Inflation beyond slow-roll and non-Gaussianities

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中国科学院理论物理研究所

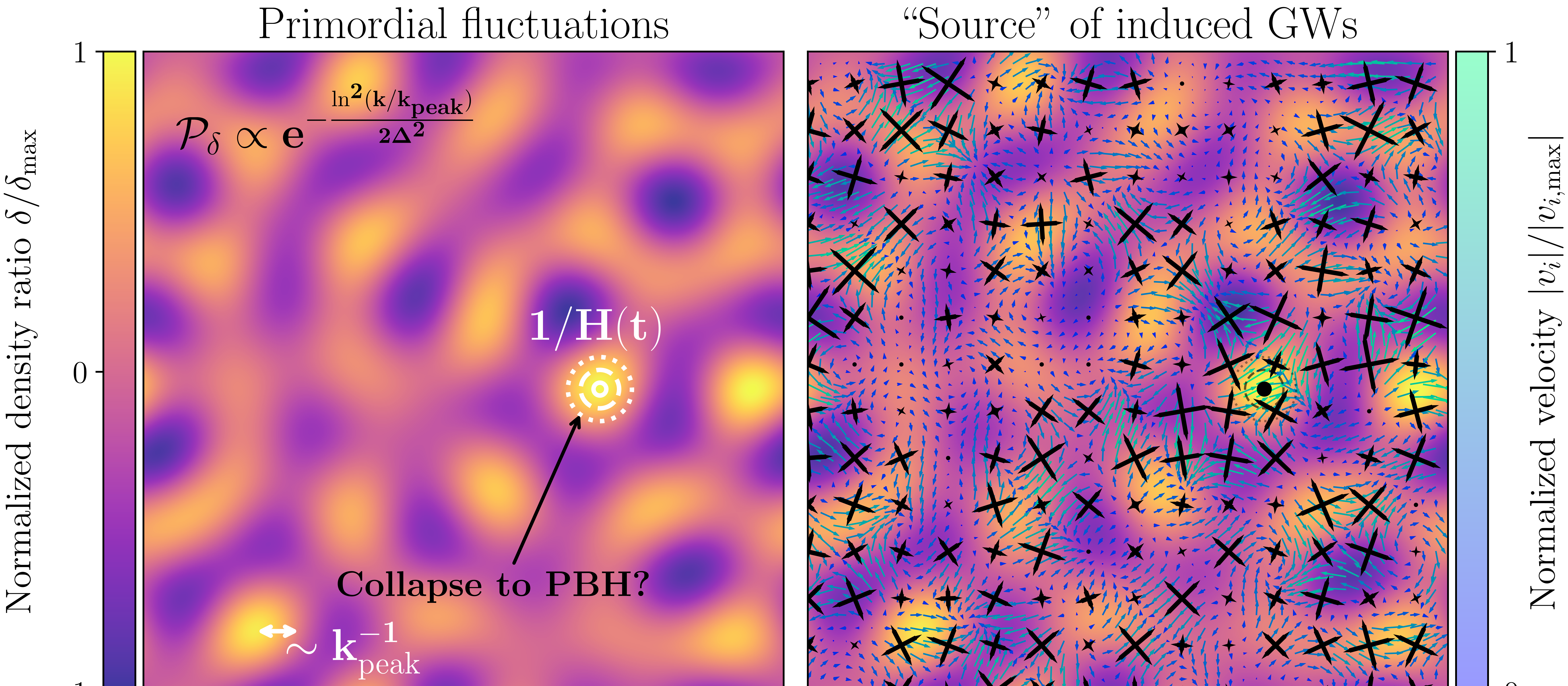
第五届量子场论及其应用研讨会

北京春晖园, Oct 31, 2025

CONTENT

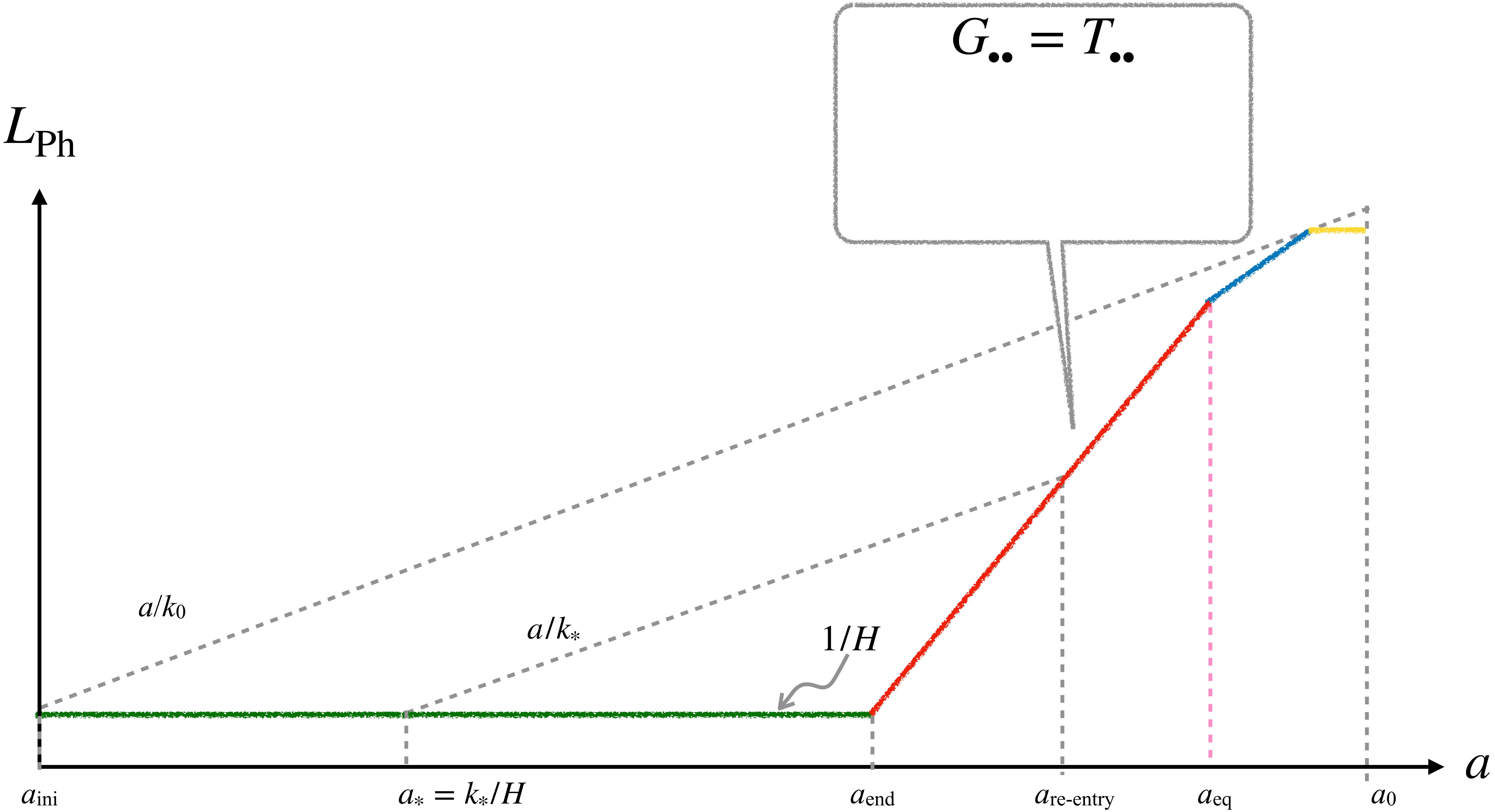
- Introduction: PBH and its detectability
- Application: δN formalism in single-field inflation
- Recent topics: gradient expansion and bubbles
- Conclusion


Angular scale
90° 18° 1° 0.2° 0.1° 0.07° 0.05°
Planck
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
10^4 10^3 10^2 10^1
2 10 30 1000 2000 3000 4000
Multipole moment ℓ
CMB anisotropy
$\mathscr{R} = \delta N \approx -\frac{H}{\dot{\varphi}}\delta\varphi$
Reconstruction
ℓ
10 100 1000
Power spectrum of $\mathscr{R}$
$\log(10^{10}\mathcal{P}_{\mathcal{R}})$
4.0 3.5 3.0 2.5 2.0
$\sim 10^{-9}$
Gaussian
adiabatic
10^{-4} 10^{-3} 10^{-2} 10^{-1}
k/Mpc^{-1}
3σ 2σ 1σ 0σ
USR,
multifield,
curvaton
$\sim 10^{-2}$
Gaussian?
Adiabatic?
nonlinear
perturbation
gravitational
collapse
Matarrese et al, PRD 47, 1311;
PRL 72, 320; PRD 58, 043504
Ananda et al, gr-qc/0612013
Bauman et al, hep-th/0703290
Scalar Perturbation
Induced GW
crosscheck
Saito, Yokoyama, PRL 102, 161101
Cai, SP, Sasaki, PRL 122, 201101
Primordial
Black Hole
PBH
Zeldovich & Novikov 1966
Hawking 1971
Carr & Hawking 1974

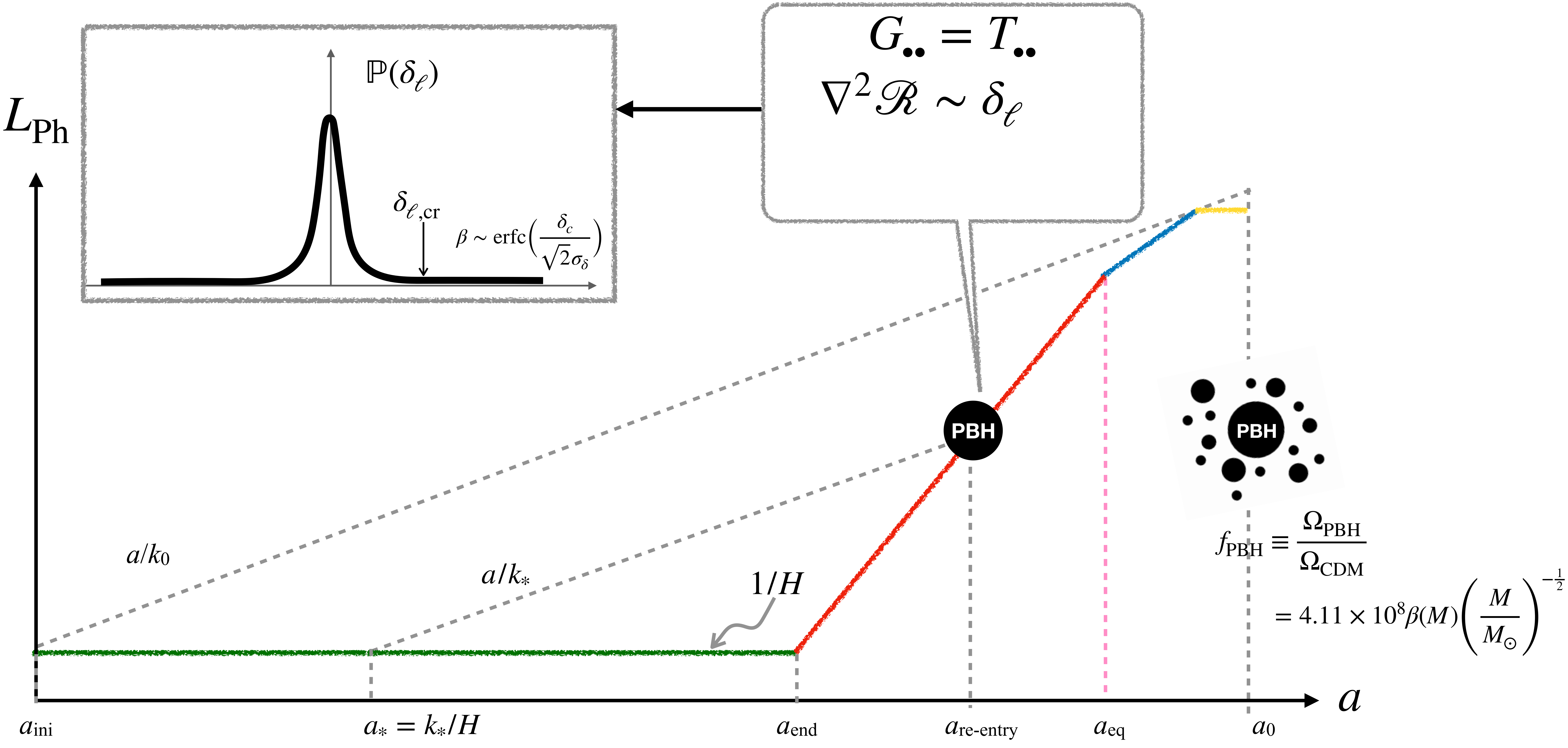


Domenech, 2402.17388, PBH book, Chapter 18

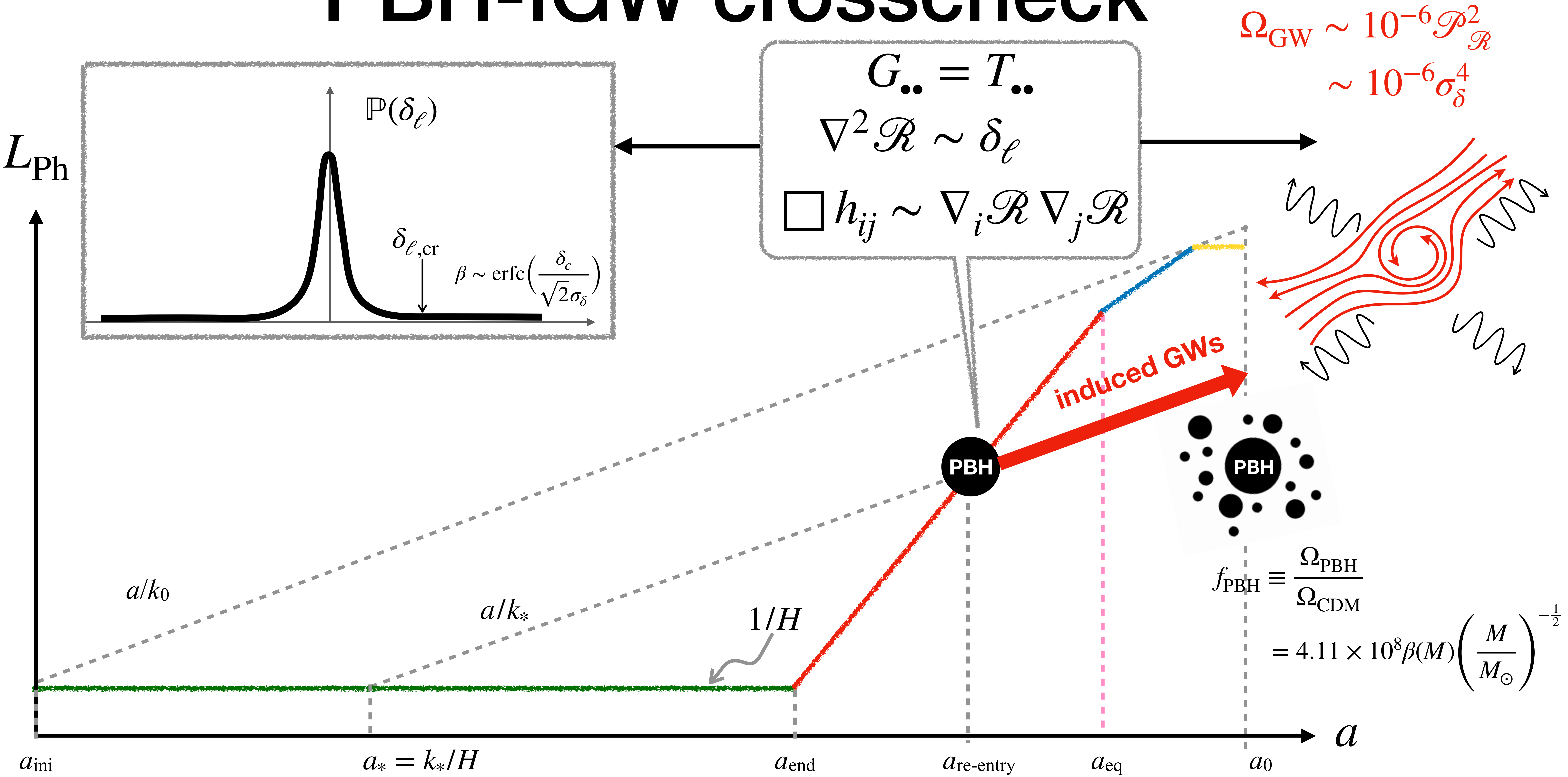
PBH-IGW crosscheck



PBH-IGW crosscheck



PBH-IGW crosscheck



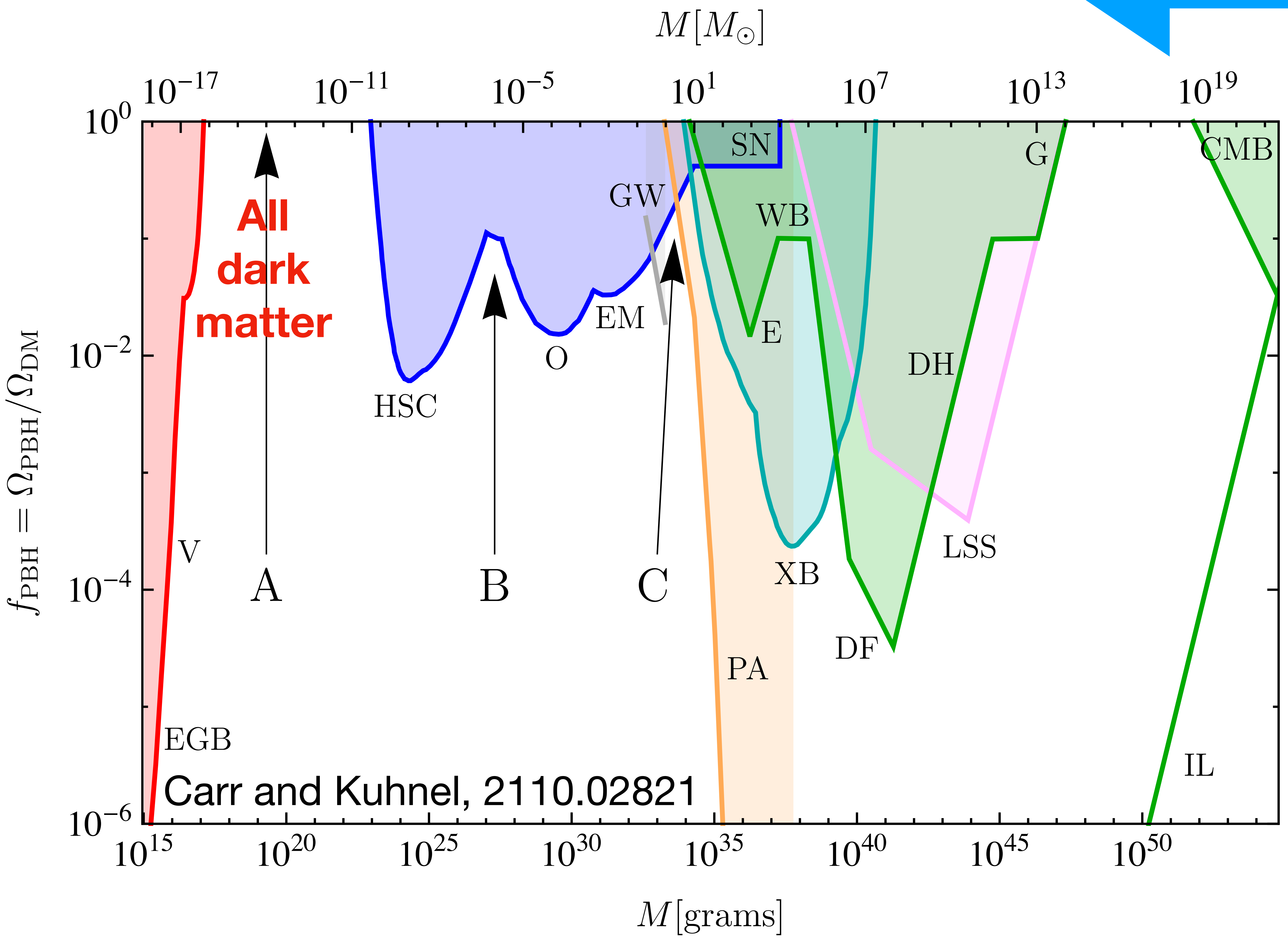
PBH-IGW crosscheck

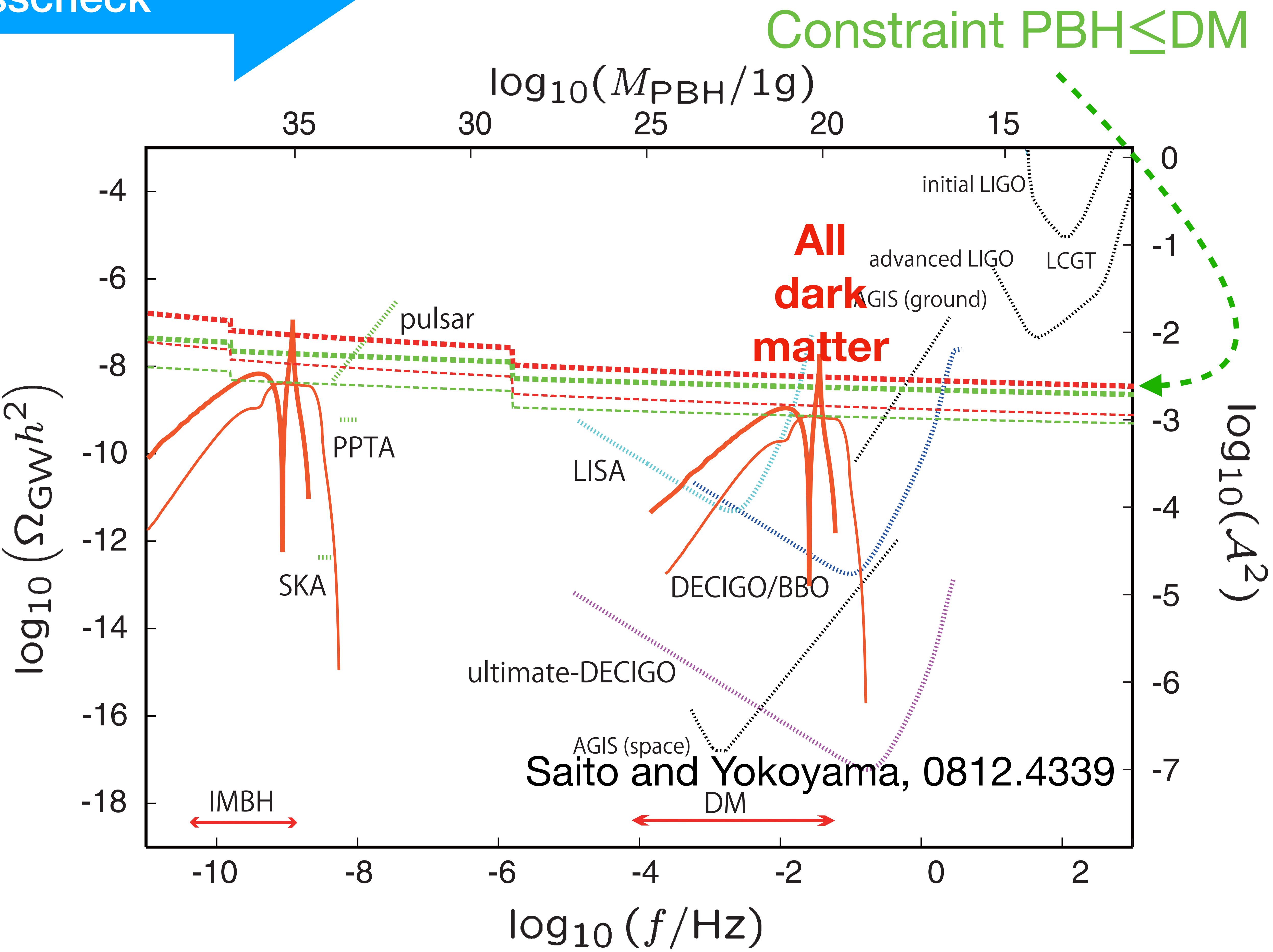
$$f_{\mathrm{IGW}} \sim 3\mathrm{Hz}\left(\frac{M_{\mathrm{PBH}}}{10^{16}\mathrm{g}}\right)^{-\frac{1}{2}}$$

PBH constraints

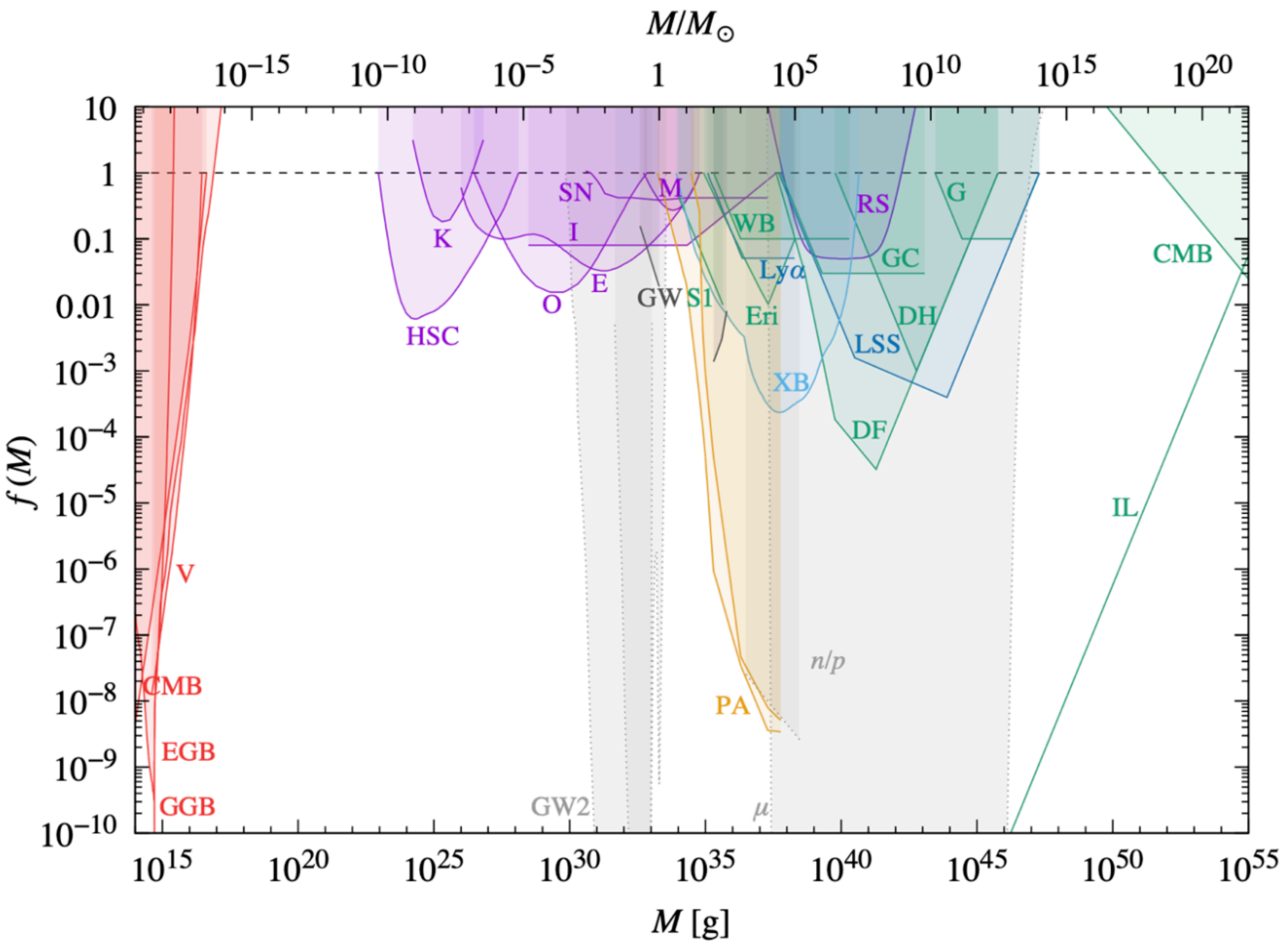
crosscheck

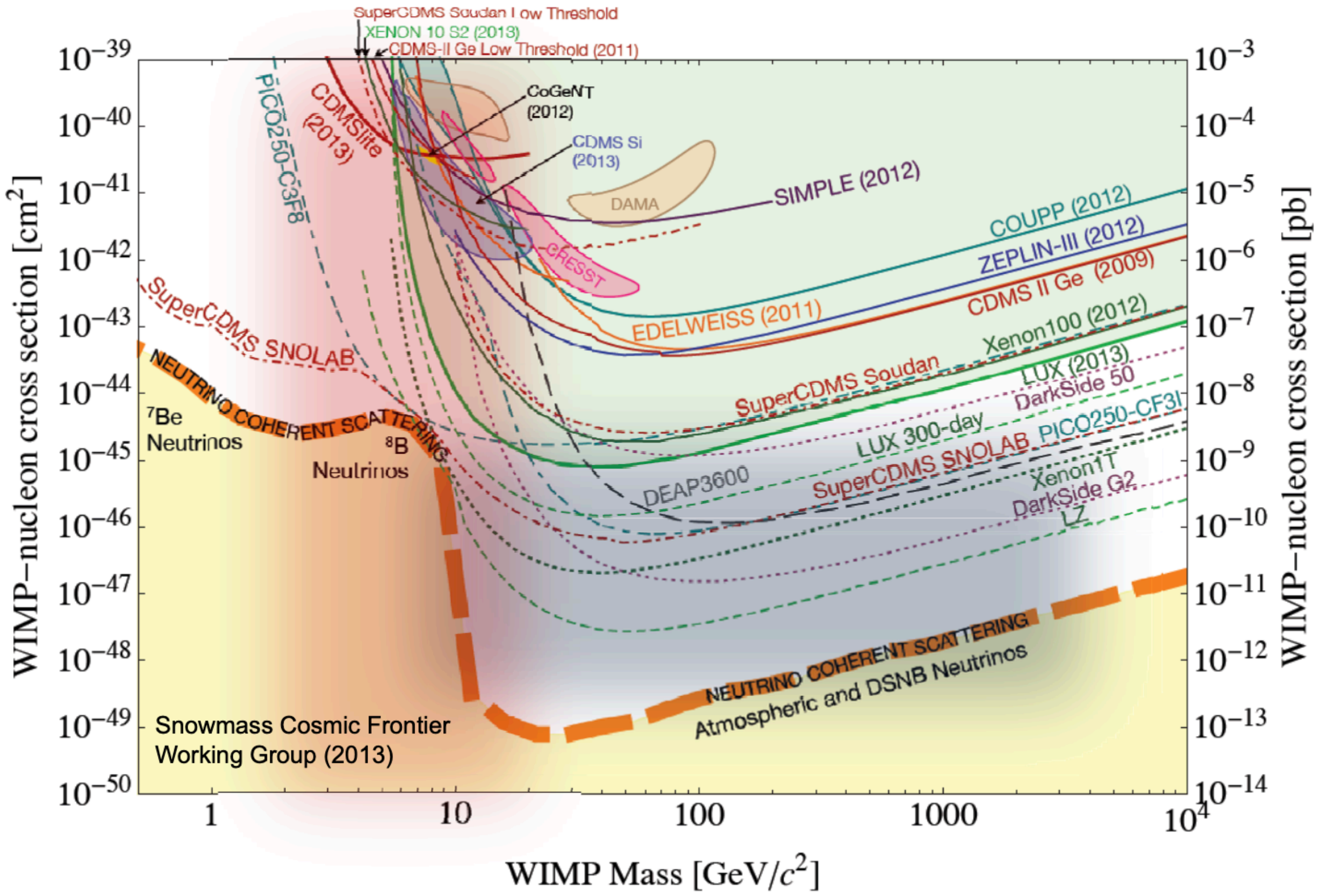
Induced GW predictions



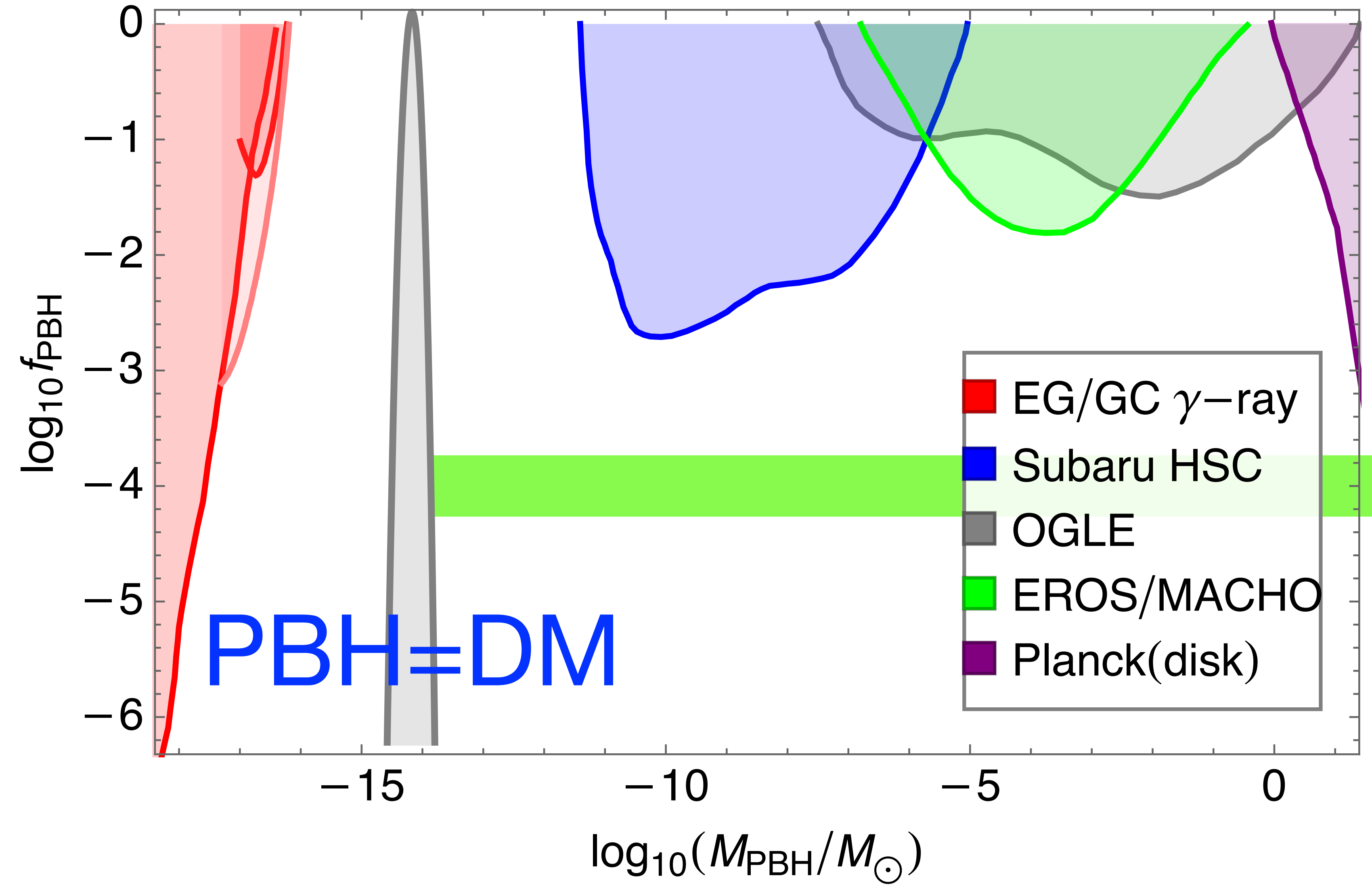


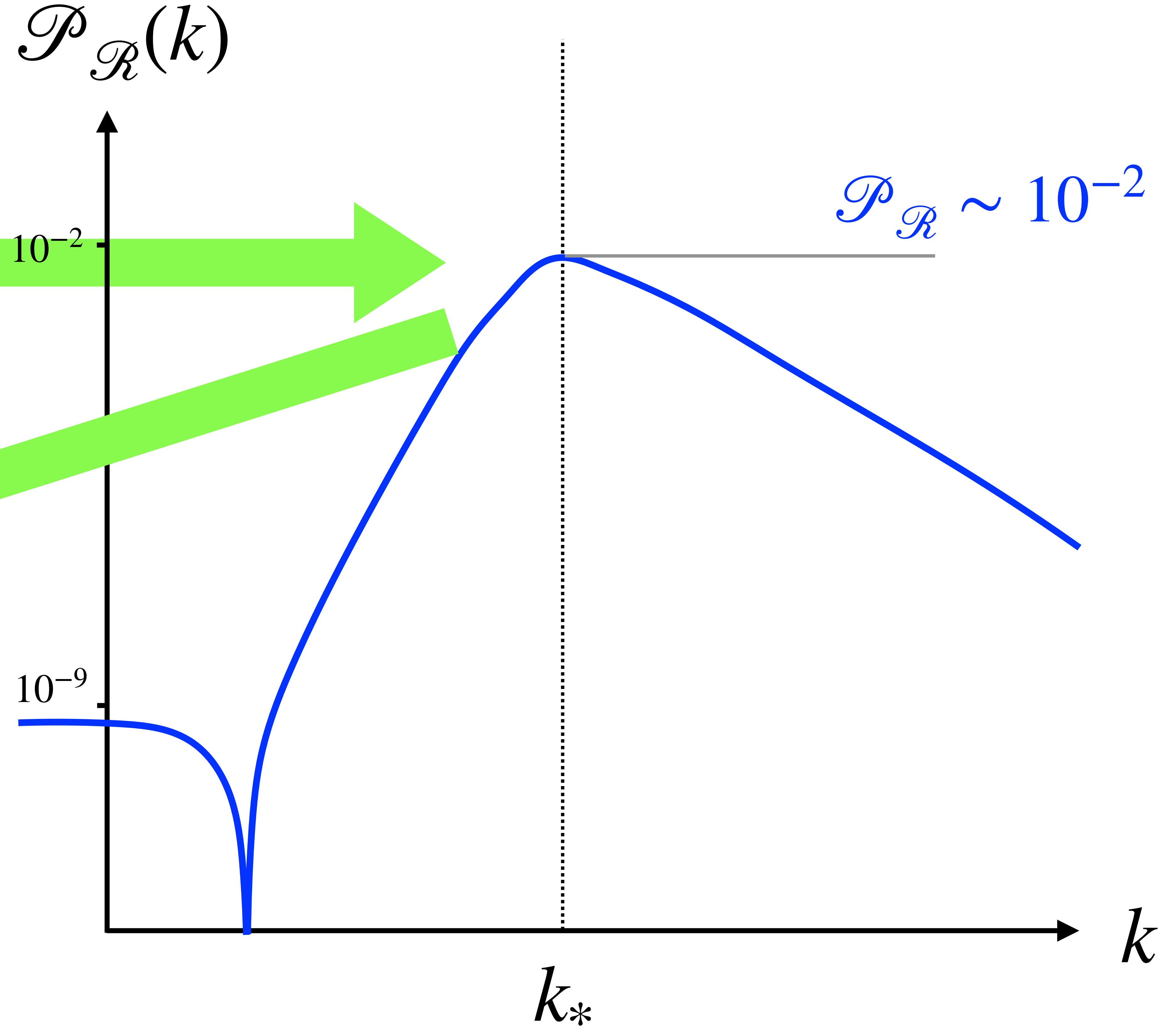
Constraints on PBH

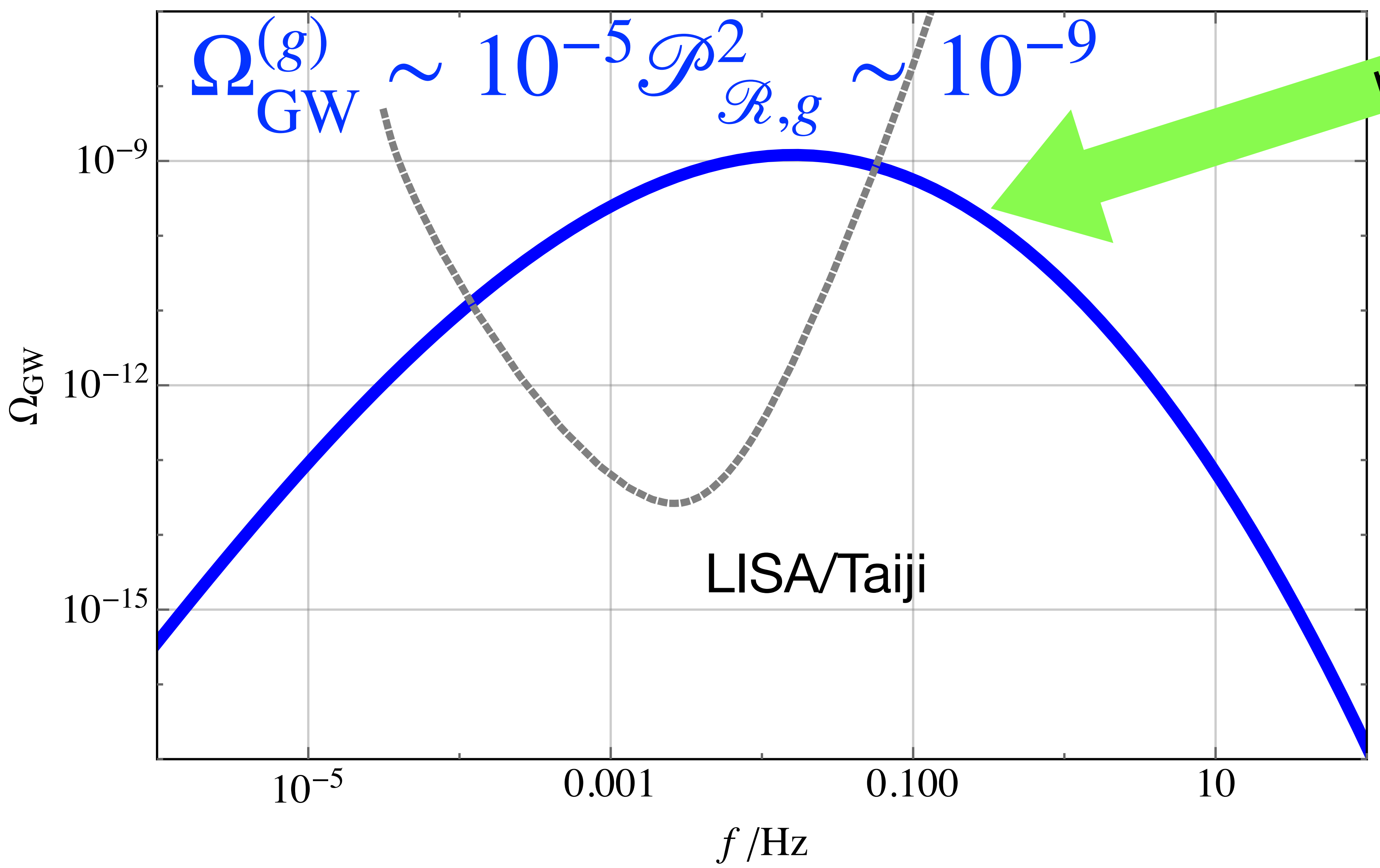




PBH-IGW crosscheck

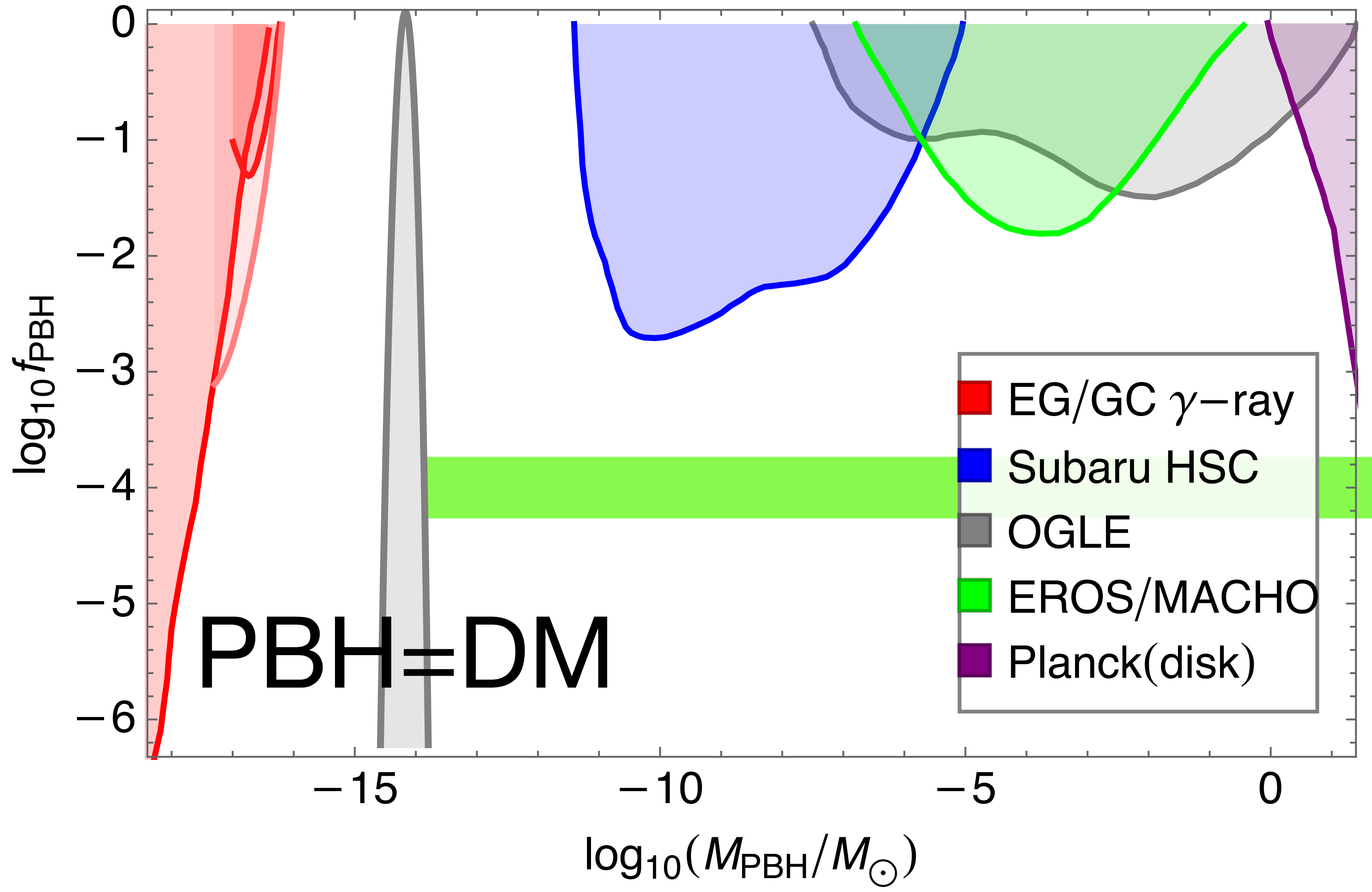


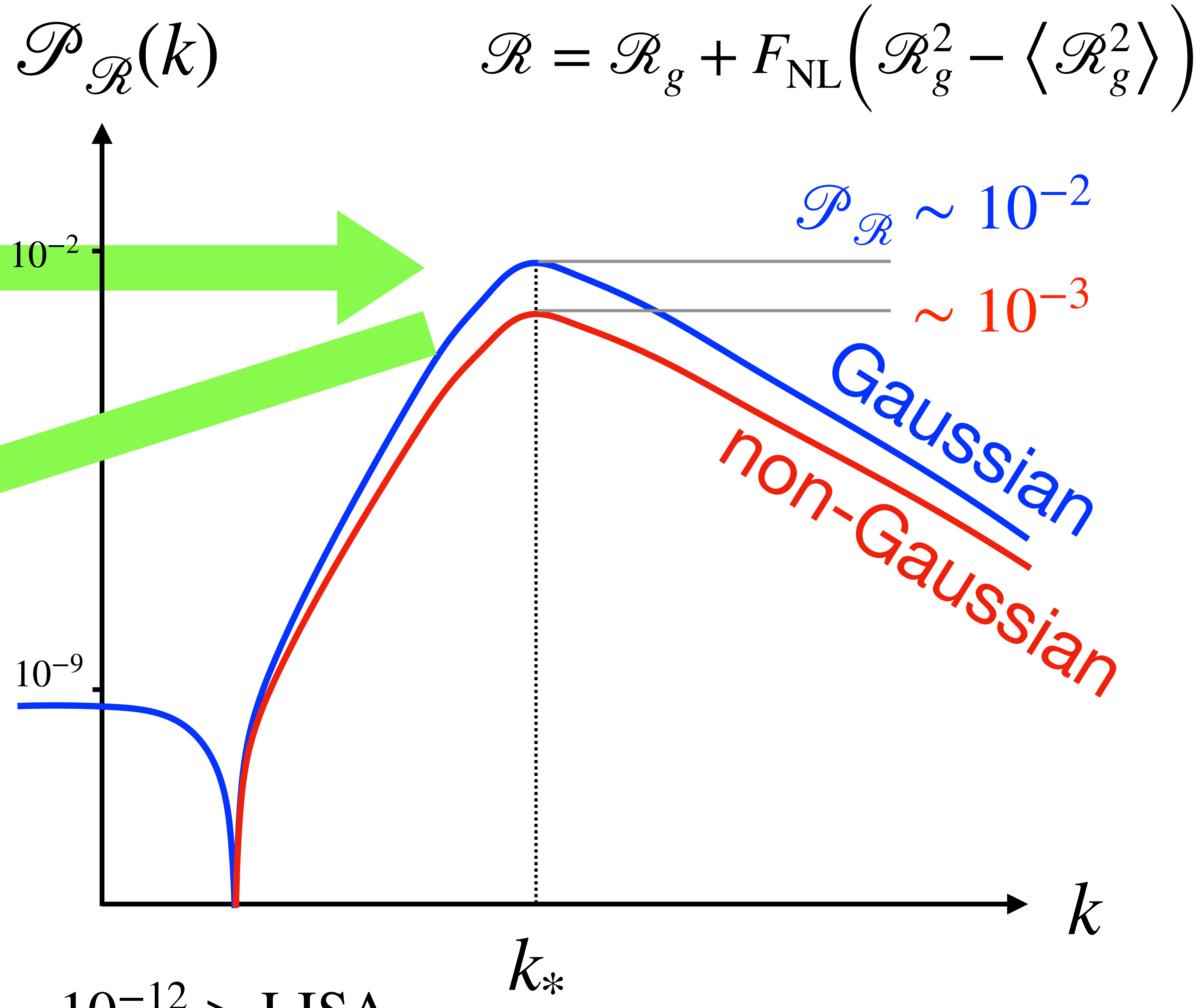


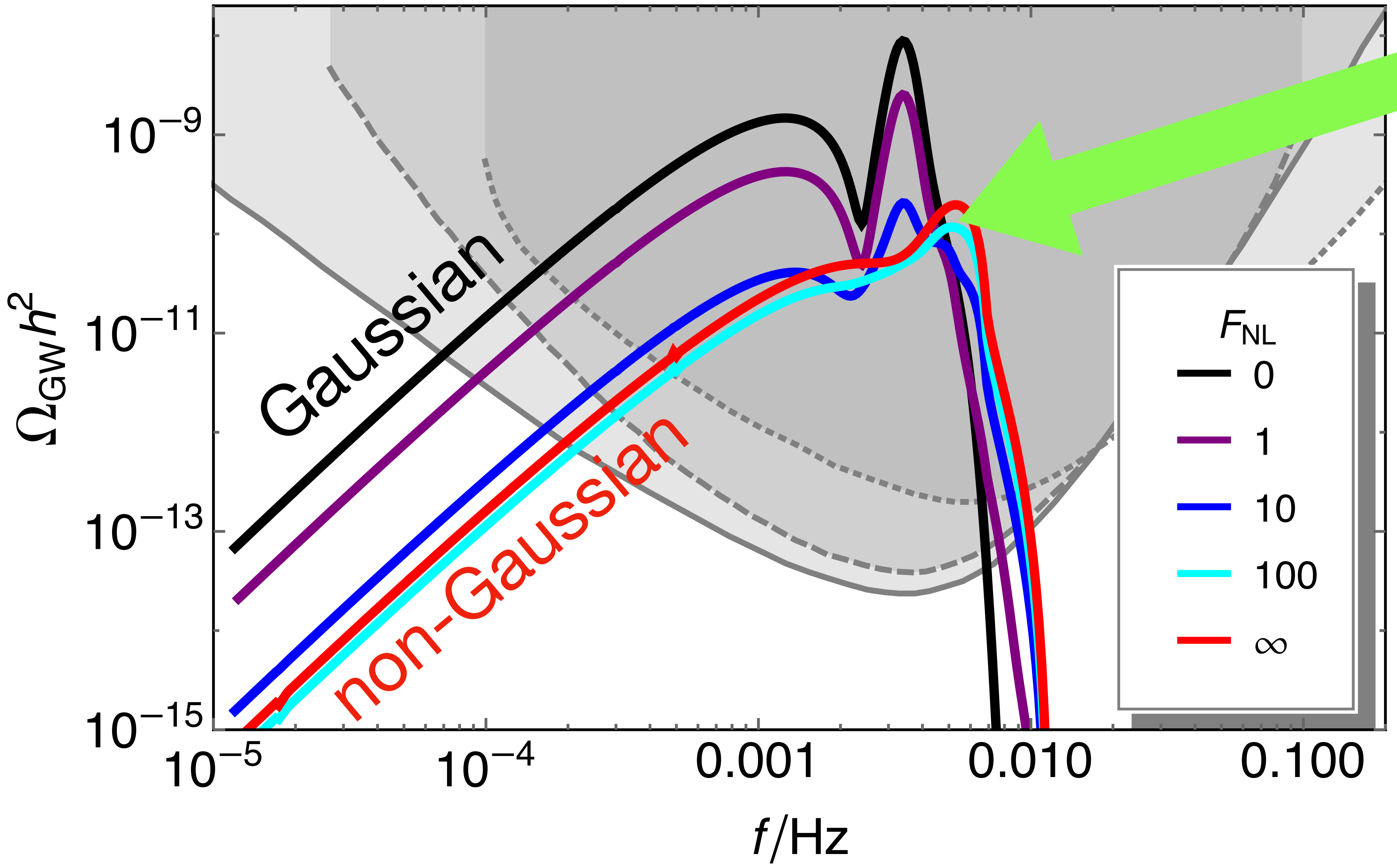


Saito & Yokoyama 0812.4339; 0912.5317
Bugaev & Klimai 0908.0664; 1012.4697
Escriva et al, 2211.05767

Including non-Gaussianity





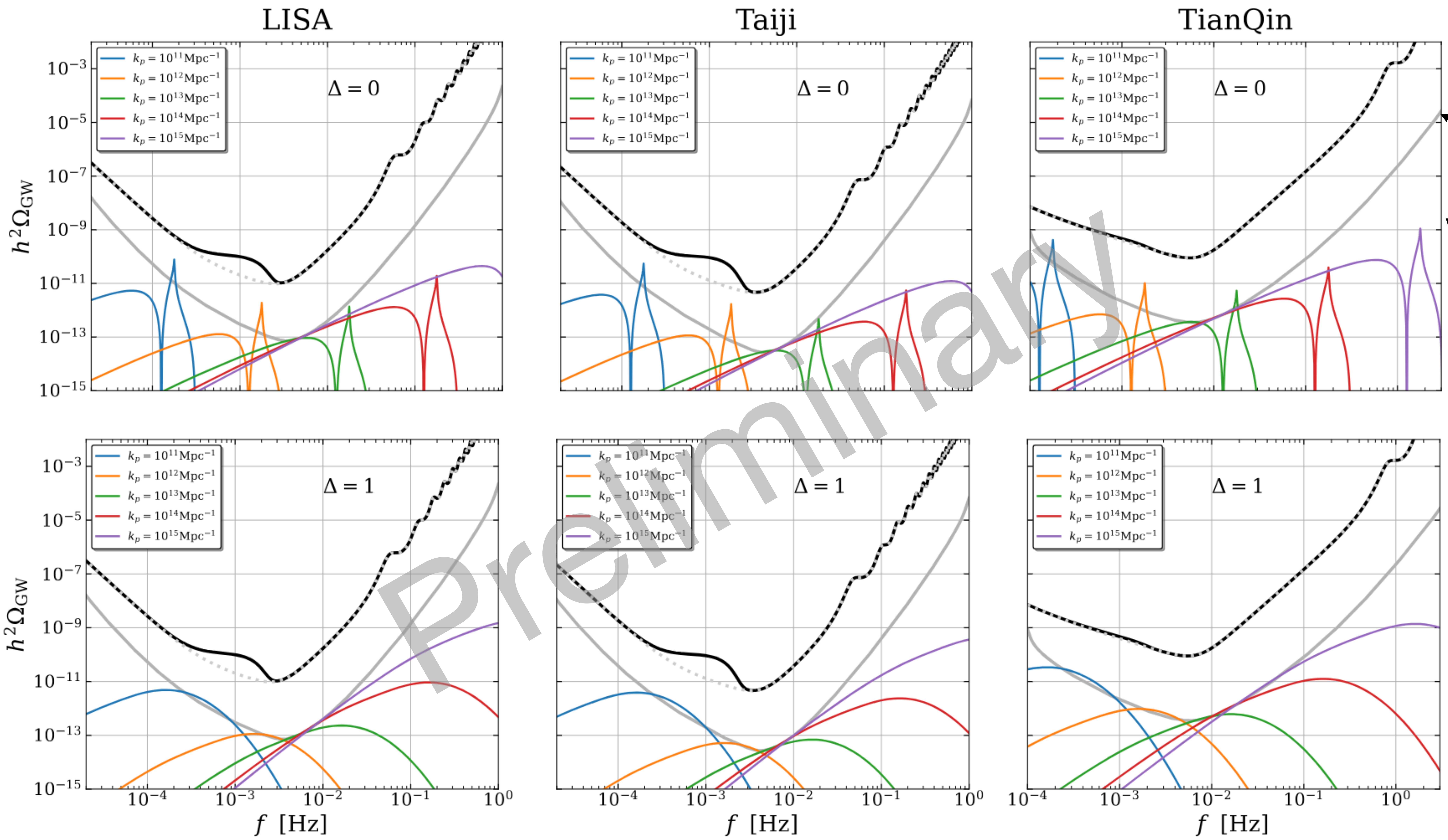


$$F_{\rm NL}\mathscr{P}_{\mathscr{R}} \sim 10^{-2}$$

$$\Omega_{\rm GW}^{\rm (NG)} \sim 10^{-5} F_{\rm NL}^4 \mathscr{P}_{\mathscr{R},g}^4 \sim 10^{-12} > \text{LISA}$$

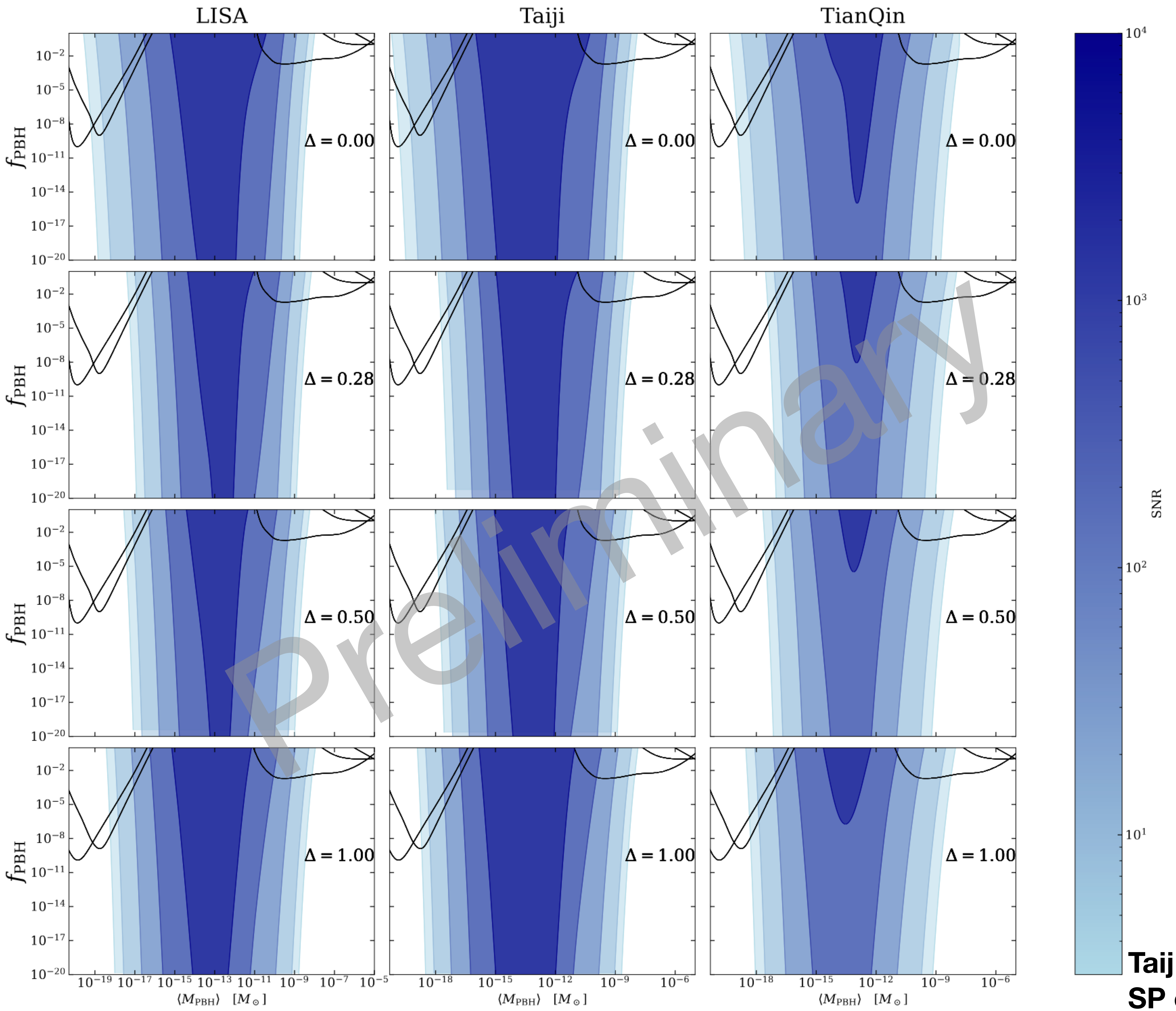
LISA/Taiji/TianQin can probe PBH-DM.

Cai, SP, Sasaki, PRL 122, 201101
SP, 2404.06151, PBH book ch.7



PLI of 3 yr w/. SNR=3

Taiji white paper, to appear
SP et al, to appear



Taiji white paper, to appear
SP et al, to appear

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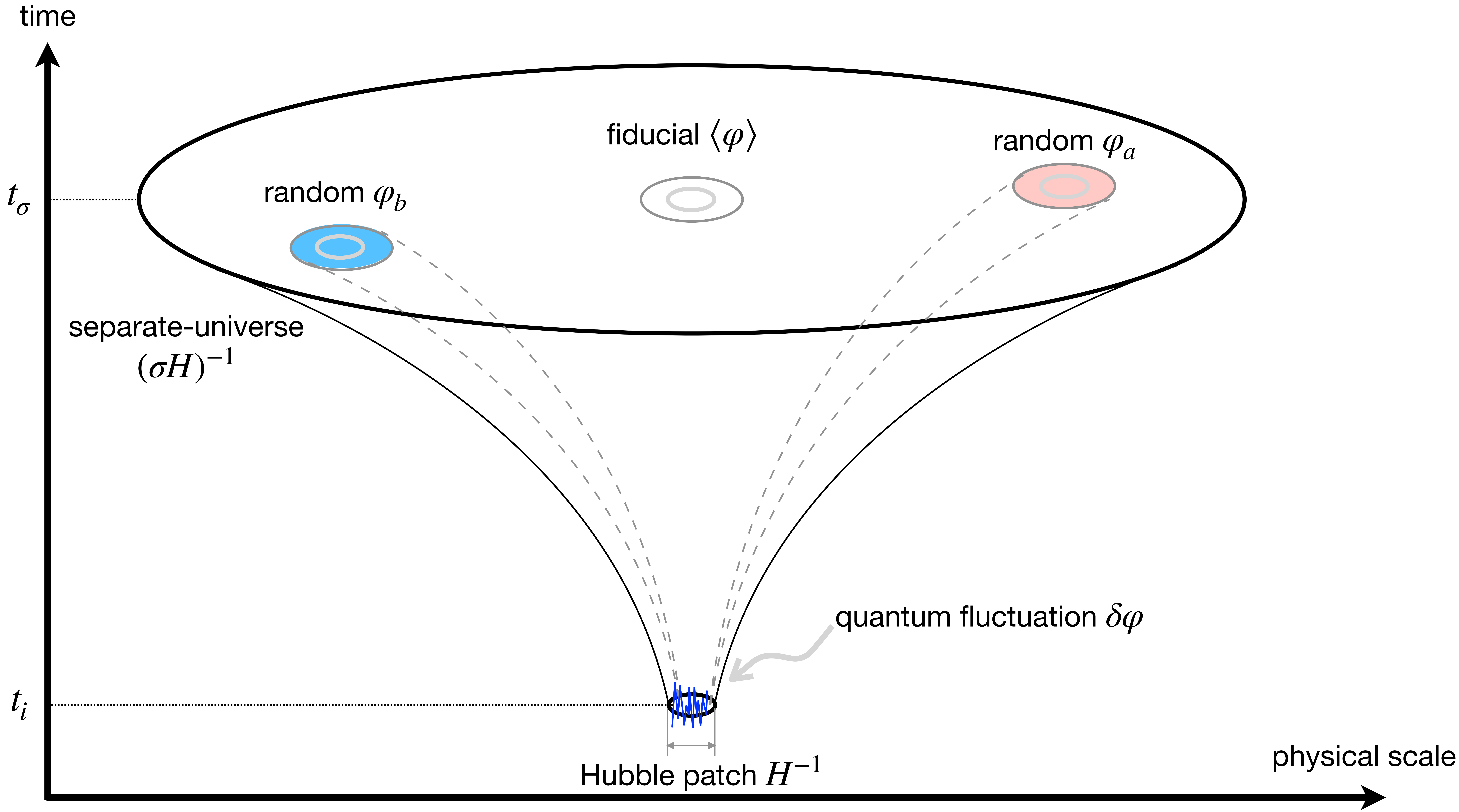
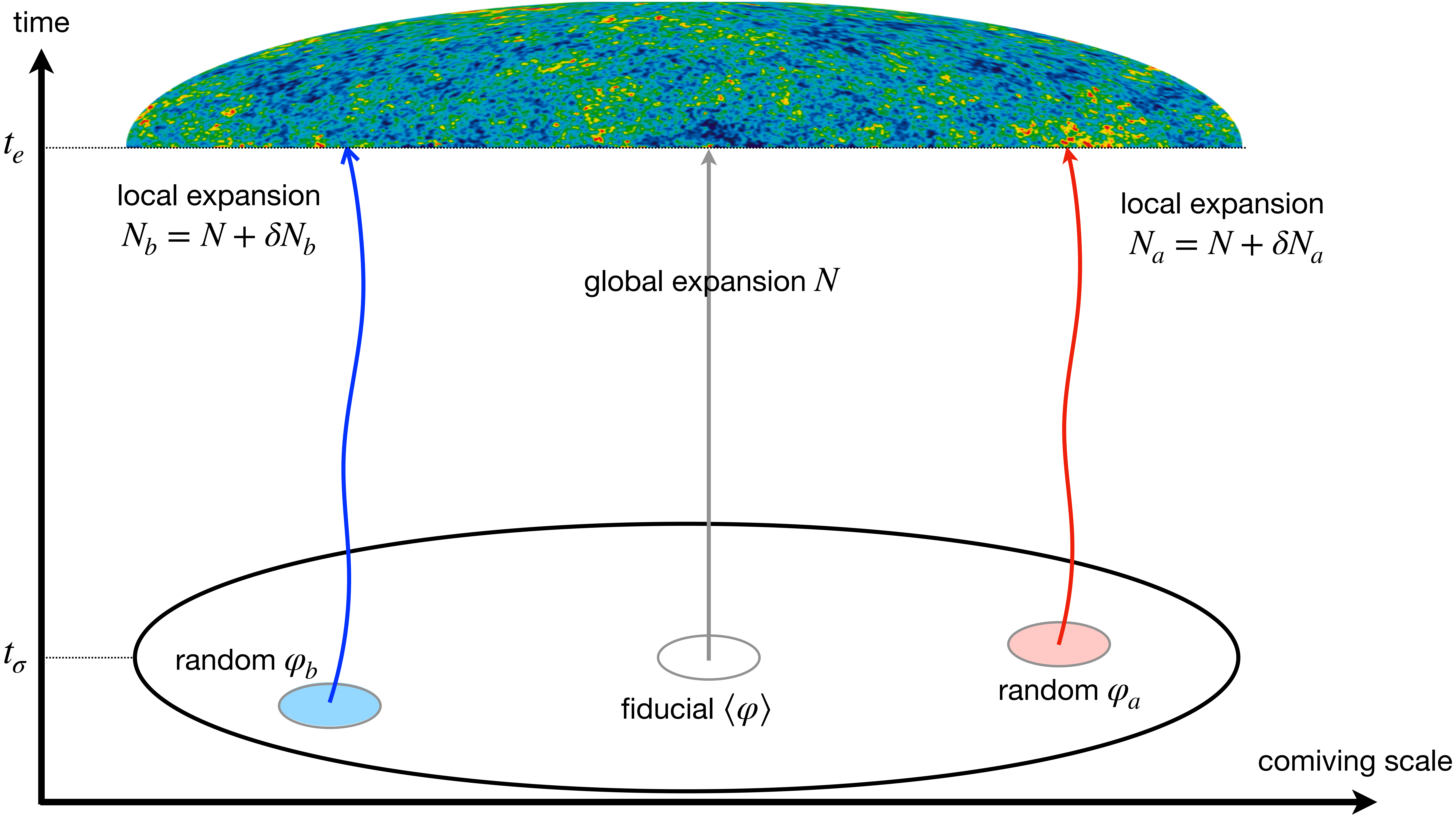

time
fiducial $\langle\varphi\rangle$
random φ_a
random φ_b
t_σ
separate-universe
$(\sigma H)^{-1}$
quantum fluctuation $\delta\varphi$
t_i
Hubble patch H^{-1}
physical scale

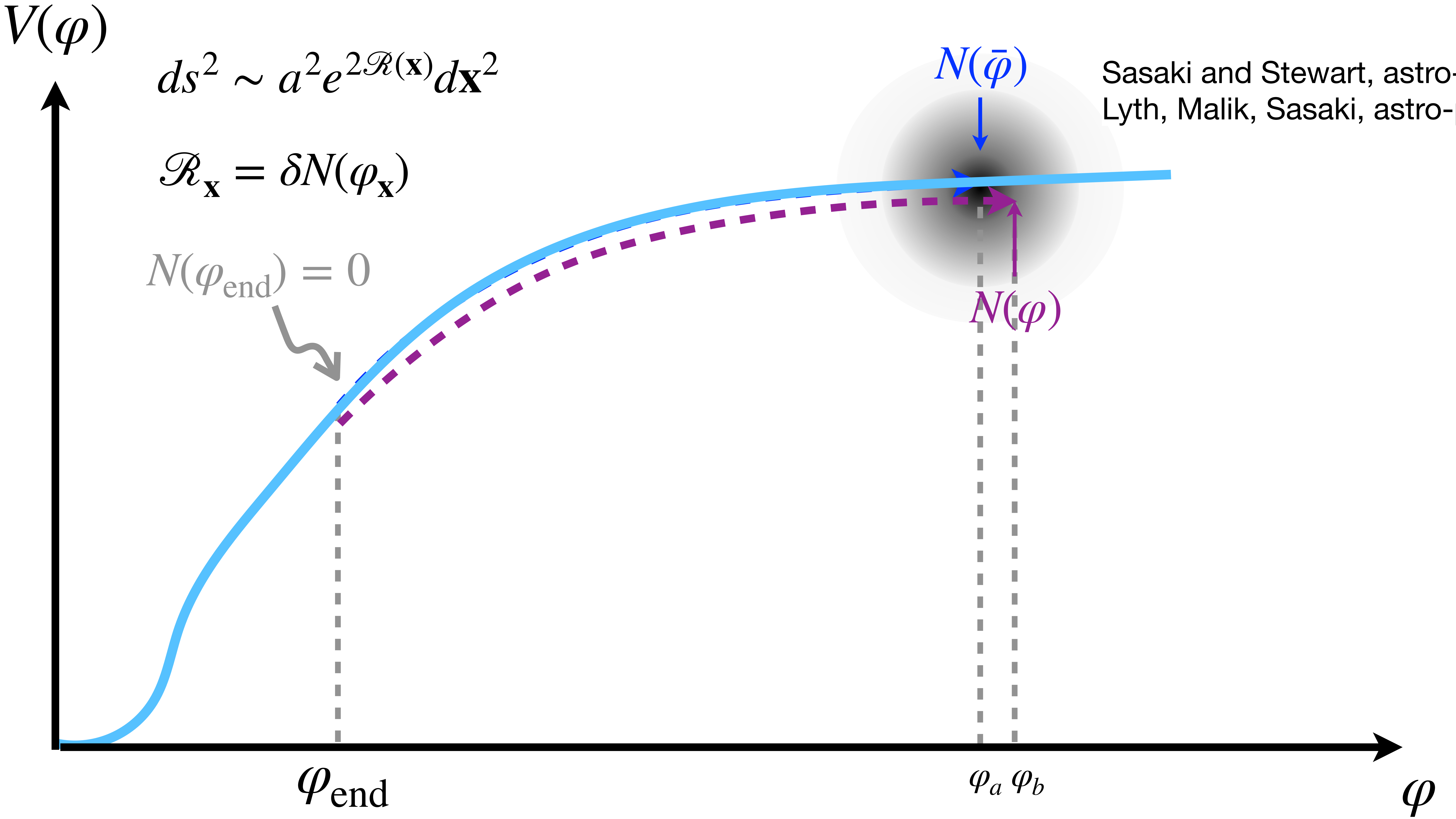

time
t_e
t_σ
local expansion
$N_b = N + \delta N_b$
global expansion N
local expansion
$N_a = N + \delta N_a$
random φ_b
fiducial $\langle \varphi \rangle$
random φ_a
comiving scale

δN formalism

$$ds^2 \sim a^2 e^{2\mathscr{R}(\mathbf{x})} d\mathbf{x}^2$$

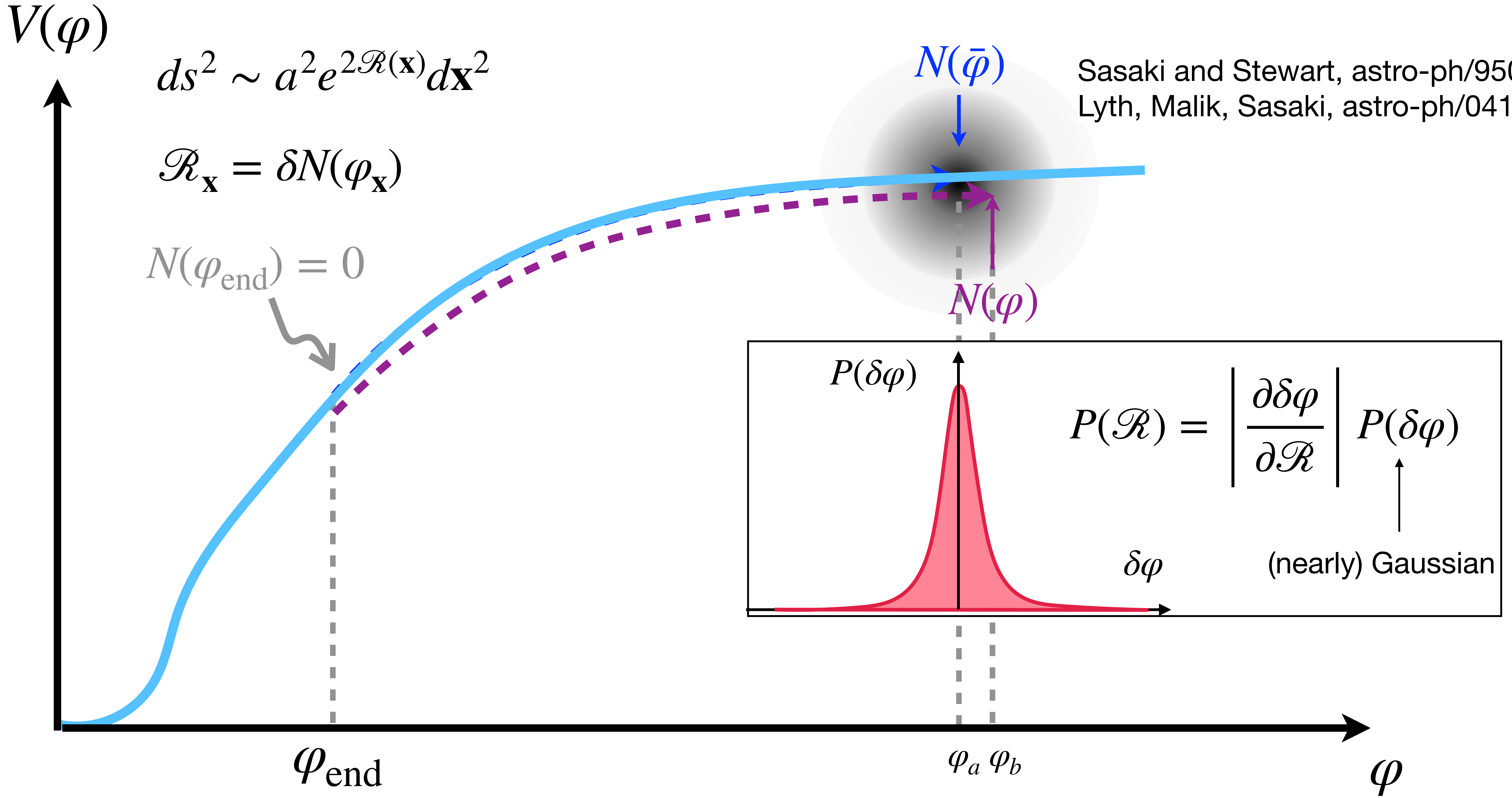
$$\mathscr{R}_{\mathbf{x}} = \delta N(\varphi_{\mathbf{x}})$$

Sasaki and Stewart, astro-ph/9507001
Lyth, Malik, Sasaki, astro-ph/0411220

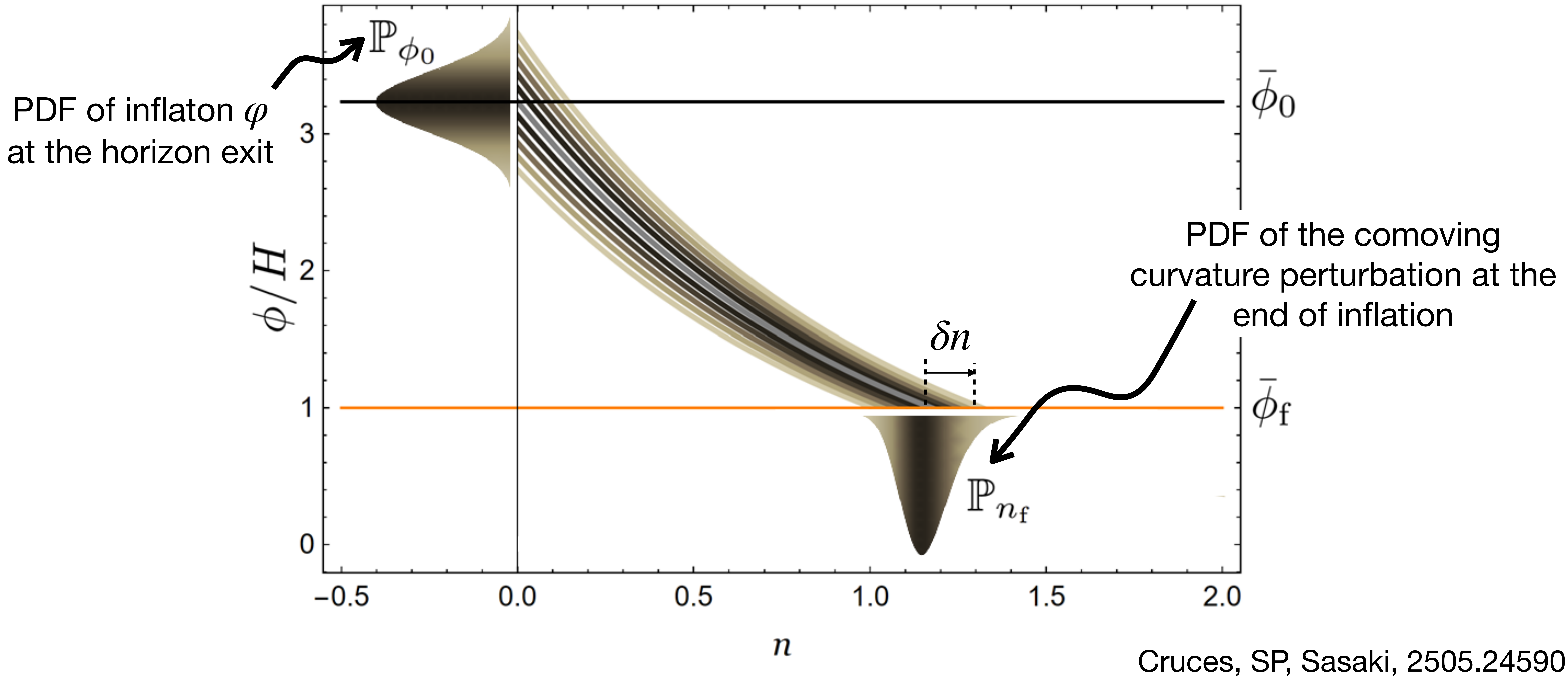


δN formalism

Sasaki and Stewart, astro-ph/9507001
Lyth, Malik, Sasaki, astro-ph/0411220

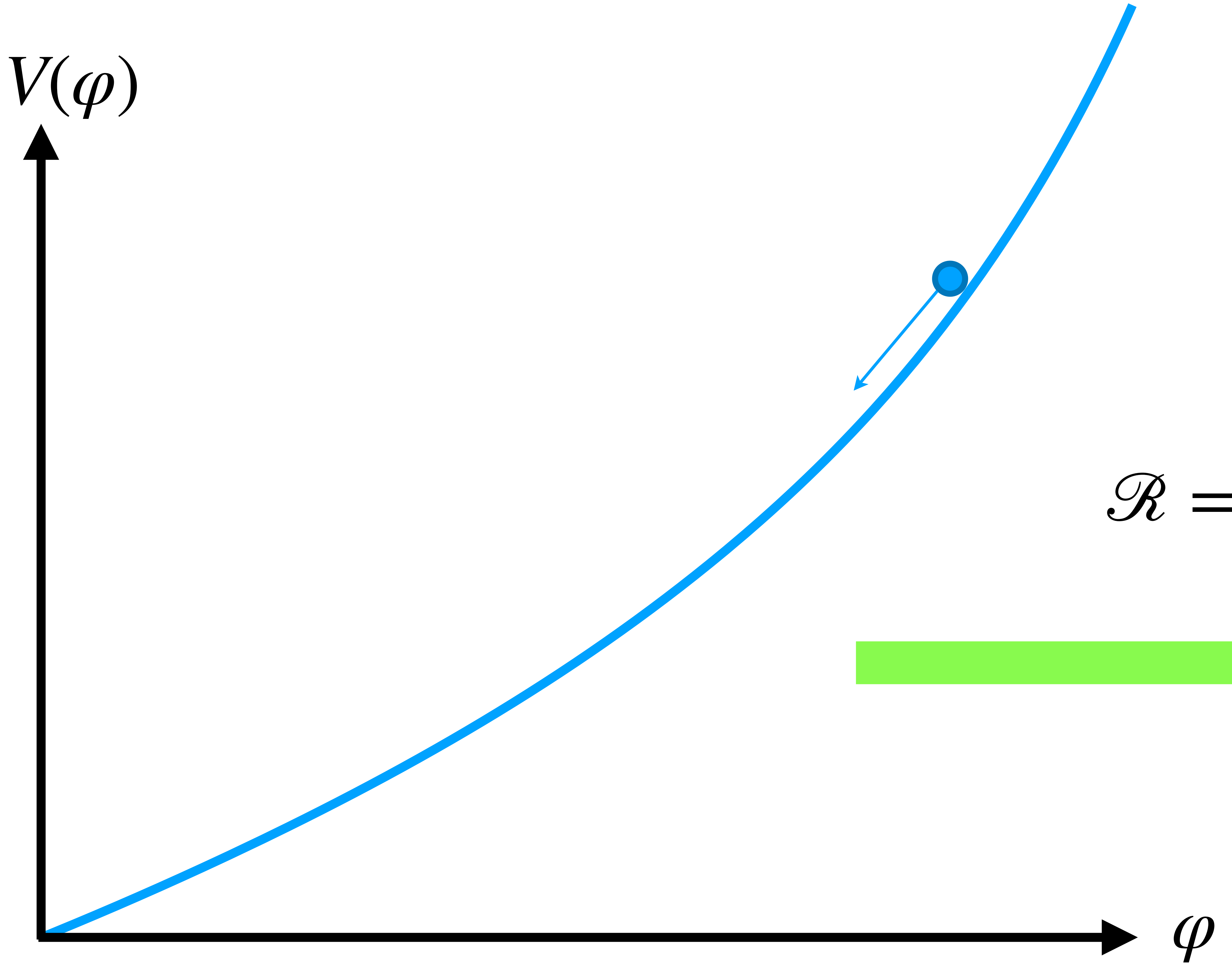


δN formalism

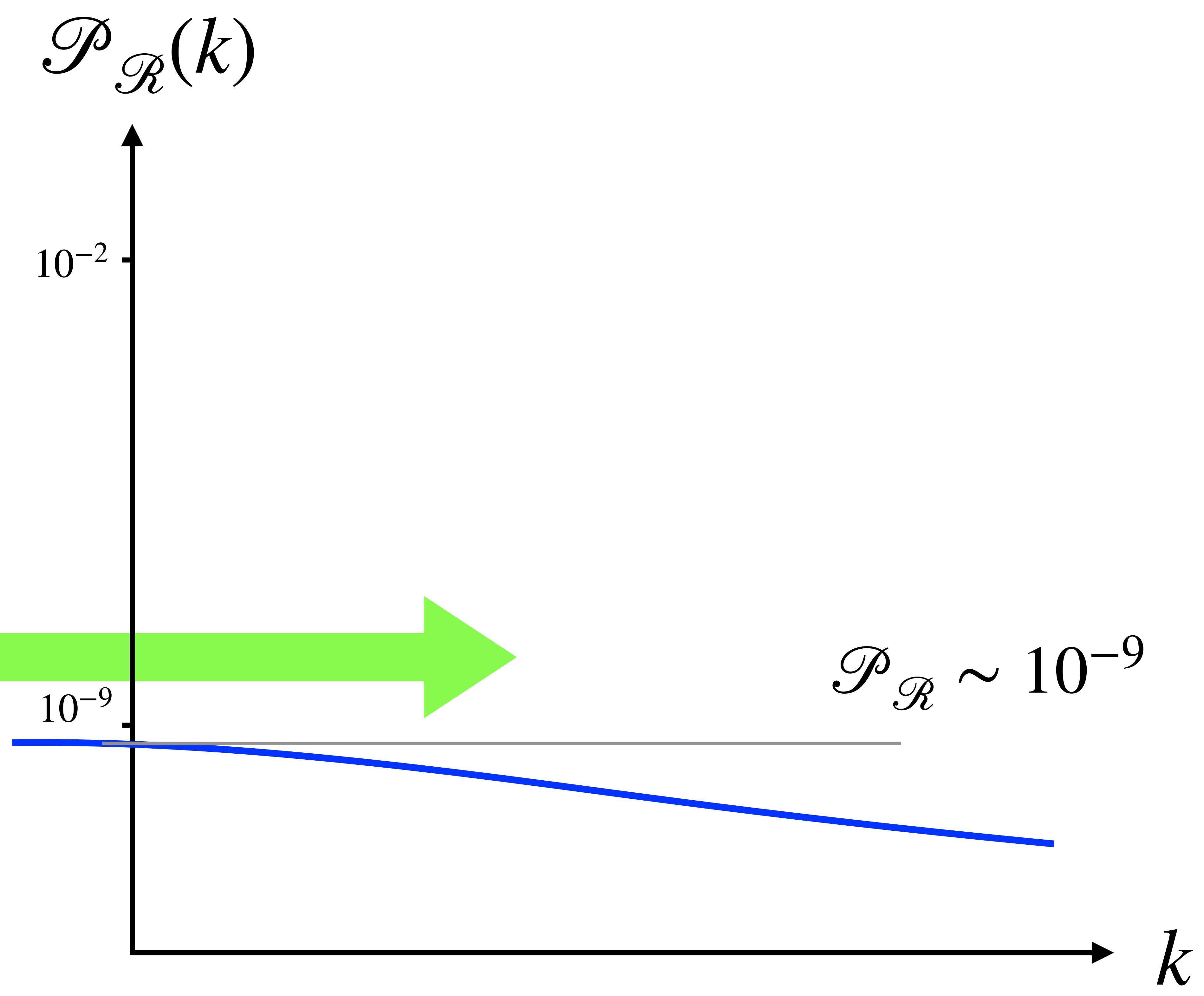


Cruces, SP, Sasaki, 2505.24590

Gaussian Curvature Perturbation

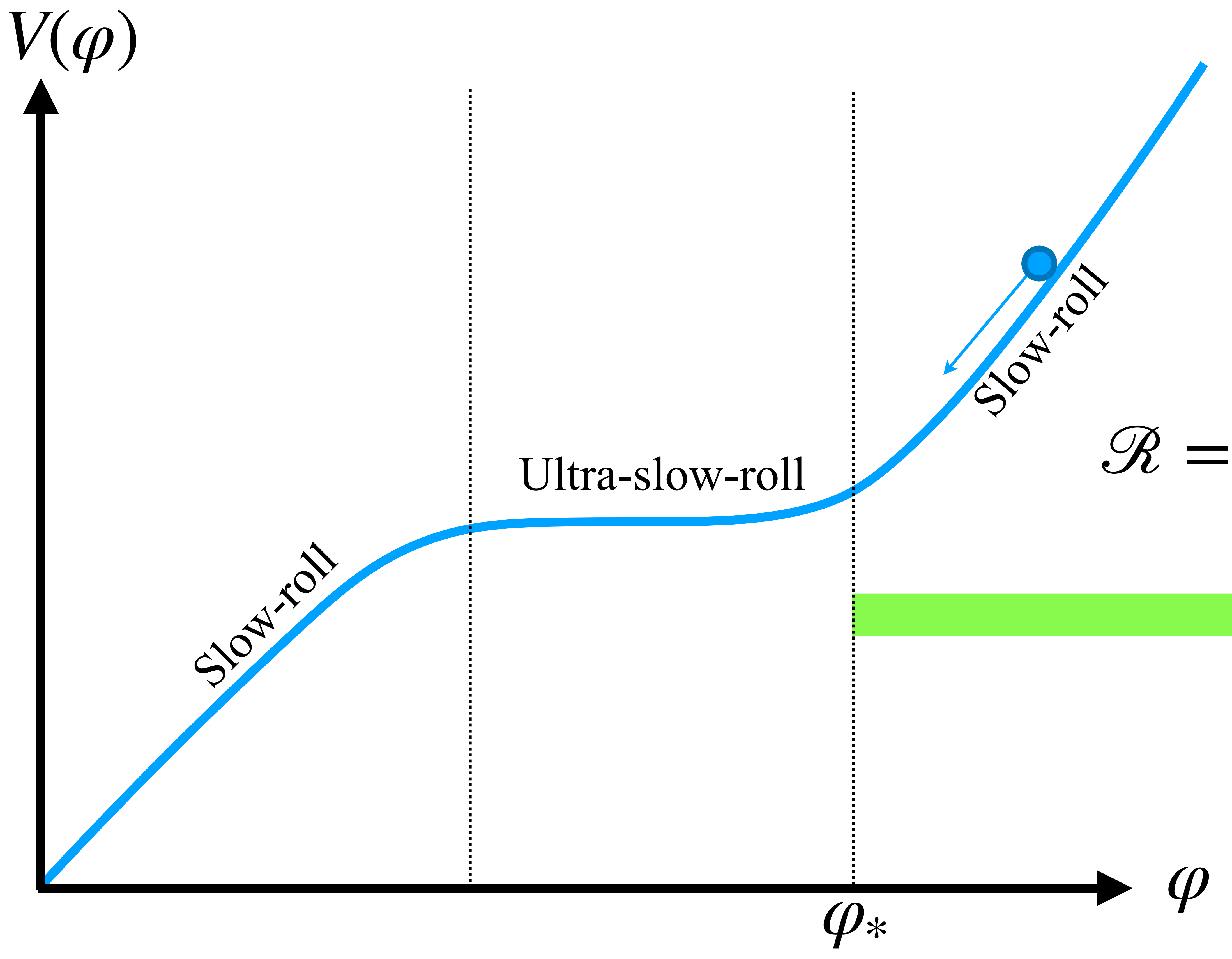


$$\mathscr{R} = \delta N \approx -\frac{H}{\dot{\varphi}}\delta\varphi$$



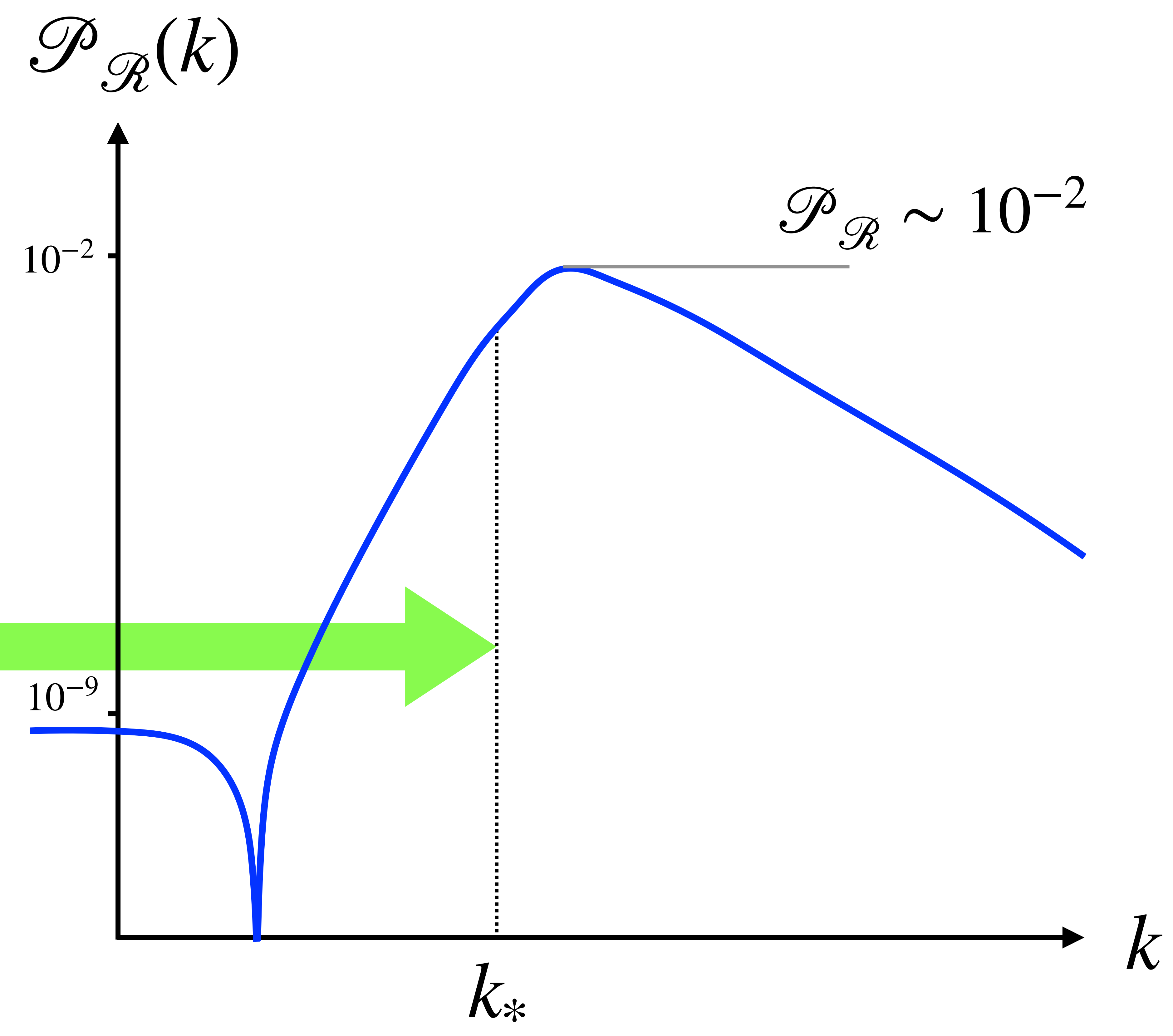
Stewart and Sasaki, astro-ph/9507001
Lyth and Roquigez, astro-ph/0504045

Ultra-slow-roll Inflation



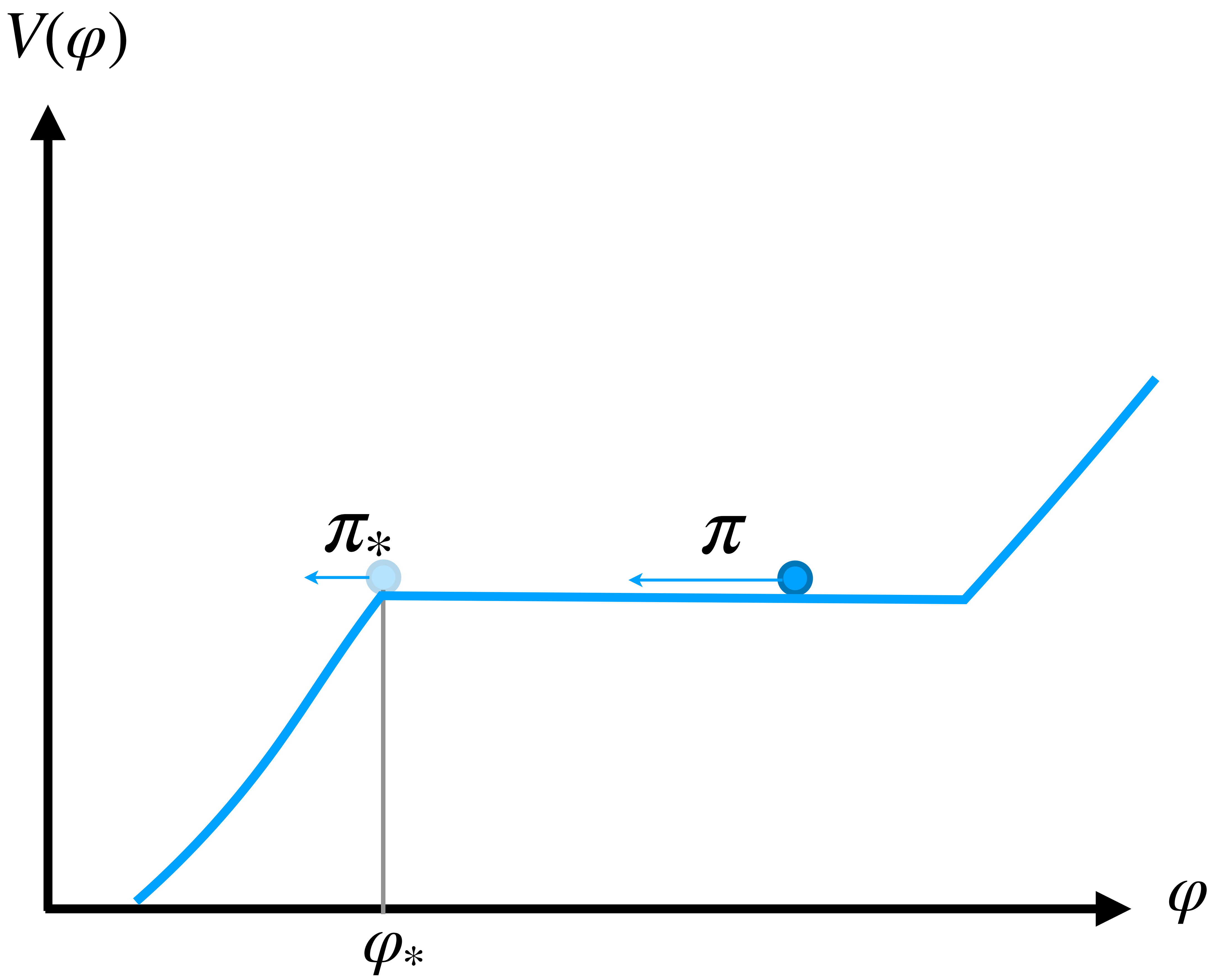
$$\mathcal{R} = \delta N \approx -\frac{H}{\dot{\varphi}}\delta\varphi$$

?



Starobinski, JETP Lett. 55, 489
Byrnes, Cole, Patil, 1811.11158
Cole, Gow, Byrnes, Patil, 2204.07573
SP & Jianing Wang, 2209.14183

Ultra-slow-roll inflation



$$\ddot{\varphi} + 3H\dot{\varphi} = 0$$

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}N^2} - 3\frac{\mathrm{d}\varphi}{\mathrm{d}N} = 0 \qquad \left(N = \int_t^{t_*} H\mathrm{d}t\right)$$

$$\pi' - 3\pi = 0 \qquad \left(\pi \equiv -\frac{\mathrm{d}\varphi}{\mathrm{d}N}\right)$$

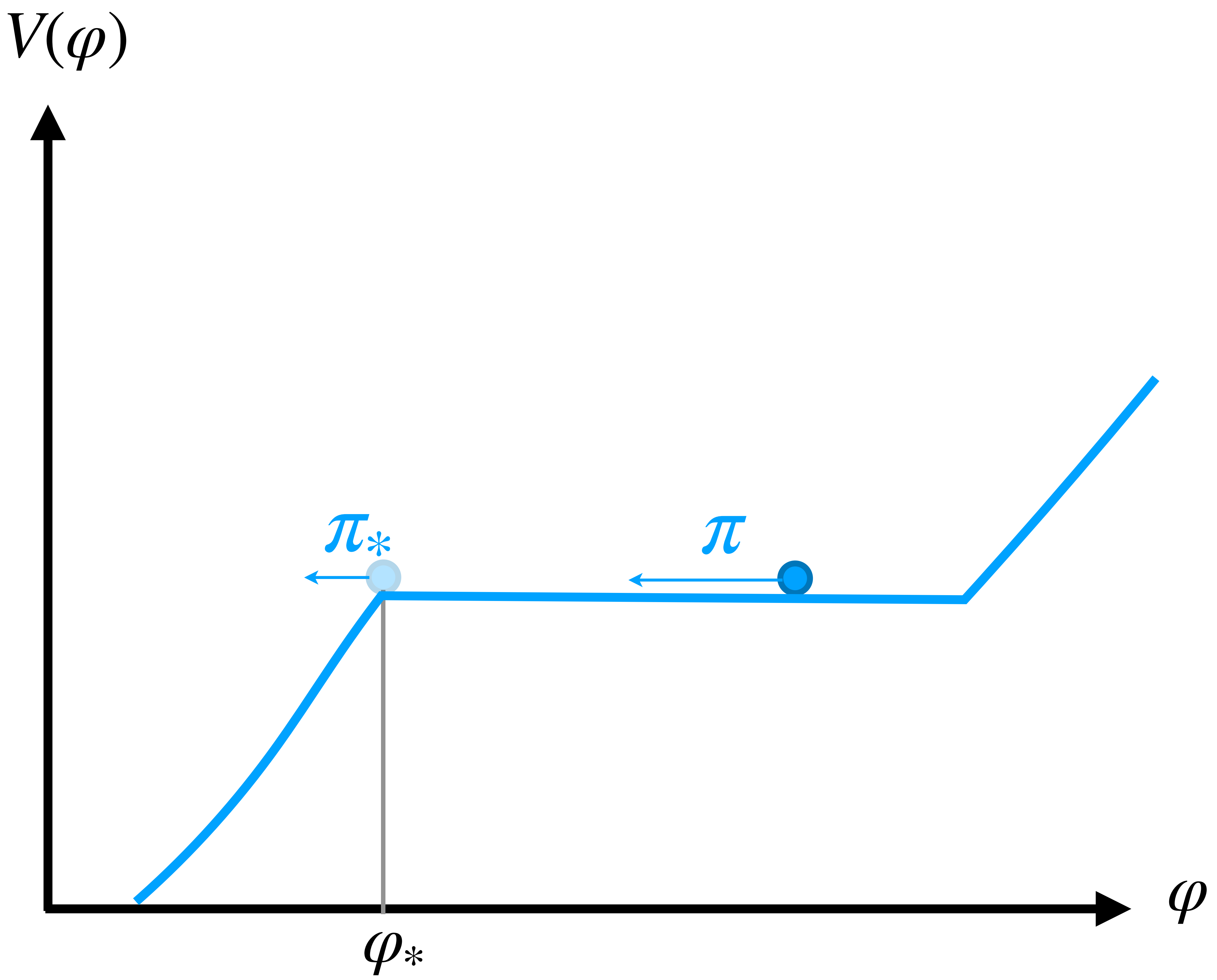
$$\pi = \pi_* e^{3N}$$

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

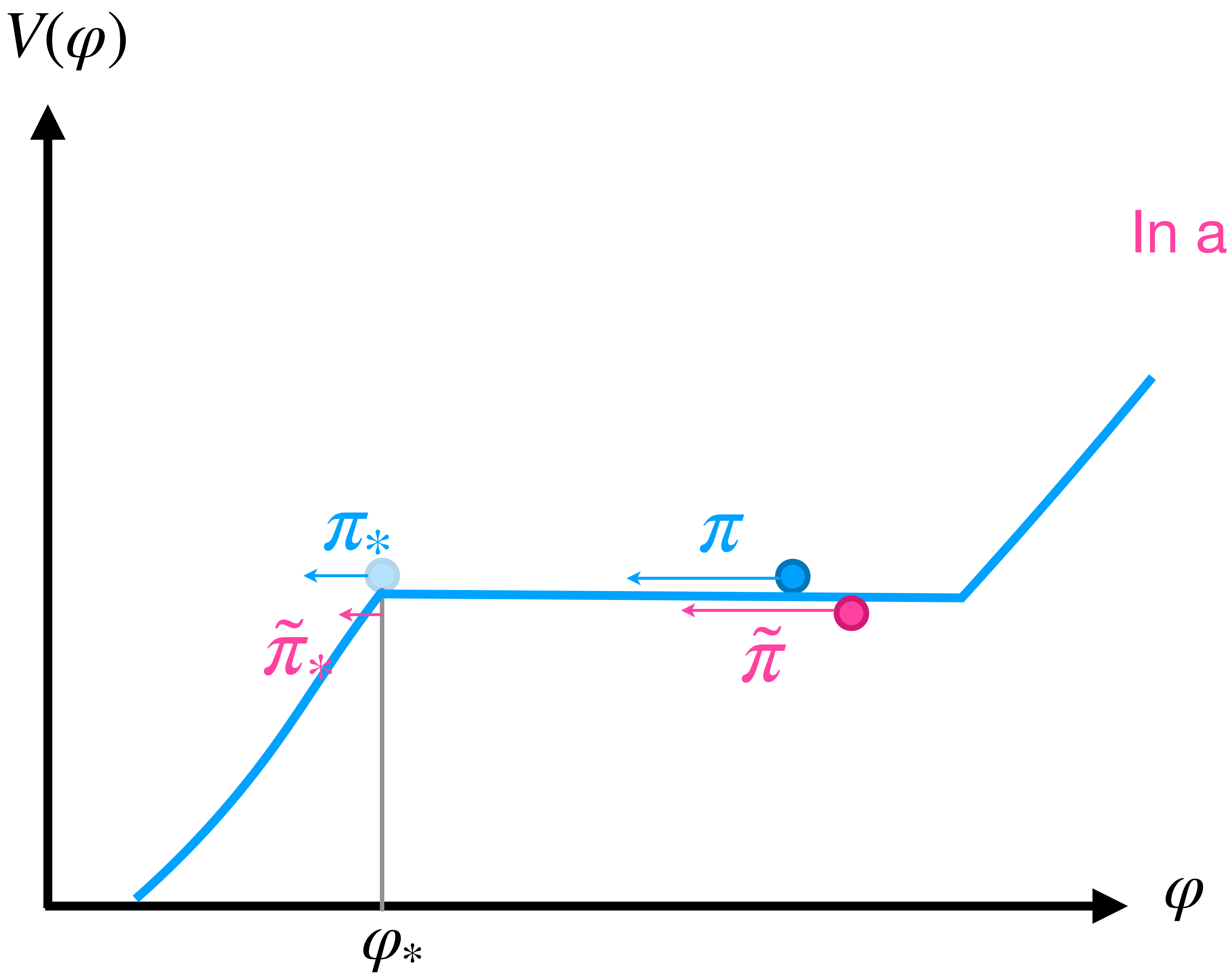
Ultra-slow-roll inflation

In the “fiducial” patch

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$



Ultra-slow-roll inflation



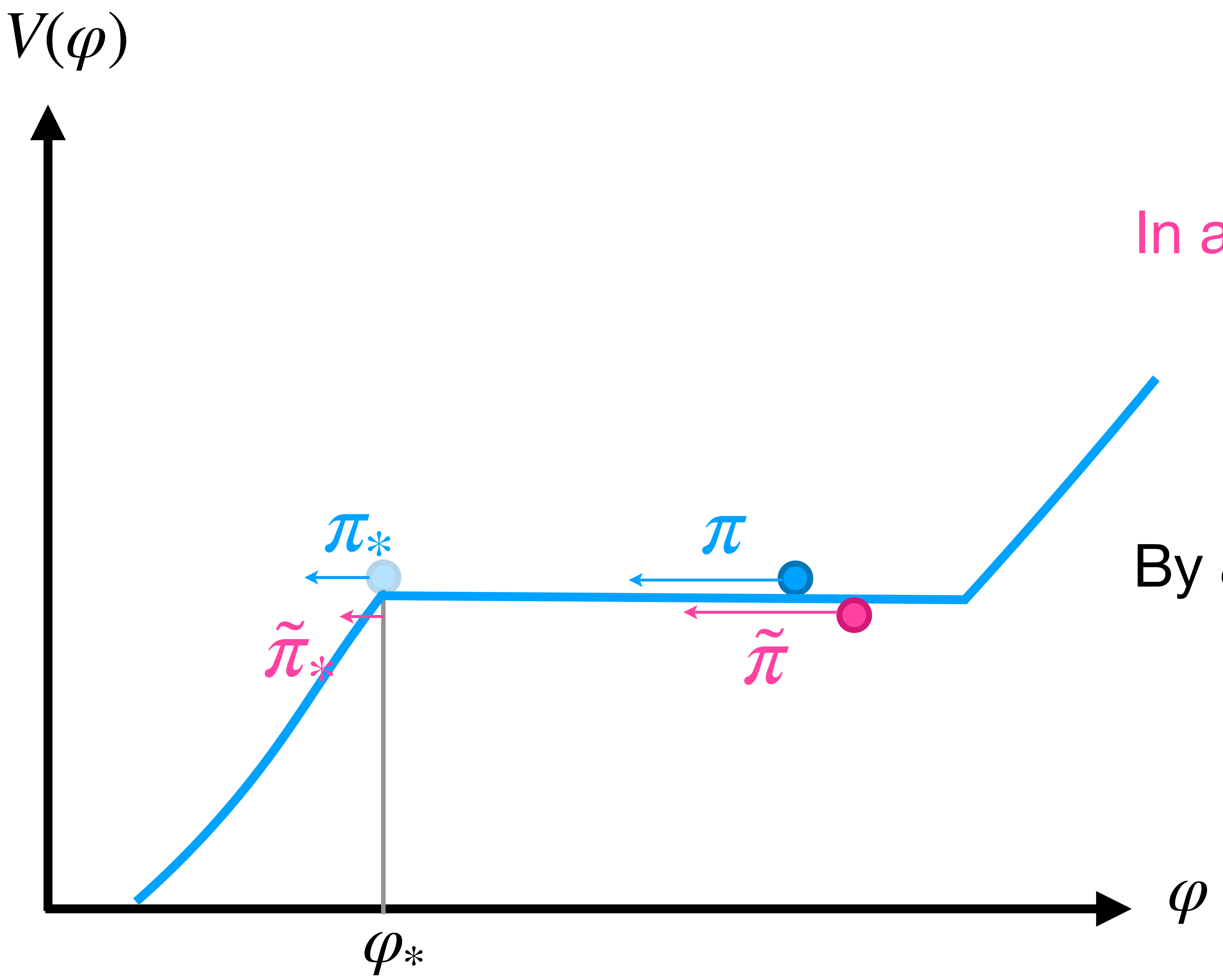
In the "fiducial" patch

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

In a perturbed patch

$$\tilde{N} = -\frac{1}{3}\ln\frac{\tilde{\pi}_*}{\tilde{\pi}}$$

Ultra-slow-roll inflation



In the "fiducial" patch

$$N = -\frac{1}{3}\ln\frac{\pi_*}{\pi}$$

In a perturbed patch

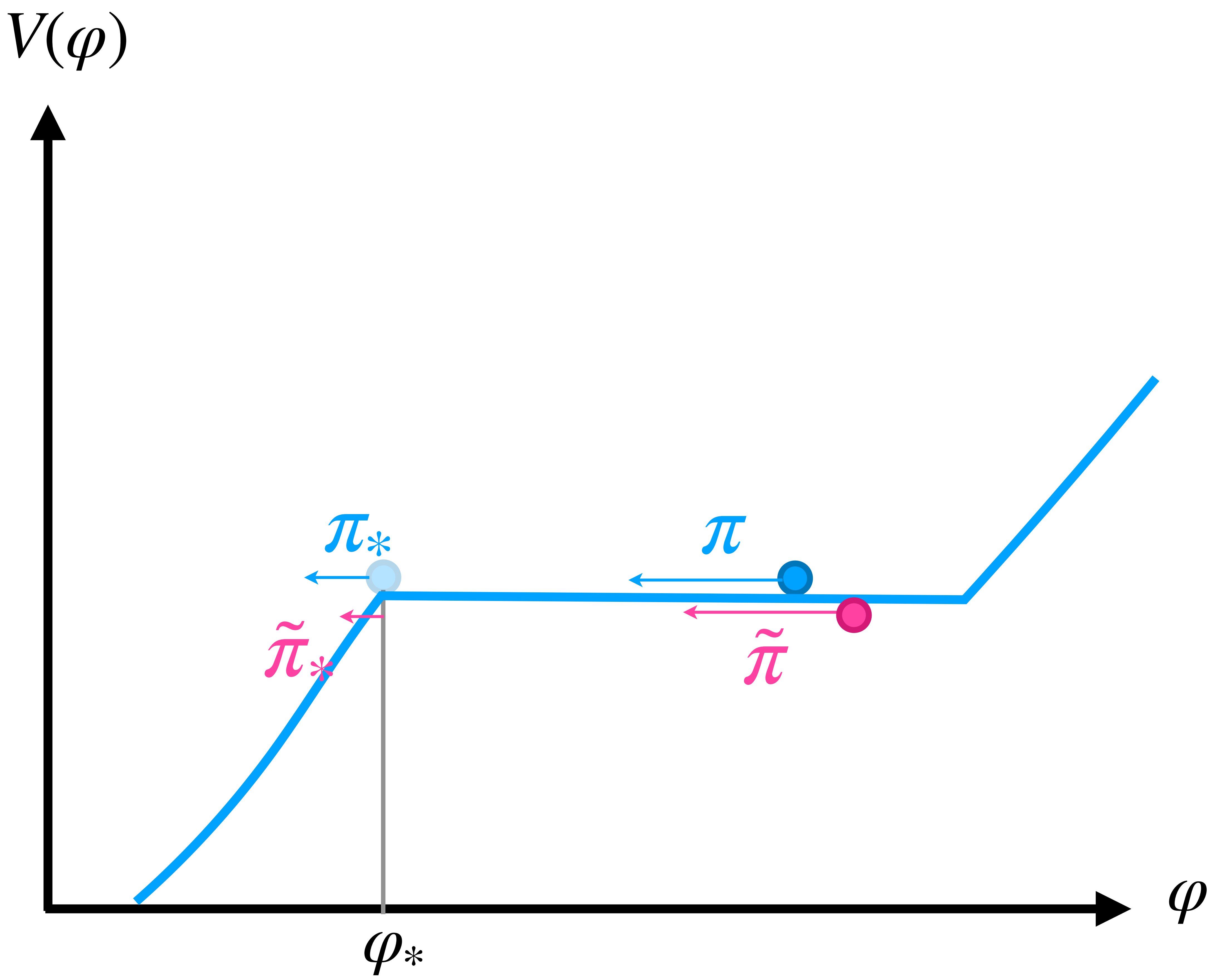
$$\tilde{N} = -\frac{1}{3}\ln\frac{\tilde{\pi}_*}{\tilde{\pi}}$$

By δN formalism, the curvature perturbation is

$$\mathscr{R} = \delta N = \tilde{N} - N = \frac{1}{3}\ln\frac{\tilde{\pi}}{\pi}\frac{\pi_*}{\tilde{\pi}_*}$$

$$= \frac{1}{3}\ln\left(1+\frac{\delta\pi}{\pi}\right) - \frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*}\right)$$

Ultra-slow-roll inflation

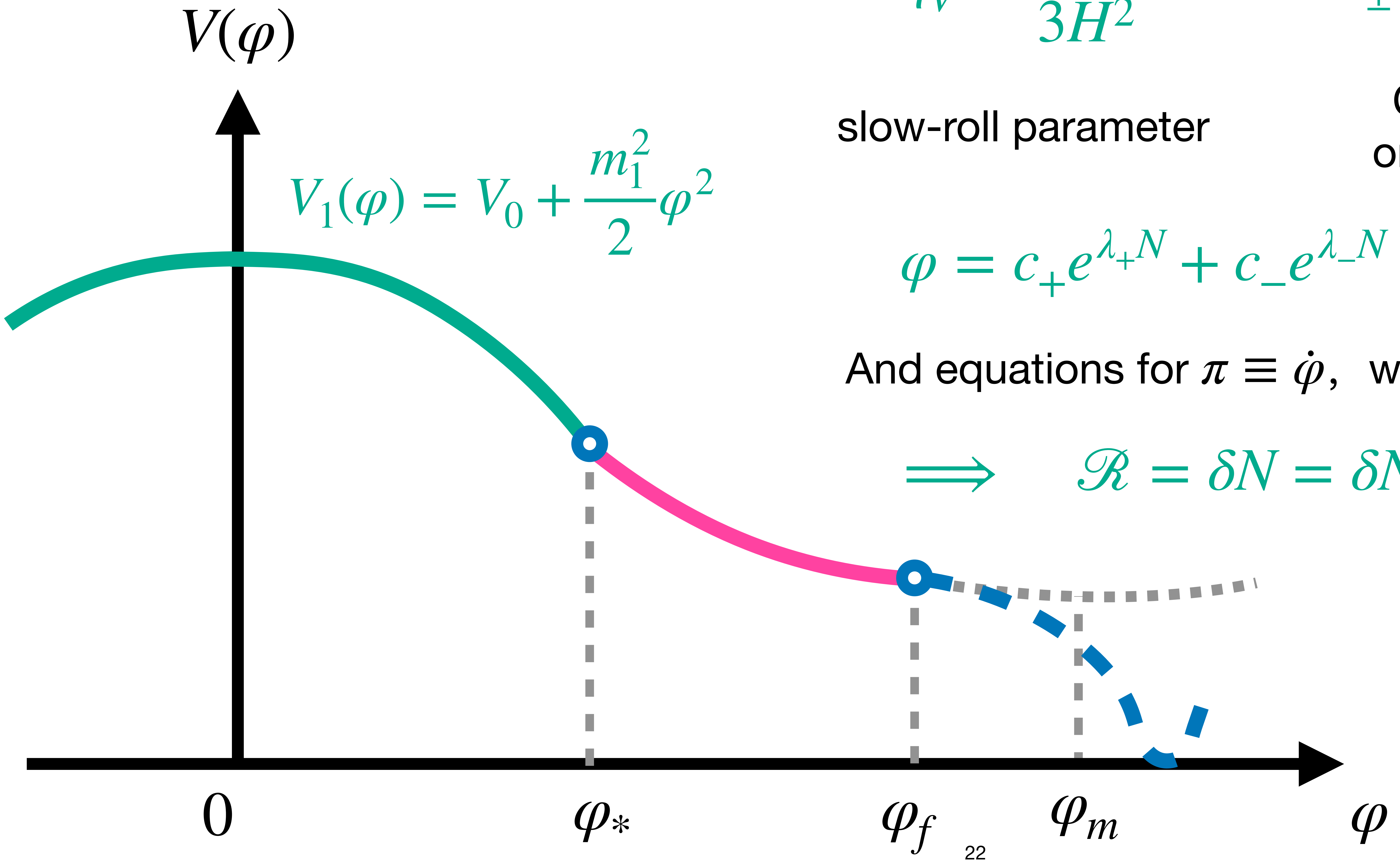


$$\mathscr{R} \approx -\frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*}\right)$$

$$\left(f_{\rm NL}=\frac{5}{2},\quad g_{\rm NL}=-\frac{25}{3},\ \cdots\right)$$

Namjoo, Firouzjahi, Sasaki, 1210.3692
Chen, Firouzjahi, Komatsu, Namjoo, Sasaki, 1308.5341
Cai, Chen, Namjoo, Sasaki, Wang, Wang, 1712.09998
Biagetti, Franciolini, Kehagias, Riotto, 1804.07124
Passaglia, Hu, Motohashi, 1812.08243
SP and Sasaki, 2211.13932
SP, 2404.06151
Relation with stochastic approach, see e.g.
Jackson et al 2410.13683, Cruces et al 2410.17987
Including intrinsic non-Gaussianity, see e.g.
Ballestores et al 2412.14106, Caravato et al 2506.11795

piecewise quadratic potential



$$\eta_V = \frac{m_1^2}{3H^2}$$

slow-roll parameter

$$\lambda_\pm = \frac{3 \pm \sqrt{9 - 12\eta_V}}{2}$$

Characteristic roots
or Liyapnov exponent

$$\varphi = c_+ e^{\lambda_+ N} + c\_e^{\lambda\_N}$$

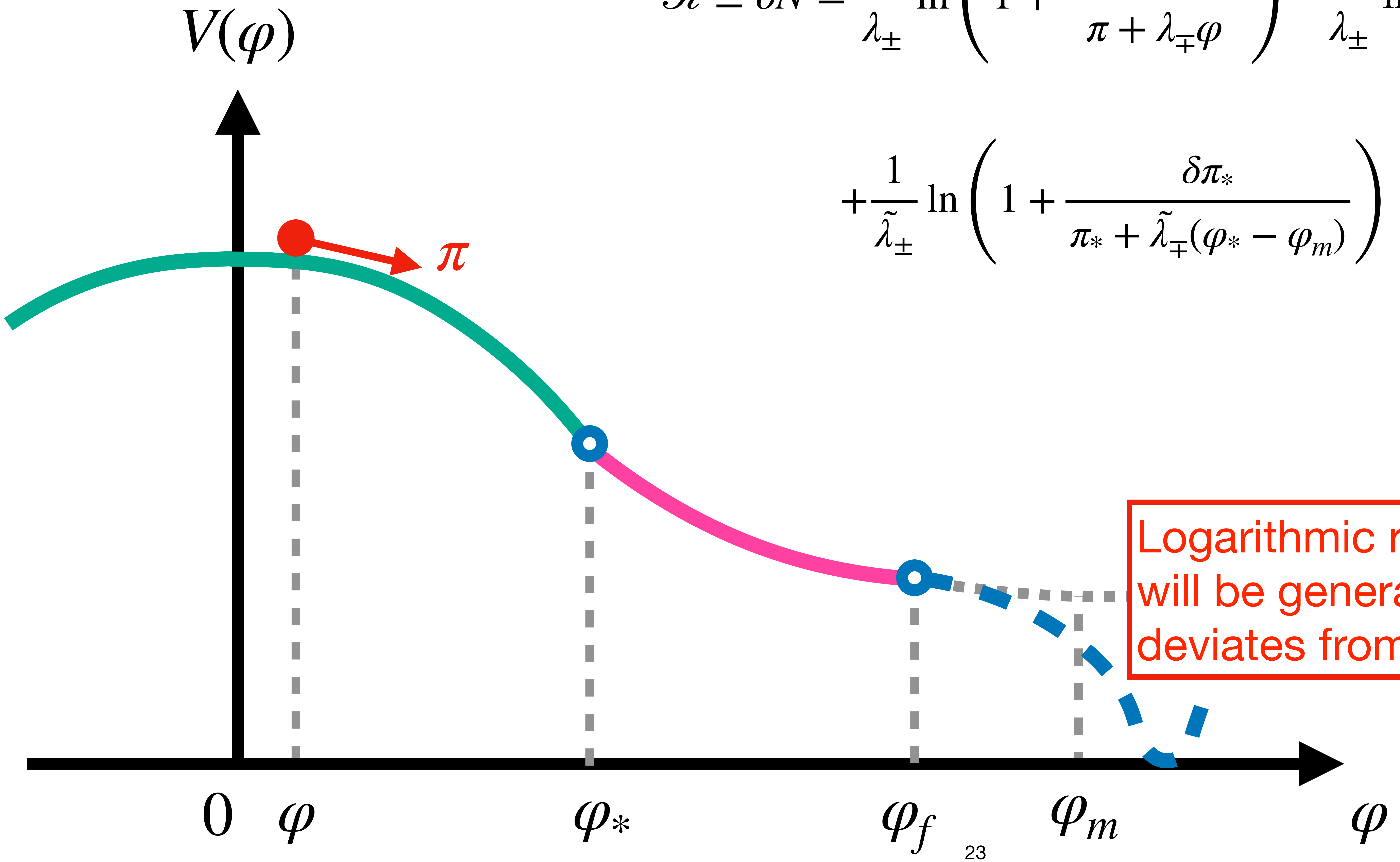
And equations for $\pi \equiv \dot{\varphi}$, which gives $N(\varphi,\ \pi)$

$$\Longrightarrow \quad \mathcal{R} = \delta N = \delta N(\delta\varphi, \delta\pi)$$

Logarithmic Duality

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi + \lambda_\mp \delta\varphi}{\pi + \lambda_\mp \varphi}\right) - \frac{1}{\lambda_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_\mp \varphi_*}\right)$$

$$+ \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \tilde{\lambda}_\mp(\varphi_* - \varphi_m)}\right) - \frac{1}{\tilde{\lambda}_\pm} \ln\left(1 + \frac{\delta\pi_f}{\pi_f + \tilde{\lambda}_\mp(\varphi_f - \varphi_m)}\right)$$



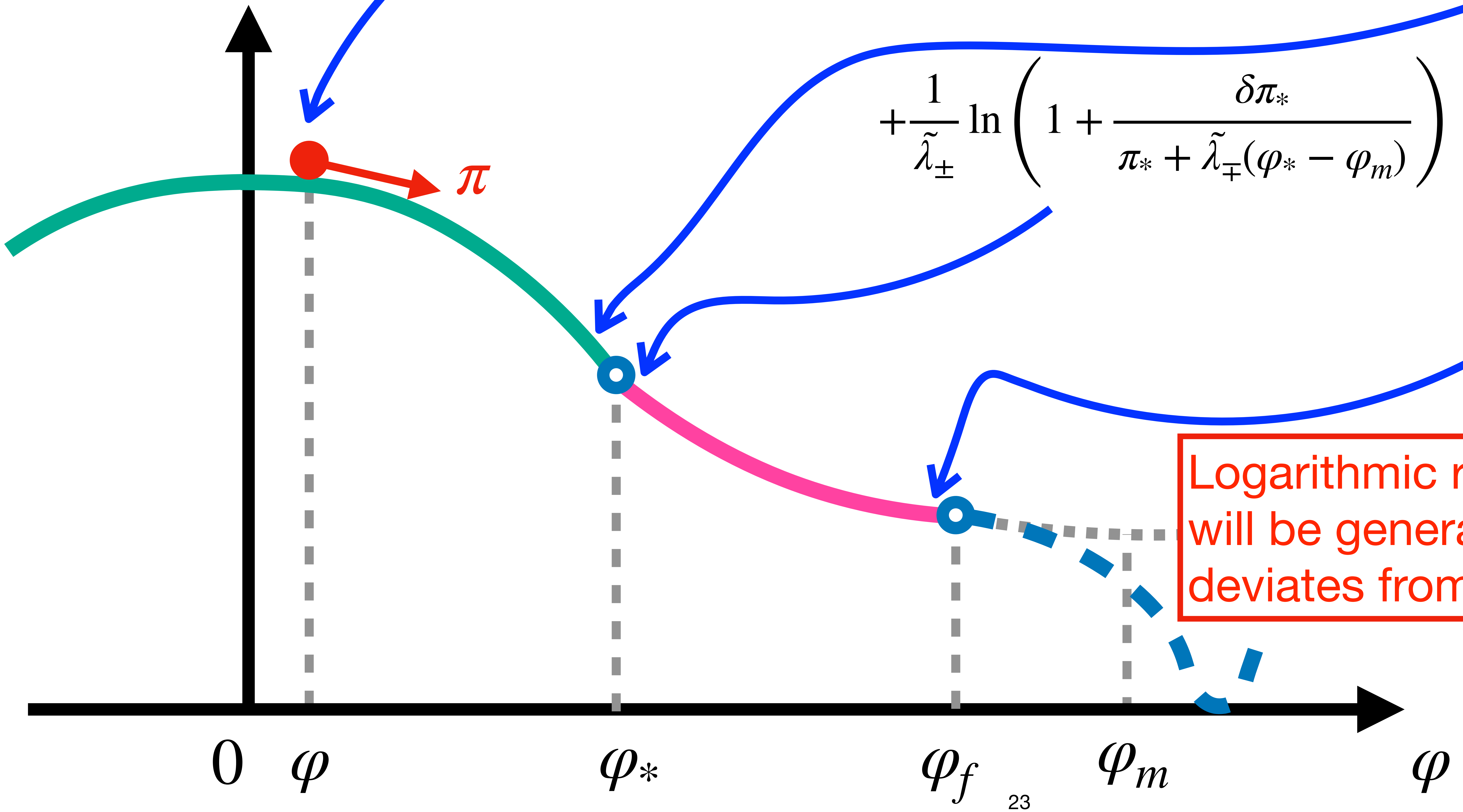
Logarithmic relation (i.e. non-Gaussianity) will be generated when the evolution deviates from attractor.

SP and Sasaki, PRL 131, 011002
SP, 2404.06151, PBH book ch.7

Logarithmic Duality

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)$$

$$+\frac{1}{\tilde\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\tilde\lambda_\mp(\varphi_*-\varphi_m)}\right) - \frac{1}{\tilde\lambda_\pm}\ln\left(1+\frac{\delta\pi_f}{\pi_f+\tilde\lambda_\mp(\varphi_f-\varphi_m)}\right)$$



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Logarithmic Duality

SP and Sasaki, PRL 131, 011002
SP, 2404.06151, PBH book ch.7

$\mathcal{R}(\delta\varphi, \delta\pi)$

$$\mathcal{R} = \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi+\lambda_\mp\delta\varphi}{\pi+\lambda_\mp\varphi}\right) - \frac{1}{\lambda_\pm}\ln\left(1+\frac{\delta\pi_*}{\pi_*+\lambda_\mp\varphi_*}\right)+\cdots$$

$\lambda_- \ll 1$

$(f_{NL} = -\frac{5}{6}\lambda_-)$

$$\mathcal{R} = -H\frac{\delta\varphi}{\dot\varphi} + \frac{3}{5}f_{\rm NL}\left(-H\frac{\delta\varphi}{\dot\varphi}\right)^2$$

Slow-roll inflation
Stewart and Sasaki, 1995
Lyth and Roquigez, 2005

$\lambda_- = -1/\mu$

$$\mathcal{R} = -\mu\ln\left(1-\frac{\mathcal{R}_g}{\mu}\right)$$

Constant-roll
Atal, Garriga, Marcos-Caballero, 1905.13202
Atal, Cid, Escrivà, Garriga, 1908.11357
Escrivà, Atal, Garriga, 2306.09990
Inui, Motohashi, SP, et al, 2409.13500

$\lambda_- = 0$

$$\mathcal{R} = -\frac{1}{3}\ln\left(1+\frac{\delta\pi_*}{\pi_*}\right)$$

Ultra-slow-roll
Namjoo, Firouzjahi, Sasaki, 1210.3692
Cai, Chen, et al 1712.09998
Biagetti et al 1804.07124
Passaglia et al 1812.08243

$\lambda_- = 3/2$

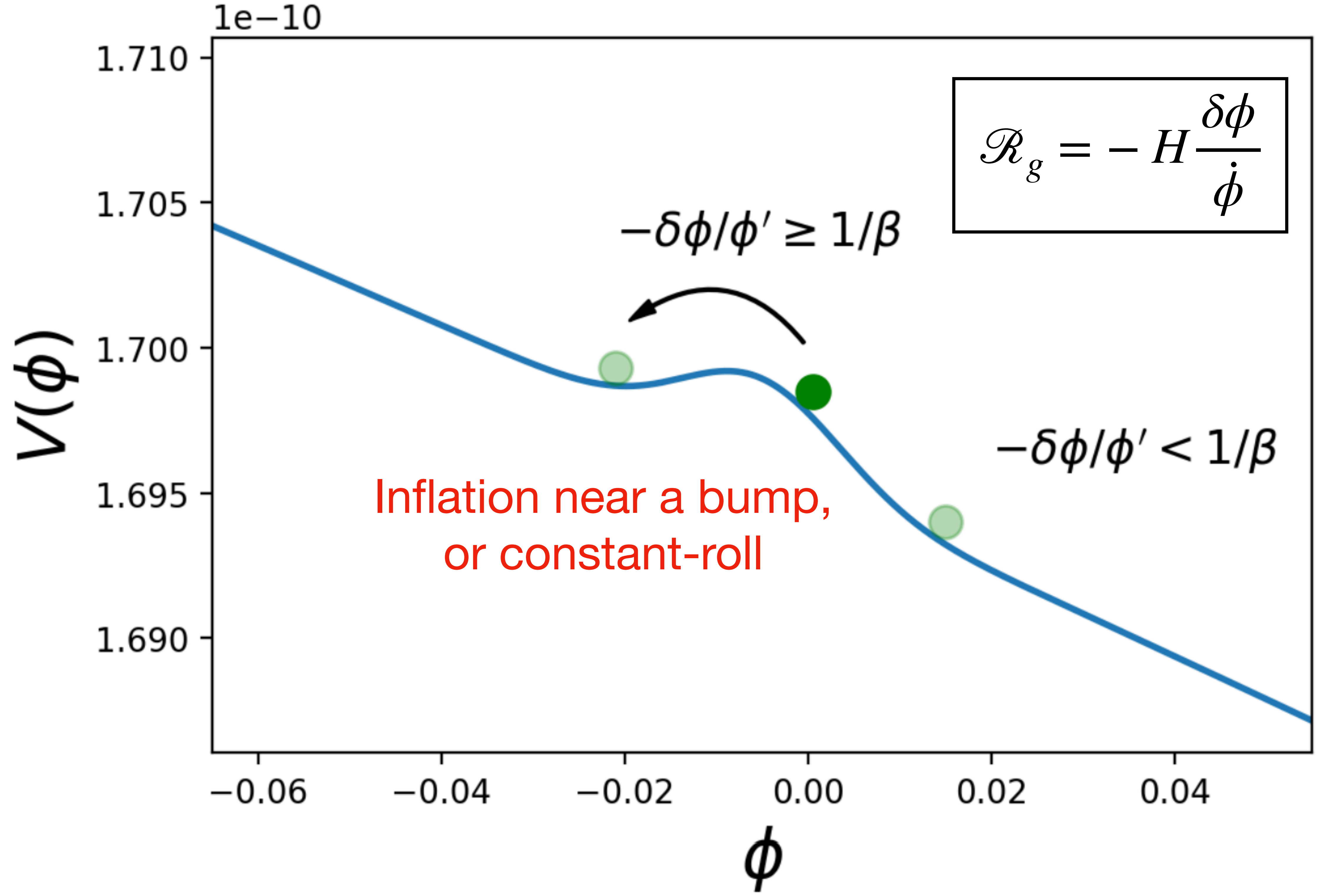
$$\mathcal{R} = \frac{2}{3}\ln(1+\delta)$$

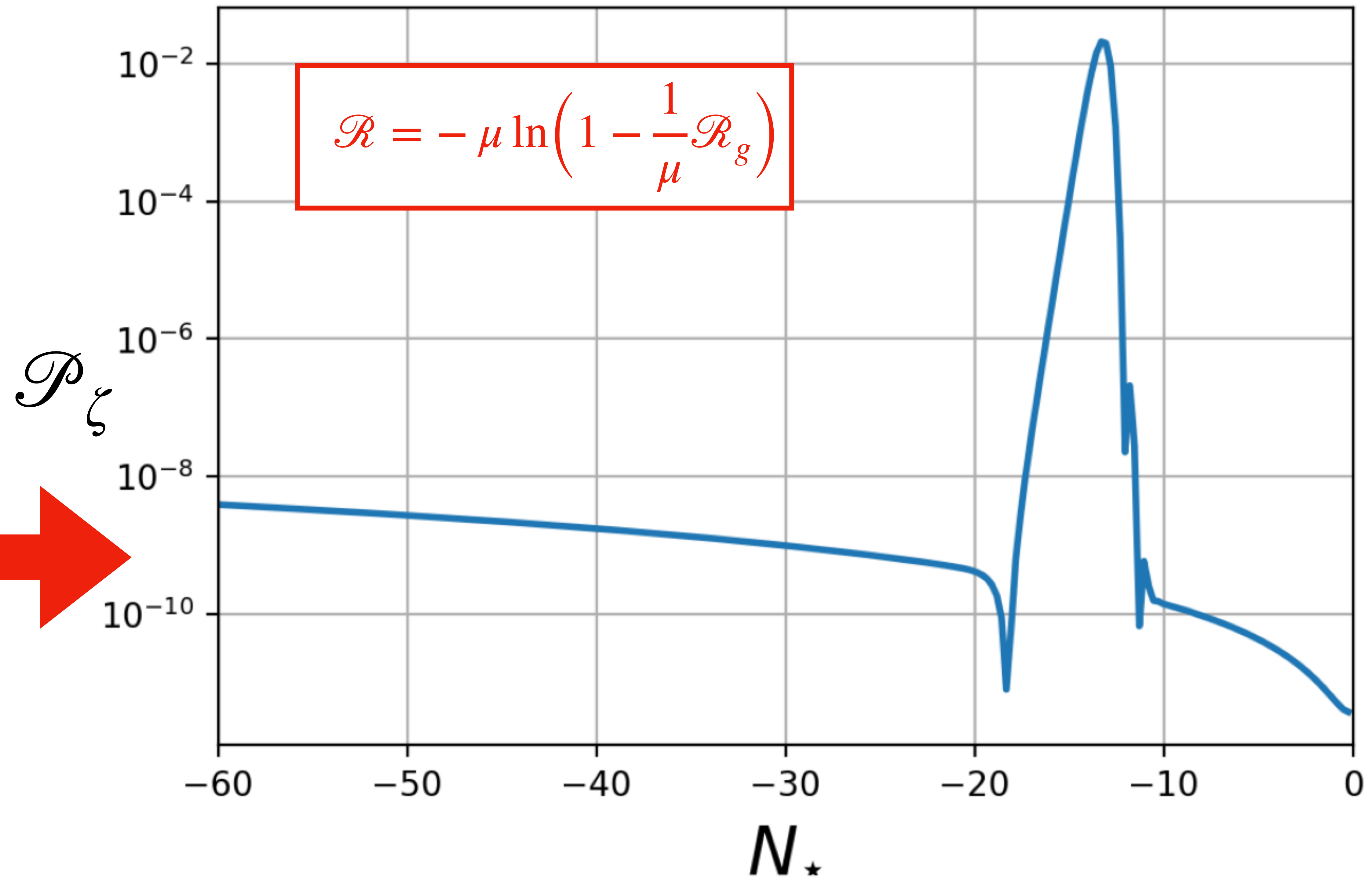
Curvaton scenario,
SP and Sasaki, 2112.12680
Ferrante et al, 2211.01728
Hooper et al. 2308.00756

$$\mathcal{R} = -\frac{1}{\lambda}\ln\left(f(\mathcal{R}_{\rm G})\right)$$

Extensions,
Kawaguchi et al, 2305.18140
SP and Yokoyama, in prep

Application to Constant-roll

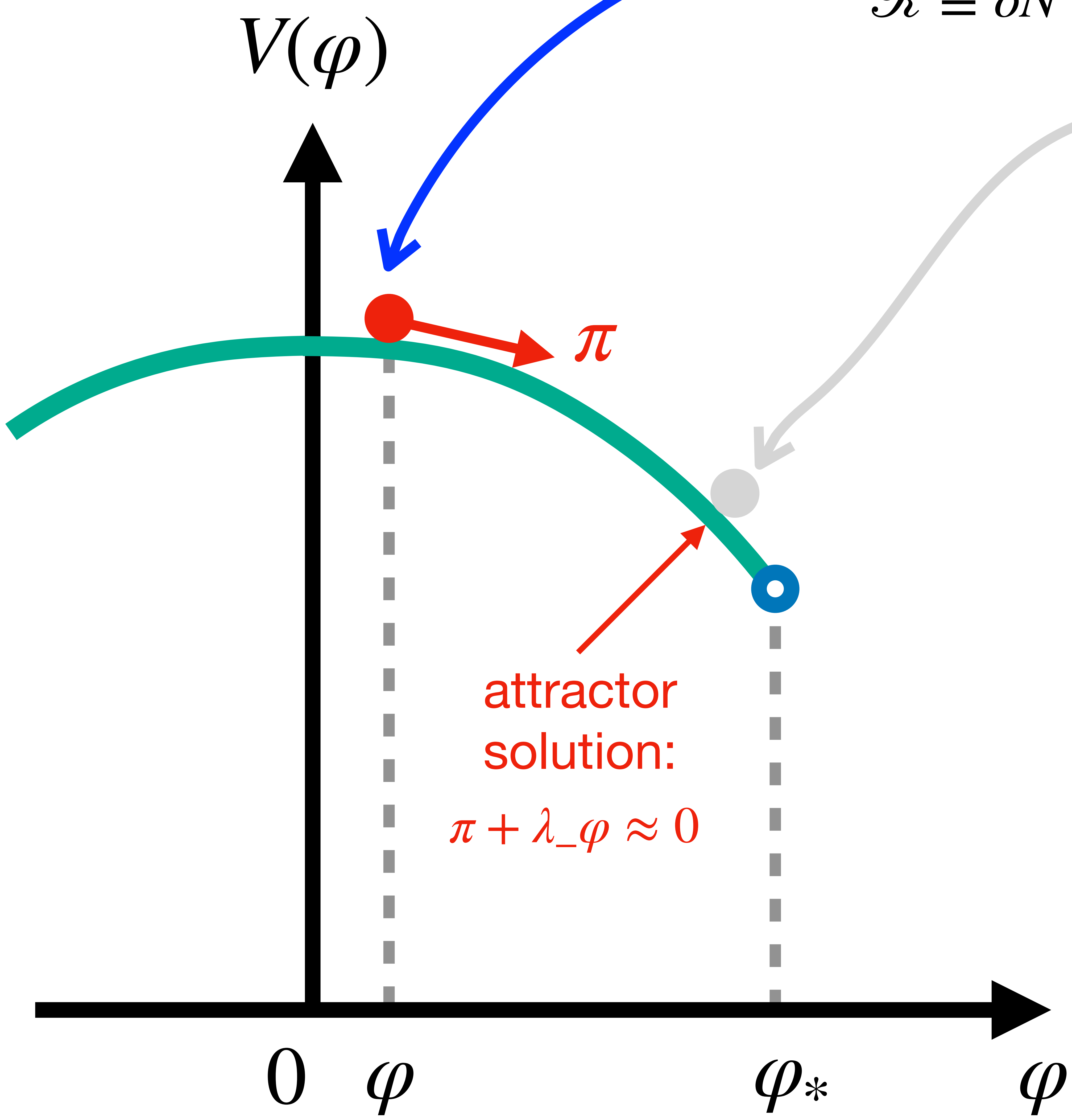


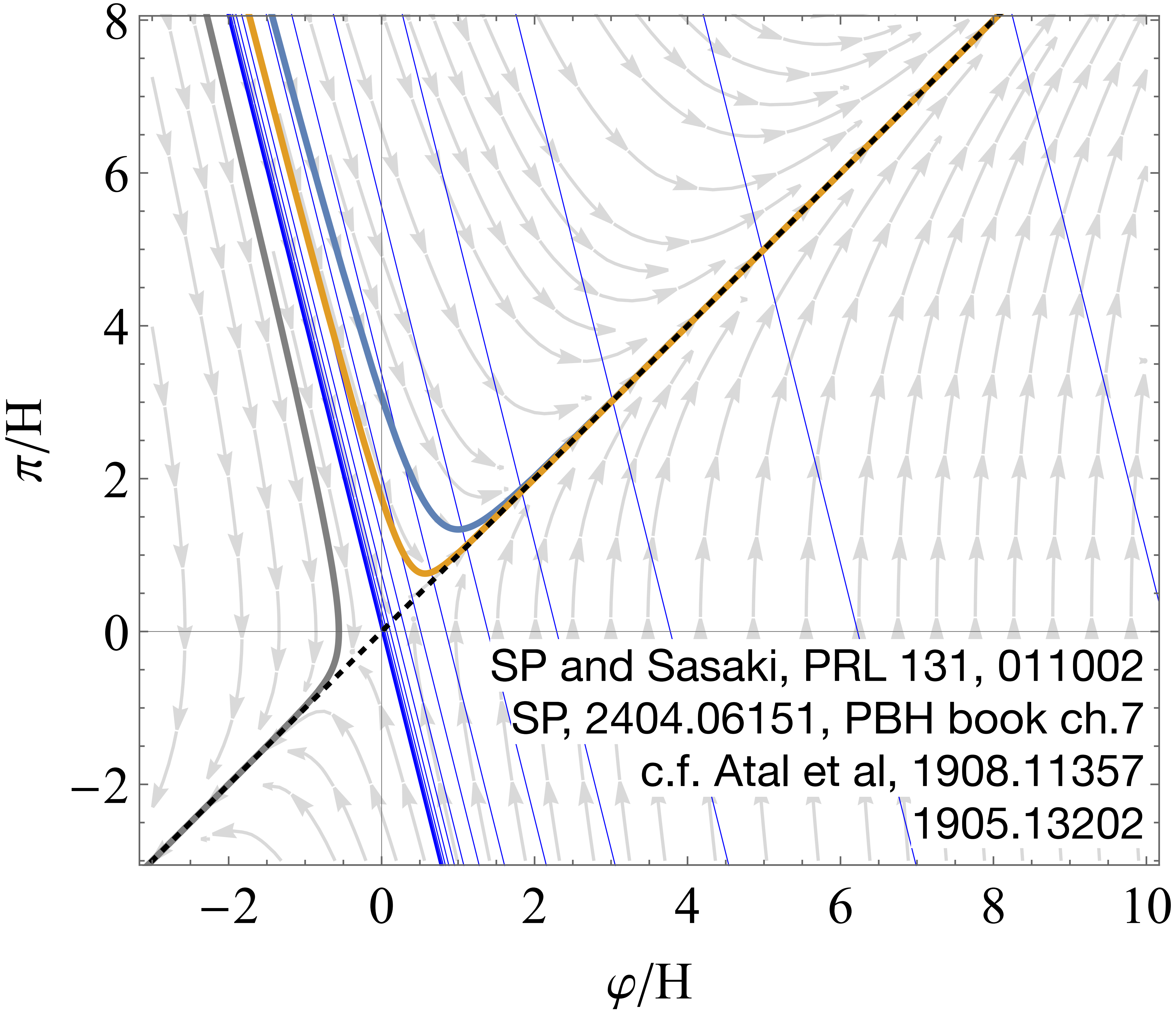


Atal, Garriga, Marcos-Caballero, 1905.13202
Atal, Cid, Escrivà, Garriga, 1908.11357
Escrivà, Atal, Garriga, 2306.09990

Application to Constant-roll

$$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi}\right) - \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi_*}{\pi_* + \lambda_+ \varphi_*}\right)$$





SP and Sasaki, PRL 131, 011002
SP, 2404.06151, PBH book ch.7
c.f. Atal et al, 1908.11357
1905.13202

Probability Distribution Function

SP and Sasaki, PRL 131, 011002
SP, 2404.06151, PBH book ch.7

For the simplest single-logarithm case: $$\mathscr{R} \equiv \delta N = \frac{1}{\lambda_-} \ln\left(1 + \frac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi}\right)$$

$$\Downarrow \quad P(\mathscr{R})d\mathscr{R} = \boxed{P(\delta\varphi)}d\delta\varphi$$

Gaussian PDF with variance $\sigma_{\delta\varphi}^2$

$$P(\mathscr{R}) = \frac{e^{\lambda_- \mathscr{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} \left|\lambda_-\right| \varphi \exp\left[-\frac{\varphi^2}{2\sigma_{\delta\varphi}^2}\left(e^{\lambda_- \mathscr{R}} - 1\right)^2\right]$$

Probability Distribution Function

SP and Sasaki, PRL 131, 011002
SP, 2404.06151, PBH book ch.7

For the simplest single-logarithm case: $\mathscr{R} \equiv \delta N = \dfrac{1}{\lambda_-} \ln\left(1 + \dfrac{\delta\pi + \lambda_+ \delta\varphi}{\pi + \lambda_+ \varphi}\right)$

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Gaussian PDF with variance $\sigma_{\delta\varphi}^2$

$$P(\mathscr{R}) = \frac{e^{\lambda_-\mathscr{R}}}{\sqrt{2\pi}\sigma_{\delta\varphi}} \left|\lambda_-\right| \varphi \exp\left[-\frac{\varphi^2}{2\sigma_{\delta\varphi}^2}\left(e^{\lambda_-\mathscr{R}} - 1\right)^2\right]$$

$\lambda_- < 0$

$P(\mathscr{R}) \sim e^{\lambda_-\mathscr{R}}$

exponential tail

$\lambda_- > 0$

$P(\mathscr{R}) \sim \exp\left(-c^2 e^{2\lambda_-\mathscr{R}}\right)$

Gumbel-distribution-like tail

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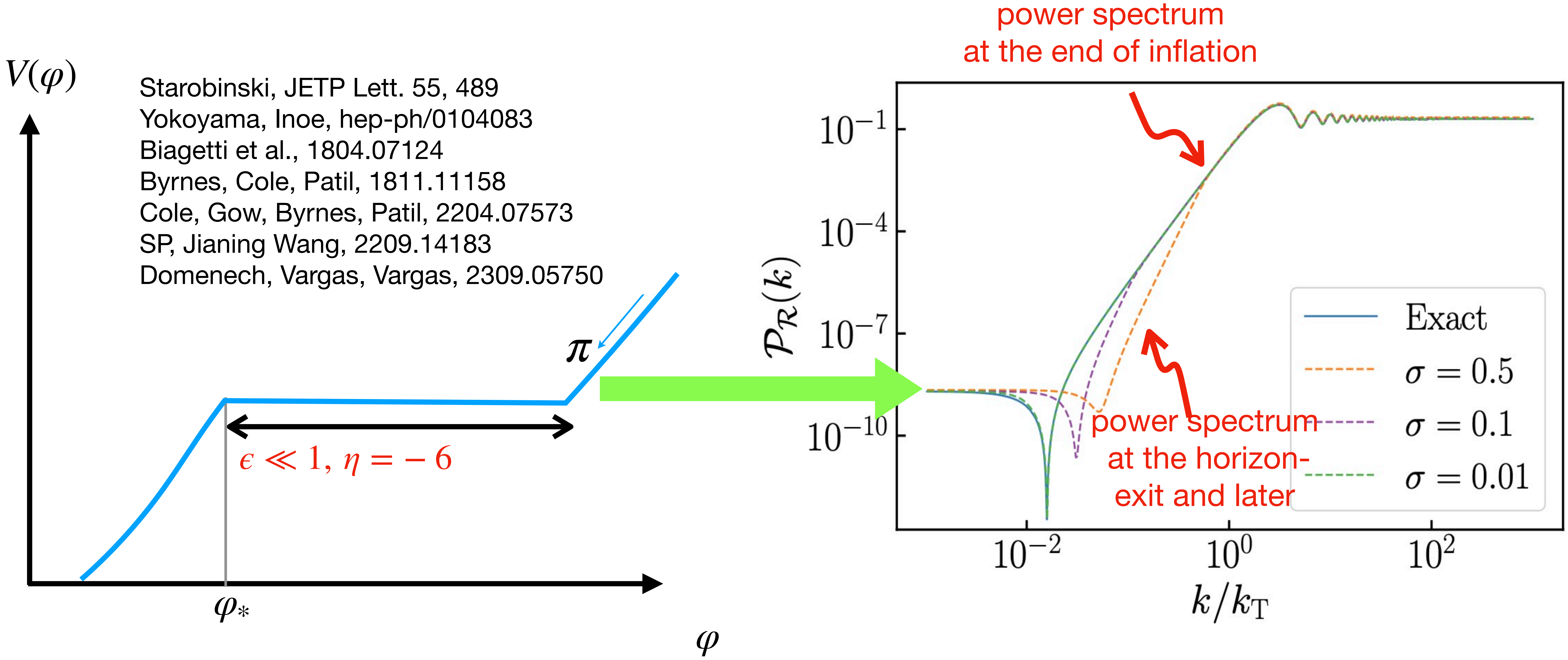
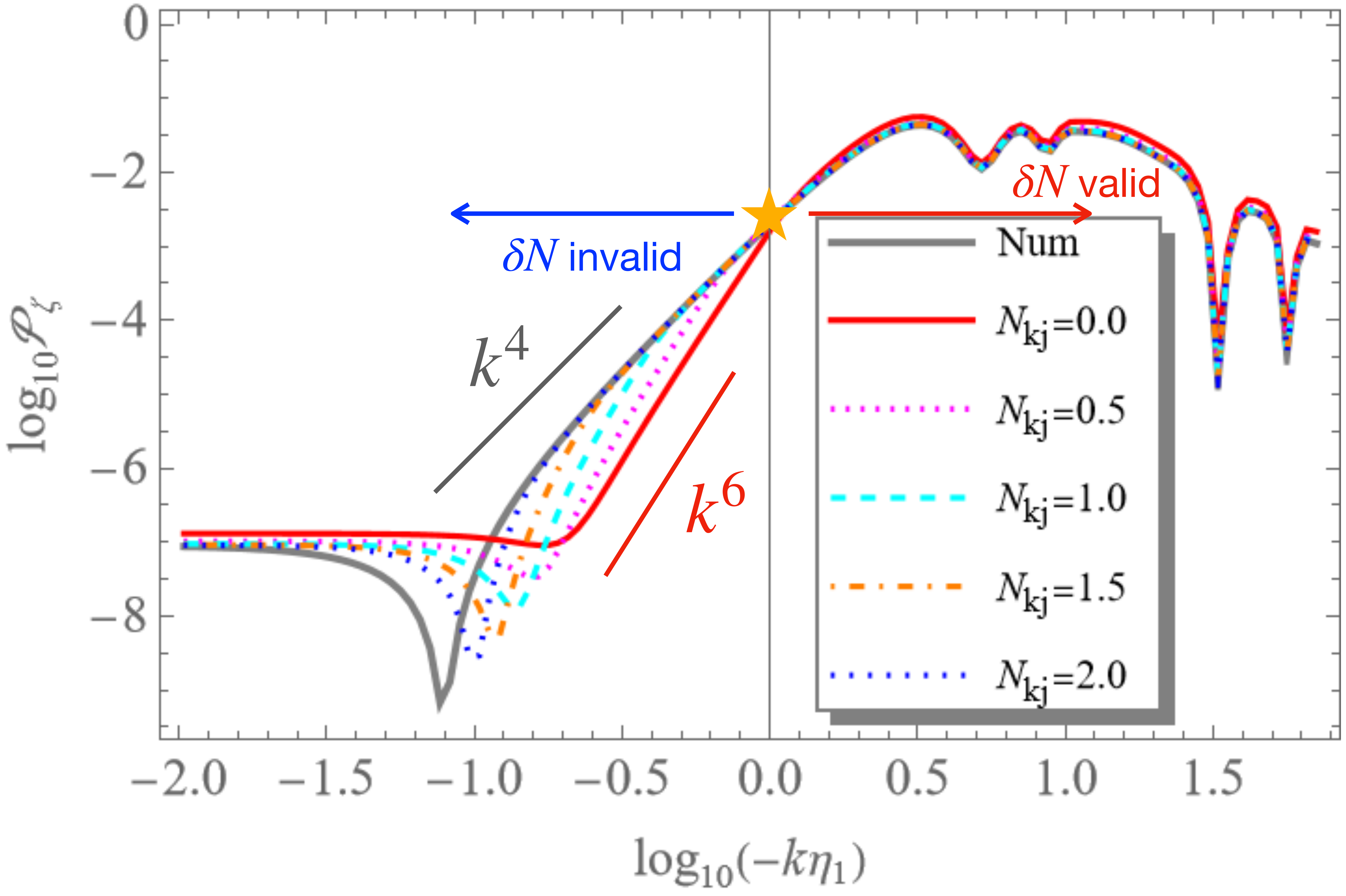
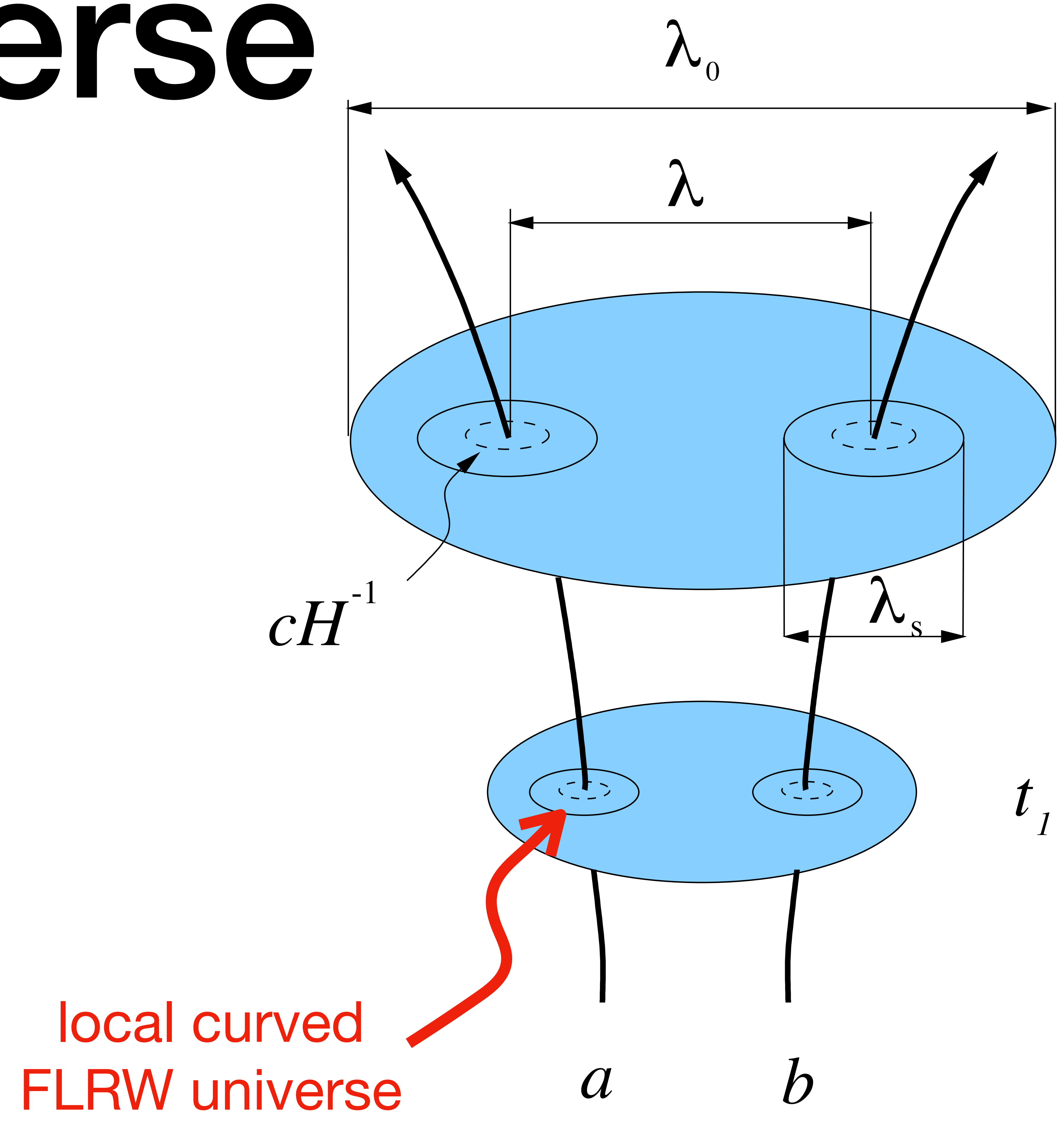
Topic 1: Gradient Expansion

Starobinski, JETP Lett. 55, 489
Yokoyama, Inoe, hep-ph/0104083
Biagetti et al., 1804.07124
Byrnes, Cole, Patil, 1811.11158
Cole, Gow, Byrnes, Patil, 2204.07573
SP, Jianing Wang, 2209.14183
Domenech, Vargas, Vargas, 2309.05750



Separate Universe

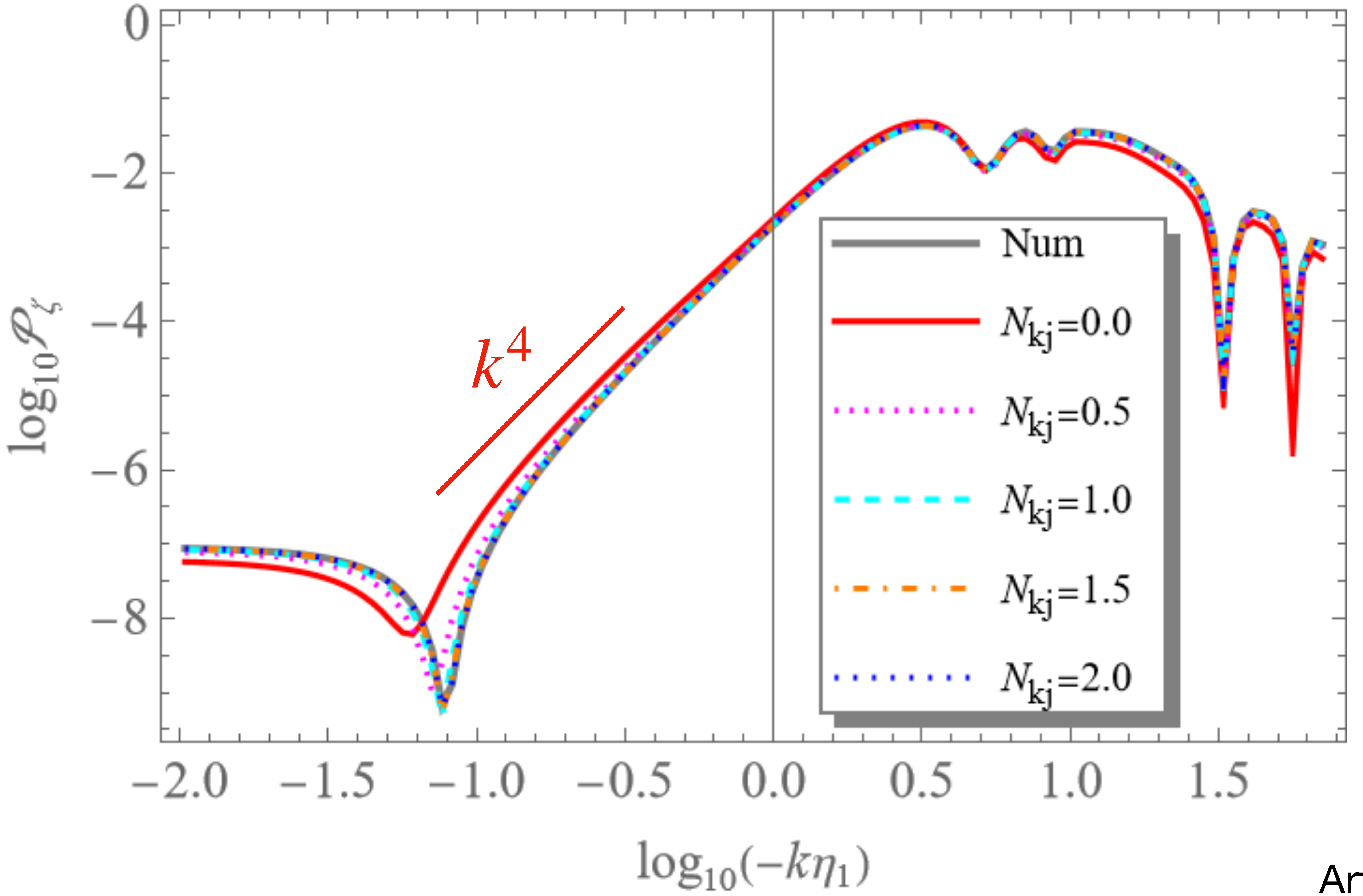


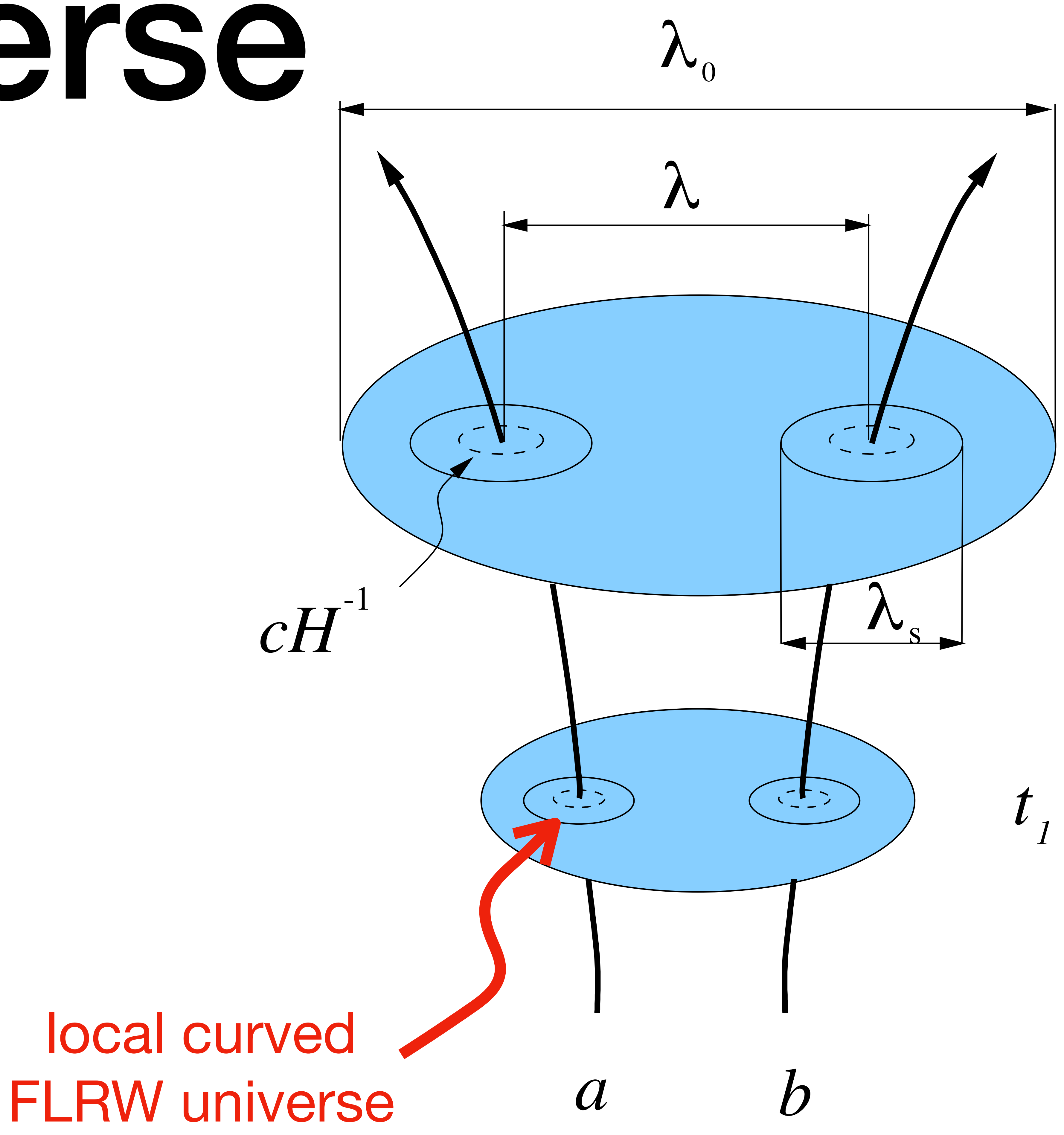


$$\mathscr{R}(N_e) = \delta N(\delta\varphi(N_i))$$

c.f.
Domenech et al., 2309.05750
Jackson et al., 2311.03281

Separate Universe



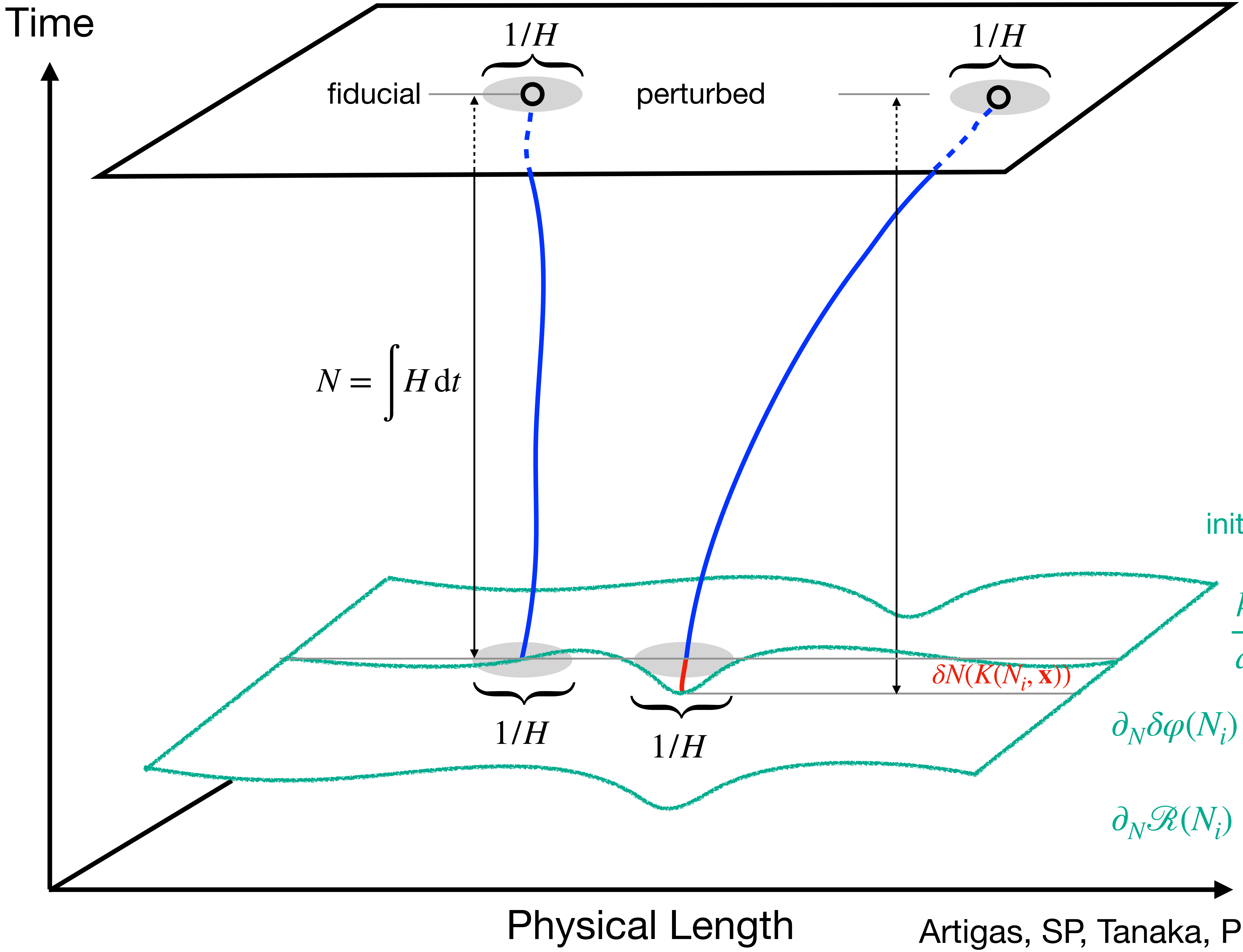


$$\mathscr{R}(N_e) = \mathscr{R}(N_i) + \delta N(k^2\mathscr{R}(N_i))$$

(Note that $k^2\mathscr{R} \sim K$)

Artigas, SP, Tanaka, PRL 134 (2025), 221001
c.f. Briaud, Kawaguchi, Vennin, 2509.05124

δN formalism



initial comoving slice

$$\delta\varphi(N_i) = 0,$$

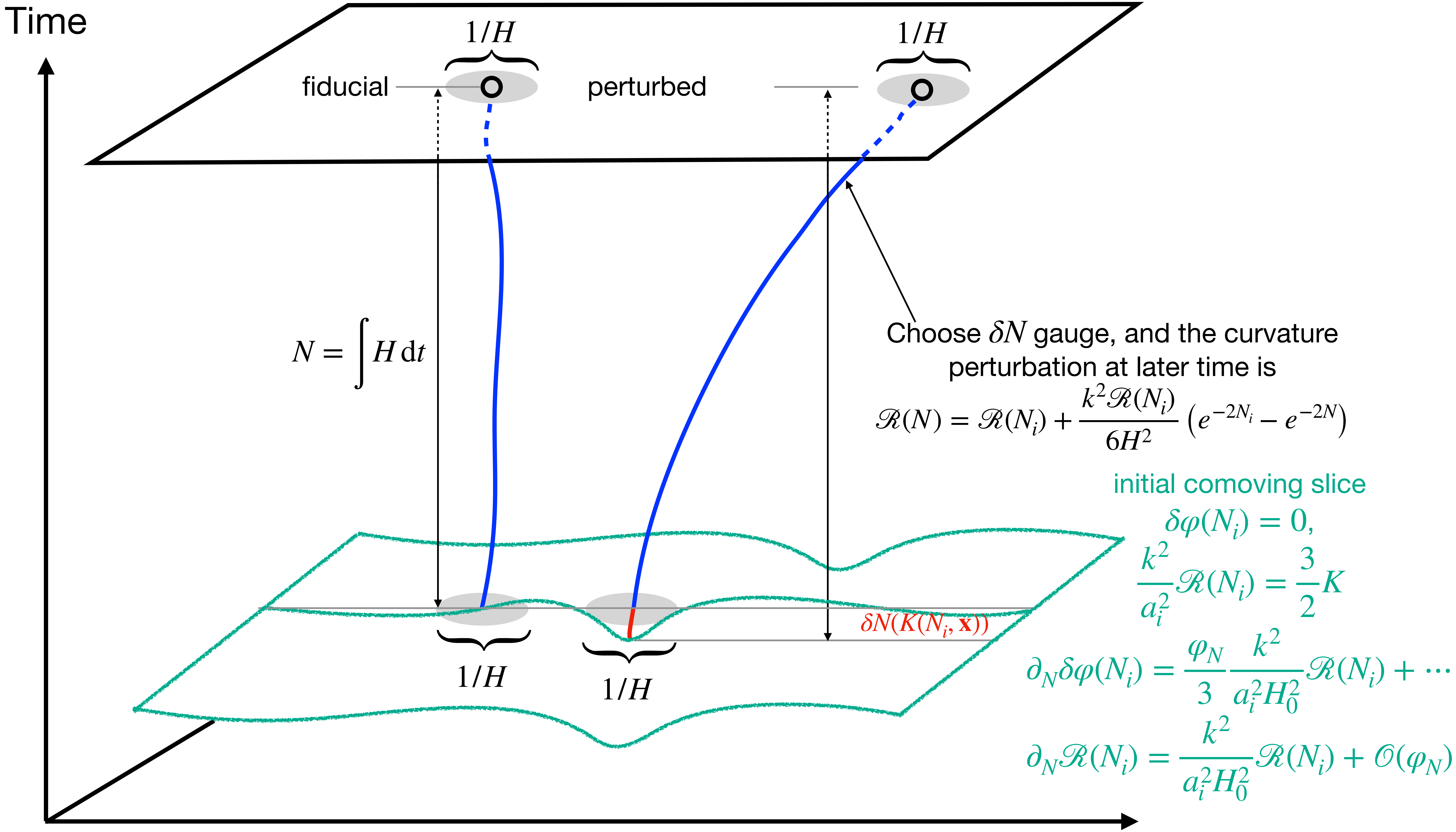
$$\frac{k^2}{a_i^2}\mathcal{R}(N_i) = \frac{3}{2}K$$

$$\partial_N \delta\varphi(N_i) = \frac{\varphi_N}{3}\frac{k^2}{a_i^2 H_0^2}\mathcal{R}(N_i) + \cdots$$

$$\partial_N \mathcal{R}(N_i) = \frac{k^2}{a_i^2 H_0^2}\mathcal{R}(N_i) + \mathcal{O}(\varphi_N)$$

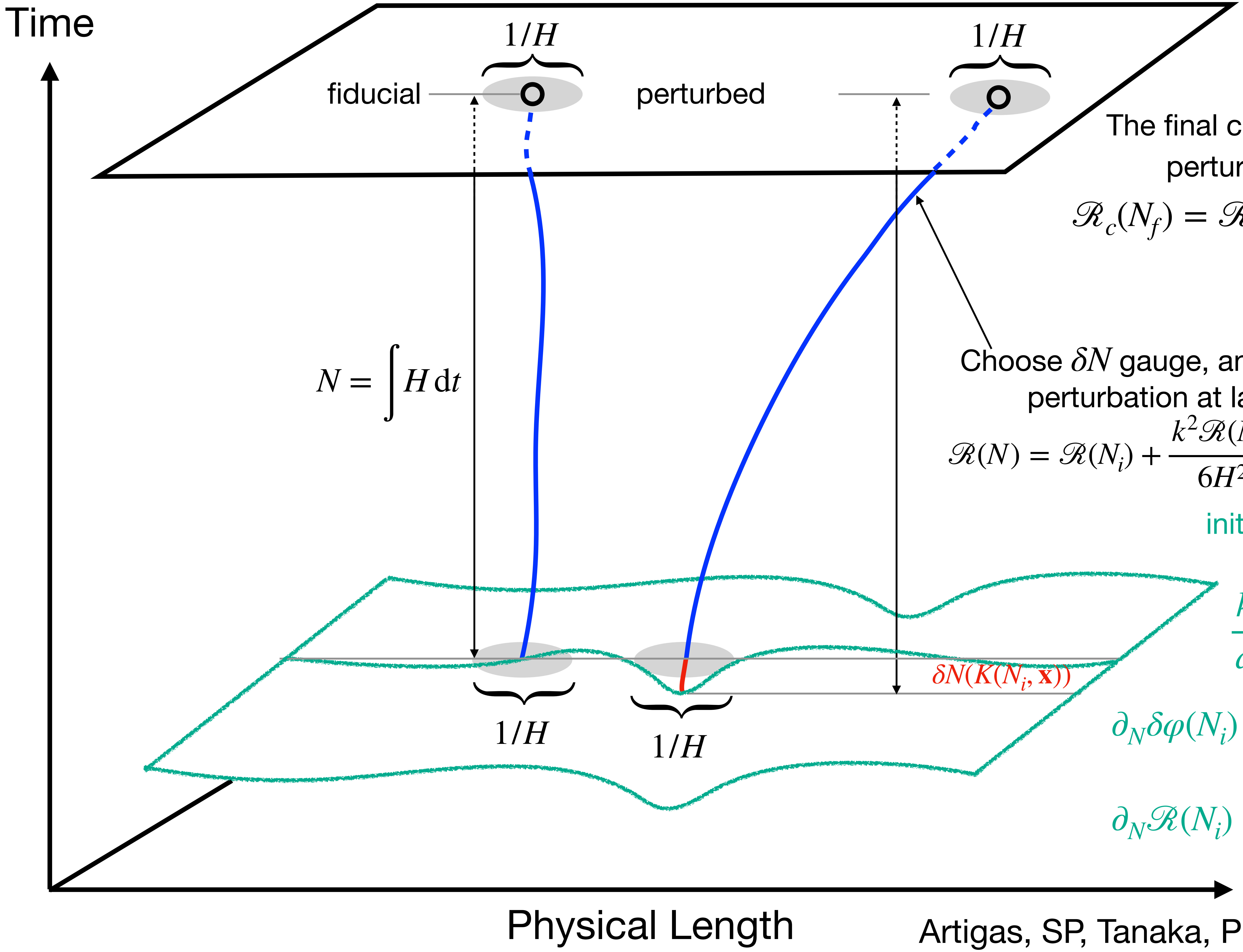
Artigas, SP, Tanaka, PRL 134 (2025), 221001

δN formalism



Artigas, SP, Tanaka, PRL 134 (2025), 221001

δN formalism



The final comoving curvature perturbation at N_f is

$$\mathcal{R}_c(N_f) = \mathcal{R}(N_f) + \delta N(K(N_i, \mathbf{x}))$$

Choose δN gauge, and the curvature perturbation at later time is

$$\mathcal{R}(N) = \mathcal{R}(N_i) + \frac{k^2 \mathcal{R}(N_i)}{6H^2}\left(e^{-2N_i} - e^{-2N}\right)$$

initial comoving slice

$$\delta\varphi(N_i) = 0,$$

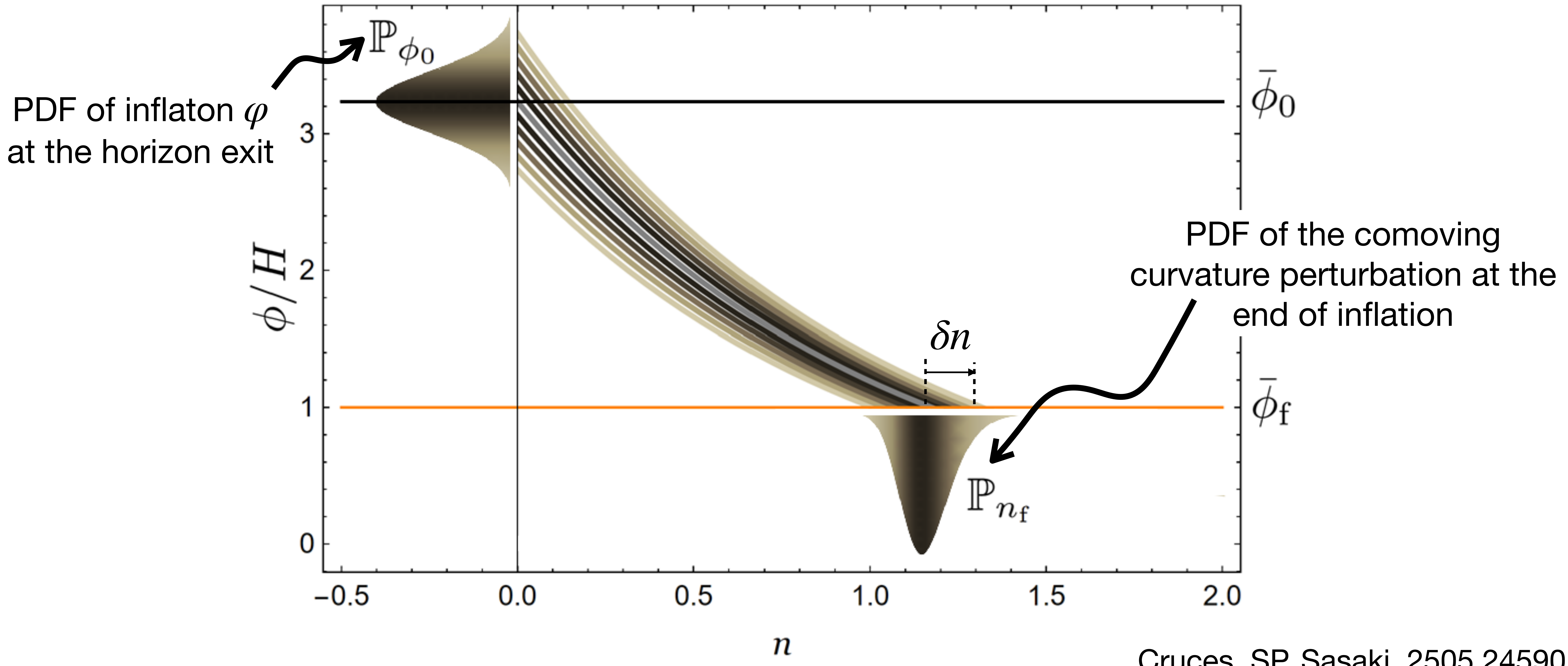
$$\frac{k^2}{a_i^2}\mathcal{R}(N_i) = \frac{3}{2}K$$

$$\partial_N \delta\varphi(N_i) = \frac{\varphi_N}{3}\frac{k^2}{a_i^2 H_0^2}\mathcal{R}(N_i) + \cdots$$

$$\partial_N \mathcal{R}(N_i) = \frac{k^2}{a_i^2 H_0^2}\mathcal{R}(N_i) + \mathcal{O}(\varphi_N)$$

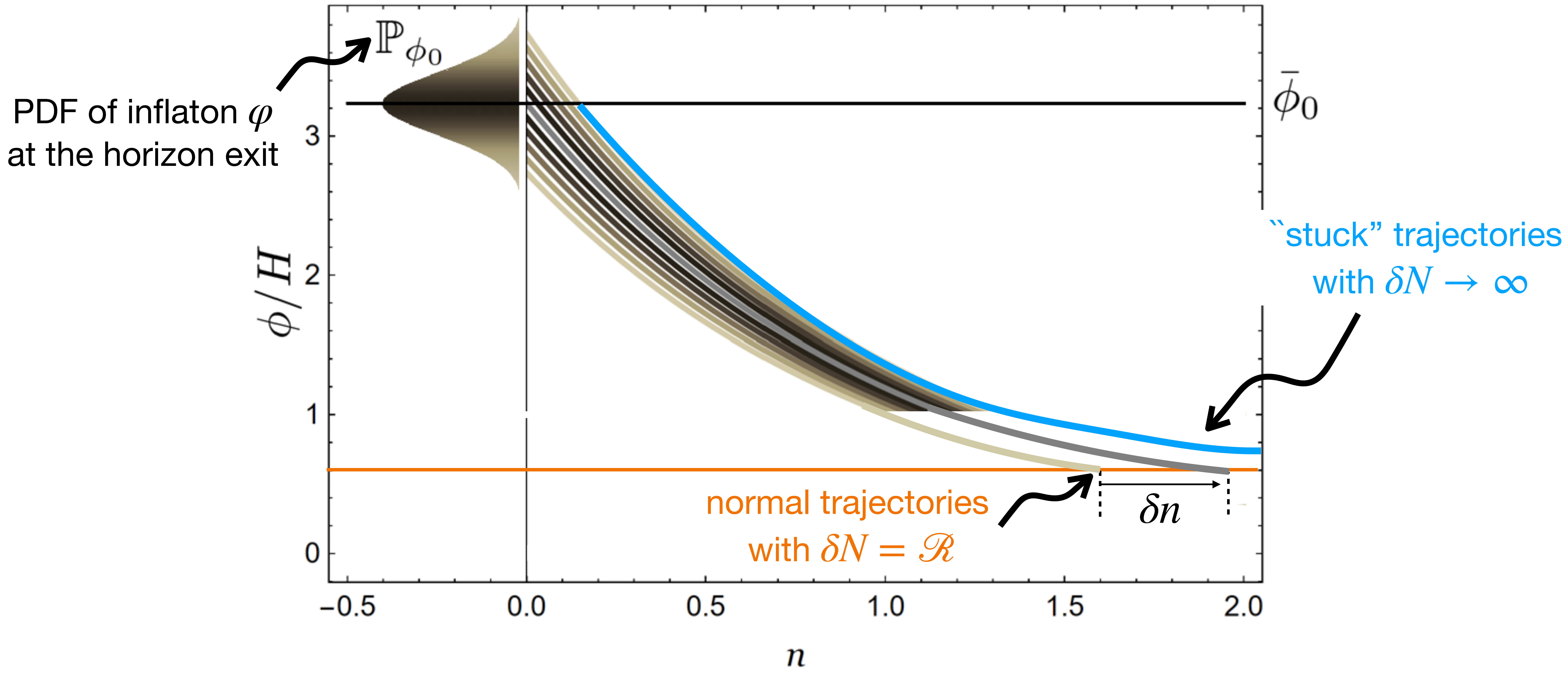
Artigas, SP, Tanaka, PRL 134 (2025), 221001

Topic 2: Probability conservation



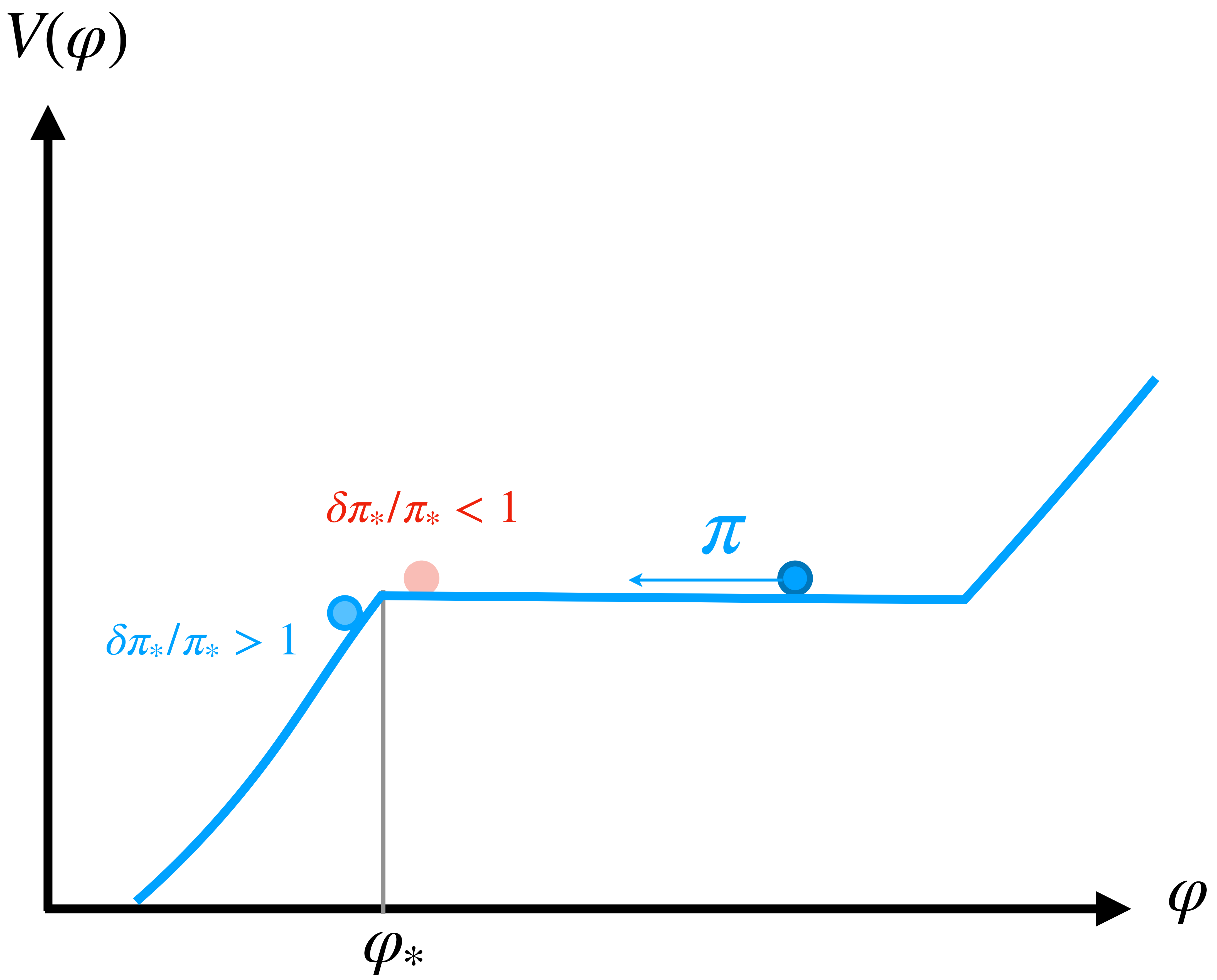
Cruces, SP, Sasaki, 2505.24590

Topic 2: Probability conservation



Topic 2: Probability conservation

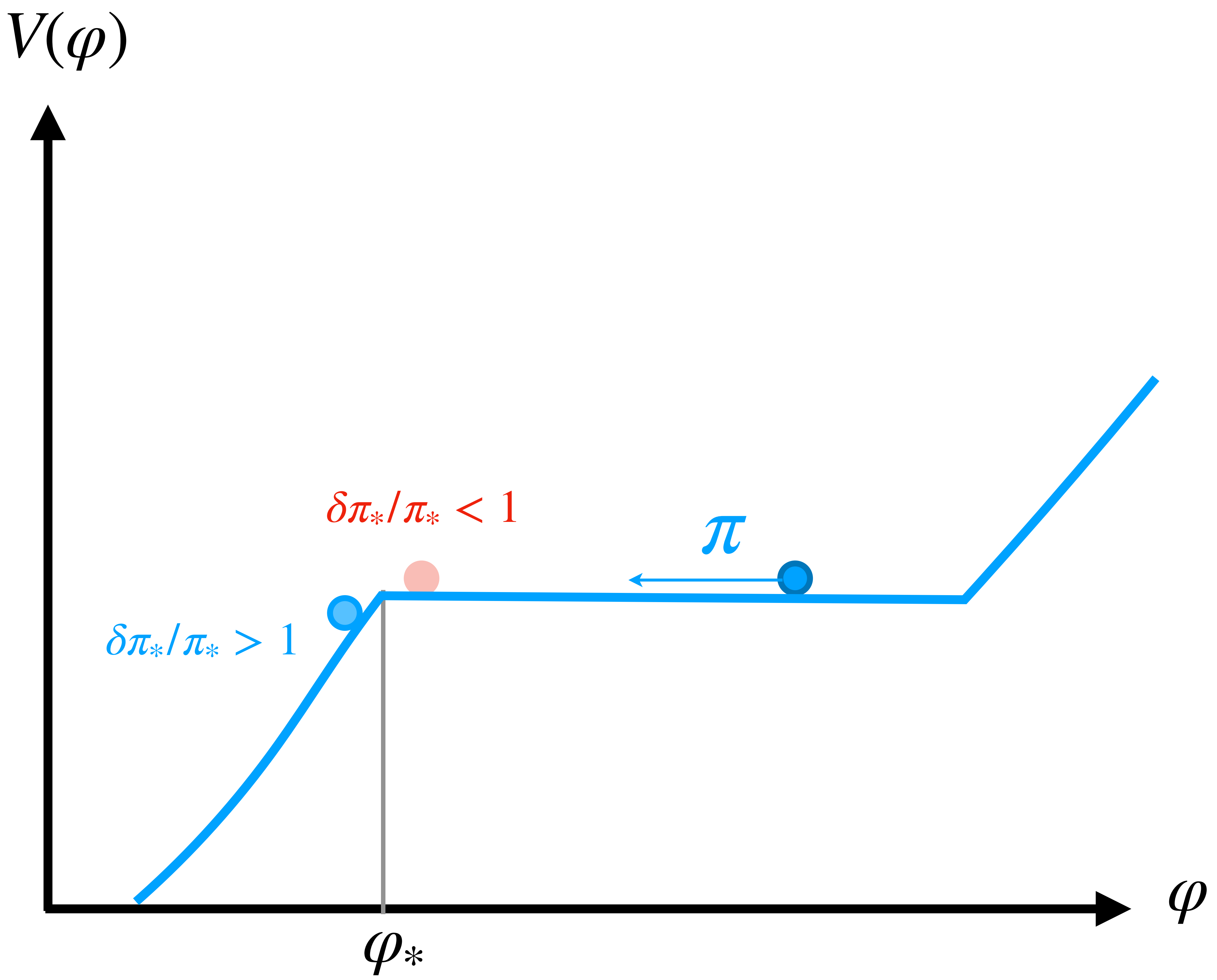
Ultra-slow-roll inflation



$$\delta N = -\frac{1}{3}\ln\left(1 - \frac{\delta\pi_*}{\pi_*}\right)$$

$$\begin{cases} = \mathscr{R} & \text{with } \delta\pi_*/\pi_* < 1 \\ \to \infty & \text{with } \delta\pi_*/\pi_* > 1 \end{cases}$$

Ultra-slow-roll inflation

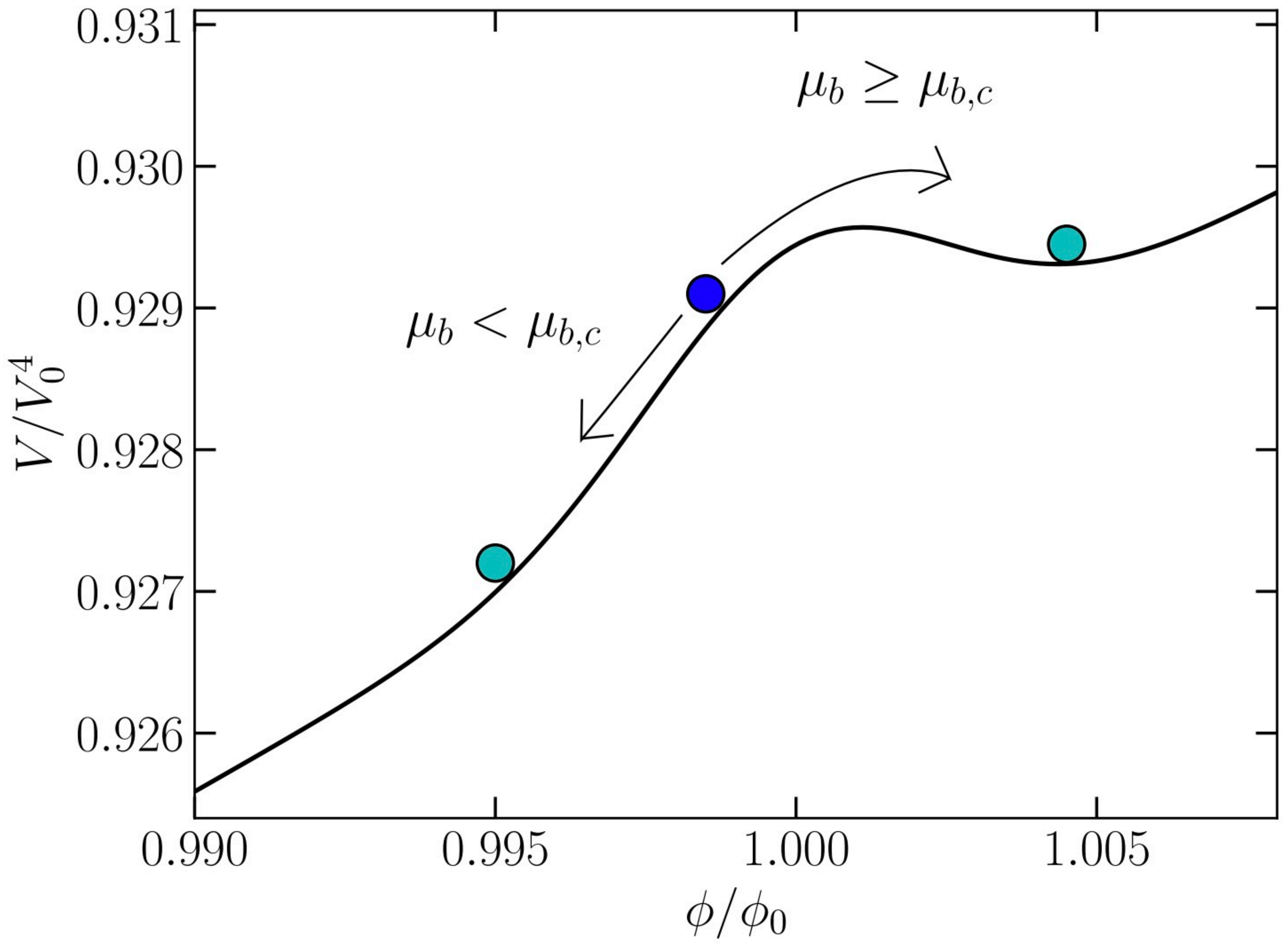


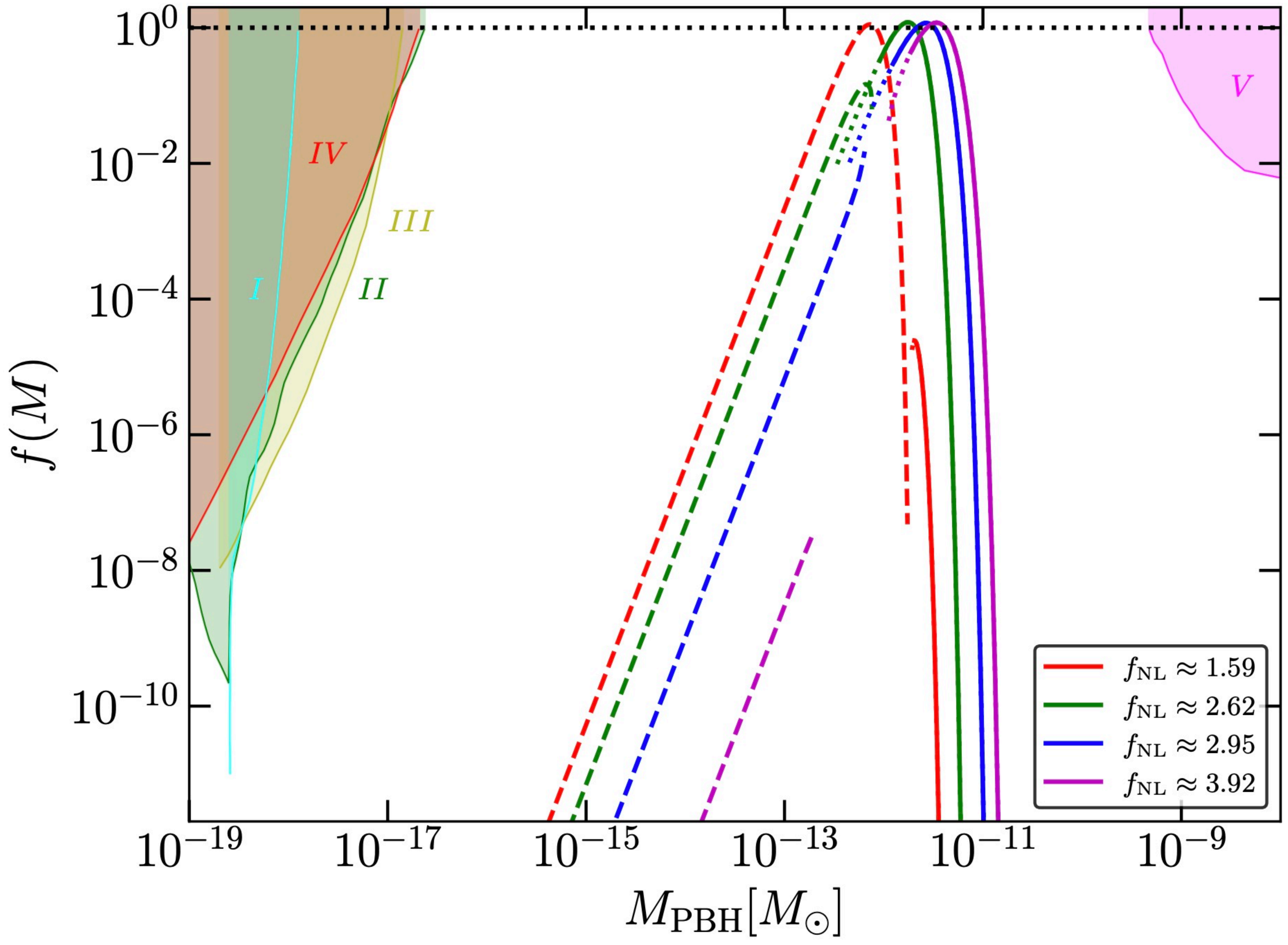
$$\delta N = -\frac{1}{3}\ln\left(1 - \frac{\delta\pi_*}{\pi_*}\right)$$

$$\begin{cases} = \mathscr{R} & \text{with } \delta\pi_*/\pi_* < 1 \\ \to \infty & \text{with } \delta\pi_*/\pi_* > 1 \end{cases}$$

Eternal inflation

Constant-roll inflation and bubble channel



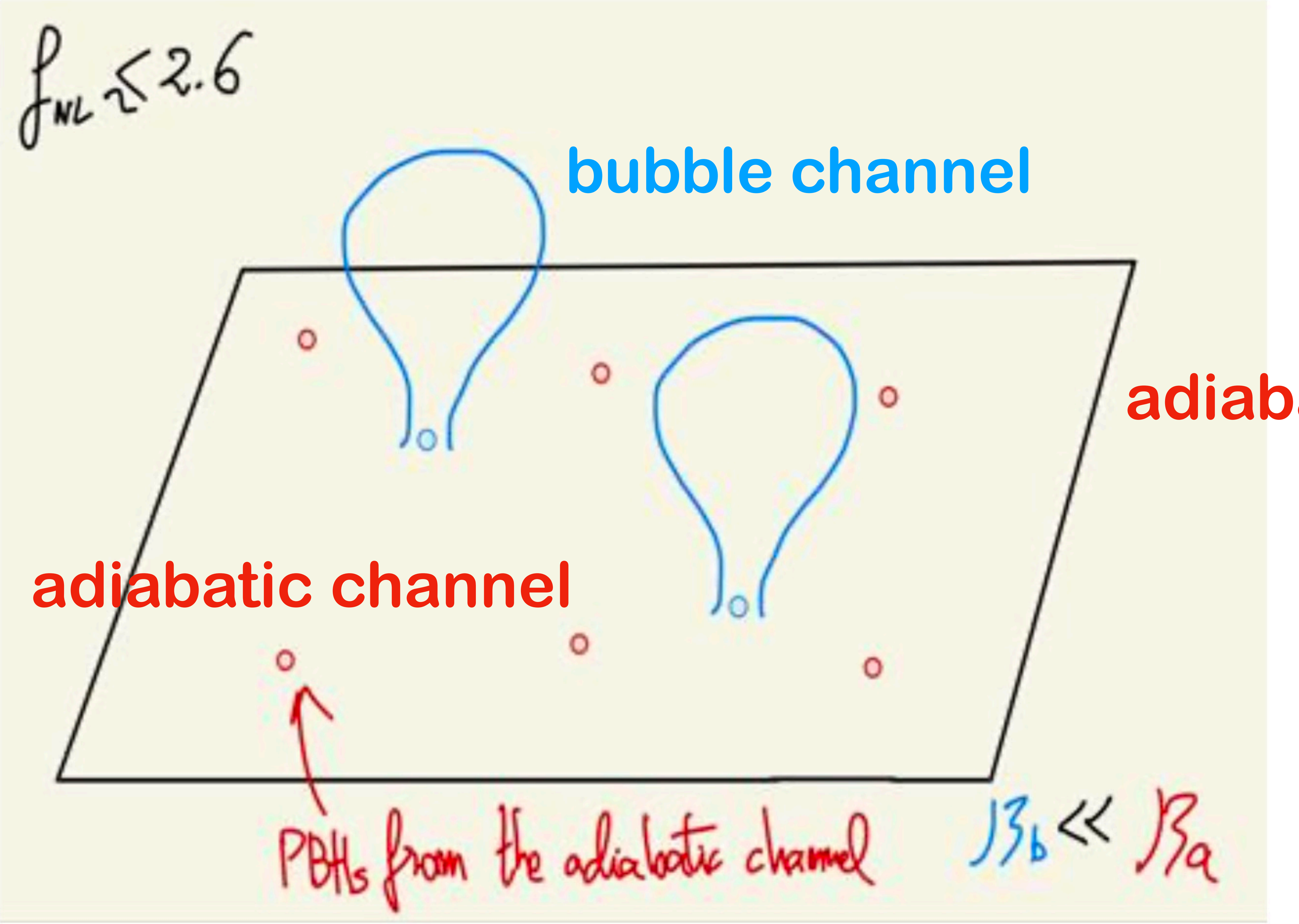


Escriva, Atal, Garriga, 2306.09990

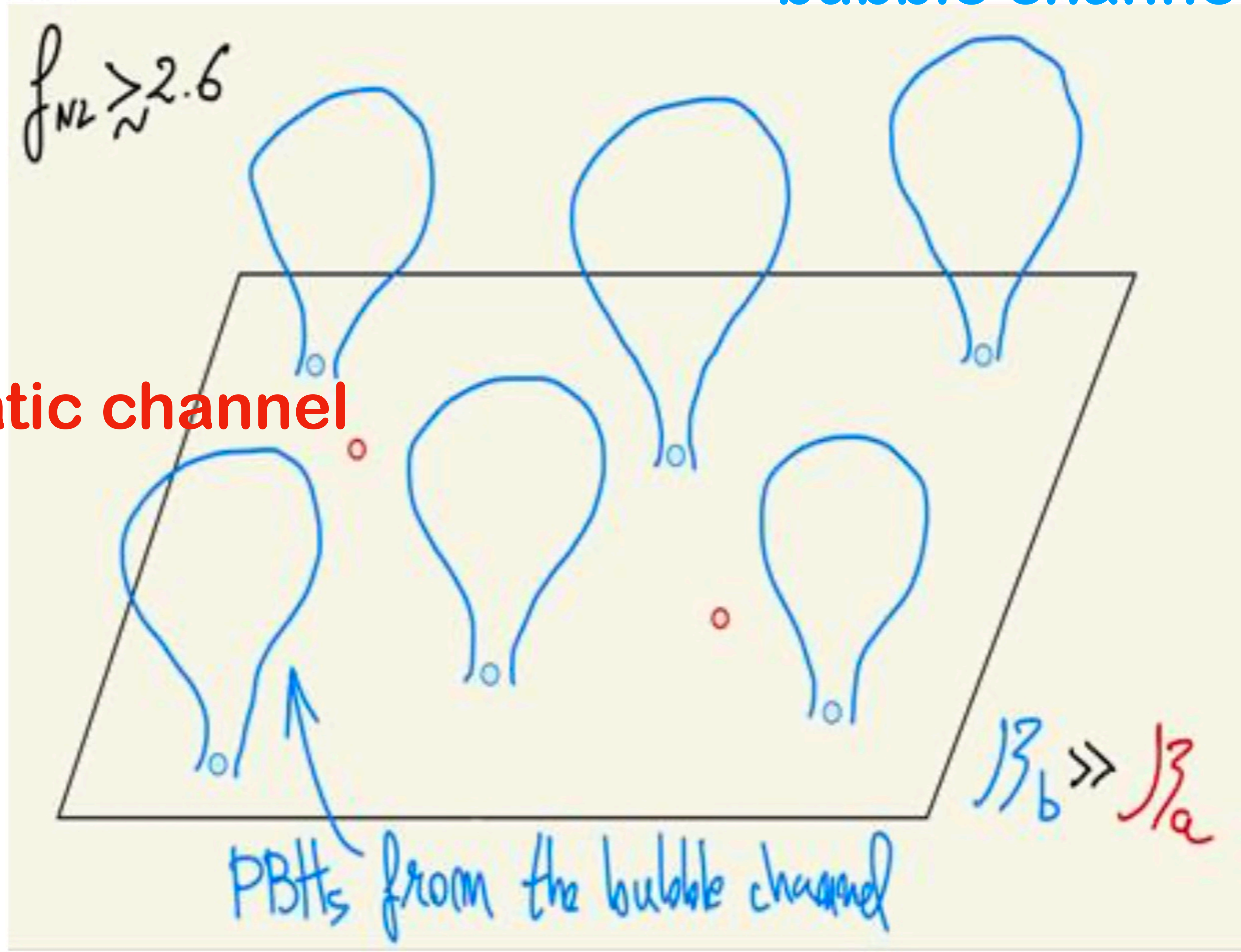
From Jaume Garriga’s slide at Paris IHT

Qualitative picture

Fluctuations type I-> dominant contribution

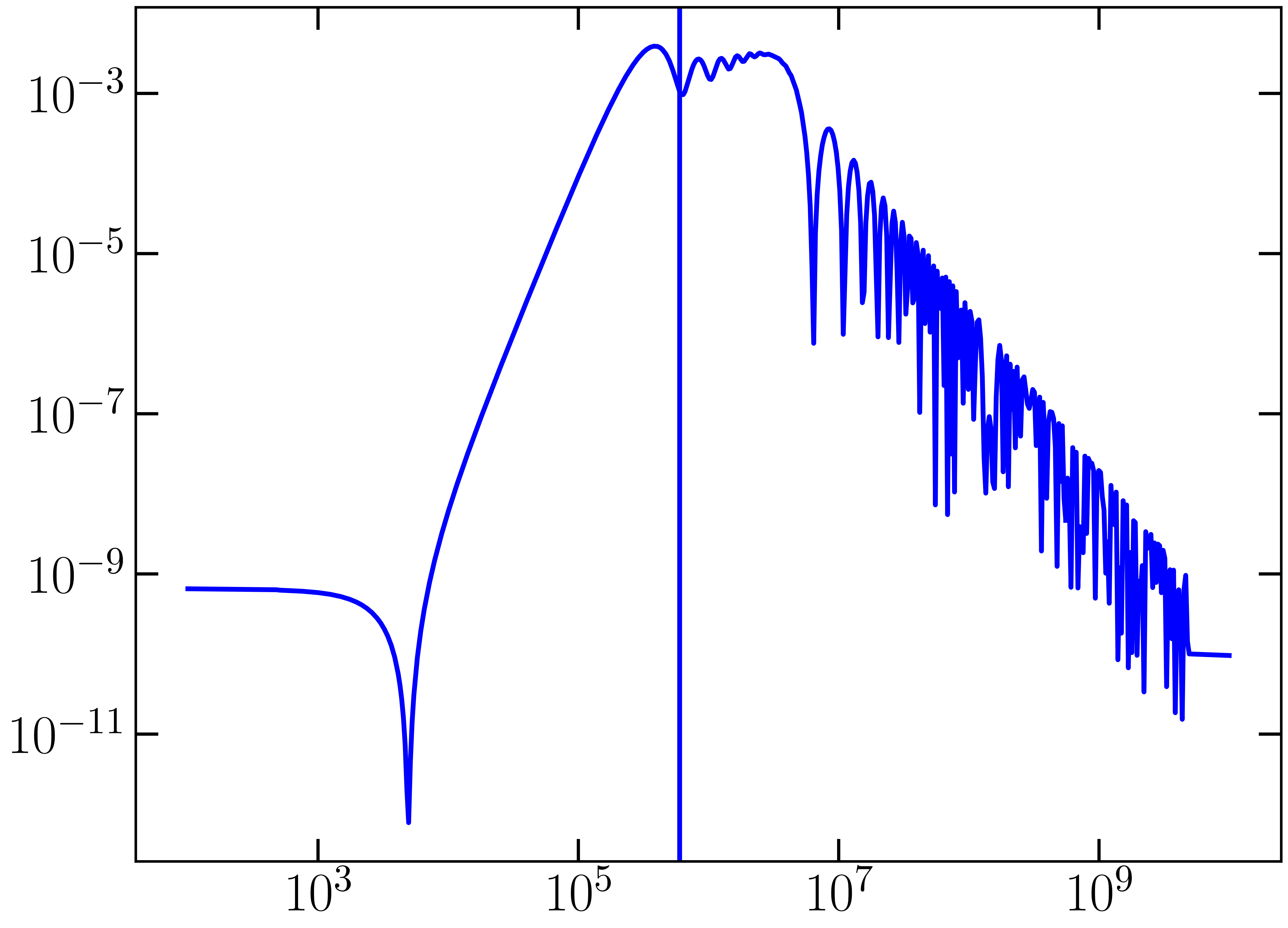


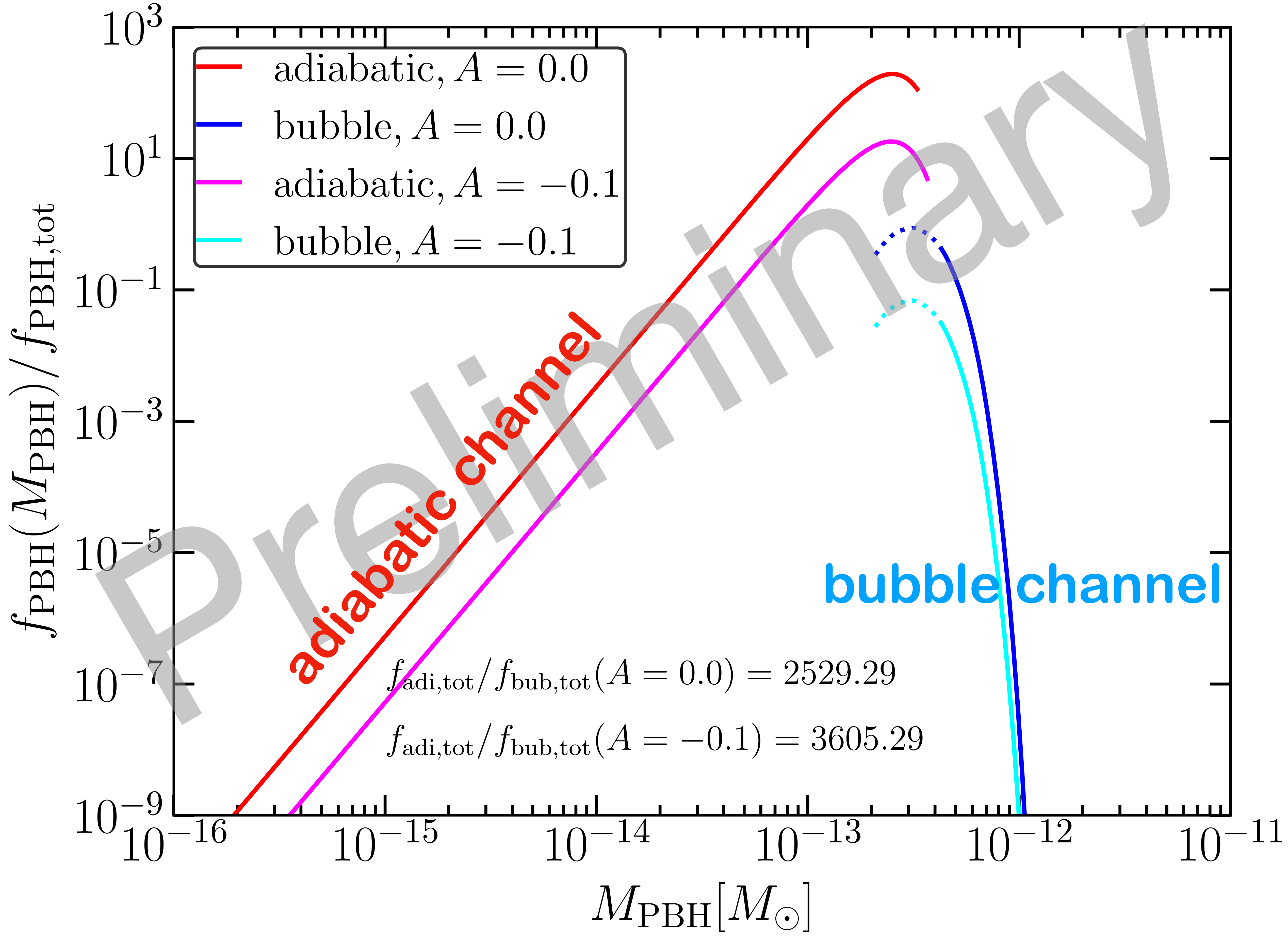
Fluctuations type II-> dominant contribution



Escriva, Atal, Garriga, 2306.09990

USR and bubble channel





Escriva, Garriga, SP, in preparation

Conclusion

- On super-horizon scales, the non-Gaussianity can be well described by δN formalism, which gives general logarithmic functions of $\mathscr{R}(\delta\varphi_k, \delta\dot{\varphi}_k)$.

- Large non-Gaussianity appears when evolution of φ deviates from the attractor.

- Original δN based on separate-universe approach fails to deal with decaying mode. We proposed the extended δN by considering K as an initial condition.

- Ultra-slow-roll inflation inevitable generates local eternal inflation patch (Type-B PBHs) formed by the bubble channel, which was omitted in the literature.

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