Toponium Formation Time

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Motivation

- A pronounced enhancement of the $t\bar{t}$ production cross section near threshold has been observed by CMS:2025kzt and ATLAS:2025mvr.
- CMS: $\sigma(\eta_t) = 8.8^{+1.2}_{-1.4}$ pb.
- ATLAS: $\sigma(\eta_t) = 9.0 \pm 1.3$ pb.
- Significance: $> \mathbf{5}\sigma$ — a new milestone in top-quark physics.

Does the $t\bar{t}$ pair require time to evolve into a physical bound state?

All references say YES.

Two Competing Pictures of Toponium Formation

1. Instantaneous Formation (Quantum)

- The $t\bar{t}$ is immediately projected onto a superposition of nS and continuum.
- Green's function: Fadin:1987wz,Beneke:1999zr..., but claim finite formation time.
- Effectively assumes no formation time; bound-state components appear promptly.

2. Finite-Time Formation (Classic)

- Bound state develops over time due to QCD interactions.
- Formation time estimate: $t_n \sim n^3 \times 1.41 \times 10^{-25}$ s.
- Consistent with classical Coulombic dynamics and relativistic causality.

Different temporal evolution leads to different experimental predictions.

Why Toponium? Unique Time Scales

Toponium is an ideal laboratory due to its extreme properties:

- Production scale: Compton wavelength $\sim 1/m_t \approx 1.14$ am (pointlike).
- Bound-state size: $\langle r \rangle_{nS} \sim 13.3\, n^2$ am.
- Formation time: Classic $t_n^C \sim 1.41\, n^3 \times 10^{-25}$ s VS Quantum $t_n^Q = 0$.
- Top quark lifetime: $\tau_t \approx 5.02 \times 10^{-25}$ s.

Key point: $\tau_t \sim t_1^C$, so the top decay acts as a built-in quantum clock — sensitive to whether the bound state had time to form.

$\Rightarrow$ Decay probability depends on formation dynamics.

Top quark and resummation

- QCD corrections to $t \to W^+ b$: CSL, Oakes, Yuan, PRD1991
- NLO FCNC t decay: JJ Zhang, CSL, J Gao, H Zhang, Z Li, PRL2009
- $t\bar{t}$ AFB and tt: Berger, QH Cao, Chuan-Ren Chen, CSL, H Zhang, PRL2011
- Top Quark Decay at NNLO: Jun Gao, CSL, Hua Xing Zhu, PRL2013
- p_T resummation for $t\bar{t}$: HX Zhu, CSL, HT Li, DY Shao, LL Yang, PRL2013
- Resummation HH at the LHC, Ding Yu Shao, CSL, Hai Tao Li, JHEP2013
- $t\bar{t}$ at small p_T: HT Li,CSL, DY Shao, LL Yang, HX Zhu, PRD2013
- ...

Discriminating Observable: R_b Ratio

The cross-section ratio near threshold is highly sensitive to interference effects:

$$R_b = \frac{\sigma(e^+e^- \to b\bar{b})}{\sum_{q=u,d,s,c,b} \sigma(e^+e^- \to q\bar{q})} \quad \text{at } \sqrt{s} \approx 343 \text{ GeV}$$

- Sensitive to interference between $t\bar{t}$ and γ/Z amplitudes [?].
- The **instantaneous** model predicts strong coherent enhancement.
- The **finite-time** model suppresses early coherence due to delayed formation.
- $\Rightarrow$ Distinct line shapes and R_b values.

$$\text{Pure } \sigma(e^+e^- \to J_t(1S) \to b\bar{b}) \propto e^{-4t_n/\tau_t}$$

Coulomb-like potential and Hamiltonian

- The $t\bar{t}$ interaction is governed by a Coulomb-like QCD potential:

$$V(r) = -\frac{\lambda}{r}, \qquad \lambda = 0.258.$$

- The dynamics are described by the nonrelativistic Hamiltonian Sumino:1992ai:

$$\hat{H} = 2m_t - i\Gamma_t - \frac{\nabla^2}{m_t} + V(r),$$

with:

$$m_t = 172.57\ \mathrm{GeV}, \qquad \Gamma_t = 1.31\ \mathrm{GeV}.$$

- The imaginary term $-i\Gamma_t$ accounts for the top-quark lifetime ($\tau_t \approx 5 \times 10^{-25}$ s).

Coulomb-like potential λ and excress production cross section $\sigma(\eta_t)$

- LO potential $\lambda = 4/3\alpha_s$ with $\mu = 22.3$ GeV:

$$\lambda = 0.210 \quad \text{result} \quad \sigma(\eta_t) = 5.45\,\text{pb}.$$

- NLO potential with $\mu = 22.3$ GeV:

$$\lambda = 0.210 \quad \text{result} \quad \sigma(\eta_t) = 6.51\,\text{pb}.$$

- Parameterized potential:

$$\lambda = 0.258 \quad \text{result} \quad \sigma(\eta_t) = 9.03\,\text{pb}.$$

Properties of nS bound states

- Spin-triplet and spin-singlet bound states:

$$J_t(nS), \quad \eta_t(nS), \qquad n = 1, 2, 3, \ldots$$

- Key parameters:

$$E_n = -\frac{\lambda^2 m_t}{4n^2} = -\frac{2.872}{n^2}\ \mathrm{GeV},$$
$$r_n = \frac{2n^2}{\lambda m_t} = \frac{n^2}{22.3}\ \mathrm{GeV}^{-1},$$
$$v_n = \frac{\lambda}{2n} = \frac{0.129}{n},$$
$$|\psi_{nS}(0)|^2 = \frac{\lambda^3 m_t^3}{8\pi n^3} = \frac{3.51\times 10^3}{n^3}\ \mathrm{GeV}^3.$$

- Nonrelativistic condition holds: $v_n^2 \approx 0.017/n^2$.

Time evolution of the wavefunction

- The time evolution includes exponential decay governed by τ_t:

$$\psi_n(\vec{r}, t) = e^{-i(2m_t+E_n)t} e^{-t/\tau_t} \psi_n(\vec{r}, 0).$$

- $\Rightarrow$ Toponium acts as a **yoctosecond-scale chronometer** of quantum coherence.
- At production $(t = 0^-)$, the system is pointlike:

$$\psi(\vec{r}, 0^-) \simeq \delta(\vec{r}).$$

- Large Γ_t keeps virtuality above $\sqrt{m_t \Gamma_t}$, suppressing soft-gluon divergences.

Quantum vs. classical formation

Quantum (instantaneous) picture

Immediately after creation ($t = 0^+$):

$$\psi(\vec{r}, 0^+) = \sum_n \psi_{nS}(0)\,|nS\rangle + \int d^3k\,\psi_{\vec{k}}(0)\,|\vec{k}\rangle.$$

The state is a coherent superposition of all bound and continuum eigenstates.

Classical (causal) picture

The $t\bar{t}$ pair evolves over a finite time into a bound state.
In the CM frame, quarks produced with $r(0) \sim 1/m_t$ and $v < \sqrt{\lambda/(m_t r(0))}$ form bound states via oscillatory motion.

Classical formation time derivation

- For a top quark with energy E_n:

$$E_n = m_t v^2(t) - \frac{\lambda}{2r(t)}, \qquad \frac{dr}{dt} = \sqrt{\frac{\lambda}{2m_t r(t)} + \frac{E_n}{m_t}}.$$

- Initial condition: $r(0) = 0$; final radius: $r(t_n^C) = 3r_n/4$.
- Solving gives:

$$t_n^C = \frac{(4\pi - 3\sqrt{3})n^3}{3m_t\lambda^2} \approx 1.41\, n^3 \times 10^{-25}\,\mathrm{s}.$$

This provides the characteristic causal formation time of the nS state.

Classical formation picture

The classical mechanics have been used in modeling the toponium formation time (Strassler:1990nw,Fabiano:1997xh). As a scale, top quark lifetime is 5.02×10^{-25} s.

Group	t_n	t_1 (10^{-25}) s	t_2 (10^{-25}) s
Strassler and Peskin	$4r_n/v_n$	9.18	73.5
Fabiano	$2\pi r_n/v_n$	14.4	115
This work	$(4\pi - 3\sqrt{3})r_n/12v_n$	1.41	11.3

Phenomenological parameterization

- Define the dimensionless parameter A to interpolate between scenarios:

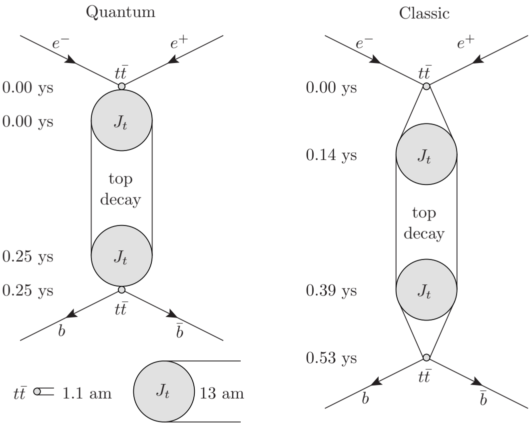
$$t_n = A\, t_n^C, \qquad A = \begin{cases} 0 & \text{Quantum (instantaneous)}, \\ 1 & \text{Classical (causal)}. \end{cases}$$

- The amplitude of the nS state is modified by

$$e^{-At_n^C/\tau_t} \quad \Rightarrow \quad \text{cross-section suppression} \;\sim e^{-2At_n^C/\tau_t}.$$

- A captures the degree of temporal coherence in toponium formation.

Quantum VS Classic formation time



Definition of the Observable R_b

We define the cross-section ratio (Fu:2025yft):

$$R_b = \frac{\sigma(e^+e^- \to b\bar{b})}{\sum_{q=u,d,s,c,b} \sigma(e^+e^- \to q\bar{q})}. \tag{1}$$

- Sensitive to interference and threshold dynamics.
- Clean observable for future lepton colliders (CEPC, FCC-ee).

Amplitude Decomposition Near Top Threshold

The amplitude of the process can be expressed as:

$$\mathcal{M}(e^+e^- \to b\bar{b}) = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}(t\bar{t}) + \cdots, \tag{2}$$

where only the top contribution $\mathcal{M}(t\bar{t})$ is relevant near threshold.

Expanding in α_s and the relative velocity v (Kawabata:2016aya,Chen:2019fla):

$$\mathcal{M}(t\bar{t}) = \mathcal{M}^{\text{One Loop}} + \mathcal{M}^{\text{Two Loop}} + \mathcal{M}^{\text{S-wave}} - \mathcal{M}^{\text{Double Counting}} + \cdots \tag{3}$$

S-wave Contribution and Green Function

The S-wave top–antitop amplitude is proportional to the Green function:

$$\mathcal{M}^{\text{S-wave}}(t\bar{t}) \propto G(0,0,E), \quad E = m_{t\bar{t}} - 2m_t. \tag{4}$$

The two-point Green function $G(\vec{r},0,E)$ satisfies the Schrödinger equation:

$$\left[-\frac{\nabla^2}{m_t} + V(r) - (E + i\Gamma_t)\right] G(\vec{r},0,E) = \delta^3(\vec{r}). \tag{5}$$

It describes production and annihilation of the $t\bar{t}$ pair at the same point.

Spectral Decomposition of the Green Function

With

$$\delta^3(\vec{r}) = \sum_n |n(\vec{r})\rangle\langle n(0)| + \int \mathrm{d}^3\vec{k}\, |\vec{k}(\vec{r})\rangle\langle\vec{k}(0)|, \tag{6}$$

Expanding in bound and continuum states:

$$G(0,0,E) = \sum_n \frac{|n(0)\rangle\langle n(0)|}{E_n - (E + i\Gamma_t)} + \int \mathrm{d}^3\vec{k}\, \frac{|\vec{k}(0)\rangle\langle\vec{k}(0)|}{E_k - (E + i\Gamma_t)}. \tag{7}$$

The continuum state wavefunction is (Sommerfeld:1931qaf,Landau:1991wop):

$$|\vec{k}(0)\rangle = \sqrt{\frac{1}{4\pi}\frac{\pi m_t \lambda/k}{1 - e^{-\pi m_t \lambda/k}}}. \tag{8}$$

Explicit Form of $G(0,0,E)$

Combining bound and continuum contributions:

$$G(0,0,E) = \sum_n \frac{(\lambda m_t)^3}{8\pi n^3} \frac{1}{E_n - E - i\Gamma_t} + \int \frac{k^2\,\mathrm{d}k}{2\pi} \frac{1}{k^2/m_t - E - i\Gamma_t} \frac{m_t\lambda/k}{1 - e^{-\pi m_t \lambda/k}}. \tag{9}$$

- Real part divergent, imaginary part finite (Beneke:2013jia).
- Discrete sum reproduces Breit–Wigner peaks for nS states.

Analytic Form via Lippmann–Schwinger Equation

The leading-order Coulomb Green function (Hoang:1997vs,Beneke:1999qg):

$$G(0,0,E) = \frac{m_t^2}{4\pi}\left\{ i\tilde{v} + \lambda\left[-\frac{\frac{1}{\epsilon} - \gamma_E + \ln 4\pi}{2} + \ln\left(\frac{-2i\tilde{v}m_t}{\mu}\right) - \frac{1}{2} + \gamma_E + \psi\left(1 - \frac{i\lambda}{2\tilde{v}}\right)\right]\right\}, \tag{10}$$

where $\tilde{v} = \sqrt{(E + i\Gamma_t)/m_t}$.

Perturbative Expansion of $G(0,0,E)$

Expanding in powers of λ (or α_s):

$$G(0,0,E) = G^{\rm LO} + G^{\rm NLO} + G^{>\rm NLO}, \tag{11}$$

$$G^{\rm LO}(0,0,E) = \frac{m_t^2}{4\pi}\, i\tilde{v}, \tag{12}$$

$$G^{\rm NLO}(0,0,E) = \frac{m_t^2}{4\pi}\lambda\left[\frac{1}{2}\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right) - \ln\left(\frac{-2i\tilde{v}m_t}{\mu}\right) + \frac{1}{2}\right]. \tag{13}$$

- $G^{\rm LO}$ and $G^{\rm NLO}$ correspond to one- and two-loop amplitudes.
- Higher terms encoded in $G^{>\rm NLO}$.

Avoiding Double Counting

To prevent overlap between perturbative and bound-state terms:

$$\mathcal{M}^{\text{Double Counting}}(t\bar{t}) \propto G^{\text{LO}}(0,0,E) + G^{\text{NLO}}(0,0,E), \tag{14}$$

and the corrected amplitude becomes

$$\mathcal{M}^{\text{S-wave}}(t\bar{t}) - \mathcal{M}^{\text{Double Counting}}(t\bar{t}) \propto G^{>\text{NLO}}(0,0,E). \tag{15}$$

Including Finite Formation Time

The finite toponium formation time modifies each nS term in $G(0,0,E)$ as

$$\frac{|n(0)\rangle\langle n(0)|}{E_n - (E + i\Gamma_t)} \longrightarrow e^{-2An^3 t_1^C/\tau_t} \frac{|n(0)\rangle\langle n(0)|}{E_n - (E + i\Gamma_t)}. \tag{16}$$

- $A = 0$: instantaneous (quantum) formation.
- $A = 1$: finite causal (classical) formation.
- Leads to observable suppression in R_b spectrum.

1. Introduction
2. Toponium formation time
3. Amplitude of $e^+e^- \to b\bar{b}$ near top threshold
4. **Collider probes**
5. Summary

Collider simulations at future lepton colliders

We perform pseudo-experiments at future lepton colliders such as CEPC and FCC-ee. The total uncertainty on R_b at each energy point $\sqrt{s_i}$ is estimated as:

$$\Delta R_b^{\mathrm{PEX}}(i) = \pm 1.1 \times 10^{-4}\,(\mathrm{sys}) \pm \frac{83.2 \times 10^{-4}}{\sqrt{\mathcal{L}_i\ [\mathrm{fb}^{-1}]}}\,(\mathrm{stat}), \tag{17}$$

Main uncertainty sources:

- Potential scheme: 9.8×10^{-5}
- Beam energy spread: 3.6×10^{-5}
- Other effects (detector, background, luminosity): smaller by an order of magnitude

Global χ^2 construction

We construct a global χ^2 function incorporating mass constraints:

$$\chi^2 = \frac{(m_t - m_t^c)^2}{(\Delta m_t)^2} + \sum_{i=1}^{2} \left(\frac{R_b^{\mathrm{PEX}}(i) - R_b^A(i; m_t, \lambda)}{\Delta R_b^{\mathrm{PEX}}(i)} \right)^2 , \tag{18}$$

with parameters:

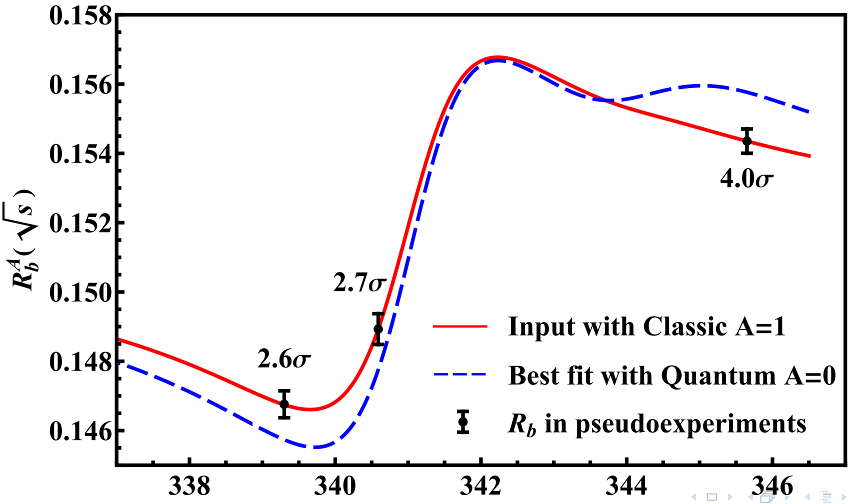
- $m_t^c = 172.57\,\mathrm{GeV}$
- $\Delta m_t = 25\,\mathrm{MeV}$ (CEPC), 40–75 MeV (FCC-ee)
- Degrees of freedom: $\mathrm{ndf} = 2$

Interpretation

The fit compares theoretical predictions under different formation scenarios:

$$A = 0 \quad \text{Quantum}, \qquad A = 1 \quad \text{Classic}.$$

Discrimination power between scenarios (tobe updated)



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Summary

- We have introduced a **collider-accessible framework** to experimentally probe the toponium formation time at the yoctosecond scale ($10^{-24}\,\mathrm{s}$).
- Test whether quantum states: **emerge *instantaneously*, or evolve over a *finite causal time***.

Discussion

Thanks