Analytic decay width of the Higgs boson to massive bottom quarks

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arxiv: 2310.20514, 2411.07493, 2511.XXXX

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第五届量子场论及其应用研讨会

2025-11-02

Motivation

Higgs is important in the Standard Model.

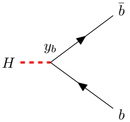
The dominant decay mode of the Higgs boson is $H \to b\bar{b}$.

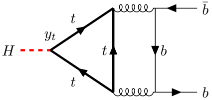
The process $H \to b\bar{b}$ has been observed in LHC [CMS 2018, ATLAS 2018].

The accuracy of y_b will be improved at future electron colliders. [CEPC group 2025].

Decay channels	$b\bar{b}$	$c\bar{c}$	gg	WW^*	ZZ^*
BR	57.7%	2.9%	8.6%	21.5%	2.6%
Rel. Stat. Un.	0.3%	2.2%	1.3%	1.1%	7.6%
Rel. Syst. Un.	0.1%	3.6%	1.8%	0.4%	4.3%
Rel. Total Un.	0.3%	4.2%	2.2%	1.2%	8.7%

Motivation





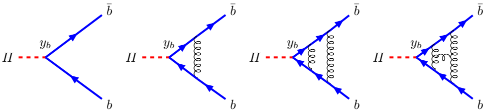
Inclusive decay width up to $\mathcal{O}(y_b^2\alpha_s^4)$ with massless bottom quarks [Gorishnii, Kataev, Larin, Surguladze 1990, Chetyrkin 1997, Baikov, Chetyrkin, Kuhn 2006]

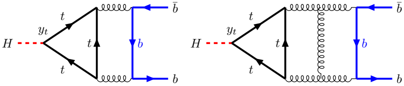
Differential decay width at $\mathcal{O}(y_b^2\alpha_s^2)$ with massive bottom quarks [Bernreuther, Chen, Si 2018, Behring, Bizoń 2020, Somogyi, Tramontano 2020]

Large m_t limit, differential decay width at $\mathcal{O}(\alpha_s^3)$ [Mondini, Schubert, Williams 2020, Chen, Jakubčík, Marcoli, Stagnitto 2023]

Large m_t limit, analytic calculations at $\mathcal{O}(\alpha_s^3)$ and $\mathcal{O}(\alpha_s^4)$ [This talk]

$H \to b\bar{b}$ with bottom quark Yukawa coupling and top quark Yukawa coupling up to $\mathcal{O}(\alpha_s^3)$.



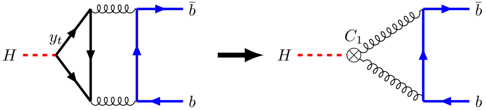


The decay width can be decomposed to

$$\Gamma_{H\to b\bar{b}} = \Gamma^{y_b y_b}_{H\to b\bar{b}} + \Gamma^{y_b y_t}_{H\to b\bar{b}} + \Gamma^{y_t y_t}_{H\to b\bar{b}}. \tag{1}$$

Effective theory

In the effective theory, the top quark can be integrated in the large top quark mass limit, $m_t \to \infty$.



And

$$\text{top quark loop}\,(y_t) \to C_1 \mathcal{O}_1, \quad \alpha_s^{(6)} \to \alpha_s^{(5)} \tag{2}$$

$$\mathcal{O}_1 = H G_{a,\mu\nu} G^{a,\mu\nu} \tag{3}$$

This approximation works exceedingly well.

Effective theory

The Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = -\frac{H}{v}\left(C_1\mathcal{O}_1^R + C_2\mathcal{O}_2^R\right) + \mathcal{L}_{\text{QCD}}\,,$$
$$\mathcal{O}_1 = (G^0_{a,\mu\nu})^2, \quad \mathcal{O}_2 = m_b^0\bar{b}^0 b^0. \tag{4}$$

$$C_1 = -\left(\frac{\alpha_s}{\pi}\right)\frac{1}{12} - \left(\frac{\alpha_s}{\pi}\right)^2\frac{11}{48} + \mathcal{O}(\alpha_s^3)$$
$$C_2 = 1 + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{5}{18} - \frac{1}{3}L_t\right) + \mathcal{O}(\alpha_s^3), \tag{5}$$

$L_t = \log(\mu^2/m_t^2)$ in the on-shell scheme. $H \to b\bar{b}$ can be decomposed into three parts

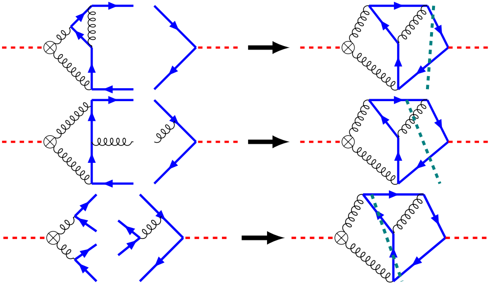
$$\Gamma_{H\to b\bar{b}} = \Gamma^{C_2C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_2}_{H\to b\bar{b}} + \Gamma^{C_1C_1}_{H\to b\bar{b}}. \tag{6}$$

Optical Theorem

$$\Gamma_{Hb\bar{b}} = \frac{\mathrm{Im}(\Sigma)}{m_H}, \tag{7}$$

where Σ represents the forward scattering amplitudes of the process $H \to b\bar{b} \to H$.

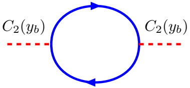
The imaginary part comes from cut diagrams. For example,



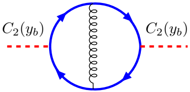
The analytical result of $\color{blue}{\Delta^{C_2C_2}_{2,b\bar{b}}}$ have been calculated in the massive m_b. [Wang, Wang, Zhang 2023]

$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3 \Delta^{C_2C_2}_{3,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^4 \Delta^{C_2C_2}_{4,b\bar{b}}\right]$$

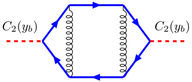
$\Delta^{C_2C_2}_{3,b\bar{b}}$ and $\Delta^{C_2C_2}_{4,b\bar{b}}$ have been calculated in the massless m_b. [Gorishnii, Kataev, Larin, Surguladze 1990, Chetyrkin 1997, Baikov, Chetyrkin, Kuhn 2006]



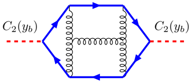
$\Delta^{C_2C_2}_{0,b\bar{b}}$



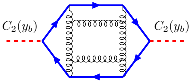
$\Delta^{C_2C_2}_{1,b\bar{b}}$



$\Delta^{C_2C_2}_{2,b\bar{b}}$



$\Delta^{C_2C_2}_{3,b\bar{b}}$



$\Delta^{C_2C_2}_{4,b\bar{b}}$

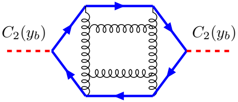
$$\Gamma^{C_2C_2}_{H\to b\bar{b}} = C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^4\Delta^{C_2C_2}_{4,b\bar{b}}\right],$$

$$\Delta^{C_2C_2}_{0,b\bar{b}} = \frac{3\overline{m}_b^2 m_H}{8\pi v^2}\left(1-\frac{4m_b^2}{m_H^2}\right)^{3/2} \approx \frac{3\overline{m}_b^2 m_H}{8\pi v^2} \tag{8}$$

At $\mathcal{O}(\alpha_s^2)$, the finite bottom quark mass effect is quite small, since $\overline{m}_b^2/m_H^2 \approx 5\times 10^{-4}$.

Therefore m_b in propagators for the corrections to $\Delta^{C_2C_2}_{3,b\bar{b}}$ and $\Delta^{C_2C_2}_{4,b\bar{b}}$ can be neglected.

$$\begin{aligned}\Delta^{C_2C_2}_{4,b\bar{b}}(\mu=m_H, m_b\to 0) = \frac{3\overline{m}_b^2 m_H}{8\pi v^2}\times\Big(&-\frac{427735\zeta(7)}{2304}+\frac{25\pi^6}{3402}+\frac{116945\zeta(3)^2}{864}\\ &+\frac{469675\zeta(5)}{432}+\frac{116945\pi^2\zeta(3)}{864}+\frac{667\pi^4}{288}-\frac{12308897\zeta(3)}{1728}-\frac{19637651\pi^2}{31104}\\ &+\frac{5005075879}{497664}\Big), \qquad C_A=3, C_F=4/3, n_f=5.\end{aligned} \tag{9}$$

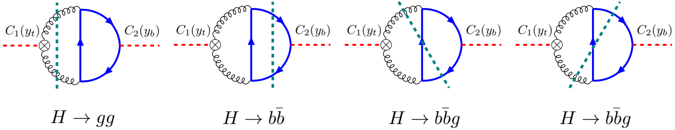


$\Gamma^{C_1C_2}_{H\to b\bar{b}}$ $\left(\Gamma^{y_by_t}_{H\to b\bar{b}}\right)$

There are four cuts at the diagram of $\Delta_1^{C_1C_2}$.

$H \to b\bar{b}$ includes $H \to b\bar{b}$, $H \to b\bar{b}b\bar{b}$, $H \to b\bar{b}g$, $H \to b\bar{b}gg$, $H \to b\bar{b}q\bar{q}$...

$H \to gg$ includes $H \to gg$, $H \to ggg$, $H \to gggg$, $H \to ggq\bar{q}$...



$$\Delta^{C_1C_2}_{1,b\bar{b}}|_{m_b\to 0} = \frac{m_H m_b \overline{m_b}(\mu)}{\pi v^2} C_A C_F \left[-\frac{1}{8}\log^2(\frac{m_b^2}{m_H^2}) - \frac{3}{4}\log\left(\frac{\mu^2}{m_H^2}\right) + \frac{\pi^2}{8} - \frac{19}{8} + \mathcal{O}(\frac{m_b^2}{m_H^2})\right]$$

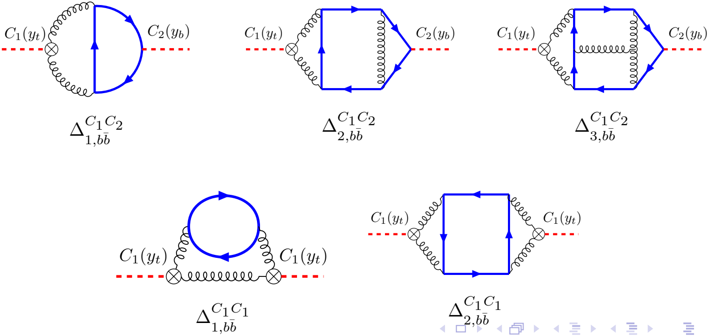
$$\Delta^{C_1C_2}_{1,gg}|_{m_b\to 0} = \frac{m_H m_b \overline{m_b}(\mu)}{\pi v^2} C_A C_F \left[\frac{1}{8}\log^2(\frac{m_b^2}{m_H^2}) - \frac{\pi^2}{8} - \frac{1}{2} + \mathcal{O}(\frac{m_b^2}{m_H^2})\right] \quad (10)$$

$H \to$ hadron has been calculated to α_s^4 [Chetyrkin, Steinhauser 1997, Davies, Steinhauser, Wellmann 2017], in which the m_b in propagators can be set to zero.

$\Gamma_{H\to b\bar{b}}^{C_1C_2}$ $\left(\Gamma_{H\to b\bar{b}}^{y_by_t}\right)$ and $\Gamma_{H\to b\bar{b}}^{C_1C_1}$ $\left(\Gamma_{H\to b\bar{b}}^{y_ty_t}\right)$

$$\begin{aligned}\Gamma_{H\to b\bar{b}}^{C_1C_2} &= C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta_{1,b\bar{b}}^{C_1C_2}+\left(\frac{\alpha_s}{\pi}\right)^2\Delta_{2,b\bar{b}}^{C_1C_2}+\left(\frac{\alpha_s}{\pi}\right)^3\Delta_{3,b\bar{b}}^{C_1C_2}\right],\\ \Gamma_{H\to b\bar{b}}^{C_1C_1} &= C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta_{1,b\bar{b}}^{C_1C_1}+\left(\frac{\alpha_s}{\pi}\right)^2\Delta_{2,b\bar{b}}^{C_1C_1}\right].\end{aligned} \tag{11}$$

The bottom mass need to be kept in the calculations.

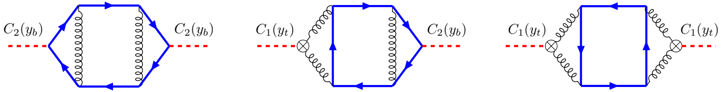


In our works, we analytically calculate three-loop self-energy with massive bottom quarks.

$$
\begin{aligned}
\Gamma^{C_2C_2}_{H\to b\bar{b}} &= C_2C_2\left[\Delta^{C_2C_2}_{0,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)\Delta^{C_2C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_2C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_2C_2}_{3,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^4\Delta^{C_2C_2}_{4,b\bar{b}}\right],\\
\Gamma^{C_1C_2}_{H\to b\bar{b}} &= C_1C_2\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_2}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_2}_{2,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^3\Delta^{C_1C_2}_{3,b\bar{b}}\right],\\
\Gamma^{C_1C_1}_{H\to b\bar{b}} &= C_1C_1\left[\left(\frac{\alpha_s}{\pi}\right)\Delta^{C_1C_1}_{1,b\bar{b}} + \left(\frac{\alpha_s}{\pi}\right)^2\Delta^{C_1C_1}_{2,b\bar{b}}\right], \qquad (12)
\end{aligned}
$$

The higher order bottom mass effects in $\Delta^{C_2C_2}_{3,b\bar{b}}$ and $\Delta^{C_2C_2}_{4,b\bar{b}}$ are small.

To obtain full $\mathcal{O}(\alpha_s^4)$ corrections, we still need to calculate $\Delta^{C_1C_2}_{3,b\bar{b}}$ in the future.



$\Delta^{C_2C_2}_{2,b\bar{b}}$ $\Delta^{C_1C_2}_{2,b\bar{b}}$ $\Delta^{C_1C_1}_{2,b\bar{b}}$

Master integrals calculations

Generating amplitudes → IBP reduction → Master integral (MIs) calculations.

Most MIs can be expressed as the multiple polylogarithms (MPLs) by canonical differential equations.

We define $z \equiv m_H^2/m_b^2$. Two square roots can be rationalized simultaneously.

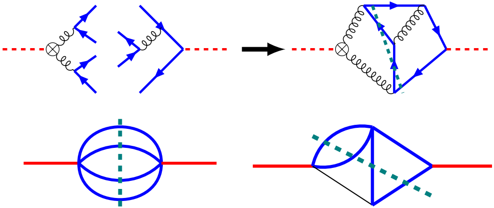
$$r_1 = \sqrt{z(z-4)}, \qquad r_2 = \sqrt{z(z+4)}. \tag{13}$$

Five symbol letters:

$$l_1 = \sqrt{z},\ l_2 = \frac{\sqrt{z}+\sqrt{z+4}}{2},\ l_3 = \sqrt{z+4},\ l_4 = \frac{\sqrt{z}+\sqrt{z-4}}{2},\ l_5 = \sqrt{z-4}. \tag{14}$$

Master integrals calculations

Some MIs contain elliptic integrals and cannot be written as MPLs



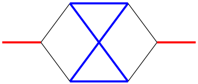
They have been calculated in [Lee, Onishchenko 2019] for the process $e^+e^- \to Q\bar{Q}Q\bar{Q}$.

After choosing a regular basis, only the $\mathcal{O}(\epsilon^0)$ parts of the MIs are needed.

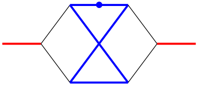
Complete elliptic integrals or one-fold integrals of them. For example,

$$F_1^{4b}(z) = (z-16)f(z) = \frac{16\pi(z-16)}{z}\left[\mathrm{K}(1-k_-)\mathrm{K}(k_+) - \mathrm{K}(k_-)\mathrm{K}(1-k_+)\right],$$
$$F_4^{4b}(z) = \frac{\beta s F_1^{4b}(z)}{s-4} - \int_{16}^{z} dz_1 \frac{4\beta_1(z_1+2)F_1^{4b}(z_1)}{(z_1-16)(z_1-4)^2} \tag{15}$$

Master integrals calculations



$F_{\text{NP-top},1}$



$F_{\text{NP-top},2}$

The two MIs are finite and only the $\mathcal{O}(\epsilon^0)$ parts are needed. $K(x)$ and $E(x)$ are the first and second kind elliptic integrals.

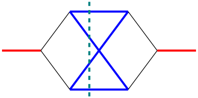
$$\frac{\partial F^{(0)}_{\text{NP-top},1}}{\partial z} = -4zF^{(0)}_{\text{NP-top},2}, \quad \frac{\partial F^{(0)}_{\text{NP-top},2}}{\partial z} = \frac{F^{(0)}_{\text{NP-top},1}}{z^3(z+16)} - \frac{F^{(0)}_{\text{NP-top},2}(2z+16)}{z(z+16)} + R(z),$$

$$F^{(0)}_{\text{NP-top},1} = \int_{4(16)}^{z} \frac{K\left(-\frac{x}{16}\right)K\left(\frac{z}{16}+1\right) - K\left(\frac{x}{16}+1\right)K\left(-\frac{z}{16}\right)}{K\left(-\frac{x}{16}\right)E\left(\frac{x}{16}+1\right) + K\left(\frac{x}{16}+1\right)\left(E\left(-\frac{x}{16}\right) - K\left(-\frac{x}{16}\right)\right)} \times \frac{x(x+16)\sqrt{xz}}{2} R(x)dx,$$

$$F^{(0)}_{\text{NP-top},2} = \int_{4(16)}^{z} \frac{K\left(\frac{x}{16}+1\right)E\left(-\frac{z}{16}\right) + K\left(-\frac{x}{16}\right)\left(E\left(\frac{z}{16}+1\right) - K\left(\frac{z}{16}+1\right)\right)}{K\left(-\frac{x}{16}\right)\left(E\left(\frac{x}{16}+1\right) - K\left(\frac{x}{16}+1\right)\right) + K\left(\frac{x}{16}+1\right)E\left(-\frac{x}{16}\right)} \times \frac{(x+16)x^{3/2}}{(z+16)z^{3/2}} R(x)dx, \qquad R(x) \text{ depends on the sub-sectors.} \tag{16}$$

Master integrals calculations

Consider $H \to b\bar{b}b\bar{b}$



The sub-sectors are

$$F^{(0)}_{\text{NP-top,1}} = \int_{16}^{z} \frac{K\left(-\frac{x}{16}\right) K\left(\frac{z}{16}+1\right) - K\left(\frac{x}{16}+1\right) K\left(-\frac{z}{16}\right)}{K\left(-\frac{x}{16}\right) E\left(\frac{x}{16}+1\right) + K\left(\frac{x}{16}+1\right)\left(E\left(-\frac{x}{16}\right) - K\left(-\frac{x}{16}\right)\right)} \times \frac{x(x+16)\sqrt{xz}}{2} R(x) dx,$$

$$R(x) = \int_{16}^{x} \text{椭圆积分 (s)} ds \tag{17}$$

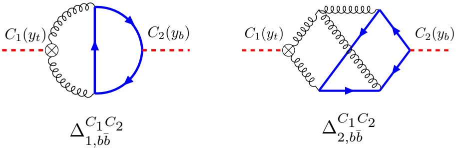
The $F^{(0)}_{\text{NP-top,1}}$ is expressed as the two-fold integrals of elliptic integrals.

Asymptotic expansion

$\Delta^{C_1C_2}_{b\bar{b}}|_{m_b\to 0}$ is induced by soft quarks, which is differs from Sudakov double logarithm.

$z = m_H^2/{m_b}^2$

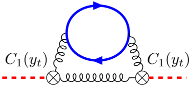
$$\Delta^{C_1C_2}_{1,b\bar{b}}|_{m_b\to 0} = -\frac{m_H m_b \overline{m_b}(\mu)}{8\pi v^2} C_A C_F \log^2(\frac{m_b^2}{m_H^2}) + \cdots$$
$$\Delta^{C_1C_2}_{2,b\bar{b}}|_{m_b\to 0} = -\frac{m_H m_b \overline{m_b}(\mu)}{192\pi v^2} C_A C_F (C_A - C_F) \log^4(\frac{m_b^2}{m_H^2}) + \cdots \quad (18)$$



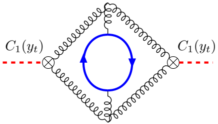
This color structure distinguishes it from the Sudakov double logarithms and shares the same features as the results for the quark-gluon splitting function [Vogt 2010], $Hb\bar{b}$ form factor [Liu, Penin 2017], and off-diagonal "gluon" thrust [Moult, Stewart, Vita, Zhu 2020, Beneke, Garny, Jaskiewicz, Szafron, Vernazza, Wang 2020].

Asymptotic expansion

$$\Delta^{C_1C_1}_{1,b\bar{b}}|_{m_b\to 0} = -\frac{m_H^3}{6\pi v^2}C_A C_F \log(\frac{m_b^2}{m_H^2}) + \cdots$$
$$\Delta^{C_1C_1}_{2,b\bar{b}}|_{m_b\to 0} = -\frac{m_H^3}{72\pi v^2}C_A^2 C_F \log^3(\frac{m_b^2}{m_H^2}) + \cdots \quad (19)$$



$\Delta^{C_1C_1}_{1,b\bar{b}}$



$\Delta^{C_1C_1}_{2,b\bar{b}}$

The logarithm in $\Delta^{C_1C_2}_{1,b\bar{b}}$ from the collinear bottom quark pair.

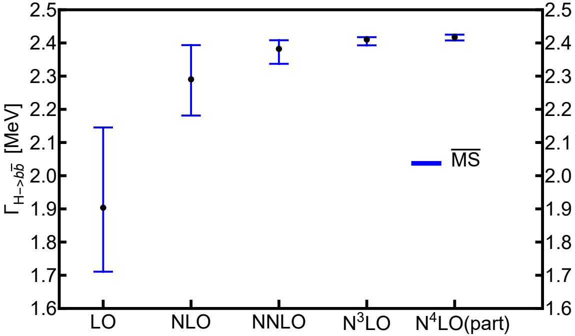
The logarithm in $\Delta^{C_1C_2}_{2,b\bar{b}}$ from the soft gluon splitting to soft bottom quark pair.

We transform all on-shell m_b to $\overline{\mathrm{MS}}$ bottom mass $\overline{m_b}$

$$\overline{m_b}(\mu) = m_b \left(1 - \left(\frac{\alpha_s}{\pi}\right) C_F \left[1 + \frac{3}{4}\log\left(\frac{\mu^2}{m_b^2}\right)\right] + \mathcal{O}(\alpha_s^2)\right) , \tag{20}$$

$\overline{m_b}(\mu = m_H) = 2.78425$ GeV, $m_H = 125.09$ GeV, $m_t = 172.69$ GeV,
$\alpha_s(\mu = m_H) = 0.112715$

$\mu = m_H$	[MeV]	$\Gamma^{C_2C_2}_{Hb\bar{b}}$	$\Gamma^{C_1C_2}_{Hb\bar{b}}$	$\Gamma^{C_1C_1}_{Hb\bar{b}}$
$\Gamma_{H\to b\bar{b}}\,(\overline{\mathrm{MS}})$	$\mathcal{O}(\alpha_s^0)$	1.9036	-	-
	$\mathcal{O}(\alpha_s^1)$	0.3868	-	-
	$\mathcal{O}(\alpha_s^2)$	0.0735	0.0183	-
	$\mathcal{O}(\alpha_s^3)$	0.0048	0.0142	0.0091
	$\mathcal{O}(\alpha_s^4)$	-0.0025	$*$	0.0095



The results in the $\overline{\text{MS}}$ schemes. The error bars denote the scale uncertainties.

$$\Gamma^{\text{QCD}}_{H\to b\bar{b}}\left(\overline{\text{MS}}\right) = 2.417^{+0.008}_{-0.010}\ \text{MeV}. \tag{21}$$

Conclusion

We have calculated the analytic result of the dominant decay channel of the Higgs boson, $H \to b\bar{b}$, at $\mathcal{O}(\alpha_s^3)$ and $\mathcal{O}(\alpha_s^4)$.

The $\mathcal{O}(\alpha_s^3)$ correction increases the NNLO decay rate by 1% due to the large logarithms

The coefficient of the double logarithm at $\mathcal{O}(\alpha_s^3)$ is proportional to $C_A - C_F$, which is a typical color structure in the next-to-leading power resummation with soft quarks.

Our analytic results provide a useful reference to check the resummation formula in future.