Third-Order QCD Corrections to Single Hadron Production in e^+e^- Annihilations

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Based on Phys.Rev.Lett. 135 (2025) 10, 10

第五届量子场论及其应用研讨会

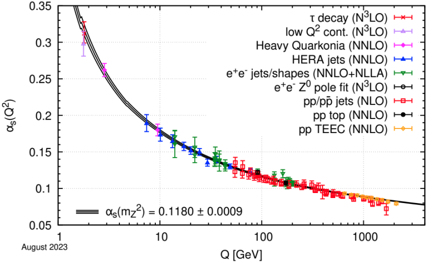
2025 年 10 月 31-11 月 2 日，北京

QCD: Asymptotic Freedom and Confinement

Evolution of $\alpha_s(Q)$

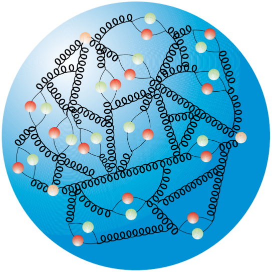
$$\frac{d\alpha_s}{d\ln Q} = -2\alpha_s\beta < 0$$

- High Q: perturbative QCD
- Low Q: non-perturbative QCD



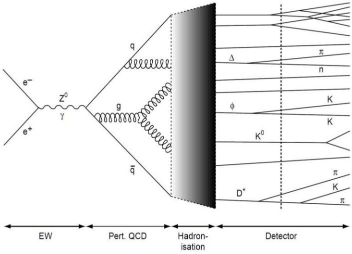
Confinement

- Partons can not be observed separately
- They are confined within hadrons



Hadronization

- Hadronization: Partons $\rightarrow$ Hadrons?
- No first-principle understanding
- Hadronization model
  - Lund string model [Andersson, Gustafson, Ingelman, Sjöstrand, 83], implemented in Pythia
  - Cluster model [Webber, 00], implemented in Herwig
- Fragmentation functions $D_i^h(z)$
  - Number density of h carrying momentum fraction z of parton i



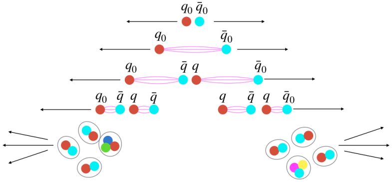


Figure from EPJC 85 (2025) 16

Fragmentation functions

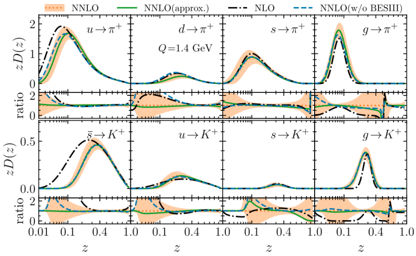
- Global fitting to determine FFs
  - Parameterize the FFs at an initial scale Q_0
  - Evolve the FFs to Q according to DGLAP evolution equation

$$dD_i^h/d\ln Q = 2D_k^h \otimes P_{ki}^T$$

  - Match the data with theory predic. based on factorization formula

$$d\sigma^h = D_i^h \otimes d\hat{\sigma}_i$$

- Rapid progress recently, FFs at NNLO precision become available
  - NPC, NNFF, BDSSV,MAP, ...

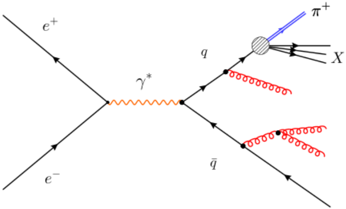


NPC, PRL 135, 041902 (2025)

Definition of SIA

- e^+e^- annihilate to produce a single hadron

$$e^+ + e^- \to \gamma^*/Z \to h + X$$



- The cleanest process for studying FFs (No beam remnants)
- The differential cross section can be written as

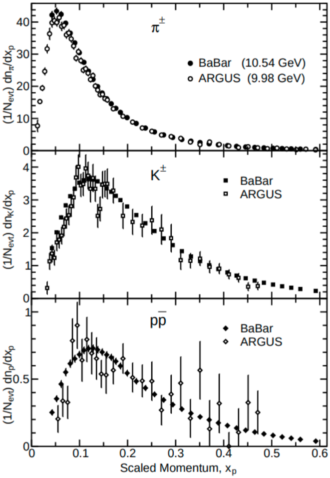
$$\frac{d^2\sigma^h}{dx\,d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma_T^h}{dx} + \frac{3}{4}\sin^2\theta\frac{d\sigma_L^h}{dx} + \frac{3}{4}\cos\theta\frac{d\sigma_A^h}{dx}\,,$$

- Focus on σ_T, σ_L, which can be extracted from γ^*-mediated SIA

Lots of experimental data

- Lots of SIA data was collected
  - BaBar, Belle, LEP, BESIII...
- Future collider will collect more data
  - CEPC, STCF, FCC-ee, ILC...
- Constraints to further improve the precision of FFs using SIA data
  - Precision of SIA
  - Precision of P^{T}_{ik}, known at NNLO [Chen, TZY, Zhu, Zhu, 20; Mitov, Moch, Vogt, Almasy, 06, 07, 11]
  - First step: SIA at N3LO

First N3LO result for hadron production



BaBar, PRD 88, 032011 (2013)

QCD factorization for SIA

- QCD factorization

$$\frac{d\sigma_k^h}{dx} = D_g^h \otimes (2C_{gq}) \sum_{q=1}^{N_f} \sigma_{q\bar{q}}^{(0)} + \sum_{q=1}^{N_f} \left(D_q^h + D_{\bar{q}}^h\right) \otimes \left(C_{qq}^V + C_{q\bar{q}}^V\right) \sigma_{q\bar{q}}^{(0)} + \sum_{q'=1}^{N_f} \left(D_{q'}^h + D_{\bar{q}'}^h\right) \otimes \left(C_{q'q} + C_{q'\bar{q}}\right) \sum_{q=1}^{N_f} \sigma_{q\bar{q}}^{(0)} + \mathcal{O}(\Lambda_{\text{QCD}}/Q)$$

- The formula is valid to all orders in α_s [Collins, Soper, Sterman, 04]
- Wilson Coefficients C_{ij} can be computed in pQCD
- This talk: determine transverse and longitudinal C_{ij} to N3LO

Theory motivations

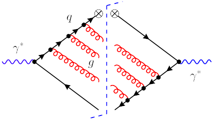
- SIA at NNLO was computed almost 30 years ago [Rijken, van Neerven, 96]
- Its cousin process, Deep Inelastic Scattering (DIS), had already reached N3LO precision back in 2005 [Vermaseren, Vogt, Moch, 05]
  - Derived from imaginary part of forward scattering
- Resummation in the threshold limit for SIA has achieved to
  - N3LL [Cacciari, Catani, 01; Moch, Vogt, 09] and N4LL [Xu, Zhu, 24]
- Resummation in small-x limit for SIA has achieved to NNLL [Albino, Bolzoni, Kniehl, Kotikov, 11; Vogt, 11]
  - Closed form to all orders in $a_s = \alpha_s/(4\pi)$ for LL

$$C_{gq}^{\text{LL}}(x) = \frac{8 C_F a_s \log(x)\, {}_1F_2\left(\frac{5}{4}; \frac{3}{2}, 2; -8 a_s C_A \log^2(x)\right)}{x}$$

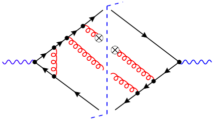
  - Providing more fixed-order data, is it possible to get closed form beyond LL?

Standard way to compute SIA

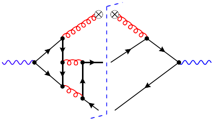
- Cut decompositions ($+$ VVV which is proportional to $\delta(1-x)$)



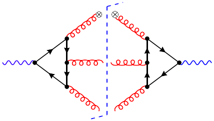
RRR (3 channels, $3 \times 5 = 15$ pieces)



VRR (3 channels, $3 \times 4 = 12$ pieces)



VVR (1 channel, $1 \times 3 = 3$ pieces)



VV*R (2 channels, $2 \times 3 = 6$ pieces)

- Need to compute 36 pieces, cumbersome and easy to make mistakes
- Towards SIA at N3LO by the standard way
  - Master integrals for all pieces available [Magerya, Fekeshazy, 25]

Do we have better approach?

Crossing relations between DIS and SIA

- Partonic DIS at LO

$$\gamma^*(q) + \text{quark}(p) \to \text{quark}(k)$$

$$x_B = \frac{-q^2}{2p\cdot q}$$

- Partonic SIA at LO

$$\gamma^*(q) \to \text{anti-quark}(p) + \text{quark}(k)$$

$$x = \frac{2p\cdot q}{q^2}$$

- Analytic continuation (AC) rule: $p \to -p,\ q \to q,\ x_B \to 1/x$
- Prescription for DIS

$$(p+q)^2 + i0^+ \to x_B = \text{Re}\,[x_B] - i0^+, x_B^* = \text{Re}\,[x_B] + i0^+$$

- Modified AC rule

$$p \to -p, x_B \to 1/x$$
$$(1-x_B)^\epsilon \to x^{-\epsilon}(1-x)^\epsilon e^{i\pi\epsilon}$$
$$(1-x_B^*)^\epsilon \to x^{-\epsilon}(1-x)^\epsilon e^{-i\pi\epsilon}$$

Breakdown of a direct analytic continuation

- The AC fail beyond two-loop order for cross-section quantities [Almasy, Mitov, Moch, Vogt, 06, 07, 11]
  - Information of $(1-x_B)^\epsilon, (1-x_B^*)^\epsilon$ is lost, rely on the AC of $\ln(1-x_B)$

    The critical part of the analytic continuation is that of powers of $\ln(1-x)$, which is given by

    $$\ln(1-x) \xrightarrow{AC_\kappa} \ln(1-x) - \ln x + \kappa i\pi \quad \text{with} \quad \kappa = 0 \text{ or } 1 \,. \tag{8}$$

    For $\kappa = 1$ the real part is taken in the end. It is not clear at all that beyond NLO Eq. (8) is applicable, in either form, to quantities such as physical evolution kernels instead of to (classes of) Feynman diagrams, see the discussions in Refs. [5,8,16].
  - Try to derive P_{ij}^T by AC of DIS, unable to completely fix 3-loop P_{qg}^T

    In summary, these considerations are still not sufficient to definitely fix the right-hand-side of Eq. (13). As an estimate of the remaining uncertainty we suggest to use the offset

    $$\delta P_{\rm qg}^{(2)T}(x) \;=\; \pm 2\zeta_2\beta_0(C_A - C_F)\,(11 + 24\ln x)\,P_{\rm qg}^{(0)T}(x) \,. \tag{38}$$

- The problem was solved in the context of EFT[Chen, TZY, Zhu, Zhu, 20]
  - Derive the complete 3-loop P_{ij}^T for the first time
- We further refine the approach and extend it to the full QCD

Threshold-limit structures

- $x_B \to 1/x$ crosses the branch cut at $1 - x_B$
- Analyse threshold-limit structures by expansion-by-region techniques
- Collinear mode: $l \parallel p \quad \longrightarrow (1 - x_B)^\epsilon$
- Ultra-soft mode: $l \sim 0 \quad \longrightarrow e^{i\pi\epsilon}$

Cuts	Threshold-limit structures for DIS (omit overall $((p+q)^2)^{-3\epsilon}$)
RRR	B_0
VRR	$B_s e^{i\pi\epsilon} + B_c(1 - x_B)^\epsilon$
VVR	$B_{s,s} e^{2i\pi\epsilon} + B_{s,c} e^{i\pi\epsilon}(1 - x_B)^\epsilon + B_{c,c}(1 - x_B)^{2\epsilon}$
VV*R	$B_{s,s} + B_{c,s} e^{i\pi\epsilon}(1 - x_B^*)^\epsilon + B_{c,s}(1 - x_B)^\epsilon e^{-i\pi\epsilon} + B_{c,c}(1 - x_B)^\epsilon(1 - x_B^*)^\epsilon$

- Issues come from terms like $E = B\, e^{i\pi\epsilon}(1 - x_B)^\epsilon$ in VVR and VV*R
  - Correct way: $\mathcal{AC}(E) = B\, e^{2i\pi\epsilon}(1 - x)^\epsilon$
  - Naive way: $\mathcal{AC}(E + c.c.) = 2Be^{i\pi\epsilon} \cos(\pi\epsilon)(1 - x)^\epsilon$
  - Diff: $[\mathcal{AC}(E) + c.c.] - \text{Re}\,[\mathcal{AC}(E + c.c)] = -2B\,(1 - x)^\epsilon \sin^2(\pi\epsilon) \neq 0$

Master formula to determine SIA

- Issues come from VVR and VV*R only instead of RRR and VRR
- Master formula to determine SIA from DIS

$$
\begin{aligned}
d\sigma^{(3),\,\mathrm{SIA}} = \mathrm{Re}\Bigg[& \mathcal{AC}\left(d\sigma^{(3),\,\mathrm{DIS}}\right) \\
& - \mathcal{AC}\left(d\sigma^{(3),\,\mathrm{DIS}}_{\mathrm{VVR}} + \mathrm{c.c.}\right) - \mathcal{AC}\left(d\sigma^{(3),\,\mathrm{DIS}}_{\mathrm{VV^*R}}\Big|_{x_B^* \to x_B}\right) \\
& + \left\{\mathcal{AC}\left(d\sigma^{(3),\,\mathrm{DIS}}_{\mathrm{VVR}}\right) + \mathrm{c.c.}\right\} + \mathcal{AC}\left(d\sigma^{(3),\,\mathrm{DIS}}_{\mathrm{VV^*R}}\right)\Bigg]
\end{aligned}
$$

  - The first term is a direct AC of the DIS result at N3LO [Vermaseren, Vogt, Moch, 05]. It contains the incorrect contributions from VVR and VV*R
  - The incorrect contributions are subtracted in the second line
  - The contributions from the correct AC are restored in the third line
- Missing pieces: VVR and VV*R, compute them directly

VVR+VV*R: Branches separation

- It's straightforward to compute ϵ-expanded VVR and VV*R
- To make AC work properly, crucial to separate branches $(1-x_B)^{n\epsilon}$
- Can be achieved using (canonical) differential equations (DE) [Gehrmann, Remiddi, 99; Henn, 13]

$$d\vec{f}/dx = \epsilon A(x)\vec{f}$$

- Ansatz for master integrals $\vec{f} = \sum_n \vec{f}_n (1-x)^{n\epsilon}$
- Substitute it into the canonical differential equations, we obtain

$$\frac{d\vec{f}_n}{dx} = \epsilon \left(A(x) + \frac{n}{1-x}\right)\vec{f}_n$$

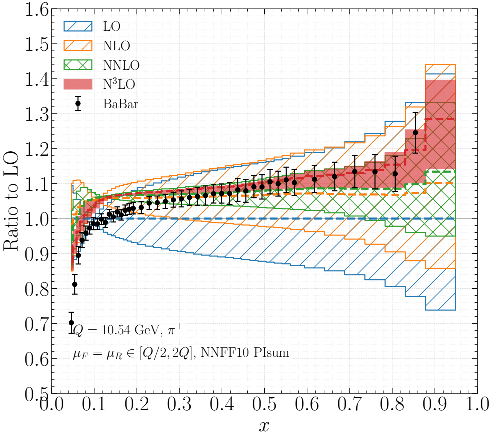
- Determine ϵ-expanded f_n by solving the above canonical DE

Analytic results and checks

- $\delta(1-x)$ term can not be determined by AC
- Determine it by sum rule: $\int_0^1 dx\, x(d\sigma_T/dx + d\sigma_L/dx) = \sigma_{\text{total}}$
- Obtain analytic results for SIA Wilson coeffient at N3LO
- The results are expressed via HPLs
- Checks
  - $\delta(1-x)$ terms agree with those from threshold resummation [Moch, Vogt, 09; Xu, Zhu, 24]
  - Poles in ϵ cancel when using three-loop P_{ij}^T [Chen, TZY, Zhu, Zhu, 20; Mitov, Moch, Vogt, Almasy, 06, 07, 11]
  - Integrating over x for VVR and VV*R, agree with inclusive results by [Jakubcık, Marcoli, Stagnitto, 23]
  - Small-x limit agree with the expansion result from small-x resummation [Albino, Bolzoni, Kniehl, Kotikov, 11; Vogt, 11]

Results

- Data: BaBar data with $\sqrt{s} = 10.54$ GeV [BaBar, PRD 88, 032011 (2013)]
- FFs: NNFF1.0 [EPJC 77, 516 (2017)]
- N3LO greatly reduce the scale uncertainties
- Improve the description of BaBar data
- Improve the description in small-x region? Consider to include small-x resummation



Conclusion and Outlook

Conclusion

- The first N3LO result for hadron production
- Computational complexity is greatly reduced by exploiting the analytic continuation relation between DIS and SIA
  - No need to compute the complicated RRR and VRR
  - Only the simpler VVR and VV*R need to be computed
- $2\times$ reduction of scale uncertainties compared to NNLO
- N3LO results improve the description of the BaBar data

Outlook

- Apply the AC formula to Higgs-mediated DIS for HSIA computation
- Perform small-x resummation at N3LL and compare with data

Thank you for your attention!