UV divergences beyond conventional renormalization: A UV-free scheme（紫外自由方案）

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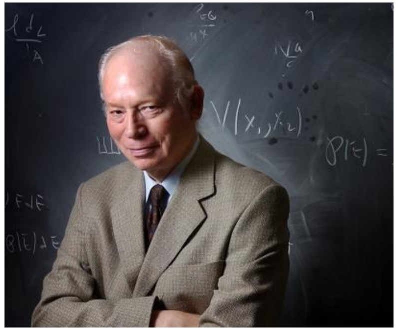
Outline:

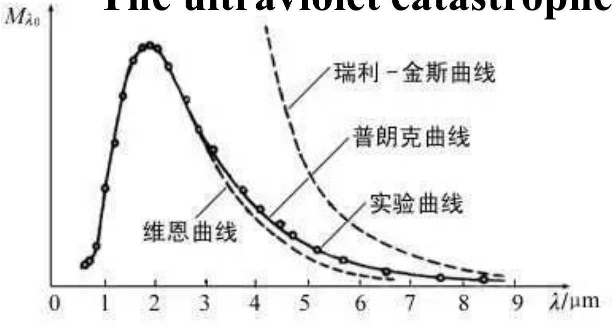
- **I. Background**
- **II. Free flow of ideas**
- **III. Graviton loop in Einstein gravity**
- **IV. Dark energy in QFT**
- **V. Summary and outlook**

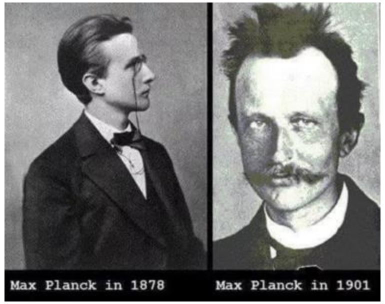
I. Background: UV problems



Physics thrives on crisis

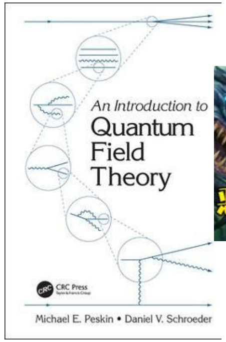
The ultraviolet catastrophe

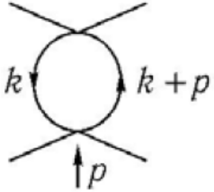




e.g. the electron g-2 $\mathcal{O}(10^{-12})$

UV divergences of loops





UV divergence!

$\mathcal{T}_{\mathrm{F}}$

Not well-defined

Paradigm procedure

Devil

∞

−

Devil

∞

Regularization Renormalization

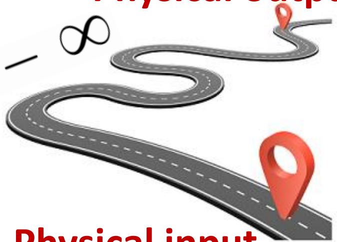
Devil or Angel?

=

Angel

$\infty - \infty$

Physical output



Physical input

Loop road

I. Background: UV problems

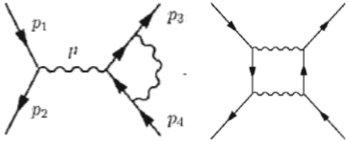
Renormalization

So far so good. **But …**

$\infty - \infty$

Log divergence is OK

The cost: making infinite bare parameters as the foundation of the theory to cancel out UV divergences of loops.



A post-hoc approach for UV divergences

Step out of the log zone

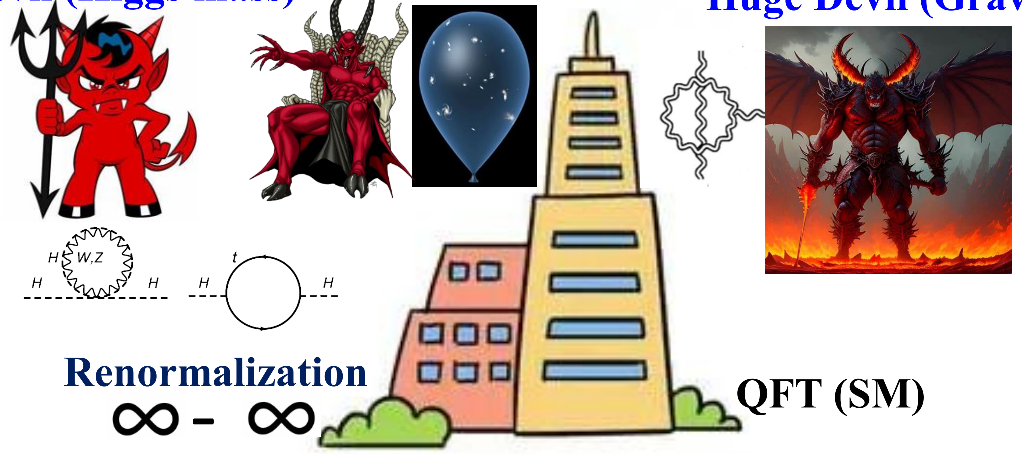
Power-law divergences are problematic

Three devils over the renormalization building

Large Devil (Higgs mass)

Big Devil (Dark energy)

Huge Devil (Gravity)



Renormalization

$\infty - \infty$

QFT (SM)

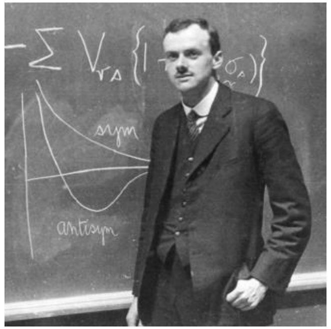
Three UV problems (power-law divergences)

(1) Higgs mass (125GeV)

(2) Einstein gravity (non-renormalizable)

(3) Dark energy (10^{120})

I. Background: Pioneers' Vision 先贤愿景



P.A.M. Dirac

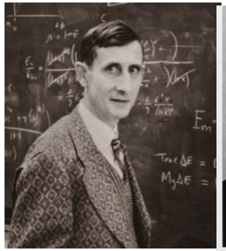
Hence most physicists are very satisfied with the situation. They say: "Quantum electrodynamics is a good theory, and we do not have to worry about it any more." I must say that I am very dissatisfied with the situation, because this so-called "good theory" does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small—not neglecting it just because it is infinitely great and you do not want it!

I disagree with most physicists at the present time just on this point. I cannot tolerate departing from the standard rules of mathematics. Of course, the proper inference from this work is that the basic equations are not right. There must be some drastic change introduced into them so that no infinities occur in the theory at all and so that we can carry out the solution of the equations sensibly, according to ordinary rules and without being bothered by difficulties. This requirement will necessitate some really drastic changes: simple changes will not do, just because the Heisenberg equations of motion in the present theory are all so satisfactory. I feel that the change required will be just about as drastic as the passage from the Bohr orbit theory to the quantum mechanics.

Feynman

QED - The Strange Theory of Light and Matter

The shell game that we play is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate.



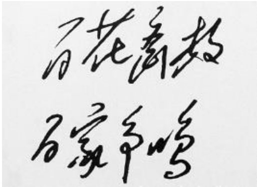
Dyson Schwinger

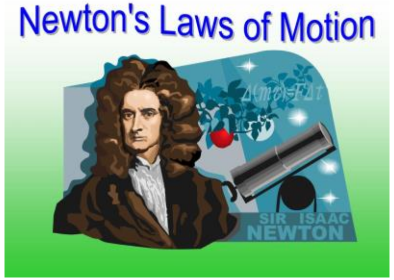
They envisioned a self-consistent quantum theory whose physical predictions were inherently finite, rendering the subtraction of infinities fundamentally unnecessary.

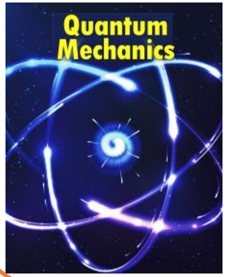
Is it possible to have a method where UV divergences are absent by construction?

Next

II. Free flow of ideas --- UV-free scheme







Negligible

Starting point: physical laws evolve locally.

Low-energy corrections

Loop

UV regions (Planck scale)

Loop

Negligible?!

Regularization & renormalization

A conceptual breakthrough:

P-V

Derivative method

UV-free scheme

arXiv:2305.18104

Commun. Theor. Phys. 78, 015201

A presumption:

Local

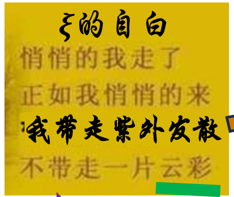
Physical contributions of loops are finite with contributions from UV regions of momenta being insignificant.

To obtain the physical results of loops, an equation is introduced

$$\mathcal{T}_{\mathrm{F}} \Rightarrow \mathcal{T}_{\mathrm{P}} = \left[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\right]_{\{\xi_1, \cdots, \xi_i\} \to 0} + C$$

(★primary antiderivative★ + boundary constant)

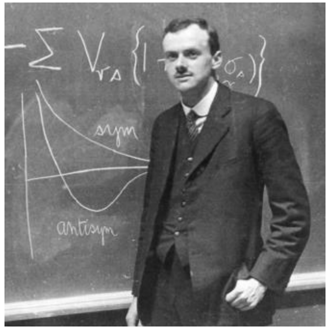
or $$\mathcal{T}_{\mathrm{P}} = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi^n}\right]_{\xi \to 0} + C,$$ (Similar to $E_p = -\frac{GMm}{r}$+C)



arXiv:2403.09487

$\mathcal{T}_{\mathrm{F}}$ → $\mathcal{T}_{\mathrm{P}}$

$e.g.$ $f(\mathrm{x}) = 1 + x^1 + x^2 + x^3 + x^4 + \ldots$ → $\frac{1}{1-x}$ Analytic continuation



P.A.M. Dirac

Mathematical beauty

$$\mathcal{T}_{\mathrm{P}} = \left[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\right]_{\{\xi_1, \cdots, \xi_i\} \to 0} + C$$

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or

$$\mathcal{T}_{\mathrm{P}} = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi^n}\right]_{\xi \to 0} + C,$$

arXiv:2403.09487

$$\boxed{\mathcal{T}_{\mathrm{P}} = \mathcal{T}_{\mathrm{F}}^{[n]}(\xi) + C}$$

Here $\left[\int (d\xi) \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi}\right]_{\xi \to 0}$ is denoted as $\mathcal{T}_{\mathrm{F}}^{[1]}(\xi)$

$\mathcal{T}_{\mathrm{P}}$ has UV transformation invariance

The loop calculation involves the following steps:

(a) Write down the transition amplitude $\mathcal{T}_{\mathrm{F}}$ by the Feynman rules.

(b) Add ξ to the denominator of a propagator in the loop and take the derivative a sufficient number of times; Feynman parameterization is required for cases involving multiple propagators.

(c) Integrate over the loop momentum to obtain a UV-convergent result.

(d) Evaluate the antiderivative with respect to ξ, then take the limit $\xi \to 0$ of the primary antiderivative, thereby obtaining the UV-convergent primary antiderivative.[1]

(e) Set the boundary constant C at specified reference energy points to derive the final result $\mathcal{T}_{\mathrm{P}}$.

To a new route: It first reproduces the successful renormalization results, and then gives an explanation to the three UV problems.

UV-free scheme:

assume that the physical transition amplitude $\mathcal{T}_{\mathrm{P}}$ with propagators can be described by an equation of

$$\mathcal{T}_{\mathrm{P}} = \Big[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\Big]_{\{\xi_1, \cdots, \xi_i\} \to 0} + C \,, \quad (1)$$

a. Tree-level:

the photon propagator $\frac{-ig_{\mu\nu}}{p^2+i\epsilon}$,

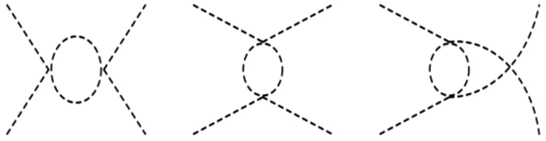
$$\mathcal{T}_{\mathrm{F}}(\xi) = \frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon}, \quad \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi} = \frac{-ig_{\mu\nu}(-1)}{(p^2+\xi+i\epsilon)^2},$$

$$\Big[\int d\xi \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi}\Big] = \frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon}, \quad \text{with } \; C = 0$$

$$\mathcal{T}_{\mathrm{P}} = \Big[\int d\xi \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi}\Big]_{\xi \to 0} = \frac{-ig_{\mu\nu}}{p^2+i\epsilon}$$

the gauge field propagator restored

b. Loop-level Log:



ϕ^4 theory

The physical scattering amplitude

$$\mathcal{T}_{\mathrm{P}}(s) = \Big[\int d\xi \frac{\partial \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi}\Big]_{\xi \to 0} + C_1$$

$$= \Big[\frac{-\lambda^2}{2}\int d\xi \int \frac{d^4k}{(2\pi)^4} \frac{-i}{(k^2-m^2+\xi)^2} \frac{i}{(k+q)^2-m^2}\Big]_{\xi \to 0} + C_1 \,,$$

$$\mathcal{T}_{\mathrm{P}}(s) = \frac{-i\lambda^2}{32\pi^2}\int_0^1 dx \log[m^2 - x(1-x)s] + C_1 \,.$$

$\boxed{\text{A freedom of } \xi \text{ in propagators}}$

Considering the renormalization conditions, $s = 4m^2$, $t = u = 0$. $\Rightarrow$ $C_1 = \frac{i\lambda^2}{32\pi^2}\int_0^1 dx \log[m^2 - 4m^2 x(1-x)] \,.$

No troublesome UV divergence in loop calculations!

In massless limit $\mathcal{T}_{\rm P}=\mathcal{T}_{\rm P}(s)+\mathcal{T}_{\rm P}(t)+\mathcal{T}_{\rm P}(u)$

$$s=-t=-u=\mu^2 \qquad =\frac{i\lambda^2}{32\pi^2}\Big(\log\frac{\mu^2}{s}+\log\frac{\mu^2}{-t}+\log\frac{\mu^2}{-u}\Big)$$

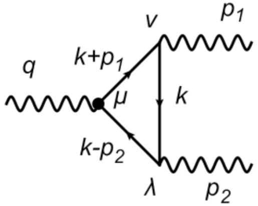
the n-point physical correlation function $G_{\rm P}^{(n)}$ can be set by the physical field $\phi_{\rm P}(x)$ with $\phi_{\rm P}(x) = Z^{1/2}\phi(x,\mu)$, and the rescaling factor Z is finite here. The local correlation function $G^{(n)}$ (shorthand for a full expression $G^{(n)}(\phi,\lambda,m,\cdots,\mu)$) in the perturbation expansion can be written as $G^{(n)} = Z^{-n/2}G_{\rm P}^{(n)}$. Considering $\frac{dG_{\rm P}^{(n)}}{d\mu}=0$, the variation of μ in the massless limit can be described by a relation

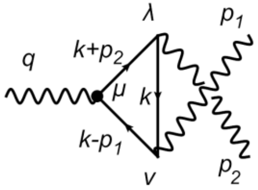
$$(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial\lambda}+n\gamma)G^{(n)}=0\,.$$

This is the form of the Callan-Symanzik equation [5, 6], and we have another picture about it in UV-free scheme. The μ-dependent term in UV-free scheme is from the boundary constant C. For the ϕ^4 theory in the massless limit, the one-loop result of the parameter γ is zero ($\mathcal{T}_{\rm P}^{2p}=0$). The beta function can be derived by Eq. (10), with the result

$$\beta=-i\mu\frac{\partial}{\partial\mu}\mathcal{T}_{\rm P} =\frac{3\lambda^2}{16\pi^2}+\mathcal{O}(\lambda^3)\,.$$

μ running in a finite picture





γ^5 the original form

$$\partial_\mu j^{\mu5}=iq_\mu\mathcal{T}_{\rm P}^{\mu\nu\lambda}\epsilon^*_\nu(p_1)\epsilon^*_\lambda(p_2) =-\frac{e^2}{16\pi^2}(\frac{2}{3}-2\log r)\varepsilon^{\alpha\nu\beta\lambda}F_{\alpha\nu}F_{\beta\lambda}$$

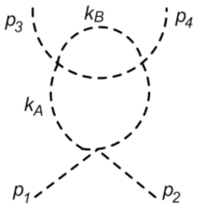
Taking $C_0=2\log r$

If $C_0=-\frac{1}{3}$

SM self-consistent

charge values of quarks coincidence, or correlation?

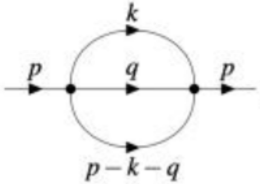
two-loop transition



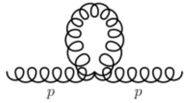
$$\mathcal{T}_{\rm P}=\Big[\int d\xi\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]_{\xi\to0}+C$$
$$=\Big[\frac{(-i\lambda)^3}{2}\int d\xi\int\frac{d^4k_A}{(2\pi)^4}\frac{d^4k_B}{(2\pi)^4}\frac{i}{k_A^2-m^2}\frac{i}{(k_A+q)^2-m^2}\times\frac{-i}{(k_B^2-m^2+\xi)^2}\frac{i}{(k_B+k_A+p_3)^2-m^2}\Big]_{\xi\to0}+C$$

with $q=p_1+p_2$

Log divergences are OK



Two-loop calculation by hand



=0

The transition:

Regularization & renormalization

bridge

UV-free scheme

Interpretation:

An integral with logarithmic divergence

$$I_4 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-\Delta)^2}$$

$$\text{Taking } \mathcal{T}_\text{F}^d = 1/(k^2-\Delta)^2$$

In the dimensional regularization

$$I_D = \int \frac{d^Dk}{(2\pi)^D}\left(\left[\int d\xi \frac{\partial \mathcal{T}_\text{F}^d(\xi)}{\partial \xi}\right]_{\xi\to 0} + C\right)$$

$$= \int \frac{d^Dk}{(2\pi)^D}\left(\left[\int d\xi \frac{-2}{(k^2-\Delta+\xi)^3}\right]_{\xi\to 0} + C\right)$$

$$I_D = \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2-\Delta)^2}$$

Renormalization is required

The momentum integration and the antiderivative are exchanged

$$I'_D = \left[\int d\xi \int \frac{d^Dk}{(2\pi)^D} \frac{\partial \mathcal{T}_\text{F}^d(\xi)}{\partial \xi}\right]_{\xi\to 0} + C$$

$$= \left[\int d\xi \int \frac{d^Dk}{(2\pi)^D} \frac{-2}{(k^2-\Delta+\xi)^3}\right]_{\xi\to 0} + C$$

$$\xrightarrow{D=4} \quad I'_4 = \left[\int d\xi \int \frac{d^4k}{(2\pi)^4} \frac{-2}{(k^2-\Delta+\xi)^3}\right]_{\xi\to 0} + C$$

Non-commutation **UV-free scheme**

A natural transition

Regularization & renormalization

Top-down structure

Top $e_0 + \Delta e$

$\infty - \infty$

Down $e = e_0 + \Delta e$

UV region

Low-energy region

UV-free scheme

Bottom-up structure

Up UV physics being free!

UV physics being free

Bottom $\mathrm{e} = e_\mu + \Delta e_\mu$

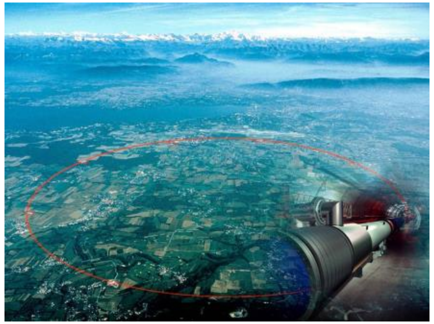
Locally finite

An illustration: electron physical charge $e = e_0 + \Delta e = e_\mu + \Delta e_\mu$

UV divergences and μ

Only μ

The hierarchy problem (c. Loop-level Λ^2, Λ^4)

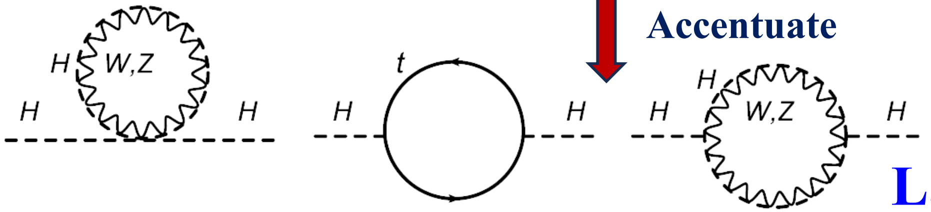


LHC

Higgs boson

125 GeV

Accentuate



The hierarchy problem

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right]$$

Fine-tuning!

A real problem for renormalization!

Large Devil (Higgs mass)

Higgs in the first diagram

$$\mathcal{T}_{\rm P}^{H1} = \left[\int d\xi_1 d\xi_2 \frac{\partial \mathcal{T}_{\rm F}^{H1}(\xi_1,\xi_2)}{\partial\xi_1\partial\xi_2}\right]_{\{\xi_1,\xi_2\}\to 0} + C$$

$$= \left[(-3i)\frac{m_H^2}{2v^2}\int d\xi_1 d\xi_2 \int \frac{d^4k}{(2\pi)^4} \times \frac{2i}{(k^2 - m_H^2 + \xi_1 + \xi_2)^3}\right]_{\{\xi_1,\xi_2\}\to 0} + C\,.$$

After integral, one has

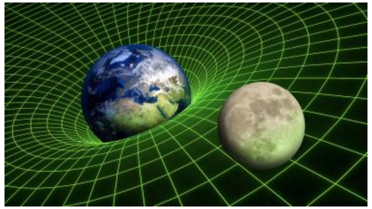
$$\mathcal{T}_{\rm P}^{H1} = i\frac{3m_H^4}{32\pi^2 v^2}\log\frac{1}{m_H^2} + C$$

$$= i\frac{3m_H^4}{32\pi^2 v^2}\log\frac{\mu^2}{m_H^2}\,.$$

125 GeV Higgs can be obtained without fine-tuning, i.e., an alternative interpretation within SM.

III. Graviton loop in Einstein gravity

$$\mathcal{S} = \int d^4X\sqrt{-g}\,\Big[-\frac{2}{\kappa^2}R + \mathcal{L}_{\rm M}\Big] \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$



Huge Devil (Gravity)

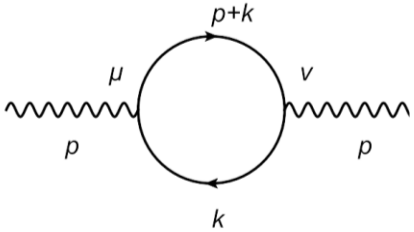
Loops: **Renormalization** → ~~→~~ **Non-renormalizable Einstein Gravity**

? → **Einstein Gravity cannot be quantized!?**

Plan B: **Another alternative method**
UV-free scheme

For the primary antiderivative $\xi-$**dependent choice**

$$\mathcal{T}_{\rm P}^{t2n} = A\Big[\frac{(\xi+\Delta)^n}{n!}(\log|\xi+\Delta| - (\sum_{l=1}^{n}\frac{1}{l}))\Big]_{\xi\to 0} + C_1$$
$$= A\frac{\Delta^n}{n!}\log|\Delta| + C\,.$$



$$\mathcal{T}_{\rm P}^{\mu\nu} = -\frac{ie^2}{2\pi^2}\int_0^1 dx(p^\mu p^\nu - g^{\mu\nu}p^2)x(1-x)$$
$$\times\log(m^2 - p^2x(1-x)) + C^{\mu\nu}\,,$$

with the Ward identity automatically preserved by the primary antiderivative.

arXiv: 2403.09487

One-loop propagator

$$\frac{i\Pi_{\mu\nu\alpha\beta}/2}{p^2+i\epsilon} \qquad \Pi_{\mu\nu\alpha\beta}=\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}-\eta_{\mu\nu}\eta_{\alpha\beta}$$

The $\mu\nu \leftrightarrow \alpha\beta$ asymmetry involved at one-loop level

$$\mathcal{T}_{\rm P}^{a}=\Big[i\kappa^2\frac{i\Pi_{\mu_3\nu_3\mu_4\nu_4}}{2}\frac{i}{16\pi^2}\Big(V^{\mu_3\nu_3\mu_4\nu_4|\lambda_1\mu\nu\lambda_2\alpha\beta}p_{\lambda_1}p_{\lambda_2}\times(\xi_1-\xi_1\log\xi_1)+\frac{V^{\mu\nu\alpha\beta|\lambda_3\mu_3\nu_3\lambda_4\mu_4\nu_4}\eta_{\lambda_3\lambda_4}}{4}\times(\xi_1^2\log\xi_1-\frac{3}{2}\xi_1^2)\Big)\Big]_{\xi_1\to 0}+C_a^{\mu\nu\alpha\beta}\,.$$
$$=0.$$

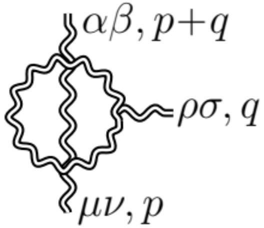
$$\mathcal{T}_{\rm P}^{b}=\frac{(2i\kappa)^2}{2}\frac{i}{16\pi^2}\int_0^1 dx(-\frac{1}{4})\Big\{\frac{1}{16}[40x^2(1-x)^2p^\mu p^\nu p^\alpha p^\beta +2p^2((1-2x)^2(15x^2-15x-2)(p^\mu p^\nu\eta^{\alpha\beta}+p^\alpha p^\beta\eta^{\mu\nu}) +(10x^4-20x^3+17x^2-7x+2)(p^\nu p^\beta\eta^{\mu\alpha}+p^\mu p^\beta\eta^{\nu\alpha} +p^\nu p^\alpha\eta^{\mu\beta}+p^\mu p^\alpha\eta^{\nu\beta}))+p^4((115x^4-230x^3+103x^2 +12x+1)\eta^{\mu\nu}\eta^{\alpha\beta}+(85x^4-170x^3+139x^2-54x+3) \times(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}))]\log\frac{1}{-p^2x(1-x)}\Big\}+C_b^{\mu\nu\alpha\beta}\,.$$

$$\mathcal{T}_{\rm P}^{c}=(-1)(i\kappa)^2\frac{4i}{16\pi^2}\int_0^1 dx(-\frac{1}{4})\Big\{\frac{1}{4}[4(4x^4-8x^3+2x^2 +2x+1)p^\mu p^\nu p^\alpha p^\beta+p^2((8x^4-16x^3+4x^2+4x-1) \times(p^\nu p^\beta\eta^{\mu\alpha}+p^\mu p^\beta\eta^{\nu\alpha}+p^\nu p^\alpha\eta^{\mu\beta}+p^\mu p^\alpha\eta^{\nu\beta}) +2x(14x^3-24x^2+13x-4)p^\mu p^\nu\eta^{\alpha\beta}+2p^\alpha p^\beta\eta^{\mu\nu} \times(14x^4-32x^3+25x^2-6x-1))+p^4(2x(11x^3-22x^2 +13x-2)(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha})+(12x^4-24x^3+16x^2 -4x+1)\eta^{\mu\nu}\eta^{\alpha\beta})]\log\frac{1}{-p^2x(1-x)}\Big\}+C_c^{\mu\nu\alpha\beta}\,,$$

$n-$loop with overlapping divergences

superficial degree of divergence $2n+2$ $\qquad \mathcal{T}_{\rm P}^{t2n}=A\frac{\Delta^n}{n!}\log|\Delta|+C$

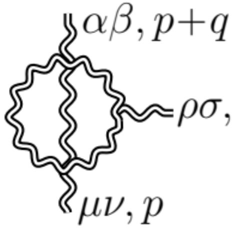
$$\mathcal{T}_{\rm P}^{\rm total}=\mathcal{T}_{\rm P}^{t2(n+1)}+\mathcal{T}_{\rm P}^{t2n}+\cdots+\mathcal{T}_{\rm P}^{t2}+\mathcal{T}_{\rm P}(\log)+\mathcal{T}_{\rm P}(\text{finite})\,,$$



$$\mathcal{T}_{\rm P}^{V}=(2i)^3\kappa^5(\frac{i}{16\pi^2})^2\int_0^1 dx\int_0^1 dy\int_0^1 dz\frac{z}{2^4(1-z)^4}\times\Big\{A_3\frac{\Delta_0^3}{3!}+A_2\frac{\Delta_0^2}{2!}+A_1\Delta_0+A_0\Big\}\log\frac{1}{\Delta_0}+C^{\mu\nu\alpha\beta\rho\sigma}$$

Here Δ_0 is $\Delta_0=b^2-ac$, with $a=z+(1-z)x(x-1)$, $b=yzq+(1-z)x(x-1)p$, $c=yzq^2+(1-z)x(x-1)p^2$. A_3, A_2, A_1, A_0 are coefficients related to sextic, quartic, quadratic, logarithmic divergence inputs respectively.

arXiv: 2403.09487



$$
\begin{aligned}
A_3 = \frac{z-1}{64a^8} \big(& [440a^2 + a(1564x^2 + 1300x + 23)(z-1) \\
& +4(281x^4 - 562x^3 + 683x^2 - 402x + 273)(z-1)^2] \\
& \times\eta^{\mu\nu}(\eta^{\alpha\rho}\eta^{\beta\sigma} + \eta^{\alpha\sigma}\eta^{\beta\rho}) + [744a^2 + a(1932x^2 + 44x \\
& +1203)(z-1) + 4(297x^4 - 594x^3 + 1563x^2 - 1266x \\
& +673)(z-1)^2]\eta^{\rho\sigma}(\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}) + [440a^2 \\
& +a(1564x^2 - 1100x + 2423)(z-1) + 4(281x^4 \\
& -562x^3 + 683x^2 - 402x + 273)(z-1)^2]\eta^{\alpha\beta}(\eta^{\mu\rho}\eta^{\nu\sigma} \\
& +\eta^{\mu\sigma}\eta^{\nu\rho}) + [1032a^2 + a(3396x^2 - 3020x + 801) \\
& \times(z-1) + 4(591x^4 - 1182x^3 + 1101x^2 - 510x + 215) \\
& \times(z-1)^2](\eta^{\alpha\rho}\eta^{\beta\nu}\eta^{\mu\sigma} + \eta^{\alpha\nu}\eta^{\beta\rho}\eta^{\mu\sigma} + \eta^{\alpha\nu}\eta^{\beta\sigma}\eta^{\mu\rho} \\
& +\eta^{\alpha\sigma}\eta^{\beta\nu}\eta^{\mu\rho} + \eta^{\alpha\rho}\eta^{\beta\mu}\eta^{\nu\sigma} + \eta^{\alpha\mu}\eta^{\beta\rho}\eta^{\nu\sigma} + \eta^{\alpha\mu}\eta^{\beta\sigma}\eta^{\nu\rho} \\
& +\eta^{\alpha\sigma}\eta^{\beta\mu}\eta^{\nu\rho}) + [1696a^2 + a(4844x^2 + 848x + 4147) \\
& \times(z-1) + 4(787x^4 - 1574x^3 + 2521x^2 - 1734x \\
& +795)(z-1)^2]\eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma}\big) .
\end{aligned}
$$

Parameter A_0 $(l = p + q)$

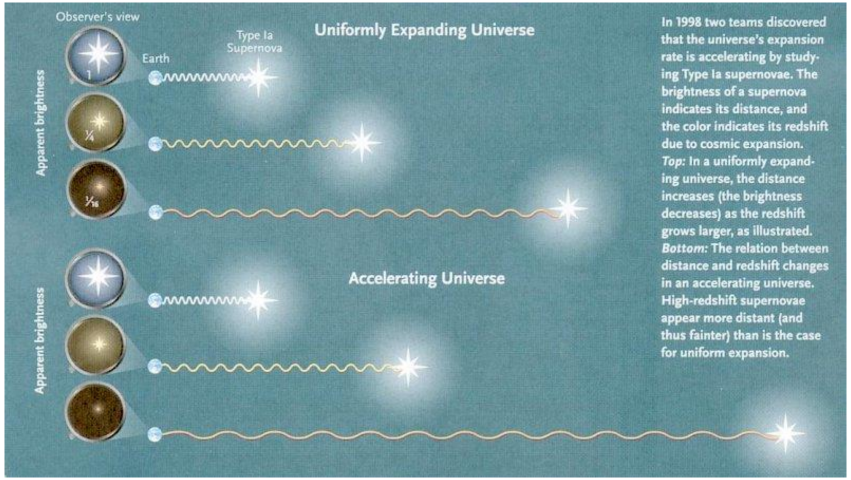
In the case of $p^2 = l^2 = 0$, the result is

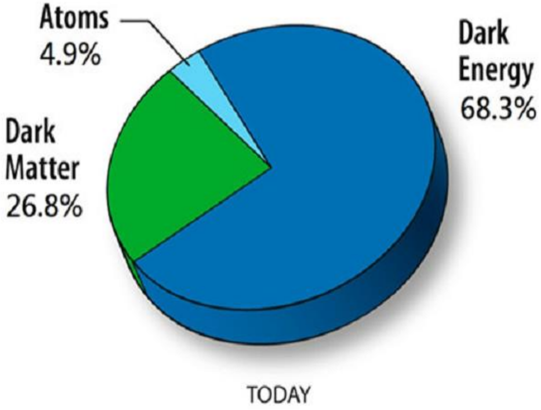
$$
\begin{aligned}
A_0 = -\frac{(z-1)^3}{64a^8}\Big\{ & 16y^3z^3[a^3(8x^2-8x+7) - 2a^2(4x^4-8x^3+16x^2-12x+11)yz + a(14x^4-28x^3+53x^2-39x+28)y^2z^2 \\
& -14(x^2-x+1)^2y^3z^3]q^\alpha q^\beta q^\mu q^\nu q^\rho q^\sigma - 8y^2z^2[a^4(6-9x+9x^2) + a^2(x-1)x(47-75x+83x^2-16x^3+8x^4)y(1-z)z \\
& -2a(x-1)x(21-25x+32x^2-14x^3+7x^4)y^2(1-z)z^2 + 28(x-1)x(1-x+x^2)^2y^3(1-z)z^3 + a^3((x-1)x(-12+41x \\
& -41x^2)(1-z) + (-7+14x-6x^2-16x^3+8x^4)yz)](p^\rho q^\alpha q^\beta q^\mu q^\nu q^\sigma + p^\sigma q^\alpha q^\beta q^\mu q^\nu q^\rho) + 8y^2z^2[a^4(7-9x+9x^2) - 28 \\
& \times(x-1)x(1-x+x^2)^2y^3(1-z)z^3 + ay^2z^2((x-1)x(49-57x+59x^2-4x^3+2x^4)(1-z) + (-14+45x-73x^2+56x^3 \\
& -28x^4)yz) + a^3((x-1)x(-3+29x-29x^2)(1-z) + (-7-9x+x^2+16x^3-8x^4)yz) + a^2yz((x-1)x(-9-21x+13x^2 \\
& +16x^3-8x^4)(1-z) + (12-17x+37x^2-40x^3+20x^4)yz)](p^\beta q^\alpha q^\mu q^\nu q^\rho q^\sigma + p^\alpha q^\beta q^\mu q^\nu q^\rho q^\sigma) - 4yz[2a^5 + 56(x-1)^2 \\
& \times x^2(1-x+x^2)^2y^3(1-z)^2z^3 - a^4(9(x-1)x(1-z) + 2(3-7x+7x^2)yz) + 2a(x-1)xy^2(1-z)z^2((x-1)x(-35+29x \\
& -17x^2-24x^3+12x^4)(1-z) + (14-45x+73x^2-56x^3+28x^4)yz) + a^2(x-1)xy(1-z)z(10(x-1)x(5-6x+6x^2) \\
& \times(1-z) + (-38+31x+9x^2-80x^3+40x^4)yz) + 2a^3(-3(x-1)^2x^2(1-z)^2 + (x-1)x(17-18x+18x^2)y(1-z)z \\
& +(2-3x+11x^2-16x^3+8x^4)y^2z^2)](p^\beta p^\rho q^\alpha q^\mu q^\nu q^\sigma + p^\alpha p^\rho q^\beta q^\mu q^\nu q^\sigma + p^\beta p^\sigma q^\alpha q^\mu q^\nu q^\rho + p^\alpha p^\sigma q^\beta q^\mu q^\nu q^\rho) - 8yz[a^5(5-6x \\
& +6x^2) + 28(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3 + 2a^4((x-1)x(7-x+x^2)(1-z) + (-21+34x-30x^2-8x^3+4x^4) \\
& \times yz) + 2a(x-1)xy^2(1-z)z^2(3(x-1)x(-7+6x-2x^2-8x^3+4x^4)(1-z) + (14-45x+73x^2-56x^3+28x^4)yz) \\
& +a^3(2(x-1)^2x^2(3+2x-2x^2)(1-z)^2 + (x-1)x(-47+107x-91x^2-32x^3+16x^4)y(1-z)z + (53-50x+30x^2 \\
& +40x^3-20x^4)y^2z^2) + a^2yz((x-1)^2x^2(-30+87x-79x^2-16x^3+8x^4)(1-z)^2 + (x-1)x(11-30x+34x^2-8x^3 \\
& +4x^4)y(1-z)z + 2(-8-11x+25x^2-28x^3+14x^4)y^2z^2)]p^\alpha p^\beta q^\mu q^\nu q^\rho q^\sigma - 8[a^6 + 28(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3 \\
& \times z^3 + a^5(2(x-1)x(1-x+x^2)(1-z) + (-2-5x+5x^2)yz) + 2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(-14+4x+15x^2 \\
& -38x^3+19x^4)(1-z) + (14-45x+73x^2-56x^3+28x^4)yz) + a^4((1-2x)^2(x-1)^2x^2(1-z)^2 + (x-1)x(7-8x+8x^2) \\
& \times y(1-z)z + (1+8x-16x^3+8x^4)y^2z^2) + a^2(x-1)xy(1-z)z((x-1)^2x^2(-1+12x-12x^2)(1-z)^2 + 3(x-1)x \\
& \times(-1-11x+39x^2-56x^3+28x^4)y(1-z)z + 2(-8-11x+25x^2-28x^3+14x^4)y^2z^2) + a^3(x-1)x(1-z)(2(x-1)^2 \\
& \times x^2(1-x+x^2)(1-z)^2 + (x-1)x(10+9x-9x^2)y(1-z)z + (18-25x+79x^2-108x^3+54x^4)y^2z^2)](p^\alpha p^\beta p^\rho q^\mu q^\nu q^\sigma \\
& +p^\alpha p^\beta p^\sigma q^\mu q^\nu q^\rho) + 8y^2z^2[8a^4(1-2x+2x^2) - 28(x-1)x(1-x+x^2)^2y^3(1-z)z^3 + ay^2z^2((x-1)x(49-88x+142x^2 \\
& -108x^3+54x^4)(1-z) + (-14+45x-73x^2+56x^3-28x^4)yz) + a^3(12(x-1)x(1-x+x^2)(1-z) + (-31+88x-104x^2 \\
& +32x^3-16x^4)yz) + 2a^2yz((x-1)x(-16+21x-29x^2+16x^3-8x^4)(1-z) + (17-51x+69x^2-36x^3+18x^4)yz)] \\
& \times(p^\nu q^\alpha q^\beta q^\mu q^\rho q^\sigma + p^\mu q^\alpha q^\beta q^\nu q^\rho q^\sigma) - 4yz[2a^5(6-11x+11x^2) + 56(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3 + a^2(x-1)x \\
& \times y(1-z)z((x-1)x(68-113x+129x^2-32x^3+16x^4)(1-z) + (-26+117x-213x^2+192x^3-96x^4)yz) + a^4((x-1) \\
& \times x(-11+52x-52x^2)(1-z) + 2(-4+7x+x^2-16x^3+8x^4)yz) + 2a(x-1)xy^2(1-z)z^2(-5(-1+x)x(7-12x+20x^2 \\
& -16x^3+8x^4)(1-z) + (14-45x+73x^2-56x^3+28x^4)yz) + a^3(-2(x-1)^2x^2(12-37x+37x^2)(1-z)^2 + (x-1)x \\
& \times(27-31x+63x^2-64x^3+32x^4)y(1-z)z - 2(2-3x+11x^2-16x^3+8x^4)y^2z^2)](p^\nu p^\rho q^\alpha q^\beta q^\mu q^\sigma + p^\nu p^\sigma q^\alpha q^\beta q^\mu q^\rho \\
& +p^\mu p^\rho q^\alpha q^\beta q^\nu q^\sigma + p^\mu p^\sigma q^\alpha q^\beta q^\nu q^\rho) - 4yz[-6a^5(3-5x+5x^2) + 56(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3 + a^4((x-1)x \\
& \times(-11-32x+32x^2)(1-z) + 2(27-44x+52x^2-16x^3+8x^4)yz) + 2a(x-1)xy^2(1-z)z^2((x-1)x(-42+67x-95x^2 \\
& +56x^3-28x^4)(1-z) + 2(14-45x+73x^2-56x^3+28x^4)yz) + a^2yz((x-1)^2x^2(7+36x-20x^2-32x^3+16x^4)(1-z)^2 \\
& -8(x-1)x(13-29x+43x^2-28x^3+14x^4)y(1-z)z + 4(10-21x+35x^2-28x^3+14x^4)y^2z^2) - 2a^3((x-1)^2x^2(-3 \\
& +31x-31x^2)(1-z)^2 + (x-1)x(-21+17x-33x^2+32x^3-16x^4)y(1-z)z + (35-57x+85x^2-56x^3+28x^4)y^2z^2)] \\
& \times(p^\beta p^\nu q^\alpha q^\mu q^\rho q^\sigma + p^\alpha p^\nu q^\beta q^\mu q^\rho q^\sigma + p^\beta p^\mu q^\alpha q^\nu q^\rho q^\sigma + p^\alpha p^\mu q^\beta q^\nu q^\rho q^\sigma) - 4[56(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3z^3 + 2a^5 \\
& \times((x-1)^2x^2(1-z) + (1-x+x^2)yz) - a^4(x-1)x(1-z)((x-1)x(1-z) + 4(x-1)x^2(1-z) - 4(x-1)x^3(1-z) \\
& +(-10-31x+31x^2)yz) + 2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(-28+39x-53x^2+28x^3-14x^4)(1-z) + 2(14-45x \\
& +73x^2-56x^3+28x^4)yz) + a^2(x-1)xy(1-z)z((x-1)^2x^2(37-41x+41x^2)(1-z)^2 - 2(x-1)x(38-71x+99x^2 \\
& -56x^3+28x^4)y(1-z)z + 4(10-21x+35x^2-28x^3+14x^4)y^2z^2) + a^3(x-1)x(1-z)((x-1)^2x^2(-5-2x+2x^2) \\
& \times(1-z)^2 + 8(x-1)x(6-x+x^2)y(1-z)z - 2(25-36x+50x^2-28x^3+14x^4)y^2z^2)](p^\beta p^\nu p^\rho q^\alpha q^\mu q^\sigma + p^\alpha p^\nu p^\rho q^\beta q^\mu q^\sigma \\
& +p^\beta p^\nu p^\sigma q^\alpha q^\mu q^\rho + p^\alpha p^\nu p^\sigma q^\beta q^\mu q^\rho + p^\beta p^\mu p^\rho q^\alpha q^\nu q^\sigma + p^\alpha p^\mu p^\rho q^\beta q^\nu q^\sigma + p^\beta p^\mu p^\sigma q^\alpha q^\nu q^\rho + p^\alpha p^\mu p^\sigma q^\beta q^\nu q^\rho) + 4[4a^6(1-x \\
& +x^2) - 56(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3z^3 + 2a^5((x-1)x(3-8x+8x^2)(1-z) + (2-21x+13x^2+16x^3-8x^4) \\
& \times yz) + 2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(35-46x+48x^2-4x^3+2x^4)(1-z) - 3(14-45x+73x^2-56x^3+28x^4) \\
& \times yz) + a^4(4(x-1)^2x^2(1-5x+5x^2)(1-z)^2 + (x-1)x(61-104x+56x^2+96x^3-48x^4)y(1-z)z + 2(1+9x+19x^2 \\
& -56x^3+28x^4)y^2z^2) - 2a^2(x-1)xy(1-z)z((x-1)^2x^2(-29+75x-67x^2-16x^3+8x^4)(1-z)^2 + (x-1)x(-35+66x
\end{aligned}
$$

21 pages

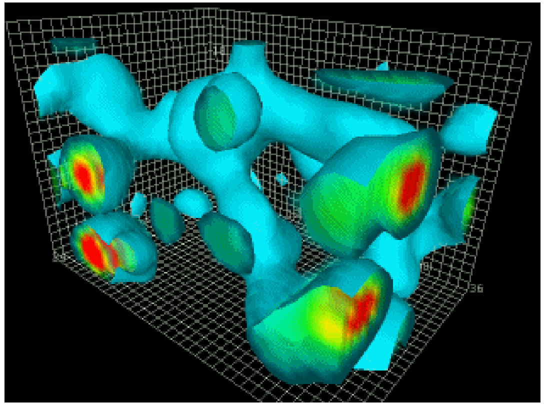
IV. Dark energy in QFT

Big Devil (Dark energy)





$\Lambda + \mathrm{CDM}$



$$\langle\rho\rangle = \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3}\,\frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

$$(2.3\ \mathrm{meV})^4$$

10^{120}

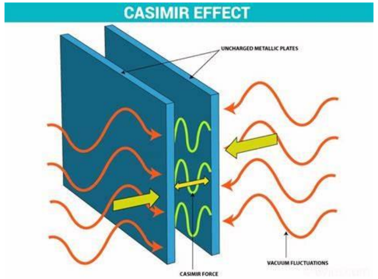
New ultraviolet catastrophe

Vacuum energy

The vacuum energy density

$$\rho_0^i = (-1)^{2j} g_i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m_i^2}$$

$$\mathcal{T}_{\rm P}^i = \left[\int (d\xi)^3 \frac{\partial^3 \mathcal{T}_{\rm F}^i(\xi)}{\partial \xi^3}\right]_{\xi\to 0} + C$$
$$= \left[(-1)^{2j} g_i \int (d\xi)^3 \int \frac{d^3k}{(2\pi)^3} \frac{3}{16} \frac{1}{\left(\sqrt{k^2+m_i^2+\xi}\right)^5}\right]_{\xi\to 0} + C$$
$$= (-1)^{2j} g_i \frac{1}{64\pi^2} m_i^4 \log(m_i^2) + C \ .$$



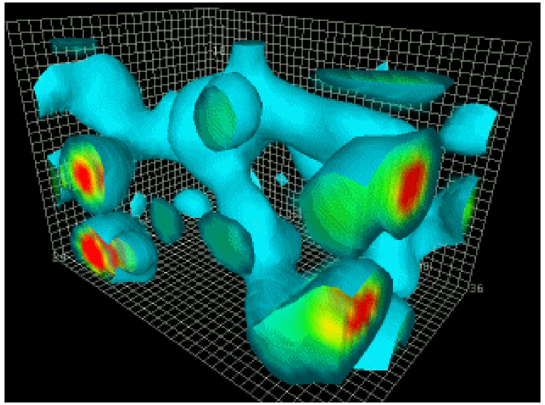
$\zeta(-1) = -\frac{1}{12}$

$$\mathcal{T}_{\rm P}^i = (-1)^{2j} g_i \frac{1}{64\pi^2} m_i^4 \log \frac{m_i^2}{\mu_\Lambda^2}$$

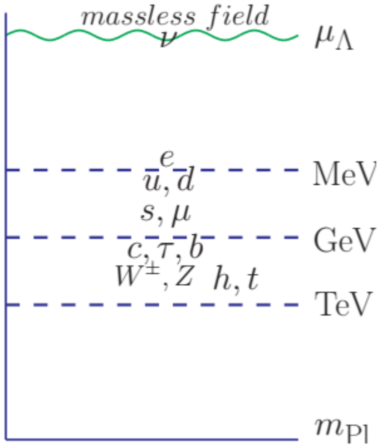
μ_Λ likes $\Lambda_{\rm QCD}$ and the electroweak scale v

arXiv:2410.06604

No longer a problem caused by the UV region



If dark energy comes from the vacuum energy, then the contributions of heavy fields should be suppressed by some unknown mechanisms.



The ripple description

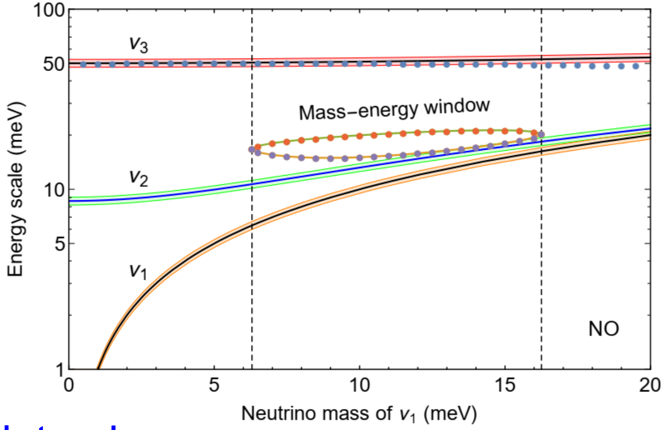
An effective active degrees of freedom $g_i^* = g_i e^{-m_i^2/\mu_\Lambda^2}$

Naturalness

Gaussian distribution

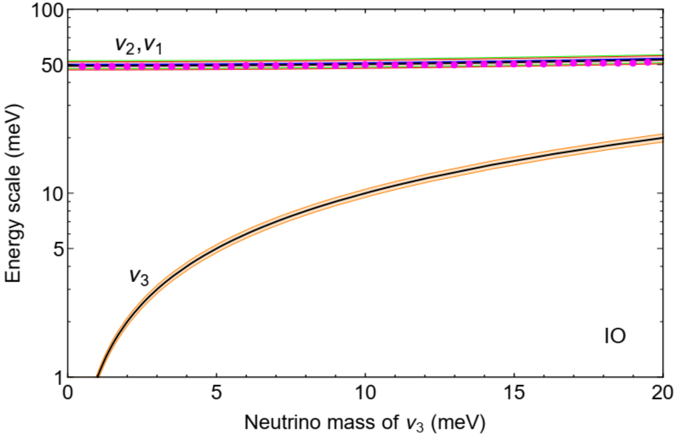
Neutrino fields play a major role

Neutrino mass and dark energy density $(2.3\ \mathrm{meV})^4$ $\quad H_0 \approx 70$ km /(s· Mpc)



Likewise, the neutrino mass window set by dark energy is 6.3 meV $\lesssim m_1 \lesssim$ 16.3 meV, 10.7 meV $\lesssim m_2 \lesssim$ 18.4 meV, 50.5 meV $\lesssim m_3 \lesssim$ 52.7 meV, and the total neutrino mass is 67.5 meV $\lesssim m_1 + m_2 + m_3 \lesssim$ 87.4 meV.

Naturalness

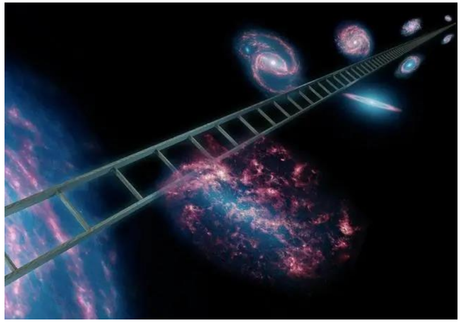


Hubble tension

The nearby universe

H_0 = (73.04 ± 1.04) km /(s·Mpc)

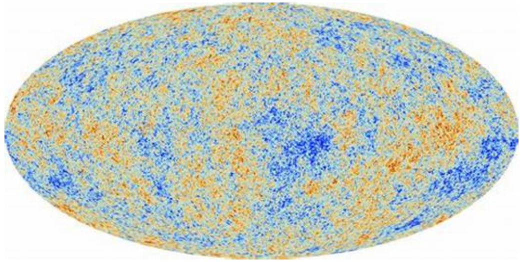
$$(2.37\ \mathrm{meV})^4$$



The early universe

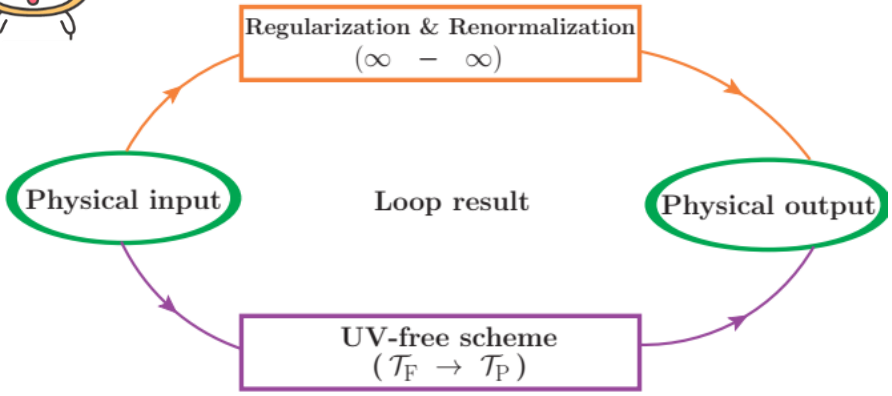
H_0 = (67.4 ± 0.5) km /(s·Mpc)

$$(2.24\ \mathrm{meV})^4$$



μ_Λ has a slow-running behavior

Why does the UV-free scheme still hold for power-law divergences?



Two alternative routes

(a) *Equivalent transformation* of the loop integral from UV divergence to UV divergence mathematically expressed form (regularization), with renormalization required to remove the UV divergence.
(b) *Analytic continuation* of the transition amplitude from UV divergent $\mathcal{T}_{\rm F}$ to UV converged $\mathcal{T}_{\rm P}$ (the UV-free scheme here), without UV divergences in calculations.

A test

The hierarchy problem of Higgs mass

(a) New particles (TeV) needed to cancel out UV contributions of loops to the Higgs mass

(b) An interpretation within SM

SELECTED BY NATURE

UV-free scheme

Analytic continuation

$$\mathcal{T}_{\rm F} \Longrightarrow \mathcal{T}_{\rm P} = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_{\rm F}(\xi)}{\partial \xi^n}\right]_{\xi\to 0} + C,$$

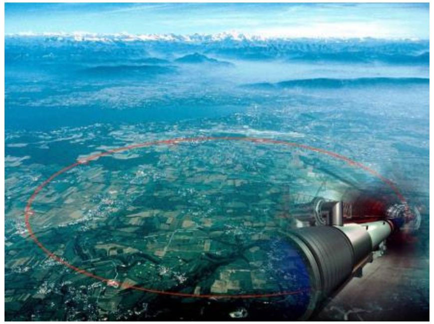
Finite input | **UV divergence input (continuation)**

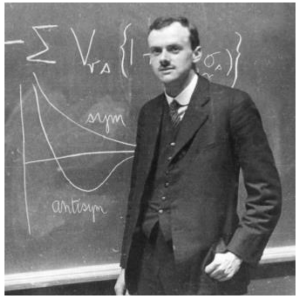
Tree level | Loop finite | Loop Log | Loop $\Lambda^2, \Lambda^4, \Lambda^6, \ldots$

Originally well-defined

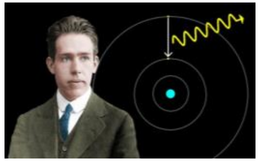
A conservative way

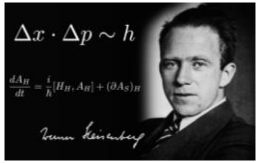
LHC



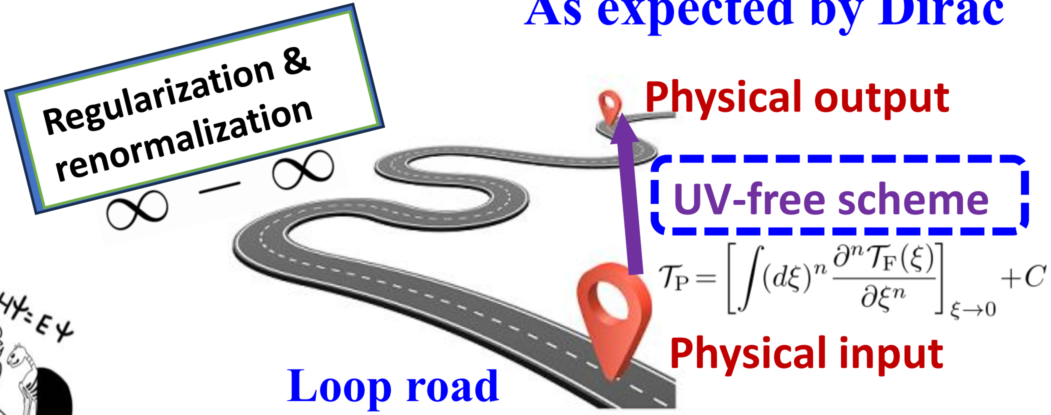


P. A. M. Dirac I believe the successes of the renormalization theory will be on the same footing as the successes of the Bohr orbit theory applied to one-electron problems.





As expected by Dirac



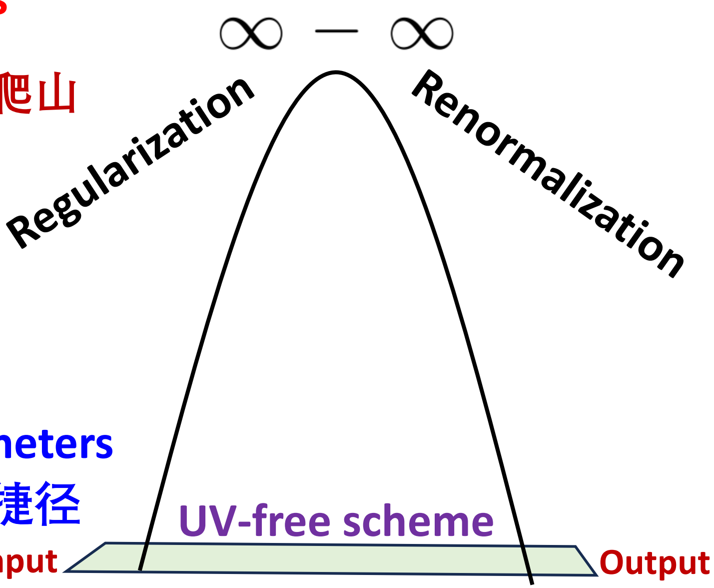
Schemes	Tree level	Loop finite	Loop Log	Loop $\Lambda^2, \Lambda^4, \Lambda^6, \ldots$
Regularization & renormalization (∞-∞)			OK	Problematic
UV-free scheme (T_F ->T_P)	OK	OK	OK	OK

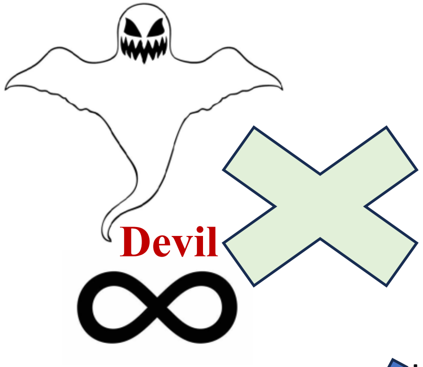
Both loops of the renormalizable Standard Model and non-renormalizable Einstein gravity being OK!

Bare parameters

UV divergences

负重爬山





UV-free scheme

$$\mathcal{T}_{\mathrm{P}} = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_{\mathrm{F}}(\xi)}{\partial \xi^n} \right]_{\xi \to 0} + C$$

In an inherently finite framework, loop diagram calculations become substantially simpler.

Finite parameters

终南捷径

The UV-free scheme applies wherever the Standard Model holds as a low-energy effective theory. Deviations are signatures of new physics.

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From the handling of UV divergences to their preclusion by design.

V. Summary and outlook

打油诗一首

A. An alternative method --- UV-free scheme: Finite loop results obtained without UV divergences, and this is effective for loop Log and power−law divergence inputs.

B. To the hierarchy problem of the 125 GeV Higgs, an alternative interpretation without fine-tuning within SM.

C. Applications to Einstein gravity and dark energy.

紫外自由方案

质本洁来去无痕，
解析延拓局域根。
紫外散尽星河阔，
有限参量见本真。

使用新方法时别忘了引用噢！

Outlook:

The beginning of a new method.

前方瀚海星辰，邀您同行共探！

恭祝李老师：耄耋新竹，风华更茂！

Thank you!