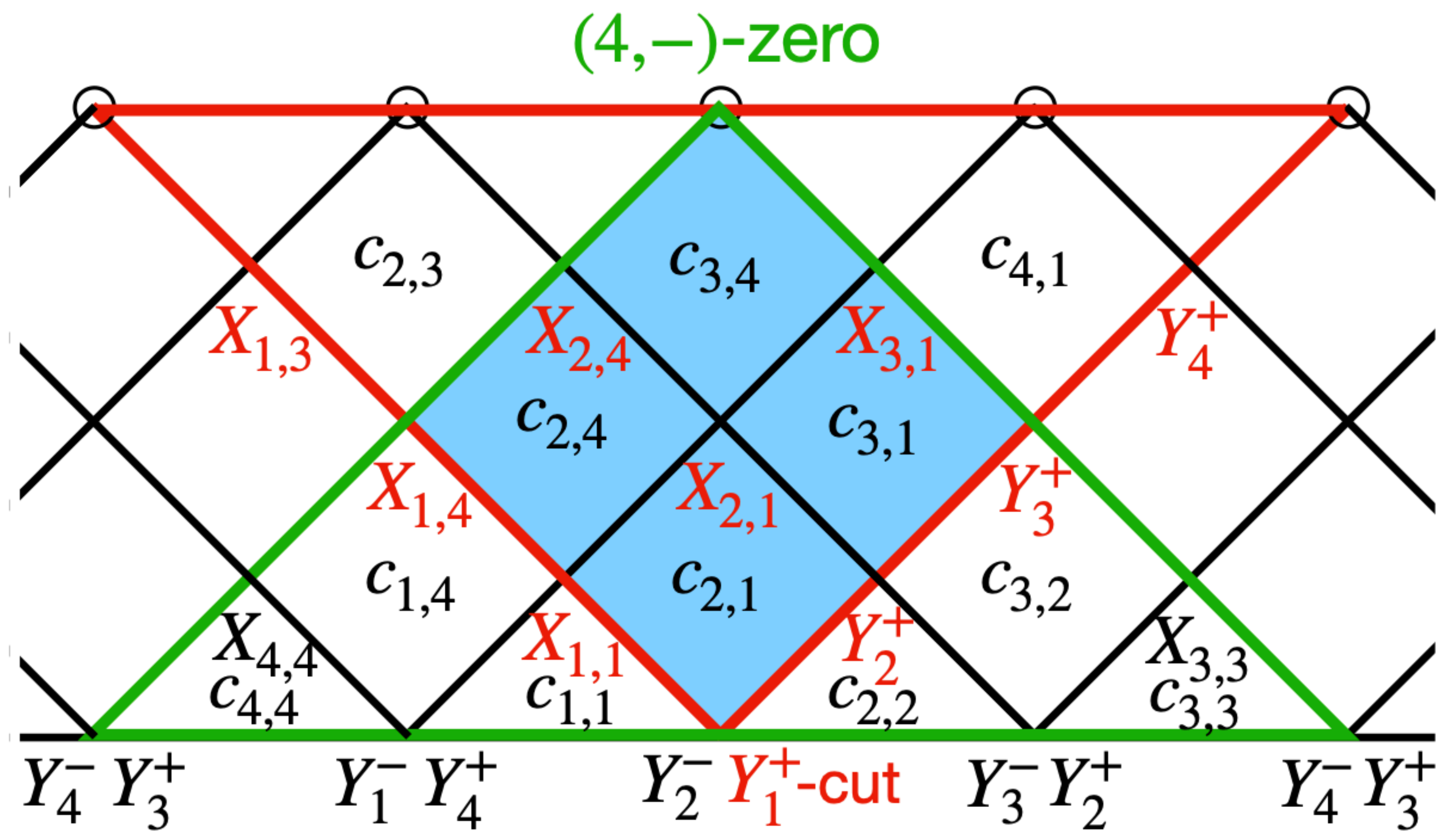


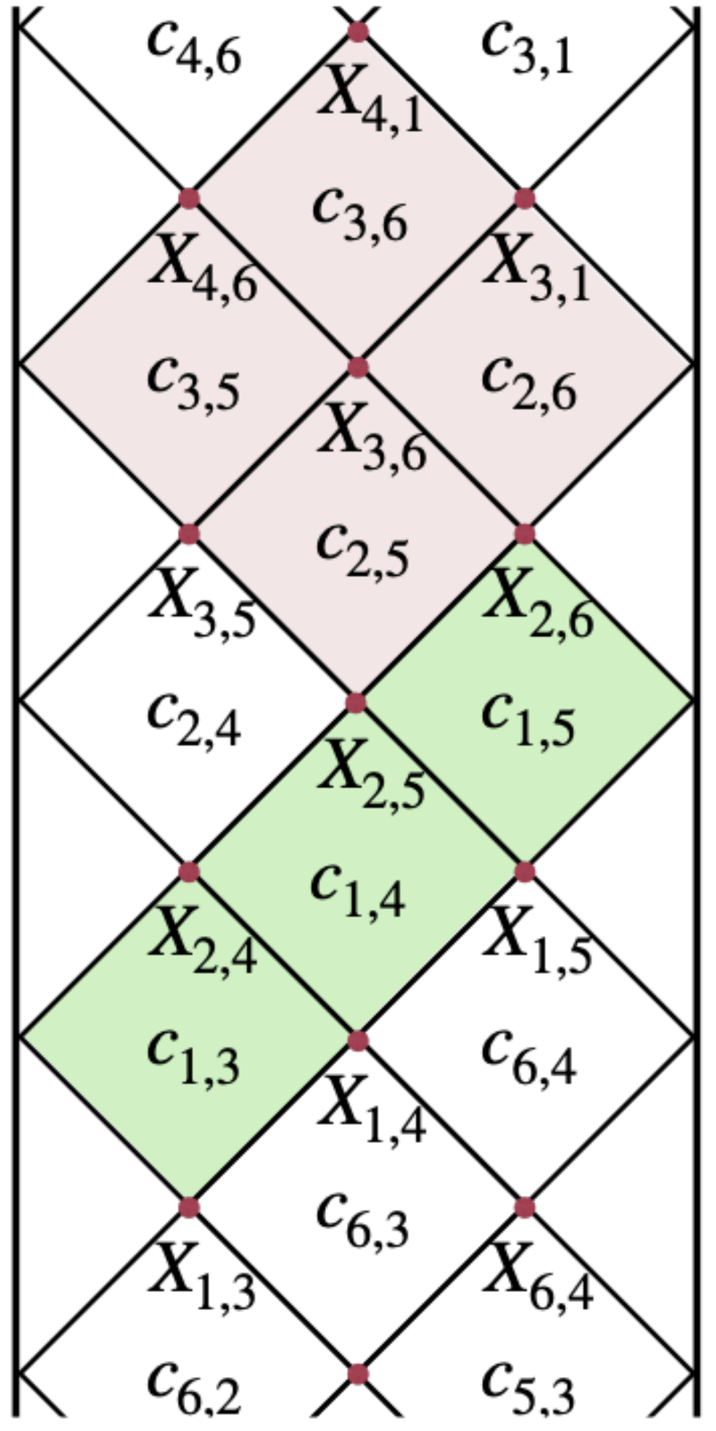
The 1-loop integrand is uniquely fixed by these zeros, assuming only locality

Loop zeros as tree zeros

- The overlap between the region defined by the cut **Res**$_Y[I_n] = B_{n+2}$ and the **mountain zero** gives precisely the different shapes of tree zeros







- Tree zeros are unified at loop level! Rare that structures simplify at loop level..

Open problems

- Proof for Zeros $\Rightarrow$ Locality & Unitarity (both tree and loop)
- Physical origin of zeros/splitting?
- Zeros at higher loop level? Zeros $\Rightarrow$ Unitarity/Locality to **all orders** in perturbation theory??
- Work in progress: **cosmological wavefunction** also uniquely fixed by hidden zeros!