# 两介子光锥分布振幅研究进展

Shan Cheng (程山)

Hunan University

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# Overview

I: 光锥分布振幅

II: 两介子光锥分布振幅和一个具体应用

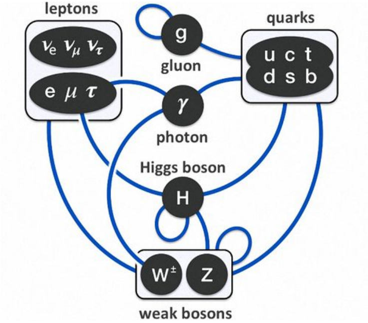
i: 两介子光锥分布振幅

ii: $B \to \pi\pi l\nu$ 和 $|V_{ub}|$ 测量

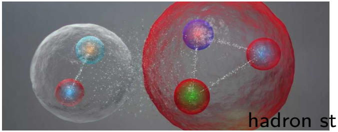
III: 总结和展望

# Emergent phenomena of QCD

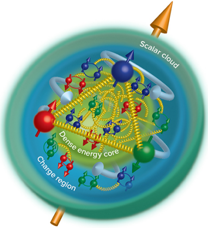
QCD is believed to confine, that is, its physical states are color singlets with internal quark and gluon degrees of freedom




Parts



hadron structure




hadron formation

Observed entity

# Emergent phenomena of QCD

- QCD allow us to study hadron structures in terms of partons
- Exclusive QCD processes: separate the hard partonic physics out of the hadronic physics (soft, nonperturbative objects), **Factorization**

- Define hadron structures by quantum field theories
- Identify theoretical observables in factorizable formulism

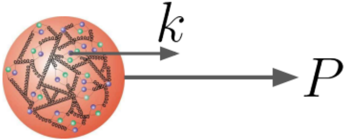
$$\frac{d\sigma}{d\Omega} = \int_x^1 \frac{d\zeta}{\zeta} \mathcal{H}(\zeta) f(\frac{x}{\zeta})$$

- The hard amplitudes can be calculated perturbatively, order by order
- The universal nonperturbative objects can be studied by QCD-based analytical (QCDSRs, $\chi$PT, instanton) and numerical approaches (LQCD)

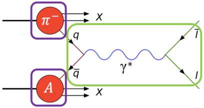
- Also can be studied by performing global QCD analysis and fit CETQ, CT, MMHT, NNPDF, ABM, JAM, et.al.

# PDF,TMD,GPD

## Definitions of pion distribution




One dimension PDF

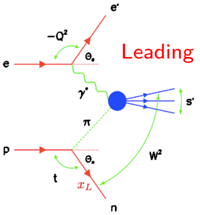



Drell-Yan

$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi,\mu) \otimes f_j^{A}(x_A,\mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Extracted from **fixed target $\pi A$ data**

- △ $f_i(\zeta) = \int \frac{dz^-}{4\pi} e^{-i\zeta P^+ z^-} \langle \pi | \bar{\psi}_i(0, z^-, \mathbf{0}_T)\gamma^+ \psi_i(0) | \pi \rangle$
- △ $\zeta = \frac{k^+}{P^+}$, the parton momentum fraction
- △ $f_i(\zeta) \sim \sum_\alpha \int dk_T^2 \langle \pi | b^\dagger_{k,\alpha} b_{k,\alpha}(\zeta P^+, k_T, \alpha) | \pi \rangle$
  number operator
- △ transversal momentum distributions (TMD) $f(\zeta, k_T)$
- △ Generalized parton distributions (GPD) $f(\zeta, b_T)$




Leading neutrons

**Deeply virtuality meson production**

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- △ TDIS at 12GeV JLab, leading proton observable, fixed target instead of collider (HERA);
- △ EIC, EIcC, great integrated luminosity to reduce the systematics uncertainties;
- △ COMPASS++/AMBER give $\pi$-induced DY data.

# LCDAs

**Exclusive QCD processes with larger momentum transfers**

Light-cone distribution amplitudes (LCDAs)

- The use of conformal symmetry inQCD [Braun, Korchemsky, Müller 2003]
- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD under conformal trans.* *VS* *rotation symmetry*
- *longitudinal* $\otimes$ *transversal* dofs *VS* *angular* $\otimes$ *radial* dofs for spherically symmetry potential
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group
  **collinear subgroup of conformal group** $SL(2, R) \cong SU(1,1) \cong SO(2,1)$ $\Rightarrow$ collinear twist

# LCDAs

- Define the LCDAs with the Lorentz and gauge invariant ME

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(p)\rangle = f_\pi \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[ip_\mu\left(\phi(u,\mu)+\frac{x^2}{4}\phi_1^4(u,\mu)\right)+\cdots\right]$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(p)\rangle = f_\rho m_\rho \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[p_\mu \frac{\epsilon^{(\lambda)}\cdot x}{p\cdot x}\left(\phi_\parallel(u,\mu)-\phi_\perp^3(u,\mu)\right)+\cdots\right]$$

$$\langle 0|\bar{q}(x)\gamma_\mu\gamma_5 q(-x)|f_0(p)\rangle = p_\mu \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[\phi(u,\mu)+\cdots\right]$$

‡ $\gamma_\mu\gamma_5, \sigma_{\mu\nu}, \cdots$ operators define other LCDAs at different twists

- LCDAs are dimensionless functions of $u$ and renormalization scale $\mu$

△ the probability amplitudes to **find the light meson** in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

△ nonlocal operators on the lhs are renormalized at scale $\mu$

$$\phi_2(u,\mu) = Z_2(\mu)\int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{\rm BS}(u,k_\perp)$$

△ decay constant $\langle 0|\bar{u}(0)\gamma_z\gamma_5 d(0)|\pi^-(P)\rangle = if_\pi p_\mu$ △ normalizations such as $\int_0^1 du\,\phi(u) = 1$

# 单粒子图像的局限性

- Pion（轻赝标量）介子 LCDA 在描述大动量转移过程中取得了巨大成功

△ 伴随着微扰 QCD 因子化方案的建立发展过程 $F_\pi(Q^2)$ 的横动量因子化方法

| | | |
|---|---|---|
| $\int_0^1 du_i \phi(u_1, \tilde{Q}_1) T_H(u_i, Q) \phi(u_2, \tilde{Q}_2)$ | 1980s | [Lepage & Brodsky 1980, Efremov & Radyushkin 1980] |
| $\Downarrow$ | | |
| $\int_b^{\bar{b}} du_1 \int_{a/u_1}^{1-a/\bar{u}_1} \phi(u_1, k_{1T}) T_H(u_i, Q) \phi(u_2, k_{2T})$ | 1990s | [Huang, Shen & Kroll 1991, Huang, Shen & Ma 1994] |
| $\Downarrow$ | | |
| $\psi(u_1, \mu_{r_1}) T_H(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b_i, Q)} \psi(u_2, \mu_{r_2})$ | 2000s | [Botts & Sterman 1989, Li & Sterman 1992, Li 1999] |
| $\Downarrow$ | | |
| $\sum_{t_i} \psi^{t_1}(u_1, \mu_{r_1}) T_H^{t_i, \mathrm{LO+NLO}}(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, \mu_{r_2})$ | 2010s | [Li, Shen, Wang & Zou 2011, **SC**, Fan & Xiao 2014] |
| $\sum_{t_i} \psi^{t_1}(u_1, b_1, \mu_{r_1}) T_H^{t_i, \mathrm{LO+NLO}}(u_i, b_i, Q) S_t(u_i) e^{-S(u_i, b, Q)} \psi^{t_2}(u_2, b_2, \mu_{r_2})$ | 2020s | [Chai & **SC** 2025] [Chen$^2$, Feng & Jia 2024], [Ji, Shi, Wang$^3$ & Yu 2025] |

- 高精度检验时代，矢量/标量介子的 LCDAs 描述开始展现出一定局限性

  △ the probability amplitudes to **find the light meson** in a state with $\cdots$

  △ the $\pi\pi$ invariant mass spectra in $[0.554, 0.996]$GeV is selected to identify candidates for the $\rho$ resonance, $f_0(980)$

- 如何准确描述不稳定中间粒子的宽度效应，不同分波的贡献和相互干涉效应，以及非共振态的 QCD 背景 ？

- 以 $|V_{ub}|$ 疑难问题为例 $\cdots$

# $|V_{ub}|$ 疑难和解决方案

- $|V_{ub}|$ tension $\quad |V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2024]

‡ $\sim 2.5\sigma$ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.70 \pm 0.16) \times 10^{-3}$ measured via the $B \to X_u l^- \bar{\nu}$ and $B \to \pi l^- \bar{\nu}$ processes, respectively.

- 更精确的测量和格点计算，更全面更系统的物理分析方法

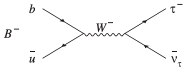
  simultaneous determination in excl. and incl. processes [Belle, PRL131. 211801 (2023)]

  $|V_{ub}| = (3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$ and $(3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$

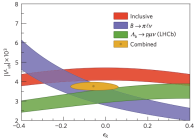
- 精细结构、丰富的 QCD 效应 high order QCD corrections, more structures

  [Khadjamirian, Mannel, Pivovarov & Wang, JHEP 02 (2013) 010, JHEP 09 (2010) 089]

- 寻找新的物理过程开展独立测量




‡ pure leptonic decay $\quad |V_{ub}| f_B = 0.72 \pm 0.09$ MeV [Belle], $1.01 \pm 0.14$ MeV [BABAR], $0.77 \pm 0.12$ MeV [FLAG2021]




‡ baryon decay [LHCb, Nature Physics 11, 743-747 (2015)]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu})} R_{\rm FF} \xrightarrow{|V_{cb}|} |V_{ub}| = (3.72 \pm 0.23) \times 10^{-3}$$

$*$ consistent with determinations in exclusive $B \to \pi l \bar{\nu}$ decay

# $|V_{ub}|$ 疑难和解决方案

- $|V_{ub}|$ in the $B \to \rho l \nu$ channel   the same $b \to u l \nu$ transition as in $B \to \pi l \nu$ channel
- simultaneous measurements of the $\frac{d\mathcal{B}}{dq^2}$ for $B \to \pi^- l^+ \nu_l$ and $B \to \rho^0 l^+ \nu_l$
  [Belle-II PRD 111. 112009(2025)] [HFLAV 2023] [P. Bharucha and et.al., JHEP 08 (2016) 098]

$$|V_{ub}|_{B\to\pi l\nu} = \left(3.73 \pm 0.10 \pm 0.16|_{\rm LQCD+LCSRs}\right) \times 10^{-3},$$
$$|V_{ub}|_{B\to\rho l\nu} = \left(3.19 \pm 0.22 \pm 0.26|_{\rm LCSRs}\right) \times 10^{-3}.$$

- $B \to V$ FFs updated via $B$-meson LCSRs [Gao, et.al., PRD 101. 074035 (2020)]

$$|V_{ub}|_{B\to\rho l\nu} = \left(3.05^{+1.34}_{-1.30}|_{\rm theo}\ {}^{+0.19}_{-0.20}|_{\rm data}\right) \times 10^{-3},$$
$$|V_{ub}|_{B\to\omega l\nu} = \left(2.54^{+1.09}_{-1.05}|_{\rm theo}\ {}^{+0.18}_{-0.19}|_{\rm data}\right) \times 10^{-3}$$

- The QCD studies (LCSRs, LQCD) treat $\rho$ as a stable single particle
- While the signal channel in a $B \to \rho l \nu$-type decay is $B \to \pi\pi l \nu$ ($B_{l4}$)

  [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]
  [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van. Dyk PRD 89. 014015 (2014)]

- 如何准确描述不稳定中间粒子的宽度效应，不同分波的贡献和相互干涉效应，以及非共振态的 QCD 背景 ？

# $2\pi$DAs

- Chiral-even LC expansion with gauge factor $[x, 0]$

$$\langle\ \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\rho^-(p)\rangle = f_\rho m_\rho \int_0^1 du\, e^{i(2u-1)p\cdot x}\left[p_\mu \frac{\epsilon^{(\lambda)}\cdot x}{p\cdot x}\left(\phi_\parallel(u,\mu) - \phi_\perp^3(u,\mu)\right) + \cdots\right]$$

- $2\pi$DAs is decomposed in terms of $C_n^{3/2}(2u-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=0,\mathrm{even}}^{\infty}\ \sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)\, C_n^{3/2}(2u-1)\, C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(u,\zeta,k^2,\mu) = 6u(1-u)\sum_{n=1,\mathrm{odd}}^{\infty}\ \sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)\, C_n^{3/2}(2u-1)\, C_\ell^{1/2}(2\zeta-1)$$

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass $k^2$** ?

‡ Watson theorem of $\pi$-$\pi$ scattering amplitudes

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\, \frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ **$2\pi$DAs in a wide range of energies is given by $\delta_\ell^I$ and a few subtraction constants**

# $2\pi$DAs

- The subtraction constants of $B_{n\ell}(k^2)$ at low $k^2$ (around the threshold)

| (nl) | $B^{\parallel}_{n\ell}(0)$ | $c_1^{\parallel,(nl)}$ | $\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$ | $B^{\perp}_{n\ell}(0)$ | $c_1^{\perp,(nl)}$ | $\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$ |
|---|---|---|---|---|---|---|
| (01) | 1 | 0 | 1.46 $\to$ 1.80 | 1 | 0 | 0.68 $\to$ 0.60 |
| (21) | -0.113 $\to$ 0.218 | -0.340 | 0.481 | 0.113 $\to$ 0.185 | -0.538 | -0.153 |
| (23) | 0.147 $\to$ -0.038 | 0 | 0.368 | 0.113 $\to$ 0.185 | 0 | 0.153 |
| (10) | -0.556 | - | 0.413 | - | - | - |
| (12) | 0.556 | - | 0.413 | - | - | - |

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new $a_2^{\pi}$, $a_2^{\rho}$ [**SC** 2019, 2023]

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- $2\pi$DAs were introduced in 1990s at leading twist [Polyakov 1999, Diehl 1998]

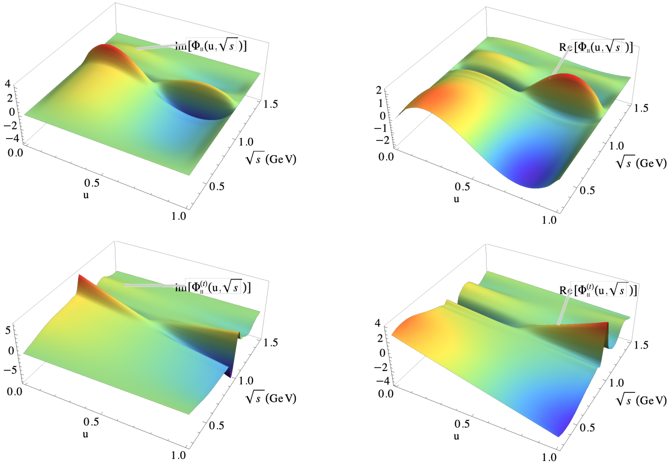
$$\langle\, \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu\tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

- twist-three $2\pi$DAs are studied very recently [**SC** 2502.07333]

$$\langle\pi^+(k_1)\pi^-(k_2)|\bar{q}(0)\gamma^\mu q(x)|0\rangle = \int du e^{i\bar{u}k\cdot x}\Big[k^\mu\Phi_{\parallel}(\bar{u},\zeta,k^2) + \bar{k}^\mu\Phi_{\perp}^{(v)}(\bar{u},\zeta,k^2)\Big]$$

$$\Phi_{\perp}^{(v)}(u,\zeta,k^2) = 6u\bar{u}\Big[B_{01}^{\perp}(k^2,\mu)C_1^{1/2}(2\zeta-1) + \frac{1}{6}\Big(B_{21}^{\perp}(k^2,\mu)C_1^{1/2}(2\zeta-1) + B_{23}^{\perp}(k^2,\mu)C_3^{1/2}(2\zeta-1)\Big)C_2^{3/2}(2u-1)\Big]$$

# $2\pi$DAs $[\pi\pi]_S$




- Leading twist and twist-three $2\pi$DAs for isospin scalar two-pion system
- asymmetry of the twist-three $2\pi$DAs in the parton momentum fraction $u$

- symmetric twist-three LCDAs of $f_0$ obtained under the narrow-width approximation
- where QCD sum rules dictate that the asymmetric component vanishes

# $B \to \pi\pi$ 形状因子

- 回到 $|V_{ub}|$ 的疑难问题，$B \to \rho l\nu$ 测量的信号道是 $B \to \pi\pi l\nu$ $(B_{l4})$

$$i\langle\pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma$$
$$+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right)$$
$$+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)$$

† $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function, $q\cdot k = (m_B^2 - q^2 - k^2)/2$,
$q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta-1)$, $\beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$

- **QCDF** (QCD factorization) in the large dipion mass
  - [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)] $T_I \propto F_{Bto\pi}\otimes\phi_\pi$,
- **SU(3) flavor symmetry/breaking** with the intermediate resonance
  - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng and et.al., 2301.00090, PRD 112, 033002 (2025)]

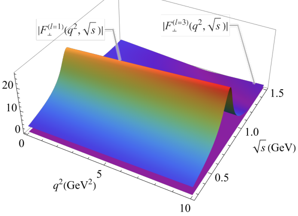
# $B \to \pi\pi$ 形状因子

- **LQCD** (Lattice QCD) in the $\rho$ resonance region with a simple BW model
  - [L. Leskovec and et.al, PRL 134.161901 (2025), **Editors' Suggestion**]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large $q^2$ by taking dispersive methods in terms of Omnés functions
  - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]

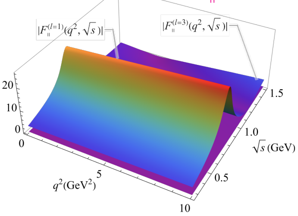
  in the full phase-space by a novel parameterization with unitarity
  - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025), **Editors' Suggestion**]
- **LCSRs** (Light-cone sum rules) in the small and intermediate $q^2$
  - [**SC**, A. Khodjamirian and J. Virto, JHEP 05(2017)157] $B$-meson LCSRs , [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12(2019)083, 06(2023)034] $B \to K\pi$
  - [C. Hambrock and A. Khodjamirian, NPB 905(2016)379-390] $2\pi$DAs LCSRS of $F_{\parallel,\perp}$
  - [**SC**, A. Khodjamirian and J. Virto, PRD(R) 96 (2017)051901] timelike-helicity FF $F_t$ and $F_0$
  - [**SC**, PRD 99 (2019) 053005] $2\pi$DAs updates and $B \to [\pi\pi]_{\rm S,P}$ FFs
  - [**SC** and J.M Shen, EPJC(2020)6:554, **SC** and S.L Zhang, EPJC (2024)84:379] Pheno
  - [**SC**, arXiv: 2502.07333] first study of twist-three $2\pi$DAs and $|V_{ub}|$ extraction
  - [**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, arXiv:2509.15659] $D_s \to [\pi\pi]_{\rm S}\, e\nu$, minor $q\bar{q}$ contribution

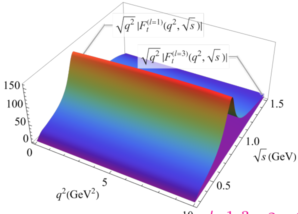
# $B \to \pi\pi$ 形状因子

$P$, $F$-wave components $F_{\perp}^{l=1,3}(q^2, k^2)$

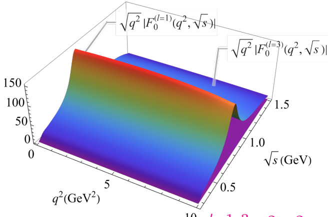



$P$, $F$-wave components $F_{\parallel}^{l=1,3}(q^2, k^2)$



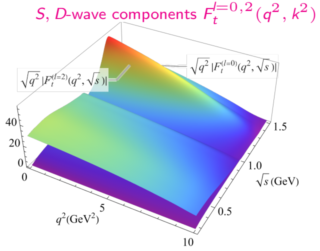



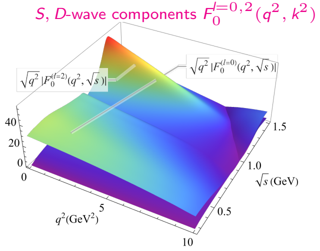

$P$, $F$-wave components $F_t^{l=1,3}(q^2, k^2)$




$P$, $F$-wave components $F_0^{l=1,3}(q^2, k^2)$

# $B \to \pi\pi$ 形状因子




- twist-three contribution vanish in $F_{\perp,\parallel}^{(l=1,3,\cdots)}$
- persist in $F_{t,0}^{(l=0,1,\cdots)}$, give a significant correction $\sim 40\%$ to leading twist result
- $\rho$ resonance is dominate in $F_{\perp,\parallel,t,0}^{(l=1)}(q^2,k^2)$, $F$-wave contributions are negligible
- $S$-wave component is dominate in the small $k^2$, $D$-wave component is comparable in the large $k^2$ and small $q^2$

# 通过 $B_{l4}$ 过程测量 $|V_{ub}|$

- 2D partial differential decay width

$$\frac{d^2\Gamma}{dq^2\,dk^2} = G_F^2|V_{ub}|^2 \frac{\beta_\pi\sqrt{\lambda}q^2}{3\,(4\pi)^5\;m_B^3}\left[\left(|F_0^{(S)}|^2 + |F_0^{(P)}|^2\right) + \beta_\pi^2\left(|F_\parallel^{(P)}|^2 + |F_\perp^{(P)}|^2\right) + \cdots\right]$$

- $|V_{ub}|$ extraction in the regions of $\rho$ and $f_0$ resonances [Belle 2021]

$$\boxed{|V_{ub}|_{B^+\to[\rho^0\to]\pi^+\pi^-l^+\nu_l} = (4.27 \pm 0.49|_{\rm Data} \pm 0.55|_{\rm LCSRs}) \times 10^{-3}}$$

$$\boxed{|V_{ub}|_{B^+\to[f_0\to]\pi^+\pi^-l^+\nu_l} = (3.96 \pm 0.47|_{\rm Data} \pm 0.52|_{\rm LCSRs}) \times 10^{-3}}$$

[**SC**, 2502.07333]

- simultaneous measurements of the $\frac{d\mathcal{B}}{dq^2}$ for $B \to \pi^- l^+\nu_l$ and $B \to \rho^0 l^+\nu_l$

[Belle-II PRD 111. 112009(2025)] [HFLAV 2023] [P. Bharucha and et.al., JHEP 1608. 098]

$$|V_{ub}|_{B\to\pi l\nu} = \left(3.73 \pm 0.10 \pm 0.16|_{\rm LQCD+LCSRs}\right) \times 10^{-3},$$
$$|V_{ub}|_{B\to\rho l\nu} = (3.19 \pm 0.22 \pm 0.26|_{\rm LCSRs}) \times 10^{-3}.$$

- 解释了通过 $B \to \rho l\nu$ 过程测量的较小结果。虽然当前误差较大，但是前景光明。 † Belle-II 积分亮度未来五年有望达到 $3ab^{-1}$，实验精度将提高三倍； † LCSRs 和 LQCD 对形状因子的联合分析，可以将理论精度提高两倍。

# 总结和展望

- $2\pi$DAs 提供了 $\pi\pi$ 质量谱的一般描述

  † 准确厘清不同分波和同一个分波中不同共振态的贡献， † 有效分离、清晰描述非共振态背景的贡献， † 基于交换对称性来理解 SPD/GPD,SPA [Lorcé, Pire & Song PRD 106, 094030 (2022), 109, 074016 (2024)]

- $2\pi$DAs 是重介子四体半轻衰变 ($H_{l4}$) 研究的关键输入

  † 为 $|V_{ub}|$ 疑难和 FCNC 反常研究提供了新动能， † 为重味强子分布振幅的研究提供了并行检测

- 在三扭度水平研究了 $2\pi$DAs，实现了在 $B_{l4}$ 过程抽取 $|V_{ub}|$ [**SC**, 2502.07333]
- 理解重介子衰变中的同位旋标量介子 (对) 的部分子结构和多粒子图像
  [**SC**, L.Y Dai, J.M Shen and S.L Zhang 2509.15659]
- 在 $D \to \pi\pi l^+ l^-$ 衰变中深入理解 FCNC 过程可能的反常行为 wishlists
- 在 $e^+e^-$ 湮灭 $\gamma^* \to \pi\pi\gamma, \gamma^*\gamma \to \pi\pi$ 过程中进一步理解 $2\pi$DAs
- $B$ 介子多体强子衰变过程 Dalitz 图中非共振态区域的局域 CP 破坏

Thank you for your patience.

# Backup slides  Pion LCDAs

- DA is expressed by the MEs of gauge invariant non-local operators

$$\langle 0|\bar{u}(x)\Gamma[x,-x]d(-x)|\pi^-(P)\rangle$$

- The path-ordered gauge factor along the straight line

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt(x-y)_\mu A^\mu(tx+\bar{t}y)\right]$$

- Introduce light-like vectors $p$ and $z$: $P^2 = m_\pi^2,\ p^2 = 0,\ z^2 = 0$
- $P \to p$ in the limit $m_\pi^2 = 0$ and $x \to z$ for $x^2 = 0$

- Expansion in power of large momentum transfer is governed by contributions from small transversal separations $x^2$ between constituents

$$z_\mu = x_\mu - \frac{P_\mu}{m_\rho^2}\left[xP - \sqrt{(xP)^2 - x^2 m_\pi^2}\right] = x_\mu\left[1 - \frac{x^2 m_\pi^2}{4(z\cdot p)^2} - \frac{p_\mu}{2}\frac{x^2}{z\cdot p} + \mathcal{O}(x^4)\right]$$

$$p_\mu = P_\mu - \frac{z_\mu}{2}\frac{m_\pi^2}{p\cdot z} \quad\Rightarrow\quad z\cdot P = z\cdot p = \left[(xP)^2 - x^2 m_\pi^2\right]^{1/2}$$